

Feedback Trading

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ABSTRACT

Order flow has been found to carry information to the market. When assessing *how* informative order flow is, the VAR methodology is typically employed, using impulse response functions. However, in such analyses, the direction of causality runs explicitly from order flow to asset return. If data are sampled at anything other than at the highest frequencies then any feedback trading may well appear contemporaneous; trading in period t depends on the asset return in that interval. The implications of contemporaneous feedback trading are examined in the spot USD/EUR currency market and we find that when data are sampled at the one and five minute frequency, such trading strategies cause the price impact of order flow to be significantly larger than when feedback trading is ruled out.

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Order flow, one way buying or selling pressure, has a contemporaneous impact on prices, while at the very highest frequencies, the converse is not true. However, when aggregated over time, order flow and prices can be expected to impact on each other simultaneously, a phenomenon we call *contemporaneous feedback trading*. The existence of contemporaneous feedback trading implies that such models cannot be estimated with traditional techniques. This is unfortunate, since empirical models with feedback trading can be expected to give firmer support for theoretical models than models that ignore such trading strategies. In this paper we argue that feedback trading is an inevitable consequence of time aggregation of order flow models, and propose an estimator of such models by using instrumental variable techniques. We find that the price impact of trades is much stronger when feedback trading is incorporated, further supporting market microstructure theories generally, and the validity of the order model specifically.

It is well known in the theoretical microstructure literature that order flow conveys private information to the market as a whole. In this way, information is aggregated via the trading process implying that order flow has permanent effects on prices. See Kyle (1985), Glosten and Milgrom (1985), Easley and O'Hara (1987), and Evans and Lyons (2002b) for examples of such models. These models imply that if trades carry private information then the informativeness of trades can be accessed by their price impact. Furthermore, in empirical models when data are employed at the very highest frequencies, order flow, by definition, can only be affected by the lags of price changes. However, when data are aggregated, transactions and order entry are simultaneous, frustrating empirical investigations.

The existence and profitability of feedback trading strategies has been considered in a number of papers. De Long, Shleifer, Summers, and Waldmann (1990) build a model of feedback trading with rational speculators who will buy (sell) when the price rises (falls). The profitability of a number of feedback trading strategies in stock markets is considered in Jegadeesh and Titman (1993) and the existence of high frequency positive feedback trading in the US treasury market is documented in Cohen and

Shin (2003).

The most common methodology for empirically assessing the informativeness of order flow, is the vector autoregressive (VAR) model of Hasbrouck (1991). His model was originally applied to data at the tick-by-tick frequency, where the direction of causality runs explicitly from order flow to asset price returns. Hasbrouck introduces a shock to the trading process, representing private information, and computes the cumulated effect on the asset return. The greater the cumulated effect, or impulse response, the more information trades are argued to carry. These VAR models have become standard in the microstructure literature; recent examples include Dufour and Engle (2000) and Engle and Patton (2004) for stocks, Evans (2002) and Payne (2003) for currencies and Cohen and Shin (2003) and Green (2003) for treasuries. In the VAR framework, the asset returns in period t are regressed on contemporaneous order flow (date t) as well as lagged returns and order flows (dated $t - 1$ or earlier), whereas order flows are only regressed on lagged returns and flows; order flows at date t do not depend on contemporaneous asset returns.

These models therefore rule out contemporaneous feedback trading, an assumption which is overly restrictive when the data are sampled at anything other than at the highest frequencies. If traders have the ability to respond to price changes and trade before the end of the time interval used in the empirical investigation, then order flows will indeed cause asset price changes within that period, but these price changes may then feed back into order flows in the same interval. A shock to order flows within period t causes a change in the asset price within that interval. If other traders react to this price change by buying or selling the asset themselves in that period, perhaps because they expect a wave of trading activity that pushes the price in one direction or another, this significantly affects market dynamics and estimates of trade informativeness. Therefore, a model which does not allow contemporaneous feedback trading will bias any estimates of how much information trades actually carry. If a positive order flow shock causes an increase in the asset price, which in turn causes an *increase* in order flows via feedback trading within that period, the

total effect/price impact of the order flow shock will be higher than when feedback trading is ruled out. Alternatively, if there exists negative feedback trading, perhaps because of expected return reversals of the initial asset price change, the ultimate price impact of the trade will be smaller than the non-feedback case.

The reason for ruling out feedback trading in empirical models with aggregated data is that without such a restriction, the model cannot be estimated since the VAR becomes unidentified. By allowing returns to depend on order flow but ruling out the converse, the two equation VAR in returns and order flows becomes a recursively ordered structural VAR, which is just identified when the variance/covariance matrix of the residuals is restricted to be diagonal. However, as we show in Section 1, when data are aggregated from tick-by-tick to any lower frequency, contemporaneous feedback trading is inevitable. Imposing the restriction that order flows do not depend on contemporaneous asset returns then represents a mis-specification of the empirical model and is therefore likely to result in biased parameter estimates and incorrect inference.

However, it is not possible to estimate the simultaneous impact of order flows and prices on each other in the VAR model since, in the standard setup, not enough information is available for the VAR's identification. In order to identify both structural parameters, additional information is required and we suggest that sufficient information can be obtained from statistics (typically order flows or returns) from related assets or markets. This additional information can be used to identify and estimate the structural VAR in a very efficient manner through standard instrumental variables methods. In so far as many assets, whether they are currency, stocks or bonds, have a number of assets related to them, it seems likely that information should be readily available. This could then be used by the econometrician in order to estimate an otherwise unidentified model and hence allow for contemporaneous feedback trading. In the case of stocks, when trying to estimate a feedback trading VAR for IBM, one could try using statistics based on Hewlett Packard flows and returns for example, or any stock in the same or related industry. The question of

which instruments to use is simply an empirical one. One may simply choose those instruments found to be strongly correlated with the endogenous right hand side regressors.

We apply our instrumental variables VAR methodology to the spot USD/EUR (US dollar per euro) foreign exchange market since this is a very active market and provides a natural testing ground for our hypotheses of contemporaneous feedback trading. The data are taken from the Reuters D2000-2 electronic trading system, one of the two dominant brokered trading platforms used in the inter-dealer spot FX market and cover the eight month period from 1st December 1999 to 24th July 2000.¹ In order to estimate the structural, feedback VAR for the USD/EUR market we use instrumental variables, where the instruments are statistics obtained from the closely linked markets of USD/GBP (US dollar per pound sterling) and GBP/EUR (pound sterling per euro). It is clear that the USD/EUR rate should, in the absence of arbitrage, equal the USD/GBP rate multiplied by the GBP/EUR rate. Statistics obtained from these markets, in particular returns and order flows, may then be correlated with the endogenous variables for which we are trying to instrument, i.e. USD/EUR returns and flows.

In order to evaluate the importance of the feedback trading parameter, we consider two sampling frequencies; one minute and five minutes.² From both frequencies we estimated the VARs with and without feedback trading and calculated impulse response functions following an order flow shock, representing private information, in order to assess the informativeness of trades.

We find that for the one minute frequency, the feedback trading parameter in the structural VAR is positive and significant at the 1% level and the impulse response function following an order flow shock is larger when feedback trading is permitted. However the difference between the restricted and unrestricted IRFs is not signifi-

¹The other electronic trading system is that of EBS and together they account for between 85% and 95% of all interdealer trading. See Bank for International Settlements (2001).

²When the data were sampled at lower frequencies the abilities of other market statistics to instrument for USD/EUR endogenous variables deteriorated to such an extent that such analysis became pointless.

cantly different, at the 5% level. At the five minute frequency, the feedback trading parameter in the VAR is quantitatively large and significant at the 5% level, and the impulse response without feedback trading is significantly below that when feedback trading is permitted. This suggests that in the case of spot FX markets, feedback trading is prevalent and has significant implications when examining the price impact/informativeness of order flows, especially when the data are sampled at the lower/five minute frequency. For the spot FX market considered in this exercise, feedback trading is positive, i.e. order flow in one period depends positively on the asset return in that period.

We demonstrate in a theoretical context in Section 1 that omitting feedback trading in aggregated data will result in a mis-specified model and will bias the estimated impact of order flow on prices. This is confirmed in our empirical exercise where we show that feedback trading is a significant concern for empirical order flow models. By employing appropriate instruments, we show that in high frequency foreign exchange data, there exists positive contemporaneous feedback trading: order flow in one period depends positively on the return experienced within that period. This results in the price impact of an order flow shock being significantly greater than when one imposes a recursive ordering of the VAR. Private information, in the form of unanticipated order flow shocks, then has a larger impact on returns than previously believed, i.e. trades carry more information than previous estimates suggest.

The rest of the paper is organised as follows. Section 1 motivates the need to model contemporaneous feedback trading when data are aggregated at any level. Section 2 introduces the model to be estimated and describes the standard techniques to be employed as well as explanations as to how to obtain analytical confidence bounds for the impulse response functions. Section 2 also discusses the choice of instruments used. Section 3 presents the regression results and reports the impulse response functions. Section 4 discusses our findings, placing them within the existing finance literature and Section 5 concludes.

1 The Inevitability of Contemporaneous Feedback Trading in Aggregated Data

In this section we show how contemporaneous feedback trading can result simply from considering aggregated data. By definition, contemporaneous feedback trading, whereby order flows at date t depend on date t asset returns, cannot occur in tick-by-tick data; a trader can only respond to a price change once the price has indeed changed. Assume, without loss of generality, that returns and flows can be characterised by a VAR with only 1 lag. The VAR model, originally introduced by Sims (1980) and implemented in the microstructure literature by Hasbrouck (1991), is a simple statistical framework that allows us to examine the relationships between asset returns and trading activity; more specifically, order flows. Hasbrouck originally applied the model to US equity data where the data were sampled in ‘transaction time’, i.e. tick-by-tick data were used. In what follows, assume the data are sampled at a sufficiently high frequency so that contemporaneous feedback trading is ruled out. This could be tick-by-tick, ten second or even one second data for very active markets.³ Since the possibility of contemporaneous feedback trading is ruled out, the system of equations can be written as:

$$\begin{aligned} R_t &= \alpha_1 + \beta F_t + \phi_{11}R_{t-1} + \phi_{12}F_{t-1} + \epsilon_t^R \\ F_t &= \alpha_2 + \phi_{21}R_{t-1} + \phi_{22}F_{t-1} + \epsilon_t^F \end{aligned} \tag{1}$$

where R_t is the return on the asset in period t , defined as the log first difference of the price, F_t is the order flow in period t ; the number of buyer less seller initiated trades in that interval and ϵ_t^R and ϵ_t^F are serially uncorrelated, independent errors with variances $\sigma_{\epsilon^R}^2$ and $\sigma_{\epsilon^F}^2$ respectively. The system can be written as a structural

³For the purpose of this study we simply wish to examine data at a frequency where no contemporaneous feedback trading can occur and then see what happens when we aggregate the data at a lower frequency.

VAR:

$$\underbrace{\begin{bmatrix} 1 & -\beta \\ 0 & 1 \end{bmatrix}}_Q \underbrace{\begin{bmatrix} R_t \\ F_t \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_\alpha + \underbrace{\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}}_\phi \underbrace{\begin{bmatrix} R_{t-1} \\ F_{t-1} \end{bmatrix}}_{y_{t-1}} + \underbrace{\begin{bmatrix} \epsilon_t^R \\ \epsilon_t^F \end{bmatrix}}_{\epsilon_t} \quad (2)$$

$$\Rightarrow Qy_t = \alpha + \phi y_{t-1} + \epsilon_t \quad \text{Var}(\epsilon_t) = \Omega = \begin{bmatrix} \sigma_{\epsilon^R}^2 & 0 \\ 0 & \sigma_{\epsilon^F}^2 \end{bmatrix}$$

where $y_t = [R_t \ F_t]'$ and $\epsilon_t = [\epsilon_t^R \ \epsilon_t^F]'$, $t = 1, \dots, 2T$. What we wish to do is to move from a structural VAR at a high frequency, t , to a structural VAR where the frequency is halved (frequency τ).⁴ However, in order to do this, (2) must be converted into a reduced form and put into state space representation. Aggregation can then be performed using the methods in Harvey (1989). The reduced form of (2), at the frequency, t , is clearly

$$y_t = \mu + Q^{-1}\phi y_{t-1} + Q^{-1}\epsilon_t \quad t = 1, \dots, 2T \quad (3)$$

where $\mu = Q^{-1}\alpha$ and the corresponding reduced form at the frequency, τ , where $\tau = 2t$, is shown in Appendix A.1 to be

$$y_\tau = \mu^+ + A y_{\tau-1} + e_\tau \quad \tau = 1, \dots, T \quad (4)$$

where $A = Q^{-1}\phi(I_2 + Q^{-1}\phi)$. Since returns and order flows are both flow variables, as opposed to stocks, then the period τ (low frequency) return is simply the sum of the two t period returns in that interval, and similarly for order flows. The variance

⁴For example, t could represent data sampled at the ten second frequency and τ would represent data sampled at the twenty second frequency.

of e_τ is given by

$$Var(e_\tau) = G\Omega G' + Q^{-1}\Omega(Q^{-1})' \quad (5)$$

where $G = (I_2 + Q^{-1}\phi)Q^{-1}$. To convert (4) into a structural VAR, we first factorise the variance of e_τ . This is done in (6).

$$Var(e_\tau) = \begin{bmatrix} G & Q^{-1} \end{bmatrix} \begin{bmatrix} \Omega & 0 \\ 0 & \Omega \end{bmatrix} \begin{bmatrix} G' \\ (Q^{-1})' \end{bmatrix} = P\tilde{\Omega}P' \quad (6)$$

If we premultiply (4) by $[I_2 \ I_2]P^+$, where P^+ is the Moore-Penrose inverse of the 2×4 matrix P , $P^+ = (P'P)^{-1}P'$, then the structural form can be written as:

$$[I_2 \ I_2]P^+y_\tau = [I_2 \ I_2]P^+\mu^+ + [I_2 \ I_2]P^+Ay_{\tau-1} + [I_2 \ I_2]P^+e_\tau \quad (7)$$

The variance of the error vector in the structural form is now

$$Var([I_2 \ I_2]P^+e_\tau) = 2\Omega \quad (8)$$

(7) therefore has the appealing property that the variance of the error (return or flow) at the τ period frequency is twice that of the error at the t period frequency.⁵ In order to solve for the structural parameters in (7), note that if we let $x_\tau = P^+y_\tau$,

⁵Note that the errors in the return and flow equations at the t period frequency are serially uncorrelated and independent, as too are those at the τ period frequency.

then x_τ is the solution to

$$\underbrace{\begin{bmatrix} g_{11} & g_{12} & 1 & \beta \\ g_{21} & g_{22} & 0 & 1 \end{bmatrix}}_{[G \ Q^{-1}] = P} \underbrace{\begin{bmatrix} x_{1\tau} \\ x_{2\tau} \\ x_{3\tau} \\ x_{4\tau} \end{bmatrix}}_{x_\tau} = \underbrace{\begin{bmatrix} R_\tau \\ F_\tau \end{bmatrix}}_{y_\tau} \quad (9)$$

There are clearly an infinite number of solutions for x_τ and this is to be expected when considering the literature on simultaneous equation models; premultiply any structural form by a non-singular matrix and the reduced form, (4), will be unaffected. However, $[I_2 \ I_2] P^+ y_\tau$, the right hand side of the structural equation, (7), will have the general form

$$\begin{bmatrix} \left(\frac{1}{g_{11}} + \frac{g_{12}g_{21}}{g_{11}|G|} \right) & - \left(\frac{g_{12}}{|G|} \right) \\ - \left(\frac{g_{21}}{|G|} \right) & \left(\frac{g_{11}}{|G|} \right) \end{bmatrix} \begin{bmatrix} R_\tau \\ F_\tau \end{bmatrix} \quad (10)$$

$$+ \begin{bmatrix} \left(1 - \frac{1}{g_{11}} + \frac{g_{21}}{|G|} \right) & - \left(\frac{\beta}{g_{11}} + \frac{(g_{11} - g_{21}\beta)}{|G|} \right) \\ \left(\frac{g_{21}}{|G|} \right) & \left(1 - \frac{(g_{11} - g_{21}\beta)}{|G|} \right) \end{bmatrix} \begin{bmatrix} m_\tau \\ n_\tau \end{bmatrix} \quad (11)$$

for any real values of m_τ and n_τ , $\tau = 1, \dots, T$. For a proof, see Appendix A.2. Therefore there are an infinite number of structural VARs at the τ period frequency that are consistent with the recursively ordered structural t period VAR in (2). The top rows of (10) and (11) can be interpreted as the left hand side of the structural return equation and the bottom rows can be interpreted as the left hand side of the structural flow equation. However, a common choice of structural form in (7) is the Choleski solution. This essentially chooses the arbitrary values of m_τ and n_τ so that the R_τ term in the structural flow equation drops out. The recursively ordered structural VAR which results is just identified if the variance/covariance matrix of the residuals is assumed to be diagonal. Therefore, by choosing the Choleski solution,

standard econometric methods can be used to estimate the model (OLS for example). For all other solutions though, the coefficient on contemporaneous returns in the flow equation will depend on g_{21} . Only if g_{21} equals zero will there be no contemporaneous feedback trading. Assume t represents data sampled at the ten second frequency and τ represents data at the lower, twenty second frequency. In Appendix A.1, g_{21} is shown to be equal to ϕ_{21} , the coefficient on lagged returns in the flow equation in the ten second VAR. This is perfectly intuitive. If the order flow from seconds 11 to 20 depends on the return from seconds 1 to 10, then part of the order flow from seconds 1 to 20 will depend on part of the return from seconds 1 to 20, i.e. contemporaneous feedback trading exists! The question then becomes, on what grounds should the Choleski solution be chosen? The justification for choosing the Choleski solution is always on the basis that it makes life easier; the model becomes just identified and estimation can take place simply using OLS. However, by imposing a recursive structure on the VAR, the Choleski solution rules out contemporaneous feedback trading. In this paper, we suggest that one should use the data to tell us what the coefficient on returns in the flow equation should be, rather than assuming it to be zero. In our empirical application below we show that using data to calculate the effect of contemporaneous returns on flows, rather than assuming the coefficient to be zero, can have serious implications when calculating the price impact of trades and hence on estimates of their information content.

2 The VAR Model with Feedback Trading

The VAR model is a convenient statistical framework that allows one to analyse the relationships between, among other things, asset returns and order flow. By interpreting order flow shocks as private information, one can examine the price impact of such shocks, via impulse response functions, and therefore give a quantitative estimate of the information content of trades. The greater the price impact, the more information trades are argued to carry.

2.1 Model design

The standard VAR model allows asset returns to depend on contemporaneous order flows but not the converse. From Section 1 we saw that when data are aggregated, even at still very high frequencies, this recursive ordering may not be valid. Here we allow both returns and order flows to depend on each other contemporaneously. The model we wish to estimate can be written as:

$$y_t + By_t = c + \sum_{j=1}^p \phi_j y_{t-j} + \epsilon_t \quad t = 1, \dots, T \quad (12)$$

In our example, y_t is simply the 2×1 vector of endogenous variables at date t , i.e. returns and flows; $y_t = [R_t \ F_t]'$. In the appendix we generalise the analysis from the 2 to the n variable case.⁶ c is a 2×1 vector of constants, the summation term contains the lags of the VAR and ϵ_t is a 2×1 vector of residuals with zero mean and variance matrix Ω , assumed to be diagonal. B is the 2×2 matrix of structural parameters with zeros along the main diagonal.⁷

$$B = \begin{bmatrix} 0 & -b_{12} \\ -b_{21} & 0 \end{bmatrix} \quad (13)$$

b_{12} represents the contemporaneous effect of flows on returns and b_{21} represents the contemporaneous feedback trading parameter. For each equation, $i = 1, 2$, we can stack the k exogenous and $n - 1 (= 1)$ endogenous regressors into a $(1 + k) \times 1$ vector, z_{it} . Stacking these vectors across the T observations allows us to write

$$y_i = z_i \Pi_i + \epsilon_i \quad i = 1, 2 \quad (14)$$

⁶In the model of Engle and Patton (2004), returns of ask prices and returns of bid prices are considered separately, so y_t need not be restricted to be a 2×1 vector. In the original Hasbrouck (1991) setting, y_t also contained a number of trade related variables including trade sign and the interactions between trade sign and volume and spread.

⁷We separate y_t and By_t in (12) as this simplifies the notation in the appendix when we calculate the distribution of the impulse response functions.

Π_i is a $(1 + k) \times 1$ vector of parameters. y_i is the $T \times 1$ vector of the scalar y_{it} s ($y_{it} \in \{R_t, F_t\}$) and similarly for ϵ_i . z_i is the $T \times (1 + k)$ matrix formed by stacking the $T, (1 + k) \times 1, z_{it}$ vectors. In matrix form, (14) can be written:

$$\underbrace{\begin{bmatrix} R \\ F \end{bmatrix}}_Y = \underbrace{\begin{bmatrix} z_1 & 0_{T \times (1+k)} \\ 0_{T \times (1+k)} & z_2 \end{bmatrix}}_Z \underbrace{\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}}_\pi + \underbrace{\begin{bmatrix} \epsilon^R \\ \epsilon^F \end{bmatrix}}_\epsilon \quad (15)$$

Y is $2T \times 1$, Z is $2T \times 2(1+k)$, π is $2(1+k) \times 1$ and ϵ is $2T \times 1$. Writing the system in this form will help us calculate the distribution of the impulse response functions in Section 2.3, since they will be functions of the distribution of the $2(1+k) \times 1$ vector, π .

2.2 Instrumental variables

Since each of the equations in (14) contain endogenous variables on the right hand side, we estimate using instrumental variables. For the $1 + k$ variables in z_{it} we use the $g + k$ instruments w_{it} , $i = 1, 2$ and $g \geq 1$. For a greater explanation of the use of these variables as instruments, see Section 2.5. Using two stage least squares, the IV estimator for Π_i , $i = 1, 2$, is denoted $\hat{\Pi}_i$ and calculated as:

$$\hat{\Pi}_i = \left[z_i' w_i (w_i' w_i)^{-1} w_i' z_i \right]^{-1} z_i' w_i (w_i' w_i)^{-1} w_i' y_i \quad (16)$$

where w_i is simply the $T \times (g + k)$ matrix formed by stacking the $T w_{it}$ vectors of instruments. Using standard instrumental variables methods, the distribution of $\hat{\pi}$, the $2(1+k) \times 1$ vector of parameters in (15) is given by (see Appendix A.3):

$$\sqrt{T}(\hat{\pi} - \pi) \xrightarrow{d} N(0, \Sigma_\pi) \quad (17)$$

2.3 Impulse response functions

In order to evaluate the informativeness of trades, a common approach is to use impulse response functions (IRFs). To calculate these impulse response functions, we convert the VAR of (12) into its MA(∞) representation. It is simple to show that (12) can be written as⁸

$$y_t = \mu + \Psi_0 (I_2 + B)^{-1} \epsilon_t + \Psi_1 (I_2 + B)^{-1} \epsilon_{t-1} + \Psi_2 (I_2 + B)^{-1} \epsilon_{t-2} + \dots \quad (18)$$

where μ is the unconditional mean of the vector y_t . B is given in (13) and Ψ_S , $S = 0, 1, 2, \dots$, is given by

$$\Psi_S = (I_2 + B)^{-1} \phi_1 \Psi_{S-1} + (I_2 + B)^{-1} \phi_2 \Psi_{S-2} + \dots + (I_2 + B)^{-1} \phi_P \Psi_{S-P} \quad (19)$$

where ϕ_j , $j = 1, \dots, P$, are the coefficients on the lags in the VAR in (12) and $\Psi_0 = I_2$, the 2×2 identity matrix. $\Psi_k = 0_{2 \times 2} \forall k < 0$. The impacts of ϵ_t on y_{t+S} are shown by the impulse response functions, H_S , where H_S is the 2×2 coefficient matrix on ϵ_{t-S} in (18).

$$H_S = \Psi_S (I_2 + B)^{-1} \quad (20)$$

We therefore introduce a one unit order flow shock to each of the VARs and examine the effect of the feedback trading parameter by comparing the IRFs with and without this feedback trading. In order to determine whether the non-feedback IRF is significantly different from the IRF of the unrestricted VAR, we have to calculate the distribution of this feedback impulse response and we do so analytically using

⁸See Hamilton (1994) for example.

the delta method. From (20), H_S depends only on the parameters in the structural VAR, the distribution of which is shown in (17). If $\hat{H}_S = \hat{\Psi}_S (I_2 + \hat{B})^{-1}$, where the caret denotes parameter estimate, and if $\hat{h}_S = \text{vec}(\hat{H}'_S)$ then the distribution of the IRF parameters is given, using (17), as

$$\sqrt{T}(\hat{h}_S - h_S) \xrightarrow{d} N(0, G_S \Sigma_\pi G'_S) \quad (21)$$

where G_S is $4 \times 2(1+k)$ and equals $\frac{\partial h_S}{\partial \pi'}$. However, to calculate G_S , one cannot use the results of Hamilton (1994). The structural VARs considered in Hamilton (1994) are estimated from the reduced form and the structural parameters are backed out from the variance/covariance matrix of the residuals. In that way, the distribution of the IRFs depends, not only on the distribution of the reduced form parameters, but also on the distribution of the variance/covariance matrix of the errors. Since we use instrumental variables to estimate the structural parameters directly, the distribution of the IRFs in (21) will depend only on the distribution of the π parameters and *not* on the distribution of the variance/covariance matrix of residuals. In Appendix A.4 we show, using methods similar to those of Hamilton (1994) and Lütkepohl (1990), that G_S can be written as

$$G_S = \frac{\partial h_S}{\partial \pi'} = [I_4 + (I_2 \otimes B')]^{-1} \left[\frac{\partial \psi_S}{\partial \pi'} - (H_S \otimes I_2) S_{B'} \frac{\partial \Theta_{B'}}{\partial \pi'} \right] \quad (22)$$

where $S_{B'}$ and $\frac{\partial \Theta_{B'}}{\partial \pi'}$ are shown to be matrices of zeros and ones, and $\frac{\partial \psi_S}{\partial \pi'}$ is given by

$$\begin{aligned} \frac{\partial \psi_S}{\partial \pi'} = & - (I_2 \otimes [\phi_1 \Psi_{S-1} + \dots + \phi_P \Psi_{S-P}]) \left((I_2 + B)^{-1} \otimes ((I_2 + B)^{-1})' \right) S_{B'} \frac{\partial \Theta_{B'}}{\partial \pi'} \\ & + \sum_{j=1}^P \left\{ ((I_2 + B)^{-1} \otimes \Psi'_{S-j}) \frac{\partial \Theta_{\phi'_j}}{\partial \pi'} + ((I_2 + B)^{-1} \phi_j \otimes I_2) \frac{\partial \psi_{S-j}}{\partial \pi'} \right\} \end{aligned} \quad (23)$$

where, again, $\frac{\partial \Theta_{\phi_j}}{\partial \pi^j}$, $j = 1, \dots, P$, are matrices of zeros and ones. Using (21), (22) and (23) we can then calculate the distribution of the impulse response functions and therefore see whether the restricted/non-feedback IRF is significantly different from the unrestricted impulse response.⁹

2.4 Data

The market we consider is that of the spot USD/EUR (US dollar per euro) inter-dealer foreign exchange market, taken from the Reuters D2000-2 electronic trading system. The data we consider as instruments are those from the USD/GBP (US dollar per pound sterling) and GBP/EUR (pound sterling per euro). When sampling the data we record the last transaction price in each period (one minute or five minutes) and the order flow, defined as the number of buyer initiated trades minus the number of seller initiated trades. Unfortunately we have no information on traded quantities. However, to the extent that earlier work has shown little size variation in trades on this dealing system (Payne 2003) and that in other applications it is the number rather than aggregate size of trades that has been shown to matter for prices and volatility (Jones, Kaul, and Lipson 1994), we expect that this limitation will not distort our results. Furthermore, even when both the number and size of trades have been available, research has often focussed on the former measure of trading activity (Hasbrouck 1991). We also decide to remove certain sparse trading periods from our sample. These include weekends, the overnight period, defined as 1800 to 0600 GMT (BST in the summer months) where trading activity was found to be very thin and some public holidays including Christmas, New Year, Easter (Good Friday and Easter Monday) and the May Day bank holiday. Periods where the D2000-2 data feed broke down were also excluded. These periods are defined as those where no transactions (and hence no price changes) occurred for at least thirty minutes during the day in any of the three FX markets, i.e. if the data feed broke down on GBP/EUR or

⁹The distribution of the cumulative IRFs can be calculated quite easily from (21). See Lütkepohl (1990).

USD/GBP but not USD/EUR then those data are still excluded, purely because the GBP/EUR and USD/GBP data are needed in the construction of the instruments. This filtering process reduced the total number of observations to 90949 at the one minute frequency and 18401 at the five minute frequency. Table 1 contains statistical information on exchange rate returns, defined as 100 times the logarithmic difference in prices, transaction frequencies and order flows for our filtered data sample.

2.5 Instrumenting the endogenous variables

The IV estimator and its distribution, reported in Section 2.2 are standard results. The main question at this point concerns what instruments one can use and how good they are at instrumenting the endogenous regressors. Since the data available to us include not only USD/EUR returns and transactions but also those from the USD/GBP and GBP/EUR markets, the statistics from these other two markets seem prime candidates for use as instruments. Previous research has documented the cross effects of order flow on exchange rates. Evans and Lyons (2002a) document the role that order flow in one currency has in determining exchange rates in other markets. In particular DEM/USD (Deutsche mark-dollar) and CHF/USD (Swiss franc-dollar) order flows have significant effects on a number of other dollar exchange rates. These cross market effects are also documented in Daniélsson, Luo, and Payne (2002) which considers the USD/EUR, GBP/EUR, USD/GBP and JPY/USD markets. Theoretical explanations as to why cross effects of order flow exist are also presented in Lyons and Moore (2003), which examines the triangle of rates between the US dollar, euro and yen.

Since the triangle of rates between the dollar, sterling and euro form a strict cointegrating system, using contemporaneous USD/GBP and GBP/EUR returns as instruments for USD/EUR returns is likely to be problematic. Following a shock to USD/EUR returns at date t for example, this will affect not only the USD/EUR rate but also one or both of the sterling rates, otherwise clear arbitrage opportunities would result. In which case date t USD/GBP and GBP/EUR returns will

be correlated with the date t error in the USD/EUR return equation. The use of contemporaneous USD/GBP and GBP/EUR returns as instruments for the endogenous USD/EUR variables will then result in biased parameter estimates just as OLS estimates would. Instead, we consider lags of sterling returns which should not be correlated with the errors and hence stand a good chance of being valid instruments.

Also, under conditions of no arbitrage, it is clear that the USD/EUR return at time t will equal the sum of the returns in the USD/GBP and GBP/EUR markets. Therefore, using lags of *both* sterling returns as instruments will be problematic since, unless the coefficients on the sterling returns are different in the first stage regression in the 2SLS procedure, we will essentially be using a ‘synthetic’ lagged USD/EUR return to instrument for contemporaneous returns. However, the lagged USD/EUR return is effectively being used as an instrument for itself, since it too is included in the VAR. For this reason we only use one of either USD/GBP or GBP/EUR returns as instruments. Which return series we use will depend on how good they are at instrumenting for the endogenous regressors. Since the no arbitrage problem does not hold for our order flow series; USD/EUR order flow in period t does not have to equal the sum of the USD/GBP and GBP/EUR flows, we consider both USD/GBP and GBP/EUR order flows as candidate instruments for the contemporaneous USD/EUR flow regressor.

Researchers have often pointed to the pitfalls of using weak instruments and the bias that such instruments introduce. See for example Buse (1992), Bound, Jaeger, and Baker (1995), Wang and Zivot (1998) and Staiger and Stock (1997). It is therefore vital that we examine the quality of our instruments. In our two equation case, in returns and flows, we only have one endogenous regressor and so test the quality of our instruments using the procedure discussed in Pagan and Robertson (1998). They suggest an easily implementable test for the quality of potential instruments by means of a Wald test. For cases with multiple endogenous regressors, see Shea (1996) and Hall, Rudebusch, and Wilcox (1996).¹⁰ The results of these Wald tests

¹⁰To test the quality of our instruments we run a regression of the endogenous regressor in each of the structural equations in (12) on all the exogenous variables (lagged USD/EUR flows and returns)

are reported in Table 2 for the one and five minute frequency VARs.

For the one minute frequency VAR, three lags of USD/GBP and GBP/EUR flows were chosen to instrument for USD/EUR flows in the return equation, while two lags of USD/GBP returns were chosen as instruments for USD/EUR returns in the flow equation. Testing the quality of the instruments for USD/EUR flows in the return equation produced a Wald test of 28.09. For returns in the flow equation, the instrument Wald test was 236.13. The 1% critical values for the corresponding χ^2 distributions are 16.81 and 9.21 respectively, suggesting that the chosen variables are good instruments for the endogenous regressors. When considering the five minute frequency VAR, contemporaneous and one lag of both USD/GBP and GBP/EUR flows were chosen to act as instruments for USD/EUR flows in the return equation, while in the flow equation, two lags of GBP/EUR returns were chosen to instrument for USD/EUR returns. The regression of USD/EUR flows on all exogenous variables and instruments produced a Wald test of 7109.01, while testing how good the GBP/EUR returns are as instruments for returns in the flow equation produced a Wald test of 62.26. Again both of these are significant at the 1% level, suggesting that these variables make good instruments.¹¹ We also considered sampling the data at lower frequencies. However, at anything lower than the five minute frequency, the instrument Wald test became insignificant, even at the 5% level, suggesting that neither USD/GBP or GBP/EUR variables would be good at

as well as the candidate instruments. A Wald test is then performed on the coefficients of those instruments. For the endogenous USD/EUR return regressor we begin by using the first lag of either USD/GBP or GBP/EUR returns and continue increasing the lag length of the instruments until no more explanatory power is added by their inclusion. The choice as to which sterling series to use, is made based on the overall fit of this first stage regression. For the USD/EUR flow regressor, we start by considering contemporaneous USD/GBP and GBP/EUR flows and increase the lag length in a similar fashion.

¹¹It may appear strange that different variables were chosen to act as instruments at the one and five minute frequencies. This may be reconciled when one considers the different FX market dynamics at the different frequencies. With different dynamics and cross correlations at one and five minute frequencies, it may be unsurprising to find different choices of instruments. The question of which variables to include as instruments is, after all, an empirical one. The huge Wald statistic on the instruments for USD/EUR flows (7109.01) comes primarily from the use of contemporaneous USD/GBP and GBP/EUR flows as instruments. Surprisingly, contemporaneous flows were not of any use for the one-minute VAR, suggesting possibly delayed information spill-overs from one market to another.

instrumenting for the endogenous USD/EUR returns and flows. Since only lagged returns are suggested as instruments, due to the problems of using contemporaneous USD/GBP and GBP/EUR returns explained above, as soon as one considers lower frequency data, the ability of these lagged variables to instrument for USD/EUR returns is likely to fall. At the hourly or daily frequency for example, returns and flows will no longer be serially correlated. If USD/EUR returns are not correlated with its own lag, it is highly unlikely that they will be correlated with the lags of USD/GBP or GBP/EUR returns. At the lower frequencies, even the fifteen minute level, the candidate instruments became very weak. Therefore the estimations were only performed for the one and five minute VARs.

To our knowledge, only one other paper has tried to examine contemporaneous feedback trading in the foreign exchange market. Evans and Lyons (2003) use a VAR model, as we do, in returns and order flows but is not as general as the procedure outlined above. Evans and Lyons split order flows into two types, both having different roles. They assume returns depend on contemporaneous ‘informational’ trades while ‘feedback’ trades depend on contemporaneous returns. The order flow measure available from the data is simply the sum of these two components. This is more restrictive than our approach since we do not, in any way, split trades into different motives. By using variables obtained from other FX markets as instruments we are able to estimate an otherwise unidentified model.¹²

3 Estimation Results

The estimation results of the one minute VAR with and without feedback trading are shown in Table 3. The lag lengths of the VAR were chosen using the Schwartz Information Criterion and this resulted in seven lags of returns and three for order flows. Each VAR was estimated equation by equation and heteroscedasticity and autocor-

¹²At the daily frequency Evans and Lyons (2003) find evidence of negative feedback trading!

relation consistent standard errors were calculated using the Newey-West method.¹³ The left panel shows the estimation results of the feedback VAR. There are a number of important findings. Firstly, as one would expect, returns depend positively on contemporaneous order flow. One way buying (selling) pressure causes positive (negative) returns intra minute. Returns also display negative serial correlation, as have been found in Payne (2003) and Evans (2002), both of whom consider the Deutsche mark-dollar market. Returns also depend negatively on lagged own order flow, although the explanatory power of these variables in the determination of returns is surprisingly low compared to previous studies; the R^2 is only 11.2%.¹⁴ Of more interest in this paper are the results for the order flow equation. Order flow appears to depend positively on contemporaneous returns, with a coefficient that is significant at the 1% level. This suggests that following a positive return in one minute, traders ‘buy into’ the currency in that same period, possibly because they expect further price changes in the same direction. However, this is not consistent with the negative serial correlation observed in one minute returns. On the other hand, positive intra minute feedback trading is consistent with the positive effect of lagged returns on order flows, seen in both versions of the VAR. This lagged feedback trading phenomenon is considered in more detail by Cohen and Shin (2003) in the US treasury market. Without feedback trading, the R^2 in the flow equation is only 8.2%. However, when contemporaneous feedback trading is allowed, the R^2 increases to 26.5%, suggesting that contemporaneous price changes are an important determinant of order flows.

Table 4 gives the VAR results when the data are sampled at the five minute frequency. Again the VARs were estimated equation by equation and the Newey-West method was used to correct for heteroscedasticity and serially correlated errors. The

¹³The feedback VAR was estimated equation by equation using instrumental variables, described above, while the recursively ordered/non-feedback VAR was estimated equation by equation using OLS. In this way, we compare the ‘true’, unrestricted VAR to that which would have been estimated using current best practice.

¹⁴This raises the question of how good lagged sterling flows are at instrumenting for contemporaneous USD/EUR flows at the one minute frequency. The dramatic reduction in R^2 from non-feedback to feedback VAR, along with the dramatic reduction in the t-stat on contemporaneous flows, suggests either a huge mis-specification in the non-feedback VAR, or the use of instruments which are not as strong as the Wald test suggests. However, this problem is not apparent in the 5 minute frequency VAR, in which the feedback trading parameter is larger and makes more of a difference.

Schwartz Information Criterion suggested using three lags of returns and one of flows. The results are similar, but more pronounced, than those from the higher/one minute frequency VAR. Again, order flows have a positive and significant effect on contemporaneous returns, as one would expect, and returns display negative serial correlation and depend negatively on lagged flows. The explanatory power of these variables for five minute returns is quite high, with an R^2 of 31.1%. When examining the flow equation, we again find evidence of feedback trading. Flows depend positively, not only on lagged returns, but also on contemporaneous five minute returns. The coefficient on contemporaneous returns in the flow equation is significant at the 5% level and quantitatively very large; the size of the contemporaneous feedback trading parameter is more than two and a half times larger than that on the first lag of returns (56.27 versus 21.23).¹⁵ Indeed, if there is positive feedback trading (lagged and contemporaneous) at the one minute frequency, this, by definition, will be shown as contemporaneous feedback trading at the five minute frequency, as demonstrated in Section 1. The results therefore suggest that positive feedback trading is present in the spot USD/EUR market and significant at high frequencies. Intra minute feedback trading is significant but not large, possibly because of the time it takes for traders to react to the price movements. At the five minute frequency, however, intra period feedback trading becomes much larger as any lagged feedback trading at higher frequencies gets incorporated into the contemporaneous feedback effect. We can interpret the effects of this feedback trading using standard VAR analysis, namely impulse response functions. This is the focus of the next section.

3.1 Implications of contemporaneous feedback trading: Impulse response functions

We follow standard practice and use IRFs to estimate the information content of trades. Shocking the system with an order flow shock, ϵ_{it} , where i corresponds to

¹⁵For the one minute VAR, contemporaneous feedback trading had a coefficient of 27.02, compared with 24.12 for the first lag of returns.

the order flow equation in (12), can be interpreted as examining the effect of private information. The larger the impact such a shock has on returns, the more informative order flows are argued to be. By comparing the impulse response functions following an order flow shock in the two VARs (feedback trading versus non-feedback trading) we can examine how important contemporaneous feedback trading is. Intuitively, by ignoring the positive feedback trading (in the recursively ordered structural VAR) it is likely that any order flow shock will have a smaller impact on returns. The existence of positive feedback trading will cause the price impact of order flow shocks/private information to be larger than when feedback trading is ignored, i.e. trades carry *more* information than previous estimates suggest.

The impulse response functions following a one unit order flow shock are shown in the top panel of Figure 1 for the one minute frequency VAR. A number of features can be noted.

- The impact of the order flow shock is almost immediate. Following the one unit shock in the feedback VAR, this causes a 1.09 basis point return and after ten minutes the cumulative return is 1.06 basis points.
- When contemporaneous feedback trading is allowed, the effect of the order flow shock is larger than when contemporaneous feedback trading is prohibited; the feedback impulse response is more than double that in the non-feedback VAR. However, the non-feedback VAR impulse response function is not significantly different from the feedback IRF, i.e. the non-feedback IRF lies within the 95% confidence bound of that from the unrestricted VAR.¹⁶

This suggests that at the one minute frequency, the difference between the two impulse response functions is *economically* significant, if not statistically so (at the 5% level). Evaluating the informativeness of order flows by considering their price im-

¹⁶Impulse responses were also done using a one standard deviation order flow shock but this had no effect on our results. This is because the standard deviations of order flow shocks were very similar in both VAR specifications; see Table 3.

pact will result in a bias if a recursively ordered VAR is considered. However, the statistical significance of this bias is questionable.

The results from the five minute frequency VAR are more pronounced and suggest a much more important role for feedback trading in the interpretation of IRFs. The impulse response functions following a one unit order flow shock are shown in the bottom panel of Figure 1. The notable features are given below.

- On impact of the order flow shock in the feedback VAR, this causes a return of 1.12 basis points. The cumulative return is 1.02 basis points after thirty minutes.
- Again, as in the one minute VAR results, the effect of a one unit order flow shock is larger when feedback trading is allowed. On impact of the order flow shock in the non-feedback VAR, the return is only 0.43 basis points, i.e. the feedback IRF is over two and a half times that of the non-feedback VAR. However, for the five minute frequency case, the non-feedback IRF is significantly different from the unrestricted impulse response, i.e. it lies outside the 95% confidence bound.

Therefore, at the five minute frequency, feedback trading appears to have important consequences when trying to calculate the price impact/informativeness of order flow. The IRF that is commonly computed (that does not allow contemporaneous feedback trading) is significantly below the ‘true’ IRF which does allow such trading strategies. The price impact of order flow, and hence proxies for the informativeness of such trades, is therefore larger than is commonly believed, implying that trades carry more information than previous studies suggest. The feedback trading that occurs both contemporaneously and also with lags at the one minute frequency, has significant repercussions when modelling five minute data without feedback trading, as is commonly done.

4 Discussion and Interpretation

We have shown that feedback trading in the USD/EUR spot FX market does exist even at high frequencies, specifically the one and five minute sampling frequencies. At the one minute frequency, even though the non-feedback impulse response is not different from the unrestricted IRF in a statistical sense, it is different in an economic sense; the feedback impulse response is more than twice that of the non-feedback IRF, implying trades carry over twice as much information than current estimates suggest. At the five minute frequency, the contemporaneous effect of returns on order flows is significant and causes the IRFs with and without feedback trading to differ significantly, economically and statistically. Indeed, if feedback trading occurs at the one minute frequency in the lags and also contemporaneously, then by definition, such trading strategies will appear contemporaneous at the five minute frequency. This positive feedback trading causes the price impact of unanticipated order flow shocks (representing private information) to be larger compared to when contemporaneous feedback trading is ruled out. The price impact of private information/order flow shocks that is commonly calculated will then be biased downwards compared with the true impact.

In this paper we have labelled the effects of contemporaneous asset returns on order flows as feedback trading effects. However, there may be other reasons why date t order flows depend on contemporaneous asset returns. Firstly, traders wishing to trade large quantities may break up their trade into a number of smaller sized transactions. By walking up and down the limit order book and splitting a large buy order into a number of smaller trades, to be executed within a short time interval, this will be shown up in the VAR as order flows depending on contemporaneous asset returns. Even though traders are not ultimately wishing to trade based on previous price changes; the decision to trade was made some time earlier, this will still manifest itself as date t order flow depending (statistically) on date t asset returns. Imposing a recursively ordered structural VAR will still be a mis-specification. Another reason why order flow may appear dependent on contemporaneous returns in aggregated

data is because of the existence of stop-loss orders (Osler 2002). If the price of an asset falls to a certain level, traders may initiate sell orders in order to *stop losses* from getting any larger. In which case, negative (positive) returns induce negative (positive) order flow immediately. In any case, what we have demonstrated in this paper is that order flow at date t depends positively on date t asset returns in the spot FX market. Whether this is due to the splitting of dealers' trades, stop loss orders or due to 'pure' feedback trading based on extrapolative expectations of future price changes, is irrelevant. In all cases, the assumption of a recursively ordered structural VAR will result in a misspecified model and in a bias in any estimate of the price impact/informativeness of trades.

This paper does not try to explain why feedback trading may occur in the foreign exchange market, or indeed how profitable such strategies may or may not be. The purpose of the paper is to analyse the effects of contemporaneous feedback trading on estimates of the price impact of order flows. Such measures are commonly used as proxies for the informativeness of trades (how much information trades carry). As shown in this study, these measures are underestimated if feedback trading is ignored and the data are sampled at anything other than at the highest frequencies. Following a positive order flow shock, representing private information, this causes a positive return due to the asymmetric information (Kyle 1985, Glosten and Milgrom 1985) or inventory control channels (Lyons 1995). If such price changes induce further trades, which in turn cause price changes, etc., then the total effect of the order flow shock will be greater than when contemporaneous feedback trading is prohibited.

At the five minute frequency we find that after a one unit order flow shock, this causes a return of approximately 1.12 basis points whereas when feedback trading is ignored, the return is only 0.43 basis point. The non-feedback IRF is significantly different from that of the unrestricted VAR, implying that feedback trading makes a difference when calculating the price informativeness of trades.

5 Conclusions

Microstructure theory suggests that trades carry information and hence have permanent effects on prices. The information content of these trades is normally quantified by examining their price impact (Hasbrouck 1991) after fitting the data to a vector autoregression. However, common practise is to allow returns to depend on contemporaneous order flows but not the converse. The recursively ordered structural VAR that results can then be estimated quite easily. Although intuitive at ultra-high frequencies, such as tick-by-tick, as soon as one starts aggregating the data, any feedback trading (that by definition can only occur in the lags of tick-by-tick data) will appear contemporaneous. The recursively ordered VAR then becomes misspecified and can have important repercussions when examining the price impact of order flow shocks. In this paper we use standard instrumental variables techniques in order to estimate a VAR model that allows contemporaneous feedback trading. Feedback trading is found to be significant and positive at the one and five minute frequencies, with the implication that the price impact of order flows is underestimated when such trading strategies are not allowed. Trades, in the form of order flow shocks, therefore carry more information/have a larger impact on asset prices than previously believed.

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Table 1: Summary Statistics on USD/EUR Exchange Rate Returns and Flows

Variable	Mean	Variance	Skewness	Kurtosis	Autocorrelation (lags)				
					1	5	10	20	
1 minute frequency									
<i>returns</i>	-0.000160	0.000815	0.201	28.1	-0.0259*	-0.0157*	-0.0128*	-0.00177	
<i>absolute returns</i>	0.0166	0.000540	4.35	48.2	0.224*	0.132*	0.108*	0.0875*	
<i>flows</i>	0.0380	12.1	-0.379	20.3	0.232*	0.0299*	-0.00214	-0.000590	
<i>trades</i>	4.02	19.1	2.41	16.2	0.567*	0.412*	0.357*	0.294*	
5 minute frequency									
<i>returns</i>	-0.000820	0.00374	-0.00431	7.84	-0.0448*	0.00631	-0.000700	0.00559	
<i>absolute returns</i>	0.0422	0.00196	2.42	13.0	0.195*	0.101*	0.0738*	0.0553*	
<i>flows</i>	0.190	94.2	-0.106	9.91	0.124*	0.0171*	-0.00752	0.00198	
<i>trades</i>	19.9	295.1	1.99	11.1	0.685*	0.451*	0.312*	0.108*	

Notes: The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the first difference of the logarithm of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. * denotes significance at the 5% level or less.

Table 2
Instrumenting USD/EUR returns and flows

Return equation

$$R_t^{USD/EUR} = c + b F_t^{USD/EUR} + \sum_{i=1}^m u_i R_{t-i}^{USD/EUR} + \sum_{j=1}^n v_j F_{t-j}^{USD/EUR} + \epsilon_t^R$$

		1 min frequency	5 min frequency
		$F_{t-1}^{USD/GBP}$ $F_{t-1}^{GBP/EUR}$ $F_{t-2}^{USD/GBP}$ $F_{t-2}^{GBP/EUR}$ $F_{t-3}^{USD/GBP}$ $F_{t-3}^{GBP/EUR}$ $\underbrace{\hspace{10em}}_{w_t}$	$F_t^{USD/GBP}$ $F_{t-1}^{USD/GBP}$ $F_t^{GBP/EUR}$ $F_{t-1}^{GBP/EUR}$ $\underbrace{\hspace{10em}}_{w_t}$
Frequency	Wald test	1% critical value	Degrees of freedom
1 min	28.09	16.81	6
5 min	7109.01	13.28	4

Flow equation

$$F_t^{USD/EUR} = c + b R_t^{USD/EUR} + \sum_{i=1}^m u_i R_{t-i}^{USD/EUR} + \sum_{j=1}^n v_j F_{t-j}^{USD/EUR} + \epsilon_t^F$$

		1 min frequency	5 min frequency
		$R_{t-1}^{USD/GBP}$ $R_{t-2}^{USD/GBP}$ $\underbrace{\hspace{10em}}_{w_t}$	$R_{t-1}^{GBP/EUR}$ $R_{t-2}^{GBP/EUR}$ $\underbrace{\hspace{10em}}_{w_t}$
Frequency	Wald test	1% critical value	Degrees of freedom
1 min	236.13	9.21	2
5 min	62.26	9.21	2

Notes: R_t^x is $100 \times$ the log first difference of exchange rate x at date t . F_t^x is the order flow for exchange rate x , defined as the number of buyer less the number of seller initiated transactions in period t .

Table 3: USD/EUR VAR results (1 minute frequency)

	Feedback VAR		Non-feedback VAR	
	R_t equation	F_t equation	R_t equation	F_t equation
<i>constant</i>	-0.000519 ^a (-4.21)	0.0435 ^a (3.88)	-0.000376 ^a (-4.75)	0.0381 ^a (3.44)
<i>flow_t</i>	0.00841 ^a (4.17)		0.00468 ^a (57.61)	
<i>return_t</i>		27.02 ^a (3.13)		
<i>return_{t-1}</i>	-0.249 ^a (-5.28)	24.12 ^a (26.55)	-0.165 ^a (-15.89)	22.52 ^a (28.86)
<i>return_{t-2}</i>	-0.0684 ^a (-4.37)	7.31 ^a (12.83)	-0.0420 ^a (-6.01)	7.07 ^a (12.47)
<i>return_{t-3}</i>	-0.0258 ^a (-2.99)	2.91 ^a (4.09)	-0.0152 ^b (-2.19)	2.86 ^a (4.02)
<i>return_{t-4}</i>	-0.0346 ^a (-4.58)	2.83 ^a (5.05)	-0.0255 ^a (-4.85)	2.45 ^a (4.44)
<i>return_{t-5}</i>	-0.0366 ^a (-5.20)	2.65 ^a (5.47)	-0.0286 ^a (-5.41)	2.16 ^a (4.67)
<i>return_{t-6}</i>	-0.0267 ^a (-4.47)	2.16 ^a (4.64)	-0.0197 ^a (-4.82)	1.87 ^a (4.09)
<i>return_{t-7}</i>	-0.0171 ^a (-3.64)	0.994 ^b (2.04)	-0.0146 ^a (-3.74)	0.688 (1.44)
<i>flow_{t-1}</i>	-0.000399 ^c (-1.76)	0.0954 ^a (12.64)	0.00000878 (0.16)	0.109 ^a (17.94)
<i>flow_{t-2}</i>	-0.000191 ^a (-2.88)	0.0213 ^a (3.60)	-0.000113 ^a (-2.74)	0.0209 ^a (3.53)
<i>flow_{t-3}</i>	-0.000268 ^a (-3.37)	0.0313 ^a (3.33)	-0.000152 ^a (-2.88)	0.0311 ^a (3.31)
R^2	0.112	0.265	0.302	0.082
$\hat{\sigma}^2$	0.0285	3.33	0.0239	3.33

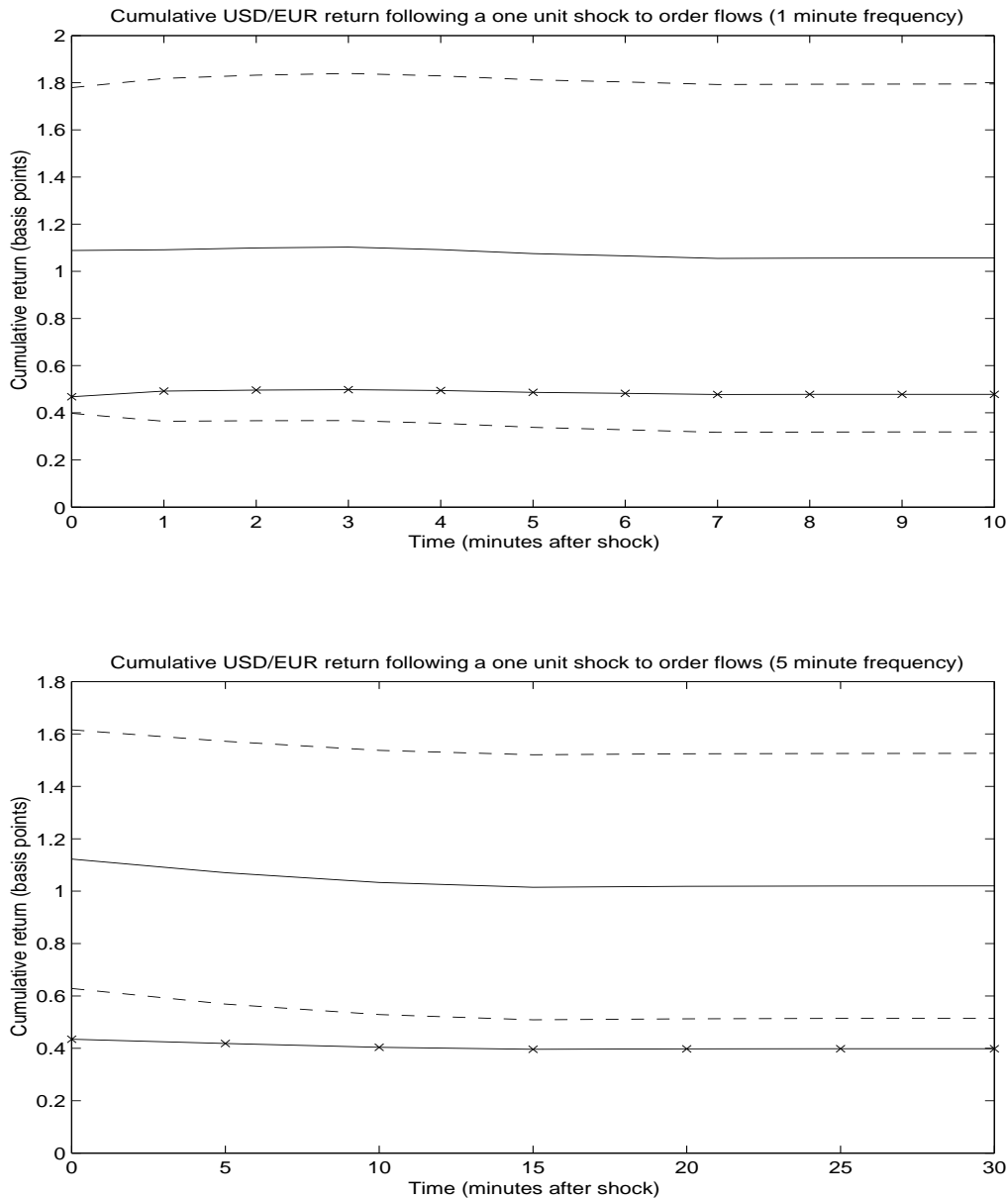
Notes: The data cover the eight month period from 1st December 1999 to 24th July 2000. The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the log first difference of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. a, b, c denote significance at the 1%, 5% and 10% levels respectively. Newey-West corrected T-stats in parentheses.

Table 4: USD/EUR VAR results (5 minute frequency)

	Feedback VAR		Non-feedback VAR	
	R_t equation	F_t equation	R_t equation	F_t equation
<i>constant</i>	-0.00232 ^a (-6.30)	0.254 ^a (3.48)	-0.00181 ^a (-5.48)	0.201 ^a (2.85)
<i>flow_t</i>	0.00688 ^a (63.47)		0.00435 ^a (45.95)	
<i>return_t</i>		56.27 ^b (2.44)		
<i>return_{t-1}</i>	-0.183 ^a (-12.28)	21.23 ^a (9.41)	-0.138 ^a (-9.82)	17.83 ^a (10.33)
<i>return_{t-2}</i>	-0.0660 ^a (-7.72)	6.15 ^a (3.72)	-0.0559 ^a (-6.79)	3.98 ^a (2.90)
<i>return_{t-3}</i>	-0.0337 ^a (-4.53)	3.08 ^b (2.29)	-0.0288 ^a (-4.41)	1.94 (1.54)
<i>flow_{t-1}</i>	-0.000213 ^a (-2.78)	0.0398 ^b (2.57)	-0.0000979 (-1.20)	0.045 ^a (2.98)
R^2	0.311	0.375	0.469	0.022
$\hat{\sigma}^2$	0.0501	9.60	0.0446	9.60

Notes: The data cover the eight month period from 1st December 1999 to 24th July 2000. The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the log first difference of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. a, b, c denote significance at the 1%, 5% and 10% levels respectively. Newey-West corrected T-stats in parentheses.

Figure 1. Impulse Response Functions for Feedback and Non-feedback VARs



Notes: The figures plot the impulse response functions following a one unit order flow shock. The shock was introduced into the estimated VAR of (12) and the cumulative return calculated. In both plots, the solid line gives the impulse response function from the feedback VAR and the dashed lines trace out a 95% confidence interval for the IRF derived from (21). The crossed line gives the impulse response from the non-feedback VAR.

A Appendix

A.1 Aggregation of the ultra-high frequency VAR in model (1)

The methods used here are taken from Harvey (1989). Let τ denote timing at the twenty second frequency, $\tau = 1, \dots, T$, and t denote timing at the ten second frequency, $t = 1, \dots, 2T$. The reduced form of the ten second frequency VAR is then given in (3). For convenience, this is given in (A.1) below.

$$y_t = Q^{-1}\alpha + Q^{-1}\phi y_{t-1} + Q^{-1}\epsilon_t \quad t = 1, \dots, 2T \quad (\text{A.1})$$

Let μ denote the unconditional mean of the stationary vector process, y_t . In which case we can write

$$y_t - \mu = Q^{-1}\phi(y_{t-1} - \mu) + \underbrace{Q^{-1}\epsilon_t}_{u_t} \quad (\text{A.2})$$

Putting (A.2) into state space form, gives the state and observation equations as (A.3) and (A.4) respectively.

$$\underbrace{\begin{bmatrix} y_{t+1} - \mu \\ y_t - \mu \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} Q^{-1}\phi & 0 \\ I_2 & 0 \end{bmatrix}}_F \underbrace{\begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix}}_{v_{t+1}} \quad (\text{A.3})$$

$$\text{Var}(v_{t+1}) = \Sigma = \begin{bmatrix} Q^{-1}\Omega(Q^{-1})' & 0 \\ 0 & 0 \end{bmatrix}$$

$$y_t = \mu + \underbrace{\begin{bmatrix} I_2 & 0 \end{bmatrix}}_{H'} \begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \end{bmatrix} \quad (\text{A.4})$$

If y_t^f denotes the cumulator variable, i.e. $y_t^f = y_t$ for the first ten seconds of a twenty second period and $y_t^f = y_t + y_{t-1}$ for the second ten second period, then:

$$\begin{aligned} y_{2(\tau-1)+1}^f &= y_{2(\tau-1)+1} \\ y_{2(\tau-1)+2}^f &= y_{2(\tau-1)+1} + y_{2(\tau-1)+2} \end{aligned} \quad (\text{A.5})$$

The cumulator variable at the ten second frequency, but at times $t = 2, 4, 6, \dots$, can therefore be given, using the observation equation, (A.4), as

$$y_t^f = y_{t-1}^f + \mu + H'F\xi_{t-1} + H'v_t \quad (\text{A.6})$$

At times $t = 1, 3, 5, \dots$, the cumulator variable is given by

$$y_{t-1}^f = \mu + H'\xi_{t-1} \quad (\text{A.7})$$

Substituting y_{t-1}^f into (A.6) and using $\xi_{t-1} = F\xi_{t-2} + v_{t-1}$ gives

$$y_t^f = 2\mu + H'(F + F^2)\xi_{t-2} + \underbrace{H'(I_2 + F)v_{t-1} + H'v_t}_{H'\bar{v}_t^f} \quad (\text{A.8})$$

Since the cumulator function, y_t^f , at times $t = 2, 4, 6, \dots$, is the same as y_τ , the data sampled at the twenty second frequency, then we can write

$$y_\tau = 2\mu + H'(F + F^2) \begin{bmatrix} y_{\tau-1} - 2\mu \\ y_{\tau-2} - 2\mu \end{bmatrix} + H'\bar{v}_\tau^f \quad (\text{A.9})$$

Noting that $H'(F + F^2) = [Q^{-1}\phi(I_2 + Q^{-1}\phi) \quad 0]$ then the reduced form model at

the twenty second frequency can be written as

$$y_\tau = \underbrace{(I_2 - Q^{-1}\phi(I_2 + Q^{-1}\phi))}_{\mu^+} 2\mu + \underbrace{Q^{-1}\phi(I_2 + Q^{-1}\phi)}_A y_{\tau-1} + e_\tau \quad (\text{A.10})$$

where $e_\tau = [I_2 \ 0] \bar{v}_t^f$. $Var(e_\tau)$ is therefore given by

$$\begin{aligned} Var(e_\tau) &= [I_2 \ 0] Var(\bar{v}_t^f) \begin{bmatrix} I_2 \\ 0 \end{bmatrix} = [I_2 \ 0] [(I_2 + F) \Sigma (I_2 + F)' + \Sigma] \begin{bmatrix} I_2 \\ 0 \end{bmatrix} \\ &= (I_2 + Q^{-1}\phi) Q^{-1} \Omega (Q^{-1})' (I_2 + Q^{-1}\phi)' + Q^{-1} \Omega (Q^{-1})' \\ &= G \Omega G' + Q^{-1} \Omega (Q^{-1})' \end{aligned} \quad (\text{A.11})$$

where $G = (I_2 + Q^{-1}\phi) Q^{-1}$. Expanding this gives us the elements of G .

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 1 + \phi_{11} + \beta\phi_{21} & (1 + \phi_{11} + \beta\phi_{21})\beta + \phi_{12} + \beta\phi_{22} \\ \phi_{21} & \phi_{12} + \beta\phi_{22} \end{bmatrix} \quad (\text{A.12})$$

A.2 Derivation of the structural form of the twenty second VAR

The structural VAR at the twenty second frequency is given in (7) and $x_\tau = P^+ y_\tau$, is the solution to the equation in (9). The general solution for $x_{i\tau}$, $i = 1, \dots, 4$, can

be found using Gaussian elimination, resulting in

$$\begin{aligned}
x_{1\tau} &= \left(\frac{1}{g_{11}} + \frac{g_{12}g_{21}}{|G|} \right) R_\tau - \left(\frac{g_{12}}{|G|} \right) F_\tau - \left(\frac{1}{g_{11}} - \frac{g_{21}}{|G|} \right) m_\tau - \left(\frac{\beta}{g_{11}} + \frac{(g_{11} - g_{21}\beta)}{|G|} \right) n_\tau \\
x_{2\tau} &= - \left(\frac{g_{21}}{|G|} \right) R_\tau + \left(\frac{g_{11}}{|G|} \right) F_\tau + \left(\frac{g_{21}}{|G|} \right) m_\tau - \left(\frac{g_{11} - g_{21}\beta}{|G|} \right) n_\tau \\
x_{3\tau} &= m_\tau \\
x_{4\tau} &= n_\tau
\end{aligned} \tag{A.13}$$

for any real values of m_τ and n_τ , $\tau = 1, \dots, T$, implying an infinite number of solutions. The right hand side of (7) can be written as $[I_2 \ I_2] x_\tau$, and the coefficients on R_τ and F_τ are the structural parameters of interest. Noting that $[I_2 \ I_2] x_\tau = [(x_{1\tau} + x_{3\tau}) \ (x_{2\tau} + x_{4\tau})]'$ gives us (10).

A.3 Distribution of $\hat{\pi}$

The notation and methods used here are similar to those of Hamilton (1994) but we allow IV estimation rather than simple OLS. In this appendix we also generalise the 2 variable case, in returns, R_t , and flows, F_t , to the n variable case, since one may wish to model a number of returns and trade characteristics. From (16)

$$\begin{aligned}
\sqrt{T} (\hat{\Pi}_i - \Pi_i) &= \left[\left(\frac{1}{T} \sum_{t=1}^T z_{it} w'_{it} \right) \left(\frac{1}{T} \sum_{t=1}^T w_{it} w'_{it} \right)^{-1} \left(\frac{1}{T} \sum_{t=1}^T w_{it} z'_{it} \right) \right]^{-1} \\
&\quad \left(\frac{1}{T} \sum_{t=1}^T z_{it} w'_{it} \right) \left(\frac{1}{T} \sum_{t=1}^T w_{it} w'_{it} \right)^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{it} \epsilon_{it}
\end{aligned} \tag{A.14}$$

$$\text{Let } Q_{(g+k) \times (g+k)}^{ii} = \frac{1}{T} \sum_{t=1}^T w_{it} w'_{it} \quad \text{and also} \quad Q_{ww}^{ii} \longrightarrow E[w_{it} w'_{it}] \quad (\text{A.15})$$

$$Q_{(n-1+k) \times (g+k)}^{ii} = \frac{1}{T} \sum_{t=1}^T z_{it} w'_{it} \quad Q_{zw}^{ii} \longrightarrow E[z_{it} w'_{it}] \quad (\text{A.16})$$

$$Q_{(g+k) \times (n-1+k)}^{ii} = \frac{1}{T} \sum_{t=1}^T w_{it} z'_{it} \quad Q_{wz}^{ii} \longrightarrow E[w_{it} z'_{it}] \quad (\text{A.17})$$

Also assume that these expectations exist and are finite. This basically states that the instruments are correlated with the right hand side regressors in (14). The instruments, w_{it} , have the property that

$$plim \frac{1}{T} \sum_{t=1}^T w_{it} \epsilon_{it} = 0 \quad i = 1, \dots, n \quad (\text{A.18})$$

so that the instruments are uncorrelated with the errors in the original VAR. Let $Q^{ii} = [Q_{zw}^{ii} Q_{ww}^{ii-1} Q_{wz}^{ii}]^{-1} Q_{zw}^{ii} Q_{ww}^{ii-1}$, then from (A.14),

$$\sqrt{T} (\hat{\Pi}_i - \Pi_i) = Q^{ii} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{it} \epsilon_{it} \quad i = 1, \dots, n \quad (\text{A.19})$$

Stacking these up for $i = 1, \dots, n$, noting that $\pi = [\Pi'_1 \ \Pi'_2 \ \dots \ \Pi'_n]'$, i.e. $\pi = \text{vec}(\Pi')$ where $\Pi' = [\Pi_1 \ \Pi_2 \ \dots \ \Pi_n]$, then we obtain

$$\sqrt{T} (\hat{\pi} - \pi) = \begin{bmatrix} Q^{11} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{1t} \epsilon_{1t} \\ Q^{22} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{2t} \epsilon_{2t} \\ \vdots \\ Q^{nn} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{nt} \epsilon_{nt} \end{bmatrix} \quad (\text{A.20})$$

which can be written

$$\sqrt{T}(\hat{\pi} - \pi) = \underbrace{\begin{bmatrix} Q^{11} & 0 & 0 \\ 0 & Q^{22} & \\ & & \ddots \\ 0 & & & Q^{nn} \end{bmatrix}}_Q \begin{bmatrix} \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{1t} \epsilon_{1t} \\ \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{2t} \epsilon_{2t} \\ \vdots \\ \frac{1}{\sqrt{T}} \sum_{t=1}^T w_{nt} \epsilon_{nt} \end{bmatrix} \quad (\text{A.21})$$

$$= Q \frac{1}{\sqrt{T}} \sum_{t=1}^T \zeta_t \quad (\text{A.22})$$

where

$$\zeta_t = \begin{bmatrix} w_{1t} \epsilon_{1t} \\ w_{2t} \epsilon_{2t} \\ \vdots \\ w_{nt} \epsilon_{nt} \end{bmatrix} \quad (\text{A.23})$$

Let $S = \frac{1}{T} \sum_{t=1}^T \zeta_t \zeta_t'$ and let $S \rightarrow E[\zeta_t \zeta_t'] = S^P$ which we assume exists and is finite.

If the population analogues of Q and S are denoted with superscript P s then

$$\sqrt{T}(\hat{\pi} - \pi) \xrightarrow{d} N\left(0, \underbrace{Q^P S^P Q^{P'}}_{\Sigma_\pi}\right) \quad (\text{A.24})$$

In the empirical application of the paper, S^P is estimated using the Newey-West method, i.e.

$$\hat{\pi} \approx N\left(\pi, \frac{Q \hat{S} Q'}{T}\right) \quad (\text{A.25})$$

where Q is as in (A.21) and \hat{S} is given by

$$\hat{S} = \hat{S}_0 + \sum_{v=1}^q \left[1 - \frac{v}{q+1} \right] \left(\hat{S}_v + \hat{S}'_v \right) \quad (\text{A.26})$$

where

$$\hat{S}_v = \begin{bmatrix} \frac{1}{T} \sum_{t=v+1}^T w_{1t} \epsilon_{1t} \epsilon_{1t-v} w'_{1t-v} & 0 & \cdots & 0 \\ 0 & \frac{1}{T} \sum_{t=v+1}^T w_{2t} \epsilon_{2t} \epsilon_{2t-v} w'_{2t-v} & & \\ \vdots & & \ddots & \\ 0 & & & \frac{1}{T} \sum_{t=v+1}^T w_{nt} \epsilon_{nt} \epsilon_{nt-v} w'_{nt-v} \end{bmatrix} \quad (\text{A.27})$$

This is the same as estimating the system using instrumental variables equation by equation and making the Newey-West correction on each equation in turn.

A.4 Distribution of the impulse response functions

The results outlined here are similar to those given in Lütkepohl (1990) and Hamilton (1994). However, the structural forms of the VAR that they consider are estimated from reduced forms, with the structural parameters backed out of the variance/covariance matrix of the residuals. In which case the distribution of the IRFs will be functions of the distribution of the parameters of the VAR but also of the distribution of the variance/covariance matrix of the errors. Since our structural VAR is estimated directly, using instrumental variables, the distribution of our IRFs will depend only on the distribution of the VAR parameters. The differences between our results and those in Lütkepohl (1990) and Hamilton (1994) are non-trivial.

The impulse response functions are given by H_S in (20). Using $\hat{h}_S = \text{vec} \left(\hat{H}'_S \right)$, the distribution of the IRFs can be calculated from (21). For convenience, this is given

below in (A.28).

$$\sqrt{T} \left(\hat{h}_S - h_S \right) \xrightarrow{d} N \left(0, G_S \Sigma_\pi G'_S \right) \quad (\text{A.28})$$

G_S is the matrix formed from the derivatives of each of the elements in the vector h_S with respect to each of the elements in the vector π , i.e. $G_S = \frac{\partial h_S}{\partial \pi'}$. Below, we show how these are calculated using $H_S = \Psi_S (I_n + B)^{-1}$ and noting that Ψ_S and B depend on the π parameters, the distribution of which is given in (17).

$$\begin{aligned} H_S (I_n + B) &= \Psi_S \\ \Rightarrow (I_n + B') H'_S &= \Psi'_S \end{aligned} \quad (\text{A.29})$$

Letting η denote an element of π , then differentiating (A.29) with respect to η gives

$$(I_n + B') \frac{\partial H'_S}{\partial \eta} + \frac{\partial B'}{\partial \eta} H'_S = \frac{\partial \Psi'_S}{\partial \eta} \quad (\text{A.30})$$

Using the result that $vec(ABC) = (C' \otimes A) vec(B)$ and letting $\psi_S = vec(\Psi'_S)$, then (A.30) can be written as

$$\frac{\partial h_S}{\partial \eta} = [I_{n^2} + (I_n \otimes B')]^{-1} \left[\frac{\partial \psi_S}{\partial \eta} - (H_S \otimes I_n) \frac{\partial vec(B')}{\partial \eta} \right] \quad (\text{A.31})$$

We then need expressions for $\frac{\partial \psi_S}{\partial \eta}$ and $\frac{\partial vec(B')}{\partial \eta}$. Start by considering $\frac{\partial vec(B')}{\partial \eta}$. From

(13), and using the n variable case,

$$B = \begin{bmatrix} 0 & -b_{12} & \cdots & -b_{1n} \\ -b_{21} & 0 & \cdots & -b_{2n} \\ \vdots & & & \vdots \\ -b_{n1} & -b_{n2} & \cdots & 0 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} 0 & -b_{21} & \cdots & -b_{n1} \\ -b_{12} & 0 & \cdots & -b_{n2} \\ \vdots & & & \vdots \\ -b_{1n} & -b_{2n} & \cdots & 0 \end{bmatrix} \quad (\text{A.32})$$

Write

$$vec(B') = S_{B'} \Theta_{B'} \quad (\text{A.33})$$

where $vec(B')$ is the $n^2 \times 1$ vector formed by stacking the n^2 elements of B' . Since B has zeros along the diagonal, there are only $n^2 - n$ structural parameters that are estimated. Hence $\Theta_{B'}$ is the $(n^2 - n) \times 1$ vector of the $-b_{ij}$ parameters, $i, j = 1, \dots, n$, $i \neq j$. $S_{B'}$ is the $n^2 \times (n^2 - n)$ matrix of zeros and ones that maps the elements of $\Theta_{B'}$ onto $vec(B')$. Therefore

$$\frac{\partial vec(B')}{\partial \eta} = S_{B'} \frac{\partial \Theta_{B'}}{\partial \eta} \quad (\text{A.34})$$

Since $\Pi' = [\Pi_1 \ \Pi_2 \ \dots \ \Pi_n]$, then Π' can be written as

$$\Pi' = \underbrace{\left[\begin{array}{cccc} b_{12} & b_{21} & \cdots & b_{n1} \\ b_{13} & b_{23} & & b_{n2} \\ \vdots & \vdots & & \vdots \\ b_{1n} & b_{2n} & \cdots & b_{nn-1} \\ & c' & & \\ & \phi'_1 & & \\ & \vdots & & \\ & \phi'_P & & \end{array} \right]}_{n \text{ columns}} \left. \begin{array}{l} \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \\ \vphantom{\left[\right.} \end{array} \right\} \begin{array}{l} n-1 \text{ rows} \\ \\ \\ \\ \\ \\ \\ \\ k \text{ rows} \end{array} \quad (\text{A.35})$$

Noting that $\pi = \text{vec}(\Pi')$, it then becomes clear that $\frac{\partial \Theta_{B'}}{\partial \pi'}$ is an $(n^2 - n) \times n(n - 1 + k)$ matrix of zeros and ones, i.e.

$$\frac{\partial \Theta_{B'}}{\partial \pi'} = \left[\begin{array}{ccc} -I_{n-1} & & 0_{(n-1) \times (n-1)(n-1+k)+k} \\ 0_{(n-1) \times (n-1+k)} & -I_{n-1} & 0_{(n-1) \times (n-2)(n-1+k)+k} \\ 0_{(n-1) \times 2(n-1+k)} & & -I_{n-1} \\ \vdots & & \vdots \\ 0_{(n-1) \times (n-1)(n-1+k)} & -I_{n-1} & 0_{(n-1) \times k} \end{array} \right] \quad (\text{A.36})$$

Since $\frac{\partial \Theta_{B'}}{\partial \pi'}$ is formed by stacking the $n(n - 1 + k)$ $\frac{\partial \Theta_{B'}}{\partial \eta}$ vectors, each of which are $(n^2 - n) \times 1$ and $\frac{\partial h_S}{\partial \pi'}$ is formed by stacking the $n(n - 1 + k)$ $\frac{\partial h_S}{\partial \eta}$ vectors, then we have our first expression, the matrix analogue of $\frac{\partial \text{vec}(B')}{\partial \eta}$. We now need an expression for $\frac{\partial \psi_S}{\partial \eta}$. From (19) we can write

$$\Psi'_S = \Psi'_{S-1} \phi'_1 ((I_n + B)^{-1})' + \Psi'_{S-2} \phi'_2 ((I_n + B)^{-1})' + \dots + \Psi'_{S-P} \phi'_P ((I_n + B)^{-1})' \quad (\text{A.37})$$

Differentiating this with respect to η , an element of π , and rearranging results in

$$\begin{aligned}
\frac{\partial \Psi'_S}{\partial \eta} &= [\phi_1 \Psi_{S-1} + \phi_2 \Psi_{S-2} + \dots + \phi_P \Psi_{S-P}]' \frac{\partial ((I_n + B)^{-1})'}{\partial \eta} \\
&\quad + \Psi'_{S-1} \frac{\partial \phi'_1}{\partial \eta} ((I_n + B)^{-1})' + \dots + \Psi'_{S-P} \frac{\partial \phi'_P}{\partial \eta} ((I_n + B)^{-1})' \\
&\quad + \frac{\partial \Psi'_{S-1}}{\partial \eta} ((I_n + B)^{-1} \phi_1)' + \dots + \frac{\partial \Psi'_{S-P}}{\partial \eta} ((I_n + B)^{-1} \phi_P)'
\end{aligned} \tag{A.38}$$

and implementing the *vec* operator, where $\psi_S = \text{vec}(\Psi'_S)$, gives us

$$\begin{aligned}
\frac{\partial \psi_S}{\partial \eta} &= (I_n \otimes [\phi_1 \Psi_{S-1} + \phi_2 \Psi_{S-2} + \dots + \phi_P \Psi_{S-P}]') \frac{\partial \text{vec}((I_n + B)^{-1})'}{\partial \eta} \\
&\quad + ((I_n + B)^{-1} \otimes \Psi'_{S-1}) \frac{\partial \text{vec}(\phi'_1)}{\partial \eta} + \dots + ((I_n + B)^{-1} \otimes \Psi'_{S-P}) \frac{\partial \text{vec}(\phi'_P)}{\partial \eta} \\
&\quad + ((I_n + B)^{-1} \phi_1 \otimes I_n) \frac{\partial \psi_{S-1}}{\partial \eta} + \dots + ((I_n + B)^{-1} \phi_P \otimes I_n) \frac{\partial \psi_{S-P}}{\partial \eta}
\end{aligned} \tag{A.39}$$

Stack the elements of the $n \times n$ matrix ϕ'_j in an $n^2 \times 1$ vector, $\Theta_{\phi'_j}$, i.e. $\Theta_{\phi'_j} = \text{vec}(\phi'_j)$ then from (A.35), it can be seen that $\frac{\partial \Theta_{\phi'_j}}{\partial \pi'}$ can be written as¹⁷

$$\frac{\partial \Theta_{\phi'_j}}{\partial \pi'} = \begin{bmatrix} 0_{n \times nj} & I_n & 0_{n \times (n-1)(n-1+k) + (P-j)n} \\ 0_{n \times (n-1+k) + nj} & I_n & 0_{n \times (n-2)(n-1+k) + (P-j)n} \\ \vdots & & \vdots \\ 0_{n \times (n-1)(n-1+k) + nj} & I_n & 0_{n \times (P-j)n} \end{bmatrix} \tag{A.40}$$

Again, since $\frac{\partial \psi_S}{\partial \pi'}$ is formed by stacking the $n(n-1+k)$ $\frac{\partial \psi_S}{\partial \eta}$ vectors, then (A.40) gives us the matrix analogue terms of $\frac{\partial \text{vec}(\phi'_j)}{\partial \eta}$. The last term we need is $\frac{\partial \text{vec}((I_n + B)^{-1})'}{\partial \eta}$.

¹⁷If we have different lag lengths of returns and flows, as in our empirical model, these $\frac{\partial \Theta_{\phi'_j}}{\partial \pi'}$ matrices need to be altered slightly. Also $\text{vec}(\phi'_j) = S_{\phi'_j} \Theta_{\phi'_j}$ and $S_{\phi'_j}$ will not in general equal the identity matrix.

Using the results of Magnus and Neudecker (1988), pages 96, 148 and 151, we can see that

$$\frac{\partial \text{vec} \left((I_n + B)^{-1} \right)'}{\partial \eta} = - \left[(I_n + B)^{-1} \otimes \left((I_n + B)^{-1} \right)' \right] S_{B'} \frac{\partial \Theta_{B'}}{\partial \pi'} \quad (\text{A.41})$$

where $S_{B'}$ and $\frac{\partial \Theta_{B'}}{\partial \pi'}$ are defined as before. Substituting (A.41) into (A.39) and summing the elements in the second and third rows will give us our definition of $\frac{\partial \psi_S}{\partial \pi'}$ when we stack the $n(n-1+k) \frac{\partial \psi_S}{\partial \eta}$ vectors. This is (23) in the text. Stacking the $n(n-1+k) \frac{\partial h_S}{\partial \eta}$ vectors in (A.31) will give the result in (22), which we can then use to calculate the distribution of the IRFs.