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Unemployment Dynamics with International Capital Mobility*

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Abstract

We study the response of domestic unemployment rates to shocks in total factor productivity for economies with high capital mobility and low labour mobility. We show that rapid capital movements across national borders, like those experienced by developed nations in the last twenty years, substantially amplify the impact on the domestic unemployment rate of fluctuations in total factor productivity relative to what would have happened in a closed economy, shorten the duration of the responses and raise the variability of employment. Capital flows increase the riskiness of labour income and reduce the riskiness of capital income.

Keywords: unemployment, foreign direct investment, fluctuations, capital mobility

JEL Classification: E24, E32, F21, J64

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1. Introduction

One of the most striking recent changes in the world economy is the speed with which the capital markets of industrial countries have become integrated. Capital flows have surpassed trade flows and attempts to explain their rise usually fall short of predicting the extent of the rise (see for example de Menil, 1999). One of the manifestations of the integration is the relocation of industrial plants from one country to another, usually greeted by the receiving country as good news for job creation and by the losing country as bad for jobs. Does international capital mobility really make a difference to the job creation and job destruction flows, and through them to the equilibrium unemployment rate of a country? This is the question addressed in this paper.

The paper shows first how unemployment and the capital stock can be jointly obtained from the same equilibrium model. In this respect the paper borrows heavily from the model of Bean and Pissarides (1993). Its second and main objective is to show how more international capital mobility affects the dynamics of unemployment, although not, generally, the mean level of unemployment. We show that as more international capital mobility takes place, unemployment responds faster and with more amplitude to shocks to TFP, so over long periods of time both unemployment and workers’ incomes have more variance than in an economy without international capital mobility. The reasons for this are simple. If an economy is hit by a negative TFP shock it reduces its demand for labour, but its inherited capital stock acts as a cushion against the fall in demand. If capital can leave the country in the pursuit of higher rates of return elsewhere the cushion is not as effective and unemployment rises more.

The implications of our analysis is that workers’ incomes and jobs become less secure and capitalists’ incomes more secured. We do not pursue the analysis of welfare-improving policy but an obvious response is that there should be more protection of workers’ incomes against shocks. We also do not carry out a complete empirical test of our model, but show that there has been an increase in the volatility of unemployment as capital mobility increased in recent years. The changes in the data come close to matching the predictions of a calibrated version
Section 2 is a formal statement of a lifecycle model of savings with labour market matching. Unemployment in this model is an equilibrium outcome of costly job creation. Sections 3 to 5 study the dynamics of employment and unemployment in small and large economies where total factor productivity shocks are imperfectly correlated. Section 6 provides numerical examples of how capital mobility affects the dynamic response of the unemployment rate to fluctuations in factor productivity, and gives some evidence from the OECD on recent trends in capital mobility and unemployment. We conclude with a discussion of some testable implications in section 7.

2. A Model of capital mobility and unemployment

We study a world consisting of many small countries, which are identical in all respects except for their total factor productivity (TFP). Countries are indexed by $i = 1, \ldots, N$ and time is discrete and indexed by $t = 1, \ldots, \infty$. Production in each country takes place with the use of capital and labour. There is only one final good which is storable and which can be used either for consumption in the period that it is produced or for production (as capital) in the following period. Capital is completely mobile across countries. In contrast, labour is completely immobile and each country has a fixed endowment of labour $L_i$.

Product and capital markets are competitive but the labour market is characterized by frictions. Employment is determined by matching and wages by bargaining. The model is a version of models previously suggested by Merz (1995), den Haan et al. (1997) and Bean and Pissarides (1993) for economies without capital mobility. It also has common features with models sometimes used to illustrate the effects of international capital mobility on savings and investment, such as the one by Obstfeld (1986).²

¹ For econometric testing of the link between the volatility of employment and capital movements see Vallanti (2003).
² Obstfeld’s (1986) objective was to show that perfect international capital mobility is consistent with the kind of correlations between investment and savings reported by Feldstein and Horioka (1980). His mechanism is also present in our model: a positive TFP shock increases
The model is one of two-period overlapping generations of workers and jobs that last for one period. At every period $t$ a fixed number of workers $L_i$ is born in each country $i$. In period 1 of their lives workers can work, consume and save, but in period 2 they are retired and only consume. Employment in period 1 is determined by a matching game between young workers and jobs, which are offered by firms. Firms make zero profits in equilibrium and they are owned by individuals who have the same utility functions as their workers. Jobs enter at the beginning of the period and if they are matched to a worker they produce, if not they exit. If workers are matched they become employed and earn wage $w_{it}$ and if not they remain unemployed and earn unemployment income $bw_{it}$. Consumption and savings are chosen to maximize a utility function and savings are in the form of claims on next period’s output. However, workers can have shares on the capital stock of any country in the world, and they will choose to put their savings in the country that maximizes their rate of return.

Households of each generation $t$ have a common utility function

$$u(c^t_{it}, c^{t+1}_{it}) = \log c^t_{it} + (1 + \rho')^{-1} \log c^{t+1}_{it} \quad \rho' > -1$$

(2.1)

expressed over their lifecycle consumption vector $(c^t_{it}, c^{t+1}_{it})$. The savings that maximize utility are a constant fraction $s = 1/(2 + \rho')$ of period 1 income. In the first period of their lives households consume either $(1 - s)w_{it}$ or $(1 - s)bw_{it}$ and in the second period they sell their capital stock to firms at the rental $r_{t+1}$ and consume the income $sw_{it}r_{t+1}$ or $sbw_{it}r_{t+1}$. Capital depreciates completely in production.\(^3\)

A single good is produced in each country from capital and labour via a stochastic constant returns Cobb-Douglas technology

$$y_{it} = \theta_{it} k^\alpha_{it}$$

(2.2)

which relates output per worker, $y_{it}$, with capital per worker, $k_{it}$, and a stochastic total factor productivity term, $\theta_{it}$. We assume that $\ln \theta_{it}$ follows an autoregressive
domestic savings and attracts capital from abroad, further increasing domestic investment.

\(^3\)Blanchard (1985) shows that equilibria in overlapping generations economies with identically homothetic preferences and potentially infinite lifecycles are qualitatively similar to those of economies with two-period lifecycles.
process
\[ \ln \theta_{it} = \rho \ln \theta_{it-1} + \varepsilon_{it} \quad (2.3) \]
with \( 0 \leq \rho < 1 \), \( E(\varepsilon_{it}) = E(\varepsilon_{it}\varepsilon_{is}) = 0 \ \forall t, s \neq t, E(\varepsilon_{it}^2) = \sigma^2 \).

Firms create new jobs at a unit cost \( \kappa > 0 \) which measures the resources absorbed in setting up the job and hiring a worker. It is a constant for all periods and countries. A fraction of the jobs are matched to workers and the remaining fraction remains idle. All jobs are destroyed at the end of the period and a new job creation round begins next period. The firm’s income is 0 for each idle job and \( \pi_{it} \) for each occupied job.

Jobs are matched with available workers by a constant-returns-to-scale search technology which connects the fraction \( m_t \) of labour force matches with the ratio \( n_t \) of jobs to total labour supply. Formally
\[ m_t = m(n_t). \quad (2.4) \]
This is an increasing, concave function with the following properties:
\[ \lim_{n \to 0} [m(n)/n] = 1 \quad \lim_{n \to \infty} [m(n)] = 1 \quad (2.5) \]
We use a log-linear approximation to this function, which proved useful in empirical work (see Petrongolo and Pissarides, 2001):
\[ m_{it} = n_{it}^{1-\eta}, \quad (2.6) \]
with \( \eta \in (0, 1) \). Of course, the log-linear function does not satisfy the restrictions in (2.5), but it has proved a useful local approximation to an interior solution when the unemployment rate is not arbitrarily close to zero. The absence of a constant in (2.6) can be justified by the choice of units, such that \( n \) is always a number less than 1 for all realistic equilibria.

We now have all the tools we need to describe how factor markets operate. In each period \( t \), once the state of nature \( \theta_{it} \) is known, entrepreneurs create \( L_t n_{it} \) jobs which are matched with the \( L_t \) young workers to produce \( L_t m_{it} \) occupied jobs. A wage bargain takes place which determines the wage rate \( w_{it} \) and profit rate \( \pi_{it} \). Entrepreneurs then buy capital stock \( k_{it} \) for each occupied job at the
rental \( r_{it} \), which either exhausts domestic savings or is equal to a world rental rate. Production occurs next, and at the end of the period wage payments and consumption take place. Both output and capital markets are competitive.

2.1. Domestic labour markets

Domestic labour markets are symmetric, so we drop the subscript \( i \) from this analysis. Once a match occurs in period \( t \) between a worker and a new job, the wage rate \( w_t \) paid by the firm is set as the outcome of a static Nash bargaining game between each worker and the firm. Formally

\[
  w_t = \arg \max \left\{ (V_t - V_0)^{\beta'} (\Pi_t - \Pi_0)^{1-\beta'} \right\}. \tag{2.7}
\]

Here \((V_t, \Pi_t)\) are payoffs to the worker and firm in a successful match, \((V_0, \Pi_0)\) are the corresponding payoffs for an unsuccessful match, and \( \beta' \in [0,1] \) is the worker’s bargaining power. Payoffs are conditional on the current TFP shock,\(^4\) and are expressed as current income equivalents of lifetime utility for each party.

Suppose wages are taxed at the rate \( \tau_t \in [0,1] \) and unemployed workers receive unemployment compensation \( bw_t(1-\tau_t) \), proportional to the net economy-wide wage rate, financed by wage taxes. If \( \pi_t \) is each firm’s variable profit from a match and \((1-\tau_t)w_t \) is each worker’s after-tax income, payoffs turn out to be proportional to income for any homothetic utility function. In particular, the indirect utility function that corresponds to a two-period homothetic utility is the product of a function that is increasing in lifecycle income and another that depends only on prices. Formally, under the assumption that the owners of the firm’s income stream have the utility function (2.1),

\[
  V_t = (1-\tau_t)w_t Z_t \tag{2.8}
\]

\[
  V_0 = bw_t Z_t (1-\tau_t)
\]

\[
  \Pi_t = \pi_t Z_t
\]

\[
  \Pi_0 = 0
\]

\(^4\)Another possibility, which we do not analyze here, is to bargain over expected payoffs before the state of nature is realized. This bargain, which spreads TFP risks over the entire population, may be of particular interest to workers who are unable to trade contingent claims in domestic or international markets.
The proportionality constant $Z_t$ will not affect the outcome of the bargain; its precise form depends on the utility function and the one-period ahead interest rate.

In view of equation (2.8), the bargaining problem becomes

$$w_t = \arg\max \left\{ \left[ (1 - \tau_t)w_t - bw_t(1 - \tau_t) \right]^{\beta'} \pi_t^{1-\beta'} \right\}$$

(2.9)

Solving this, one obtains for the symmetric Nash equilibrium,

$$w_t - bw_t = \beta'(w_t - bw_t + \pi_t).$$

(2.10)

Firms take as given the outcome of this bargain and the rental rate, $r_t$, of capital. We assume that capital depreciates completely after one period, so $r_t$ is one plus the rate of interest. Profit maximization then implies that the capital-labour ratio $k_t$ satisfies

$$r_t = \alpha \theta_t k_t^{\alpha-1}.$$  

(2.11)

From (2.2) and (2.11) we obtain profits in terms of wages

$$\pi_t = \theta_t k_t^\alpha - r_t k_t - w_t$$

$$= (1 - \alpha) \theta_t k_t^\alpha - w_t.$$  

(2.12)

Therefore, the wage bargain yields, from (2.10) and (2.12),

$$w_t = \beta \theta_t (1 - \alpha) k_t^\alpha$$

(2.13)

$$\pi_t = (1 - \beta) \theta_t (1 - \alpha) k_t^\alpha,$$

(2.14)

where $\beta$ is defined by the generalized share of labour in the wage bargain

$$\beta \equiv \frac{\beta'}{1 - (1 - \beta')b}.$$  

(2.15)

Job creation continues until profits from successful matches cover the costs of attracting new workers, that is, until

$$m_t \pi_t - \kappa n_t = 0.$$  

(2.16)
Substitution from (2.6) and (2.14) into (2.16) yields the job creation condition

\[(1 - \alpha)(1 - \beta)\theta_t k_t^\alpha n_t^{-\eta} = \kappa.\] (2.17)

We can also combine the matching function with the job creation condition in a different way to obtain a relation between employment and the capital stock per worker:

\[m_t = \left(\frac{(1 - \alpha)(1 - \beta)}{\kappa} \theta_t k_t^\alpha\right)^{(1-\eta)/\eta}.\] (2.18)

2.2. International capital markets

Asset markets direct household savings to firms. In economies without public debt or currency, aggregate household wealth equals the value of the capital stock. To see how this equality applies in our model, we recall that \(s\), the savings rate out of wage income, is independent of the interest rate:

\[s = 1/(2 + \rho').\] (2.19)

We suppose in what follows that unemployment compensation is financed by wage taxes in each \(t\). Then, transfers do not influence aggregate household wealth and so, at time \(t\), aggregate household wealth in country \(i\) is

\[W_{it} = sL_im_{it}w_{it}\] (2.20)

while capital stock at \(t + 1\) is

\[K_{it+1} = L_im_{it+1}k_{it+1}.\] (2.21)

Equating world capital with world wealth implies

\[\sum_{i=1}^{N} L_i[m_{it+1}k_{it+1} - sm_{it}w_{it}] = 0\] (2.22)

for all \(t\). Perfect capital mobility requires equality of the rate of return to capital in all countries, which from (2.11) yields,

\[\theta_t k_t^\alpha - 1 = \theta_j k_j^\alpha - 1.\] (2.23)
for all \((i, j, t)\).

Equilibrium with international capital mobility satisfies equations (2.13), (2.18), (2.22), and (2.23) simultaneously. Without capital mobility, equation (2.23) does not apply, and equilibrium in each country satisfies instead the following equations:

\[
m_{it+1}k_{it+1} = sw_{it}m_{it}
\]  

Equation (2.22) is of course still satisfied because each and every term in the summation is equal to zero by (2.24).

3. Equilibrium with immobile capital

The evolution of a closed economy is described by equations (2.24) and (2.3), where the state variable is the probability distribution of tomorrow’s capital-labour ratio conditional on today’s capital-labour ratio and on today’s realized value of the TFP shock. Substitution of wages from (2.13) and employment from (2.18) into (2.24) gives the equation determining the dynamics of the capital-labour ratio in the absence of capital mobility

\[
\ln k_{t+1} = \frac{\eta}{\alpha(1 - \eta) + \eta} \ln s\beta(1 - \alpha) + \frac{\alpha}{\alpha(1 - \eta) + \eta} \ln k_t - \frac{1 - \eta}{\alpha(1 - \eta) + \eta} \ln \theta_{t+1} + \frac{1}{\alpha(1 - \eta) + \eta} \ln \theta_t.
\]

The capital-labour ratio in period \(t+1\) depends negatively on the contemporaneous shock because a positive \(\theta_{t+1}\) does not change the total capital stock but increases desired employment. So the existing capital stock is spread more thinly among a larger number of jobs. By contrast, capital accumulation increases one period after a shock has taken place, so if say, \(\theta_t\) is positive, there is more capital in period \(t + 1\) and more capital per job as well.

The employment dynamics for the closed economy are derived by making use of (2.18) and (3.1). Substitution of \(k_t\) from (2.18) into (3.1) gives the difference
equation
\[
\ln m_{t+1} = \frac{\alpha(1-\eta)}{\alpha(1-\eta)+\eta} B + \frac{\alpha}{\alpha(1-\eta)+\eta} \ln m_t + \frac{1-\eta}{\alpha(1-\eta)+\eta} \ln \theta_{t+1}
\]
(3.2)
\[
B \equiv \ln s\beta(1-\alpha) + \frac{1-\alpha}{\alpha} \ln \left(1-\alpha\right)(1-\beta) \kappa.
\]
(3.3)
In contrast to the equation for the dynamics of the capital stock, employment in period \(t + 1\) depends only on the realization of the contemporaneous shock and on lagged shocks through a distributed lag on employment.

We consider some of the properties of this difference equation. First, suppose the value of the shock \(\theta\) is 1 for a sufficiently long period of time for equation (3.2) to practically reach its steady state and in period \(t\) jumps to a higher positive value, say \(\theta'\), which it keeps for ever. Employment is practically at its steady-state value of \(\exp(B\alpha(1-\eta)/\eta(1-\alpha))\) up to \(t - 1\), and starts rising at \(t\). In period \(t\) it rises for given aggregate capital stock, so the capital-labour ratio falls. Savings rise in period \(t\) so in period \(t + 1\) the capital stock begins to rise, providing another reason for the rise in employment. The biggest single rise in employment takes place in the period of the shock, \(t\), and subsequent rises are in decreasing amounts. Eventually, employment converges to the new steady state, where it takes the value,
\[
\ln m' = \frac{1-\eta}{\eta(1-\alpha)}(\alpha B + \ln \theta').
\]
(3.4)
The unconditional expected value of employment in this economy satisfies
\[
E(\ln m) = \frac{\alpha(1-\eta)}{\eta(1-\alpha)} \ln s\beta(1-\alpha) + \frac{1-\eta}{\eta} \ln \left(1-\alpha\right)(1-\beta) \kappa.
\]
(3.5)
The Appendix shows that its variance is
\[
var(\ln m_t) = \frac{(1-\eta)^2}{\eta(1-\alpha)\left[\eta(1-\alpha)+2\alpha\right]} \frac{\eta(1-\alpha)+\alpha(1+\rho)}{\eta(1-\alpha)+\alpha(1-\rho)} \frac{\sigma^2}{1-\rho^2}.
\]
(3.6)
Higher savings rate increase mean employment because they imply more capital and more job creation (see also Bean and Pissarides, 1993). A higher share of labour in the division of job surplus, represented by higher \(\beta\), has two effects on
mean employment that work in opposite direction. On the one hand, it implies more savings, which increases employment, but on the other hand it implies less expected profit and so less job creation. For a small increase in $\beta$ the overall effect is

$$\frac{\partial E(\ln m)}{\partial \beta} = 1 - \frac{\eta}{\beta \eta} \left( \frac{\alpha}{1 - \alpha} - \frac{\beta}{1 - \beta} \right).$$

(3.7)

For plausible parameter values this expression is negative and sufficiently far from zero that even small variations in these coefficients would still imply that higher share of labour reduces employment. The share of labour in the wage bargain, $\beta$, is usually fixed at 1/2, or, with unemployment insurance at an even higher value. The share of capital, $\alpha$, is never as high as 1/2, and usually it is closer to 1/3. So the model implies that higher UI benefits, which increase $\beta$, increase unemployment, as we would expect.

### 4. The small economy with international capital mobility

In the small economy with capital mobility the supply of capital is infinitely elastic. By the law of large numbers the equality of rates of return across countries in (2.23) reduces to an equality of each rate of return to a constant world rental rate $r_w$. In logs,

$$\ln k_t = -\ln r^w + \ln \frac{\alpha}{1 - \alpha} + \frac{1}{1 - \alpha} \ln \theta_t.$$  

(4.1)

Combining as before the relation between the capital stock and employment in (2.18) with the capital market equilibrium condition (4.1) we get the equation for the dynamics of employment in the small economy with international capital mobility:

$$\ln m_t = \frac{1 - \eta}{\eta(1 - \alpha)} (\alpha C + \ln \theta_t)$$

(4.2)

$$C \equiv \ln \frac{\alpha}{r^w} + \frac{1 - \alpha}{\alpha} \ln \frac{(1 - \alpha)(1 - \beta)}{\kappa}.$$  

The main feature that stands out of the employment equation for this economy is that it is free of internal dynamics. Adjustment is instantaneous, by our
assumption that the capital stock is infinitely elastic and it will enter or leave the country in response to the productivity shocks. If $\theta$ changes from 1 to some positive value $\theta' > 1$, employment instantly rises by $(1 - \eta) \ln \theta'/\eta(1 - \alpha)$. This rise is the same as the maximum rise achieved in the economy without capital mobility, under similar circumstances but only if $\theta$ stays at the higher value for a long time. Again, the reason for this difference in the dynamics is that the economy without capital mobility needs to accumulate the capital before it can create the new jobs needed to increase employment whereas with capital mobility the economy imports it.

The comparison of the moments of employment under the autoregressive assumptions on $\theta$ made in (2.3) reveals some interesting patterns. In the economy with capital mobility the mean of the log of employment, when $\theta_t$ obeys (2.3), is $\alpha(1 - \eta)C/\eta(1 - \alpha)$. Comparison with (3.5) shows that the difference between the expected value of the log employment in the economy without capital mobility, now distinguished by superscript $c$, and the expected value of the log employment in the economy with capital mobility, denoted $E(\ln m^o)$, is

$$E(\ln m^c - \ln m^o) = -\frac{\alpha(1 - \eta)}{\eta(1 - \alpha)} \ln \frac{s\beta(1 - \alpha)}{\alpha} r^w. \quad (4.3)$$

If the world consists of many countries like the one that we are discussing, which receive independent shocks, the marginal product of capital in each is equal to a constant. Capital mobility removes all uncertainty on the rate of return to capital, and so the capital stock of the representative country is, on average, the one derived from (3.1) when $\ln \theta_t$ is set equal to 0 (its mean value) for all $t$. The result is

$$\ln k^o = \frac{1}{1 - \alpha} \ln s\beta(1 - \alpha) \quad (4.4)$$

and combining this with (4.1) we get the value of the world rental rate

$$r^w = \frac{\alpha}{s\beta(1 - \alpha)}. \quad (4.5)$$

Substitution from (4.5) into (4.3) then gives the following result for the unconditional employment means:

$$E \ln m^c = E \ln m^o. \quad (4.6)$$
Over long periods of time, and in a world of many small economies and free capital mobility, the mean of the log of employment will be the same as the mean of the log in the economy without the capital mobility.

If now \( u \) is a small number, a good approximation yields \( \ln m = \ln(1-u) \approx -u \), so the average unemployment rates are also equal. But if there are large differences in the variances of employment in the two regimes, a better approximation yields

\[
E \ln m = \ln E m - \frac{1}{2} \sigma_m^2 \approx -Eu - \frac{1}{2} \sigma_m^2, \tag{4.7}
\]

where \( \sigma_m^2 \) is the variance of the log of \( m \). So, if the variance of employment with capital mobility is bigger, as we will argue presently, the convexity of the log function implies that the mean unemployment rate with capital mobility may actually be a little smaller than the mean unemployment rate without capital mobility.

We can easily obtain a result similar to the one in (4.6) for the capital-labour ratio with and without capital mobility, by taking unconditional expected means of (4.1) and (3.1) respectively. The value of output is \( m t \theta_t k_t^\alpha \) and so it follows that a similar result also holds for output.

The key difference between economies with and without capital mobility is the variance of the asymptotic unemployment rate. In the open economy adjustment to any shock is faster (instantaneous under our extreme assumptions) so variance will be greater. From (4.2) we see that the variance of employment in the open economy satisfies

\[
var(\ln m^o) = \frac{(1 - \eta)^2}{\eta^2(1 - \alpha)^2} \frac{\sigma^2}{1 - \rho^2}. \tag{4.8}
\]

Comparison with the variance of the log in the closed economy, (3.6) and now denoted \( var(\ln m^c) \), unambiguously gives

\[
var(\ln m^o) > var(\ln m^c) \tag{4.9}
\]

for all values of \( \rho \) strictly less than 1.
Using a Taylor approximation around \( u = 0 \) and noting that \( m = 1 - u \), yields

\[
\left[ \ln m - E \ln m \right]^2 = \left[ -u - \frac{1}{2} u^2 + Eu + \frac{1}{2} Eu^2 \right]^2
\]

\[= u^2 + (Eu)^2 - 2Eu + \text{higher order terms.} \tag{4.10}\]

Taking expected values on both sides of (4.10), we obtain

\[\text{var}(\ln m) = Eu^2 - (Eu)^2 = \text{var}(u). \tag{4.11}\]

In other words, capital mobility raises the variability of the unemployment rate.

We can rewrite (4.9) as follows

\[\text{var}(u^o) > \text{var}(u^c). \tag{4.12}\]

By making use of (4.1) and (3.1) we can easily show that the variance of capital intensities is higher when there is international capital mobility (intuitively, the higher variance in employment is due to the higher variance in the capital stock). Therefore output \( m_t \theta_t k_t^\alpha \) also has more log-variance for a given distribution of \( \theta \) when there is international capital mobility. Since the log is a convex function it follows that the mean value of output with capital mobility is higher than the mean of value of output in the absence of capital mobility. To a first approximation, given plausible realizations of the employment rate (say in the range 0.9-1.0), the higher mean output is due to higher mean capital stock over long periods of time.

5. The large economy with capital mobility

Before looking at numerical examples, we consider the implications of capital mobility for a large economy by studying equilibrium in a two-country world, with the two countries assumed to be of equal size to facilitate computations. Country variables are distinguished by subscript 1 or 2, as appropriate. Each country is hit by a shock \( \theta_t \) in each period \( t \). The shocks have common variance and non-zero covariance.

In the absence of capital mobility between the two countries, equilibrium satisfies the same properties as before, with capital accumulation given by (3.1) and
employment dynamics by (3.2). But equilibrium with capital mobility is now different, because the absence of many small countries does not allow us to apply the law of large numbers to set the rate of return to capital equal to a constant. As before, however, the rate of return to capital in the two countries is equalized. To find the equilibrium we aggregate to get world savings in period $t$:

$$s\beta(1 - \alpha) (m_{1t}\theta_{1t}k_{1t}^{\alpha} + m_{2t}\theta_{2t}k_{2t}^{\alpha}).$$

(5.1)

World capital stock in period $t + 1$ is

$$m_{1t+1}k_{1t+1} + m_{2t+1}k_{2t+1}$$

(5.2)

and the difference equation governing the evolution of capital is

$$m_{1t+1}k_{1t+1} + m_{2t+1}k_{2t+1} = s\beta(1 - \alpha) (m_{1t}\theta_{1t}k_{1t}^{\alpha} + m_{2t}\theta_{2t}k_{2t}^{\alpha}) .$$

(5.3)

Each country still satisfies the job creation condition (2.18) which, when substituted into (5.3), gives a single difference equation in the two capital stocks. In order to solve this equation we need a condition for the distribution of world capital between the two countries. But this is provided by the condition that the rate of return to capital in each country should be equalized. Since the rate of return to capital in each country is given by $\alpha\theta_{it}k_{it}^{-(1-\alpha)}$, this implies that the world distribution of capital satisfies

$$\frac{k_{1t}}{k_{2t}} = \left(\frac{\theta_{2t}}{\theta_{1t}}\right)^{-\frac{1}{1-\alpha}} \forall t.$$  

(5.4)

The four equations (5.3), (5.4) and (2.18) for each country reduce to two equations in the capital stocks and employment levels. It turns out that the four difference equations are similar to the ones for the closed economy, except for the “shock” terms. We write here the equation for employment in country 1, the others following immediately:

$$\ln m_{1t+1} = \frac{\alpha(1 - \eta)}{\alpha(1 - \eta) + \eta}B + \frac{\alpha}{\alpha(1 - \eta) + \eta}\ln m_{1t}$$

$$+ \frac{1 - \eta}{\alpha(1 - \eta) + \eta}\left(\ln \theta_{1t+1} + \alpha \ln \frac{1 + R_{t+1}^{1/\eta(1-\alpha)}}{1 + R_{t+1}^{1/\eta(1-\alpha)}}\right)$$

(5.5)
where \( R_t \equiv \theta_{2t}/\theta_{1t} \). Comparing this equation with (3.2) shows that all terms are the same as in the economy without capital mobility, except that now there is a spillover from the other country, which depends on the ratio of the two shocks. The spillover is transmitted via capital movements.

The two-country case illustrates a richer model of world equilibrium. Both the saving rate and the relative return to capital influence employment, the former through the availability of world-wide capital and the latter through the international mobility of capital. In the absence of capital mobility, the key influence on employment is the availability of capital through savings; in the small economy with capital mobility it is the relative rate of return, since the supply of world capital is by definition large vis-a-vis the small country. The influence of the two productivity shocks on capital and employment in the large economy reflects the joint impact of capital availability and capital mobility. From (5.5) it immediately follows that the contemporaneous productivity shock in the domestic economy \( (\theta_{1t+1}) \) increases capital and employment through the importation of capital from the other country and the contemporaneous shock in the other country \( (\theta_{2t+1}) \) decreases it, through the export of capital. Last period’s shocks \( (\theta_{1t} and \theta_{2t}) \) should both increase capital this period through the higher savings that they imply. But the own-country shock \( \theta_{1t} \) has an ambiguous effect in our equation because the domestic capital stock \( k_{1t} \) already accounts for some of the influence of last period’s shocks. For example, imagine a temporarily high \( \theta_{1t} \) relative to \( \theta_{2t} \). This implies that capital is imported from country 2 in period \( t \) and that there is more saving in country 1. In period \( t + 1 \) the higher savings will increase the capital stock \( k_{1t+1} \) but the capital that was imported in period \( t \) will now be re-exported. This will act to reduce the domestic capital stock in period \( t + 1 \) relative to period \( t \), i.e. the positive effect of \( k_{1t} \) on \( k_{1t+1} \) is counteracted by a negative influence from the subsequent fall in domestic TFP.

The Appendix shows that when the two productivity shocks are AR1 with common autocorrelation \( \rho \) and cross-correlation coefficient \( r \), the variance of each country’s shock with perfect capital mobility is given (to a first-order approximation) by
The mean of the shock is 0 in both economies. In the absence of international capital mobility the mean is also zero but the variance is \( \frac{\sigma^2}{1 - \rho^2} \). Therefore the ratio of variances is

\[
1 + \frac{\alpha[\alpha + \eta(1 - \alpha)]}{\eta^2(1 - \alpha)^2}(1 - \rho)(1 - r) \geq 1,
\]

with equality holding only in the case where the two shocks are perfectly (positively) correlated. In this case the rates of return to each country are always equal without capital mobility, so each country operates as if capital were not mobile. In the absence of a perfect correlation in the shocks, the variance of employment with capital mobility is always higher than the variance without capital mobility. The difference is greater the less covariance there is between the shocks in each country: with less correlation, rates of return to capital in the absence of capital mobility are less correlated as well, so more capital flows between the countries. The maximum variance is achieved when the shocks are perfectly negatively correlated. In this case when a country gets a positive shock the other gets a negative one, maximizing the difference in the rates of return to capital.

6. A numerical example and some evidence

Our motivation for studying international capital mobility is the recent increase in international capital movements in the OECD and the rise in unemployment rates observed in many European countries. We have shown that our model does not imply that the mean value of unemployment should increase with international capital mobility but that the variance of cyclical unemployment should increase. We present here some evidence to illustrate the extent of the penetration of domestic economies by international capital flows and to look at the properties of cyclical unemployment. We also construct some numerical examples to illustrate the potential quantitative significance of the mechanisms that we have identified in the preceding sections of this paper.
Table 1 gives sample means for a measure of the penetration of foreign capital in OECD economies. The measure used is the inflow of foreign capital (net of outward movements of foreign capital) as a fraction of total domestic investment.\(^5\) The striking feature of the data is the big increase in international capital mobility after the mid 1980s. There is no convincing explanation in the literature yet for this large increase, which is about four times as large as the increase in trade flows. The increase is world-wide, although it is bigger in European countries (except Germany), following the single market process that started in 1986 and culminated in 1992 and 1999 (see de Menil, 1999). The rise is much bigger in small economies than in the G7.

Table 2 gives the standard deviations in the cyclical component of unemployment for two sub-samples, one before 1985 and one after. On average the standard deviation of unemployment is higher in the more recent sub-sample, as the model would predict. The rise is much bigger in small economies than in the G7. The un-weighted average standard deviation in the G7 is the same in the two sub-periods but for the small economies it is larger by a substantial factor.

The question naturally arises whether the rise in the standard deviations of unemployment from the first to the second sample has anything to do with the rise in international capital flows. It is beyond the scope of this paper to undertake a full empirical test of this proposition, however, which would require a more complete model of unemployment. But we illustrate the potential importance of international capital mobility for the dynamics of unemployment by studying the properties of a calibrated model.

Suppose that \(\alpha = \frac{1}{3}\) and \(\eta = \frac{1}{2}\), as is usually assumed in the quantitative literature. The ratio of the variance of unemployment with and without capital mobility implied by (3.6), (4.8) and (4.11) is

\[
\frac{\text{var}(u^o)}{\text{var}(u^c)} = \left(1 + \frac{2\alpha}{\eta(1 - \alpha)}\right) \left(\frac{\eta(1 - \alpha) + \alpha(1 - \rho)}{\eta(1 - \alpha) + \alpha(1 + \rho)}\right),
\]

\(\text{(6.1)}\)

\(^5\)This measure is likely to understate what we are trying to measure but we use it because it is readily available on a comparable basis and we need it only to illustrate recent developments. FDI measures only foreign investment when the control of the investment remains in the hands of the foreign investor. Our model includes investment made by foreigners and under the control of domestic producers, as, for example, when a foreign resident buys shares in a new issue.
so choosing the value \((1/3, 1/2)\) for the parameter vector \((\alpha, \eta)\) yields

\[
\frac{\text{var}(u^e)}{\text{var}(u^c)} = 3 \left( \frac{2 - \rho}{2 + \rho} \right).
\]  \hspace{1cm} (6.2)

The parameter \(\rho\) measures the persistence in the productivity shock. If we approximate it by the AR1 coefficient on a Solow residual, it is usually high, exceeding 0.9, in quarterly data. But our period of analysis is longer. It is the time that it takes for savings to transform into capital and into new jobs. The ratio of variances depends crucially on this parameter. For \(\rho = 0\), the variance in the economy with capital mobility is three times as large as in the closed economy. For \(\rho = 0.9\) it is only 1.7 times as large. For a middle value of 0.4 the variance when there is capital mobility is twice as large.

If we approximate the lognormal asymptotic probability distribution of employment with a symmetric, one-parameter distribution on some bounded interval, then capital mobility amplifies deviations of the unemployment rate from its expected value \(\bar{u}\) by a factor equal to the square root of the ratio of the variances, that is, we can write

\[
u^0 - \bar{u} = \sqrt{\frac{\text{var}(u^e)}{\text{var}(u^c)}} (u^c - \bar{u}).
\]  \hspace{1cm} (6.3)

For example, in the case of \(\rho = 0.4\), if in the trough of a symmetric business cycle the unemployment rate in the economy without capital mobility is 3 percentage points above the average unemployment rate, in the economy with capital mobility it will be about 4.2 percentage points above the average rate.

For the large economy the parameter vector \((\alpha, \eta) = (1/3, 1/2)\) gives the ratio of variances \(1 + 2(1 - \rho)(1 - r)\). For independent shocks \(r = 0\), this yields a value very close to the ratio of variances in the case of a small economy. For \(\rho = 0\) the ratio is the same, 3, and for \(\rho = 0.4\) it is 2.2. But if, in addition, we have \(r = 0.5\), which is reasonable given the degree of synchronization of business cycles in modern open economies, the ratio of variances when \(\rho = 0.4\) drops to 1.4. So the greater variability associated with international capital mobility is likely to be more a feature of small open economies than of large ones.
Now applying the parameter vector $(1/3, 1/2)$ to the economy without capital mobility in (3.2) we get

$$\ln m_t = 0.25B + 0.5 \ln m_{t-1} + 0.75 \ln \theta_t. \tag{6.4}$$

Although our main interest is in the standard deviations of unemployment, which are unaffected by the value of $B$, we can guess a reasonable value for $B$ by taking the sample means of (6.4) to obtain

$$B \approx -2\bar{u}. \tag{6.5}$$

Substituting back into (6.4) we obtain

$$\ln m_t = 0.5(-\bar{u} + \ln m_{t-1}) + 0.75 \ln \theta_t. \tag{6.6}$$

In contrast, the dynamic equation for employment in the economy with capital mobility, (4.2), implies, given (4.5) which makes $C$ identical to $B$,

$$\ln m_t = \bar{u} - 1.5 \ln \theta_t. \tag{6.7}$$

We calibrate (6.6) and (6.7) with annual data from several OECD countries for the period 1970-98. The shock $\ln \theta$ is the cyclical component of labour productivity (ratio of output to employment) derived by applying the Hodrick-Prescott filter to the series. The series obtained is very close to the Solow residual of real business cycle theory. We convert the calibrated series for $\ln m_t$ to unemployment by taking the exponential. Table 2 presents the results of the calibrations. In the majority of cases the ratio of the standard errors for unemployment with and without capital mobility is in the range implied for very small autocorrelations in the shocks in our numerical examples.

Comparing the standard deviations of actual and calibrated unemployment in Table 2 shows that, on average, actual standard deviations are smaller than calibrated ones. This provides an interesting contrast with real business cycle models, which usually under-predict the variance of employment (this is especially true of models with classical labour demand and supply functions but also of infinite horizon search models, as in Merz, 1985). But for the small economies
(i.e. excluding the G7) the rise in the standard deviations before and after the mid 1980s is approximately of the same order of magnitude as the rise predicted by our model when capital is allowed to move between countries. Although we cannot at this stage test whether the observed rise in the standard deviations is due to the higher international capital mobility, this result is supportive of further empirical research in this direction.

7. Conclusions

We have shown that international capital mobility can substantially amplify fluctuations in unemployment and output. Calibrations show that the variance of unemployment with perfect international capital mobility can be up to three times as large as the variance of unemployment without capital mobility, implying that cyclical peaks and troughs in unemployment overshoot those in economies without capital mobility by up to 1.7 percentage points of unemployment. Small OECD economies have experienced a rise in the variance of their unemployment rates of this order of magnitude sometime in the mid 1980s, which coincided with a fast rise in international capital mobility. Such effects imply that small economies trading in a world with large international capital flows need to devise ways to insure the income of workers whose wealth is poorly diversified across countries, because international capital flows tend to shift income risk from capital to labour.

Future work in this area needs to address this policy question within reasonably estimated or calibrated models. Estimation should test whether the dynamic properties of unemployment are different when there is international capital mobility from those without (see Vallanti, 2003, for preliminary estimates). In particular, is adjustment to domestic shocks faster when the economy is small? Are foreign shocks transmitted to the domestic economy through capital flows when the economy is large? In the case of large economies, trade flows need also to be taken into account in empirical tests, because foreign shocks may influence trade prices and thereby be transmitted to the domestic economy. A higher foreign shock would then increase the domestic demand for labour, working against the effect of higher capital mobility. This trade channel should be less important in
small economies as they are more likely to be faced with a perfectly elastic demand for their exports.

8. Appendix

8.1. The variance of employment in the closed economy

Let \( \zeta \equiv \alpha / [\alpha (1 - \eta) + \eta] < 1 \). Then, the employment equation (3.2) becomes

\[
\ln m_t = \zeta (1 - \eta) B + \zeta \ln m_{t-1} + \frac{\zeta (1 - \eta)}{\alpha} \ln \theta_t \tag{8.1}
\]

\[
B \equiv \ln s \beta (1 - \alpha) + \frac{1 - \alpha}{\alpha} \ln \left( \frac{(1 - \alpha)(1 - \beta)}{\kappa} \right).
\]

where

\[
\ln \theta_t = \rho \ln \theta_{t-1} + \varepsilon_t \tag{8.2}
\]

with \( 0 \leq \rho < 1 \), \( E(\varepsilon_t) = E(\varepsilon_t \varepsilon_s) = 0 \ \forall t, s \neq t, E(\varepsilon_t^2) = \sigma^2 \). We immediately find

\[
E \ln m_t = \frac{\zeta (1 - \eta)}{1 - \zeta} B, \quad E \ln \theta_t = 0, \quad \text{var}(\ln \theta_t) = \frac{\sigma^2}{1 - \rho^2} \quad \forall t. \tag{8.3}
\]

It also follows from (8.1) that

\[
\text{var}(\ln m_t - \zeta \ln m_{t-1}) = \frac{\zeta^2 (1 - \eta)^2 \text{var}(\alpha B + \ln \theta_t)}{\alpha^2} = \frac{\zeta^2 (1 - \eta)^2 \sigma^2}{\alpha^2 (1 - \rho^2)}. \tag{8.4}
\]

But

\[
\text{var}(\ln m_t - \zeta \ln m_{t-1}) = \text{var}(\ln m_t) + \zeta^2 \text{var}(\ln m_{t-1}) - 2 \zeta \text{cov}(\ln m_t, \ln m_{t-1})
\]

\[
= (1 + \zeta^2) \text{var}(\ln m_t) - 2 \zeta \text{cov}(\ln m_t, \ln m_{t-1}). \tag{8.5}
\]
To find the covariance note that

\[
cov(\ln m_t, \ln m_{t-1}) = E \left( \ln m_t - \frac{\zeta(1 - \eta)}{1 - \zeta} B \right) \left( \ln m_{t-1} - \frac{\zeta(1 - \eta)}{1 - \zeta} B \right)
\]

\[
= E \left( \zeta \ln m_{t-1} + \frac{\zeta(1 - \eta)}{\alpha} \ln \theta_t - \frac{\zeta(1 - \eta)}{1 - \zeta} B \right)
\]

\[
\left( \ln m_t - \frac{\zeta(1 - \eta)}{1 - \zeta} B \right)
\]

\[
= \zeta E (\ln m_t - 1)^2 - 2 \zeta \frac{\zeta(1 - \eta)\alpha}{1 - \zeta} E \ln m_{t-1} + \zeta \left( \frac{\zeta(1 - \eta) B}{1 - \zeta} \right)^2
\]

\[
+ \frac{\zeta(1 - \eta)}{\alpha} E(\ln \theta_t)(\ln m_{t-1})
\]

\[
= \zeta \text{var}(\ln m_{t-1}) + \frac{\zeta(1 - \eta)}{\alpha} E(\ln \theta_t)(\ln m_{t-1}). \quad (8.6)
\]

To find \( E(\ln \theta_t)(\ln m_{t-1}) \) we expand to get

\[
E(\ln \theta_t)(\ln m_{t-1}) = E(\rho \ln \theta_{t-1} + \varepsilon_t)(\zeta(1 - \eta) B + \zeta \ln m_{t-2} + \frac{\zeta(1 - \eta)}{\alpha} \ln \theta_{t-1})
\]

\[
= \rho \zeta E(\ln \theta_{t-1})(\ln m_{t-2}) + \frac{\zeta(1 - \eta)}{\alpha} E(\ln \theta_{t-1})^2, \quad (8.7)
\]

giving

\[
E(\ln \theta_t)(\ln m_{t-1}) = \frac{\rho \zeta(1 - \eta) \sigma^2}{\alpha(1 - \rho \zeta)} \frac{1 - \rho^2}{1 - \rho^2}. \quad (8.8)
\]

Substitution from (8.8) into (8.6) and from (8.6) into (8.5) gives

\[
\text{var}(\ln m_{t+1} - \zeta \ln m_t) = (1 - \zeta^2) \text{var}(\ln m_t) - 2 \frac{\zeta^2(1 - \eta)^2}{\alpha^2} \frac{\rho \zeta}{1 - \rho \zeta} \frac{\sigma^2}{1 - \rho^2}. \quad (8.9)
\]

Therefore, making use of (8.4) we get

\[
(1 - \zeta^2) \text{var}(\ln m_t) - 2 \frac{\zeta^2(1 - \eta)^2}{\alpha^2} \frac{\rho \zeta}{1 - \rho \zeta} \frac{\sigma^2}{1 - \rho^2} = \frac{\zeta^2(1 - \eta)^2}{\alpha^2} \frac{\sigma^2}{1 - \rho^2}. \quad (8.10)
\]

and so

\[
\text{var}(\ln m_t) = \frac{1 + \rho \zeta \frac{\zeta^2(1 - \eta)^2}{\alpha^2} \frac{\sigma^2}{1 - \rho^2}}{\frac{1}{\rho \zeta} \frac{\zeta^2(1 - \eta)^2}{\alpha^2} \frac{\sigma^2}{1 - \rho^2}}. \quad (8.11)
\]

Substituting out the \( \zeta \) we obtain the final result

\[
\text{var}(\ln m_t) = \frac{(1 - \eta)^2}{\eta(1 - \alpha)[\eta(1 - \alpha) + 2\alpha]} \frac{\eta(1 - \alpha) + \alpha(1 + \rho)}{\eta(1 - \alpha) + \alpha(1 - \rho)} \frac{\sigma^2}{1 - \rho^2}. \quad (8.12)
\]
8.2. Properties of shocks, two large economies

The shock to country 1 employment is

\[ \ln \phi_{1t+1} = \ln \theta_{1t+1} + \alpha \ln \frac{1 + R_t^{1/\eta(1-\alpha)}}{1 + R_{t+1}^{1/\eta(1-\alpha)}} \]  

(8.13)

where \( R_t = \theta_{2t}/\theta_{1t} \). In the absence of international capital mobility the shock is \( \ln \theta_{1t+1} \) and in country 2 shocks are symmetric to those of country 1. Let

\[ \ln \theta_{it} = \rho \ln \theta_{i(t-1)} + \varepsilon_{it} \quad i = 1, 2 \]  

(8.14)

and \( 0 \leq \rho < 1, \ E(\varepsilon_{it}) = 0, \ E(\varepsilon_{it}^2) = \sigma^2, \ E(\varepsilon_{1t}\varepsilon_{2t}) = r\sigma^2, \ E(\varepsilon_{it}\varepsilon_{jt-s}) = 0 \) for \( i, j = 1, 2 \) and all \( s \geq 1 \) and \(-1 \leq r \leq 1\) is the correlation coefficient between the two shocks. It follows that

\[ E(\ln \theta_{it}) = 0 \] \[ E(\ln \theta_{it}^2) = \frac{\sigma^2}{1 - \rho^2}. \]  

(8.15)

Expand now \( \ln(1 + R_t^{1/\eta(1-\alpha)}) \) around \( R = 1 \) as a function of \( \ln R \):

\[ \ln(1 + R_t^{1/\eta(1-\alpha)}) = \ln 2 + \frac{\partial \ln(1 + R_t^{1/\eta(1-\alpha)})}{\partial \ln R} \ln R \]

\[ = \ln 2 + \frac{1}{2\eta(1-\alpha)} \ln R. \]  

(8.16)

Therefore,

\[ \ln \left( \frac{1 + R_t^{1/\eta(1-\alpha)}}{1 + R_{t+1}^{1/\eta(1-\alpha)}} \right) = \frac{1}{2\eta(1-\alpha)} (\ln R_t - \ln R_{t+1}) \]

\[ = \frac{1}{2\eta(1-\alpha)} (\ln \theta_{2t} - \ln \theta_{2t+1} - (\ln \theta_{1t} - \ln \theta_{1t+1})) \]  

(8.17)

Define now \( \xi \equiv \alpha/\eta(1-\alpha) \). From (8.13) it follows that

\[ \ln \phi_{1t} = \left( 1 + \frac{\xi}{2} \right) \ln \theta_{1t+1} - \frac{\xi}{2} \ln \theta_{1t} + \frac{\xi}{2} (\ln \theta_{2t} - \ln \theta_{2t+1}) \]  

(8.18)
It follows that to the first approximation employed in (8.16) the means of the shocks in the two-country case are zero

\[ E(\ln \phi_{it}) = 0 \quad \forall t \text{ and } i = 1, 2. \] (8.19)

The variance of the shock is

\[
E(\ln \phi_{it})^2 = E\left(\left(1 + \frac{\xi}{2}\right) \ln \theta_{it+1} - \frac{\xi}{2} \ln \theta_{it}\right)^2 + E\left(\frac{\xi}{2} (\ln \theta_{2t} - \ln \theta_{2t+1})\right)^2 + 2E\left(\left(1 + \frac{\xi}{2}\right) \ln \theta_{it+1} - \frac{\xi}{2} \ln \theta_{it}\right)\left(\frac{\xi}{2} (\ln \theta_{2t} - \ln \theta_{2t+1})\right) \quad (8.20)
\]

Taking each term in turn and making use of (8.14) we get

\[
E\left(\left(1 + \frac{\xi}{2}\right) \ln \theta_{it+1} - \frac{\xi}{2} \ln \theta_{it}\right)^2 = \left(1 + \frac{\xi}{2}\right)^2 E\left(\ln \theta_{2t} - \ln \theta_{2t+1}\right)^2 (8.21)
\]

\[
E\left(\left(1 + \frac{\xi}{2}\right) \ln \theta_{it+1} - \frac{\xi}{2} \ln \theta_{it}\right)\left(\frac{\xi}{2} (\ln \theta_{2t} - \ln \theta_{2t+1})\right) = (1 - \rho) \frac{\xi^2}{2} \frac{\sigma^2}{1 - \rho^2} (8.22)
\]

\[
E\left(\frac{\xi}{2} (\ln \theta_{2t} - \ln \theta_{2t+1})\right)^2 = (1 - \rho) \frac{\xi^2}{2} \frac{\sigma^2}{1 - \rho^2} \quad (8.23)
\]

given that

\[ E(\ln \theta_{1t} \ln \theta_{2t}) = \rho^2 E(\ln \theta_{1t-1} \ln \theta_{2t-1}) + r \sigma^2 \]

\[ = r \frac{\sigma^2}{1 - \rho^2}. \]
Substituting each term from (8.21), (8.22) and (8.23) into (8.20) we get the variance

\[ E(\ln \phi_{1t})^2 = (1 + \xi (1 + \xi) (1 - \rho)(1 - r)) \frac{\sigma^2}{1 - \rho^2}. \]  

(8.24)

Substituting the value of \( \xi \) out we get

\[ E(\ln \phi_{1t})^2 = \left( 1 + \frac{\alpha [\alpha + \eta (1 - \alpha)]}{\eta^2 (1 - \alpha)^2} (1 - \rho)(1 - r) \right) \frac{\sigma^2}{1 - \rho^2}. \]  

(8.25)

9. Data sources and definitions


Labour productivity. The ratio of GDP to employment. Source, OECD as above.

References


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Table 2
Standard deviations of cyclical unemployment series

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