Aggregate Implications of Defined Benefit and Defined Contribution Systems*

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Abstract

We use a general equilibrium life-cycle model with incomplete markets and heterogeneous agents to evaluate the macroeconomic and welfare implications of Defined Benefit (DB) versus Defined Contribution (DC) systems, and to investigate the effects of incremental reform within a particular system. Extensive calibrations illustrate the trade-off between efficiency and redistribution that a tax-financed, DB social security system generates. We find that social welfare is maximized for small but positive levels of DB because of the redistributive value associated with these systems. On the other hand, steady-state within-DC system comparisons reveal that a zero DC tax rate maximizes social welfare.

JEL Classification: E21, E62, H55.

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1 Introduction

Financing retirement benefits is probably the most significant fiscal challenge that governments in industrial economies will be facing in the next few decades. Social security reform has therefore become an important public policy issue for many countries and various reform proposals have been recently put forth. Given the importance of understanding the aggregate and welfare implications of different social security systems existing in the OECD, a number of recent papers have investigated the implications of social security reform from the perspective of a general equilibrium, heterogeneous agent model with reasonable levels of idiosyncratic labor income uncertainty and market frictions (see, among others, İmrohoğlu, İmrohoğlu and Joines (1995), De Nardi, İmrohoğlu and Sargent (1999), Bohn (1999), Conesa and Krueger (1999), Gertler (1999), Huggett and Ventura (1999), Storesletten, Telmer and Yaron (1999), Diamond and Geanakoplos (2002), Fuster, İmrohoğlu and İmrohoğlu (forthcoming) and Caballé and Fuster (forthcoming)).[1]

We follow this general equilibrium literature to analyze the aggregate and welfare implications of social security arrangements in the presence of empirically relevant market frictions and individual heterogeneity, taking care to explicitly embed in the model the main institutional, national social security arrangements observed in OECD economies. Specifically, we compare the aggregate implications of defined benefit (DB) versus defined contribution (DC) systems and also investigate the economic outcomes from varying the generosity of a particular system. That is, we perform a comparison both between DB and DC systems but also within a particular system. We do this because an incremental reform (like an increase in payroll social security taxation or a reduction in benefits) might be more easily implemented in a politico-economic equilibrium than a more radical reform that replaces a pay-as-you-go (PAYGO) defined benefit (DB) system with a fully-funded, defined contribution (DC) one.

The closest paper to ours is Storeslsetten, Telmer and Yaron (1999) and it is useful to point out certain important differences. First, we perform the analysis within a particular

[1] A parallel literature analyzes similar issues by taking the interest rate and/or the rate of return on equity as exogenous (see, among others, Campbell et. al. (2001), Feldstein and Ranguelova (2001), Deaton, Gourinchas and Paxson (2002) and Miles and Sefton (2002), for instance).

system (DB or DC) allowing us to make welfare statements about more gradual reforms within an existing system. Second, our specification of social security arrangements is different. For instance, we include a flat pension that arises regardless of prior contributions but is earned simply by being a citizen in the particular economy, an arrangement that seems to be pervasive in OECD social security systems. Third, we introduce accidental and intentional bequests as determinants of the initial wealth distribution in the economy. Either of these assumptions is defensible according to recent evidence on wealth accumulation over the life cycle (Dynan, Skinner and Zeldes (2002) argue for accidental bequests while Laitner (2001) and De Nardi (fortcoming) argue for the explicit imposition of a bequest motive). Fourth, in an attempt to generate more realistic wealth distribution profiles, we experiment with different levels of ex ante heterogeneity in discount rates, following the Krusell and Smith (1998) infinite horizon results that small discount rate heterogeneity can match the skewed U.S. wealth distribution.

Given the desire to incorporate observed social security arrangements in the model, we first broadly describe different social security systems that exist in OECD economies and attempt to classify them into categories with broadly similar institutional features. We then embed aspects of these institutional arrangements in a realistically calibrated general equilibrium life-cycle model to quantify the implications for aggregate saving and capital formation. By varying the institutional features of the model, we can then better understand the potential effects of social security reform.

A normative question of interest concerns the balance between the distortionary effect of Social Security taxation and the risk sharing benefit of a Social Security system. We find that the insurance provided by a DB system can outweigh the efficiency cost from higher taxes to finance the DB payments. As a result, social welfare is maximized at positive DB provision levels. On the other hand, the fully-funded DC system that taxes an individual and offers the benefits during retirement depending on the interest rate and the individual's contributions, does not improve social (aggregate) welfare for any positive tax rate. There are two main reasons for this surprising result. First, the constraint that forces young workers to save through the DC account distorts the consumption-saving allocation sufficiently to generate consumption profiles for the poor that are substantially different from what they would have preferred in the absence of forced saving. Second, the models generate higher capital accumulation and a lower interest rate implying that saving for retirement (either through the DC or non-DC account) earns a lower rate of return that outweighs the positive effect of higher mean wages in the economy. Equivalently, in incomplete market (liquidity constraint) models similar to the ones studied in this paper there is over-accumulation of capital (Aiyagari, 1995). Introducing compulsory saving makes the liquidity constraint more binding for the more unlucky agents in the economy who experience lower earnings draws, generating additional precautionary saving over and above the already high capital accumulation implied by the zero DC tax rate.

The paper is organized as follows. Section 2 summarizes some of the key facts about social expenditure in the OECD and also categorizes the different social security systems in three main groups. Section 3 goes through the theoretical model and describes the equilibrium in the economy. Section 4 analyzes the defined benefit (DB) system and section 5 the defined contribution (DC) one. Section 6 concludes.

2 Stylized Facts on Pension Spending in the OECD

2.1 Old-Age Social Spending in the OECD

It is instructive to briefly review the evidence on the amount of public social expenditure in the OECD, and in particular, the amount of old-age social spending. To that effect, we use the OECD Social Expenditure Database: 1980-1998, 2001 edition. Table 1 reports the public social expenditure in the OECD between 1980 and 1998 as a percentage of GDP. Table 1 shows that the average social expenditure over all economies for this period is quite high (around twenty percent of GDP). Moreover, there is substantial variation in public spending both across countries and, for the same country, across time. Greece, for instance, starts 1980 with a share equal to 11.5 percent but rises to 22.7 percent by 1998. On the other hand, certain countries have kept total social expenditure relatively constant over the period: Austria and Belgium have varied their social expenditure between 25 and 27 percent of GDP in the last decade. Furthermore, continental Europe has a substantially higher social spending expenditure than the U.S. or Canada; in 1998 France and Germany spent 29 and 27 percent of their GDP, respectively, while the U.S. spent 15 percent. The last column reports the average expenditure for each country over these years and shows how most European countries (including the U.K.) have an average public spending expenditure between 22 and 28 percent over this period while, by comparison, the U.S. is at around 14 percent.

Even if social expenditure as a percent of GDP is high, old-age cash benefits might not be necessarily high given that other expenditures (namely health care) might be dominating the government budget. Table 2 shows that this is not so: old-age cash benefits are substantial and growing over time. In 1980, for instance, the average old-age expenditure as a percentage of GDP in the OCJ-829.2(-37.9(a0-2.3(n)]TJ/TT111Tf11.47210TD0Tc(15)Tj/13T21Tf4847210T.1(d)Tj/

grams, dividing them by two criteria: defined contribution versus defined benefit and funded (based on accumulated assets) versus unfunded (pay-as-you-go, PAYGO). An OECD citizen can actually be covered from a combination of such programs at a given point in time.

Most private/corporate pension plans are either funded, defined contribution (DC) or funded, defined benefit (DB) plans. In the former, employees have individual investment accounts to which they and/or their employer make periodic payments. The accumulated funds are usually invested in a range of securities and, at retirement, withdrawals are made depending on the value of the assets in the accounts, which reflect both the original contribution and the accumulated return. In funded, defined benefit plans, on the other hand, the final benefit is typically linked to the number of years the employee has been with the firm and the level of working-year earnings but not with the investment return or contribution in the pension fund. Private organizations are increasingly moving towards the defined contribution system since the risk associated with defined benefit systems seems to be quite high.

In this paper we will instead focus on the social security arrangements that exist at the national level which have, until recently, tended to be unfunded. The most usual system seems to be an unfunded, defined benefit program. Within this system, there are two main subcategories that are ubiquitous in OECD Social Security arrangements. The first class is meant to cater for the poorer individuals in a society and involves means-tested pensions that typically postulate a flat amount that eligible households can expect to receive. This category of pension payments seems to encompass Beveridge's guiding principles to social security design, namely that the aim of a social security system is to provide a universal, minimum, income level. Table 4 summarizes some of the main flat pension provisions in the OECD and illustrates how a minimum level of pensions seems to be guaranteed in most OECD economies, most of the time by virtue of being a citizen/resident in the country for a minimum number of years. Table 4 also illustrates that the flat pension component is quite small for wealthier individuals but can provide decent support for poorer individuals in an economy. Indeed, Hubbard, Skinner and Zeldes (1995) have argued that such social insurance mechanisms can prevent some individuals from saving for retirement during their working lives. More recently, the same crowding out of private saving by social insurance provisions has been found by Castañeda, Diaz-Giménez and Rios-Rull (2003) as necessary to match the poorest quintile of the U.S. wealth distribution.

The second typical arrangement is an earnings-related pension provided by the state and is more similar to the model pioneered by Bismarck in Germany in the nineteenth century. The retirement benefit is determined by a formula relating past earnings to current benefits and the current payments to retirees are typically financed through mandatory employer and employee contributions of the current workers. A similar arrangement is an earnings-related pension provided by the employer. The last two arrangements seem to make the bulk of replacement income during retirement. We will abstract from employer contributions in what follows and focus on a flat pension, DB system that is funded by proportional taxation on labor income on the currently working.

The third main system we will consider arises from recent suggestions to reform social security and introduce individual private accounts. Individual retirement accounts have been suggested because of their potentially welfare-enhancing effects by using the equity premium to reduce the amount of money that needs to be set aside to achieve the same level of benefits that could be achieved through a PAYGO system (see Feldstein and Ranguelova (2001), for instance).

3 The Model

This section presents the model of individual, life-cycle, consumption-savings behavior under uncertainty.

3.1 Preferences

Time is discrete and t denotes adult age. Utility is time separable and there is one non-durable good. We assume that the period-by-period felicity function is of the constant relative risk aversion (CRRA) form

$$\mathsf{U}(\mathsf{C}_t) = \frac{{\mathsf{C}_t}^{1-\rho}}{1-\rho}, \quad \rho \neq 1, \quad \rho > 0 \tag{1}$$

$$\mathsf{U}(\mathsf{C}_t) = \ln \mathsf{C}_t, \quad \text{when } \rho = 1. \tag{2}$$

The agents live for a maximum of T periods. We allow for uncertainty in T in the manner of Hubbard, Skinner and Zeldes (1995). The probability that a consumer is alive at time $(\mathsf{t}+1)$ conditional on being alive at time t is denoted by p_t $(\mathsf{p}_0 = 1)$. Given these survival probabilities, the weight of each cohort in the population can be computed as $\mathsf{w}_t = \mathsf{p}_t \mathsf{w}_{t-1}$ where $\mathsf{w}_0 = \mathsf{p}_0$ and the demographic weights are normalized to sum to one.

We will allow for both accidental and intentional bequests in the calibration. Accidental bequests take place unintentionally because of unexpected death. They are partly motivated by the work of Dynan, Skinner and Zeldes (2003) who argue that saving for retirement and for emergencies usually accounts for around two thirds of households as the main reason for saving in the U.S. Survey of Consumer Finances. Saving for an estate on the other hand features in less than five percent of the cases. Nevertheless, saving either for retirement or for precautionary reasons can be used for bequests in the (low probability) event of death reconciling the low incidence of citing bequests as a reason for saving and the fact that very few househols die with no assets. Moreover, using accidental bequests as an initial gift to the first age group in the model, allows us to have an initial distribution of wealth that does not put a probability weight of one in the zero wealth state. Nevertheless, we do experiment with the explicit incorporation of an intentional bequest motive because it has recently been argued that a reasonably calibrated bequest motive is necessary to capture the wealth distribution in the U.S. economy (Laitner (2001) and De Nardi (forthcoming)). For the case where a bequest motive exists, we need to specify a functional form for the value of bequests. We assume that the functional form for the bequest motive is identical to the felicity function and its strength is controlled by the parameter γ which is greater than or equal to zero. Letting A denote assets at the time of death, the felicity from bequests is given by

$$\mathsf{b}(\mathsf{A}) = \frac{\gamma}{1-\rho}\left(\frac{\mathsf{A}}{\gamma}\right)^{1-\rho} = \frac{\gamma^{\rho}}{1-\rho}(\mathsf{A})^{1-\rho}$$

When $\gamma = 0$, the bequest motive does not exist.

3.2 Labor Endowment

Retirement occurs at time K, $K < T$. For simplicity K is assumed to be exogenous and deterministic ($K = 65$). Before retirement, each agent supplies an exogenous quantity of labor that differs in individual productivity over the life cycle. Specifically, the exogenous stochastic process for individual labor supplied is given by

$$L_{it} = \exp(f(t, Z_{it}))P_{it}U_{it} \tag{3}$$

$$P_{it} = P_{it-1}^{\phi}N_{it} \tag{4}$$

where $f(t, Z_{it})$ is a deterministic function of age and household characteristics Z_{it} that captures the observed hump-shape in labor income over the life cycle and $\phi < 1$ implying that idiosyncratic labor income shocks have a persistent component.

The labor income uncertainty implied by this process is identical to the one used by Hubbard, Skinner and Zeldes (1995), and is decomposed into a persistent (but not permanent) component, P_{it}, a transitory component, U_{it}. We assume that the $\ln U_{it}$, and $\ln N_{it}$ are each independent and identically distributed with mean zero and variances σ_u^2, and σ_n^2, respectively. The log of P_{it} therefore evolves as an AR(1) with a deterministic drift, $f(t, Z_{it})$.

Given these assumptions and ignoring the deterministic drift, individual labor income follows

$$\ln L_{it} = \phi \ln L_{it-1} + \ln N_{it} + \ln U_{it} - \phi \ln U_{it-1}, \tag{5}$$

It should be pointed out that the preferred specification for most (but not all) empirical labor income studies involves a permanent ($\phi = 1$) component (see, among others, MaCurdy (1982), Abowd and Card (1989) and Pischke (1995)). Instead, we follow Hubbard, Skinner and Zeldes (1995) and Storesletten, Telmer and Yaron (1999) and use an AR(1) with a very persistent parameter ($\phi = 0.95$) that can potentially allow us to more easily investigate non-linear taxation and benefit schedules.

The final wage for individual i at age t will be determined as the product of an aggregate component that will arise from the production function (we call this wage W) and the idiosyncratic labor supply L_{it} : $Y_{it} = WL_{it}$. Earnings during retirement will be determined by the relevant pension arrangements in the economy.

3.3 Production Function

There is a continuum of firms with Cobb-Douglas production functions and they behave competitively in product and factor markets. Let K denote the aggregate capital stock and L the total labor supply. There is no aggregate uncertainty in this economy therefore generating a stationary wealth distribution. Aggregate production is given by $\mathsf{Y}_t = \mathsf{F}(\mathsf{K}_t, \mathsf{L}_t) = \mathsf{K}_t^{\alpha} \mathsf{L}_t{}^{1-\alpha}$. Factor prices earn the marginal product of capital so that $\mathsf{r} = \alpha(\frac{K}{L})^{\alpha-1} - \delta$ where δ is the depreciation rate of capital. The wage rate is given by the marginal product of labor $\mathsf{W} = (1-\alpha)(\frac{K}{L})^{\alpha}$.

3.4 The government

The government taxes both factors of production to cover government expenditures and the interest on outstanding debt. The government budget constraint is

$$\mathsf{G}_t + \mathsf{r}\,\mathsf{D}_t = \mathsf{D}_{t+1} - \mathsf{D}_t + \mathsf{T}_t + \mathsf{E}_t$$

where G is government consumption, D is public debt and T denotes the taxation proceeds. E denotes accidental bequests. In the baseline model we assume that these accidental bequests are fully taxed by the government and are therefore treated as tax revenue. Government expenditures do not enter the agents' utility function. Taxation proceeds arise from proportional taxation on capital and labor income. We set the capital tax rate (τ_K) to be equal to the labor income tax rate (τ_L); given an assumption about the level of government expenditures (twenty percent of GDP) these tax rates are determined endogenously and will also play a role in evaluating the differences between the different economies. We could have instead allowed the government expenditure to vary so that the budget was always balanced given fixed tax rates τ_K and τ_L. However, keeping government expenditures constant across different institutional features is an important component of the welfare analysis, as it eliminates the wealth effect resulting from changes in government consumption.

3.5 Social Security Arrangements

There are three different types of social security arrangements, two of which might be operational and valid at any given point in time for a particular individual. We now describe how the three different arrangements can be mapped into the calibrated economy.

(i) The flat pension system has the simplest features and is the easiest to describe and make operational. We assume that all agents in the economy receive the small, fixed benefit regardless of contributions. This benefit is financed by a proportional tax on the labor income of the currently employed agents in a PAYGO fashion. Therefore, given the exogenously assumed benefit level, we can solve for the tax rate that can support the assumed benefit level. We name the endogenous equilibrium tax rate that is associated with this system τ_{flat} and the pension payment Π.

(ii) The earnings-related pension provided by the state is unfunded and the benefit usually depends on the average maximum earnings for a certain subset of the total number of years spent working. For simplicity here, we define the benefit as depending on the average earnings over the whole working life, a lower bound on the benefits implied by the actual retirement rules. We view this as a good short-cut representation of the more complicated model agents might be solving. During working life, benefits evolve according to $(\mathsf{DB}_{i0} = 0)$:

$$\mathsf{DB}_{it} = \mathsf{DB}_{it-1} + \frac{\mathsf{Y}_{it}}{\mathsf{wkyrs}}$$

During retirement, the benefit is defined as a certain percentage (λ) of DB_{iK}, where $\mathsf{K} = 65$ is the exogenous retirement age. For every age from retirement until death, the individual receives a pension payment from the DB system equal to $\lambda * \mathsf{DB}_{iK}$. We experiment over a range of replacement rates (λ) to understand the dynamics of the model and the aggregate and distributional implications of different DB replacement rates. Given a value for λ, a proportional social security tax rate on labor income (denoted by τ_{db}) is determined endogenously to ensure that taxes to fund the DB system equal the outlays on benefits to the retirees, taking into account the demographic weights and the probability of death.

(iii) The DC pension provided by the state is described by the following benefit formula:

$$\mathsf{DC}_{it} = (1 + \mathsf{r})\mathsf{DC}_{it-1} + (\tau_{dc})\mathsf{Y}_{it}$$

with $DC_{i0} = 0$ once more. Each worker has her own individual account and faces a tax rate τ_{dc} of mandatory contribution and cannot access the money in the account before retirement. Moreover, in the benchmark model, the interest rate applied to the accumulated benefits is free of any capital gains tax, a benefit that is usually shared by many defined contribution programs. At retirement, the pension payment in each period ($t > K$) equals $\frac{DC_{it}}{EXPLife_{it}}$ where $EXPLife_{it}$ denotes the number of years the retired individual expects to live at age t while taxes are not paid on the pension received to make the comparison between the DB and DC economies symmetric. During this period, the account evolves as $DC_{it} = (1+r)DC_{it-1} - \frac{DC_{it}}{EXPLife_{it}}$. It is important to note that, contrary to the assumption made in the DB system where the tax was endogenous, in this system τ_{dc} is set exogenously and the pension payment becomes the endogenous variable. We take this approach because setting $\tau_{dc} = \tau_{db}$ facilitates a more fair comparison between the DB and DC systems.

3.6 Assets and wealth accumulation

There are two perfectly substitutable assets in the economy: government debt and the claim to the capital stock. Since the two assets are perfectly substitutable, they earn the same rate of return and individuals are therefore indifferent between holding government debt or saving through holding claims on the capital stock. It is important to also note that the distribution of ages (demographics), asset wealth and individual productivity shocks will be affecting asset price dynamics in the economy. We let μ denote the cross-sectional distribution of wealth, possible idiosyncratic shocks, ages and wealth in DC or DB accounts. Since we assume that there are no aggregate shocks in the economy, factor prices (wages and interest rates) are constant and a law of large numbers can be applied to show that μ will be constant over time (even though there is substantial individual mobility between the different states).

We follow Deaton (1991) and define cash on hand (total liquid wealth) as $X_{it} = C_{it} + B_{it+1}$ where B_{it+1} can denote either government bonds or claims to the capital stock. Liquid wealth can either be consumed or saved and liquid wealth in the next period can arise either from labor income, or from prior saving. At time t, an agent enters the period with invested wealth in the bond market B_{it} and receives after-tax capital income and after tax labor income from inelastically supplying one unit of labor. Labor income and social security taxes are paid on this amount. Cash on hand evolves as

$$\mathsf{X}_{it+1} = \mathsf{B}_{it}(1 + (1 - \tau_K)\mathsf{r}) + (1 - \tau_L - \tau_{ss} - \tau_{flat})\mathsf{L}_{it+1}\mathsf{W}_{t+1} \tag{6}$$

where $\tau_{ss} = \tau_{db}$ in the DB system and $\tau_{ss} = \tau_{dc}$ in the DC system. We impose an exogenous restriction on borrowing, rationalized by adverse selection and moral hazard problems. The constraint implies that $\mathsf{B}_{it} \geq 0$ for all t.

3.7 Equilibrium Definition

Our definition of equilibrium will include budget balance on the pay-as-you-go social security system as well as on the government budget. Given our goal to investigate the equity-efficiency trade-off that arises from higher taxation to finance larger benefit payments, we set the pension payment exogenously and derive the social security tax that will be needed to generate a sustainable social security payment. We therefore define the equilibrium to consist of these three endogenously-determined variables ($\tau_K = \tau_L$, τ_{ss} and τ_{flat}), market clearing prices $\mathsf{R} = 1 + \mathsf{r}$ and W, and a set of cohort specific value and bond functions, $\{\mathsf{V}_t, \mathsf{B}_t\}_{t=1}^{T+1}$, such that,

1. Firms maximize profits by equating marginal products of capital and labor to their respective marginal costs. Specifically,

$$\mathsf{r} = \mathsf{F}_K(\mathsf{K}, \mathsf{L}) - \delta$$

and

$$\mathsf{W} = \mathsf{F}_L(\mathsf{K}, \mathsf{L})$$

2. Individuals pick optimal consumption-saving under earnings uncertainty and liquidity constraints, given the interest rate, wages and the economic environment. The recursive optimization problem faced by each household can be written down as follows

$$V_t(X_{it}, A_{it}, L_{it}; W, r) = MAX_{C_{it}, B_{it+1}} U(C_{it}) + \beta\{p_t E_t V_{t+1}(X_{it+1}, A_{it+1}, L_{it+1}; W, r) + (1 - p_t) E_t b(X_{it+1}, L_{it+1}, A_{it+1}; W, r)\}$$

where A_{it} denotes the amount accumulated in the retirement account, and all other variables have been previously defined.

3. Markets clear and aggregate quantities result from individual decisions. Specifically,

$$D + K = \int_\mu \int_i B_i d\mu$$

$$L = \int_\mu \int_i L_i did\mu$$

$$G + K' + \int_\mu \int_i C_i did\mu = F(K, L) + (1 - \delta)K$$

while the first equation changes to the following when DC accounts exist

$$D + K = \int_\mu \int_i (B_i + A_i) d\mu$$

where A_i are the assets of household i accumulated through the DC account.

4. The government budget constraint is satisfied. Specifically,

$$G + rD = \int_\mu \int_i (\tau_K B_i r + \tau_L L_i W) did\mu + E$$

where E denotes accidental bequests:

$$E = \int_\mu \int_i B_i [1 + r(1 - \tau_K)] did\mu$$

5. The pay-as-you-go components of the social security system are balanced at all times. In the flat pension component of social security this means that

$$\int_\mu \int_i \tau_{flat} L_i W did\mu = \int_\mu \int_i \Pi did\mu$$

For the DB pension system this means that

$$\int_{\mu}\int_{i\in I_W}\tau_{db}\mathsf{L}_i\mathsf{W}\,\mathsf{d}i\mathsf{d}\mu = \int_{\mu}\int_{i\in I_R}[\lambda\exp(\mathsf{f}(\mathsf{K},\mathsf{Z}_{iK}))\mathsf{P}_{iK}]\mathsf{d}i\mathsf{d}\mu$$

where the integrations on the left hand side over i are done over all workers (I_W) while the integrations over i on the right hand side are done over retirees (I_R).

For the DC system, by construction, the individual retirement accounts are balanced for each individual.

3.8 Solution Method

Analytical solutions to this problem do not exist. We therefore use a numerical solution method based on the Euler equations associated with this problem to derive optimal policy functions for total savings as a function of age, liquid assets and either the defined benefit level or the amount of wealth accumulated in the defined contribution account. We discretize the state-space (cash-on-hand) and use a cubic spline interpolation (with a more dense grid point for the low levels of cash on hand where the kink in the consumption-saving policy function arises). We use a linear interpolation along the second continuous state variable (DB or DC account). We use Gaussian quadrature to approximate the distributions of the innovations to the labor income process. In every period t prior to T, we find the optimal saving function that solves the non-linear Euler equation and use a quadrature procedure to approximate the exogenous AR(1) state variable. We offer more details of this numerical implementation in the appendix.

3.9 Welfare

We are interested in making welfare evaluations both within and between systems. To do so, we compute the value functions as functions of cash on hand, the benefit level depending on the social security system, the idiosyncratic persistent shock and age at the equilibrium interest and taxation rates. We then evaluate aggregate welfare by monte carlo integration: we simulate 2000 individual life histories and compute for every age, cash on hand and idiosyncratic shock the value associated with the optimal program using the appropriate demographic weights. This is an utilitarian social welfare function (denote it by U). Utilitarian welfare increases if consumption rises, if inequality is reduced (since the welfare function is concave) or if uncertainty is reduced (since agents are risk averse).

To compare two different economies, we find the percent of life-time consumption that agents in one system are prepared to give up to accept the policy change. We report all our results with the $\lambda = 0$ case as the benchmark (call this economy A). It can be shown that the proportional, percentage life-time consumption agents are prepared to give up to move from one regime (A) to another (B) is given by[2]

$$100 * [(\frac{\mathsf{U}_B}{\mathsf{U}_A})^{1/(1-\rho)} - 1]$$

This is the metric we use in our evaluations.

3.10 Parameter Calibration

The model is solved at an annual frequency. The coefficient of relative risk aversion is set equal to 2, the annual rate of time preference, δ, equal to 0.04. For the individual labor income process, we use parameters almost identical to those used by Hubbard, Skinner and Zeldes (1995): 21 percent per year for σ_u, 15 percent per year for σ_n and 0.95 for the AR(1) parameter. The deterministic labor income profile reflects the hump shape of earnings over the life-cycle, and the corresponding parameter values are taken from Cocco, Gomes and Maenhout (1999). The production function parameter α is set to 0.36, and capital depreciates at seven percent. The flat pension benefit is set at 0.1 (implying a replacement rate of the last working period mean, net earnings of around ten percent). The government share of GDP is set at 20 percent, while government debt is set at 60 percent of GDP. There is no Social Security debt in equilibrium in this economy.

For the DB economy, we vary the DB pension provision (λ) from zero to 0.4 which generates an endogenous replacement rate that encompasses empirically observed magnitudes. To finance these benefits, an endogenous social security tax rate is found by equilibrating

[2] See Flodén (2001) for an detailed discussion the components of this welfare metric and results from an infinite horizon economy that is, in most other dimensions, similar to the economy studied here.

social security tax revenues and benefits. For the DC economy, we instead set the social security tax rates first and pension payments from the system are made endogenous.

4 Defined Benefit and Flat Pension Economy

4.1 No ex ante Heterogeneity and Accidental Bequests Fully Taxed

We first describe the policy functions and wealth accumulation profiles in the DB-flat pension economy. Figures 1 and 2 show how consumption is positively related with the expected benefit level and that this relationship is more important as the household approaches retirement. Early on in the life-cycle (figure 1 presents policy functions for age 21) the household saving decision is dominated by the precautionary savings motive and the expected benefits during retirement do not affect the consumption-saving decision significantly. As the retirement date approaches, however, the expected benefit level (that depends on the resolution of labor income uncertainty) becomes important and individuals with lower expected benefits make up the shortfall by higher private saving (figure 2). Figure 3 shows the same economic intuition using the life-cycle wealth accumulation profiles: the more generous social security system crowds out private saving resulting in a lower wealth accumulation over the life cycle.

At this stage it is instructive to present the quantitative economy-wide effects of varying the DB level (table 5). As the DB level is increased, private saving is being crowded out, resulting in a lower equilibrium capital stock and a lower output level. Equivalently, the investment to GDP ratio decreases and the consumption to GDP ratio rises (since the government to GDP ratio is exogenously set at 20 percent). Given that a more generous DB system crowds out private saving, the interest rate rises; in particular the gross interest rate rises from 5.60 percent when $\lambda = 0$ to 9.46 percent when $\lambda = 0.40$. For the more generous system, a higher steady-state DB tax rate is needed to fund the higher DB payments: this tax rate rises from zero (when λ is zero) to 10.6 percent (when $\lambda = 0.40$). The tax rate on labor and capital gains ends up rising as well given the fall in the capital stock (which determines the tax base) and the assumption that the government to GDP ratio stays constant. In particular, the tax rate on labor income (and capital) rises from 19.3 to 22.7 percent when

λ rises from zero to 0.40.

We also report the replacement rate of net, last working period, mean wages from the two different pension systems. The replacement from the flat pension rises from around 8.6 to 12.3 percent as λ is increased from zero to 0.40, mostly due to the reduction in aggregate wages as the capital stock falls. The replacement ratio of net mean wages from the DB system becomes the important source of retirement income since it rises to around 75 percent for the highest benefit level.

Wealth inequality is an interesting variable to think about in the presence of redistributive systems. We use the Gini coefficient to calculate wealth inequality in the model economies[3]. This coefficient varies from 0.53 when $\lambda = 0$ to 0.56 when $\lambda = 0.40$ illustrating that the wealth distribution is becoming (surprisingly) a little more unequal as the DB system becomes more generous. A closer look indicates that the inequality is mostly driven by what happens during retirement. Retirees receive more unequal benefits when λ is higher, generating a slightly higher wealth inequality during retirement.

Table 5 also reports aggregate consumption-equivalents in percentage terms (our welfare measure). Given the substantial crowding out of capital that a more generous DB system implies, we might expect that aggregate social welfare is decreasing for higher DB benefits. This is not necessarily the case, however. Aggregate social welfare is a non-monotonic function of benefits and is maximized at different DB levels depending on the structural parameters of the model. In general, young workers (older workers/retirees) tend to dislike (like) a more generous welfare system.

Why does this non-monotonicity arise? To answer this question it is useful to consider Figure 4 which plots the standard deviation of consumption for the lowest and highest DB economies along with the standard deviation of after-tax earnings for the same economies. The earnings distribution is more compact for the higher DB economy because taxes are higher to fund the given government expenditure. As a result, the standard deviation of

[3]The Gini coefficient ranges between zero and one. If all agents in the economy have equal wealth, then the coefficient is zero. If one agent in the economy has all the wealth, then the coefficient is one. In the U.S., the wealth distribution is heavily skewed to the right: the Gini coefficient associated with the U.S. economy is 0.78 (see, for instance, Castañeda et. al. (2003)).

consumption over working life is slightly lower for the high DB economy during working life. Figure 5 plots the effects on mean consumption and illustrates that mean consumption is allowed to be higher during the working life cycle for the low benefit economies. This reflects the lower amount of taxation and crowding out of capital that lower benefits imply. Thus, a lower standard deviation of consumption is associated with a lower mean consumption during working life. On the other hand, both the mean and the standard deviation of consumption during retirement are higher for the larger DB levels.

Figures 4 and 5 illustrate the conflict between equity and efficiency that the redistributive DB system imparts on the economy. A more generous DB system implies a higher mean consumption during retirement due to the higher social security benefits but also higher inequality as these payments exacerbate a given level of initial inequality (from age 65). On the other hand, current (young) workers are penalized by lower equilibrium wages due to the crowding out of aggregate capital. The cost in terms of foregone consumption for the workers is seen in figure 5 where mean consumption is highest in the lowest benefit economy since the inefficiency from higher taxation is minimized.

The welfare calculations on table 5 illustrate the tradeoff between equity and efficiency since the welfare function is rising in the DB pension level at low pension benefits, reaches a maximum at around 0.10 units of promised DB replacement rate and then is reduced as the inefficiency from higher taxation becomes more important than the equity gain. A decomposition of this welfare change from the zero benefit economy between workers and retirees to argue that a positive tax on capital gains can be rationalized since the tax can move the capital stock closer to its complete markets equivalent (by reducing the equilibrium capital stock). A similar mechanism seems to operate in our artificial economy making the DB value that maximizes social welfare positive[4].

4.2 No Ex Ante Heterogeneity and Accidental Bequests Providing Initial Wealth Distribution

We have assumed until now that accidental bequests arising from unexpected death are fully taxed by the government and enter the balanced budget government constraint to determine the capital (labor) tax rate. A related assumption is that all agents start life with zero initial assets. Alternatively (and perhaps more naturally) we may assume that accidental bequests are given as gifts to the youngest agents in the economy. As a result, the initial distribution of assets does not also start out at zero but instead reflects the random death process inherent in the model.

Table 6 reports the results from varying the DB level in this new setting. There are three main changes relative to the results from fully taxing accidental bequests. First, the tax rate on capital rises to balance the government budget since accidental bequests (around six percent of GDP in table 5) can now not be relied upon to balance the budget. In the baseline $\lambda = 0$ case the change is from 19.3 percent to 28.1 percent. Second, output, capital and the capital/output ratio all increase relative to table 5. To understand this result, it is useful to compare the wealth accumulation profiles for this economy (figure 6) relative to the previous economy (figure 3). The main difference arises from the initial asset assumption: when bequests are received as gifts, assets for very young agents are much higher than when initial assets start from zero. Given that the policy functions are the same in the two economies, the interest earned on these higher initial assets implies that a slightly

[4] When individuals are made more patient ($\delta = 0.02$), higher saving arises for any given level of the interest rate. This implies that equilibrium saving (and the capital stock) rises and the interest rate falls (by around two percent). Given the higher capital stock, output and wages in the economy (the tax base), the equilibrium tax rates are reduced by around one percent (for all DB variations). Positive levels of DB provision continue to maximize social welfare.

higher total wealth accumulation is possible, as reflected in the aggregate capital stock in table 6. Third, aggregate wealth inequality decreases between workers in the two economies. Specifically, the gini coefficient is between 0.53 and 0.56 when accidental bequests are fully taxed but falls to between 0.47 and 0.55 when initial assets are determined by accidental bequests. A decomposition of this inequality between workers and retirees illustrates that the gini coefficients for retirees are much closer between the two economies, implying that the changes are coming from wealth inequality among the workers. Comparing the mean wealth accumulation profiles from figures 3 and 6, the results look natural since total wealth accumulation reaches similar levels by age 65, yet initial assets are widely skewed to the left in figure 3 because of the zero initial assets assumption.

How does welfare change relative to the no DB system case? Our mean aggregate welfare computation in table 6 illustrates that total welfare is maximized when $\lambda = 0.2$. The same reasons that drove this result in the previous economy become more important. First, a decomposition of the welfare change between workers and retirees once more illustrates that retirees prefer a more generous DB system while workers prefer smaller levels. A further decomposition over the life-cycle (omitted for space considerations) illustrates that workers over the age of around 40 on average prefer the more generous system. Aggregate social welfare is maximized at a higher level of benefits than in the previous economy. This arises from having a higher equilibrium capital stock implying that a higher efficiency cost can be paid before welfare is reduced. Second, the even higher wealth accumulation in the economy implies that capital has been over-accumulated to a greater amount than in the previous subsection. A more generous system can crowd out the capital stock more and take it closer to its complete markets counterpart.

4.3 No Ex Ante Heterogeneity and Bequest Motive

What happens when in addition to accidental bequests, agents care about their offspring and save directly to provide them with intentional bequests? We use a parameter for the bequest motive $\gamma = 10$, possibly an upper bound for this parameter given the wealth accumulation profiles that are generated by the model. Table 7 reports the results from varying the DB level in this new setting. The two main changes relative to the results from fully taxing accidental bequests that arose in the previous subsection persist: the taxation rate of capital rises to balance the government budget while output, capital and the capital/output ratio all increase relative to table 5. To understand this result, it is again useful to compare the wealth accumulation profiles for this economy (figure 7) relative to the previous economies (figures 3 and 6). The main difference from figure 3 continues to arise from the initial asset assumption: when bequests are received as gifts, assets for very young agents are much higher than when initial assets start from zero. There is now a second difference from both figures 3 and 6, however. Specifically, the intention to leave bequests forces death at age 100 with positive assets. This is a lot more pronounced for the highest benefit economy in figure 7, since the textbook life-cycle wealth accumulation does not seem to take place but instead is being replaced by an almost horizontal line over the life cycle. This is reflected in the wealth gini coefficient which varies between 0.35 and 0.26 in table 7 but is between 0.53 and 0.56 in table 5 and 0.47 and 0.55 in table 6.

What is striking in table 7 relative to the previous economies is that aggregate welfare is maximized at even more generous welfare levels. This result depends on the fact that wealth accumulation is much higher than in the previous two economies, so that the inefficiency cost (or the crowding out of capital) is not very high in an abundant-capital economy.

Figure 8 sheds further light on the welfare effects between the three economies. Specifically, for the middle benefit cases ($\lambda = 0.2$) the graph plots the standard deviation of consumption over the life-cycle varying the bequest assumption. Inequality (as measured by the standard deviation of consumption over the life-cycle) rises the most when bequests are fully taxed. The closest case is the one where bequests are not taxed but accidental bequests are given as gifts to the young. Nevertheless, the difference between the two measures is not substantial. What is more significant is the inequality of consumption for the economy with the bequest motive which rises over the life-cycle but does not fluctuate to the degree observed for the other two economies.

4.4 Ex Ante Heterogeneity

We have also considered ex ante discount rate heterogeneity to potentially generate even more skewed wealth distributions that can more closely match the U.S. data. This follows the recent suggestion by Krusell and Smith (1998) that small discount rate heterogeneity can account well for the skewed wealth distribution in the U.S. in an economy where agents have infinite horizons. For these comparative statics, we assumed that there are three discount rates in the economy (two, four and six percent) and the weight given to each one of them is one third.

The direction of the comparative statics results is identical to what has been reported for the single discount rate economy. When $\lambda = 0$, the capital stock is 5.32 (implying a gross interest rate of 5.35%), a slightly higher capital stock from the single discount rate case in table 5 (5.15). As the generosity of the pension system is increased, private saving is crowded out generating a lower equilibrium capital stock which reaches 3.52 when $\lambda = 0.4$ (compare to 3.40 in the single discount rate economy). Mean aggregate welfare is maximized at $\lambda = 0.1$ with our welfare measure implying 0.39% higher mean consumption in that steady state than when $\lambda = 0.0$. Moreover, the non-monotonic shape of the aggregate social welfare function remains intact. We conclude that the qualitative (and, for the case considered, quantitative) implications of the model do not substantially change with discount rate heterogeneity.

A remaining issue is whether the substantial discount rate heterogeneity improves substantially the model's ability to generate a more skewed wealth distribution. The Gini coefficients now range between 0.54 when $\lambda = 0$ to 0.58 when $\lambda = 0.4$. These numbers are not significantly higher than their counterparts from the single discount rate economy (0.53 and 0.56 respectively). We conclude that even though wealth inequality increases with the introduction of discount rate heterogeneity, the change is quantitatively very small. We conclude, that in a finite horizons model, discount rate heterogeneity alone might not be sufficient to generate the wealth inequality observed in U.S. data.

5 Defined Contribution and Flat Pension Economy

5.1 No ex ante Heterogeneity and Accidental Bequests Fully Taxed

The DC system provides a pension depending on an individual's contribution in an account, and therefore there is no risk sharing between the young and the old. The system is fully-funded and every individual owns an account as an inalienable property right: variants of such a system are being considered as an alternative to the DB systems currently in operation in the OECD. In the artificial economies we construct, total savings are deposited in two accounts: the DC account with the contribution being determined exogenously and a private savings (non-DC) account. The DC contribution rate is set exogenously to the steady-state tax rate that was needed to fund a given pension benefit in the DB economy. We think this is a useful way of comparing the two economies. All other parameters are the same as in the DB economy.

Figures 9 and 10 plot the consumption policy functions varying the generosity of the system and the results are similar to the DB system. The precautionary savings motive dominates for young households and the policy function is not affected significantly by the generosity of the pension system (figure 9 for households with age equal to 21). As retirement approaches, however, a higher accumulated DC implies higher consumption for a given level of cash on hand (figure 10 for households with age equal to 55) since saving for retirement takes place. Figures 11 and 12 present life-cycle wealth accumulation profiles for the private savings (non-DC) and the DC account, respectively. Figure 11 shows that individual saving and wealth accumulation over the life cycle is higher in a less generous system. On the other hand, the DC account is increasing in the tax rate used to accumulate contributions. The sum of savings in the two accounts (which equal the capital stock and government debt in equilibrium) are slightly higher as the DC tax rate is increased, reflecting the preferential tax treatment that saving through the DC account has and is consistent with the results in İmrohoğlu, İmrohoğlu and Joines (1998).

Table 8 reports the equilibrium results from varying the DC tax contribution rate[5]. A

[5] It is useful to note that when the DB scheme is absent ($\lambda = 0$), the DC tax rate is also zero. As a result, the first column of table 8 should give identical results with the first column of table 5.

higher tax rate is associated with a higher capital stock accumulation and thus a higher output level. The capital output ratio is increased, while the share of consumption in GDP falls and the investment to GDP ratio rises. The higher capital stock means that a lower equilibrium interest rate is needed to induce saving and the gross interest rate is therefore reduced.

A number of important differences relative to the DB economy (table 5) arise. First, the higher capital stock as the DC tax rate is increased generates a higher capital-output ratio but at the same time generates a lower steady-state consumption to GDP ratio. Specifically, consumption varies from 60 to 55 percent of GDP as the tax rate varies from zero to 10.6% but in the DB model the change is in the opposite direction from 60 to 65%. Moreover, the gross (and net) interest rates also move in opposite directions from 5.60% to 9.46% in the DB model and from 5.60% to 2.86% in the DC one.

Second, the labor (capital) tax rate varies much less than the tax rate in the DB experiments. A higher output level implies that a higher government outlay exists in the economy but a higher capital stock means that the tax base (wages and capital) have increased (despite the fall in the equilibrium interest rate that mitigates this effect). Moreover, the lower interest rate implies lower expenses to service the government debt. The combined effect of all these changes is to allow the tax rates on labor and capital to remain approximately constant as the DC tax rate is changed, contrary to the DB results where taxes changed substantially more as the generosity of the system was being altered.

Third, and contrary to the DB results, the after-tax earnings distribution during working life does not change much when the DC tax rate is altered. Figure 13 illustrates the amount of inequality in consumption and labor income for the highest and lowest DC tax rates (compare to figure 4 for the DB results). Why is this happening? After-tax earnings are determined by the product of proportional taxes, the aggregate wage and the idiosyncratic shocks. The idiosyncratic shocks are not affected when moving between regimes. The tax rate on labor remains relatively constant (moves from 19.3% to 20.5% from the lowest to highest DC rate). Nevertheless, the DC tax rate rises from zero to 10.6% leading to more equalization of after-tax earnings. On the other hand, we have seen that the aggregate capital stock rises as the DC rate is increased, leading to higher aggregate wages that lead to a more wide distribution of after-tax earnings[6]. Figure 13 illustrates that the balance of these effects is to reduce slightly the after-tax earnings distribution but the difference is nowhere near the change in after-tax earnings found for the DB economy. As a result, the cross-sectional standard deviation of consumption is not affected significantly for the workers (relative to what was the case in the DB economy). During retirement earnings inequality is higher for the higher DC rates and is being gradually eliminated as the individuals reduce their DC accounts gradually to zero. There is now a pronounced difference between the standard deviation of consumption with the highest DC rate generating a more compact distribution, the effect of being able to utilize the savings through the DC account to finance retirement consumption.

Given that the DC account encourages saving and leads to higher wealth accumulation while at the same time generating a more compact distribution of consumption during retirement, one might expect the higher DC tax rate regime to lead to a higher social welfare. Table 8 illustrates that this is not the case however: social welfare is monotonically decreasing in the tax rate and aggregate social welfare is maximized when the DC tax rate is set equal to zero. There are two main reasons for these welfare results. First, in incomplete markets models with undiversifiable labor income risk and liquidity constraints there is over-accumulation of capital as agents save for precautionary reasons. This effect is probably compounded in this model as agents save for retirement. In the DB model, private saving is crowded out as benefits are increased, moving the economy closer to its complete markets counterpart. In the DC economy, on the other hand, total saving is increased since the liquidity constraint becomes more severe as a higher percentage of labor income is being comitted in the illiquid (during working life) DC account. As a result, the economy accumulates even more capital than the already high capital stock associated with the original model. Second, and related, the fact that the liquidity constraint becomes more binding as the DC rate is increased, generates a re-allocation of consumption over the life-cycle that is neither optimal nor desired. This is illustrated in figure 14, which plots mean consumption

[6] Note that in the DB economy the aggregate capital stock falls as the DB tax rate is increased, leading to lower aggregate wages. This goes in the same direction as increased taxes in generating a lower standard deviation of after-tax earnings over the life cycle.

for the tenth percentile of the wealth distribution and illustrates that there is a significant spike for this group of the population during retirement. This is paid for by lower mean consumption during the early part of the life-cycle as the individual is forced to save through the illiquid DC account. Given that the individual expects a rising labor income profile early in life and would rather borrow than save or, worse still, be forced to save for retirement, the additional constraint leads to a reallocation of consumption-savings over the life cycle that is suboptimal[7].

5.2 No Ex Ante Heterogeneity and Accidental Bequests Providing Initial Wealth Distribution

Following closely the experiments performed for the DB economy, we next assume that accidental bequests are given as gifts to the youngest agents in the economy. Table 9 reports the results from varying the DC tax rate in this new setting. Consistent with the results in the DB economy, there are two main changes relative to the results from fully taxing accidental bequests. First, the taxation rate of capital rises to balance the government budget since accidental bequests (around 7.5 percent of GDP in table 8) can now not be relied upon to balance the budget. In the baseline case when the DC tax rate is zero, the change is from 19.3 percent to 28.1 percent. Second, output, capital and the capital-output ratio all increase relative to table 8. To understand this result, it is useful to compare the wealth accumulation profiles for this economy (figure 15) relative to the previous economy (figure 11)[8]. The main difference arises from the initial asset distribution assumption: when bequests are received as gifts, assets for very young agents are much higher than when initial assets start from zero. Given that the policy functions are the same in the two economies, the interest earned on these higher initial assets implies that a slightly higher total wealth accumulation is

[7] A similar graph can be constructed comparing the mean consumption of the tenth percentile for the most generous DB and DC systems. The DB system does not exhibit a spike since savings that can be accessed to smooth consumption fluctutations for the poorer people in the DB system are much higher over the life cycle, even though they face the same tax rates for social security.

[8] The accumulation through the DC account has the same shape (but different level) since contributions are exogenous.

possible during most working years and this is reflected in the aggregate capital stock in table 9. This change in wealth accumulation also affects the wealth distribution statistics. The wealth distribution for workers is more equal now as very few agents start with zero assets in the economy: the gini coefficients for workers vary between 0.38 and 0.34 in table 9 (compare with the 0.51 to 0.49 range in table 8). The gini coefficients for retirees are not substantially changed however as the usual decumulation takes place in the retirement stage of the life cycle.

How does welfare change relative to the no-DC system case? Our mean aggregate welfare computation in table 9 confirms the conclusions from the previous subsection, namely that the overaccumulation of capital associated with higher DC tax rates does not enhance aggregate welfare: a zero DC tax rate remains optimal.

5.3 No Ex Ante Heterogeneity and Bequest Motive

We next assume that in addition to accidental bequests, agents care about their offspring and save directly to provide them with inherited wealth. We use a parameter for the bequest motive $\gamma = 10$. Table 10 reports the results from varying the DC tax rate in this new setting. The two main changes relative to the results from fully taxing accidental bequests that arose in the previous subsection persist: the taxation rate of capital rises to balance the government budget while output, capital and the capital-output ratio all increase relative to table 8. To understand this result, it is again useful to compare the wealth accumulation profiles for this economy (figure 16) relative to the previous economies (figures 11 and 15). The main difference from figure 11 continues to arise from the initial asset distribution assumption: when bequests are received as gifts, assets for very young agents are much higher than when initial assets start from zero. There is now a second difference from both figures 11 and 15, however. Specifically, the intention to leave bequests forces death at age 100 with positive assets. The effect of this altruistic motive on wealth accumulation dramatically reduces the wealth inequality in the economy since both very few agents start with zero initial assets and very few die at age 100 with zero assets. Specifically, the gini coefficient for the whole economy varies between 0.35 and 0.33, some of the lowest measures of wealth inequality in all the simulations performed.

Given the reduction in wealth inequality that takes place, the possibility arises that the welfare rankings for the different DC tax rates might change. Nevertheless, a higher capital stock arises in equilibrium than in the previous two economies, and the over-accumulation of capital persists. Moreover, the direction of the comparative statics as the DC tax rate is increased illustrates that the nature of our conclusions does not change. Indeed, aggregate welfare is still maximized at zero DC tax rates.

5.4 Ex Ante Heterogeneity

Consistent with the experiments performed for the DB economy, we next considered the effects of ex ante discount rate heterogeneity, once again assuming that there are three discount rates in the economy (two, four and six percent) and the weight given to each one of them is one third. The direction of the comparative statics results is identical to what has been reported for the single discount rate economy. When $\lambda = 0$, the capital stock is 5.32 (implying a gross interest rate of 5.35%), a slightly higher capital stock from the single discount rate case in table 5 (5.15). As the DC tax rate is increased, total saving is once again increased generating a higher equilibrium capital stock which reaches 7.66 when $\lambda = 0.4$ (compare to 7.57 in the single discount rate economy). Mean aggregate welfare is maximized at the zero DC tax rate and the monotonic decrease of the aggregate social welfare function remains intact. We conclude that the qualitative (and, for the case considered, quantitative) implications of the model do not substantially change with discount rate heterogeneity.

6 Conclusion

Our results indicate that positive levels of DB social security systems can be welfare enhancing but this does not turn out to be the case for DC systems. In particular, the over-accumulation of capital that exists in economies with liquidity constraints implies that the crowding out of capital in DB systems, coupled with its social insurance features, can be welfare enhancing. On the other hand, DC accounts generate a further increase in capital accumulation through the tightening of the liquidity constraint for the poorer segment of the population. This increase reduces aggregate welfare.

A number of extensions are the subject of current research. First, we would like to enhance the ability of the model to capture better the wealth distribution: our results indicate that ex ante heterogeneity in discount rates does not achieve this goal, and incorporating institutional constraints as in Castañeda et. al. (2003) might improve the model's predictions along that dimension. Second, incorporating aggregate uncertainty and differentiating between government debt and risky capital may help us to further understand the implications of alternative social security systems.

A Computational Method

The model is solved numerically by discretizing the two continuous state variables X_{it} (cash on hand) and A_{it} (amount accumulated in DB or DC account), using a more dense grid at the lower range where the liquidity constraint might be binding. We use an Euler equation (rather than a value function approach) because the saving policy function is smoother than the value function and therefore interpolation at points not on the grid can be performed at greater accuracy for the same number of grid points. We use a cubic spline interpolation procedure along the X_{it} dimension and a linear interpolation along the A_{it} dimension. The range of the two state variables is endogenous and depends on the equilibrium values of the other variables in the model: we use a trial and error method to ensure that the simulated variables fall within the postulated range.

The Euler equation with a bequest motive in the DB model takes the following form (set $\gamma = 0$ for the accidental bequest model)[9]:

$$\mathsf{U}'(\mathsf{X}_t - \mathsf{B}_{t+1}) = \mathsf{MAX} \begin{bmatrix} \mathsf{U}'(\mathsf{X}_t), \mathsf{p}_t \frac{1+(1-\tau)r}{1+\delta} \mathsf{E}_t \mathsf{U}'(\mathsf{C}_{t+1}(\mathsf{X}_{t+1})) \\ +(1-\mathsf{p}_t)\gamma^{\rho} \frac{1+(1-\tau)r}{1+\delta} \mathsf{E}_t \mathsf{U}'(\mathsf{X}_{t+1}) \end{bmatrix} \tag{7}$$

where the identity $\mathsf{C}_t + \mathsf{B}_{t+1} = \mathsf{X}_t$ holds for all t and the non-linear equation is solved at all grid points of X_t to find $\mathsf{B}_{t+1}(\mathsf{X}_t, \mathsf{A}_t)$. The law of motion for X_{t+1} was given in the text and

[9] The subscript i denoting an individual is suppressed in what follows for notational simplicity.

it depends on the life-cycle stage (whether a worker or a retiree and the other institutional features of the pension system). The law of motion for A_{t+1} depends on the pension system being studied and was given in the text. The numerical problem is solved backwards from the end of life. The last period policy function in the presence of the bequest motive can be shown to be $\mathsf{B}_{t+1} = \mathsf{X}_t * (1 + [\frac{1+\delta}{\gamma^\rho(1+(1-\tau)r)}]^{\frac{1}{\rho}})$, which implies that $\mathsf{B}_{t+1} = 0$ in the absence of the bequest motive ($\gamma = 0$). Given this last period saving function, the equation can be solved forward to the beginning of life.

In the DC model, we assume that at the time of death the individual's account can be liquidated and bequathed to their estate. The Euler equation therefore becomes

$$\mathsf{U}'(\mathsf{X}_t - \mathsf{B}_{t+1}) = \mathsf{MAX}\left[\begin{array}{c} \mathsf{U}'(\mathsf{X}_t), \mathsf{p}_t \frac{1+(1-\tau)r}{1+\delta} \mathsf{E}_t \mathsf{U}'(\mathsf{C}_{t+1}(\mathsf{X}_{t+1})) \\ +(1-\mathsf{p}_t)\gamma^\rho \frac{1+(1-\tau)r}{1+\delta} \mathsf{E}_t \mathsf{U}'(\mathsf{X}_{t+1} + \mathsf{DC}_{t+1}) \end{array}\right] \quad (8)$$

The only difference between the two Euler equations arises from the treatment of the accumulated assets in the DC account since these assets are now passed on to the household's descendant. The rest of the computational details remain the same as in the DB system.

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Table 1: PUBLIC SOCIAL EXPENDITURE in the OECD
MEASURE: As a percentage of GDP (SNA93)

	1980	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**Average 1980-1998**
Australia	11.32	11.33	12.34	12.96	13.30	13.50	13.38	13.19	12.45	12.78	14.36	15.43	16.38	16.51	16.25	17.79	17.92	17.62	17.81	14.56
Austria	23.33	-	-	-	-	25.13	-	-	-	-	25.00	25.26	25.89	27.28	27.98	27.88	27.91	27.04	26.80	26.32
Belgium	24.18	25.80	26.48	26.78	26.07	26.99	26.77	26.45	26.04	25.09	24.60	25.35	25.43	26.50	25.96	25.07	25.53	24.21	24.54	25.68
Canada	13.26	13.84	16.18	16.38	16.60	16.97	17.01	16.73	16.53	16.92	18.25	20.64	21.28	21.21	20.17	19.23	18.41	17.84	18.03	17.66
Czech Republic	-	-	-	-	-	-	-	-	-	-	16.81	18.18	18.52	19.02	18.99	18.64	18.59	19.41	19.42	18.62
Denmark	29.06	29.37	29.57	29.94	28.65	27.87	26.99	27.77	29.20	29.48	29.32	30.17	30.72	32.35	33.06	32.41	31.69	30.66	29.81	29.90
Finland	18.51	19.03	20.29	20.93	21.83	22.92	23.49	23.98	23.29	23.06	24.78	29.91	33.92	33.88	33.04	31.24	30.97	28.72	26.54	25.81
France	21.14	22.24	22.87	23.09	23.46	26.62	26.41	26.24	25.99	25.22	26.45	27.09	27.88	29.34	29.07	28.98	29.31	29.27	28.82	26.29
Germany	20.28	20.96	21.12	20.71	20.40	20.98	20.88	21.17	21.21	20.48	20.29	24.17	25.56	26.37	26.19	26.70	28.06	27.74	27.29	23.19
Greece	11.48	13.82	16.23	16.86	17.22	17.89	17.74	17.70	17.27	18.11	21.64	20.88	19.98	20.91	20.95	21.15	21.83	21.92	22.73	18.75
Iceland	-	-	-	-	-	-	-	-	-	-	-	-	-	-	18.03	18.63	18.32	18.12	18.44	18.31
Ireland	16.92	17.05	17.71	17.82	17.24	22.04	22.20	21.38	20.00	18.37	19.02	19.80	20.12	20.05	20.21	19.61	18.52	17.16	15.77	19.00
Italy	18.42	19.82	20.33	21.41	20.91	21.27	21.25	21.49	21.56	21.82	23.87	24.33	25.52	25.73	25.45	23.75	24.38	24.94	25.07	22.70
Japan	10.12	10.56	10.97	11.24	11.06	10.96	11.34	11.52	11.20	10.97	10.80	10.94	11.45	12.07	12.67	13.47	13.91	14.32	14.66	11.80
Korea	-	-	-	-	-	-	-	-	-	-	3.16	2.96	3.25	3.33	3.38	3.67	3.90	4.27	5.94	3.76
Luxembourg	23.28	24.97	24.20	24.37	22.98	22.81	22.26	23.08	22.25	21.45	21.74	22.22	22.48	23.03	22.58	23.30	23.51	22.33	22.09	22.89
Mexico	-	-	-	-	-	1.76	1.70	1.78	2.03	2.72	3.23	3.58	3.93	4.24	4.68	7.44	7.54	8.02	8.22	4.35
The Netherlands	27.26	28.23	29.85	30.10	28.89	27.43	27.09	27.32	26.96	26.44	27.92	28.10	28.56	28.76	26.85	25.92	25.29	24.88	23.90	27.36
New Zealand	19.15	18.18	19.22	18.82	18.15	19.43	19.47	19.89	20.96	21.96	22.53	22.56	22.40	21.02	19.90	19.32	19.67	20.76	20.97	20.23
Norway	18.55	-	-	-	-	19.68	-	-	24.74	25.93	26.00	27.15	28.41	28.13	27.95	27.62	26.48	26.16	26.97	25.67
Poland	-	-	-	-	-	-	-	-	-	-	16.19	23.02	27.31	26.64	25.44	24.74	24.86	24.21	22.83	23.92
Portugal	11.63	12.56	12.03	12.21	12.11	12.30	13.15	13.41	13.56	13.09	13.80	14.81	15.50	16.96	17.00	17.51	18.20	17.83	18.21	14.52
Slovak Republic	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13.53	13.43	13.13	13.57	13.42
Spain	15.78	16.90	16.85	17.50	17.19	18.03	17.73	17.63	18.17	18.34	19.29	20.01	20.89	21.71	21.53	20.94	20.92	20.16	19.71	18.91
Sweden	29.00	29.98	30.18	30.49	29.19	30.18	30.22	30.35	30.87	30.17	31.02	33.18	36.39	36.66	35.21	33.03	32.99	32.26	30.98	31.70
Switzerland	15.17	14.88	15.81	16.26	16.43	16.25	16.46	16.68	16.77	16.49	19.80	21.28	23.35	25.18	25.44	26.20	27.53	28.54	28.28	20.36
Turkey	4.33	3.18	3.38	3.66	4.54	4.21	4.40	4.56	5.35	6.36	6.44	7.99	7.35	7.19	7.86	7.46	10.41	11.72	11.59	6.42
United Kingdom	18.19	19.77	20.24	21.00	21.15	21.27	21.38	20.71	19.38	18.88	21.62	23.41	25.75	26.46	26.10	25.84	25.79	25.33	24.70	22.47
United States	13.13	13.43	13.70	14.04	13.12	12.87	13.04	12.99	12.92	12.93	13.36	14.41	15.11	15.36	15.34	15.41	15.30	14.93	14.59	14.00
Average over Countries	17.98	18.38	19.03	19.36	19.07	19.14	18.83	18.91	19.07	19.00	19.31	20.62	21.60	22.07	21.69	21.26	21.42	21.16	20.98	

NOTES to Table 1:

Public Social Expenditure is defined as the sum of old-age cash benefits, disability cash benefits, occupational injury and disease, sickness benefits, services for the elderly and disabled people, survivors, family cash benefits, family services, active labor market programs, unemployment, health, housing benefits and other contingencies.

Source: OECD Social Expenditure Database: 1980/1998 2001 Edition

OECD Social Expenditure web site -- for general information on the SOCX database

OECD Social Expenditure manual -- gives a description of the main notions, definitions, characteristics and structure of the SOCX database

Table 2: OLD AGE CASH BENEFITS
MEASURE: As a percentage of GDP (SNA93)

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average 1980-1998
Australia	3.19	3.20	3.31	3.26	3.17	3.03	2.94	2.90	2.72	2.69	2.91	3.02	3.03	3.13	3.02	4.24	4.31	4.22	4.30	3.29
Austria	8.48	-	-	-	-	9.36	-	-	-	-	9.45	9.54	9.57	10.00	10.18	10.24	10.26	10.11	9.94	9.74
Belgium	6.09	6.49	6.55	6.70	6.56	6.58	6.64	6.61	6.69	6.49	6.45	6.76	6.87	7.12	7.10	7.26	7.36	7.23	7.37	6.79
Canada	2.83	2.96	3.20	3.26	3.78	3.92	4.03	4.12	4.12	4.22	4.45	4.88	5.07	5.13	5.10	5.09	5.13	5.11	5.10	4.29
Czech Republic	-	-	-	-	-	-	-	-	-	-	5.44	5.98	6.09	5.63	5.46	5.36	5.54	6.31	6.37	5.80
Denmark	5.79	5.82	5.79	5.74	5.79	5.74	5.62	5.73	6.02	6.31	6.32	6.48	6.49	6.56	7.56	7.42	7.23	6.96	6.82	6.33
Finland	4.70	4.88	5.04	5.47	6.29	6.44	6.43	6.64	6.29	6.16	6.41	7.44	8.30	8.23	8.01	7.74	7.94	7.39	6.99	6.67
France	7.59	7.83	8.06	8.23	8.46	8.67	8.67	8.70	8.84	8.38	9.16	9.44	9.69	10.12	10.12	10.50	10.67	10.68	10.59	9.18
Germany	8.65	8.74	8.93	8.88	8.81	8.74	8.64	8.73	8.70	8.60	8.47	9.19	9.37	9.68	9.75	10.12	10.27	10.41	10.46	9.22
Greece	5.14	5.64	7.13	7.14	7.63	8.04	8.16	8.54	8.36	8.57	9.10	8.62	8.64	8.87	8.76	9.00	9.36	9.53	10.22	8.23
Iceland	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3.60	3.70	3.68	3.71	3.80	3.70
Ireland	4.02	4.16	4.44	4.47	4.33	4.21	4.26	4.20	4.02	3.78	3.72	3.80	3.76	3.65	3.51	3.19	2.91	2.68	2.54	3.77
Italy	7.38	8.06	8.37	8.99	8.86	9.16	9.40	9.32	9.33	9.51	10.97	11.36	12.29	12.50	12.76	12.18	12.52	12.94	12.84	10.46
Japan	2.92	3.17	3.39	3.54	3.60	3.74	4.00	4.05	3.99	3.98	3.97	4.01	4.23	4.45	4.72	5.05	5.15	5.34	5.67	4.16
Korea	-	-	-	-	-	-	-	-	-	-	0.62	0.66	0.80	0.93	0.99	1.11	1.05	1.13	1.92	1.02
Luxembourg	6.48	6.87	6.61	6.51	6.15	6.10	6.00	6.11	5.80	5.83	5.93	8.57	8.69	8.61	8.46	8.69	8.58	8.21	8.02	7.17
Mexico	-	-	-	-	-	0.09	0.11	0.19	0.21	0.30	0.28	0.32	0.36	0.41	0.44	3.21	3.72	4.46	4.54	1.33
The Netherlands	6.52	6.51	6.77	6.70	6.54	6.60	6.74	6.94	6.91	6.81	7.24	7.15	7.14	7.12	6.67	6.47	6.35	6.28	6.21	6.72
New Zealand	7.01	7.05	7.95	7.47	7.16	7.60	6.86	6.64	6.66	6.92	7.35	7.78	7.08	6.45	6.03	5.71	5.59	5.47	5.47	6.75
Norway	4.54	-	-	-	-	4.77	-	-	5.71	5.79	5.85	5.89	6.01	5.99	5.94	5.82	5.62	5.52	5.96	5.65
Poland	-	-	-	-	-	-	-	-	-	-	4.45	7.03	8.30	8.51	8.42	8.28	8.13	8.41	7.97	7.72
Portugal	3.42	3.61	3.35	3.81	3.69	3.62	3.82	4.11	4.02	3.97	4.21	4.52	4.86	5.33	5.41	5.89	6.40	6.22	6.31	4.56
Slovak Republic	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5.13	5.15	5.04	5.20	5.13
Spain	4.62	5.07	5.18	5.41	5.57	5.80	5.74	5.67	5.65	5.67	7.15	7.34	7.66	8.10	8.10	8.19	8.42	8.35	8.12	6.62
Sweden	6.65	7.17	7.17	7.31	7.00	7.13	7.22	7.19	7.25	7.25	7.23	7.61	8.23	8.38	8.19	7.87	7.85	7.68	7.46	7.47
Switzerland	5.61	5.28	5.76	5.67	6.02	5.82	5.92	5.93	5.94	5.65	8.22	8.69	9.34	9.86	9.99	10.58	10.90	11.22	11.16	7.77
Turkey	1.29	1.29	1.50	1.62	1.50	1.41	1.39	1.40	1.67	1.95	2.21	2.58	2.79	2.81	2.88	2.89	3.37	4.14	4.22	2.26
United Kingdom	5.10	5.58	5.86	5.76	5.76	5.61	5.62	5.44	5.09	5.04	8.52	9.24	9.81	9.93	9.77	9.66	9.80	10.00	9.77	7.44
United States	4.99	5.23	5.44	5.55	5.32	5.22	5.19	5.12	5.04	5.04	5.08	5.27	5.30	5.34	5.27	5.26	5.24	5.20	5.15	5.22
Average over countries	5.35	5.46	5.70	5.79	5.81	5.73	5.61	5.65	5.61	5.60	5.97	6.41	6.66	6.77	6.65	6.75	6.84	6.89	6.91	

Notes to Table 2: See Table 1.

Table 3: Old-age Expenses divided by total social expenditures in the OECD (combines the information in tables 1 and 2)

	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	Average over 1980-1998
Australia	28	28	27	25	24	22	22	22	22	21	20	20	18	19	19	24	24	24	24	23
Austria	36	--	--	--	--	37	--	--	--	--	38	38	37	37	36	37	37	37	37	37
Belgium	25	25	25	25	25	24	25	25	26	26	26	27	27	27	27	29	29	30	30	26
Canada	21	21	20	20	23	23	24	25	25	25	24	24	24	24	25	26	28	29	28	24
Czech Republic	--	--	--	--	--	--	--	--	--	--	32	33	33	30	29	29	30	33	33	31
Denmark	20	20	20	19	20	21	21	21	21	21	22	21	21	20	23	23	23	23	23	21
Finland	25	26	25	26	29	28	27	28	27	27	26	25	24	24	24	25	26	26	26	26
France	36	35	35	36	36	33	33	33	34	33	35	35	35	34	35	36	36	36	37	35
Germany	43	42	42	43	43	42	41	41	41	42	42	38	37	37	37	38	37	38	38	40
Greece	45	41	44	42	44	45	46	48	48	47	42	41	43	42	42	43	43	43	45	44
Iceland	--	--	--	--	--	--	--	--	--	--	--	--	--	--	20	20	20	20	21	20
Ireland	24	24	25	25	25	19	19	20	20	21	20	19	19	18	17	16	16	16	16	20
Italy	40	41	41	42	42	43	44	43	43	44	46	47	48	49	50	51	51	52	51	46
Japan	29	30	31	31	33	34	35	35	36	36	37	37	37	37	37	37	37	37	39	35
Korea	--	--	--	--	--	--	--	--	--	--	20	22	25	28	29	30	27	26	32	27
Luxembourg	28	28	27	27	27	27	27	26	26	27	27	39	39	37	37	37	36	37	36	31
Mexico	--	--	--	--	--	5	6	11	10	11	9	9	9	10	9	43	49	56	55	21
The Netherlands	24	23	23	22	23	24	25	25	26	26	26	25	25	25	25	25	25	25	26	25
New Zealand	37	39	41	40	39	39	35	33	32	32	33	34	32	31	30	30	28	26	26	34
Norway	24	--	--	--	--	24	--	--	23	22	23	22	21	21	21	21	21	21	22	22
Poland	--	--	--	--	--	--	--	--	--	--	27	31	30	32	33	33	33	35	35	32
Portugal	29	29	28	31	30	29	29	31	30	30	31	31	31	31	32	34	35	35	35	31
Slovak Republic	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	38	38	38	38	38
Spain	29	30	31	31	32	32	32	32	31	31	37	37	37	37	38	39	40	41	41	35
Sweden	23	24	24	24	24	24	24	24	23	24	23	23	23	23	23	24	24	24	24	24
Switzerland	37	35	36	35	37	36	36	36	35	34	42	41	40	39	39	40	40	39	39	38
Turkey	30	41	44	44	33	33	32	31	31	31	34	32	38	39	37	39	32	35	36	35
United Kingdom	28	28	29	27	27	26	26	26	26	27	39	39	38	38	37	37	38	39	40	33
United States	38	39	40	40	41	41	40	39	39	39	38	37	35	35	34	34	34	35	35	37
Average over Countries	30	31	31	31	31	30	30	30	29	29	30	31	31	31	30	32	32	33	33	

Notes to Table 3: See Table 1.

Table 4: Public flat-rate pensions in OECD countries

Country	Single (US$)	Couple (US $)	Single (PPP US$)	Couple (PPP US$)	State subsidy	Eligibility
Australia	7234	12068	6961	11613	All cost	Means-test & Res (10 yrs)
Austria	8643	12332	6969	9943	All cost	Means-test
Belgium	7687	10250	6681	8908	All cost	Means-test
Canada	3548	7097	4102	8205	All cost	Means-test & Res (40 yrs)
Czech Republic	1080	2160	2525	5050	All cost	Means-test
Denmark	7697	15394	5490	10980	All cost	Means-test & Res (40 yrs)
Finland	6052	12103	4770	9541	36% (since Jan 1996)	Means-test & Res (40 yrs)
France	7862	15724	6268	12536	Taxes on Alcohol-Tobacco	Means-test
Greece	1652	3304	2630	5261	All cost	Partly means test
Hungary	80% of minimal old age pension (40% of net average earnings).				All cost	Means-test
Iceland	2447	4894	2132	4265	Any deficit	Residence (3 years)
Ireland	6552	13104	5799	11597	Any deficit	Min contribution
Italy	4042	7103	3904	6861	All cost	Means test
Japan	6772	13543	4860	9721	One third	Min contribution
Netherlands	10636	14756	9049	12555	Some funding	Residence (49 years)
New Zealand	9212	13611	8778	12970	All cost	Residence (10 years)
Norway	6599	9899	4663	6995	Any deficits	Residence (40 years)
Portugal	1611	3223	2061	4121	All cost	Means test
Slovak Republic	820	1448	2145	3788	All cost	Means test
Spain	Decided annually by the Law on the General State				All cost	Residence (10 years)
Sweden	4985	9969	3538	7077	One fourth	Residence (40 years)
Switzerland	21102	35378	13883	23275	All cost	Means test
Turkey	10	15	82	122	All cost	Means test
UK	5521	11041	5043	10085	None	Min contribution
US	5640	8460	5640	8460	All cost	Means test

Notes:
1) PPP from OECD (http:/www.oecd.org/std/ppp/). Note that for EMU countries, the euro PPP is given in the OECD table.
2) Public flat rate pension calculations based on table 24, in the Appendix from Social Security Pension Reform in Europe, edited by Martin Feldstein and Horst Siebert, The University of Chicago Press, 2002. The data are posted at http://www.nber.org/pensioncrisis/.
3) Numbers denote annual benefits in 1997 values.
4) For countries that do not report the benefit for couples, it was assumed that the spouse receives the same benefit level.
5) Certain changes that took place after 1997 need to be noted. Since 1997, eleven OECD economies including Italy, Japan, Spain and the UK have implemented or are in the process of implementing reforms aimed to increase time spent in employment.

Table 5
Aggregate Implications of Varying the Promised Benefit level in the DB system: No Bequest Motive, Accidental Bequests are fully taxed

Defined Benefit (λ)	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	1.80	1.72	1.65	1.60	1.55
Capital	5.15	4.52	4.05	3.69	3.40
Capital/Output Ratio	2.86	2.63	2.45	2.31	2.19
Consumption (% of GDP)	60	62	63	64	65
Investment(% of GDP)	20	18	17	16	15
Gross Interest Rate (%)	5.60	6.71	7.70	8.61	9.46
Net Interest Rate (%)	4.52	5.33	6.06	6.71	7.31
Accidental Bequests (% of GDP)	7.2	6.5	5.96	5.51	5.15
SS Tax Rate (%)	1.81	1.90	1.98	2.04	2.11
Labor (Capital) Tax rate (%)	19.3	20.4	21.3	22.1	22.7
SS Def Ben Tax rate	0.0	2.66	5.32	7.99	10.6
Replacement Rate Of Net Mean Wages (%)	8.6	22.9	38.5	55.7	74.5
Replacement Rate of Net Mean Wages (%) from Flat Pension	8.6	9.55	10.4	11.3	12.3
Mean Aggregate Welfare (%)	0	0.07	-0.32	-1.35	-2.80
Mean Workers' Welfare (%)	0	-1.65	-3.63	-5.86	-8.25
Mean Retirees' Welfare (%)	0	10.3	21.9	32.1	41.9
Gini Coefficient	0.53	0.54	0.55	0.56	0.56
Gini Coefficient for Workers	0.51	0.50	0.50	0.49	0.49
Gini Coefficient for Retirees	0.56	0.59	0.62	0.64	0.65

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Table 6
Aggregate Implications of Varying the Promised Benefit level in the DB system: No Bequest Motive, Accidental Bequests are received as an initial exogenous endowment

Defined Benefit (λ)	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	1.91	1.80	1.74	1.68	1.63
Capital	6.02	5.11	4.68	4.20	3.87
Capital/Output Ratio	3.16	2.84	2.69	2.51	2.38
Consumption (% of GDP)	58	61	62	63	64
Investment(% of GDP)	22	19	18	17	16
Gross Interest Rate (%)	4.41	5.67	6.40	7.37	8.14
Net Interest Rate (%)	3.17	4.08	4.61	5.30	5.85
Accidental Bequests (% of GDP)	5.96	5.22	4.6	4.2	3.87
SS Tax Rate (%)	1.71	1.82	1.88	1.95	2.01
Labor (Capital) Tax rate (%)	28.1	28.0	28.0	28.1	28.2
SS Def Ben Tax rate	0.0	2.66	5.32	7.99	10.6
Replacement Rate Of Net Mean Wages (%)	9.2	25.0	41.9	60.4	80.6
Replacement Rate of Net Mean Wages (%) from Flat Pension	9.2	10.1	10.9	11.9	12.8
Mean Aggregate Welfare (%)	0	3.0	5.6	4.4	4.2
Mean Workers' Welfare (%)	0	0.5	1.18	-1.8	-3.3
Mean Retirees' Welfare (%)	0	15.3	29.5	42.9	55.3
Gini Coefficient	0.47	0.49	0.50	0.53	0.55
Gini Coefficient for Workers	0.38	0.38	0.36	0.39	0.40
Gini Coefficient for Retirees	0.59	0.63	0.67	0.69	0.72

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Table 7
Aggregate Implications of Varying the Promised Benefit level in the DB system: Bequest Parameter =10, Accidental Bequests are received as an initial exogenous endowment

Defined Benefit (λ)	**0.00**	**0.10**	**0.20**	**0.30**	**0.40**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	2.18	2.12	2.07	2.03	2.00
Capital	8.73	8.02	7.52	7.15	6.88
Capital/Output Ratio	4.00	3.79	3.64	3.52	3.43
Consumption (% of GDP)	52	53.5	55	55.5	56
Investment(% of GDP)	28	26.5	25	24.5	24
Gross Interest Rate (%)	1.99	2.50	2.90	3.22	3.48
Net Interest Rate (%)	1.42	1.78	2.07	2.31	2.50
Accidental Bequests (% of GDP)	8.21	8.03	7.98	8.01	8.11
SS Tax Rate (%)	1.50	1.55	1.58	1.61	1.63
Labor (Capital) Tax rate (%)	29.0	28.7	28.5	28.4	28.3
SS Def Ben Tax rate	0.0	2.66	5.32	7.99	10.65
Replacement Rate Of Net Mean Wages (%)	8.12	23.6	40.3	58.3	77.88
Replacement Rate of Net Mean Wages (%) from Flat Pension	8.12	8.68	9.23	9.79	10.36
Mean Aggregate Welfare (%)	0	3.96	7.17	9.67	11.6
Mean Workers' Welfare (%)	0	1.53	2.42	2.87	2.96
Mean Retirees' Welfare (%)	0	12.3	25.2	38	50.5
Gini Coefficient	0.35	0.32	0.29	0.28	0.26
Gini Coefficient for Workers	0.33	0.32	0.31	0.30	0.29
Gini Coefficient for Retirees	0.36	0.31	0.27	0.24	0.22

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Table 8
Aggregate Implications of Varying the Tax Rate Determining the Contribution in the DC system
No Bequest Motive, Accidental Bequests are fully taxed

DC Tax Contribution Rate (%)	**0.0**	**2.66**	**5.32**	**7.99**	**10.6**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	1.80	1.84	1.92	2.00	2.07
Capital	5.15	5.45	6.10	6.83	7.57
Capital/Output Ratio	2.86	2.96	3.18	3.42	3.66
Consumption (% of GDP)	60	60	58	56	55
Investment(% of GDP)	20	20	22	24	25
Gross Interest Rate (%)	5.60	5.17	4.31	3.52	2.86
Net Interest Rate (%)	4.52	4.08	3.39	2.79	2.27
Accidental Bequests (% of GDP)	7.2	7.3	7.5	7.7	7.9
SS Tax Rate (%)	1.81	1.78	1.71	1.64	1.58
Labor (Capital) Tax rate (%)	19.3	20.9	21.2	21.0	20.5
Replacement Rate (%) Of Net Mean Wages at Retirement	8.6	42.9	65	80.4	92.2
Replacement Rate of Net Mean Wages (%) from flat pension	8.6	8.9	8.9	8.8	8.8
Share of Savings in DC Account (%)	0	37	57	67	74
Mean Aggregate Welfare (%)	0	-9.1	-9.7	-10	-10.5
Mean Workers' Welfare (%)	0	-5.3	-6.0	-6.4	-6.9
Mean Retirees' Welfare (%)	0	-25.1	-25.5	-25.3	-25.5
Gini Coefficient	0.53	0.58	0.61	0.62	0.62
Gini Coefficient for Workers	0.51	0.50	0.50	0.49	0.49
Gini Coefficient for Retirees	0.56	0.66	0.71	0.69	0.64

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Table 9
Aggregate Implications of Varying the Tax Rate Determining the Contribution in the DC system: No Bequest Motive, Accidental Bequests are received as an initial exogenous endowment

DC Tax Contribution Rate (%)	**0.0**	**2.66**	**5.32**	**7.99**	**10.6**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	1.91	1.92	1.98	2.06	2.13
Capital	6.02	6.15	6.70	7.43	8.21
Capital/Output Ratio	3.16	3.20	3.38	3.61	3.85
Consumption (% of GDP)	58	58	56.5	55	53
Investment(% of GDP)	22	22	23.5	25	27
Gross Interest Rate (%)	4.41	4.26	3.66	2.98	2.36
Net Interest Rate (%)	3.17	2.99	2.53	2.04	1.61
Accidental Bequests (% of GDP)	5.96	6.03	6.31	6.61	6.87
SS Tax Rate (%)	1.71	1.70	1.65	1.59	1.53
Labor (Capital) Tax rate (%)	28.1	29.8	30.9	31.4	31.6
Replacement Rate (%) Of Net Mean Wages at Retirement	9.2	40.8	66.4	85.3	100
Replacement Rate of Net Mean Wages (%) from Flat Pension	9.2	9.7	10.0	10.1	10.3
Share of Savings in DC Account (%)	0	28.0	46.6	57.1	63.5
Mean Aggregate Welfare	0	-8.8	-9.5	-9.6	-10.0
Mean Workers' Welfare (%)	0	-4.9	-6.0	-6.7	-7.5
Mean Retirees' Welfare (%)	0	-22.3	-21.7	-20.1	-19.1
Gini Coefficient	0.47	0.51	0.55	0.56	0.57
Gini Coefficient for Workers	0.38	0.37	0.35	0.34	0.35
Gini Coefficient for Retirees	0.59	0.66	0.71	0.68	0.62

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Table 10
Aggregate Implications of Varying the Tax Rate Determining the Contribution in the DC system: Bequest Parameter =10, Accidental Bequests are received as an initial exogenous endowment

DC Tax Contribution Rate (%)	**0.0**	**2.66**	**5.32**	**7.99**	**10.6**
Flat Pension Benefit	**0.10**	**0.10**	**0.10**	**0.10**	**0.10**
Government (% of GDP)	**20**	**20**	**20**	**20**	**20**
Government Debt (% of GDP)	**60**	**60**	**60**	**60**	**60**
Output	2.18	2.18	2.20	2.22	2.26
Capital	8.73	8.75	8.89	9.19	9.66
Capital/Output Ratio	4.00	4.01	4.05	4.14	4.27
Consumption (% of GDP)	52	52	52	51	50
Investment(% of GDP)	28	28	28	29	30
Gross Interest Rate (%)	1.99	1.98	1.90	1.70	1.43
Net Interest Rate (%)	1.42	1.40	1.33	1.18	0.99
Accidental Bequests (% of GDP)	8.21	8.22	8.24	8.28	8.37
SS Tax Rate (%)	1.50	1.50	1.49	1.47	1.45
Labor (Capital) Tax rate (%)	29.0	29.5	30.0	30.5	30.9
Replacement Rate (%) Of Net Mean Wages at Retirement	8.12	26.1	45.0	63.8	81.5
Replacement Rate of Net Mean Wages (%) from Flat Pension	8.12	8.5	8.9	9.2	9.5
Share of Savings in DC Account (%)	0	17.6	27.8	39.4	48.5
Mean Aggregate Welfare	0	-1.0	-2.2	-3.8	-5.6
Mean Workers' Welfare (%)	0	-0.7	-1.7	-2.9	-4.6
Mean Retirees' Welfare (%)	0	-2.0	-4.1	-6.4	-8.6
Gini Coefficient	0.35	0.34	0.34	0.33	0.33
Gini Coefficient for Workers	0.33	0.33	0.33	0.34	0.34
Gini Coefficient for Retirees	0.36	0.34	0.32	0.31	0.29

Notes: Bold variables are set exogenously and all other quantities are endogenous. The analysis is done at an annual frequency. There is no ex ante heterogeneity, but there does exist ex post heterogeneity due to the realization of idiosyncratic shocks over the life cycle. The CRRA coefficient is equal to two, the AR(1) coefficient is set to 0.95, the standard deviation of the persistent innovation equals 15 percent and of the transitory innovation 21 percent. The discount rate is set at 4 percent, the depreciation rate of capital equals seven percent and alpha=0.36. The interest rate on capital is taxed at the same rate as labor income. Mean aggregate welfare is computed as described in the text. There is no secular or population growth in the economy.

Comparative Statics in Defined Benefit system

Fig.5: Mean Consumption in DB system

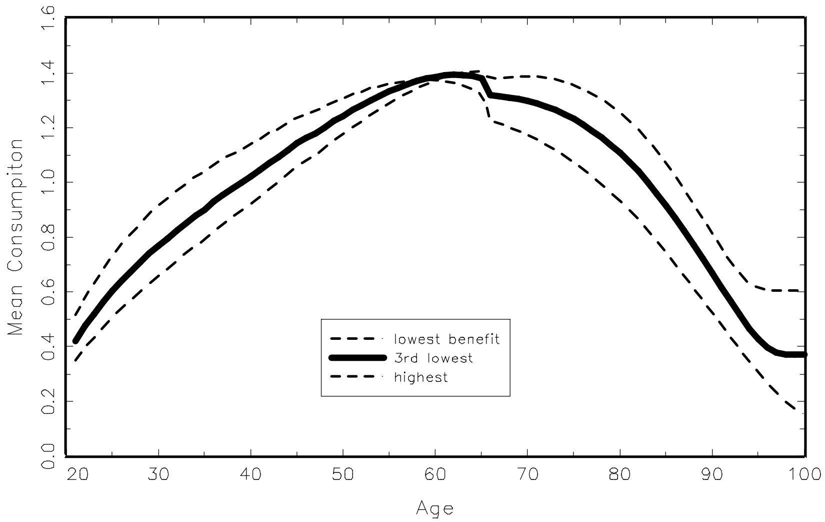


Fig.6: Life-Cycle Wealth in DB with no bequest motive

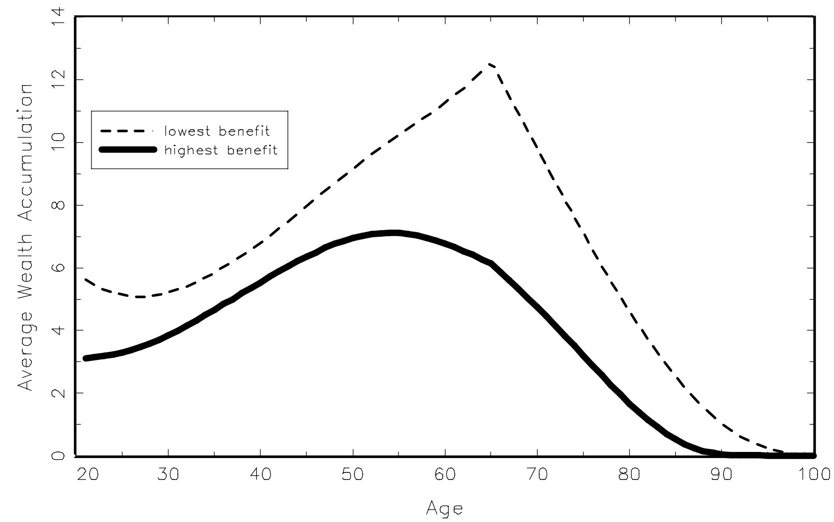


Fig.7: Life-Cycle Wealth in DB with bequest motive

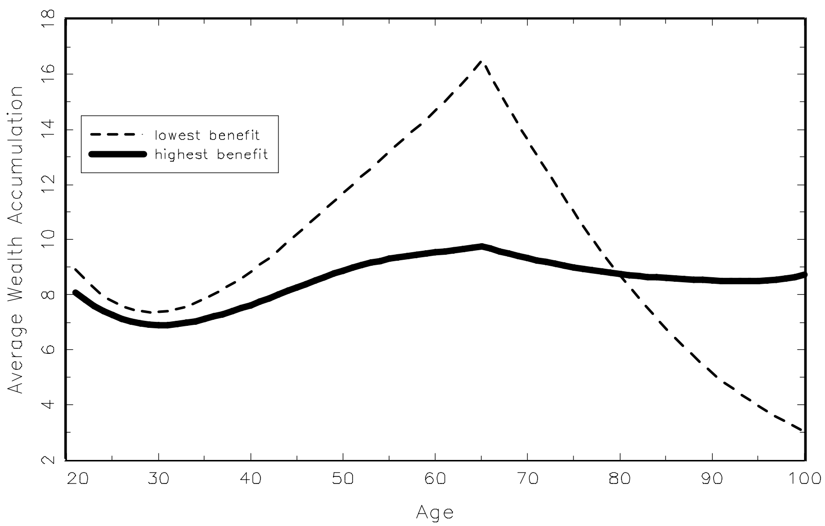
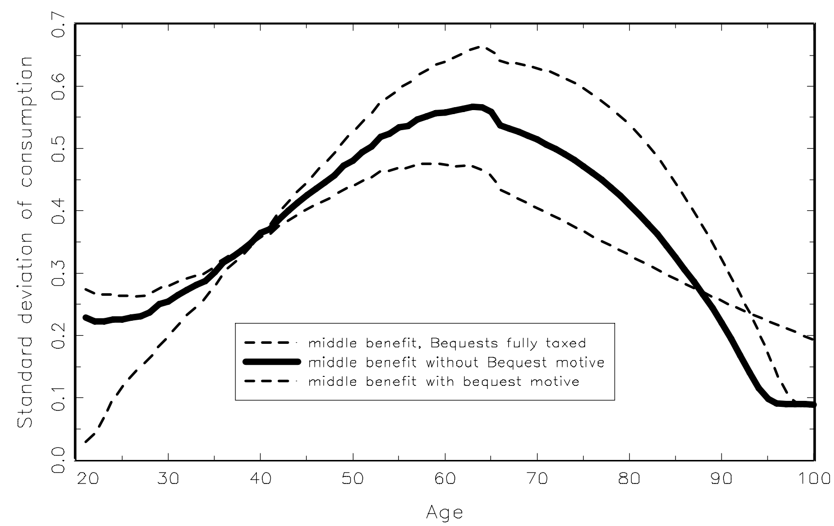


Fig.8: Inequality over the Lifecycle varying Bequest Assumptions



Comparative Statics in Defined Contribution system

Fig.9: Consumption conditional on DC pension, Age 21

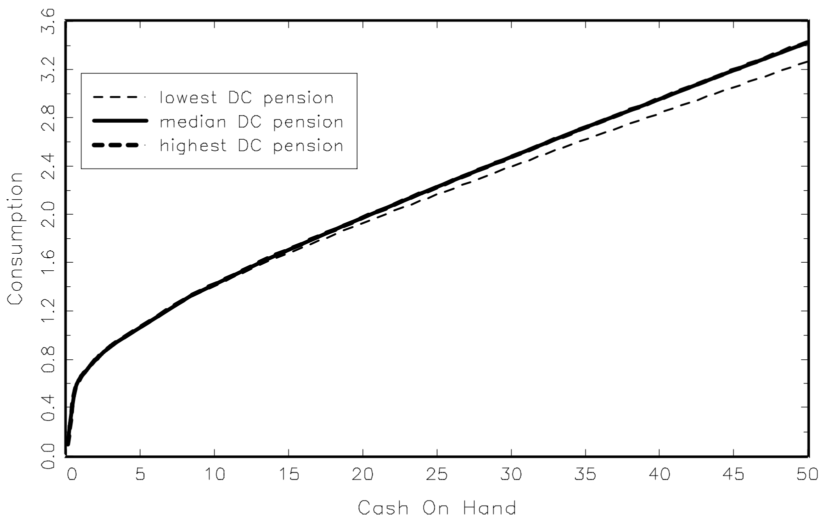


Fig.10: Consumption conditional on DC pension, Age 55

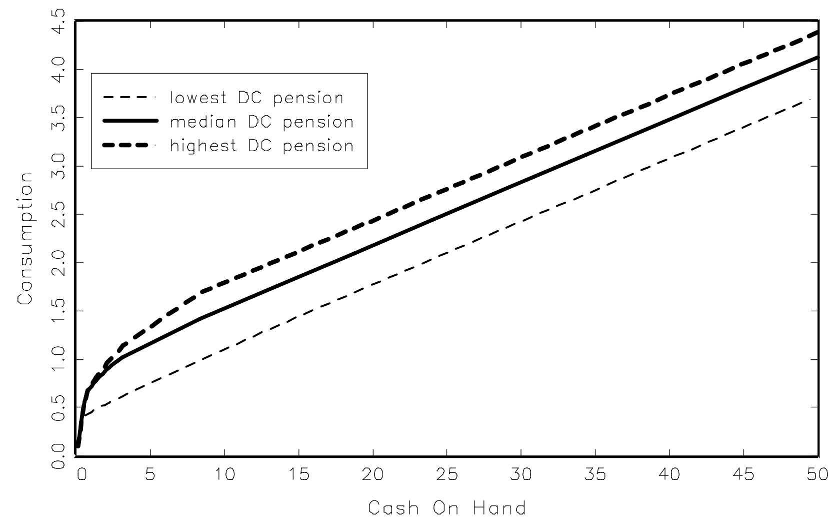


Fig.11: Average Non–DC Wealth Accumulation Varying DC system

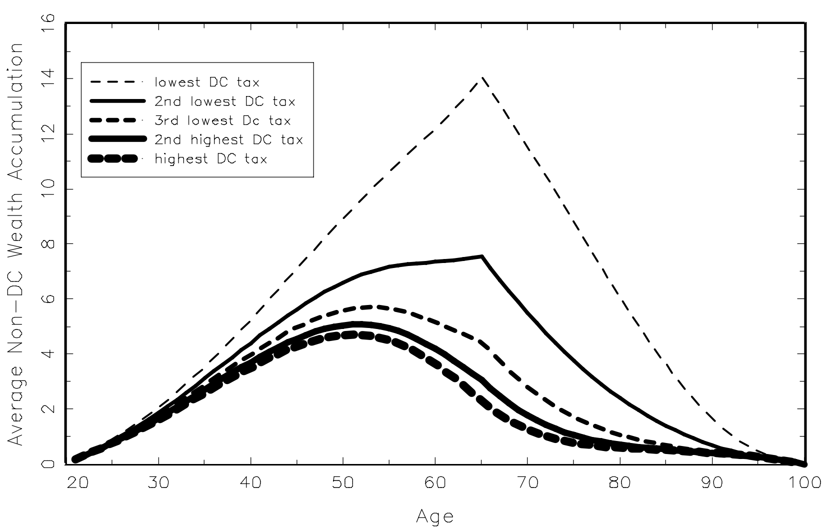
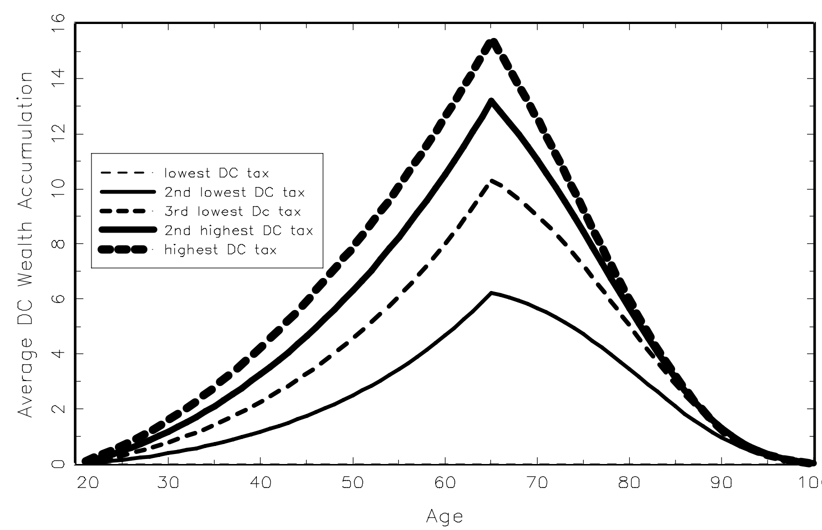


Fig.12: Average DC Wealth Accumulation Varying DC system



Comparative Statics in Defined Contribution system

Fig.13: Risk Sharing in DC system

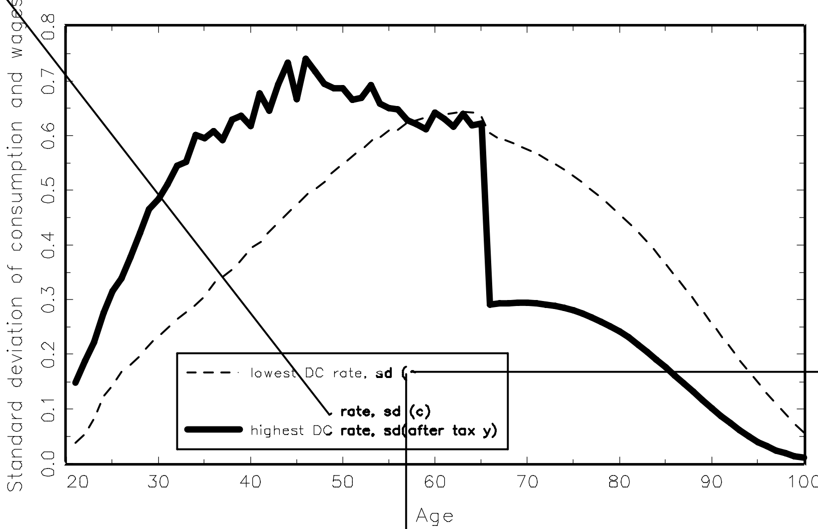


Fig.14: Mean Consumption over the Lifecycle in DC system

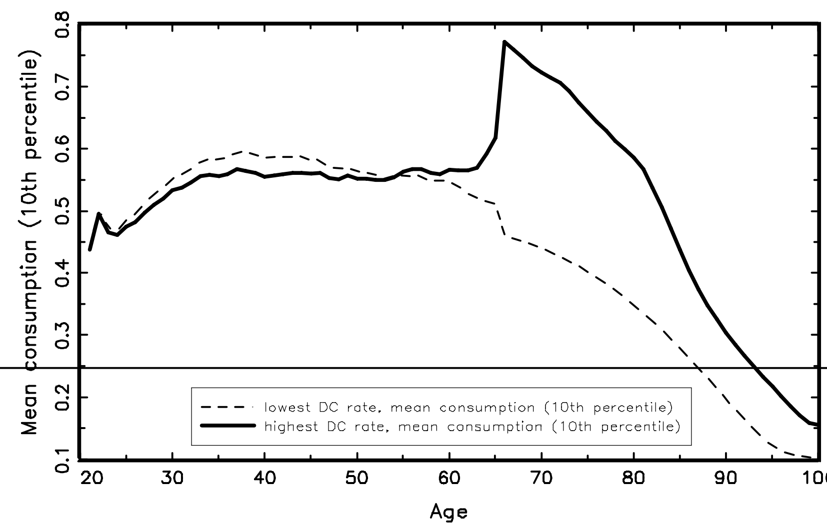


Fig.15: Non–DC Wealth with No Bequest Motive

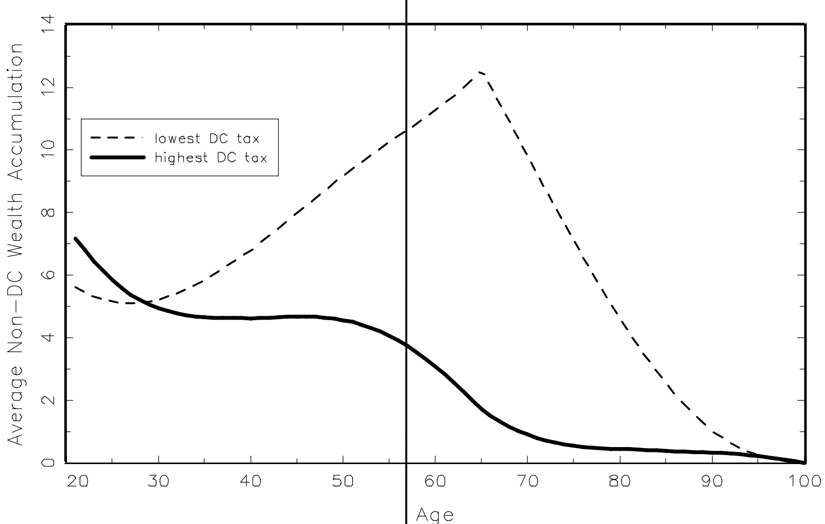


Fig.16: Non–DC Wealth with No Bequest Motive