Pension Plan Funding, Risk Sharing and Technology Choice

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Abstract

This paper presents a general equilibrium analysis on the interactions between pension plan funding, capital structure, technology choice and the equity premium. The paper shows that economies with large funded defined pension schemes may be biased towards safe production. The pricipal results derive from the existence of borrowing and short sales constraints. In the first scenario workers are constrained in the capital market and debt is risk-free. If pension benefits are sufficiently high, then the capital market constraint may be binding. Then, leveraging the risky technology gives workers an adjustment channel through which they may undo an over-exposure to risk-free investment. This results in more risky production and a fall in the equity premium. In the second scenario workers are constrained in the capital market and debt is subject to default risk. If the level of the resulting pension plan shortfall risk is low and if pension benefits are sufficiently high, then the previous is at work. If the level of shortfall risk is high, workers hedge themselves by holding risk-free assets and the constraint in the capital market is no longer binding. Then the risky firm does not benefit from leveraging and there is more safe production in the economy.

1 Introduction

This paper examines the question of how the level of pension plan benefits and funding affect risk bearing and the composition of investment in the economy as a whole and how this relationship interacts with the financial leverage of the corporate sector?

Economies such as the United Kingdom and the United States have large funded occupational pension schemes, including both defined benefit and defined contribution plans, which are part of workers' overall compensation packages. The returns on plan assets are used to pay pensions. The principal difference between the two types of plan is in who bears the return risk for solvent plans. In either case the plans are the principal source of retirement income for many workers. Defined contribution schemes are frequently life-style, so that at retirement they are predominantly invested in bonds, but are balanced between equities and bonds in the earlier accumulation phase. Defined benefit plans typically have high equity weightings (70 percent is common) but offer beneficiaries a fixed (indexed) promise at retirement. In the United Kingdom both defined contribution and defined benefit plans must buy an annuity at some stage before the retiree is 75, or forfeit the tax exemptions for payments into the plan. The annuities are usually bought from insurance companies who in order to meet payments demand longer maturity government bonds and high-grade (low risk of default) corporate bonds. Abstracting from issues relating to the supply and maturity structure of government bonds, the role of the corporate sector in supplying these income streams will have implications for corporate capital structure and the investment behaviour of the economy.

A central concern of this paper is the allocation of risk-bearing between households. In a number of papers (for example Storesletten et al (2007)) older middle aged workers who have relatively high net worth and relatively low human capital have a relatively high appetite for holding risky assets. Younger workers on the other hand, have high levels of human capital, which exposes them to background risk (for example, labour endowment shocks) and thereby limits their appetite for holding risky assets. They assume that retired workers only hold riskless assets. Models such as that of Storesletten et al (2007), with overlapping generations of three-period lived individuals allow us to understand the balance of risk bearing between generations in the economy, linking it to the age distribution of the population and the empirical wealth distribution across generations. These models are notoriously difficult to analyse but they are at the heart of research aimed at capturing the empirical interactions of life-cycle asset accumulation patterns and asset pricing.

The present paper is more modest than the above. Its aim is to understand aspects of the interaction of asset demand and investment behaviour and in particular the impact of funded pension plans on economic equilibrium. In order to do this, rather than having risk bearing evolve over the life-cycle, we take an alternative static approach. In particular, we assume that at each date there are two types of individual, namely rentiers and workers. In the model the two types of individual have strictly differentiated roles. The rentiers have high capital endowments (like the older middle aged) and have a high appetite for risk bearing. The workers have a labour endowment, which is not subject to background risk and no capital endowment; they are assumed to be poorer than rentiers and so have a low appetite for risk bearing.

The paper then provides an analysis of the impact of the type of occupational pension plan and its funding on workers' saving and portfolio behaviour on risk sharing between workers and rentiers; and the implications for the economy's technology choices. The basic framework is similar to that in Diamond and Geanakoplos (2004). In our model there are two technologies: a safe and a risky technology. Only the risky technology employs workers, who are paid wages and receive a pension promise. The pension can be a funded defined contribution plan or a defined benefit plan, but the principal focus of this paper is on the latter. We abstract from issues to do with intergenerational risk sharing, social security and the role of long-lived assets.[1] In our economy agents cannot short-sell assets and workers are unable to borrow against pension plan assets. Financial claims are held by rentiers, workers and the pension plan. Since in the model there is a single risky technology there is no distinction between aggregate and idiosyncratic risk. Workers exposure to risk is therefore either though their choosing to hold shares in the risky technology or through the pension plan.

In the basic model in the paper, without company financial policy, the failure of equivalence propositions between defined benefit and defined contribution pension plans derives from the existence of capital market constraints. The two types of plan force workers against the constraints differently (the defined contribution plan may itself be neutral) yielding an asymmetric impact on risk taking, the aggregate risk premium and technological choices.

In the case of a defined benefit pension, the plan sponsor bears the risk of any shortfall between the return on the plan's investments and the pension benefit. If

[1] Abel (2001) and Bohn (1997) both provide models with technology risk but focus on social security and intergenerational risk sharing.

the plan sponsor issues risky debt, then in the event of default, if the pension plan is not fully funded (or insured), this shortfall risk is borne by the plan members. We show that when capital market constraints on workers are binding, leveraging the risky technology, even with risky debt, raises risky production and can reduce the aggregate risk premium.

When the pension plan is defined benefit, it is exposed to shortfall risk, if the sponsor is financed with risky debt and defaults on risky debt leaving an unfunded deficit. This means that the workers are in turn exposed to technology risk. The workers' (pension plan trustees) effort to hedge this risk, essentially transferring it back to rentiers, is a key concern of the paper. A key prediction of the paper is to show that when pension plan shortfall risk is large and the dominant factor is that of hedging pension plan shortfalls, workers' demand for levered-equity declines with increases in leverage, so that the cost of capital of the risky technology does not benefit from further debt issues. The existence of high levels of pension benefits and shortfall risk then biases the economy towards safe production. This may be a partial explanation of the low cost of capital in the United Kingdom afforded companies with stable income streams and significant collateral assets such as major retailers like Marks and Spencer and Tesco, which have been able to issue low yield long-dated debt for which annuity providers have a strong appetite.

The plan of the paper is as follows: The first Section of the paper outlines the basic model focussing on the technologies of the economy and the simple lifetime allocation problems of rentiers and workers, where the behaviour incorporates the contributions and returns to pension plans. Section 2 outlines the equilibrium properties of the model, including the determination of asset prices, investment in the economy's two technologies and the aggregate risk premium under both defined contribution and defined benefit pensions. Section 3 shows the impact of the defined benefit pension plan on the equilibrium of the model when workers do not save on personal account and when they do but are subject to borrowing and short sales constraints. Section 4 introduces corporate financial policy. The focus of this section is on understanding how leveraging the risky technology may alleviate the constraint of the defined benefit pension on the workers allocation problem and reduce the distorting effect that it has on the production decisions of the economy. To do this we first show what is necessary to achieve neutrality of corporate financial policy with riskless debt. Assuming that there is no pension plan shortfall risk, we show that neutrality breaks down when the defined benefit pension constraint is binding. The implications for asset prices, technology choices and the aggregate risk premium are then illustrated. The same exercise is then repeated for the case of risky (presence of default risk) debt. Section 5 of the paper considers pension plan shortfall risk and how this interacts with company default risk. The effect of shortfall risk depends crucially upon its relative magnitude and the probability of the risky company defaulting. In particular, if the level of shortfall risk is high, workers hedge themselves by holding risk-free assets and the borrowing constraint is no longer binding. Then the risky firm does not benefit from leveraging and there is more safe production in the economy. The final section of the paper is the conclusions.

2 Basic Framework

Production takes place though two technologies, a safe technology and a risky technology. Labour is only employed in the risky technology. Claims to the two technologies are sold as shares. At each date there are two groups of agents with two-period lives, workers and rentiers; each represented by a single risk-averse member. Agents are assumed to be rational and able to make optimal financial decisions.[2] The economy does not permit the short-selling of claims on technologies's income streams.

Rentiers have an initial endowment that can be consumed or allocated to production in the two technologies. If the risky technology is active, rentiers must employ workers at the cost of wages and pensions. They either consume or save the residual to finance retirement consumption. Saving takes the form of accumulating claims on the two technologies.

The worker's endowment takes the form of a fixed amount of labour that they supply (inelastically) when young. They divide their income between current consumption and saving to finance retirement consumption, which takes the form of accumulating claims on the two technologies. These claims are purchased from rentiers. Workers retirement consumption is also partly financed by a funded occupational pension plan. There is no pay as you go pension scheme, so that we abstract from any trade between generations.

Production decisions are made by rentiers before any shares are sold to the pension plan or to workers, but with rentiers anticipating the market clearing prices for shares in the safe and risky technologies.

The labour contract has two components, a wage and an (occupational) pension.

[2] An important difference between the present set up and that of Diamond and Geanakoplos (2004) is that we do not assume that individual's second period preferences are defined over composite safe and composite risky consumption. However, that said the framework employed and the technology assumptions are close to theirs.

The specific features of the relationship of capital to labour in production and the labour and capital market conditions that determine the mix of wages and pension are not modelled. Indeed, the particular type of pension plan is exogenous. If one particular type of pension is given exogenously, this can be justified by a pension plan contracting cost. If this cost were to go to zero, the more efficient contract from a risk sharing perspective would imply lower employment costs and dominate on efficiency grounds.

2.1 Technologies

The two technologies:

A. A safe technology that only invests capital. An investment at date t of k_{0t} in the safe technology yields $\phi(k_{0t})$ at date $t+1$ with $\phi' > 0$ and $\phi'' < 0$. The return to investors per unit of capital invested in this technology will be given by $R_{t+1} = \phi(k_{0t})$.

B. A risky technology that employs an inelastic supply of labour at a fixed cost of production, z_t, and capital as a variable cost. Payment of z_t and an investment of k_{1t} yields output net of fixed labour costs of $R^*_{t+1}g(k_{1t})$, where R^*_{t+1} is stochastic. The production function $g(k_{1t})$ is stochastic with $g' > 0$, and $g'' < 0$. Notice that with this technology, labour must be employed at its reservation compensation level before capital is applied at the profit maximising level. An alternative way of making the same point is that rentiers transfer part of their endowment to workers so as to induce them to participate in risky production and so make it active. This technological assumption is a strong one but allows us to focus on the allocation of capital. Moreover, it means that our model does not require a labour market clearing condition. This assumption has little cost to the analysis in the paper as there is no labour risk in the form of labour endowment shocks.

In addition, we assume that there is a range of investments in the two technologies in which the risky technology yields higher expected returns than the riskless technology.

The cost of labour, z_t, comprises the wage cost, w_t, plus the pension contribution, f_t. In this set up the workers compensation package is a fixed cost of production with the package of wages and pension benefit, b_{t+1}, determined to satisfy the worker's exogenous labour market participation condition. The employment level and the compensation paid to labour is independent of the technology shock. This is a strong assumption that greatly simplifies the analysis and allows us to focus entirely on shocks to the productivity of capital. The cost of labour, z_t, could, however, be made a function of the type of pension plan. Then, depending upon exogenous costs, noted in the introduction, the cost of the employment contract can itself include transaction costs that depend upon the type of pension plan chosen. However, here we simply take the type of pension plan as given. Finally, at this stage we introduce a term for the pension deficit, d_{t+1}, which arises when the return on a defined benefit pension fund does not cover the pension plan liabilities.

2.2 Individuals

2.2.1 Rentiers

The representative rentier, indexed by s, has an additive, strictly concave, utility function, U. The subscript t refers to the individual's birth date; y and o refer respectively to youth and old age. The rentier has an exogenously given positive endowment of e^s_{yt}. Income at date t is used to hire workers for z_t; to finance young consumption, c^s_{yt}; and savings of a^s_{yt}. The savings are used to purchase claims on the safe and risky technology. The income stream from these investments is used to finance consumption when old, c^s_{ot}. We denote the rentier' shares in the income streams of the two technologies by α^s_{jt}, $j = 0, 1$; which due to the short-sales constraints are non-negative. The representative rentier chooses a^s_{yt} and $\{\alpha^s_{0t},\alpha^s_{1t}\} \geq 0$ to solve the following optimisation problem that follows along lines similar to Diamond and Geanakopolos (2003). Let $p_t = 1/R_{t+1}$ denote the date t price of one unit of riskless consumption at date $t+1$ in terms of date t consumption; and let q_t denote the date t price of risky capital and q^*_t the price of risky consumption date $t+1$. Then we can write

$$\max\{U(c^s_{yt}) + E_t U(c^s_{ot})\} \tag{1}$$

subject to

$$e^s_{yt} - z_t = c^s_{yt} + a^s_{yt}, \tag{2}$$

$$a^s_{yt} = \alpha^s_{0t} p_t k_{0t} + \alpha^s_{1t} q_t k_{1t}, \tag{3}$$

and

$$c^s_{ot} = \alpha^s_{0t}\phi(k_{0t}) + \alpha^s_{1t}(R^*_{t+1} g(k_{1t}) - d_{t+1}). \tag{4}$$

Note that the rentier has an obligation to the pension at date t through the term z_t but also through the deficit term d_{t+1}.

Consider the unconstrained optimisation problem of the representative rentier who holds both riskless and risky assets. The first-order conditions for the asset shares are,

$$p_t U'(c^s_{yt}) = E_t[U'(c^s_{ot})], \tag{5}$$

and

$$q_t U'(c^s_{yt}) = E_t[U'(c^s_{ot})(R^*_{t+1} g(k_{1t}) - d_{t+1})]/k_{1t}. \tag{6}$$

Defining $m^s_t = U'(c^s_{ot})/U'(c^s_{yt})$ as the stochastic discount factor, we can write

$$p_t = E_t m^s_t \tag{7}$$

and

$$\begin{aligned} q_t &= E_t[m^s_t(R^*_{t+1} g(k_{1t}) - d_{t+1})]/k_{1t} = \\ & [p_t E_t(R^*_{t+1} g(k_{1t}) - d_{t+1}) + cov(m^s_t, R^*_{t+1} g(k_{1t}) - d_{t+1})]/k_{1t} \end{aligned} \tag{8}$$

Hence when the agent buys a risky asset he is buying expected income plus covariance. Since c^s_{ot} and R^*_{t+1} are perfectly (positively) correlated, $U'(c^s_{ot})$ and R^*_{t+1} are negatively correlated the covariance is negative.

If the pension is always fully funded so that $d_t = 0$ in all states, we can write

$$\begin{aligned} q_t k_{1t} &= q^*_t g(k_{1t}) - E_t m^s_t(d_{t+1}) \\ \text{where } q^*_t &= E_t m^s_t(R^*_{t+1}) \end{aligned} \tag{9}$$

Note here that this allows us to substitute $q^*_t g(k_{1t})$ for $q_t k_{1t}$ in the subsequent analysis. With additive utility, if the utility function exhibits decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA), then as shown in Aura et al (2002), current consumption and the riskless and risky assets are normal goods and net substitutes.

2.2.2 Workers

The representative worker has an additive, strictly concave, utility function, W. The utility function again exhibits decreasing absolute risk aversion. Workers are poorer than rentiers. But in the present paper the holding patterns of safe and risky assets are driven by constraints

The worker supplies a unit of labour inelastically to the risky technology. He receives wages of w_t and a pension benefit of b_{t+1}. The package of wages and pension benefits is determined to satisfy an exogenous participation condition, so that utility must be at least equal to $\overline{W}$.[3] If the pension plan is defined benefit and is exposed to sponsor default risk, the package of wages and benefits should adjust to reflect this risk to ensure worker participation.

The worker's saving on personal account is denoted by a^{ω}_{yt}, which is divided between holding a fraction of the economy's safe assets, α^{ω}_{0t}, and risky assets, α^{ω}_{1t}. Current income net of savings is used to finance consumption when young, c^{ω}_{yt}. Old consumption is determined by the return on personal saving and the pension plan benefit, b_{t+1}. The worker's optimisation problem involves choosing a^{ω}_{yt} and again because of the short-sales constraint, $\{\alpha^{\omega}_{0t}, \alpha^{\omega}_{1t}\} \geq 0$. In solving the optimisation problem the pension plan is taken as given. The worker's problem is stated as:

$$\max\{W(c^{\omega}_{yt}) + E_t[W(c^{\omega}_{ot})]\} \tag{10}$$

subject to

$$w_t = c^{\omega}_{yt} + a^{\omega}_{yt}, \tag{11}$$

$$a^{\omega}_{yt} = \alpha^{\omega}_{0t} p_t k_{0t} + \alpha^{\omega}_{1t} q_t k_{1t}, \tag{12}$$

and

$$c^{\omega}_{ot} = \alpha^{\omega}_{0t}\phi(k_{0t}) + \alpha^{\omega}_{1t}[(R^*_{t+1} g(k_{1t}) - d_{t+1})] + b_{t+1}. \tag{13}$$

The first-order conditions for an interior optimum for the worker's asset shares take the same general form as in (5) and (6). Moreover, with the same assumptions on the form of the worker's utility function as that of the rentiers, all goods and assets are normal and net substitutes.

In this framework workers will demand stocks. However, Mankiw and Zeldes (1991) find that only a small portion (about 27 percent) of investors participate in the stock market; in their sample, of those with liquid assets in excess of $100,000, only 47.7 percent hold stocks. Limited participation can to some extent be explained by transaction costs (Allen and Gale (1994)) or non-expected utility (Dow and Werlang (1992)).[4] The basic analysis here lets workers hold positive amounts of stock.

[3] In the model we present, workers do not share production risk directly with rentiers through their wages being affected by shocks, as is the case in much of the real business cycle literature. Danthine and Donaldson (2001), for example, examine an infinite horizon model in which workers have no access to capital markets but in which shocks are shared with rentiers though variations in wages and labours' share in income.

[4] There are also behavioural arguments, such as misperception of risks or limited information regarding opportunities.

However, later in this paper it is shown that if they are exposed to significant company risk through possible default on the pension, they will demand low amounts of equity. Clearly if there are participation costs in the equity market, risk averse workers may choose not to participate in the equity market at all.

2.3 The Pension Plan

There are two types of pension: defined contribution and defined benefit. Here the type of pension is exogenously determined but as already noted, in a more complete model the type of pension plan will be endogenous.[5] For simplicity, we assume that workers themselves make no contributions to their (occupational) pension plan.

In the case of a defined contribution pension, rentiers make an immediate transfer to workers of wages, w_t, and make a contribution, f_t^{DC}, to the pension plan. The total cost to rentiers of employing labour is $z_t = w_t + f_t^{DC}$. The investment policy of the pension plan is chosen either by the workers themselves or their agents. In this case, with a defined contribution pension plan, pension plan risk is borne by the worker.

With a defined benefit plan, rentiers make an immediate transfer of wages, w_t, and promise workers a future pension benefit of $\bar{b}_{t+1}$ and pay f_t^{DB} into a pension plan. The package of wages and pension benefits is determined to ensure that the worker supplies labour. The plan sponsor determines the plan's investment policy.[6] If the return on the pension plan assets is insufficient to meet the pension promise, the sponsor must make up the deficit with an additional contribution at date $t+1$ of d_{t+1}, which is deducted from the firm's income before distribution to investors. If there is a surplus, this will be captured by the sponsor. Note that the risk to the pension plan sponsor rises with the holding of risky assets. In the event that the sponsor defaults then any pension plan shortfall is a loss to the beneficiaries.

[5] For example, defined benefit pension plans can act as a commitment mechanism for bargaining with workers over current levels of wages. The firm can promise to put more money into the pension plan in return for workers foregoing current wages, but it may not be able to commit to giving improved levels of compensation in future wages (see Ippolito, 1985). A defined benefit pension is deferred compensation, the value of which is tied to the economic success of the plan sponsor. Workers are then committed to supplying high effort levels over the long-run and do not gain from quitting or collectively shirking. If this is true, then firms have an incentive to let the company pension plan run a deficit and lever the firm's capital structure, so as to subject workers to the risk of default on their pensions should the firm perform poorly. However, an efficient trade-off between risk sharing and long-run effort incentives can be achieved with a combination of a defined benefit pension plan and a long-term labour contract with efficiency wages (see Lazear (1985)).

[6] We also abstract from the conflicts that may arise, resulting from whether it is the plan sponsor or the beneficiaries that determine the plan's investment policy.

Let α_{0t}^f and α_{1t}^f denote the shares of the economy's safe and risky technologies held by the pension plan, so that

$$f_t = \alpha_{0t}^f p_t k_{0t} + \alpha_{1t}^f q_t k_{1t}. \tag{14}$$

where $f_t \in \{f_t^{DC}, f_t^{DB}\}$. The benefits of a defined contribution pension plan are given by the return on this portfolio,

$$b_{t+1} = \alpha_{0t}^f \phi(k_{0t}) + \alpha_{1t}^f (R_{t+1}^* g(k_{1t}) - d_{t+1}). \tag{15}$$

In the case of a defined benefit pension plan, the benefit and date $t + 1$ and additional contributions for the plan are respectively:

$$\begin{aligned} b_{t+1} &= \bar{b}_{t+1}, \\ \text{and } d_{t+1} &= \bar{b}_{t+1} - \alpha_{0t}^f \phi(k_{0t}) - \alpha_{1t}^f (R_{t+1}^* g(k_{1t}) - d_{t+1}). \end{aligned} \tag{16}$$

Here, if $\alpha_{1t}^f > 0$, d_{t+1} is a random variable. From an actuarial perspective, even if $E_t d_{t+1} = 0$, then if $\alpha_{1t}^f > 0$, the pension plan will impose unexpected losses on the sponsor. At this stage of the paper we assume that with a defined benefit pension the benefit, $\bar{b}_{t+1}$, is always paid, so that if $\alpha_{1t}^f > 0$, d_{t+1} can always be covered by the income from the risky technology.

2.4 Equilibrium

In our model production decisions are made by rentiers before any shares are sold to the pension plan or to workers. However, rentiers anticipate the market clearing prices for shares in the safe and risky technologies before these decisions are made. In presenting the equilibrium we assume that in the case of a defined benefit pension plan the plan does not incur a deficit, $d_{t+1} = 0$ for all states.

First consider the safe technology. Consider the unconstrained optimisation problem of the representative rentier who holds risky assets. $p_t = 1/R_{t+!}$ is the present value of a unit of date $t+1$ safe production in terms of numeraire current consumption. Let V_{0t} denote the market value of the safe technology, then

$$V_{0t} = p_t \phi(k_{0t}). \tag{17}$$

Assuming competitive behaviour, k_{0t} is chosen to maximise $V_t - k_{0t}$:

$$p_t \phi'(k_{0t}) - 1 = 0. \tag{18}$$

Now consider the risky technology. As we have stochastic constant returns to scale we can write the value of the risky technology as $V_{1t} = q_t^* g(k_{1t})$. Assuming competitive behaviour, value maximisation implies that k_{1t} maximises $V_{1t} - k_{1t}$ and satisfies:[7]

$$q_t^* g'(k_{1t}) - 1 = 0. \tag{19}$$

The technology will be operated at the optimal level provided the exogenous labour costs, z_t, are covered.

Equilibrium requires that the markets for current (date t) consumption and asset markets clear and that firms are making optimal investment decisions. Because of Walras' Law, we can drop the current consumption goods market and consider only the markets for safe and risky assets. In stationary equilibrium, prices and young worker's and rentier's consumption and asset holdings are constant through time and across states of nature. All that varies is output, consumption of the old workers and old rentiers and the return on the pension plan. With a single consumption good and stationary and independent productivity shocks, stationary equilibrium will exist. Each period a new steady state is reached, starting with the generation born immediately after a permanent parameter change. That is our two period economy repeats itself.[8]

It will be seen immediately that the equilibrium of the model yields a positive (expected) aggregate risk-premium. The expected return on equity is given $E_t(R_{t+1}^* g(k_{1t}))/k_{1t}$. The (expected) aggregate risk premium is defined as

$$E_t(R_{t+1}^* g(k_{1t}))/k_{1t} - R_{t+1}/k_{0t}, \tag{20}$$

From the rentier's optimisation problem,

$$E_t[U'(c_{ot}^s)(R_{t+1}^* g(k_{1t}))/k_{1t}] = E_t[U'(c_{ot}^s)R_{t+1}]. \tag{21}$$

[7] Note that with constant stochastic returns to scale, $E_t(d_{t+1})$ only depends upon the composition the investment policy of the pension plan and not on the scale of investment in the risky technology. Moreover, if the pension plan is fully funded $E_t(d_{t+1}) = 0$.

[8] Note that in the paper we only compare allocations given exogenous changes in pension plan policy. We do not consider anticipated changes in pension policy, that would occur in dynamic models that allow sponsors to react to anticipated shortfalls.

Since c^s_{ot} and R^*_{t+1} are perfectly (positively) correlated, $U'(c^s_{ot})$ and R^*_{t+1} are negatively correlated. Hence $E_t(g(k_{1t})R^*_{t+1})/k_{1t} > R_{t+1}/k_{0t}$.

In a complete financial market, for any given contribution policy to the pension plan, f_t, the asset allocation policy of the plan $(\alpha^f_{0t}, \alpha^f_{1t})$ will will not affect the equilibrium prices or consumption and investment. In an incomplete financial market, however, an exogenously determined pension plan asset allocation policy can have real implications for asset holding in the economy and consequently for the prices of assets and the choice of technology. There are two polar cases: If the plan is defined benefit, provided workers consolidate the plan in their budget sets and are at an interior optimum, we obtain an equilibrium in which the funding and benefit level of the pension plan is irrelevant. On the other hand, if the worker is forced to a corner solution though the holding of riskless assets by the pension plan, the neutrality property breaks down. If on the other hand, the pension plan is defined contribution, holding contribution levels fixed, if the worker chooses the asset allocation policy of the pension plan, he will consolidate this with his private portfolio decisions and we obtain neutrality. But if the contribution level and asset allocation decision of the pension plan are not made by the workers but by plan managers, the workers may want to undo on personal account the asset allocation of the plan. Again, due to the possibility of corner solutions, neutrality can break down. However, our primary focus in the paper is on defined benefit plans.

2.5 Equilibrium with Defined Contribution Pensions

The economy has three markets at date t; namely for current consumption, and for safe and risky assets. Current consumption and safe and risky assets are to be normal goods and net substitutes. In the case of a defined contribution pension plan, the workers and rentiers demand functions for safe assets are

$$\begin{aligned} \alpha^\omega_{0t} p_t k_{0t} &= H^\omega_t(p_t, q^*_t, z_t, \alpha^f_{0t} k_{0t}) \\ \text{and } \alpha^s_{0t} p_t k_{0t} &= H^s_t(p_t, q^*_t, e^s_t - z_t). \end{aligned} \tag{22}$$

The demand functions for risky assets are:

$$\begin{aligned} \alpha^\omega_{1t} q^*_t g(k_{1t}) &= J^\omega_t(p_t, q^*_t, z_t, \alpha^f_{1t} k_{1t}) \\ \text{and } \alpha^s_{1t} q^*_t g(k_{1t}) &= J^s_t(p_t, q^*_t, e^s_t - z_t) \text{ for the risky assets;} \end{aligned} \tag{23}$$

where we have used (9). In these conditions $z_t = w_t + f_t^{DC}$; with $H_{t1}^i < 0$, $H_{t2}^i > 0$, $H_{t3}^i > 0$ and $J_{t1}^i > 0$, $J_{t2}^i < 0$, $J_{t3}^i > 0$; $H_{t1}^i + J_{t1}^i = 0$, $H_{t2}^i + J_{t2}^i = 0$ and $H_{t3}^i + J_{t3}^i = 1$ $i = \omega, s$. Finally, H_{t4}^ω and J_{t4}^ω are both equal to -1, so that holdings of riskless and risky assets in the defined contribution pension plan lead to equal offsets in non-pension plan demand. We do not write down the demand functions for current consumption but note that they take the same form as the asset demand functions.

The equilibrium with defined contribution pensions the values $\{p_t, q_t^*, k_{0t}, k_{1t}\}$ satisfy (18) and (19), as well as

$$H_t^\omega(p_t, q_t^*, z_t, \alpha_{0t}^f k_{0t}) + H_t^s(p_t, q_t^*, e_t^s - z_t) + \alpha_{0t}^f p_t k_{0t} = p_t k_{0t}, \tag{24}$$

and

$$J_t^\omega(p_t, q_t^*, z_t, \alpha_{1t}^f k_{1t}) + J_t^s(p_t, q_t^*, e_t^s - z_t) + \alpha_{1t}^f q_t^* g(k_{1t}) = q_t^* g(k_{1t}), \tag{25}$$

where the asset holdings of the pension plan, $\alpha_{0t}^f p_t k_{0t}$ and $\alpha_{1t}^f q_t^* g(k_{1t})$, are taken as given.[9]

If the worker is at an interior optimum, then $H_t^\omega > 0$ and $J_t^\omega > 0$. Hence, if f_t^{DC} is constant and its composition changes, then $dH_t^\omega + d(\alpha_{0t}^f k_{0t}) = 0$ and $dJ_t^\omega + d(\alpha_{1t}^f k_{1t}) = 0$. Changes in contributions to the pension plan, f_t^{DC}, lead to changes in wages, w_t, such that $dw_t = -df_t^{DC}$. As z_t does not change, the rentiers asset demand functions are unaffected. Hence without writing down the market equilibrium conditions we conclude that in this environment in which the defined benefit pension plan is fully integrated with other elements of the worker's intertemporal problem we obtain neutrality.

If as noted above there is an agency problem or some other friction which prevents the above consolidation, then a defined contribution plan could force capital market constraints, either borrowing or short-sales constraints to bind for workers. Then the above neutrality property will break down. However, as the principal concern of this paper is with defined benefit pensions, we do not demonstrate this here.

2.6 Equilibrium with Defined Benefit Pensions

With a defined benefit pension plan, the worker's demand functions are

[9] Conditions (18) and (19) can be substituted into (24) and (25) to form two equations in two unknowns, k_{0t} and k_{1t}. These equations can be linearised in the neighbourhood of equilibrium. For k_{0t} and k_{1t} both positive, the Jacobian matrix is invertible with a negative determinant. The comparative statics of the system with respect to the exogenous pensions parameters then follow.

$$\begin{aligned}\alpha_{0t}^{\omega}p_t k_{0t} &= H_t^{\omega}(p_t, q_t^*, w_t, \bar{b}_{t+1}) \text{ for safe assets} \\ \text{and } \alpha_{1t}^{\omega}q_t^* g(k_{1t}) &= J_t^{\omega}(p_t, q_t^*, w_t, \bar{b}_{t+1}) \text{ for the risky assets.}\end{aligned} \tag{26}$$

The rentier's demand functions for safe and risky assets are the same as (22) and (23), with $z_t = w_t + f_t^{DB}$. Changes in the composition of the pension plan investments have no effect on the asset demands of workers if $\bar{b}_{t+1}$ is unchanged. At an interior optimum, changes in the benefit level, $\bar{b}_{t+1}$, affect asset demand through changes in wages, such that $H_3^{\omega}dw_t + H_4^{\omega}d\bar{b}_{t+1} = 0$, with $H_4^{\omega} = -1$. In a constrained optimum in which $\alpha_{0t} \geq 0$ binds and $H_3^{\omega} = 0$, so that the worker demand for riskless assets is zero and does not vary with marginal changes in wages. Then if $\bar{b}_{t+1}$increases, cuts in wages will result in reduced demand for the risky asset not the riskless asset as none is held. Alternatively, if $\bar{b}_{t+1}$ is cut all increases in wages will be used to purchase the risky asset.

Hence, in the equilibrium with defined benefit pensions the values $\{p_t, q_t^*, k_{0t}, k_{1t}\}$ satisfy (18) and (19), as well as

$$H_t^{\omega}(p_t, q_t^*, w_t, \bar{b}_{t+1}) + H_t^s(p_t, q_t^*, e_t^s - z_t) + \alpha_{0t}^f p_t k_{0t} = p_t k_{0t}, \tag{27}$$

and

$$J_t^{\omega}(p_t, q_t^*, w_t, \bar{b}_{t+1}) + J_t^s(p_t, q_t^*, e_t^s - z_t) + \alpha_{1t}^f q_t^* g(k_{1t}) = q_t^* g(k_{1t}), \tag{28}$$

where again the asset holdings of the pension plan, $\alpha_{0t}^f p_t k_{0t}$ and $\alpha_{1t}^f q_t^* g(k_{1t})$, are taken as given. In the next section we examine in more detail the the impact of the defined benefit pension plan on the equilibrium of the model with borrowing and short-sales constraints.

3 The Defined Benefit Pension Constraint and Equilibrium

The impact of the defined benefit pension plan on the equilibrium of the model is understood by first considering the case when workers do not save on personal account and then when they do but are subject to borrowing and short sales constraints.

3.1 Workers do not Save on Personal Account

We begin by assuming workers do not save on personal account, $a^{\omega}_{yt} = 0$.[10]. Suppose that the pension plan were to invest only in safe assets then workers bear no risk and rentiers bear all the risk. Moreover, this means that $d_{t+1} = 0$ for all R^*_{t+1}. To pay for this insurance workers' wages are relatively lower at date t. The pension plan holds only riskless claims against the safe technology, or the riskless part of the risky technology's returns (a decomposition which we ignore for the time being). To support this allocation as an equilibrium, p_t must be sufficiently high ($\phi'(k_{0t})$ low) and q^*_t sufficiently low ($g'(k_{1t})$ high). So the aggregate risk-premium, $E_t(R^*_{t+1}g(k_{1t}))/k_{1t} - R_{t+1}/k_{0t}$, has to be correspondingly high.[11],[12]

3.2 Workers do Save on Personal Account

Assume a defined benefit pension plan and that workers can save on personal account, and let $a^{\omega}_{yt} > 0$. The pension benefit, $\bar{b}_{t+1}$, is a riskless asset held by the worker. Suppose, given $\bar{b}_{t+1}$, workers are at a portfolio optimum. Now let $\bar{b}_{t+1}$ be cut, so that f^{DB}_t declines also. This will be met with an increase in w_t, $dw_t = -df^{DB}_t$ and $da^{\omega}_{yt} = -df^{DB}_t$, with da^{ω}_{yt} being invested in riskless assets, thereby restoring the original equilibrium. In terms of equation (27), $H^{\omega}_{t3}dw_t + d\alpha^f_{0t}p_tk_{0t} = 0$ (with $H^{\omega}_{t3} = -1$), so p_t does not change. If, for example, a^{ω}_{yt} does not increase to offset the decline in pension contributions, the demand for riskless and risky assets declines. In turn, aggregate safe and aggregate risky investment decline.[13] This leads to the following

[10] Mankiw (1986) was one of the first papers to investigate the implications of borrowing constraints on the expected equity risk premium. Lucas (1994) provides an detailed analysis the impact of borrowing and short sales constraints on the risk premium, when labour income is subject uninsurable shocks.

[11] Notice that here the equity premium is high when p_t is high. This means that a high equity premium corresponds to a low value of the riskless rate of interest.

[12] Applying a well-known result of Arrow (1971), Diamond and Geanakoplos (2004) show, holding the contribution rate, $f_{t,}$ fixed, substituting a small amount of defined contribution for a riskless defined benefit pension, raises the welfare of both rentiers and workers. The defined contribution plan is the same as a savings account for the worker, with the worker choosing the investment allocation. The introduction of the defined contribution plan increases the holding of risky assets by pension plans, $\alpha^f_{1t+1}p_tk_{1t}$, at the expense of safe assets, $\alpha^f_{0t+1}p_tk_{0t}$. Then p_t decreases and q_t rises, so the expected risk premium declines. Workers who are old at the time of the substitution, whose pension plan allocation is already determined, benefit from the substitution only through a rise in the safe rate of interest. Young workers gain by holding some small amount of the risky asset. Let $d\alpha^f_{1t}p_tk_{1t} = -d\alpha^f_{0t}p_tk_{0t}$, then $dW/d\alpha^f_{1t}p_tk_{1t} = E_t[W'(c^{\omega}_{ot})(R^*_{t+1} - R_{t+1})] > 0$. In the neighbourhood of $\alpha^f_{0t}p_tk_{0t} = 0$, $E_t[W'(c^{\omega}_{ot})(R^*_{t+1} - R_{t+1})] = W'(c^{\omega}_{ot})(E_t(R^*_{t+1}) - R_{t+1}) > 0$, and so the date t workers benefit from the aggregate risk-premium.

[13] Hemming and Harvey (1983) examined the extent to which (defined benefit) pension plan and non-pension retirement saving are either substitutes or indeed complements. They found some

proposition:

Proposition 1. If $\bar{b}_{t+1}$ is sufficiently high that $\alpha_{0t}^{\omega} \geq 0$ binds, then q_t^* must be lower and p_t must be higher. This leads to a higher value of k_{0t} and lower value of k_{1t}. The aggregate risk premium must be correspondingly higher.

Proof. For sufficiently high $\bar{b}_{t+1}$, workers will choose $\alpha_{0t}^{\omega} = 0$, so that a_{yt}^{ω} is invested entirely in risky assets. For higher values of $\bar{b}_{t+1}$, unless workers can short-sell the safe technology, they are forced to a corner solution and the economy will under-invest in the risky technology. To see the implications of this, in equations (23) and (24) set $\alpha_{0t}^{f} p_t k_{0t} > 0$ and $H_t^{\omega} = 0$ and $\alpha_{1t}^{f} q_t^* g(k_{1t}) = 0$ and raise $\alpha_{0t}^{f} p_t k_{0t}$. In equation (19), as $H_t^{\omega} = 0$, the higher $\alpha_{0t}^{f} p_t k_{0t}$, the higher p_t. Hence, to satisfy (24), q_t^* must be lower. From (18) and (19), k_{0t} increases and k_{1t} declines This dampens but it does not offset the effects on p_t and q_t^*. Thus the aggregate risk premium must be correspondingly higher.[14]

This last result has similarities with that in Constantanides et al (2002). They develop a three-period overlapping generations model, in which the first (junior) generation is constrained not to be able to borrow to buy equity. This means that equity prices are lower and bond prices higher than otherwise, which raises the risk-premium. Their model thus offers an explanation of the Mehra and Prescott (1985) equity-premium puzzle. In the present model, the same result arises if workers cannot undo the excess holding of the safe asset by the pension plan, because they cannot borrow against pension plan assets to buy shares in the risky technology.

4 Company Financial Policy

We have seen how the investment policy of a defined benefit pension plan can affect the technological choices of the economy. We now examine the role of the financial policy of the risky technology and how this impacts upon the previous results. We propose only to examine simple capital structures made up of debt and equity. The

evidence of complementarity, which of course runs entirely counter to the perfect substitutes requirement needed for neutrality. They do not, however, consider how this relationship is affected by the extent to which the occupational plan is funded through sponsor or worker contributions.

Borsch-Supan and Reil-Held (1997) review what is known about the relationship between the level of risk and substitution among components of retirement income.

[14] Exley, Mehta and Smith (1997) discuss the relationship of the asset allocation of DB plans relative to shareholders' personal portfolios. They argue that offsetting transactions needed to obtain neutrality are easy to implement. The present paper does not take this view.

risky technology produces a composite good, which can be unbundled into separate debt and equity return packets that can be marketed to investors separately. The risky technology is a portfolio of two claims, levered-equity, S_{1t}, and debt, B_{1t}:

$$V_{1t} = B_{1t} + S_{1t}. \tag{29}$$

We begin by considering the case of riskless debt.

Production decisions are again made by rentiers before any shares and debt are sold to the pension plan or to workers, but with rentiers anticipating the market clearing prices for shares and debt issued by the risky technology and shares in the safe technology. Returns to the risky technology are given by $R^*_{t+1}g(k_{1t}) - d_{t+1}$, which are packaged as riskless debt, which pays D_{1t+1}, and equity which pays $R^*_{t+1}g(k_{1t}) - d_{t+1} - D_{1t+1}$. Both the defined benefit pension and debt are riskless if $R^*_{t+1}g(k_{1t}) - d_{t+1} - D_{1t+1} \geq 0$ for all realisations of $R^*_{t+1}g(k_{1t}) - d_{t+1}$ so that

$$B_{1t} = p_t D_{1t+1}. \tag{30}$$

An important addition to the model is that we allow young rentiers and workers to borrow and lend with each other. Let $L^\omega_t \gtrless 0$ and $L^s_t \gtrless 0$ represent the holdings of personal loans (borrowing is negative) by workers and rentiers respectively. There are no direct loan market transactions with the pension plan. We assume that the market borrowing rate equals the lending rate, denoted by r_{t+1}, and that $L^\omega_t + L^s_t = 0$.

Let α^s_{1t} and β^s_{1t} represent the shares of the risky technology's equity and debt held in the portfolio of the representative rentier. The rentier's optimisation problem is to choose c^s_{yt}, a^s_{yt}, L^s_t, α^s_{0t}, α^s_{1t} and β^s_{1t} to solve:

$$\max\{U(c^s_{yt}) + E_t U(c^s_{ot})\} \tag{31}$$

subject to

$$e^s_{yt} = c^s_{yt} + a^s_{yt} + L^s_t, \tag{32}$$

$$a^s_{yt} = \alpha^s_{0t} p_t k_{0t} + \alpha^s_{1t} S_{1t} + \beta^s_{1t} B_{1t}, \tag{33}$$

and

$$c^s_{ot} = \alpha^s_{0t}\phi(k_{0t}) + \alpha^s_{1t}(R^*_{t+1}g(k_{1t}) - d_{t+1} - D_{1t+1}) + \beta^s_{1t} D_{1t+1} + (1 + r_{t+1})L^s_t. \tag{34}$$

Similarly, let $\{\alpha^\omega_{1t}, \beta^\omega_{1t}\}$ be the shares of equity and debt issued by the risky technology that are held in the worker's saving account. The worker's problem is to choose

a^{ω}_{yt}, c^{ω}_{yt}, L^{ω}_{t}, α^{ω}_{0t}, α^{ω}_{1t} and β^{ω}_{1t} to solve:

$$\max\{W(c^{\omega}_{yt}) + E_t W(c^{\omega}_{ot})\} \tag{35}$$

subject to

$$w^{\omega}_{yt} = c^{\omega}_{yt} + a^{\omega}_{yt} + L^{\omega}_{t}, \tag{36}$$

$$a^{\omega}_{yt} = \alpha^{\omega}_{0t} p_t k_{0t} + \alpha^{\omega}_{1t} S_{1t} + \beta^{\omega}_{1t} B_{1t}, \tag{37}$$

and

$$\begin{aligned} c^{\omega}_{ot} &= \alpha^{\omega}_{0t}\phi(k_{0t}) + \alpha^{\omega}_{1t}[R^{*}_{t+1} g(k_{1t}) - d_{t+1} - D_{1t+1}] \\ &\quad + \beta^{\omega}_{1t} D_{1t+1} + (1 + r_{t+1}) L^{\omega}_{t} + \bar{b}_{t+1}. \end{aligned} \tag{38}$$

Finally, let $\{\alpha^{f}_{1t}, \beta^{f}_{1t}\}$ be the shares of equity and debt issued by the risky technology held by the pension plan, then

$$f^{DB}_{t} = \alpha^{f}_{0t} p_t k_{0t} + \alpha^{f}_{1t} S_{1t} + \beta^{f}_{1t} B_{1t}. \tag{39}$$

The return on the pension plan is

$$\alpha^{f}_{0t}\phi(k_{0t}) + \alpha^{f}_{1t}(R^{*}_{t+1} g(k_{1t}) - d_{t+1} - D_{1t+1}) + \beta^{f}_{1t} D_{1t+1}. \tag{40}$$

The benefits from the defined benefit pension plan are:

$$\begin{aligned} b_{t+1} &= \bar{b}_{t+1} \\ \text{and } d_{t+1} &= \bar{b}_{t+1} - \alpha^{f}_{0t}\phi(k_{0t}) - \alpha^{f}_{1t}(R^{*}_{t+1} g(k_{1t}) - d_{t+1} - D_{1t+1}) - \beta^{f}_{1t} D_{1t+1}, \end{aligned} \tag{41}$$

If the risky technology issues riskless debt, the debt is sold as a perfect substitute for shares in the safe technology and the levered-equity is sold separately. Then assuming that the rentier holds both assets, from the first-order conditions we obtain

$$B_{1t} = E_t[m^{s}_{t}] D_{1t+1} \tag{42}$$

so that the price of a unit of debt income, q^{D}_{t}, is

$$q^{D}_{t} = B_t / D_{1t+1} = p_t \tag{43}$$

and

$$S_{1t} = E_t[m_t^s(R_{t+1}^* g(k_{1t}) - d_{t+1} - D_{1t+1})] = \tag{44}$$
$$E_t[m_t^s(R_{t+1}^* g(k_{1t}) - d_{t+1})] - B_{1t} = q_{1t}^* g(k_{1t}) - E_t[m_t^s d_{1t+1}] - B_{1t}$$

In writing the value of equity this way, we see that there is no need for a separate equity price, as the equity price is linearly dependent on q_{1t}^* and B_{1t}, where q_t^* is given in (9).

The value of k_{0t} is still chosen to maximise the value of the riskless technology, so that (18) is satisfied, $p_t\phi'(k_{0t}) - 1 = 0$. Assuming $d_{1t+1} = 0$ for all realisations of R_{t+1}^* so that $E_t[m_t^s d_{1t+1}] = 0$, then the value of k_{1t} is chosen to maximise the value of the risky technology, so adding (42) and (44), $V_{1t} = B_{1t} + S_{1t}$, , then maximising $V_{1t} - k_{1t}$ we obtain (19), $q_t^* g'(k_{1t}) - 1 = 0$. Of course we have not established at this stage whether the prices p_t and q_t are invariant to the leveraging of the risky technology, we turn to this in the next subsection.

When the risky technology issues riskless debt, the debt is sold as a perfect substitute for shares in the safe technology and the levered-equity is sold separately. As the value of shares in the risky technology is equal to the total value of that technology less the value of debt, as already noted, we can think of the equity market as being a market for this quantity and use the price q_t^* rather than q_t^S. The equilibrium values of $\{p_t, q_t^*, k_{0t}, k_{1t}\}$ as well as the value of loans L_t satisfy:

$$H_t^\omega(p_t, q_t^*, w_t, \overline{b}_t) + H_t^s(p_t, q_t^*, e_t^s - z_t) + B_t^\omega(p_t, q_t^S, w_t, \overline{b}_t) \tag{45}$$
$$+B_t^s(p_t, q_t^*, e_t^s - z_t) + \alpha_{0t}^f p_t k_{0t} + \beta_{1t}^f B_{1t} = p_t k_{0t} + B_{1t},$$

$$L_t^\omega(p_t, q_t^*, w_t, \overline{b}_t) + L_t^S(p_t, q_t^*, w_t, \overline{b}_t) = 0 \tag{46}$$

and

$$S_t^\omega(p_t, q_t^*, w_t, \overline{b}_t) + S_t^s(p_t, q_t^*, e_t^s - z_t) = S_{1t}, \tag{47}$$

and the value maximisation conditions, (18) and (19). The functions B_t^ω and B_t^s are respectively the demand functions for debt held by workers and rentiers. S_t^ω and S_t^s are their respective demands for levered-equity. The asset demand functions retain the net substitutes property and other properties of those in (27) and (28). Finally, the loans market, for borrowing and lending between workers and rentiers is in zero net supply. These loans will be a perfect substitute for other riskless assets with a price of p_t.

4.1 A Modigliani-Miller Proposition

We now examine the effect of changes in the composition of the liabilities financing the risky technology on the equilibrium of the economy. In the absence of binding capital market constraints, when transactions on personal account are perfect substitutes for borrowing on corporate account, we obtain a variant of Modigliani-Miller's(1958) Proposition 1.[15] This result is a useful benchmark for the subsequent analysis.

Before examining the impact of company financial policy we note the following: Individuals will not simultaneously make riskless loans and hold shares in the safe technology. First $L_t^\omega > 0$ ($L_t^s < 0$) only if $\bar{b}_{t+1} > 0$ is low, $\alpha_{0t}^\omega > 0$ and $\alpha_{0t}^s = 0$. If $\bar{b}_{t+1} > 0$ and high, then $L_t^\omega < 0$ ($L_t^s > 0$) only if $\alpha_{0t}^\omega = 0$ and $\alpha_{0t}^s > 0$. This means that loans are only used to purchase risky assets. We assume that workers and rentiers can borrow and lend at the same rate as firms.

Given the type of pension plan and its financing and taking $\{p_t, q_t^*, k_{0t}, k_{1t}\}$ as given, workers and rentiers solve the above optimisation problems. Let k_{0t}, k_{1t},p_t, q_t^*, B_{1t}, S_{1t}, $\{\alpha_{0t}^f, \alpha_{1t}^f, \beta_{1t}^f\}$, $\{\alpha_{0t}^\omega, \alpha_{1t}^\omega, \beta_{1t}^\omega, L_t^\omega\}$, $\{\alpha_{0t}^s, \alpha_{1t}^s, \beta_{1t}^s, L_t^s\}$ denote the equilibrium values of the relevant variables.

Now consider an alternative allocation in which, other things equal, the risky technology has an increased amount of debt. Let $\widehat{k}_{0t}$, $\widehat{k}_{1t}$, $\widehat{p}_t$, $\widehat{q}_t$ $\widehat{B}_{1t}$, $\widehat{S}_{1t}$, $\{\widehat{\alpha}_{0t}^f, \widehat{\alpha}_{1t}^f, \widehat{\beta}_{1t}^f\}$, $\{\widehat{\alpha}_{0t}^\omega, \widehat{\alpha}_{1t}^\omega, \widehat{\beta}_{1t}^\omega, \widehat{L}_t^\omega\}$, $\{\widehat{\alpha}_{0t}^s, \widehat{\alpha}_{1t}^s, \widehat{\beta}_{1t}^s, \widehat{L}_t^s\}$ denote the corresponding equilibrium values of the relevant variables. For simplicity, at this stage we assume $\widehat{\alpha}_{1t}^f = \alpha_{1t}^f = 0$, so that the pension plan is riskless and $d_{t+1} = 0$ for all realisations of $R_{t+1}^* g(k_{1t})$. We will relax this assumption later. We state the basic Modigliani Miller proposition and supply a straightforward proof in the Appendix.

Proposition 2. Case 1. Let $L_t^\omega > 0$, $\alpha_{0t}^\omega > 0$ and $\alpha_{0t}^s = 0$, then the equilibrium of the economy is invariant to the pension plan policy. Case 2. Alternatively let $L_t^\omega < 0$, $\alpha_{0t}^\omega = 0$ and $\alpha_{0t}^s > 0$, then again neutrality obtains.

4.2 The Defined Benefit Pension Constraint and Company Financial Policy: Riskless Debt

The defined benefit pension plan promises workers benefits of $b_{t+1} = \bar{b}_{t+1}$. Corporate borrowing itself allows workers access to more risk exposure per unit of equity investment. This will only matter if the defined benefit plan is so large that it is constraining the workers to hold excess riskless assets, with the constraint $\alpha_{0t}^\omega \geq 0$

[15] See Stiglitz (1969).

binding, and workers not being able to undertake the loans described at the end of the last section, so $L_t^\omega = 0$ also.

The next proposition shows that when the pension plan forces the constraint $\alpha_{0t}^\omega \geq 0$ to bind, workers have an increased demand for levered-equity in the risky technology. Hence, the price of risky production is higher as is the volume of risky production.

Proposition 3. If $\alpha_{0t}^\omega \geq 0$ binds and $L_t^\omega = 0$, then an increase in the risky technology's debt level causes a substitution of production towards the risky technology and a fall in the aggregate risk premium.

Proof. Begin at an allocation with no leverage, then the risky technology splits its capital structure into debt and equity. Given $\alpha_{0t}^\omega \geq 0$ is binding, workers will voluntarily undertake a secondary trade in which they reduce their holding of bonds issued by the risky technology, $\Delta B_t^\omega = \Delta\beta_{1t}^\omega B_{1t}$, and buy the now levered-equity. Rentiers who are at an interior optimum must buy $\Delta\beta_{1t}^\omega B_{1t}$ from workers and sell shares in the safe technology. However, at current prices they will retain their demand for shares in the risky technology and so must be induced to sell shares. In (45) and (47), p_t falls and q_t^* must rise. The fall in p_t will lead to a decrease in safe production. The increase in q_t^* in turn creates an incentive to increase risky production. In this case, $E_t(R_{t+1}^* g(k_{1t}) - d_{t+1})/k_{1t}$ declines and R_{t+1}/k_{0t} rises, and the aggregate risk premium falls. QED.

Thus we see that, absent corporate borrowing, in order to obtain risky assets, workers have to buy a composite asset in the form of unlevered-equity, the embedded riskless asset being locked in because of constraints. Corporate borrowing mitigates the constraining effect that limited borrowing to finance security market transactions has on the aggregate risk-premium, so the aggregate risk premium decreases with corporate borrowing so long as the workers' borrowing constraint is binding. Corporate borrowing will take place until the constraint $\alpha_{0t}^\omega \geq 0$ no longer binds and workers are indifferent to the risky technology's financial policy.[16]

[16]This result means that institutional asset holding through the pension plan is constraining the economy's risk bearing. However, the evidence is that individuals who are not covered by pension plans have only limited participation in the equity market. This may reflect low levels of wealth or risk tolerance and not constraints on capital market transactions.

4.3 The Defined Benefit Pension Constraint and Company Financial Policy: Risky Debt

Now let the risky technology issue defaultable bonds that pay $\min(D_{t+1}, R^*_{t+1}k_{1t})$. Equity now pays $\max(D_{t+1} - R^*_{t+1}k_{1t}, 0)$. We begin by assuming that the pension plan is not exposed to shortfall risk, so the pension benefit is always met, so that d_{t+1} is zero for all realisations of R^*_{t+1}. Therefore, we consider the case then when the pension plan holds only riskless assets, $\{\alpha^f_{1t}, \beta^f_{1t}\} = 0$.

If individuals can undertake the necessary transactions on personal account, the Modigliani-Miller result generalises to firms issuing risky debt. With unrestricted borrowing and lending and if call and put options on the risky technology can be traded at all exercise prices, the Modigliani-Miller proposition holds. Let C_{1t} denote the value of a call option on the risky technology with exercise price D_{t+1} and with payoff function $\max(R^*_{t+1}g(k_{1t}) - D_{1t+1}, 0)$. P_{1t} denotes the value of a put option with payoff $\max(D_{t+1} - R^*_{t+1}g(k_{1t}), 0)$. Combining levered-equity, a short put option on the risky technology with exercise price D_{1t+1}, and riskless borrowing with face value D_{1t+1}, we obtain the unlevered return on the safe technology

$$\max(R^*_{t+1}g(k_{1t}) - D_{1t+1}, 0) - \max(D_{t+1} - R^*_{t+1}g(k_{1t}), 0) + D_{1t+1} = R^*_{t+1}g(k_{1t}). \quad (48)$$

The ability to trade options for all values of D_{t+1} means that the payoff functions created by corporate borrowing are available through trade in options. For a given debt-equity ratio only two options are necessary to span the payoffs of the levered firm through holding shares in the unlevered firm. This no-arbitrage argument means that leverage does not create a wedge between the levered, V_{Lt}, and unlevered, V_{Ut}, values of the risky technology. Then we have the well-known result that

$$V_{Lt} = C_t - P_t + p_t D_{1t+1} = V_{Ut}. \quad (49)$$

This condition implies $C_{1t} = V_{Ut} - B_{1t}$, where $B_{1t} = p_t D_{1t+1} - P_{1t}$. An alternative interpretation of this condition, which is useful for current purposes, is that the equity issued by the levered technology is replicated by an investment in the equity of the unlevered technology financed by a loan, which is collateralised by unlevered-equity. That is if $D_{t+1} > R^*_{t+1}g(k_{1t})$, the borrower forfeits $R^*_{t+1}g(k_{1t})$. The debt in the risky technology is riskless debt less a put option.[17]

[17] Hellwig (1981) contains an extensive discussion of these issues. Fama and Miller (1972) and Grossman and Stiglitz (1980) also discuss these issues. The latter point out that the ability to buy

The next step is to re-examine the worker's and rentier's optimiation problems with a riskless defined benefit plan with benefit level $\bar{b}_{t+1}$, when the risky technology issues risky debt. Initially we assume that borrowing and lending takes place between rentiers and workers and call and put options can be traded between rentiers and workers. The value of the call is denoted by C_{1t} and the put by P_{1t}. Rentiers and workers shares of these options are γ^s_{1t} and γ^ω_{1t} for the call; and δ^s_{1t} and δ^ω_{1t} for the put. Long and short positions must be possible if options are traded, however, an equilibrium condition is that there is zero net trade: $\gamma^s_{1t} + \gamma^\omega_{1t} = 0$ and $\delta^s_{1t} + \delta^\omega_{1t} = 0$. The rentier's optimisation problem is to choose c^s_{yt}, a^s_{yt}, L^s_t, α^s_{0t}, α^s_{0t}, β^s_{1t} γ^s_{1t} and δ^s_{1t} to solve:

$$\max\{U(c^s_{yt}) + E_t U(c^s_{ot})\} \tag{50}$$

subject to

$$e^s_{yt} = c^s_{yt} + a^s_{yt} + v^s_t, \tag{51}$$

$$a^s_{yt} = \alpha^s_{0t} p_t k_{0t} + \alpha^s_{1t} S_{1t} + \beta^s_{1t} B_{1t}, \tag{52}$$

$$v^s_t = L^s_t + \gamma^s_{1t} C_t + \delta^s_{1t} P_t, \tag{53}$$

and

$$\begin{aligned} c^s_{ot} \quad = \quad & \alpha^s_{0t} R_{t+1} \phi(k_{0t}) + \alpha^s_{1t}(R^*_{t+1} g(k_{1t}) - D_{1t+1}, 0) \\ & + \beta^s_{1t} \min(D_{1t+1}, R^*_{t+1} g(k_{1t})) + \gamma^s_{1t} \max(R^*_{t+1} k_{1t} - D_{1t+1}, 0) \\ & + \delta^s_{1t} \min(D_{1t+1} - R^*_{t+1} g(k_{1t}), 0) + (1 + r_{t+1}) L^s_t. \end{aligned} \tag{54}$$

The worker's problem is to choose a^ω_{yt}, c^ω_{yt}, L^ω_t, α^ω_{0t}, α^ω_{1t}, β^ω_{1t} γ^ω_{1t} and δ^ω_{1t} to solve:

$$\max\{W(c^\omega_{yt}) + E_t W(c^\omega_{ot})\} \tag{55}$$

subject to

$$w^\omega_{yt} = c^\omega_{yt} + a^\omega_{yt} + v^\omega_t, \tag{56}$$

$$a^\omega_{yt} = \alpha^\omega_{0t} p_t k_{0t} + \alpha^\omega_{1t} S_{1t} + \beta^\omega_{1t} B_{1t}, \tag{57}$$

$$v^\omega_t = L^\omega_t + \gamma^\omega_{1t} C_t + \delta^\omega_{1t} P_t, \tag{58}$$

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shares on margin in this way means that the market is essentially complete. They also argue that unless shares are held in escrow, lack of verifiability may lead to multiple pledging of securities and a breakdown of this mechanism.

and

$$
\begin{aligned}
c_{ot}^{\omega} &= \alpha_{0t}^{\omega} R_{t+1}\phi(k_{0t}) + \alpha_{1t}^{\omega} \max(R_{t+1}^{*} g(k_{1t}) - D_{1t+1}, 0) \\
&+\beta_{1t}^{\omega} \min(D_{1t+1}, R_{t+1}^{*} g(k_{1t})) + (1 + r_{t+1}) L_t^{\omega} \\
&+\gamma_{1t}^{\omega} \max(R_{t+1}^{*} g(k_{1t}) - D_{1t+1}, 0) + \delta_{1t}^{\omega} \min(D_{1t+1} - R_{t+1}^{*} g(k_{1t}), 0) + \bar{b}_{t+1}.
\end{aligned} \tag{59}
$$

Before proceeding note, that in the above formulation, the call option is nothing more than levered-equity; so the only function of the call option market is to overcome a short-sales constraint in the equity market. It follows that if both workers and rentiers hold long positions in equity there will be no trade in call options.

Then assuming that the rentier holds both risky debt and equity, from the first-order conditions we obtain

$$B_{1t} = E_t[m_t^s \min(D_{t+1}, R_{t+1}^{*} g(k_{1t}))] \tag{60}$$

and

$$S_{1t} = E_t[m_t^s \max(R_{t+1}^{*} g(k_{1t}) - D_{t+1}, 0)] = q_{1t}^{*} g(k_{1t}) - B_{1t} \tag{61}$$

Risky debt is not a perfect substitute for shares in the safe technology, so it is traded in a separate market with its own price per unit of income, q_{1t}^D. The difference between q_{1t}^D and p_t reflects the default risk premium given by the value of the embedded put option. The value of the put option with exercise price D_{t+1} on the whole of the risky technology is

$$P_{1t} = E_t[m_t^s \max(D_{t+1} - R_{t+1}^{*} g(k_{1t}), 0)] \tag{62}$$

However, we note here that in our economy if out options are traded, it will be for a fraction δ_{1t} of this amount. This value in turn can be seen to be increasing in D_{t+1}.

As in the multi-asset case of Aura et al (2002), the asset demand functions retain the net substitutes property and other properties of the demand functions in (27) and (28). The equilibrium solution, $\{p_t, q_t^{*}, k_{0t}, k_{1t}\}$ as well as the value of loans L_t and the put option P_t, satisfy:

$$H_t^{\omega}(p_t, q_t^{*}, q_t^D, w_t, \bar{b}_t) + H_t^s(p_t, q_t^{*}, q_t^D, e_t^s - z_t) + \alpha_{0t}^f p_t k_{0t} = p_t k_{0t}, \tag{63}$$

$$B_t^{\omega}(p_t, q_t^{*}, q_t^D, w_t, \bar{b}_t) + B_t^s(p_t, q_t^{*}, q_t^D, e_t^s - z_t) + \beta_{1t}^f B_{1t} = B_{1t}, \tag{64}$$

$$L_t^{\omega}(p_t, q_t^{*}, w_t, \bar{b}_t) + L_t^S(p_t, q_t^{*}, w_t, \bar{b}_t) = 0, \tag{65}$$

$$S_t^{\omega}(p_t, q_t^{*}, q_t^D, w_t, \bar{b}_t) + S_t^s(p_t, q_t^{*}, q_t^D, e_t^s - z_t) = S_{1t}, \tag{66}$$

and

$$\delta_{1t}^{\omega} P_{1t}(p_t, q_t^*, w_t, \bar{b}_t, D_{1t}) + \delta_{1t}^{s} P_{1t}(p_t, q_t^*, w_t, \bar{b}_t, D_{1t}) = 0 \tag{67}$$

and because of value maximisation (18) and (19) also. Although the put option is a derivative security, in this set up it is not necessarily redundant as it allows agents to trade a particular component of the corporate bond on a secondary basis after having purchased the bond in the primary market. That is closing down the options market will result in a different equilibrium of the economy unless markets are complete.[18]

The proposition below shows that corporate leverage is irrelevant if the appropriate options are traded.

Proposition 4. If the defined benefit promise, $\bar{b}_{t+1}$, is not forcing the constraint $\alpha_{0t}^{\omega} \geq 0$ to bind, so $\alpha_{0t}^{\omega} > 0$ and $\alpha_{0t}^{s} > 0$, and individuals can trade the options contracts we have described, changes in the risky technology's financial policy have no effect.

Proof. This is a variant of the Modigliani-Miller proposition. If it applies, then both workers and rentiers simply hold equity and corporate debt issued by the risky technology in the proportions issued and make offsetting transactions in loans and put options to replicate the initial equilibrium. To see this, note that in (59), the sum of terms

$$\begin{aligned} &\alpha_{1t}^{\omega} \max(R_{t+1}^* g(k_{1t}) - D_{1t+1}, 0) + \beta_{1t}^{\omega} \min(D_{1t+1}, R_{t+1}^* g(k_{1t})) \\ &+(1 + r_{t+1}) L_t^{\omega} + \delta_{1t}^{\omega} \min(D_{1t+1} - R_{t+1}^* g(k_{1t}), 0) + \bar{b}_{t+1} \end{aligned} \tag{68}$$

can be made independent of ΔD_{1t+1} provided $(1+r_{t+1})\Delta L_t^{\omega} = (\beta_{1t}^{\omega} - \alpha_{1t}^{\omega})\Delta D_{1t+1}$ and $\delta_{1t}^{\omega} = -(\beta_{1t}^{\omega} - \alpha_{1t}^{\omega})$. Similarly, in (53)

$$\begin{aligned} &\alpha_{1t}^{s}(R_{t+1}^* g(k_{1t}) - D_{1t+1}, 0) + \beta_{1t}^{s} \min(D_{1t+1}, R_{t+1}^* g(k_{1t})) \\ &+\delta_{1t}^{s} \min(D_{1t+1} - R_{t+1}^* g(k_{1t}), 0) + (1 + r_{t+1}) L_t^{s} \end{aligned} \tag{69}$$

is independent of ΔD_{1t+1} provided $\Delta L_t^{s} = -\Delta L_t^{\omega}$ and $\delta_{1t}^{s} = -(\beta_{1t}^{s} - \alpha_{1t}^{s}) = -\delta_{1t}^{\omega}$. QED.

Now consider the implications of workers being constrained by the pension plan. They will have a demand for levered-equity and so the risky technology's value will be

[18] See Ross (1976).

affected by its financial policy and it will have an incentive to issue bonds, lowering its cost of capital and expanding production.

Proposition 5. If the constraint $\alpha_{0t}^{\omega} \geq 0$ binds and $L_t^{\omega} = 0$, then an increase in the leverage of the risky technology leads to a decrease in safe production, an increase in risky production and a decline in the aggregate risk premium.

Proof. The corporate bond is a package of a riskless bond and a short put option on the risky technology. If they could strip the riskless bond out of the corporate bond, to overcome the constraint, $\alpha_{0t}^{\omega} = 0$, workers would sell the riskless part of the bond and use the proceeds to buy levered-equity in the risky technology, now levered by risky debt. They must also adjust their put option exposure by trading with rentiers. From the workers perspective, this is equivalent to borrowing on margin to buy shares in levered-equity. However, rentiers substitute the riskless component of corporate debt for holding shares in the safe technology but retain their demand for shares in the risky technology. The value of p_t falls and this leads to a decrease in safe production and via an increase in q_t^* to an increase in risky production. Thus $E_t(R_{t+1}^* g(k_{1t}))/k_{1t}$ declines and R_{t+1}/k_{0t} rises, so that the aggregate risk premium falls. QED.

Finally, if the riskless part of the bond cannot be sold separately, the same result as the above could be obtained if put options can be traded, so the same exposure to the riskless element in the bond and the put can be achieved. Matters are different if the riskless part of the bond cannot be sold separately and put options cannot be traded. Then the risky corporate bond cannot be hedged with put options, so investors have to bear the default risk exposure. Because of the embedded short put that cannot be hedged rentiers pay less for the bond. Then the workers will stop short of selling enough bonds to buy the same amount of levered-equity as in the case above. Workers are effectively retaining more safe assets; so p_t falls by less and q_t^* rises by less, and hence the aggregate risk premium is relatively higher.

5 Corporate Financial Policy and Pension Plan Risk

With a defined benefit pension plan, if the pension plan holds risky assets and the risky technology issues defaultable debt, the pension promise, $\bar{b}_{t+1}$, is exposed to plan sponsor risk. It itself has the features of a risky corporate bond. If the pension benefit is exposed to plan sponsor risk in this way, workers either demand higher wages, higher pension benefits, or insurance against the pension plan shortfall. But this insurance can only be paid for sure if the economy has achieved this income level for sure. It is equivalent to a one-hundred percent funding requirement for the plan itself.[19] In the current model, either the risky technology must hold a buffer of riskless assets, such that the pension promise can be met for sure; or if insurance is purchased from rentiers, they themselves hold sufficient riskless assets. If the risk of the pension is covered by insurance, the analysis is the same as with a riskless pension. Therefore, suppose not and that the adjustment is through wages. Corporate bonds and pensions now both have risk characteristics that vary with the risky technology's debt-equity ratio.

In the event of default, we assume that corporate bondholders recover any residual value in the risky technology, but this could in principle be shared directly with the pension plan, depending upon insolvency laws and rules governing the winding up of pension plans. Assume that the firm cannot default on the pension without declaring bankruptcy.[20] If $R^*_{t+1}g(k_{1t}) - d_{t+1} - D_{1t+1} < 0$ the firm declares bankruptcy and pays debt holders $\min[D_{1t+1}, R_{t+1}g(k_{1t})]$. The payment to pension plan in the event of bankruptcy is then $\max[R^*_{t+1}g(k_{1t}) - D_{1t+1}, 0]$. [21]

The pension shortfall itself is given by

$$\begin{aligned} d_{t+1} &= \bar{b}_{t+1} - \alpha^f_{0t}R_{t+1}\phi(k_{0t}) - \alpha^f_{1t}\max[R^*_{t+1}g(k_{1t}) - d_{t+1} - D_{1t+1}, 0] \\ &\quad -\beta^f_{1t}\min[D_{1t+1}, R^*_{t+1}g(k_{1t})]. \end{aligned} \tag{70}$$

If the firm is solvent, then d_{t+1} is covered out of the firm's income income. In the event of the firm defaulting and the plan being in deficit the workers are exposed to a loss of $\bar{b}_{t+1} - \alpha^f_{0t}R_{t+1}\phi(k_{0t}) - \beta^f_{1t}R^*_{t+1}g(k_{1t})$. From this we can see that the workers can fully hedge the risk of being exposed to an uncovered shortfall by holding (possibly through the pension plan) a put option that pays $\bar{b}_{t+1} - \alpha^f_{0t}R_{t+1}\phi(k_{0t}) - \beta^f_{1t}R^*_{t+1}g(k_{1t})$ in the event of the firm defaulting.

Suppose that in an initial allocation the pension benefit $\bar{b}_{t+1}$ is riskless, in the sense that any shortfall will be covered by the solvent plan sponsor with probability one. Now let the risky technology raise debt at date t to a level that exposes the pension

[19] Here it should be recognised that with only one risky technology, there is no scope in the model for risk pooling that allows transfers from in-surplus to in-deficit schemes.

[20] In the UK, this has been the case since Pension Plan Winding Up Arrangements of 2003.

[21] If $d_{t+1} < 0$, then the pension plan surplus can preserve solvency of the firm when $R^*_{t+1}g(k_{1t}) - D_{1t+1} < 0$.

plan to shortfall risk. If the pension benefit is not constraining the worker's portfolio choice, so that $\bar{b}_{t+1}$ lies within the span of his optimal exposure to the riskless asset, he will demand additional wages at date t that are sufficient to finance the above set of transactions, thereby fully insuring $\bar{b}_{t+1}$. This replicates the situation in the previous subsection, the riskless assets acquired on personal account replacing those not held through the pension plan, with options transactions replicating the exposure to the risky technology.

We maintain the assumption that the pension plan investment policy is given and examine the effect of the pension being exposed to shortfall risk through the financial policy of the risky technology. However, we assume that workers do not have access to the above hedging opportunities. Hence, to adjust their risk exposure workers have to trade claims on the technology directly. For example, instead of selling call options on the risky technology they have to sell levered equity directly. There are two principal cases to consider:

Case1: The constraint $\alpha_{0t}^{\omega} \geq 0$ is not binding. The following proposition shows that in this case, issuing risky debt at date t and thereby exposing the pension plan to shortfall risk, reduces the value of the risky technology and will therefore not happen in equilibrium.

Proposition 6. Let the firm issue risky debt and expose the pension plan to shortfall risk. Under the conditions of Case 1, this leads to an increase in safe production and to a decrease in the price of risky production; and thereby to a lower level of risky production.

Proof. Given the pension plan investments, if the risky technology substitutes risky debt for equity and exposes the pension plan to company default, to maintain the value of the pension plan, workers will demand higher contributions, f_t^{DB}. But as the default risk of the pension plan is (in this case) one-hundred percent correlated with the default risk of the risky technology, to hedge the pension plan risk they will demand less levered-equity. They are, however, unable to buy a long put option, but will sell risky debt (which has an embedded short put). However, they can increase their demand for the riskless asset, so α_{0t}^{ω} is higher. To induce rentiers to participate in offsetting transactions, p_t must be higher. The higher value of p_t implies higher safe production and via a fall in q_t^*, lower risky production. QED.

Case 2: The constraint $\alpha_{0t}^{\omega} \geq 0$ is binding and $L_t^{\omega} = 0$. In this case matters are quite different to the above. Now exposing the pension plan to shortfall risk through the issuance of risky debt, will increase the demand for levered-equity and raises the price of risky production, so the amount of risky debt issued and risky production will be relatively high in equilibrium.

Proposition 7. Let the pension plan be exposed to shortfall risk. Under the conditions of Case 2, this will lead to a decrease in safe production and an increase in risky production and consequent decrease in the aggregate risk premium. However, for sufficiently high levels of company risky debt, the risk of the pension crosses a threshold, such that the constraint, $\alpha_{0t}^{\omega} \geq 0$ no longer binds and the argument of Case 1 applies.

Proof. Consider an initial allocation with no risky debt, where the defined benefit plan is constraining the worker's choice, with the constraint $\alpha_{0t}^{\omega} \geq 0$ binding and $L_t^{\omega} = 0$. The risky technology now issues risky debt, thereby exposing the pension plan to the risk of default. Workers will demand higher contributions, f_t^{DB}. With the constraint $\alpha_{0t}^{\omega} \geq 0$ binding, workers sell debt in the risky technology and increase their holding of levered-equity in that technology. Rentiers participate in offsetting transactions. The riskless part of the risky debt substitutes for their holding of shares in the safe technology. This causes a fall in p_t. The lower value of p_t leads to a decrease in safe production and through a rise in q_t^* to an increase in risky production and a consequent decrease in the aggregate risk premium. This applies so long as the constraint on the worker's problem is binding and so is monotonic. Therefore, at some level of risky debt issuance, the risk of the pension crosses a threshold such that the constraint $\alpha_{0t}^{\omega} \geq 0$ no longer binds. Then, Case 1, applies and there is no incentive to issue more debt. QED.

Proposition 6 shows that if the defined benefit pension is large and is at risk, resulting from the leverage of the sponsoring technology, workers will want to undo this exposure. In this case, the non-neutrality of company financial policy is not the result of binding borrowing constraints but arises from an inability of workers to fully hedge the resulting risk exposure. Hence, workers may hold low amounts of equity because that is necessary to reduce exposure to sponsor risk. Indeed if participation costs in the equity market are added to the model, the risk of default on pension plan obligations may cause workers not to participate in the equity market. [22]

[22] This case becomes stronger if wages are also subject to risks correlated with those of the risky technology.

6 Conclusions

This paper has presented a simple yet illustrative general equilibrium analysis of the impact of pension plan funding and asset allocation on the economy's technology choices and asset prices. The results depend crucially upon the constraints facing workers in the capital market. If (occupational) pension plans make up a significant proportion of workers retirement income, then defined benefit plans can distort the economy's portfolio composition and technology choices towards the safe technology. This will be undone to the extent that workers take offsetting transactions with the other elements of their portfolio, which theory predicts they will. However, this would mean that we observe members of defined benefit pension plans holding significant amounts of equity on personal account that may involve borrowing against pension plan assets to purchase them. There is little evidence of this.

The analysis in the paper shows that if workers are constrained in the capital market, then large pension deficits will interact with company financial policy. In particular, leveraging the risky technology may allow workers to undo an over-exposure to riskless investments, which improves risk sharing in the economy, leading to more risky production than otherwise and a reduced aggregate risk premium.

If the risky technology issues defaultable debt, matters are more complex. This is particularly so if the pension plan holds risky assets and is exposed to shortfall risk. For a given amount of defaultable debt and relatively small pension benefits, the shortfall risk to the pension plan will be small. If this is the case, then capital market constraints on the workers will be the dominant factor and the implications of leveraging the risky technology are the same as in the case of fully funded pensions. However, when pension plan shortfall risk is large, the worker's borrowing constraint no longer applies and the dominant motive is that of hedging pension plan shortfalls. Workers demand compensation for bearing this risk and for undertaking the hedging. Then the risky firm does not benefit from debt issue, so the economy will undertake relatively more safe production.

Corporate default risk will affect the value of pensions. The correlation between pension plan shortfall and company default only arises if the pension is underfunded. Introducing more than one risky technology into the economy creates diversification opportunities. This means that the risk-return opportunities for investors are improved. This will have implications for the balance of investment in the economy between safe and risky investment. However, it also means that the pension plan can deliver a given benefit with lower risk. For example, assume a joint normal probability distribution of returns on the multiple risky technologies and suppose that we create a composite risky asset that holds risky assets in the proportions issued, which is the market portfolio. For a given pension plan portfolio composition between safe and risky assets, the value at risk due to unexpected loss (or shortfall) will be decreasing in the number of assets in the risky portfolio. However, unless the pension plan is fully funded with riskless assets, the leverage of the plan sponsor affects the risk of the pension plan, so that the principal arguments of the paper still apply.

In the case described above the correlation between pension plan shortfall and plan sponsor solvency plays a key role. Pension plan diversification reduces this correlation. However, default on the plan deficit still depends upon the risk of the plan sponsor. It is this risk that workers must hedge. Ideally they do this with the appropriate level of insurance or put options. But if these opportunities are not available, it must take place through a policy that optimises shortfall risk exposure as part of the general portfolio problem.

The results in the paper on the impact of the scale of defined benefit pension plan benefits on the general equilibrium of the economy and the interaction with the plan sponsor's financial policy are essentially macroeconomic. These results can be generalised to the case of many risky technologies, where propositions concern aggregates of defined benefit pensions and the composition of aggregate balance sheets. Propositions about risk premia will then also relate to index risk premia and not individual stocks.

Finally, we turn to the absence of trade between generations. The model presented only allows trade within a generation and does not allow for trade between generation. Hence it is a stochastic variant of the type of economy examined by Diamond (1965), but with no social security system or government bonds. If we were to extend the analysis to consider its dynamic properties more fully, there will exist the possibility of over-accumulation of real assets and hence the possibility of a social security system or other forms of intergenerational transfers raising welfare.[23] There may also be welfare gains to insurance schemes that reallocate resources at a moment in time between generations. These will in turn be more complex if we introduce labour income risk by allowing, for example, shocks to the endowment of labour.

[23] Although in our model we allow workers limited participation in equity markets, the Diamond and Geanakoplos (2004) argument that the social security system itself should have a funded element that is invested in equities may stiill apply. That will depend upon the nature of the constraints that bind in the economy absent the system.

7 Appendix

Proof of Proposition 2. Case 1. Suppose to begin with that in the new allocation, $r_{t+1} = \widehat{r}_{t+1}$; $k_{0t} = \widehat{k}_{0t}$; and $k_{1t} = \widehat{k}_{1t}$. Keeping the funding level of the pension plan, f_t, fixed, the plan simply substitutes debt for equity. With $\widehat{\alpha}^f_{0t} = \alpha^f_{0t}$ and $\widehat{\alpha}^f_{1t} = \alpha^f_{1t} = 0$ fixed,

$$f^{DB}_t = \widehat{\alpha}^f_{0t}\widehat{k}_{0t} + \widehat{\beta}^f_{1t}\widehat{B}_{1t}, \tag{71}$$

and this yields

$$\bar{b}_{t+1} = \widehat{\alpha}^f_{0t}R_{t+1}\phi(\widehat{k}_{0t}) + \widehat{\beta}^f_{1t}\widehat{D}_{1t+1} \tag{72}$$

at date $t+1$. This return on the plan is unchanged if $\widehat{\beta}^f_{1t}\widehat{D}_{1t+1} = \beta^f_{1t}D_{1t+1}$, which at given r_{t+1} implies that $\widehat{\beta}^f_{1t}\widehat{B}_{1t+1} = \beta^f_{1t}B_{1t+1}$, so that given $\widehat{B}_{1t+1} > B_{1t+1}$, $\widehat{\beta}^f_{1t} < \beta^f_{1t}$.

In the original situation, in (38) the worker's exposure to D_{1t+1} is $-(\alpha^\omega_{1t} - \beta^\omega_{1t})$. Given the new value of $\widehat{\beta}^f_{1t} < \beta^f_{1t}$ and $\widehat{D}_{1t+1} > D_{1t+1}$, the representative worker maintains $\widehat{\alpha}^\omega_{0t} = \alpha^\omega_{0t}$ and $\widehat{\alpha}^\omega_{1t} = \alpha^\omega_{1t}$ and reduces lending, $\widehat{L}^\omega_t$, and also increases his holding of corporate bonds, $\widehat{\beta}^\omega_{1t}$, such that

$$\widehat{L}^\omega_t = L^\omega_t + [(\widehat{\alpha}^\omega_{1t} - \widehat{\beta}^\omega_{1t})\widehat{B}_{1t} - (\alpha^\omega_{1t} - \beta^\omega_{1t})B_{1t}]. \tag{73}$$

Workers consumption in the new situation is given by

$$\begin{aligned}\widehat{c}^\omega_{ot} &= \widehat{\alpha}^\omega_{0t}\phi(\widehat{k}_{0t}) + \widehat{\alpha}^\omega_{1t}[R^*_{t+1}\widehat{k}_{1t} - \widehat{D}_{1t+1}] \\ &+ \widehat{\beta}^\omega_{1t}\widehat{D}_{1t+1} + (1 + r_{t+1})\widehat{L}^\omega_t + \bar{b}_{t+1},\end{aligned} \tag{74}$$

but on substituting (73), $\widehat{\alpha}^\omega_{0t} = \alpha^\omega_{0t}$ and $\widehat{\alpha}^\omega_{1t} = \alpha^\omega_{1t}$, this is the same as (38).

Now consider rentiers. In the original situation, with $L^s_t < 0$, they will not borrow to purchase shares in the safe technology or riskless corporate bonds: $\alpha^s_{0t} = 0$ and $\beta^s_{1t} = 0$, so

$$c^s_{ot} = \alpha^s_{1t}[R^*_{t+1}g(k_{1t}) - D_{1t+1}] + (1 + r_{t+1})L^s_t. \tag{75}$$

In the new situation

$$\widehat{c}^s_{ot} = \widehat{\alpha}^s_{1t}[R^*_{t+1}g(\widehat{k}_{1t}) - \widehat{D}_{1t+1}] + (1 + r_{t+1})\widehat{L}^s_t. \tag{76}$$

To replicate the original situation, the rentier should borrow

$$\widehat{L}^s_t = L^S_t - [(\widehat{\alpha}^\omega_{1t} - \widehat{\beta}^\omega_{1t})\widehat{B}_{1t} - (\alpha^\omega_{1t} - \beta^\omega_{1t})B_{1t}], \tag{77}$$

and so decrease borrowing, $\widehat{L}_t^s$, and keep their share of equity in the risky technology constant. But if $\widehat{\alpha}_{1t}^s = \alpha_{1t}^s$, using (77), condition (76) is equivalent to (75). Thus, the budget sets of both workers and rentiers are unchanged, so that the same choice of both risky and safe assets is obtained as in the original equilibrium. Moreover, the market equilibrium conditions remain the same. In particular, the increase in the demand for riskless corporate bonds by workers exactly matches the increase in the supply, with borrowing and lending between workers and rentiers netting out. Hence, our original assumption that $r_{t+1} = \widehat{r}_{t+1}$ is fixed is justified and so r_{t+1} and hence p_t remain unchanged, as are q_t^*, k_{0t}, and k_{1t}, which completes the argument.

Case 2. The alternative case, where $L_t^\omega < 0$, $\alpha_{0t}^\omega = 0$ and $\alpha_{0t}^s > 0$ is symmetric to the above, provided workers can borrow on personal account to buy shares in the risky technology. Even though workers are (almost certainly) more risk averse than rentiers, this case is realistic if the riskless defined benefit pension promise, $\bar{b}_{t+1}$, is sufficiently high. Then, even if the short-sales constraint on the safe technology binds, if workers can borrow from rentiers to buy shares in the risky technology, the Modiglian-Miller proposition still applies.

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