A Theory of Sovereign Debt Roll-over Crisis*

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Abstract

In this paper, we attempt to provide theoretical investigation to debt roll-over crisis in government bond market. By using the global game techniques, we analyse coordination problem in debt auction. The approach in this paper allows us to have insight on the relation between occurrence of sovereign debt roll-over crisis and the fundamentals of economy which is not clearly explained in preceding works. This paper also makes it clear how the amount of debt for refinancing affects occurrence of sovereign debt roll-over crisis. Implications to policymakers are that they should pay attention to possible serious consequence of debt-bunching which usually grows gradually over time. In public debt management, the government decisions today affect the situation facing the government in the future via change in redemption schedules. Although the cost of debt roll-over crisis is not easy to gauge beforehand, its possibility should be noted by policymakers.

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1 Introduction

Public debt management is a serious policy issue for policymakers. The definition of public debt management could vary but, as Goodhart (1999) quotes, one of clear definitions can be found in *The Report of the Debt Management Review 1995* published by Her Majesty's Treasury of the U.K. and the Bank of England;

"The objective of debt management policy is to minimise over the long term the cost of meeting the government's financing needs, taking account of risk, whilst ensuring that debt management policy is consistent with monetary policy."

Although some factors are of interest for public debt management, choice of debt maturity is always a policy debate. Also in academic literatures on public debt management, it has been given much attention. Some of previous works on optimal debt maturity in public debt management are Lucas and Stokey (1983), Alesina, Prati and Tabellini (1990), Missale and Blanchard (1994), Falcetti and Missale (2002) and Missale, Giavazzi and Benigno (2002).

In relation to optimal debt maturity, there is another literature analysing the issue of "confidence crisis" in the government bond markets. The seminal paper in this research area is Calvo (1988). The paper analyses a situation in which a government have a certain amount of debt to roll over and it can repudiate some portion of the debt in real terms directly by declaring partial default or, more realistically, by inflation. The analysis in the paper shows the possible mutiple-equilibria due to self-fulfilling expectation. A realised equilibrium can be a pareto-inferior one with high rate of repudiation (inflation) or a pareto-superior one with low rate of repudiation (inflation). It depends on the expectation on the side of bond investors which of those equilibria actually realises. The result in the paper deepens our understanding of the dynamics in the government bond markets. However, implications to public debt management is not clearly elaborated upon in the paper. Alesina *et al.* (1990) analyses the issue of confidence crisis *a la* Calvo and relates the issue to public debt management. Alesina *et al.* (1990) shows that choice of debt maturity is important in public debt management because it changes the amount of redemption facing a government each fiscal year. The more amount of redemption is in one fiscal year, the more likely a confidence crisis, stop of roll-over of debt by investors, will emerge. The paper clearly shows the idea that a government should spread the amount of redemption over years by carefully designing redemption schedule to avoid occurrence of confidence crisis. Relying on short-term debt for finance is risky in this sense, though it usually contributes to lower cost of servicing debt.

Calvo (1988) and Alesina *et al.* (1990) provoke convincingly the risk of confidence crisis in government bond market. Its implication to the public debt management is rich. However, they leave room for analysis to obtain more solid implications to policy debates. In both papers, the range of parameter values, which makes an economy vulnerable to possible confidence crisis, is shown but the relation between fundamentals of an economy and occurrence of confidence crisis in it is not clearly pinned down. Both papers show that a confidence crisis can occur if all of investors think it will. But a investor's belief about other investors' belief is unsatisfactorily described so that the analysis on the motivation for each investor to purchase government debt can not be developed. This in turn leads to a vague relation between fundamentals of an economy and occurrence of confidence crisis.

By employing the global game techniques introduced by Carlsson and van Damme (1993) and refined by Morris and Shin (1998, 2001, 2002), we will revisit confidence crisis issue and provide more theoretical back ground. The most related analysis is one in Morris and Shin (1998) about currency attack regarding theoretical framework. In this theoretical framework, currency attack by a speculator in Morris and Shin (1998) is equivalent to an investor's action not to purchase government debt in the model in this paper. Coordination problem among investors in our model emerges due to market microstructure in government bond auction, which is described later.

Moreover, we will analyse the relation between the amount of government debt for refinancing and occurrence of confidence crisis. Given certain strength of fundamentals, it is intuitive that confidence crisis is more likely to occur if the government faces heavy refinancing pressure. The results of our analysis give justification to this intuition. Based on those results, we will discuss implications for policymakers in public debt management.

The organisation of this paper is as follows. In Section 2, we introduce the model in which payoff function for participants in government bond auction and the loss function of government are specified. In Section 3, equilibrium in perfect information game is derived and we discuss its unsatisfactory features. In Section 4, imperfect information game is analysed and a sufficient condition for the existence of unique equilibrium in the game is derived. Based on the results in Section 4, we discuss the effect of change in the amount of debt for refinancing to the unique equilibrium in Section 5. In Section 6, we elaborate upon policy implications of the results in preceding sections. We have concluding remarks in Section 7.

2 The Model

The government in this paper has a certain amount of debt which has been incurred in the past. The debt is owned by the public in the form of government bond. In a certain fiscal year, a certain amount of government bonds, b, is to be redeemed. b is a portion of total government debt. Being given the loss function for the government which is specified later, the best choice for the government is to roll over the whole amount of maturing bonds, b. We analyse the outcome in the government bond auction for this roll-over.

2.1 Market Microstructure and Specialist's payoff

2.1.1 Market Microstructure of the Government Bond Auction

In the preceding works like Calvo (1988) and Alesina *et al.* (1990), an occurrence of confidence crisis is due to a sudden coordinated change in investors' expectation about government action. However, it is not explained how such a sudden change is triggered. In addition, it is not clearly explained how fundamentals of economy and debt level matter for its occurrence. In other words, mapping from the formers to the latter event is left vague.

Recognising those unsatisfactory treatment of important coordination issue, we will explore its cause by looking at market microstructure of government bond auction. By doing so, we can build a model with more realistic features. Thereby, we can derive results with clearer implications for policymakers.

In preceding works mentioned above, investors are agents who participate in government bond auction and hold government bonds as far as its return is not less than alternative investment opportunity. This setup is authentic and clear but a modification to this will make our model more appropriate for analysing coordination problem in government bond auction. The market microstructure set in our model makes the payoff of each investor dependent on the action of other investors. This gives rise to coordination motivation among investors.

The specification of market microstructure of government bond auction is as follows. While *investor* in preceding works refers to whole buy-side of government bond, we divide the buy-side of the government bond into two groups in this paper. One is *final investor* and another is *specialist*. *Final investor* is an agent who holds government bond eventually. Final investors are at the same time a player in government bond auction in previous works. In our model, we have a different agent acting as a player in government bond auction. It is *specialist* who buys government bond in auction and sell it to final investors.

Final investors buy the government bond as long as the government does not declare default. On the other hand, specialists pay attention to their payoff from their activity. Division between specialist and final investor is a novel set-up in literature about confidence crisis and, as we will see in details later, the payoff for specialist and government actions in response to auction result yield coordination problem in government bond auction.

The results of auction affect the government's action. In other words, the government will decide what to do as their best response to auction results. This set-up in our model below is realistic especially for countries who have strong worry on a confidence crisis in government bond markets. In heavily indebted countries, the results of auction may lead the government to default. Suppose auction results in a complete failure and the government can not finance from bond markets for redemption in a fiscal year. It needs to raise money for redemption by resorting to taxation or other external resource such as international organisations. If the government thinks the cost of raising finance by those means are extremely high or it is impossible, it would be rational for them to declare default. We will analyse the situation like this in the model below.

Before we move on to derive equilibrium in a game of perfect information and imperfect information, we specify the specialist's payoff, the loss function of government and the timing of the game.

2.1.2 Specialist's Payoff

There is continuum of specialists and the measure of specialists is normalised to one. In addition, we assume that each specialist decides either to buy b of government bond or not[1]. Therefore, if all the specialists decide to buy, the total amount bought by them is b.

Specialists gain profit from business margin accompanying their activity. The business margin for a specialist from dealing with one unit of government bond is expressed by m. We assume m is constant unless the fundamentals of the economy is extremely weak and there is no demand for the government bonds issued by the government[2]. We will elaborate upon this below. In addition, we assume $m < 1$. This assumption means that the business margin

[1] The setup that actions of a specialist are either to purchase b of government bond or not is consistent with the equilibrium concept in this paper. We will derive the range of fundamentals divided into two ranges. In one range above a threshold, the government does not declare default and all specialists decide to purchase government bonds as much as they can. This is because the profit for them is proportional to the amount which they deal. Since specialists are identical, it is reasonable to think they will end up purchasing the same amount. In another range under the threshold, the government declares default and no specialists will purchase the government bond.

[2] The assumption of constant business margin for specialist activity is for simplicity. However, the business margin will vary depending on various factors. One factor will be demand for government bonds in a country. Apparently, change in demand for government bonds in a country in different phase of business cycles is not reflected in this assumption. It is usual that the demand for government bonds is higher in a downturn phase in business cycles than in a recovering phase. This cyclical fluctuations and other short-run movements such as investors' reaction to announcement of economic statistics are not considered in our setups. Ignoring these aspects in this paper is for simplification which enables us to focus the relation among fundamentals of economy, amount of debt to be refinanced and occurrence of confidence crisis in government bond markets. By virtue of the simplification, our analysis in this paper sheds some light on the relation which was left unclear.

for specialist activity is not unrealistically so big as 100%.

We express a state of fundamentals characterising the economy by a parameter θ. The larger θ is the stronger the fundamentals are. In the context of our analysis about government bond roll-over crisis, the fundamentals mean capability of a government to collect tax revenue. This will be described in detail when we discuss the form of loss function for a government in the next subsection.

We define a certain extremely low value of θ for our interest. We assume there is a certain θ at which the value of m drops into negative and m is negative below the certain value of θ. We express this specific θ as θ_{-}. The intuitive interpretation of θ_{-} is that people do not demand government bond issued by a country of extremely weak fundamentals. Some plausible reasons for almost evaporation of demand for government bonds in a country of extremely weak fundamentals can be elaborated upon. In such an economy, capital-flight may reduce financial resources for domestic investment opportunity including government bonds. It is also probable that domestic saving is not large in such a country. As for international investors, they would face internal position restrictions based on sovereign ratings. Even without such restrictions, portfolio managers will hesitate to include the government bonds issued by such a country in their funds because its justification is difficult in describing to their clients. If there is no demand from final investors, specialists can not yield any revenue from specialist activity. Some cost for their activity turns the profit into negative in such a case.

For the range of θ below θ_{-} and itself, all of specialists will withdraw from their business giving them negative payoff. Therefore, each specialist will not make a bid in government bond auction irrespective of actions of other specialists.

It is possibly arguable that we are not sure if we have observed an economy characterised by such weak fundamentals as θ_-. But this is consistent with our analysis in this paper. Actually, we will derive a result that confidence crisis will be observed before fundamentals deteriorate to the level of θ_-. As we will see later, the important thing is that each specialist faces uncertainty about other specialists' belief about fundamentals. In other words, they can never be totally convinced that no other specialist thinks that fundamentals are as weak as θ_-.

It is also imaginable that a specialist would determine its own action by taking its effect on the result of government bond auction. In other words, a specialist may behave as a large player in the market who decides auction results. Although this possibility is important, this is not realistic. Specialists in an auction market for bond issued by a heavily indebted government will not boldly bid huge amount to enable the government to roll over its debt. Therefore, we stick to a usual assumption that each specialist is small in the sense it does not affect auction result decisively.

Having the setups described above we can specify the payoff of a specialist in the diagram below.

		Government's Action	
		Not Default	Default
Specialist's Action	Buy	mb	$-b$
	Not Buy	0	0

2.2 The Loss Function of the Government

The objective of a benevolent government in Calvo (1988) and Alesina *et al.* (1990) is maximising *per capita* consumption level. The calculations for determining the government's action in them are somewhat complex to yield corresponding *per capita* consumption level. But the government's strategies in Calvo (1988) and Alesina *et al.* (1990) are essentially attributable to comparison between two cost factors. They are, namely, the cost of distortionary taxation to repay debt not rolled over and the one of default or repudiation of debt. We also keep this critical feature in this paper. But, to make the discussion simple and thereby as illuminating as possible about the main analysis on how deterioration in fundamentals and increase in government debt for refinancing trigger a confidence crisis, we will set a very simple loss function for the government. The government's loss function has the form below.

$$\begin{aligned} L &= L(\tau, \theta, D) \\ &= L(b, \ell, \theta, D) \\ &= f(\tau, \theta) + D \\ &= f(b, \ell, \theta) + D \end{aligned}$$

Here we denote the ratio of specialists among all not purchasing government bond in auction by ℓ. Obviously, $0 \leq \ell \leq 1$.

Following the preceding works, the first cost term, $f(\cdot)$ is cost accompanying taxation. The more tax revenue the government raises for redemption the more costly it is in its loss function. We may interpret this term as the degree of difficulty to finance for repaying debt in any specific situation. In this sense, the cost is a general idea including not only loss due to distortionary taxation but also other difficulties facing the government such as political tension due to taxation. For convenience, we call it taxation cost in this paper.

The taxation cost term is a function of the amount of finance to be raised, τ, and fundamentals, θ. In the term, we have $\tau = b\ell$. That is, the amount of the government bond to be rolled over times the ratio of specialists not buying the government bond in auction. If all of b is rolled over, i.e., $\ell = 0$, we have $\tau = 0$. Therefore, $f = f(0, \theta)$. If some specialist do not purchase the government bond, we have $\ell > 0$. Then, the cost of taxation is $f = f(\tau, \theta) = f(b\ell, \theta) = f(b, \ell, \theta)$ as far as the government chooses to repay $b\ell$ rather than declare default. In this way, the value of $f(\cdot)$ depends on ℓ and b.

We set the following realistic assumptions about the effect of change in τ and θ on the first term in the government's loss function.

$$f(\tau, \theta) \geq 0,\ f_\tau(\tau, \theta) > 0,\ f_\theta(\tau, \theta) < 0$$

$$f(0, \theta) = 0,\ f(\infty, \theta) = \infty$$

$$f(\tau, -\infty) = f_\tau(\tau, -\infty) = \infty \text{ and } f(\tau, \infty) = f_\theta(\tau, \infty) = 0$$

Hereafter, a function with a subscript denote partial derivative of the function with respect to a variable in the subscript.

As for the second term in the government's loss function, this term means that declaring default by the government is costly for the government. And its cost is a fixed value regardless of the amount of defaulted debt. The cost of default $D = c$ if the government declares default and $D = 0$ otherwise.

A peculiarity in this loss function is the lump-sum nature of cost of default. This is the same specification as in Alesina *et al.* (1990) and this feature allows our analysis to focus on the main aim in this paper.

Having this loss function, the best for the government is that it can roll over all of its debt coming due in a fiscal year in the government bond market. If it can do so, the government does incur neither the cost of taxation for repayment nor that of default. That is $L(\cdot) = 0$. But, if a confidence crisis occurs in the government bond market, the government has to decide on whether to choose taxation or declare default. Whatever action it chooses, the loss is positive; $L(\cdot) > 0$. Obviously, equilibrium with full roll-over is pareto-superior to one with undersubscription in auction.

For discussion in later sections we define a certain value of fundamentals, θ^-, for a certain value of b. We define θ^- is the minimum value of θ which hold $f(b, 1, \theta) < c$. The intuition of having θ^- is that very strong fundamentals make it so easy for the government to raise tax revenue for redemption even if all of debt, b, needs to be financed by taxation. For the range of θ larger than θ^- and θ^-, the government will never declare default.

In relation with another specific value of θ defined earlier, θ_-, it is plausible to assume $\theta_- < \theta^-$ for any given value of b because θ_- expresses extremely weak fundamentals. This implies the following as well.

$$f(b, 1, \theta_-) \geq c$$

To avoid unnecessary complexity, we assume that a government will choose default if its cost is equal to the cost of taxation. Thus, with $\theta = \theta_-$, a government surely chooses to declare default if no specialists purchase the government bond in auction.

2.3 The Timing of the Game

We consider the following timing of the game. In a certain fiscal year the government needs to repay a certain amount of debt by the end of the fiscal year. At the beginning of the year the government tries to finance for repayment. It has only one chance to do auction for rolling over the debt. In this budgetary accounting framework, the timing of events is described as follows.

1. At the beginning of a fiscal year, the government announces the amount of government bond for sale. This amount is the same as one to be

rolled over, b. We assume its current deficit is zero for simplicity.

2. Each specialist decides whether to buy the amount b of the government bond or not.

3. The government knows how much bonds are bought by specialists in auction. If the government can not sell a portion of b, it needs to decide either to raise tax revenue for repaying its debt, $b\ell$, or to declare default. The choice of action by the government is determined as the result of minimization of its loss function. That is, it compares the cost of taxation and that of declaring default.

4. Each specialist tries to find final investors buying the government bond. If he can sell the government bond to final investors, the specialist obtains profits from its business activity. But if he can not find final investors buying the government bond, he will bear loss. Final investors decide if they buy the government bond from specialists or not. Final investors buy the government bond as long as the government does not declare default. Therefore, the possible loss incurred by a specialist is b in the case of default.

For setting-up the game in this manner, we have an important assumption on the effect of default which is realistic. When the government declares default, not only the government bonds issued in the past but also ones newly issued in the fiscal year are not paid back. This assumption can be justified by a practice in reality which is called *cross-default.* This cross-default assumption is important because this allows us to set up a model without need for intertemporal consideration. Contrary to the setups in this paper, investors' strategies involve intertemporal consideration in Calvo (1988) and Alesina *et al.* (1990). Namely, investors worry if the government bond will be redeemed or not in future fiscal years which will be affected by investors in the future years. However, this seems to be counter-intuitive when we observe the practices in reality. When investors worry about the possibility of default of government bond, they take it into account if the government has enough finance for the coming quarter or half-year. That is, their worry is about the state of the government in the quite near future and how urgently in the near future the government needs finance for repaying debt depends how successful the roll-over. Based on this observation we set up the game as one shot game[3].

3 Perfect Information Game

We first derive the results in perfect information game. In this game the value of θ is perfectly know to all specialists. This means that θ is common knowledge to them.

In such a game, there are uncountable equilibria and there are two notable stable equilibria. One is that all specialists buy the government bond in auction and the fundamentals are as strong as expressed by θ^-. Another is that all specialists do not buy the government bond in auction and the fundamentals are as weak as expressed by θ_-.

If θ is equal to or larger than θ^-, the government will never declare default even in the situation where they can not sell any bond to roll over its debt in auction. For $\theta \geq \theta^-$, cost of taxation for repaying all maturing bonds is less than that of default. Acknowledging this fact, all specialists will buy the government bond in auction, irrespective of actions of other specialists. Therefore, auction will never be undersubscribed.

[3]Although the author thinks the static setups are realistic and allow us to focus the main aim of our work here, extension with dynamic setups is referred as a possible direction of future research in the concluding remarks of this paper.

If θ is equal to or less than θ_-, the business margin for buying the government bond is negative. All specialists will not buy the government bond in auction, irrespective of actions of other specialists. Because $\theta \leq \theta_- < \theta^-$, the government will declare default in this case. The action of specialist is rational in light of government's response to the result of auction.

In this perfect information game, any combination of $\ell = 0$ and θ in the range of $\theta \geq \theta^-$ and any combination of $\ell = 1$ and θ in the range of $\theta \leq \theta_-$ are equilibrium. In addition, for any θ in the range of $\theta_- < \theta < \theta^-$, either $\ell = 0$ or $\ell = 1$ can be in equilibrium. We can not say anything about the realisation of a specific equilibrium. We can not even discuss the probability of realisation of each equilibrium.

4 Imperfect Information Game

4.1 Information Structure

In imperfect information game in this paper, uncertainty is both in realisation of fundamentals and in signal which each specialist observes about the realised fundamentals. This realistic information structure converts the perfect information game above into the global game introduced by Carlsson and van Damme (1993) and refined by Morris and Shin (1998, 2001, 2002). Using this approach, we can analyse an issue bearing coordination problem which is inherent in the government bond markets. With regard to solution procedures we employ ones in Morris and Shin (2001).

In the global game setups, the timing of the game summarised in Subsection 2.3 needs to be revised slightly. First of all in the timing of game, nature chooses θ from normal distribution with mean y, and precision α, that is, variance $1/\alpha$. We can interpret higher y as ex-ante stronger fundamentals.

Then each specialist observes a noisy signal about the realised fundamentals. Specialist i receives his signal

$$x_i = \theta + \varepsilon_i$$

where ε_i is normally distributed with mean 0 and precision β, that is, variance $1/\beta$. For $i \neq j$, ε_i and ε_j are independent.

Receiving the specific signal to him, specialist i's posterior distribution of θ is normal distribution with mean

$$\xi_i \equiv \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

and precision $\alpha + \beta$, that is, variance $1/(\alpha + \beta)$.

The realisation of those information structures is to be added before the timing of the game in Subsection 2.3. By doing so, we have the global game setups for our analysis and we will explore the characteristics of the equilibrium.

4.2 Switching Strategy and Specialists' Payoff

Suppose every specialist buys the government bond in auction if and only if his posterior belief about θ is higher than a certain value ξ. We prove the existence of unique equilibrium when every specialist employs this switching strategy. In the discussion below, we consider the expected payoff of buying the government bond in auction when a specialist's posterior belief is exactly equal to ξ. We denote the expected-payoff as $U(\xi)$.

As in Morris and Shin (2001), we will derive the following lemmas.

Lemma 1 *If ξ solves $U(\xi) = 0$, then there is an equilibrium in which every specialist employs the switching strategy around ξ. If there is a unique ξ which solves $U(\xi) = 0$, then there is no other switching equilibrium.*

Lemma 2 *$U_\xi(\xi) > 0$ for all ξ if and only if $\frac{\alpha}{\sqrt{\beta}}$ is sufficiently small to keep $\frac{\alpha}{\sqrt{\beta}} < -\sqrt{2\pi}\ \frac{f_\psi}{f_\ell}$ everywhere on the support of the function $U(\bullet)$ and $f(\bullet)$.*

Recent global game literatures provide us with the logic behind Lemma 1. In this paper, we prove the existence of unique ξ in switching strategies solving $U(\xi) = 0$. Actually, the results in the literatures allow us further to claim that the unique switching equilibrium around ξ is the only equilibrium which survives the iterated deletion of dominated strategies. The detailed proof is provided by Morris and Shin (1998, 2001, 2002). In this paper, application of the results is aimed so that we avoid the repetition of the proof.

In the following sections, we first prove Lemma 2 and solve out the unique equilibrium with the switching strategy. Then, we will do comparative statics with respect to the unique equilibrium. The comparative statics will yield some important implications to public debt management.

Following the switching strategy around ξ, specialists buy the government bond if and only if their private signal x is greater than

$$x(\xi, y) \equiv \frac{\alpha + \beta}{\beta}\xi - \frac{\alpha}{\beta}y$$

Conditional on θ, x has normal distribution with mean θ and precision β. The proportion of specialists who do not buy the government bond is calculated as the area under the distribution up to x. Therefore, it is

$$\ell = \Phi\left(\sqrt{\beta}\,(x(\xi, y) - \theta)\right)$$

As a preliminary, we prove the existence of unique θ corresponding to each value of ξ. We know the government compares the cost of taxation and the cost of declaring default. Therefore, the equilibrium default equation is

$$f(b, \ell, \theta) = f\left(b, \Phi(\sqrt{\beta}\,(x(\xi, y) - \theta)), \theta\right) = c \tag{1}$$

We denote the solution for θ in this equation as ψ. Setting $F \equiv f\left(b, \Phi(\sqrt{\beta}\,(x(\xi,y)-\psi)), \psi\right)$ and $K(\psi) \equiv \sqrt{\beta}(x(\xi,y)-\psi)$, we have

$$\begin{aligned} F_\psi &= f_\ell \Phi_K K_\psi + f_\psi \\ &= -\sqrt{\beta} f_\ell \Phi_K + f_\psi \end{aligned}$$

We know $f_\ell > 0$, $\Phi_K > 0$ and $f_\psi < 0$. Therefore, $F_\psi < 0$. This proves that we have a unique ψ to solve the equilibrium default equation, because the right-hand side of the equation, c, is constant.

The payoff with the switching strategy, $U(\xi)$, for a specialist from buying the amount b of the government bond in auction is

$$\begin{aligned} U(\xi) &= m \int_\psi^\infty \sqrt{\alpha+\beta}\,\phi\left(\sqrt{\alpha+\beta}\,(\theta-\xi)\right) d\theta \\ &\quad - b \int_{-\infty}^{\psi} \sqrt{\alpha+\beta}\,\phi\left(\sqrt{\alpha+\beta}\,(\theta-\xi)\right) d\theta \\ &= m - (m+b)\,\Phi\left(\sqrt{\alpha+\beta}\,(\psi-\xi)\right) \end{aligned}$$

We check the sign of the differentiation of $U(\xi)$ with respect to ξ. The differentiation is expressed as

$$U_\xi = -(m+b)\sqrt{\alpha+\beta}\,\phi\left(\sqrt{\alpha+\beta}\,(\psi-\xi)\right)\left(\frac{\partial\psi}{\partial\xi} - 1\right)$$

Therefore, a sufficient condition for $U_\xi > 0$ is $\frac{\partial\psi}{\partial\xi} < 1$.

Applying the expression for $x(\xi,y) = \frac{\alpha+\beta}{\beta}\xi - \frac{\alpha}{\beta}y$ in the equilibrium default equation (1), we have implicit differentiation of the equation with respect to ξ as

$$f_\ell \sqrt{\beta}\left(\frac{\alpha+\beta}{\beta} - \frac{\partial\psi}{\partial\xi}\right)\phi\left(\sqrt{\beta}\,(x-\psi)\right) + f_\psi \frac{\partial\psi}{\partial\xi} = 0$$

By rearranging this equation we have

$$\frac{\partial\psi}{\partial\xi} = \frac{(\alpha+\beta)\,\phi\left(\sqrt{\beta}\,(x-\psi)\right)}{\sqrt{\beta}\left(\sqrt{\beta}\phi\left(\sqrt{\beta}\,(x-\psi)\right) - \frac{f_\psi}{f_\ell}\right)}$$

Since $f_\ell > 0$ and $f_\psi < 0$, the denominator is positive. In addition, we have $\frac{\partial \psi}{\partial \xi} > 0$.

We would like to obtain a sufficient condition for $\frac{\partial \psi}{\partial \xi} < 1$. $\frac{\partial \psi}{\partial \xi} < 1$ can be expressed as

$$(\alpha + \beta)\,\phi\left(\sqrt{\beta}\,(x - \psi)\right) < \sqrt{\beta}\left(\sqrt{\beta}\phi\left(\sqrt{\beta}\,(x - \psi)\right) - \left(\frac{f_\psi}{f_\ell}\right)\right)$$

This is boiled down to

$$\frac{\alpha}{\sqrt{\beta}}\phi\left(\sqrt{\beta}\,(x - \psi)\right) < -\left(\frac{f_\psi}{f_\ell}\right)$$

The left-hand side is maximised at $\phi\left[\sqrt{\beta}\,(x - \psi)\right] = \frac{1}{\sqrt{2\pi}}$ when $x = \psi$. Therefore, a sufficient condition to hold this inequality is

$$\frac{\alpha}{\sqrt{\beta}} < -\sqrt{2\pi}\left(\frac{f_\psi}{f_\ell}\right)$$

The right-hand side is positive since $f_\ell > 0$ and $f_\psi < 0$. This condition requires that $\frac{\alpha}{\sqrt{\beta}}$ is very small which holds if β is much larger than α. In other words, private signal to specialists has much higher precision than one about realization of fundamentals.

4.3 Unique Equilibrium

In this section, we prove the existence of unique symmetric equilibrium when specialists employ switching strategy.

We have two equations for two unknown variables ξ and ψ to yield the equilibrium. The first equation is the equilibrium default equation for the government (1). That is

$$f\,(b, \ell, \psi) = c \qquad (1)$$

here

$$\ell = \Phi\left(\frac{\alpha}{\sqrt{\beta}}(\xi - y) + \sqrt{\beta}(\xi - \psi)\right)$$

Another is the payoff equation for specialists in which both of buying and not-buying the government bond give them the same return. It is the equation (2) below.

$$U(\xi) = m - (m+b)\,\Phi\left(\sqrt{\alpha+\beta}\,(\psi-\xi)\right) = 0 \tag{2}$$

Equation (2) gives us

$$\xi = \psi - \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{m}{m+b}\right)$$

By inserting this expression for ξ in the first equation, we have

$$f\left(b, \ell^*, \psi\right) = c$$

here

$$\ell^* = \Phi\left(H(b,\psi)\right)$$

and

$$H(b,\psi) \equiv \frac{\alpha}{\sqrt{\beta}}\left(\psi - \frac{1}{\sqrt{\alpha+\beta}}\Phi^{-1}\left(\frac{m}{m+b}\right) - y\right) - \sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\frac{m}{m+b}\right)$$

Then, the differentiation of $H\left(b,\psi\right)$ with respect to ψ is

$$H_\psi = \frac{\alpha}{\sqrt{\beta}}$$

We are now ready to check the value of $\frac{\partial \ell^*}{\partial \psi}$. We have

$$\frac{\partial \ell^*}{\partial \psi} = \Phi_H H_\psi = \frac{\alpha}{\sqrt{\beta}}\Phi_H$$

We come back to the equation for equilibrium default (1). We denote the left-hand side of the equation $F^* \equiv f(b, \ell^*, \psi)$ and we have the differentiation of F^* with respect to ψ as

$$F^*_\psi = f_{\ell^*} \frac{\partial \ell^*}{\partial \psi} + f_\psi$$

We know $f_{\ell^*} > 0$ and $f_\psi < 0$ from the characteristics of $f(\cdot)$. Since $\frac{\partial \ell^*}{\partial \psi} = \frac{\alpha}{\sqrt{\beta}} \Phi_H$, we have $lim_{\frac{\alpha}{\sqrt{\beta}} \to 0} \frac{\partial \ell^*}{\partial \psi} = 0$. Therefore, $lim_{\frac{\alpha}{\sqrt{\beta}} \to 0} F^*_\psi = f_\psi < 0$. Alternatively, if $\frac{\alpha}{\sqrt{\beta}} < -\frac{f_\psi}{\Phi_H f_{\ell^*}}$, we have $F^*_\psi < 0$. Because the right-hand side of the first equation is a constant, there is a unique ψ to satisfy the equilibrium conditions if $\frac{\alpha}{\sqrt{\beta}}$ is very small. We have obtained a sufficient condition for the existence of unique equilibrium; $\frac{\alpha}{\sqrt{\beta}}$ is very small. This is the same as a sufficient condition for $U_\xi > 0$, which is proven in Subsection 4.2.

We summarise the result obtained in this subsection as a theorem.

Theorem 1 *If $\frac{\alpha}{\sqrt{\beta}} \to 0$, there is a unique equilibrium. Roll-over of debt stops if $\theta \leq \psi$ and it continues if $\theta > \psi$.*

5 Effect of the Amount of Debt for Refinancing

In the preceding sections, we have proven that there is a unique value of θ, that is ψ, above which all of specialists buy the government bond in auction and below which they don't. In the proof of the existence of unique equilibrium ψ, we have taken the value of b as fixed. In this section, we will investigate how a change in b affects ψ. The result will give us useful implications to public debt management.

We can find the relation between b and ψ, that is the sign of $\frac{d\psi}{db}$, by analysing the equilibrium default equation for the government (1) with ℓ^* in it. It is

$$f(b, \ell^*, \psi) = c \tag{3}$$

here

$$\ell^* = \Phi(H(b, \psi))$$

and

$$\begin{aligned} H(b, \psi) \equiv & \frac{\alpha}{\sqrt{\beta}} \left(\psi - \frac{1}{\sqrt{\alpha+\beta}} \Phi^{-1} \left(\frac{m}{m+b} \right) - y \right) \\ & - \sqrt{\frac{\beta}{\alpha+\beta}} \Phi^{-1} \left(\frac{m}{m+b} \right) \end{aligned}$$

As a preliminary, we will check the sign of ℓ^*_b. With $N \equiv \frac{m}{m+b}$ and $N_b = -\frac{m}{(m+b)^2} < 0$, we have

$$\begin{aligned} H_b &= -\frac{\alpha}{\sqrt{\beta}\sqrt{\alpha+\beta}} \Phi_N^{-1} N_b - \sqrt{\frac{\beta}{\alpha+\beta}} \Phi_N^{-1} N_b \\ &= - \left(\frac{\alpha}{\sqrt{\beta}\sqrt{\alpha+\beta}} + \sqrt{\frac{\beta}{\alpha+\beta}} \right) \Phi_N^{-1} N_b > 0 \end{aligned}$$

since $\Phi_N^{-1} > 0$ and $N_b < 0$. We also know $\Phi_H > 0$. Therefore,

$$\ell^*_b = \Phi_H H_b > 0$$

Obviously, we also have

$$\ell^*_\xi = 0$$

because ℓ^* does not include ξ.

The preliminaries are all obtained to know the sign of $\frac{d\psi}{db}$ now. By expressing ℓ^* as a function of b and ψ like $\ell^* = \ell^*(b, \psi)$, we have the total implicit differentiation of the equilibrium default equation for the government (3) as

$$f_{\ell^*} \ell^*_\psi d\psi + f_{\ell^*} \ell^*_b db + f_b db + f_\psi d\psi = 0$$

From this we have

$$\frac{d\psi}{db} = -\frac{f_{\ell^*}\ell^*_b + f_b}{f_{\ell^*}\ell^*_{\psi} + f_{\psi}}$$

All the terms in the numerator are positive and the sign of $\frac{d\psi}{db}$ depends on the sign of the denominator.

Actually, the denominator is the expression for F^*_{ψ} in Subsection 4.3. Therefore, we already know that we have F^*_{ψ} 3 < 0 if $\frac{\alpha}{\sqrt{\beta}}$ is very small. It needs to be at least so small to hold $\frac{\alpha}{\sqrt{\beta}} < - \frac{f_{\psi}}{\Phi_H f_{\ell^*}}$. If $F^*_{\psi} < 0$, we have $\frac{d\psi}{db} > 0$. A sufficient condition for the existence of unique equilibrium works as a sufficient condition for $\frac{d\psi}{db} > 0$.

This final inequality tells us that ψ is increasing in b. This result fits our intuition very much. The higher the refinancing pressure is, the stronger the fundamentals need to be for keeping specialists purchasing the government bonds.

6 Implications

The results of our analysis in this paper suggest that a welfare-enhancing policy measure is worth considering by the government in a heavily indebted country. Namely, debt-maturity lengthening to avoid concentration of redemption in a certain fiscal year.

Actually, it is not a novel idea that debt-maturity lengthening would be welfare-enhancing. Alesina *et al.* (1990) points to possible welfare-enhancement by scattering debt redemption evenly over time. Calvo (2000) explicitly recommends this practice. However, as we have already investigated, their recommendation is from a different assessment of the same problem. The results of their analysis show that increase in debt for refinancing in a fiscal year aggravates the welfare loss in confidence crisis. Therefore, debt-maturity lengthening by which redemption schedules are flattened and averaged down lessens the welfare loss in the worst case in their theory.

In contrast, the results in this paper conclude that increase in debt for refinancing triggers confidence crisis even at a state with stronger fundamentals. Debt-maturity lengthening can push the threshold fundamentals farther from current fundamentals. By using this practice a government can reduce the likelihood of confidence crisis. Public debt management works in this important sense.

It is often observed that a country facing worry about its solvency from international investors calls for debt restructuring. In some cases a debt restructuring plan holding the present value of debt unchanged is offered by the government. The aim of such debt restructuring plan can be understood to ameliorate the possibility of confidence crisis by reducing the amount of redemption and lengthening the average maturity of issued bonds. By doing so, the government will be able to buy time to improve the fundamentals of the economy. It is sometimes carried out with the help of external authorities such as IMF. In other words, it tries to change ψ by changing b because it knows that y, ex-ante strength of fundamentals, can not be changed in a short period of time.

Like the case of Italy in the 1980s, refinancing pressure becomes a very serious problem in a heavily indebted country. As Townend (1997) reflects, refinancing pressure was a very hot issue in public debt management in the U.K. in the 1980s as well. Attempts by a government to lengthen the average maturity of debt observed in Italy and Britain in the past can be understood as reflection of worry about confidence crisis. In Italy, it was done by issuing floating-interest rate bond whose coupon rate was linked with short-term interest rate. In Britain the attempt was done by introducing inflation-indexed bond. These examples will serve as historical lessens for policymakers to learn how important it is to pay attention to debt-bunching problem although foreseeing the cost is difficult. The Japanese government might have learnt from those examples. Facing debt-bunching in coming years, they decided to repurchase their debt from the government bond markets in order to ameliorate debt-bunching. More recently they announced to introduce inflation-indexed bonds which would be used for issuing long-term bonds and help the government to average down the redemption schedules.

Occurrence of confidence crisis in one period may force a government to be stuck in "Bunching Trap." Once confidence crisis occurs it would be difficult for the government to issue long-term bonds. This would force the government to issue short-term bonds. As a result, the accumulation of short-term bonds give birth to another bunching of redemption in the near future. In relation with this conjecture, we can cite an example from Italy in the 1980s. In the 1980s Italian government worried about the short average maturity of issued bonds and they tried to lengthen it. It was successful until 1987 but investors' resentment to hold the government bond was observed in that year. Although the turmoil receded in a couple of months, it became very difficult for the government to issue long-term bonds after the incident. As a result, the average maturity of issued bonds started shortening again and it continued to shorten for years. Alesina *et al.* (1990) and Passacantando (1996) describe the incident in details.

Of course it is an issue for discussion that there is usually trade-off between debt-maturity lengthening and increase in debt-servicing costs. Forward-premium in yield curve makes issuance of long-term bonds more costly than short-term bonds and issuance of long-term bonds is necessary for levelling off redemption schedule (lengthening debt-maturity). It is reasonable to think that forward-premium reflecting future uncertainty is not negligible in a heavily indebted country. The bottom-line is how seriously the government gauges the cost of confidence crisis. As Calvo (2000) pointed to, it can be very large.

In international contexts, our results also can give us a hypothesis about how economic crisis in one country spreads to other heavily indebted countries. Suppose a negative shock hits Country A and it suffers economic doldrums due to it. Suppose another country, Country B is a net-exporter to Country A and the doldrums of Country A's economy reduces the net-export from Country B. This linkage in real economy could trigger confidence crisis in government bond markets in Country B even if the net-export from Country B to Country A does not significantly drop. If the fundamentals of Country B is slightly stronger than the threshold in our analysis, a slight deterioration due to a small drop in net-export could be enough to push the fundamentals below the threshold. In such a situation, contagion of a small degree in real economy could be a reason for a violent financial collapse in a contagion-hit country. From this point of view as well, averaging redemption schedules is recommended. It will work to keep a heavily indebted country immune to contagion.

Finally, we discuss what will be consequences if the government can resort to monetization of its debt. In our model in this paper, we have not included that possibility. Extension of the model will be necessary for rigorous analysis but it is possible to discuss the issue in the framework of our model. Monetization in confidence crisis context is a practice of the central bank in a country to purchase all the government bonds unsold in auction. If monetization is surely conducted by the central bank, a specialist in our model can know that his payoff is always positive, irrespective of actions of other specialists. Therefore, all the specialists will buy the government bond in auction. No undersubscription will happen. Then, the possibility of confidence crisis is to be ignored. The government can pursue minimization of cost of debt-servicing by shortening debt maturity, because the trade-off between debt-maturity lengthening and increase in debt-servicing cost does not exist. But this conjecture is correct as far as the cost of monetization is zero or very small in the loss function of the government. If monetization is costly, it is rational for the government to compare the cost with the cost accompanying other alternatives, namely the cost of taxation and declaration of default. The cost of monetization will be different among the countries but cost of excessive inflation is most likely to be included. Expectation of continuing monetization in the government bond auction will lead to acceleration of inflation and the cost of monetization could exceed the cost of default at some stage. This possibility will make specialists worry about their payoff in buying the government bond. The worry, in imperfect information environment, gives rise to coordination problem among specialists shown in this paper. Therefore, monetization as an alternative action by the government does not eliminate completely the possibility of confidence crisis. Keen attention to debt-bunching is still recommended in public debt management.

7 Concluding Remarks

In this paper we attempt to provide theoretical investigation to confidence crisis in government bond markets. By using the global game techniques we analyse coordination problem in government bond auction. The approach in this paper allows us to have insight on the relation between occurrence of confidence crisis and the fundamentals of economy which is not clearly explained in preceding works. This paper also makes it clear how the amount of debt for refinancing affects occurrence of confidence crisis. Implications to policymakers are that they should pay keen attention to possible serious consequence of debt-bunching. Debt-bunching usually grows gradually over time and one of its main causes is myopic policy in public debt management. Policymakers in a certain period is subject to lure of tipping the balance to short-term debt in their liability because it usually works for reduction in debt-servicing costs. However, repetition of this practice yields debt-bunching as time goes. In public debt management, the government decisions today affect the situation facing the government in the future via change in redemption schedules. Although the cost of confidence crisis is not easy to gauge beforehand, it should be noted by policymakers.

As the final remark, caveats and ways for possible extension should be noted. The model in this paper is static. The author thinks that the approach with static game reflects reality and successfully illuminates the mechanism relating fundamentals and occurrence of confidence crisis. However, it is also true that extension with dynamic consideration would deepen the analysis on confidence crisis. For example, unlike our setups in the model, we can assume the government has multiple number of auction opportunities in a fiscal year. The government needs to raise finance from auctions for the amount to be refinanced in the fiscal year. The less the government can sell its bonds in an earlier auction, the more they need to sell in later auctions. This is one possible extension and there will be more. Such extensions are left for future works.

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