A Human Capital Explanation for an Asset Allocation Puzzle*

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Abstract

We show that a life-cycle asset allocation model with liquidity constraints and realistically calibrated uninsurable labor income risk rationalizes the asset allocation puzzle of Canner, Mankiw and Weil (1997). Based on empirical estimates of the correlation between stock returns and individual earnings, labor income is a closer substitute to long-term bonds than to stocks. As a result, more risk averse investors hold a smaller proportion of their risky portfolio in equities. Moreover, this explanation is consistent with the recommendation that younger households should be more heavily invested in stocks than older households.

JEL Classification: G11.

Key Words: Life-Cycle Models, Portfolio Choice, Liquidity Constraints, Uninsurable Labor Income Risk.

1 Introduction

Popular financial advisors recommend that more risk-averse investors should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. Canner, Mankiw and Weil (1997) point out that this prediction is inconsistent with the Capital Asset Pricing Model (CAPM) which implies that all investors should hold the same combination of risky assets, while risk aversion should only determine the size of the investment to the risky assets as a whole. Canner, Mankiw and Weil (1997) explore various possible explanations of this puzzle and find them unsatisfactory.

Recently, a number of possible explanations for this divergence between theory and practice have been put forth. Campbell and Viceira (2001), Brennan and Xia (2002) and Campbell, Chan and Viceira (2003) rationalize this advice in the context of intertemporal asset allocation models with time-varying expected returns. Bajeux-Besnainou, Jordan and Portait (2001) explain the puzzle by assuming that the investor's horizon may exceed the maturity of the cash asset. Shalit and Yitzhaki (2003) instead use conditional stochastic dominance arguments to illustrate that advisors, acting as agents for numerous clients, recommend portfolios that are not inefficient for all risk averse investors.

In this paper we present a human capital explanation for the asset allocation puzzle identified by Canner, Mankiw and Weil (1997). To do so, we develop a life-cycle asset allocation model with intermediate consumption and stochastic uninsurable labor income in which households can invest in three financial assets: Treasury Bills, stocks and long-term government bonds. The model integrates two main motives that have been identified as quantitatively important in explaining individual and aggregate wealth accumulation. First, a precautionary savings motive in the presence of undiversifiable labor income risk generates asset accumulation to smooth unforeseen contingencies (Zeldes (1989), Deaton (1991) and Carroll (1997)). Second, pension income is lower than mean working-life labor income implying that saving for retirement becomes important at some point in the life cycle. The combination of precautionary and retirement saving motives has recently been shown to generate realistic wealth accumulation profiles over the life cycle.[1]

[1] See, for instance, Hubbard, Skinner and Zeldes (1995), Carroll (1997), Attanasio, Banks, Meghir and

In a model with undiversifiable labor income risk, the optimal asset allocation is determined by two elements. First, the presence of background risk decreases the investor's willingness to hold risky assets.[2] Second, since households cannot borrow against their future labor income, the degree of substitutability between human capital and all available financial assets also matters. In particular, households will tend to invest less in assets that are close substitutes to human capital, and as the ratio of financial wealth to future human capital increases (decreases) this effect becomes weaker (stronger). We show that both factors contribute to explain the asset allocation puzzle.

First, the more risk-averse investors care more about background risk and therefore they invest a smaller fraction of their portfolio in equities. Second, given the correlation structure between the different asset returns and individual earnings shocks, labor income is a closer substitute for long-term bonds than for stocks.[3] Therefore households that accumulate less financial wealth, thus for whom human capital is a larger fraction of their total wealth, will invest more in equities and less in "human capital substitutes". In the presence of uninsurable income risk, the more-risk averse households are the ones who accumulate more wealth over the life-cycle, and therefore invest a larger fraction of their wealth in (safer) labor income substitutes such as long-term bonds.

Our explanation is also consistent with another common feature of popular advice: younger households should be more heavily invested in equities than older households. In fact, both the asset allocation puzzle and the horizon effect are explained by the substitutability between long-term bonds and human capital. Specifically, non-traded human capital is a large component of young households' total assets, and to the extent that this asset is a closer substitute to long term bonds and Treasury bills, these households invest a larger fraction of their portfolios in equities. As they get older, the present value of future labor income is falling and they optimally decrease their equity holdings.

Within the power utility set-up, decreasing risk aversion (and prudence) implies a si-

Weber (1999), Gourinchas and Parker (2002) and Cagetti (2003).

[2] See, for example, Pratt and Zeckhauser (1987), Kimball (1993), or Gollier and Pratt (1996).

[3] This extends the results in the two-asset case, that find that human capital is a closer substitute for Treasury bills than for stocks (Heaton and Lucas (1997)).

multaneous increase of the elasticity of intertemporal substitution (EIS). Since wealth accumulation is determined by both risk aversion and EIS, it is not clear whether our results are actually driven by differences in former, rather than by differences in the latter. We therefore extend the model by considering Epstein-Zin preferences (Epstein and Zin (1989)) which allow us to separate the two, and find that the result remains valid, even when we change risk aversion while keeping the EIS constant.

The rest of the paper is organized as follows. Section 2 outlines the model and solution method, while the corresponding results for the baseline case are presented in Section 3. Section 4 discusses certain comparative statics experiments that illustrate the robustness of the conclusions while section 5 concludes.

2 The Model

2.1 Preferences

Time is discrete and t denotes adult age that corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of $T^* = 81$ periods (age 100). The probability that a consumer/investor is alive at time $(t+1)$ conditional on being alive at time t is denoted by p_t ($p_0 = 1$). Households have CRRA utility functions defined over one single non-durable consumption good,

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \tag{1}$$

where C_t is the consumption level at time t and ρ is the coefficient of relative risk aversion. With this preference specification, the elasticity of intertemporal substitution is given by $1/\rho$.

2.2 Labor Income Process

The labor income process before retirement is the same as the one used by Gourinchas and Parker (2002) and Cocco, Gomes and Maenhout (1999)[4] and it is given by

$$Y_{it} = P_{it} U_{it} \tag{2}$$

$$P_{it} = \exp(f(t, Z_{it})) P_{it-1} N_{it} \tag{3}$$

where $f(t, Z_{it})$ is a deterministic function of age and household characteristics Z_{it}, P_{it} is a permanent component, and U_{it} is a transitory component. We assume that $\ln U_{it}$, and $\ln N_{it}$ are independent and identically distributed with mean $\{-.5 * \sigma_u^2, -.5 * \sigma_n^2\}$,[5] and variances σ_u^2, and σ_n^2, respectively. The log of P_{it}, evolves as a random walk with a deterministic drift, $f(t, Z_{it})$.

For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period K, corresponding to age 65 ($K = 46$). Earnings in retirement ($t > K$) are given by $Y_{it} = \lambda P_{iK}$, where λ is the replacement ratio (a scalar between zero and one).

2.3 Financial Assets

The investment opportunity set is constant and there are three financial assets, one relatively riskless asset (treasury bills) and two risky assets: stocks and long-term (government) bonds. The return on Treasury bills is given by

$$R_{t+1}^T = R^f + \varepsilon_{t+1}^T$$

while the returns on risky assets (denoted by R_t^S and R_t^B) are given by

$$R_{t+1}^S - R^f = \mu^S + \varepsilon_{t+1}^S \tag{4}$$

and

$$R_{t+1}^B - R^f = \mu^B + \varepsilon_{t+1}^B \tag{5}$$

[4] Deaton (1991) and Carroll (1997) use a similar process.

[5] With this specification the mean of the level of the log random variables equals 1.

where $\varepsilon_t^T \sim N(0, \sigma^2_{\varepsilon^T})$, $\varepsilon_t^S \sim N(0, \sigma^2_{\varepsilon^S})$ and $\varepsilon_t^B \sim N(0, \sigma^2_{\varepsilon^B})$.

In the more general formulation we allow for correlation across asset returns, and between asset returns and earnings shocks. Campbell and Viceira (2001), Brennan and Xia (2002) and Campbell, Chan and Viceira (2003) have shown that time variation in expected returns can help to explain the Canner, Mankiw and Weil (1997) asset allocation puzzle. In our paper we explicitly consider a constant investment opportunity set, so that all results are driven by the combination of liquidity constraints and undiversifiable labor income risk.

2.4 Wealth accumulation

Following Deaton (1991) we denote cash on hand as the liquid resources available for consumption and saving. Next period's cash on hand ($X_{i,t+1}$) is given by

$$X_{i,t+1} = S_{it}R^S_{t+1} + B_{it}R^B_{t+1} + T_{it}R^T_{t+1} + Y_{i,t+1} \tag{6}$$

where S_{it}, B_{it} and T_{it} denote respectively stock holdings, holdings of long-term bonds and relatively riskless asset holdings (Treasury Bills) at time t. Since the household must allocate her cash-on-hand (X_{it}) between consumption expenditures (C_{it}) and savings we also have

$$X_{it} = C_{it} + S_{it} + B_{it} + T_{it} \tag{7}$$

Finally, we prevent households from borrowing against their future labor income. More specifically we impose the following restrictions:

$$T_{it} \geq 0 \tag{8}$$

$$B_{it} \geq 0 \tag{9}$$

$$S_{it} \geq 0 \tag{10}$$

2.5 The optimization problem and solution method

The complete optimization problem is then

$$\underset{\{S_{it},T_{it},B_{it}\}_{t=1}^{T^*}}{MAX} E_0 \sum_{t=1}^{T^*} \beta^{t-1} \left(\prod_{j=0}^{t-1} p_j \right) U(C_{it}) \tag{11}$$

where β is the time discount factor; subject to the constraints given by equations (4) to (10), and to the stochastic labor income process given by (2) and (3) if $t \leqslant K$, and $Y_{it} = \lambda P_{iK}$ if $t > K$.

Analytical solutions to this problem do not exist. We therefore use a numerical solution method based on the maximization of the value function to derive the optimal decision rules. The details are given in the Appendix, and here we just present the main idea. We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income (P_{it}). The laws of motion and the value function can then be rewritten in terms of the normalized variables, and we use lower case letters to denote them (for instance, $x_{it} \equiv \frac{X_{it}}{P_{it}}$). This allows us to reduce the number of state variables to two: age (t) and normalized cash-on-hand (x_{it}). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. Therefore, we can use this value function to compute the policy rules for the previous period and, given these, obtain the corresponding value function. This procedure is then iterated backwards. We optimize over the different choices using grid search. After solving for the optimal policy functions, we can simulate the model to replicate the behavior of a large number of households and compute the corresponding allocations.

2.6 Parameter Calibration

2.6.1 Preference parameters

We consider a large range of values for the coefficient of relative risk aversion (ρ): from 2 to 8. For most of the paper we will use a discount factor of $\beta = 0.95$ but the results are not sensitive to this choice. We use the mortality tables of the *National Center for Health Statistics* to parameterize the conditional survival probabilities.

2.6.2 Labor income process

Carroll (1997) estimates the standard deviation of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and our baseline values are close to his: 10 percent per year for σ_u and 8 percent per year for σ_n. The deterministic labor income profile reflects the hump shape of earnings over the life-cycle, and the corresponding parameter values, just like the retirement transfers (λ), are taken from Cocco, Gomes and Maenhout (1999). It is common practice to estimate different labor income profiles for different education groups (college graduates, high-school graduates, households without a high-school degree). We only report results obtained with the parameters estimated from the sub-sample of high-school graduates, as the results for the other two groups are very similar.

2.6.3 Asset returns

We use the identical distribution of returns used by Canner, Mankiw and Weil (1997, table 2) which are based on annual data between 1926 and 1992 from the CRSP data base.[6] The mean real return from holding Treasury Bills is 0.6 percent, and its standard deviation is 4.3%. Long-term government bonds have an expected return of 2.1% and a standard deviation of 10.1%, while stocks earn an average return equal to 9.0% with a standard deviation of 20.8%. The correlation between Treasury Bill and bond (stock) returns is 0.63 (0.09) while the correlation between bond and stock returns equals 0.23.

Davis and Willen (2001) estimate a correlation between stock returns and income shocks between zero and 0.3 for different subgroups of the population. We therefore use as baseline correlation between stocks and permanent earnings shocks a value of 0.15, but we also present results with this correlation equal to zero. Given that long term bonds and Treasury Bills are viewed as safer instruments than stocks, we set their correlations with permanent shocks equal to zero in the baseline case but also offer comparative statics results for positive values.

3 Baseline Case

3.1 Consumption and wealth accumulation

Figure 1 plots mean normalized consumption, mean normalized wealth and mean normalized income, for ρ equal to 5, and all other parameters set equal to their baseline values. Early

[6] Extending the sample to 2000 does not substantially change these numbers.

in life the household is liquidity constrained and saves only a small buffer stock of wealth, which serves as insurance against labor income uncertainty. From approximately ages 30 to 35 onwards, she starts saving for retirement and wealth accumulation increases significantly. During the retirement period consumption decreases at a fast pace, as a result of the very high effective discount rate (high mortality risk).

Tables 1 and 2 show, respectively, the mean consumption to wealth ratio and the mean wealth to income ratio for different values of the preference parameters and for different age groups. A higher ρ increases risk aversion (and prudence) and decreases the elasticity of intertemporal substitution.[7] Increasing risk aversion generates more wealth accumulation by increasing the demand for precautionary savings. The impact of a change in the elasticity of intertemporal substitution (EIS) depends on the difference between the investor's discount rate and the (endogenous, through the optimal asset allocation) rate of return on her invested wealth. If this difference is positive (low rate of return) then a lower EIS will increase retirement savings and vice-versa. Since this difference is not very large the EIS has a small impact on wealth accumulation, and as a result, the optimal consumption to wealth ratio is generally a decreasing function of ρ for almost all the relevant range that we consider.[8] In particular, the consumption to wealth ratio is decreasing for all values of ρ greater than 2, while for the age groups between 36 and 100 this monotonicity happens for all values of ρ that we consider.

3.2 Asset allocation

Figure 2 graphs the unconditional mean asset allocation in equities ($\overline{\alpha}_t^S$) and long-term bonds ($\overline{\alpha}_t^B$), for the same preference parameters as in figure 1 ($\rho = 5$).[9] In the two-asset case, even though earnings risk is uninsurable, future labor income crowds-out cash holdings rather than

[7]Later on we will consider Epstein-Zin preferences (Epstein and Zin (1989)) which allow us to separate these two effects.

[8]The difference between the discount rate and the return on invested wealth is a function of risk aversion itself, since the less risk-averse investors invest a larger fraction of their portfolio in stocks.

[9]Cash holdings are not plotted since they are just the residual of the other two, and because they are quite small.

stock holdings (see Heaton and Lucas (1997)). We show that this result remains unchanged when we add long-term bonds to the model. Moreover, just like cash, the long-term bond is also as a close substitute for labor income and therefore young households should invest almost all of their portfolios in equities, as in the life-cycle model with two-assets (Cocco, Gomes and Maenhout (1999)). Young households are "overinvested in their human capital" and view this non-tradeable asset as an implicit relatively riskless asset in their portfolio. Given that the holdings of this relatively riskless asset are larger in the early part of the life-cycle, they allocate most of their financial wealth to stocks.[10] As retirement approaches, and financial wealth increases relative to the present value of future labor income (the implicit relatively risk free asset), households start investing in labor income substitutes: long-term bonds and Treasury bills. When retirement savings is at its peak, almost 30% of total wealth is now being invested in long-term bonds and Treasury Bills.

Tables 3.1 through 3.3 quantify this intuition by computing the average optimal share invested in stocks (α^S), the average optimal share invested in long-term bonds (α^B), and the share of stocks in the portfolio of risky assets ($\alpha^S/(\alpha^S+\alpha^B)$) for different values of the coefficient of relative risk aversion.[11] Consistent with the previous intuition, as we increase ρ the portfolio of risky assets is tilted away from stocks (table 3.1). Early in life (ages $20-35$), the ratio of financial wealth to the present value of future labor income is extremely low, and consequently households invest heavily in stocks (more than 99% of their portfolio for all values of risk aversion). In the next age group ($36-65$) there is a monotonic decrease of the exposure in the stock market as risk aversion rises, the average exposure is 98.7% when $\rho = 2$ and decreases to 58.4% when $\rho = 8$, and the same qualitative pattern is also visible during retirement (last row of table 3.1).

Table 3.2 shows the optimal asset allocation to long-term bonds. For the youngest group, the share of wealth invested in stocks is close to 100% for almost all ρ generating very little changes in bond holdings as risk aversion rises. For the $36-65$ age group, however, the

[10] During the very first years of adult life households hold a small fraction of their wealth in cash since the present value of future labor income is actually still increasing.

[11] The average optimal share invested in Treasury Bills (α^T) is just the residual of the other two, and therefore we do not report it in the tables.

share of wealth in long-term bonds monotonically increases from 0.8% when $\rho = 2$ to 24.2% when $\rho = 8$.

During retirement both future labor income (the present value of the pension transfers) and wealth are falling, so that the optimal asset allocation is determined by the relative speed with which these two decrease. In our case wealth decreases at a faster rate and therefore the share of wealth invested in stocks slightly decreases with age.[12]

3.3 Explaining the Canner, Mankiw and Weil puzzle

Canner, Mankiw and Weil (1997) show that the recommendations of popular financial advisors, who suggest that more risk-averse investors should allocate a higher fraction of their overall risky portfolio (stocks plus bonds) to bonds, constitute a puzzle in the context of the CAPM model and mean-variance optimization. Indeed, the CAPM predicts that all investors should hold the same combination of risky assets, and that risk aversion should only determine the overall size of the investment in those assets, without affecting the composition (mutual fund separation theorem).

In a complete markets setting, the optimal asset allocation is exclusively determined by the risk aversion coefficient and the distribution of asset returns. As previously discussed, in the presence of undiversifiable human capital, two other factors become important: the background risk effect and the ratio of financial wealth to the present value of future labor income.

Both of these factors contribute to explain the Canner, Mankiw and Weil puzzle. First, more risk averse households accumulate more financial wealth over the life-cycle (see tables 1 and 2), and as a result they invest a larger fraction of this wealth in (safer) labor income substitutes such as long-term bonds (table 3.2). Second, since prudence and risk aversion are both determined by ρ, the more risk averse households are also the ones that care more about background risk, and this again leads to a smaller investment in stocks (as a share of total risky assets). The results in Table 3.3, for instance, illustrate that higher risk aversion generates consistently a monotonically negative relationship between risk aversion and the

[12] Except during the very last years, when most households have very little financial wealth left.

average share invested in stocks relative to total risky assets (stocks plus long-term bonds).

Finally, it is important to note that in our setting the human capital explanation is consistent with the investment horizon recommendation: young households invest a higher fraction of their portfolio in stocks than older households. In fact, the driving force is the same for both results: the substitutability between long-term bonds and human capital. Specifically, non-traded human capital is a large componet of young households' total assets, and therefore these households invest a larger fraction of their portfolios in equities. As they get older, the present value of future labor income is falling and they optimally decrease their equity holdings.

4 Extensions and Robustness Checks

4.1 Changing the Correlation Structure

4.1.1 Correlation Between Labor Income Shocks and Stock Returns

The empirical evidence on the magnitude of the correlation between individual labor income shocks and asset returns is relatively mixed. As a result, we want to confirm that our results are not sensitive to the levels of correlation assumed in the model. In the first case, reported in Tables 4.1 through 4.3, we now set the correlation coefficient between stock returns and permanent labor income shocks equal to zero. Decreasing this correlation makes equities even more attractive and increases the share of wealth invested in stocks (compare Table 4.1 with Table 3.1). Table 4.2 shows that the share of wealth invested in long-term bonds monotonically increases as risk aversion rises during working life, and also during retirement for most risk aversion parameters.

Finally, table 4.3 shows that the main result in the paper remains unchanged: the average share of wealth invested in stocks relative to stocks and long-term bonds decreases with risk aversion. Moreover, the horizon effect is still present: the optimal allocation to stocks (bonds) decreases (increases) with age as households compensate for the reduction in future labor income by holding more of its closest substitutes (long-term bonds and cash).

4.1.2 Correlation Between Labor Income Shocks and Treasury Bill Returns

We next increase the correlation between Treasury Bill returns and stock returns from zero (baseline) to 0.15. The change leaves the share of wealth invested in stocks virtually unchanged from the baseline case (compare table 5.1 with table 3.1). Nevertheless, the positive correlation between Treasury Bills and labor income shocks makes long-term bonds a more attractive asset relative to T-Bills and generates a substantial re-allocation of the portfolio from bills to bonds that is increasing in the risk aversion coefficient. Most importantly, our main results persist: the stock to bond ratio decreases with risk aversion, and the share of wealth in stocks monotonically decreases with age.

4.1.3 Correlation Between Labor Income Shocks and Long-Term Bond Returns

We next increase the correlation between long-term bond returns and stock returns from zero (baseline) to 0.15. The change slightly increases the share of wealth invested in stocks from the baseline case (compare Table 6.1 with Table 3.1). Nevertheless, the positive correlation between long bonds and labor income shocks makes long bonds a less attractive asset relative to T-Bills and generates a substantial re-allocation of the portfolio from bonds to bills when the reduction in stock market exposure begins to take place later in life. As before, the main results are unchanged.

4.2 Separating risk aversion from the elasticity of intertemporal substitution

Within the CRRA framework, decreasing risk aversion (and prudence) implies a simultaneous increase of the EIS. Therefore, it is not clear whether our previous results are actually driven by differences in risk aversion, or by differences in the elasticity of intertemporal substitution. To answer this question, we now assume Epstein-Zin preferences (Epstein and Zin (1989)) which allow us to separate the two. These preferences are given by the following recursion

$$V_t = \{(1-\beta p_t)C_t^{1-1/\psi} + \beta p_t E_t\left[[V_{t+1}^{1-\rho}]\right]^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \tag{12}$$

where ρ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and β is again the intertemporal discount factor. The previous preferences (power utility) correspond to the special case in which $\rho = 1/\psi$.

Table 7.1 shows the consumption to wealth ratio for different values of ρ and ψ.[13] For any value of ψ, increasing risk aversion generates more wealth accumulation by increasing the demand for precautionary savings. On the other hand, the impact of an increase in the elasticity of intertemporal substitution (EIS) depends on the difference between the investor's discount rate and the rate of return on her invested wealth. For high values of risk aversion the share of wealth invested in stocks is smaller and the expected return on wealth falls below the (motality-adjusted) discount rate. As a result, a lower EIS decreases savings, which is the pattern observed for ρ equal to 5 and 8. On the other hand, for $\rho = 2$ we have the reverse effect, as the rate of return from a more aggressive asset allocation now exceeds the discount rate and therefore a lower value of ψ actually increases savings.

In Table 7.2 we report the share of stocks in the portfolio of risky assets $(\alpha^S/(\alpha^S+\alpha^B))$.[14] As expected from the results in Table 7.1, we find that the more risk-averse investors hold a smaller fraction of their portfolio of risky assets in stocks, regardless of the value of the EIS that we are considering. Therefore, the main result of the paper remains valid even when we disentangle risk aversion from the elasticity of intertemporal substitution.

5 Conclusion

We show that the presence of undiversifiable human capital rationalizes the asset allocation puzzle identified by Canner, Mankiw and Weil (1997) even if expected returns are not time-varying. More risk-averse households accumulate more financial wealth and are more keen to compensate for the decrease in human capital (as retirement approaches) with a larger investment in relatively safe assets: Treasury bills and long-term bonds. As a result, more risk averse households at similar points in the life cycle generally hold a smaller proportion

[13] To keep the number of tables small we now only report life-cycle averages.

[14] Again we only report life-cycle averages and we have also omitted the individual shares, so as to keep the number of tables small.

of their portfolio in equities. The same mechanism generates a life-cycle investment pattern that is consistent with the standard professional investment advice: young households should invest a higher fraction of their wealth in stocks.

Appendix: Numerical Solution Method

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income (P_{it}). The laws of motion and the value function can then be rewritten in terms of these normalized variables, and we use lower case letters to denote them (for instance, $x_{it} \equiv \frac{X_{it}}{P_{it}}$). This allows us to reduce the number of state variables to two: one continuous state variable (cash on hand, x_{it}) and one discrete state variable (age, t). We discretize the state-space along the cash-on-hand dimension (the only continuous state variable), so that the relevant policy functions can now be represented on a numerical grid.

We solve the model using backward induction. In the last period ($t = T$) the policy functions are trivial, as the agent consumes all available wealth, $c_T = x_T$. As a result the value function corresponds to the indirect utility function, $V_T(x_T, .) = V(x_T)$. For every age t prior to T, and for each point in the state space, we optimize using grid search. So we need to compute the value associated with each level of consumption, and the share of liquid wealth invested in both stocks and long-term bonds. From the Bellman equation these values are given as current utility plus the discounted expected continuation value ($E_t V_{t+1}(.,.)$), which we can compute since we have just obtained V_{t+1}. We evaluate the value function, for points which do not lie on state space grid, using a cubic spline interpolation.

We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the risky asset returns. To compute the joint distribution of M correlated random variables, we use the Cholesky decomposition of the M by M variance-covariance matrix to rotate the quadrature points, keeping the weights (probabilities) the same. A clear exposition of this technique can be found in Burnside (1999, p. 104)

Once we have computed the value of all the alternatives we just pick the maximum, thus obtaining the policy rules for the current period (S_t, B_t and T_t). Substituting these decision rules in the Bellman equation we obtain this period's value function ($V_t(.,.)$), which is then used to solve the previous period's maximization problem. This process is iterated until $t = 1$.

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Table 1: Average Consumption-Wealth Ratio (C/X) for different values of the coefficient of relative risk aversion (ρ) and different age groups in percentage terms.

	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$
$20 - 35$	75.0	77.1	75.4	71.5	66.3	55.5	52.7
$36 - 65$	25.5	25.1	23.0	20.1	17.5	14.4	12.8
$66 - 100$	48.4	38.8	32.0	27.9	25.2	23.6	22.0

Notes to Table 1: X is the sum of accumulated financial wealth and labor income. The baseline parameter values are $\sigma_u = 0.1$, $\sigma_n = 0.08$, and $\beta = 0.95$. The mean real return from holding Treasury Bills is 0.6% (standard deviation 4.3%). Long-term government bonds have $\mu^B = 2.1\%$ and $\sigma_{\varepsilon^B} = 10.1\%$ while stocks earn an average μ^S equal to 9.0 percent and a standard deviation (σ_{ε^S}) of 20.8 percent. The correlation between Treasury Bill and bond (stock) returns is 0.63 (0.09) while the correlation between bond and stock returns equals 0.23. The correlation of stock returns with permanent individual labor income shocks is 0.15 and the correlation of permanent labor income shocks and long bond/T-Bill returns is zero.

Table 2: Average Wealth-Income Ratio (X/Y) for different values of the coefficient of relative risk aversion (ρ) and different age groups in percentage terms.

	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$
$20 - 35$	35.2	30.0	33.5	41.9	55.0	88.1	100.2
$36 - 65$	404.9	422.5	481.8	570.2	663.0	829.6	890.9
$66 - 100$	416.2	546.1	710.3	868.2	1005.0	1078.8	1242.6

Notes to Table 2: See Table 1 notes. Y denotes labor income.

Table 3.1: Average share of wealth invested in stocks (α^S) for different values of the coefficient of relative risk aversion (ρ) and different age groups. All values are in percentage terms.

	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$
$20 - 35$	100.0	100.0	100.0	100.0	100.0	99.8	99.1
$36 - 65$	98.7	97.2	93.9	87.2	79.0	67.4	58.4
$66 - 100$	98.0	93.7	84.4	74.1	64.7	55.5	50.0

Table 3.2: Average share invested in long-term bonds (α^B) for different values of the coefficient of relative risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$
$20 - 35$	0.0	0.0	0.0	0.0	0.0	0.18	0.90
$36 - 65$	0.8	1.5	4.0	9.1	14.7	21.0	24.2
$66 - 100$	1.6	4.9	12.2	18.9	21.7	19.9	17.7

Table 3.3: Average share invested stocks relative to total risky asset holdings ($\alpha^S/(\alpha^{LB} + \alpha^S)$) for different values of the coefficient of relative risk aversion (ρ) and different age groups. All values are in percentage terms.

	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$
$20 - 35$	100.0	100.0	100.0	100.0	100.0	99.8	99.1
$36 - 65$	99.2	98.5	95.9	90.6	84.3	76.3	70.7
$66 - 100$	98.4	95.1	87.3	79.7	74.9	73.6	75.6

Notes to Table 3: See Table 1 notes.

Table 4.1: Average share invested in stocks (α^S) for different values of the coefficient of relative risk aversion (ρ) and different age groups. Results for the case with zero correlation between stock returns and permanent labor income shocks.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100	100	99.9
$36 - 65$	98.6	89.8	67.4
$66 - 100$	98.1	74.1	49.7

Table 4.2: Average share invested in long-term bonds (α^B) for different values of the coefficient of relative risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	0.0	0.0	0.1
$36 - 65$	0.9	6.6	16.6
$66 - 100$	1.5	18.9	17.5

Table 4.3: Average share invested stocks relative to total risky asset holdings ($\alpha^S/(\alpha^{LB} + \alpha^S)$), for different values of the coefficient of risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100	100	99.9
$36 - 65$	99.1	93.0	79.5
$66 - 100$	98.5	79.7	74.0

Notes to Table 4: See notes to table 1 for parameter assumptions. The comparative statics in Tables 4.1-4.3 are for the case with zero correlation between stock returns and permanent labor income shocks. All numbers are reported in percentage terms.

Table 5.1: Average share invested in stocks (α^S) for different values of the coefficient of relative risk aversion (ρ) and different age groups. Results for the case with correlation between Treasury Bill returns and permanent labor income shocks equals 0.15.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100	100	99.1
$36 - 65$	98.7	87.2	58.6
$66 - 100$	98.0	74.1	49.6

Table 5.2: Average share invested in long-term bonds (α^B) for different values of the coefficient of risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	0.0	0.0	0.89
$36 - 65$	0.7	9.2	27.3
$66 - 100$	1.6	18.9	17.5

Table 5.3: Average share invested stocks relative to total risky asset holdings ($\alpha^S/(\alpha^{LB} + \alpha^S)$) for different values of the coefficient of risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100.0	100.0	99.1
$36 - 65$	99.3	90.2	67.5
$66 - 100$	98.4	79.7	74.0

Notes to Table 5: See notes to table 1 for parameter assumptions. The comparative statics in Tables 5.1-5.3 are for the case where the correlation between Treasury Bill returns and permanent labor income shocks equals 0.15. All numbers are reported in percentage terms.

Table 6.1: Average share invested in stocks (α^S) for different values of the coefficient of risk aversion (ρ) and different age groups. Results for the case where the correlation between long bond returns and permanent labor income shocks equals 0.15.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100.0	100.0	99.9
$36 - 65$	98.6	88.2	61.0
$66 - 100$	98.0	73.9	49.9

Table 6.2: Average share invested in long-term bonds (α^{LB}) for different values of the coefficient of risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	0.0	0.0	0.2
$36 - 65$	1.0	7.1	3.5
$66 - 100$	1.6	19.0	17.8

Table 6.3: Average share invested stocks relative to total risky asset holdings ($\alpha^S/(\alpha^{LB} + \alpha^S)$), for different values of the coefficient of risk aversion (ρ) and different age groups.

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$20 - 35$	100.0	100.0	99.8
$36 - 65$	99.0	94.6	92.6
$66 - 100$	98.4	79.5	73.7

Notes to Table 6: See notes to table 1 for baseline parameter assumptions. The comparative statics in Tables 6.1-6.3 are for the case where the correlation between long bond returns and permanent labor income shocks equals 0.15. All numbers are reported in percentage terms.

Table 7.1: Average Consumption-Wealth Ratio (C/X) for different values of the coefficient of risk aversion (ρ) and different values of the elasticity of intertemporal substitution (ψ).

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$\psi = 0.5$	44.7	37.0	35.8
$\psi = 0.2$	63.2	33.2	26.9
$\psi = 0.125$	69.6	32.6	24.3

Table 7.2: Average share invested stocks relative to total risky asset holdings ($\alpha^S/(\alpha^{LB} + \alpha^S)$) for different values of the coefficient of risk aversion (ρ) and different values of the elasticity of intertemporal substitution (ψ).

	$\rho = 2$	$\rho = 5$	$\rho = 8$
$\psi = 0.5$	99.3	99.2	81.6
$\psi = 0.2$	99.9	87.8	78.4
$\psi = 0.125$	99.9	87.5	78.6

Notes to Table 7: See notes to table 1 for baseline parameter assumptions. All numbers are reported in percentage terms. The diagonal elements denote the CRRA parameter specification reported in tables 1-3.

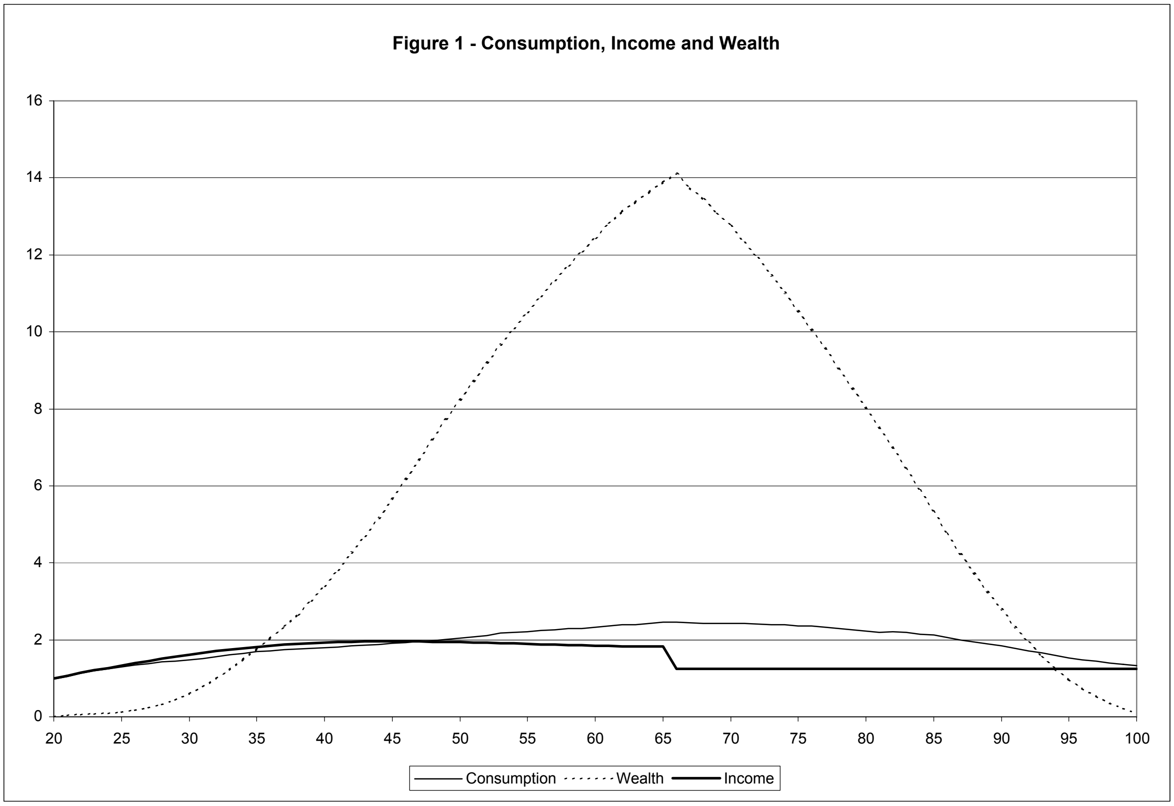
Figure 1 - Consumption, Income and Wealth
16
14
12
10
8
6
4
2
0
20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Consumption
Wealth
Income

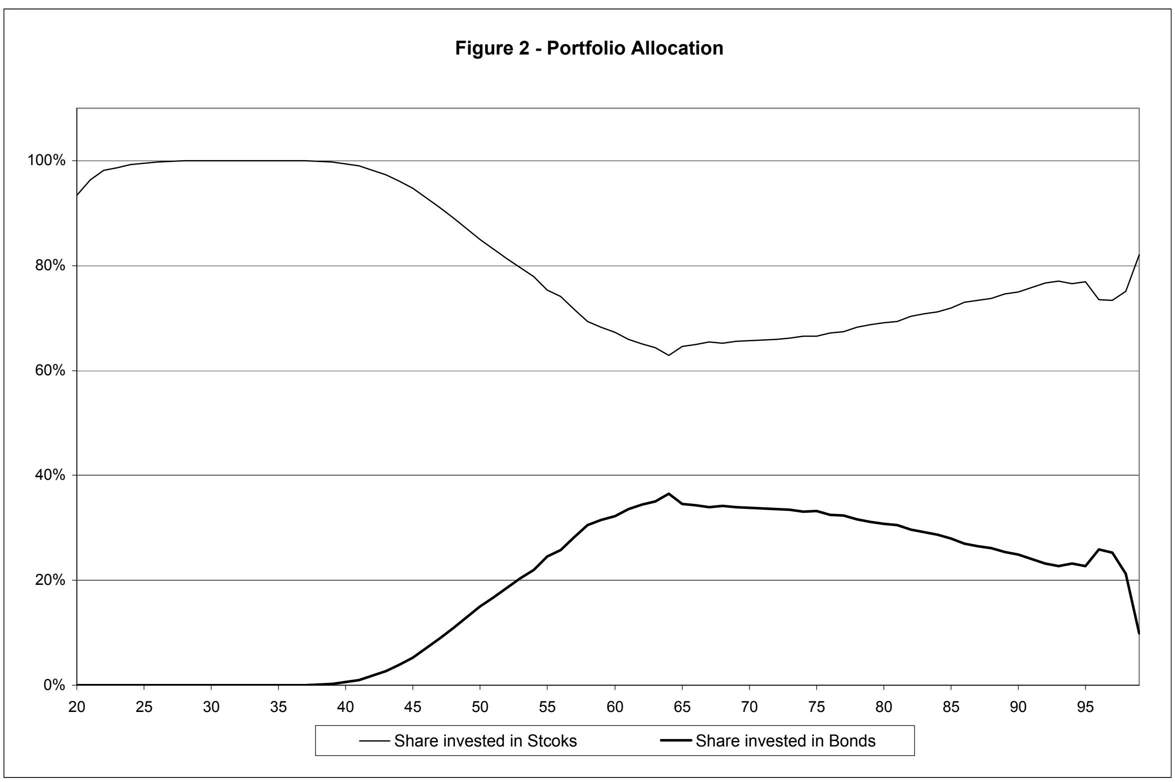
Figure 2 - Portfolio Allocation
100%
80%
60%
40%
20%
0%
20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95
Share invested in Stcoks
Share invested in Bonds