# Liquidity and Capital Structure *

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December 3, 2006

**Abstract**

This paper solves for a firm's optimal cash holding policy within a continuous time, contingent claims framework that has been extended to incorporate most of the significant contracting frictions that have been identified in the corporate finance literature. Under the optimal policy the firm targets a level of cash holding that is a non-monotonic function of business conditions and an increasing function of the amount of long-term debt outstanding. By allowing firms to either issue equity or to borrow short-term, we show how share issue and dividends on the one hand and cash accumulation and bank borrowing on the other are all mutually interlinked. We calibrate the model and show that it matches closely a wide range of empirical benchmarks including cash holdings, leverage, equity volatility, yield spreads, default probabilities and recovery rates. Furthermore, we show the predicted dynamics of cash and leverage are in line with the empirical literature. Despite the presence of significant contracting frictions we show that the model exhibits a near irrelevance of long-term capital structure property. Furthermore, the optimal policy exhibits a state-dependent hierarchy among financing alternatives that is consistent with recent explorations of pecking order theory. We calculate the agency costs generated by the conflict of interest between shareholders and creditors regarding the firm's liquidity policy and show that bond covenants that establish an earnings restriction on dividend payments may be value increasing.

*We have benefitted from the comments of two referees and seminar participants at the LSE Financial Markets Group, Warwick University, Toulouse University, the Goethe University of Frankfurt, the Symposium on Dynamic Corporate Finance and Incentives at Copenhagen Business School, the European Finance Association 2004 Meeting, and the Skinance Conference in Hemsedal 2006. Responsibility for all views expressed and for any errors is our own.

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# Liquidity and Capital Structure

> "Kerkorian's numbers just don't add up," said Nicholas Lobaccaro, an auto analyst with S G Warburg. "Ford says it needs double-digit billions of cash to survive the next downturn in the market. General Motors says it wants to put aside $13-15 billion. How can anyone believe Kerkorian when he says $2 billion is enough?"[for Chrysler] [1]

## Introduction

The quotation above illustrates the range of opinions that can be found among practitioners about the levels of liquid assets that are appropriate for firms. This observation is not an isolated case - it is often remarked that many large corporations carry surprisingly large amounts of cash on their balance sheets. However, finance theory has given very little precise guidance as to how much cash is enough. In this paper we attempt to fill this gap by directly asking what is the optimal policy toward holding liquid assets in the firm. More specifically we will identify firm characteristics which lead certain firms to hold relatively large amounts of cash and others to hold little cash. That is, we will generate a number of *cross-sectional* predictions about cash holding. We will be even more interested in the *time series* implications of our analysis. We will identify the business conditions that lead firms to hold relatively more cash and those associated with less cash. We also look at the persistence or mean reversion of cash.

In our framework, shareholders of the levered firm accumulate cash as precaution against a sustained downturn in business conditions. In effect, cash holding grants a *survival option* to shareholders who otherwise may find their share of the firm's long-term earnings prospects wiped-out through bankruptcy. The amount of cash that is required to achieve a level of protection will be greater for firms with more long-term debt. Thus, our analysis naturally leads us to explore the link between capital structure and cash holding. Furthermore, by allowing firms to either issue equity or to borrow short-term, we show how share issue and dividends on the one hand and cash accumulation and bank borrowing on the other are all mutually interlinked.

[1] Sunday Times of London, April 23, 1995. This quote refers to the attempt by Kirk Kekorian to take over Chrysler Motors arguing that in doing so he could increase shareholder value by returning most of Chrysler's $7.5 billon cash reserve to shareholders.

Our study contributes to the literature in several significant ways. First, we answer the question set out in the first paragraph. That is, we characterize completely the properties of a *dynamically* optimal liquidity policy and identify the factors that are quantitatively important determinants of that policy. We do this within a continuous time, contingent claims framework that has been extended to incorporate most of the significant contracting frictions that have been identified in the corporate finance literature. We show that under the optimal policy the firm targets a level of liquidity that is a non-monotonic function of the firm's business conditions. Actual liquid asset holdings are dependent upon the path of realized cash flows and will tend to increase as business conditions improve from low levels. However, if expected cash flows rise to relatively high levels liquid asset holding is reduced as the survival value of cash falls. By simulating the model under the optimal policy we show that it is consistent with the observed dynamics of cash holding as documented by Opler *et al* (1999).

Second, our theory contributes to the theory of optimal capital structure by incorporating into the same model cash holding, equity issuance, and *both* short-term and long-term debt. Myers (1984) identified the two main competing lines of thought on capital structure as static "trade-off" theories and dynamic "pecking order" theories. It has been recognized that a theory of capital structure based on the static trade-off of tax shields and bankruptcy costs is difficult to square with observed cross-sectional variation in leverage. Myers conjectured that costs of adjusting capital structure might account for some empirical patterns but argued in the absence of explicitly modeling the consequences of these costs trade-off theory must be found wanting. Fischer *et al* (1989) demonstrated that in the face of discrete adjustment costs optimal level of debt would be adjusted only from time to time, implying that observed leverage would exhibit inertia. More recent explorations of *dynamic trade-off models* of capital structure have shown that these models are consistent with a number of empirical regularities. Our theory contributes to this line of research by combining in the same analysis the dynamic choice of liquid assets, short-term debt and equity issuance. We show that given a level of long-term debt, under the optimal liquidity/short-term debt/equity issuance policy observed leverage will be mean reverting and will be negatively correlated with lagged earnings which is in line with previous findings. We extend previous findings by showing that at a given time, the value of the firm will be fairly insensitive to the level of long-term debt outstanding. That is, we establish the approximate irrelevance of long-term capital structure. The reason is that for different levels of long-term debt the firm is able to adapt its liquidity/short-term borrowing and equity issuance policy so as to find a similar balance of the various contracting frictions faced by the firm. Thus we support the "neutral mutations" hypothesis of Miller (1977) that firms may fall into a particular policy for long-term debt by habit or managerial fancy and that this will not adversely affect firm value if they adapt their policy on cash and short-term debt appropriately. Furthermore, we show that the predictions of pecking order theory and dynamic trade off theory are mutually compatible. Specifically, in line with recent work on pecking order theory, we show that our model exhibits a *state dependent* hierarchy among financing alternatives and that the traditional pecking order of internal first, then debt and finally equity applies for firms that are not debt capacity constrained.

To gain confidence that the effects that we identify are economically significant we calibrate our model and show that it has quantitative predictions in line with a number of empirical benchmarks including average liquidity holdings, leverage ratios, yield spreads, expected default probabilities, expected loss given default, and equity volatilities. This lends support for a number of our modeling choices. We also explore the comparative static properties of our model and find that they are generally intuitive and consistent with empirical evidence. One noteworthy finding gives new perspective on the "asset substitution effect." Specifically, over plausible ranges of parameters and business conditions, we find that increases in cash flow volatility are value reducing *for both creditors and shareholders.* This raises a doubt about the standard teaching of static corporate finance theory and is more in line with empirical results about the asset substitution effect. We explore in some detail the question of whether there may a significant divergence of interests of creditors and shareholders regarding the firm's cash holding policy. We find that under normal and good business conditions, the liquidity policy that results from maximizing firm value is very similar to the one that results from share value maximization. Significant agency costs arise only in the face of very poor business conditions. Finally we explore bond covenants that might remedy this suboptimality and find that covenants that impose earnings restrictions on dividend payments can be value increasing.

Recently, Mello and Parsons (2000), Hennessy and Whited (2005) and Rochet and Villeneuve (2004) have provided theoretical treatments of the firm's financial policy which determine liquidity holdings. The models of these papers share some but not all of the features of the present paper. Hennessy and Whited are mainly concerned with the dynamic capital structure implications of the analysis. The papers by Mello and Parsons and by Rochet and Villeneuve are concerned with the benefits of hedging. We will compare our results with all three papers below. More generally, our modelling of the firm can be viewed as extending recent work on contingent claims analysis including Leland (1994), Anderson, and Sundaresan (1996), Mella-Barral and Perraudin (1996). It will be seen that by enriching this framework to incorporate greater institutional realism, we are led to use numerical solution techniques, and in this respect we have something in common with Broadie, Chernov, and Sundaresan

(2004) and Titman, Tompaidis and Tsyplakov (2004).

A further detailed discussion of the relation of our model and findings to the literature will be given once we have presented our model in detail. The remainder of the paper is organized as follows. In Section 1 we introduce the model and the dynamic programming technique we use to solve it. We explore in some detail a benchmark solution of the model and discuss its relation to previous empirical findings. In Section 2 we draw out the implications of our model for the firm's choice of capital structure. We also study the consequences for firm behavior of changes of the firm's technology and of contracting frictions of the economic environment where the firm operates. In Section 3 we analyze the agency conflict associated with the liquidity reserve, and the related debt covenants, and in Section 4 we summarize our results and conclusions. Finally, in an Appendix we present some technical details of our numerical procedure.

# 1 The Model

## A. Overview

Before presenting our model formally, it is useful to set out the main ideas in informal terms. We consider a firm with a fixed asset in place which has been financed by equity, variable short-term debt and fixed long-term debt. The asset generates a random cash flow according to a stochastic process whose drift is itself random and follows a mean-reverting process. Any cash flow in excess of contractual debt service and fixed operating costs is subject to proportional corporate income tax, and the after-tax residual may either be paid out as dividends, used to reduce short term debt, or retained as liquid assets within the firm. Debt is assumed to be a hard claim, and any failure to meet contractual debt service results in bankruptcy. We assume that strict priority is observed in bankruptcy, with the firm's assets in excess of bankruptcy costs being awarded to the firm's creditors. Shareholders lose all. When cash flows fall short of debt service, the firm may draw-down its liquid assets or issue short term debt. It may also issue new equity; however, this external finance is costly so that the firm receives less than the full value of the shares it issues. The asset in place is indivisible so that partial sales of the risky asset are not allowed.

In this setting, firm faces two decisions. How much of the firm's earnings should be paid out as dividends? And how many new shares should be issued? Jointly, the two decisions will determine the firm's policy toward holding liquid assets and short term debt issuance. In our framework there is no reason to hold cash and borrow short term simultaneously. Thus we simply treat short-term borrowing as negative cash. We assume that these decisions are under the control of shareholders, who maximize the value of equity, calculated as the present discounted value of the future stream of dividends. The firm's decision will depend upon two state variables: the current rate of revenue cash flow and the current level of liquid assets (which is interpreted as borrowing when negative). Since all the other features of the environment are constant, this is a stationary problem. The solution of the model involves solving for the optimal policy as a function of the two state variables.

Shareholders face different costs for alternative capital market operations. Firm insiders will to some degree extract rents from liquid assets inside the firm, so that inside cash will grow at something less than the money market rate. On the other hand, the firm borrows at a rate greater than the money market rate reflecting informational rents conceded as part of its banking relationship. Finally, issuing equity will incur floatation costs. In this context, the optimal dividend and share issuance strategy in this context will be of the 'bang-bang' type, under which the state space is divided into 3 regions: in the 'save' region zero dividend is paid and earnings are accumulated in the reserve of liquid assets or used to pay down short term debt; in the 'dividend' region, the liquid reserve is immediately paid out, until it is brought back to the 'save' region, or to abandonment or bankruptcy; and in the 'issue' region, the firm immediately issues equity until liquid reserve is brought back into the 'save' region. The solution to the problem is studied by characterizing the boundaries between these regions as free boundaries, in a dynamic program.

## B. Model Assumptions and Detailed Specification

The firm has fixed assets in place, which incur operating costs at a constant rate $f$, and which generate operating revenues at a rate $dS_t$ according to the Ito equation

$$dS_t = \rho_t dt + \sigma dW_t^\sigma . \tag{1}$$

Here expected revenue $\rho_t$ at time $t$ itself obeys the Ito equation

$$d\rho_t = \kappa(\bar{\rho} - \rho_t)dt + \sqrt{\rho_t}\eta dW_t^\rho . \tag{2}$$

In these equations, $dW_t^\sigma$ and $dW_t^\rho$ are infinitesimal increments of independent, standard Brownian motions, and $\kappa, \bar{\rho}, \sigma$ and $\eta$ are positive constants. Equation (2) reflects our assumption that the expected rate of revenue is positive and mean reverting, representing variation of business conditions over the business cycle[2]. Mean reversion of $\rho_t$ is discussed

[2]This equation also has the property that varying the volatility $\eta$ does not alter the expectation of $\rho_s$ for given $\rho_t$ with $t < s$. See Duffie (2001). This will be useful when we discuss the asset substitution effect in our model.

further in Section 1F below. Equation (1) adds an unpredictable random element to the operating income. Note that after deducting the operating costs the profitability of the fixed assets is given by $dS_t - f dt$.

The firm is financed by equity, variable short-term debt, and fixed long-term debt.[3] We assume that the firm's short-term debt is instantaneously maturing, and long-term debt takes the form of a perpetual bond promising a continuous payment at rate $q$. In practice, a typical financial structure will involve short-term secured bank loans and long-term debentures which may be protected against dilution through restrictions on the amount of debt issuance (see, Petersen & Rajan, 1994, and Bolton & Freixas, 2000). We capture this by assuming that given a fixed $q$ the firm can instantaneously borrow up to its debt capacity, determined in our model by the value of the firm in bankruptcy. This will have the effect of making short-term debt default risk-free. We assume that the firm pays an interest rate on short-term borrowing, $r_{bank}$, which is in excess of the risk-free money market rate, $r$. The amount $r_{bank} - r$ reflects the informational rents ceded by the firm as part of its banking relationship.

The firm also may issue new equity to cover interest payments and operating losses. Such equity issues will be costly, in that the firm will be able to sell new shares at a fraction $\theta < 1$ of their fair value. The parameter $\theta$ will reflect fees and pricing concessions associated with primary equity market operations and may vary systematically with the efficiency of the capital markets where the firm operates. In a highly efficient market $\theta$ will be close to unity; whereas in a very underdeveloped capital market $\theta$ may be close to zero.

In addition to its fixed asset, the firm may hold a variable amount of liquid reserves. At any time $t$, the value of these will be denoted by $C_t$. Liquid reserves held within the firm will earn an 'internal' return at rate $r_{in}$, which will be less than the riskless rate $r$ earned on outside funds. This wedge $r - r_{in}$between $r_{in}$ and $r$ reflects the moral hazard faced by the shareholders, as discussed by Myers and Rajan (1998). As already mentioned, in our framework there will be no incentive to hold cash and issue short-term debt simultaneously. Thus $C_t < 0$ will correspond to a situation where the firm is borrowing short-term.

Under these assumptions, and for the time-being ignoring the possibility of equity issues, the liquid reserve is the accumulation of total earnings net of dividends, fixed costs, and interest payments on debt, and we can write

$$dC_t = (1 - \tau)(dS_t - (f + q) + r_{C_t} C_t)dt - dD_t, \tag{3}$$

[3]As in standard trade-off models the tax deductibility of interest payments provides an incentive to issue debt. Recently, DeMarzo and Sannikov (2006) have studied the problem of security design in a continuous time model without taxes and find that the optimal capital structure involves a fixed amount of long-time debt and varying short-term debt as in our model.

where $\tau$ is the corporate tax rate, and $D_t$ is the accumulated payment of dividends to , $r_C$ is equal to $r_{in}$ when $C > 0$, and equal to $r_{bank}$ when $C \leq 0$. This equation recognizes that tax is paid on the operating income and interest on the cash reserve, net of interest payments and fixed costs, i.e. $dS_t - ((f+q) - r_{C_t} C_t)dt$. Applying this to negative earnings is an analytically tractable way to model the loss carry-back and carry-forward provisions of many tax regimes.[4]

The firm becomes bankrupt if it does not meet its debt obligations and fixed costs, either from its operating revenue, its cash reserve, or by issuing new shares. On bankruptcy, the creditors are awarded the firm's fixed assets. We assume their value equals the value of the unlevered firm less bankruptcy costs, which we take to be a fraction $\alpha$ of this value. We denote the equity value with long-term debt coupon, $q$, by $J^q(\rho, C)$. Then the debt holders get value $(1-\alpha)J^0(\rho, C)$ upon bankruptcy. We assume that the firm can borrow short-term up to the value of this collateral less the face value of long-term debt, $q/r$. Thus for given $\rho$, the short-term lending limit will be a negative cash holding denoted $\underline{\mathrm{C}}(\rho)$, given by $\underline{\mathrm{C}}(\rho) = -\min\{0, (1-\alpha)J^0(\rho, C) - q/r\}$

Finally, we assume the firm chooses the dividend and capital market policy so as to maximize equity value, which is taken to be the present value of expected dividends discounted at the risk-free rate $r$. The debt is also valued by discounting at the risk free rate the coupon payments until bankruptcy, and then the bankruptcy value. This is consistent with Equations (1), (2) referring to risk the neutral probability measure.[5]

## C. PDEs for the Solution

Under the above assumptions, and ignoring for the moment the possibility of new equity issues, the value of the firm's equity is determined at any time $t$ by the current values of profitability $\rho$ and cash reserve $C$, which represents short term borrowing, if it is negative. Denoting this value by $J_t^q(\rho, C)$, then we can write the HJB equation

$$J_t^q(\rho, C) = \max_{dD_t} \left\{ dD_t + e^{-rdt} E_t^{(\rho,C)} \left[ J_{t+dt}^q(\rho_{t+dt}, C_{t+dt}) \right] \right\}, \tag{4}$$

[4]This is similar to the treatment of corporate taxes in Hennessey and Whitehed (2005). They model tax write-back and carry-forward provisions by allowing the tax rate to be progressive in terms of earnings. However, their estimated parameters imply that the marginal tax rate is essentially constant over the economically relevant range.

[5]Note that in our formulation bankruptcy occurs when our state variable evolving along a continuous sample path attains an absorbing barrier. Thus unlike reduced-form credit risk models there is no "jump-to-default risk", e.g., as in Duffie and Singleton (1999). They discount cash flows at the rate $r + \xi_t$, where $\xi_t$ is the instantaneous hazard rate of default. In this terminology, our model assumes $\xi_t = 0$.

in which $dt$ is an infinitesimally short time step and $dD_t$ is the optimal dividend payment over this time step, which must be non-negative. Also, $E_t^{(\rho,C)}$ means the expectation at time $t$, given that $(\rho_t, C_t) = (\rho, C)$. If the liquid reserve becomes low, then the firm can increase it by issuing more equity, if this is feasible in terms of the share price.

Expanding $J^q_{t+dt}(\rho_{t+dt}, C_{t+dt})$ in Equation (4), using the Ito Formula, $E[dW_t^\sigma] = 0$, and following standard manipulations we obtain the Ito Equation

$$J_t^q(\rho, C)(1 - e^{-rdt}) = \max_{dD_t \geq 0} \left\{ dD_t + \frac{\partial}{\partial t} J_t^q + \kappa(\bar{\rho} - \rho) \frac{\partial}{\partial \rho} J_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial^2}{\partial \rho^2} J_t^q \right.$$

$$\left. + \left[(1-\tau)(\rho - (f+q)) + r_C C - dD_t\right] \frac{\partial}{\partial C} J_t^q + \frac{1}{2} \sigma^2 (1-\tau)^2 \frac{\partial^2}{\partial C^2} J_t^q \right\} dt \tag{5}$$

We emphasize that this equation holds only for the optimal choice of $dD_t$, which depends on $(\rho, C)$.

The optimal choice of $dD_t$ here is singular: if $\frac{\partial}{\partial C} J_t^q < 1$, then then it is optimal to pay dividends as quickly as possible, reducing the cash holding until either $\frac{\partial}{\partial C} J_t^q \geq 1$, or until the firm becomes bankrupt. If $\frac{\partial}{\partial C} J_t^q \geq 1$, then the firm will not pay dividends.

The optimal decision can thus be characterized in terms a "save" region $\mathcal{S}$ and a "dividend" region $\mathcal{D}$ in the state space $\{(\rho, C) : C \geq \underline{C}(\rho)\}$. In $\mathcal{S}$ we have $\frac{\partial}{\partial C} J_t^q > 1$, and also Equation (5) holds, with $dD_t/dt = 0$, i.e.

$$\frac{\partial}{\partial t} J_t^q - r J_t^q + \kappa(\bar{\rho} - \rho) \frac{\partial}{\partial \rho} J_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial^2}{\partial \rho^2} J_t^q$$

$$+ \left[(1-\tau)(\rho - (f+q)) + r_C C\right] \frac{\partial}{\partial C} J_t^q + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial C^2} J_t^q = 0. \tag{6}$$

In $\mathcal{D}$ we have

$$\frac{\partial}{\partial C} J_t^q = 1, \tag{7}$$

and Equation (6) does not apply, since the value of an extra dollar in the liquidity reserve is just its value if immediately paid as a dividend. If the liquid reserve $C_t$ becomes so high that $(\rho_t, C_t) \varepsilon \mathcal{D}$, then a dividend should immediately be paid, to take $(\rho_t, C_t)$ back into the region S , or to bankruptcy.

If the liquid reserve becomes low, then it may be optimal for the firm to issue new equity. We have not included this possibility in the above formulation. In fact it is optimal to issue more equity if [6] $\frac{\partial}{\partial C} J_t^q > \frac{1}{\theta}$. This possibility leads to there being a third, 'issue' region, which we will denote by $\mathcal{I}$, lying below $\mathcal{S}$, and in which

$$\frac{\partial}{\partial C} J_t^q = \frac{1}{\theta}. \tag{8}$$

If the liquid reserve $C_t$ becomes low, so that $(\rho_t, C_t) \varepsilon \mathcal{I}$, then new equity should immediately be issued, to take $(\rho_t, C_t)$ back into the region S . Note that until bankruptcy occurs, the process $(\rho_t, C_t)$ will always remain in the save region $\mathcal{S}$, since it is immediately pushed away, whenever it enters the region $\mathcal{D}$ or $\mathcal{I}$.

These regions must be chosen to maximize $J_t^q(\rho, C)$, which implies that there must be value matching and smooth pasting of the solution across the boundaries between $\mathcal{S}$, $\mathcal{D}$, and $\mathcal{I}$. Also, these boundaries are 'free', in that they are determined as part of the solution to Equations (6), (7), (8) with value matching and smooth pasting. The boundary condition at the lower boundary $\{\underline{\mathrm{C}}(\rho)\}_\rho$ is just $J^q(\rho, \underline{\mathrm{C}}(\rho)) \geq 0$. Our solution technique will be described in detail in the Appendix below. The approach is to develop the solution to Equation (6) numerically from a distant horizon $t = T$ to $t = 0$, and to test at every point whether the value is increased by applying Equation (7) or (8), instead of Equation (6). The horizon $T$ is taken to be sufficiently far away, that the solution is independent of $T$ for $t$ near zero.

The debt value $P_t^q(\rho, C)$ can be calculated by solving a PDE, in a similar way to the equity value $J_t^q(\rho, C)$ above. In fact, the calculation is simpler because the boundaries of the region in which the debt is defined, i.e. $\mathcal{S}$, have already been determined in the equity valuation. The PDE for the debt, in the region $\mathcal{S}$, is

$$q + \frac{\partial}{\partial t} P_t^q - r P_t^q + \kappa(\bar{\rho} - \rho) \frac{\partial}{\partial \rho} P_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial}{\partial \rho^2} P_t^q +$$

$$[(1-\tau)(\rho - (f+q) + r_C C)] \frac{\partial}{\partial C} P_t^q + \frac{1}{2} \sigma^2 \frac{\partial}{\partial C^2} P_t^q = 0. \tag{9}$$

Again, we evolve the solution backwards from a distant horizon $t = T$. The boundary conditions for the debt valuation are as follows: First $\frac{\partial}{\partial C} P_t^q = 0$, where $\mathcal{S}$ meets $\mathcal{I}$ or where $\mathcal{S}$ meets $\mathcal{D}$ and $\mathcal{D}$ is above $\mathcal{S}$. This corresponds to the reflection of the process $C_t$ at these

[6] Proof: Suppose the current cash holding $C$ is too low, and the firm raises $\delta C$ in an equity issue, to increase the cash holding. Suppose the firm initially has $N$ shares, and issues $n$ more shares. Denote by $s$ the share price after issue. Then the obtained from selling each new share is $\theta s$, and also $n = \delta C/\theta s$ and $s = J(C + \delta C)/(N + n)$. These imply that $s = [J(C + \delta C) - \delta C/\theta]/N$. Now, the firm will issue shares if it increases the share price. Before issue, the share price is $J(C)/N$, and so the firm will issue if $J(C)/N < [J(C + \delta C) - \delta C/\theta]/N$, which implies that $[J(C + \delta C) - J(C)]/\delta C > 1/\theta$. QED

boundaries[7]; Second $P_t^q = (1-\alpha)J_0^0(\rho, 0)$ where $\mathcal{S}$ meets $\mathcal{D}$ and $\mathcal{D}$ is below $\mathcal{S}$, or when $C = \underline{\mathrm{C}}(\rho)$ and $\rho$ is not high enough to induce the equity holders to maintain payments to debt and fixed costs. This corresponds to bankruptcy, under which the debt holders receive the unlevered value of the firm, net of bankruptcy costs. (Note that the restrictions on short term debt mean that the firm will not go bankrupt if it has any short term debt, and so we must have $C = \underline{\mathrm{C}}(\rho) = 0$ at bankruptcy.)

## D. Benchmark Solutions to the PDEs

We take our benchmark parameter set to be $r_{in} = 4\%, r = 6\%, r_{bank} = 8\%, f = 0.14, \bar{\rho} = 0.15, \eta = 0.09, \kappa = 0.9, \sigma = 0.0, \tau = 30\%, \theta = 0.8, \alpha = 0.3$, and $q = 0.004$. Some of these parameters have a direct economic interpretation. Notice that by setting $r = 6\%$ and $r_{in} = 4\%$ we are assuming that one third of the market return on cash is dissipated by keeping the cash inside the firm and under the control of management. We view this as a reasonably severe problem of managerial moral hazard and a rather strong disincentive to holding cash. In this sense, the levels of cash holding our model predicts might be viewed as conservative. Similarly, setting $r_{bank} = 8\%$, we are assuming a significant relationship premium on short-term borrowing.[8] By setting $\theta = 0.8$ we assume 20% of the market value of newly issued equity is lost through transactions costs of one form or another. Given the direct costs plus underpricing of equity issues, we view these costs as substantial but not unreasonable in many settings.[9] Our assumption of bankruptcy costs of 30% at the high end of estimates that can be found in the literature.[10] We explore the sensitivity of our solutions to these assumptions by examining alternative parameter values below. The realism of the technological parameters $\bar{\rho}, \eta, \kappa$ will be assessed through simulation under the optimal policy.

Figure 1 gives the regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$, defined above. In this figure,the $x$-axis represents expected revenues $\rho$, and the $y$-axis represents the cash holding, which corresponds to short term debt, at negative values. The 'save' region $\mathcal{S}$ is denoted by diamonds. As explained, the solution will not stray beyond the boundary of this region, and the firm does not pay

[7] To see that this boundary condition corresponds to reflection, i.e. to the fact that if $C_t$ hits the boundary, then it is pushed back at infinite speed, note that if we had say $\frac{\partial}{\partial C} P_t^q > 0$, and it hit the barrier from below, then a long position in $P$ would be sure to gain in value, which is an arbitrage opportunity.

[8] A relatively high relationship premium may be appropriate for a smaller or younger firm. See Petersen and Rajan (1998).

[9] Smith(1977) estimates direct underwriting costs for seasoned equity issues to exceed 6% on average rising to over 13% on smaller issues. He also documents a significant price impact and other indirect costs.

[10] Like us Leland (1998) works with a model where absolute priority is respected in bankruptcy and assumes proportional bankruptcy costs of 25%. Anderson and Sundaresan (1996) show that observed yield spreads can be replicated assuming lower bankruptcy costs in a model allowing for strategic debt service.

dividend in this region. The 'issue' region is restricted to the lower boundary of cash $\{\underline{C}(\rho)\}_\rho$. We see that the firm is able to borrow short term when $\rho$ greater than about 0.14. Recall that the firm can only borrow short term, to the extent that all borrowing is riskless.

We also see in Figure 1, that the lowest value of $\rho$ at which the firm will issue shares if the cash reserve hits the lower boundary, is about 0.08. But the region $\mathcal{S}$ bulges to the left above this value, and is above zero for $\rho$ extending down to about 0.07. At such values of $\rho$ the firm is incurring operating losses, but there are $C$ values such that $(\rho, C)$ is in the interior of $\mathcal{S}$. At such $(\rho, C)$ the firm will use the liquidity represented by $C$ to pay operating losses, in the hope of surviving until business conditions improve. But if $(\rho_t, C_t)$ strays to the boundary of $\mathcal{S}$, then the cash reserve will be paid out, and the firm will be abandoned. We will later discuss covenants preventing such a discrete liquidating dividend.

The downward triangles in Figure 1 represent where the save region $\mathcal{S}$ meets the dividend region $\mathcal{D}$ from below. This can be regarded as the liquidity target. When $(\rho_t, C_t)$ is below this target, then earnings are retained so as to increase $C_t$, and when $(\rho_t, C_t)$ is above this target dividends are paid immediately, so as to reach this target.

The solid line in Figure 1 shows the average cash holding as a function of $\rho$. This is not monotone; it is increasing in $\rho$ for low levels of profitability and decreasing for high profitability. It is far below the target when $\rho$ is below the long-term average, $\bar{\rho}$. In this range, the firm is attempting to increase its cash holdings through retentions, but the dynamics of cash for given $\rho$ are such that the firm settles down to an equilibrium cash holding short of its target level. The maximum average cash level is attained at about $\rho = 0.16$, and is about 0.01.

The non-monotonicity of cash holding as a function of $\rho$ is directly related to empirical findings in the literature where cash flow has typically been included as a control variable panel studies of the determinants of corporate liquid cash holdings. Kim *et al* (1998) find a negative influence of cash flow. Opler *et al* (1999) find a positive effect in most specifications. Ditmar *et al* (2003) and Anderson (2003) find the sign of the cash flow variable is sensitive to the data set used and the model specification. In light of our theoretical results, such an unstable pattern is precisely what we should expect. A similar remark holds for the findings of Opler *et al* in their Table 3 where they sort firms in quartiles based on cash holdings and see that cash flow rates first rise then fall with average cash holdings.

Other properties of the model can be seen in Figure 2 where we present a portion of one history of the model when simulated under the optimal policy for the benchmark parameter values. This depicts the last ten years of the firm ending with its bankruptcy in its 127'th year. After the decline in profitability in year 119 the firm has exhausted its cash reserves and survives by issuing equity. With the recovery of revenues in year 121 above about $\rho = 0.145$

(compare with Figure 1) the firm briefly takes on some short-term debt; however, the amount of such debt is so small as to be imperceptible on the chart. With the subsequent strong recovery of revenues, the firm begins to accumulate cash until attaining the cash target of something less then 0.02 at which point it begins to pay dividends. (Again, compare with Figure 1). With the sustained, sharp fall of revenues from year 125 though mid-year 127, the cash reserve is exhausted, the value of equity drops to zero, and the firm is bankrupted.

## E. Benchmark Simulation of the Model

One of the strengths of the contingent claims framework is that the model can be used to derive a great many implications for observable firm characteristics which can be used in calibrating and testing. To explore our formulation of the model we simulate it as in Figure 2 and summarize its implications for the liquid reserves, leverage ratios, equity volatility, credit spreads, default probabilities, and other relevant measures.

Specifically, for the parameters above, we solve the model for the equity and debt values as functions of $(\rho, C)$, and obtain the optimal regions $\mathcal{D}, \mathcal{I}$ and $\mathcal{S}$. Then we perform 300 simulations of the $\rho_t$ variable, the realized cash holding $C_t$, equity value $J_t$ and debt value $P_t$. Each simulation starts from $\rho = 0.2, C = 0.0$ and runs to the firm's bankruptcy, or to 1000 years, if no bankruptcy occurs before then. Based on the realizations of the simulations we also calculate the average liquidity for each grid value of $\rho$, denoted $\overline{C}(\rho)$. Taking this average liquidity value as a function of $\rho$, we study the model conditional on 4 levels of profitability: $\rho = 0.10$ ('low'), $\rho = 0.15$ ('normal'), $\rho = 0.20$ ('high'), and $\rho = 0.25$ ('very high'). At each level of $\rho$ and average liquidity $\overline{C}(\rho)$, we present the net equity[11] value $J^q(\rho, \overline{C}(\rho)) - \overline{C}(\rho)$ (i.e. equity, net of the liquid reserve), debt value $P^q(\rho, \overline{C}(\rho))$, net firm value $J^q(\rho, \overline{C}(\rho)) + P^q(\rho, \overline{C}(\rho)) - \overline{C}(\rho)$ , leverage (debt value divided by the value of the firm), and the yield spread (yield on debt less $r$). Finally, we give the equity volatility, calculated as $\sqrt{\left(\sqrt{\rho}\eta\frac{\partial}{\partial\rho}J\right)^2 + \left(\sigma\frac{\partial}{\partial C}J\right)^2}/J$.

These simulations are done with respect to the objectively realized ("statistical") probabilities, and so we need to specify a risk premium. Referring to the risk associated with $\rho$, we can represent the risk premium by a parameter $\lambda$ (assumed constant, for simplicity)[12], such that to transform from the risk neutral to the statistical measure, we should replace $dW_t^\rho$ of Equation (2) by $dW_t^\rho + \lambda dt$. This $\lambda$ can be thought of as a Sharpe Ratio: it is the extra return required, per unit of extra exposure to the risk represented by

[11] Taking net equity and firm values facilitates comparisons across different scenarios, since changing between scenarios would involve making up the difference in the liquidity reserve by cash.

[12] Such $\lambda$ has to exist, in the absence of arbitrage: see Duffie (2001).

$dW_t^\rho$. To see how the risk premium affects the return of the firm's equity, note first that under the risk neutral measure, the expected return will be just the riskless return $r$. The equity value $J_t(\rho, C)$ is a smooth function of $\rho$, and using the Ito formula, we can write $dJ_t = (drift)dt + \frac{\partial}{\partial\rho}J_t d\rho \equiv rdt + \sqrt{\rho_t}\eta\frac{\partial}{\partial\rho}J_t dW_t^\rho$. The factor $\sqrt{\rho_t}\eta\frac{\partial}{\partial\rho}J_t$ here is the equity volatility, and we will calculate this in the tables below. Substituting $dW_t^\rho$ by $dW_t^\rho + \lambda dt$, we can see that the risk premium increases the expected return by $\lambda$ times this volatility. A reasonable value for the risk premium (Sharpe Ratio) of the market itself in $\lambda = 0.5$, corresponding to a market excess return of say 8%, and market volatility of say 16%. On the other hand a completely diversifiable risk would imply $\lambda = 0$. We take $\lambda = 0.3$, which is reasonable, if we assume that the risk of the firm has a systematic component, i.e. it is somewhat correlated with the market.

In addition we calculate the yield spread on zero-coupon bonds of 5 and 20 years until maturity. For this calculation, and following Duffie and Lando (2001), we assume that the perpetual debt is made up of a continuum of zero coupon bonds, and if the firm defaults, the these bonds are paid off in proportion to their value weight in the total debt. This calculation is done by adapting the perpetual debt valuation to accommodate this default rule, a payment of one dollar if there is no default before maturity, and the coupon being zero. We also calculate the probability of bankruptcy at 1, 5 and 20 year horizons. This calculation is again done by adapting the perpetual bond valuation, and we include the risk premium $\lambda$, since this probability is not risk neutral, but objectively realized.

Our results for liquidity, debt value, equity and leverage values are given in Table IA, and the credit relevant values are given in Table IB. These tables also contain results for other values of the debt parameter, which we will discuss later. We take as the main reference for our calibration the case $q = 0.004$ and $\rho = 0.15$ which corresponds to normal business conditions. In Table II we have summarized the results for this case in a way that can be compared to financial ratios of US non-financial firms as reported by Standard and Poors for the period 1997-1999. These comparisons suggest that our benchmark firm has characteristics similar to a firm toward the bottom end of the investment-grade range, BBB.

Also from Table IA, we see that in this case, the average liquid asset of about 10.5% of total asset value which is in line with average liquidity holding for small to medium sized firms during the 1980's and 1990's as depicted in Opler *et.al* Figure 2. Opler *et.al.* also investigated the dynamic properties of cash holding by U.S. firms and found evidence of mean reversion suggesting that firm target cash holdings. We have compared our model to these results by estimating their Equation 1 using simulated data of our model under the optimal policy using the benchmark parameters. The results are presented in Figure 3 and are quite similar to those reported by Opler *et.al.* in their own Figure 3.

Our model implies an equity volatility of 40% under normal conditions ($\rho = 0.15$) which is in line with market experience. For example, Zhang *et.al.* report that for a sample in which BBB rated firms predominate, historical average daily equity volatility ranges from 40% to 50% per year.

Turning to the credit risk related indicators in Table IB, we see that this firm in the reference case has a credit spread of 90 basis points on the 5-year pure discount bond. By way of comparison, Zhang *et.al.* report that the average 5-year CDS (credit default swap) rate on names rated BBB was 116 basis points between 2001 and 2003. During the same time the average 5- year CDS rates on firms rated A or above were 45 b.p.'s; whereas, the comparable rates on high yield CDS's were 450 b.p.'s. These may be compared to the spreads implied by our model for the cases $\rho = 0.2$ (59 b.p's) and $\rho = 0.10$ (397 b.p.'s), respectively.

From Table IA we also see that, under our assumption about the risk premium, the probability of default at the five-year horizon is 1.8%. This is corresponds to Standard and Poor's historical experience for a bond rated BBB or perhaps a bit below (see, de Servigny and Renault).

Finally, our model has implications of the value of defaulted bonds which can be compared to empirical recovery rates. Specifically, Figure 4 gives the equity valuations in terms of $\rho$, at $C = 0$ for our benchmark case (circles). The corresponding value of collateral is calculated as $(1 - \alpha)$ times the value of unlevered equity and is depicted by the solid line. We see from Figure 1 that the benchmark firm will go bankrupt for $\rho$ in the neighborhood of $\rho = 0.08$ depending upon the path of liquidity. Thus from Figure 4 we see that the value of debt of the bankrupted firm is about 0.03. From the same figure we also see that the debt value is about 0.06 for $\rho \geq 0.15$. Assuming that the long-term debt was issued at par with business conditions in this range, we see that the debt recovery rate on bankruptcy is about $\frac{0.03}{0.06} = 50\%$. This number is consistent with empirical studies of BBB rated firms. Over the period 1988-2002 Standard and Poors found that the recovery rates on defaulted bonds were 30 per cent for Senior Subordinated Notes, 38 per cent for Senior Unsecured Notes, and 50 per cent for Senior Secured Notes. (See de Servigny and Renault (2004) Chapter 4.)

To summarize, by simulating our model under the optimal policy and for the benchmark parameters we see it is consistent with a very wide range of empirical benchmarks. These include cash holdings, leverage, equity volatility, yield spreads, default probabilities and recovery rates.

# 2 Some Applications of the Model

## A. The Dynamics of Leverage and "Optimal" Capital Structure

It has long been recognized that the theory of optimal capital structure based on a static trade-off of tax benefits, bankruptcy costs and other financial frictions is difficult to square with a variety of empirical regularities including mean reversion of firm leverage ratios and the negative correlation of leverage ratio and lagged profitability. The initial work on *dynamic* trade-off models by Fischer *et.al.* suggested that some of the observed inertia in capital structure might be explained by fixed adjustment costs. More recent efforts with dynamic trade-off models, notably by Hennessy and Whited and Sterbulaev, have shown that under plausible values of parameters these models are capable generating mean reversion of leverage and its negative correlation with lagged profits.

Our model may be viewed as a further development in research on dynamic trade-off theory. Like Hennessy and Whited we allow for both the accumulation of cash and for variable rates of short-term borrowing. However, unlike their study, we work in a continuous time contingent claims framework and we allow for both short-term and long-term debt. Our model is also capable of generating mean reversion of leverage and the negative dependence of leverage on lagged earnings. In Figure 5 we present simulation results for our model with a constant level of long-term debt under the optimal policy and with our benchmark parameters. Leverage, measured by the book value of total long-term and short-term debt divided by the firm value, tends to fluctuate around the mean value of 0.44. There is a significant correlation coefficient of -0.47 between leverage and expected revenues ($\rho$) lagged one year.

In our model this behavior is the by-product of the share-value maximizing policy toward dividends and short-term borrowing. As seen in Figure 1, over a range of revenues close to their long-term mean an improvement in revenues (and earnings) will result in higher levels of cash-holdings which reduces the book value of total debt while firm value is increasing. The adjustment of cash to earnings is not one-for-one for as earnings improve the *targeted* cash level is reduced and eventually the higher earnings will be paid out.

Since our model includes both long-term and short-term debt we are able to explore the trade-off between these two and also their interaction with cash holding and dividends. The key insights can be understood from Figure 6 which depicts the optimal policies for two levels of long-term debt. The "high debt" case corresponds to our benchmark where $q = 0.004$ and is depicted by lines drawn with 'x'. As in Figure 1 we present the target level of liquidity, the average liquidity, and the debt capacity and equity issuance boundary as functions of expected revenues, $\rho$. In the "low debt" case we set $q = 0.0$ with all other parameters as in the benchmark case. The solutions are depicted with circles in Figure 6.

We see from Figure 6 that the effect of higher levels of long-term debt is to raise the liquidity target for any given value of expected earnings. Similarly, at a given value of $\rho$, the equity issuance boundary is increased (i.e., debt capacity decreased) and the average level of liquidity is increased. It will be noted that in the low debt case the average level of cash holdings is negative for all $\rho$. That is, on average the firm engages in varying degrees of short-term borrowing as a function of business conditions.

It is clear from this analysis that long-term and short-term debt are highly substitutable. With a reduction in long-term, a given firm will compensate by reducing its cash holdings and, possibly, borrowing short-term. In this way it will achieve a similar balance of debt tax shields and bankruptcy costs. It will be noted from Figure 6 that for values of $\rho \geq 0.19$ the average cash and target cash levels coincide for both high-debt and low-debt firms, implying that the dividend behavior is very similar for the two firms.

What are the implications of this for the optimal "time zero" level of long-term debt as in static studies of capital structure? For a firm with our benchmark parameters the answer is given in Table I. For example, if $\rho = 0.2$ and the firm sets its value of $q$ once for all it will maximize the firm value by setting $q = 0.004$, which coincides with the case we have taken as our benchmark. Thus our model does imply an optimal capital structure in the traditional sense. However, what is more interesting in Table I is that large variations in the level of long-term debt have only a small impact on firm value. For example, at $\rho = 0.2$, setting $q = 0.002$, i.e., one half of the "optimal" level leads to a reduction of firm value by only 0.3% (0.1517 versus 0.1519).

Thus we see that there is a near irrelevance of long-term capital structure in our model. Whatever the firm sets as its long-term debt level, it can find a corresponding cash and short-term debt policy that balances off bankruptcy costs, tax shields and other costs so as to achieve approximately the same value of the firm. Through experimenting with a number of variations on our benchmark formulation we consistently obtain the same near irrelevance result. For example, as will be discussed in detail below, if we change the rate of mean reversion of our model we change some aspects of the model in important ways. In particular, with a low rate of mean reversion the static optimal level of long-term debt is higher than we have found and more in line with the findings of Leland (1994) who assumes Geometric Brownian Motion. Nevertheless, in this case as well, the value of the firm is fairly constant over a wide range of values of $q$.[13]

[13]Taking $\kappa = 0.4$ and setting other parameters as in our benchmark one finds that conditional upon $\rho = 0.2$ the net firm value is maximized at $q = 0.007$ as compared to $q = 0.004$ in our benchmark case. However with these parameters the net firm value is quite insensitive to variations of $q$. For example, $q = 0.004$ results in

This seems to us one of the deep implications of a dynamic analysis of capital structure that takes also into account cash holdings and dividend policy, and this would seem to not depend importantly upon the specific technology or parameters we have adopted. And like the classic irrelevance results of Modigliani and Miller, it gives a natural way of interpreting one of the most troubling empirical findings for any theory of capital structure, namely that for otherwise very similar firms, we can observe a very wide variety of policies on debt structure.

We have shown that a firm with a fixed a level of long term debt can operate its dividend/short-term borrowing position so as to maximize share value and attain approximately the same firm value independently of the precise level of long-term debt chosen. Notice that this stops short of saying that this combined policy is fully dynamically optimal, because we have not explicitly modeled the full problem that consists of choosing at each point in time the level appropriate level of $C$ *and* $q$. As Leland (1994) has pointed out, if the level of long-term debt can be continuously adjusted without cost then long-term debt and short term debt are equivalent. Thus the interest in considering the combination of long-term and short-term debt emerges when there are costs of adjusting the level of long-term debt. However, as will be discussed in more detail below, even when issuance costs of long-term debt are quite low, the effect of mean reversion in cash flows in our model is that there is little incentive to engage in such adjustments of long-term debt. Consequently, it is likely that the solution to the full dynamic problem would add relatively little value as compared to a solution that would fix $q$ by some reasonable rule of thumb and then proceed to dynamically manage cash and short-term debt as we have described.

This observation sheds some light on the results of Welch who finds that stock returns account for about 40% of observed variations in leverage ratios while about 60% of these variations are explained by active debt market operations.[14] Welch finds that active debt market operations are *not* directed at compensating for stock returns induced leverage changes so as to maintain a constant (and presumably "optimal") leverage ratio. As has already been noted in relation to Figure 5, our model is consistent with this observed passive adjustment of leverage ratios to earnings fluctuations and associated stock price changes. The fact that Welch is unable account for *cross sectional* differences in active debt market operations using differences in transactions costs is compatible with our near irrelevancy of long-term capital

a net firm value about 1% less than for the $q = 0.007$.

[14]It should be noted that Welch's analysis uses leverage measures based on the total book value of debt [based long term debt (Compustat item 9) and debt in current liabilities (Compustat item 34)] and does not take into account liquid asset holding either by netting against short-term debt or directly as an explanatory variable. Thus his results are not directly comparable to our model.

structure. It may be difficult to identify the determinants of active debt policy precisely because there is little value maximizing imperative that drives shareholders toward any particular policy for long-term debt. Thus they may fix long-term debt according to a rule of thumb that seems plausible and which experience suggests is no worse than others that might be considered.

While our dynamic trade-off model makes no explicit allowance for information asymmetries it is interesting to compare the model's implications to the recent re-examination of the pecking order theory by Leary and Roberts (2006) who attempt to overcome the problem of low power which characterizes earlier tests of the pecking order.[15] They develop an empirical model of the pecking order that takes into account the state dependent nature of the pecking order. Depending upon the state, the firm will select financing according to three alternative hierarchies: (a. *pecking order*) internal first, then debt, then equity; (b. *debt capacity constrained*) cash first, then equity; and (c. *cash constrained*) debt first, then equity. These cases are set out in their Figure 1. These three cases also emerge in our model where the relevant state variables are $\rho$ and $C$ as can be seen in our own Figure 1. A classic pecking order (case a) prevails in our model for relatively high values of expected revenue ($\rho$) where the equity issuance boundary corresponds to negative $C$. That is, as financing needs would increase (e.g., as a result of a larger negative cash flow shock) they would be met first by drawing down internal funds, then by borrowing short-term and then by issuing equity. Case b corresponds to lower values of $\rho$ where the equity issuance boundary coincides with $C = 0$. And the cash constrained case (c) would correspond to high values of $\rho$ where current cash is negative. Thus our dynamic trade-off model provides an alternative rationale for their empirical model which does not rely on positing asymmetric information. While they do not provide a specific test of our model, we note that they find no evidence that the predictive performance of the empirical pecking order is related to indicators of information asymmetry. However, they do find that predictive performance is improved by allowing the parameters of the model to depend upon the covariates such as cash flow volatility and a dummy variable indicating that the firm pays dividends which are consistent with our model.

We close this section by considering an issue posed by Opler *et al*(1999), p.44, whether "...cash holdings and debt are two sides of the same coin"? This is supposed by practitioners who often will value a firm based on net debt ratios where liquid assets are netted out against debt. Similarly, Rajan and Zingales find that adjusted leverage (with liquid assets netted out) quite similar across large firms in OECD countries.

Our analysis of optimal cash holdings with both long-term and short-term debt shows that cash is not the same thing as negative debt. For example, with reference to Table I

[15] Most earlier tests of pecking order followed the approach introduced by Shyam-Sunder and Myers (1999).

consider again the low debt firm ($q = 0.0$) and the high debt firm ($q = 0.005$). At $\rho = 0.2$ their net firm values differ by less than 1%. However, their total net leverage ratios are 2% and 41% respectively. Thus, while firms with high levels of long-term debt optimally hold relatively high levels of cash, when compared to firms with predominantly short-term debt, they do not hold so much cash as to result in the same net leverage. Stated otherwise, cash policy, holding net leverage constant, has value implications.

It is clear that the reason this result emerges in our analysis but not in that of Hennessy and Whited is that we have included long-term debt whereas they do not. This is despite the fact that in both of our analyses cash is formally equivalent to negative *short-term* debt and that any given firm never will hold simultaneous positive levels of cash and short-term debt.[16]

## B. Asset Substitution Effect and Hedging

Since the analysis of Jensen and Meckling focussed attention on the incentive for shareholders of the levered firm to engage in *ex post* increases in asset volatility, the "asset substitution problem" has become one of the pillars of received wisdom in corporate finance. There is a presumption that the different risk profiles of debt and equity translate into agency costs. Based on this, there is a large literature on how such problems can be remedied either by security design (with Green (1984) being an early contribution) or by choice of capital structure (with Leland (1998) giving an explicit analysis of the problem). It is important to realize that the intuition about the asset substitution effect has been derived within largely static models, and it remains to be seen whether this is also a feature within a dynamic setting such as ours.

In our model, the parameters $\eta$ and $\sigma$ represent different sources of earnings volatility. $\eta$ refers to the dynamic of the profitability $\rho$. Given the mean reverting nature of the profitability relation, $\eta$ shocks are *persistent*. In contrast, $\sigma$ refers to a white noise type of volatility which represents a *non-persistent* shock to profitability.

The effect of varying these parameters are seen in Table III where we have calculated the

[16]In a recent paper Acharya, Almeida and Campello (2005) consider a three period model with cash, debt and investment choice. In that context they show that cash and negative debt are not equivalent and one dominates the other depending upon the correlation of cash flows with investment opportunities. In a different vein, De Marzo and Sannikov consider a continuous time model where a manager can derive private benefits from the firm's non-verifiable cash flows. They show that at the same time a firm may call upon its credit line and hold a compensating cash balance. That is, the same firm will hold cash and positive levels of short-term debt. These effects obviously cannot emerge in our model without real investment and managerial agency problems.

sensitivities of variables of interest to the main economic parameters of our model. As in Table I, each column is based on simulations of our model under the parameters specified and under the corresponding optimal policy. Values of the dependent variables are calculated conditional on $\rho$ and the associated average value of $C$ in the simulation.

In Columns 2 and 3, we examine the effect of varying the level of the persistent volatility while the other parameters are at their benchmark levels. These columns should be compared with each other, and with Column 1, which repeats the results for the benchmark case. Focussing on the case of $\rho = 0.15$, we see that the value of debt *and the value of equity* are both decreasing in $\eta$. As judged by the percentage decrease in value it appears that debt is more sensitive than equity to increases in $\eta$, but the direction of the effect is the same. A similar pattern holds for high profitability business conditions, $\rho = 0.20$. Interestingly, it holds as well when the firm is not far from bankruptcy, $\rho = 0.10$.

Thus in contrast with standard learning on the asset substitution effect, in our dynamic setting both debt and equity seem to have similar incentives to avoid increases in persistent cash flow volatility. Why is this the case?[17] Note that for each $\rho$, the level of cash holdings is increasing in $\eta$. Thus in response to an increase in volatility, under the optimal cash and dividend policy, the shareholders would choose to increase the amount of cash that they hold. This increased use of internal slack comes at a cost, and equity values are reduced as a result.

The effects of changes in *nonpersistent* volatility, $\sigma$, are seen in columns 4 and 5 of Table III. We see that for each level of $\rho$ increasing $\sigma$ again gives rise to lower values of both debt and equity. The reason that shareholders are hurt as well as are creditors is that under the optimal policy shareholders hold a higher liquid reserve.

Thus for both persistent volatility and transient volatility, the interests of debt and equity are roughly aligned for the changes of the volatility parameters that we have considered. This does not mean that they are *exactly* aligned and that they would agree upon exactly the optimal value if that parameter alone were to be varied.[18] However, it does raise a question about the empirical relevance of standard teaching on the asset substitution effect. Indeed, empirical studies have been notably unsuccessful to find any evidence in support of asset substitution effect (see, Andrade and Kaplan 1998 and Rauh 2006).

These results also relate to the literature on corporate incentives to hedge that has grown

[17] De Marzo and Sannikov who also study a continuous time model with cash accumulation and short-term and long-term debt also find this convergence of interests of debt and equity. They characterize the result as "surprising" but do not pursue the issue further.

[18] Stated otherwise, for some parameter values and in some states it may be the case that equity will be increasing and debt value will be decreasing in $\eta$.

up on recent years. For example, in continuous time frameworks Mello and Parsons (2000) and Rochet and Villeneuve (2004) have characterized the benefits of hedging non-persistent shocks to cash flow. While our framework and our major focus is different than theirs, our results are similar to theirs if we assume that $\sigma dW_t^\sigma$ represents a shock to earnings that can be hedged costlessly (resulting in $\sigma = 0$). For example, if the source of transient shocks to earnings come from fluctuations in commodity prices it may be possible to eliminate these risks with positions in short-dated futures contract.[19] As in our discussion of the asset substitution effect, it is clear that benefit of hedging to shareholders comes from the fact that it would allow the firm to reduce its use of costly cash balances.

Our results on increases of $\eta$ suggest that shareholders would also have an incentive to hedge persistent fluctuations of revenues ($\rho$). However, it is not clear that suitable hedging instruments are available or that the costs of hedging this risk would be low.[20].

Finally we note that our results confirm one of the most basic findings of empirical studies of liquidity, namely that higher volatility is associated with higher liquid asset holding in cross sections. (e.g., Opler et al).

## C. Mean Reversion

Our assumption of mean reversion of the firm's revenue process stands in contrast with the assumption of Geometric Brownian Motion adopted in much of the earlier literature on contingent claims analysis which emerged as a direct outgrowth of the contribution of Black and Scholes.[21] Mean reversion is a simple way to capture business cycle effects and also the dynamics of innovation and imitation in competitive product markets. Similar assumptions about the firm's technology have been made by Gomes (2001) and Hennessy and Whited (2005) who also provide supporting empirical evidence.

The effects of changing the speed of mean reversion can be seen in Table III, columns 6 and 7 where we have varied $\kappa$. Increasing the speed of mean reversion tends to reduce the influence of shocks to expected revenues and thus might be seen as something like a decrease in $\eta$. In fact the effects of the two parameters are somewhat different. This can be seen with reference to the case $\rho = 0.15$ where debt and net firm values are both increasing in $\kappa$; whereas, net equity is decreasing. The divergence of the interests of debt and equity are

[19] Note that $\sigma dW_t^\sigma$ is a martingale difference, corresponding to the assumption that there is no risk premium associated with this risk. In imperfect hedge may result in a reduction of $\sigma$.

[20] A costless hedge of $\rho$ would not be modelled by simply taking $\eta = 0$, but rather by subtracting the martingale component of the process $\rho$.

[21] Recent contributions that make the assumption of Geometric Brownian Motion include Leland (1994) and (1998), Mello and Parsons (2000) Rochet and Villeneuve (2004).

stronger under good business conditions when equity would be aided by a decrease in the speed at which the firm returns toward the long-run mean. This is seen in the case $\rho = 0.2$ where the value of equity is strongly decreasing in $\kappa$; whereas debt is increasing. However, when the firm is close to bankruptcy, $\rho = 0.1$, both equity and debt would benefit from faster mean reversion. To summarize, faster mean reversion is good for shareholders when business conditions are poor and bad for shareholders when business conditions are good. In contrast, faster mean reversion benefits creditors in all business conditions.

One criticism that has been leveled at static trade-off models is that they predict unrealistically high optimal leverage. For example, this was the case in Leland (1994), a fact that motivated the study of refinancing boundaries in Leland (1998). In this formulation when firm value has risen sufficiently relative to the point at which the current debt structure was fixed, shareholders will increase the amount of debt issued by a discrete amount. When such financial restructuring is possible, the optimal "time zero" degrees of leverage are more in line with observed leverage ratios. Strebulaev (2004) pursues this line of reasoning and shows that potential restructuring can help to explain a number of other empirical puzzles related to leverage.

The incentive to restructure long-term debt under good business conditions will be reduced by the presence of mean reversion. For example, even under the assumption of very low restructuring costs (1%), Leland found that refinancing is implemented after the asset value has increased to about twice its initial value. In our formulation, firm value is very unlikely to increase by this amount. For example, in our benchmark simulations that were initiated at $\rho = 0.2$, revenues never increases beyond $\rho = 0.3$. From Figure 4 we see that $\rho = 0.3$ corresponds to a value increase of about 1.5 fold. In fact, from such a very high level of expected revenues, the firm's profitability will then be predicted to fall rapidly through the force of mean reversion.[22]

## D. Other Comparative Statics

The remaining columns in Table III pertain to variations in what might be thought of as "efficiency" parameters describing the environment where the firm operates.

In Columns 8 and 9 of Table III, we vary the proportional bankruptcy cost $\alpha$. In our benchmark we have set this at 30% of the value of assets in place at the time of bankruptcy.

[22] When mean reversion is weak in our model we approximate the results found under the assumption of geometric Brownian. For example, as has already been pointed out, with $\kappa = 0.4$ the static optimal level of long-term debt is much higher than in our benchmark ($q = 0.007$ versus $q = 0.004$). Also, with $\kappa = 0.4$, the likelihood that $\rho$ will persist at levels far above the long-term mean increases. Accordingly, there will be greater incentives to engage in costly adjustments of financial structure *à* la Leland (1998).

This may seem high compared to some studies in the literature, e.g., Warner (1977). However, those estimates pertain to direct bankruptcy costs typically for firms with large amounts of tangible assets and costs are expressed as a proportion of the book value of assets reflecting historical costs. Furthermore, more recent studies covering indirect costs and a wide range of industries (including those with substantial intangible assets) suggests that total bankruptcy costs may be very substantial. (See Franks and Torous (1989) and Weiss (1990)). By way of comparison, Leland (1994) assumes 50% proportional bankruptcy costs and Leland (1998) assumes 25%.

In Table III we consider a low ($\alpha = 0.05$) and high ($\alpha = 0.5$) bankruptcy cost scenarios to compare with our benchmark. As expected the effect of higher bankruptcy costs is to depress the value of long-term debt. We find that higher bankruptcy costs also hurt equity values. This may seem surprising in our model where absolute priority is enforced in bankruptcy. The reason for this effect is that variations in bankruptcy costs affect the firm's capacity to borrow short-term and therefore has an impact on the optimal cash holding policy of the firm. In general, higher bankruptcy costs induce higher levels of cash holding which tend to depress equity values.

While this link between bankruptcy costs and equity is rather less direct than its impact on debt values, the effect can be substantial. For example, with $\rho = 0.15$, in the case of very low bankruptcy costs $= 0.05$, the firm's average cash holdings are 25% less than in the benchmark (0.0104 versus 0.0136) and equity values are 2.8% higher as a result (as compared to debt values which are 3.5% higher than in the benchmark).

As has been widely noted the assumption of that absolute priority of claims is maintained in bankruptcy is contradicted by empirical studies of Chapter 11 and informal distressed debt restructuring (e.g., Franks and Torous (1989)). This has led researchers to introduce strategic debt service (Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997)) whereby shareholders can use the threat of costly bankruptcy to extract concessions from creditors. This has the effect of depressing debt values, implying that a given credit spread on debt is more compatible with lower bankruptcy costs, than in a comparable model where absolute priority is maintained. Acharya et.al. (2002)) have explored this effect under the assumption that the firm holds precautionary liquid reserves. They find that since these reserves add to collateral values, the presence of these reserves serves to reduce shareholders' ability to extract concessions from shareholders. These authors do not consider this under an optimal cash holding policy; so it is not entirely clear how on balance strategic debt service effects interact with optimal cash holding and dividends. Potentially we could study this within our framework. However, we have chosen to not further complicate our model here and leave this extension as an interesting avenue for further work.

In our model the efficiency of external capital markets is captured by the parameter $\theta$ which is the fraction of the value of securities issued by the firm which accrues to the firm. The fraction $1-\theta$ is lost through the underwriting process as commissions, fees or excessive dilution. As we have already noted our benchmark assumption of $\theta = 0.8$.

In Column 10 of Table III, we consider the extreme case of $\theta = 0.01$, which may be taken as the case when capital markets very inefficient. In Column 11 we take the opposite extreme of $\theta = 0.95$ which would seem to correspond to a highly efficient capital market and small agency costs. While $\theta$ cannot be directly interpreted as the degree of underpricing, the estimates of the underpricing of IPO's, range from 5 per cent to often greater than 20 per cent. In the U.S. seasoned issues typically involve somewhat lower costs. From the estimates of Lee et.al. (1996) using the U.S. data, reasonable estimates might be 11% for IPO's and 7% for seasoned issues. This suggests that $\theta = 0.95$ would be appropriate only for very highly developed capital markets, and where there is no information asymmetry.[23].

From Columns 10 and 11 of Table III we see that when capital market efficiency increases, the average level of liquid asset holding decreases. Focusing on the average profit case $\rho = 0.15$, liquid assets average about 50 per cent of the net firm value in the very underdeveloped capital market context. In the 95 per cent efficient capital market, optimal liquid assets correspond to only about 3 per cent of firm value. This result is consistent with Opler *et al* (1999) who document empirically that firms with greater access to capital markets carry smaller amounts of liquidity reserve. It is also in line with the a study of listed firms in twelve continental European countries by Ferreira and Vilela (2002). They find a negative association between liquidity and capital market development, proxied by the ratio of the country's free float (i.e., total value of stocks held by minority shareholders) to GDP.

In our framework the cost of holding liquidity inside the firm is represented by the $r - r_{in}$. This is a proxy for the amount of rent extraction that is obtained through various forms of managerial moral hazard. In Columns 12 and 13 of Table III we take $r_{in} = 3\%$ and 5% while we maintain our other benchmark parameters including $r = 6\%$. In our view $r_{in} = 3\%$ would indicate severe agency problems since in this case half the income flow from liquid assets would be diverted or wasted by managers.

As we would expect, the average level of liquid asset holding is increasing in $r_{in}$. The effect is quite strong. Focussing on the normal profit case ($\rho = 0.15$), when $r_{in} = 0.05$ the optimal level of liquid asset holding is about 16 per cent of net firm value as compared to 12 per cent in the benchmark case. The values of both equity and debt are increasing in $r_{in}$. The benefit to shareholders is direct since as residual claims on the firm's cash flows any

[23]Based on a sample of US, non-financial firms between 1993 and 2001 a period of very high stock market activity Hennessy and Whited (2005) estimate floatation costs of about 5.9%

increase in the return on a given level of liquid securities accrues to them. The benefit to creditors is indirect through the fact that the higher level of cash holding by the firm reduces the chances of bankruptcy. Most of the benefit of lower costs of holding liquid assets accrue to shareholders. For example, when compared to the benchmark, the value of equity is 2.2 per cent higher when $r_{in} = 0.05$ in contrast with long-term debt whose value is increased by half a per cent.

The last two columns of Table III show the effects of varying $r_{bank}$. The difference $r_{bank} - r$ is a proxy for the informational rents ceded to creditors through short-term borrowing. An increase in $r_{bank}$ discourages short-term borrowing and induces the firm to carry somewhat greater amounts of cash on average. This is associated with lower values for shareholders and higher values for long-term creditors. However, these value effects are slight and only noticeable under good business conditions $\rho = 0.2$.

Finally, we note an interesting indirect effect that applies to the efficiency parameters $\theta$, $r_{in}$ and $r_{bank}$. Holding $q$ and other parameters constant, a change in the efficiency parameter that induces greater liquid asset holding will be associated with *lower* equity volatility. So, for example, a change in stock market regulation that promotes efficiency in equity issuance (i.e., higher $\theta$) may indirectly induce greater stock market volatility.

# 3 Agency Conflicts, Debt Covenants and the Liquidity Reserve

In our model, the liquidity reserve is chosen by the shareholders to maximize the share value. This choice is made simultaneously with the default strategy, and the debt holders are assumed to have no influence in these choices. It is thus possible that the liquidity reserve strategy of the shareholders is detrimental to the debt holders and to the economic efficiency of the firm.

In this section we will address a number of issues arising from this debt-equity conflict. First we measure agency costs in our model. Specifically we ask what the consequences would be for the debt, equity and firm values of maximizing firm value, rather than equity value. The answer to this tells us the severity of the conflict in terms of how much the equity holders are gaining from the bond holders via their liquidity strategy. We find that agency costs are moderate in our model in good business conditions; that is, any loss to creditors of a share value maximizing policy is compensated in large part by an increase in share value that is of nearly equal magnitude. However, agency costs rise when business conditions deteriorate. The reason is that costs of bankruptcy, either direct costs or indirect costs in the form of lost tax shields, fall disproportionately on creditors. To gain more insight, we also investigate the cases when the bankruptcy cost $\alpha$ is lower, at 5%, and $\tau = 0$, so that there is no tax shield to protect. When lower bankruptcy costs combine with zero tax shields, maximizing the firm value rather than the equity value has a relatively small effect the firm value. Mostly it is a pure transfer of value from shareholders to creditors. Generally, we find that the firm value maximizing policy calls for the firm to hold more liquidity than under the share value maximizing policy.

Then, in light of the sub-optimality of the share value maximizing liquidity policy, we consider debt covenants which might be partial remedies. First we note that in view of the result that firm value maximization calls for greater, not lesser, cash holding on average, a covenant limiting the amount of cash held by the firm would be working in the wrong direction. Instead, we consider a restriction on short-term borrowing and find that this leads to very little increase in firm value. We then consider a second covenant prohibiting the firm from paying a dividend when it is not profitable. This covenant is shown to increase firm value. The merit of this covenant is that it curbs the firm's ability to make the discrete liquidating dividend. Eliminating this creates value for creditors because this helps to avoid triggering bankruptcy prematurely.

## A. Taking the Liquidity Reserve to Maximize the Firm Value:

The levered firm value is the discounted expectation of dividends and payments to debt, up to bankruptcy, at which time its value is $1 - \alpha$ times its unlevered value. Following our discussion in Subsection 1C, if the firm value is being maximized, then its value $F_t^q(\rho, C)$ will satisfy

$$q + \frac{\partial}{\partial t} F_t^q - r F_t^q + \kappa(\bar{\rho} - \rho) \frac{\partial}{\partial \rho} F_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial^2}{\partial \rho^2} F_t^q$$

$$+ \left[ (1 - \tau)(\rho - (f + q) + r_C C) \right] \frac{\partial}{\partial C} F_t^q + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial C^2} F_t^q = 0 \tag{11}$$

in the 'save' region $\mathcal{S}^F$;

$$\frac{\partial}{\partial C} F_t^q = 1, \tag{12}$$

in the 'dividend' region $\mathcal{D}^F$; and

$$\frac{\partial}{\partial C} F_t^q = \frac{1}{\theta}. \tag{13}$$

in the 'issue' region $\mathcal{I}^F$. Consistent with the boundary conditions in Subsection 1C, if the firm become bankrupt, then it has value $(1 - \alpha) J_0^0(\rho, 0)$, and at the horizon time $T$, we take the value to be $C + \min\{q/r, (1 - \alpha) J_0^0(\rho, 0)\}$.

The numerical procedures described in the Appendix are easily adapted to this case, and the result is summarized in Table IV. In this table, the first two columns correspond to our benchmark case; the first column repeats the relevant numbers from Table I. The second column corresponds to maximizing the firm value. In this column, we calculate the firm value as just described, and then the debt value as in Section 1C, based on the regions $\mathcal{S}^F, \mathcal{D}^F$ and $\mathcal{I}^F$, and the share value is then the firm value minus the debt value. We see, as expected, that the net firm value (i.e. net of the value of the liquidity reserve) is higher, the debt value his higher, and the net equity value is lower in the second column, than in the first. Under normal or good business conditions agency costs are moderate. For example, at $\rho = 0.15$ net firm value is 2.5% greater under the firm value maximizing policy. At $\rho = 0.2$ the agency cost is only 1%. However, at $\rho = 0.1$, which is close to the bankruptcy point under the share value maximizing policy, the difference between firm value under the two policies is substantial (9.6%).

It will be recalled that under the share value maximizing policy, at low values of $\rho$ the shareholders pay out available cash in the form of a discrete dividend and abandon the firm. Under the firm value maximizing policy, the firm would retain its cash cushion as long as possible, and it would keep the firm operating through equity issuance until $\rho$ would fall to a lower critical value. Overall, under the firm value maximizing policy, the firm holds larger, not smaller, amounts of cash than under share value maximization. This casts doubt on the wisdom of forcing shareholders to pay out cash as suggested in the quotation at the start of this article.

Columns 3 and 4 repeat the case of columns 1 and 2, but with bankruptcy cost at 5%, rather than 30%. We see that with low direct bankruptcy costs, the agency costs are less. For example under normal business conditions ($\rho = 0.15$) firm value is greater by only 1.6% under the firm value maximizing policy (as compared to 2.5% with higher bankruptcy costs). This is intuitive. Since the agency costs derive from the fact that shareholders impose greater risks of bankruptcy on creditors than is optimal, when bankruptcy costs are less, the associated agency costs are less.

The other cost of bankruptcy in our model is the indirect cost of tax shields lost in liquidation. Thus in a similar vein, we repeat our experiment in columns 5 and 6, but with $\tau = 0$, so that there is no tax shield to be lost. As expected the agency costs of a share value maximizing policy are less than under the benchmark parameters of $\alpha = 0.3$ and $\tau = 0.3$. Finally, columns 7 and 8 present the case $\alpha = 0.05$ and $\tau = 0$. In this case, the agency costs are quite small outside of financial distress. Furthermore, cash holding on average are quite similar under the two policies.

To summarize this subsection, maximizing the firm value, rather than the equity value, is particularly valuable to debt holders, and to the firm value itself, when bankruptcy costs and the tax shield are significant. In this case, the agency conflict results in an erosion of economic value overall, but the effect is substantial only under poor business conditions. But when bankruptcy costs and the tax shield are not high, the agency conflict is associated with a transfer of value between the equity and debt holders, more than a destruction of economic value[24].

## B. Two Debt Covenants:

In light of the agency costs incurred under the share-value maximizing liquidity policy, it is interesting to consider whether simple, restrictive covenants might added to the firm's debt contracts as partial remedies.

Since the firm-value maximizing policy obtained in section 3A is a complex function of the firm's state variables, $\rho$ and $C$, it may be difficult to come close to first-best using covenants that are simple enough to be readily enforceable. Also, since we found that on average firm-value maximization calls for holding greater, not lesser, amounts of cash than shareholders would otherwise wish to hold, a covenant placing an upper limit on cash holding is not likely to increase firm value.

This last observation suggests that a simple possible covenant might be to restrict the amount cash position of the firm to be non-negative; that is, the firm would be prohibited from borrowing short-term. The effective of this restriction can be seen in Table V column 2 which can be compared to the benchmark solutions in column 1. Prohibiting short-term borrowing has the effect of slightly raising the average cash holdings in the firm. However, this covenant has negligible impact on firm the values of debt, equity, and the firm overall. It is notable that it achieves virtually none of the 9.6% potential reduction of agency costs in when the firm is near financial distress ($\rho = 0.10$). The ineffectiveness of this remedy is attributable to the fact that a restriction on short-term debt impinges only for high values of $\rho$, which a distressed firm is not likely to ever see. For the firm experiencing very good business conditions ($\rho = 0.2$), the restriction on short-term borrowing slightly reduces net firm value. That is, the gain to creditors does not compensate for the loss of share value.

[24]This result is in tune with the findings of Titman, Tompaidis and Tayplakov (2004), who study the debt and equity values of a levered firm, and compare these values under an equity-optimizing investment strategy, with the values if the investment strategy were constrained to be as if the firm were unlevered. They show that the equity-optimal strategy transfers value from debt to equity, but does not destroy much value. These authors have in mind real estate project financing, and 'investment' in their context corresponds to maintenance of the real estate. They take the bankruptcy cost to be zero in their benchmark case, and they do not have a tax shield effect in their model, corresponding to our condition $\tau = 0$.

It should be noted that this discussion assumes that the level of long-term debt is set at our benchmark value, $q = 0.004$. However, in light of our analysis of the effect changing levels of long-term debt as reflected in Figure 6, it is clear what the effect of a prohibition of short-term borrowing would be for different levels of long-term debt. For higher $q$, the range of $\rho$ for which unconstrained cash holding would be negative would increase to levels above $\rho = 0.25$. These are very unlikely to occur. Thus the covenant would have virtually no effect on the firm. For the firm with less long-term debt, ($q < 0.004$), a prohibition on short-term borrowing, could have an impact on values. However, the main effect would be to prevent equity holders from benefitting from tax shields under moderate and good business conditions. It would do little to protect creditors when the firm is financially distressed. Thus on balance, the covenant is likely to reduce firm value.

The problem with a simple prohibition of short-term borrowing is that since it is not conditioned on the expected earnings of the firm, it is a very poor approximation to the state contingent optimal policy. This same defect would carry over to other possible covenants which would place a fixed lower bound on the firm's cash position, e.g., $C_t \geq \underline{C}$ where $\underline{C}$ is a constant. The problem with this is that by setting the minimum cash holding at some level that affords some protection to creditors it does so at the cost of lost earnings and tax shields at high $\rho$.[25]

This discussion suggests that a more promising covenant design would condition upon earnings and would seek to protect creditors when the firm is approaching financial distress. In this regard we analyze a covenant which prohibits the firm from paying a dividend when it is not currently sufficiently profitable. Leuz, Deller and Stubenrath (1998) note that debt covenants restricting dividend payouts when the firm is in distress are common in the US, but not in the UK. They argue that such covenants are considered unnecessary in the UK because company law already restricts the conditions under which firms can pay dividends, largely based on recent earnings.

In detail, we consider a covenant which prohibits dividend payments when $\rho_t < f + q - L$, where $L$ represents a measure of lee-way allowed under the covenant. This covenant will automatically obviate the possibility of the discrete liquidating dividend, at least if $L$ is not chosen to be so big that the firm is allowed to pay dividends when it is not worthwhile to continue operating. This covenant can be incorporated into our dynamic program, by

[25] It might be thought that this problem could be overcome by obliging the firm to hold the minimum cash in a trust account which would earn the money market rate $r$. This provision would mean that the firm's long-term debt would be equivalent to a portfolio consisting of a riskless secured bond equal to $\underline{C}$ plus a risky unsecured bond which is a claim on the flow $(q - r\underline{C})dt$. This is equivalent to simply reducing the long-term debt of the firm, and the associated optimal policy would be the share-value maximizing policy for that smaller debt. The net value of the firm would be unchanged.

imposing Equation (6) above for all $(\rho, C)$ such that $\rho < f + q - L$, irrespective of whether $\frac{\partial}{\partial C} J_t^q > 1$.

The results are given in Table V, Columns 3 and 4 corresponding respectively to $L = 0.01, 0.02$. Comparing these columns with Column 1, which corresponds to there being no covenant, we see that the covenant does achieve some of the benefits sought. Under poor business conditions ($\rho = 0.1$) firm value is about 2% greater than in the benchmark case without the covenant. At higher values of $\rho$ the net benefits are less, reflecting the fact that the covenant would have effect only after a large fall of earnings which is relatively unlikely to occur in the near future.

The enhancement of the debt at the expense of the equity might be attributed at least partially to the direct value of the prospective liquidation dividend, which the covenant has saved for the debt holders at the expense of the equity holders. However, the value of the liquidating dividend is very small, when the firm is not currently in distress. This contingent cash flow can be valued by simulating its discounted risk neutral expectation, and if currently $\rho = 0.2$, then its value is about 0.00008[26]. This is less than the wealth transfer between debt and equity, associated with the covenant.

Preventing the transfer associated with the liquidating dividend cannot in any case account for the enhancement of the firm value, and this must be attributed to the fact that the covenant makes bankruptcy less likely, since the firm will continue operating at any $\rho$ value, if there is some liquidity reserve.

It is interesting that the average amount of the liquidity reserve is actually slightly reduced by this covenant and that the lee-way value $L$ does not have much effect on the valuations. Overall, our results are consistent with the view that covenants imposing a profitability condition for dividends are value increasing.[27]

[26]Simulation also shows that the a liquidating dividend will be paid in about one quarter of bankruptcies, and if it is paid, then its average value will be about 0.01. The value at $\rho = 0.2$ is so low because at such $\rho$ bankruptcy is not expected for decades.

[27]Something like the same effect might be achieved by placing a limit on the speed with which cash can be paid out as dividends. In normal times the dividend as we have modelled it could be smoothed to have finite speed with very little erosion of value. The value of such a speed limit when the firm is in distress would be that the firm would likely have returned from distress by the time a substantial dividend were paid. There are a large number of ways such a restriction could be formulated; however, we believe that the order of magnitude of the gain is likely to be close to that achieved through earnings restriction on dividends we have studied here.

# 4 Summary and Conclusions

We have developed a structural dynamic model of a company's optimal holding of liquid reserve, together with its optimal debt and equity issuance and dividend policy. Our model allows for a stochastic mean reverting earnings rate, and we it solve numerically, as a non-linear PDE with free boundaries, with the expected earnings and cash levels as the state variables.

We have shown with this model that it is in shareholders' interests to hold relatively large amounts of cash inside the firm the even if they have access to relatively efficient capital markets and even if some of the return cash is dissipated by insiders. The reason that forcing the firm to pay out "idle" cash in relatively good times may be a bad idea, is that it is myopic– it does not properly weight the future dilution costs to shareholders of raising funds, if the firm later were to approach financial distress.

While the interests of shareholders and creditors regarding cash holding policy are not perfectly aligned, they are not totally divergent either. Our comparison of a share-value-maximizing policy with a firm-value-maximizing policy reveals that the targeted levels of cash are often quite similar under both policies. The one area where their interests clearly diverge is when the firm's earnings are persistently low. In such circumstances, it is optimal for shareholders to pay out available cash in the form of special dividend, thus exposing the firm to financial distress. In this regard we have shown that it may enhance firm value to prevent the firm from paying dividends when earnings are low, as is often done through bond covenants or as seen in some systems of corporate law.

We have seen that the firm's policy toward liquid asset holding is closely connected to the question of optimal capital structure. In particular we show that higher levels of long-term debt will result higher levels of liquid asset holding and a reduction in the optimal use of short-term debt. In adapting its liquidity policy appropriately, the firm is able to balance off its various contracting frictions in such a way as to achieve approximately the same value of the firm for a wide range of long-term debt levels.

In order to assure the relevance of our solutions we have been careful in calibrating our model. For plausible parameter values, our model is able to simultaneously hit many empirical benchmarks including: firm leverage, liquidity holdings, credit spreads, default probabilities at various time horizons, debt recovery rates given default, and equity volatility. Our model also exhibits realistic comparative statics behavior. For example, it has been documented that liquidity holdings increases when external finance becomes more costly, and our model reflects this.

# Appendix: Numerical Techniques and Boundary Conditions for Solving the PDEs for Valuing Debt and Equity

Our strategy for valuing the equity is to solve Equation (6) numerically, by finite difference procedures, evolving backwards from a distant horizon time $t = T$, at which we assume the firm is liquidated. We have also changed the variable $\rho$ to $\xi := \frac{1}{2}\rho^{\frac{1}{2}}$. This change helps the numerical stability of the scheme because after we have expressed Equation (2) in terms of $\eta$, the noise coefficient becomes constant. Also, under this transformation, the solution is more detailed for low values of $\rho$, which are more important.

We use the explicit finite difference scheme (see Ames (1992)), representing the $\rho$ space by a grid with 201 points, ranging from 0 to $\rho_{max} = 3$, and we represent $C$ by a grid with 201 points ranging from $C_{min} = -0.2$ to $C_{max} = 0.3$. With these parameters, and for all the parameters in the text, the finite difference scheme is numerically stable, if we take a time step of length 0.01. Also, we take $T = 50$ years. By experimentation, we have determined that the solutions are insensitive to first order variations of these parameters. To determine the regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$, we test, at every time step and every grid point point representing $(\rho, C)$, whether it is optimal to pay dividends, issue shares, or abandon the firm.

Our numerical scheme for valuing the debt is the same as for the equity. The debt boundary condition for bankruptcy is $(1-\alpha)J^0(\rho, C)$, and this condition is calculated using an implementation of the equity valuation for zero long term debt. Our calculations for the credit spread, the probability of default at horizon 20 years, etc, are not based on evolution to a steady state. These are obtained by taking $T = 20$ years, etc, in the above. Although it might be more usual to use the technique of Successive Over-Relaxation (SOR) to obtain our steady state solution (see Ames (1992)), our finite difference scheme is more useful for studying how quickly the steady state is achieved, and for dealing with these non-steady state calculations.

Our horizon boundary conditions for debt and equity at $t = T$ are as follows: To value unlevered equity for the valuations on bankruptcy, we take $J_T^{unlevered}(\rho, C) = \max\{C, 0\}$, reflecting abandonment at time $T$. For the debt horizon value we take $P_T^q(\rho, C) = \min\{q/r, (1-\alpha)J_0^{unlevered}(\rho, 0) + C\}$, and for the equity horizon value we take $J_T^q(\rho, C) = \max\{(1-\alpha)J_0^{unlevered}(\rho, 0) + C - q/r, 0\}$. These reflect the assumption that if bankruptcy has not occurred by time $T$, the productive asset is sold, incurring the bankruptcy costs. The face value of the debt, i.e.$q/r$, is paid out of the proceeds plus cash reserve to the extent possible, and the rest goes to the equity holders. In section 3 A, when maximizing the firm value, we take $F_T(\rho, C) = (1-\alpha)J_0^{unlevered}(\rho, 0) + C$. Note that the choice of terminal time $T$ condition does not matter, if $T$ is sufficiently far away, but a good choice allows $T$ to be taken smaller, which is more efficient, and this choice prevents arbitrage at time $T$.

As well as the boundary conditions associated with the regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$, we must also choose boundary conditions at high and low values of $\xi \equiv \frac{1}{2}\rho^{\frac{1}{2}}$ and $C$. The lowest value of $\rho$ is zero, and since $d\rho/d\xi = 0$, then any smooth boundary condition here for $\frac{\partial}{\partial\rho}J$ will translate to $\frac{\partial}{\partial\xi}J = 0$. We thus impose this condition. On the other hand, it is unclear what the boundary behavior should be for high $\xi$. We therefore define a boundary region, corresponding to $\rho$ in the interval $[\rho_{max}, \rho^{+}_{max})$ with $\rho^{+}_{max} = 3.5$, and we extend the drift coefficient of Equation (2) to be $\mu(\rho) = -2\kappa\frac{\rho^{+}_{max}-\rho_{max}}{\pi}\tan(\frac{\pi}{2(\rho^{+}_{max}-\rho_{max})}(\rho - \rho_{max}))$ in this region. This extends the drift coefficient smoothly past $\rho = \rho_{max}$ ,and has infinite slope at $\rho^{+}_{max}$, and so the boundary condition $\frac{\partial}{\partial\xi}J = 0$ holds. With the economic parameters given in the text, the state variable $\rho$ will very rarely rise as high as $\rho_{max} = 3$, and so altering the state equation above this value will not affect the solution in a measurable way. Our choice of boundary region here has the value of preserving the numerical scheme, and ensuring that no unwelcome singularities are introduced into the solution at the boundary.

## TABLE I

## Panel A - Values of Equity, Debt and Liquidity

| Payment to debt, $q$ | 0.000 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 |
|---|---|---|---|---|---|---|---|
| Measures at $\rho = 0.10$ | | | | | | | |
| Liquidity | -0.0202 | 0.0144 | -0.0041 | 0.0017 | 0.0026 | 0.0042 | 0.0058 |
| Equity val | 0.0711 | 0.0555 | 0.0395 | 0.0232 | 0.0117 | 0.0049 | 0.0014 |
| Debt val | 0.0000 | 0.0167 | 0.0333 | 0.0483 | 0.0562 | 0.0529 | 0.0468 |
| Net firm val | 0.0711 | 0.0721 | 0.0728 | 0.0715 | 0.0679 | 0.0578 | 0.0481 |
| Leverage | 0.0000 | 0.2888 | 0.4843 | 0.6597 | 0.7971 | 0.8537 | 0.8676 |
| Equity volatility | 0.4659 | 0.5982 | 0.7062 | 0.9723 | 1.5381 | 1.8705 | 1.3699 |
| Measures at $\rho = 0.15$ | | | | | | | |
| Liquidity | -0.0032 | -0.0008 | 0.0040 | 0.0108 | 0.0136 | 0.0145 | 0.0138 |
| Equity val | 0.1113 | 0.0968 | 0.0810 | 0.0652 | 0.0533 | 0.0441 | 0.0366 |
| Debt val | 0.0000 | 0.0167 | 0.0333 | 0.0490 | 0.0609 | 0.0666 | 0.0708 |
| Net firm val | 0.1113 | 0.1134 | 0.1144 | 0.1143 | 0.1142 | 0.1107 | 0.1074 |
| Leverage | 0.0000 | 0.1480 | 0.2815 | 0.3920 | 0.4766 | 0.5318 | 0.5844 |
| Equity volatility | 0.2396 | 0.2726 | 0.3104 | 0.34911 | 0.3962 | 0.4477 | 0.5138 |
| Measures at $\rho = 0.20$ | | | | | | | |
| Liquidity | -0.0042 | 0.0008 | 0.0015 | 0.0025 | 0.0044 | 0.0064 | 0.0063 |
| Equity val | 0.14827 | 0.1339 | 0.1184 | 0.1028 | 0.0908 | 0.0814 | 0.0735 |
| Debt val | 0.0000 | 0.0167 | 0.0333 | 0.0491 | 0.0614 | 0.0679 | 0.0732 |
| Net firm val | 0.1482 | 0.1505 | 0.1517 | 0.1519 | 0.1521 | 0.1493 | 0.1467 |
| Leverage | 0.0000 | 0.1106 | 0.2174 | 0.3180 | 0.3920 | 0.4364 | 0.4781 |
| Equity volatility | 0.2064 | 0.2214 | 0.2498 | 0.2861 | 0.3166 | 0.3425 | 0.3742 |
| Measures at $\rho = 0.25$ | | | | | | | |
| Liquidity | -0.0211 | -0.0131 | -0.0099 | -0.0034 | -0.0020 | 0.0011 | 0.0031 |
| Equity val | 0.1851 | 0.1709 | 0.1556 | 0.1401 | 0.1281 | 0.1186 | 0.1107 |
| Debt val | 0.0000 | 0.0167 | 0.0333 | 0.0491 | 0.0616 | 0.0685 | 0.0742 |
| Net firm val | 0.1851 | 0.1876 | 0.1889 | 0.1892 | 0.1897 | 0.1871 | 0.1848 |
| Leverage | 0.0000 | 0.0955 | 0.1862 | 0.2644 | 0.3281 | 0.3639 | 0.3946 |
| Equity volatility | 0.2024 | 0.2109 | 0.2291 | 0.2448 | 0.2655 | 0.2793 | 0.2931 |

# TABLE I

## Panel B - Credit Spreads and Bankruptcy Probabilities

| Payment to debt, $q$ | 0.000 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 |
|---|---|---|---|---|---|---|---|
| Measures at $\rho = 0.10$ | | | | | | | |
| Credit spreads: | | | | | | | |
| 5 year PDB | 0.0000 | 0.0000 | 0.0002 | 0.0072 | 0.0397 | 0.1355 | 0.2958 |
| 20 year PDB | 0.0000 | 0.0000 | 0.0001 | 0.0034 | 0.0181 | 0.0577 | 0.1136 |
| Prob. bankrupt after: | | | | | | | |
| 1 year | 0.0000 | 0.0000 | 0.0003 | 0.0144 | 0.0992 | 0.3516 | 0.6718 |
| 5 years | 0.0000 | 0.0000 | 0.0005 | 0.0211 | 0.1276 | 0.4062 | 0.7148 |
| 20 years | 0.0000 | 0.0000 | 0.0008 | 0.0301 | 0.1786 | 0.5105 | 0.7988 |
| Measures at $\rho = 0.15$ | | | | | | | |
| Credit spreads: | | | | | | | |
| 5 year PDB | 0.0000 | 0.0000 | 0.0001 | 0.0016 | 0.0090 | 0.0274 | 0.0485 |
| 20 year PDB | 0.0000 | 0.0000 | 0.0001 | 0.0020 | 0.0105 | 0.0307 | 0.05211 |
| Prob. bankrupt after: | | | | | | | |
| 1 year | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0015 | 0.0060 | 0.0150 |
| 5 years | 0.0000 | 0.0000 | 0.0001 | 0.0029 | 0.0184 | 0.0595 | 0.1098 |
| 20 years | 0.0000 | 0.0000 | 0.0003 | 0.0129 | 0.0756 | 0.2244 | 0.3719 |
| Measures at $\rho = 0.20$ | | | | | | | |
| Credit spreads: | | | | | | | |
| 5 year PDB | 0.0000 | 0.0000 | 0.0000 | 0.0011 | 0.0059 | 0.0185 | 0.0331 |
| 20 year PDB | 0.0000 | 0.0000 | 0.0001 | 0.0019 | 0.0097 | 0.0284 | 0.0482 |
| Prob. bankrupt after: | | | | | | | |
| 1 year | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0009 |
| 5 years | 0.0000 | 0.0000 | 0.0000 | 0.0017 | 0.0109 | 0.0367 | 0.0691 |
| 20 years | 0.0000 | 0.0000 | 0.0003 | 0.0116 | 0.0683 | 0.2051 | 0.3426 |
| Measures at $\rho = 0.25$ | | | | | | | |
| Credit spreads: | | | | | | | |
| 5 year PDB | 0.0000 | 0.0000 | 0.0000 | 0.0008 | 0.0046 | 0.0147 | 0.0264 |
| 20 year PDB | 0.0000 | 0.0000 | 0.0001 | 0.0018 | 0.0093 | 0.0274 | 0.0465 |
| Prob. bankrupt after: | | | | | | | |
| 1 year | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 |
| 5 years | 0.0000 | 0.0000 | 0.0000 | 0.0012 | 0.0081 | 0.0279 | 0.0533 |
| 20 years | 0.0000 | 0.0000 | 0.0003 | 0.0111 | 0.0655 | 0.1974 | 0.3309 |

# TABLE II: Comparison of Model Solution to Empirical Benchmarks

The empirical benchmarks are 1997-1999 median values for US industrial firms as reported by Standard and Poors for firms rated A, BBB and BB (Source, Standard and Poors, "Adjusted Key Ratios," *Credit Week* September 20, 2000). The Model values are based on the simulation of the model under the benchmark parameters, evaluated at the case $\rho = 0.15$. The model equivalent of EBIT interest coverage is computed as $(\rho - f)/q$. Return on capital is $(\rho - f)/(P + J)$. The Long Term Debt to Capital ratio is $P/(P + J)$. The Total Debt to Capital ratio is $(P - C)/(P + J)$. Debt/Equity is $(P - C)/J$.

| | A | BBB | BB | Model |
|---|---|---|---|---|
| EBIT interest coverage | 6.8 | 3.9 | 2.3 | 2.5 |
| Return on capital | 0.199 | 0.140 | .117 | 0.15 |
| LT Debt/Capital | 0.325 | 0.41 | 0.558 | 0.477 |
| Total Debt/Capital | 0.401 | 0.474 | 0.613 | 0.370 |
| Debt/Equity | 0.197 | 0.437 | 0.905 | 0.707 |

## TABLE III - Comparative Statics

## Values of Equity, Debt and Liquidity

| Parameter being varied | None | $\eta$ | | $\sigma$ | | $\kappa$ | | $\alpha$ | | $\theta$ | | $r_{in}$ | | $r_{bank}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Parameter value | (Repeat | 0.08 | 0.10 | 0.01 | 0.02 | 0.8 | 1.0 | 0.05 | 0.5 | 0.01 | 0.95 | 0.03 | 0.05 | 0.07 | 0.09 |
| Benchmark value | Benchmark) | 0.09 | 0.09 | 0.00 | 0.00 | 0.9 | 0.9 | 0.3 | 0..3 | 0.80 | 0.80 | 0.04 | 0.04 | 0.08 | 0.08 |
| TS average liquidity | 0.0026 | 0.0017 | 0.0039 | 0.0072 | 0.0167 | 0.0033 | 0.0028 | 0.0016 | 0.0028 | 0.0258 | 0.0000 | 0.0018 | 0.0051 | 0.0107 | 0.0113 |
| Measures at $\rho = 0.10$ | | | | | | | | | | | | | | | |
| Liquidity | 0.0026 | 0.0017 | 0.0039 | 0.0072 | 0.0167 | 0.0033 | 0.0028 | 0.0016 | 0.0028 | 0.0258 | 0.0000 | 0.0018 | 0.0051 | 0.0026 | 0.0028 |
| Net equity val | 0.0117 | 0.0123 | 0.0113 | 0.0106 | 0.0084 | 0.0089 | 0.0150 | 0.0133 | 0.0117 | 0.0042 | 0.0258 | 0.0109 | 0.0130 | 0.0117 | 0.0117 |
| Debt val | 0.0562 | 0.0606 | 0.0521 | 0.0533 | 0.0475 | 0.0475 | 0.0624 | 0.0601 | 0.0540 | 0.0122 | 0.0660 | 0.0560 | 0.0565 | 0.0562 | 0.0562 |
| Net firm val | 0.0679 | 0.0729 | 0.0633 | 0.0639 | 0.0559 | 0.0564 | 0.0774 | 0.0734 | 0.0657 | 0.0164 | 0.0918 | 0.0669 | 0.0695 | 0.0679 | 0.0679 |
| Leverage | 0.7972 | 0.8121 | 0.7739 | 0.7494 | 0.6546 | 0.7964 | 0.7775 | 0.8009 | 0.7881 | 0.2897 | 0.7186 | 0.8152 | 0.7565 | 0.7972 | 0.7946 |
| Equity volatility | 1.5382 | 1.4514 | 1.5490 | 1.3667 | 1.1971 | 1.7950 | 1.1951 | 1.5220 | 1.5120 | 0.5312 | 0.8217 | 1.7123 | 1.2263 | 1.5384 | 1.5137 |
| Measures at $\rho = 0.15$ | | | | | | | | | | | | | | | |
| Liquidity | 0.0136 | 0.0123 | 0.0152 | 0.0174 | 0.0228 | 0.0144 | 0.0140 | 0.0104 | 0.0142 | 0.0368 | 0.0038 | 0.0119 | 0.0184 | 0.0136 | 0.0141 |
| Net equity val | 0.0533 | 0.0545 | 0.0523 | 0.0513 | 0.0471 | 0.0539 | 0.0533 | 0.0548 | 0.0532 | 0.0440 | 0.0633 | 0.0524 | 0.0545 | 0.0533 | 0.0533 |
| Debt val | 0.0609 | 0.0639 | 0.0575 | 0.0594 | 0.0560 | 0.0559 | 0.0644 | 0.0631 | 0.0596 | 0.0339 | 0.0663 | 0.0608 | 0.0612 | 0.0609 | 0.0609 |
| Net firm val | 0.1142 | 0.1184 | 0.1098 | 0.1107 | 0.1032 | 0.1098 | 0.1178 | 0.1179 | 0.1128 | 0.0778 | 0.1296 | 0.1132 | 0.1157 | 0.1142 | 0.1142 |
| Leverage | 0.4766 | 0.4891 | 0.4597 | 0.4637 | 0.4448 | 0.4503 | 0.4891 | 0.4917 | 0.4691 | 0.2955 | 0.4968 | 0.4859 | 0.4566 | 0.4766 | 0.4747 |
| Equity volatility | 0.3962 | 0.3533 | 0.4348 | 0.4132 | 0.4800 | 0.4317 | 0.3564 | 0.4072 | 0.3921 | 0.3256 | 0.3846 | 0.4131 | 0.3612 | 0.3962 | 0.3927 |
| Measures at $\rho = 0.20$ | | | | | | | | | | | | | | | |
| Liquidity | 0.0044 | 0.0029 | 0.0072 | 0.0092 | 0.0160 | 0.0046 | 0.0054 | 0.0025 | 0.0064 | 0.0263 | -0.0001 | 0.0041 | 0.0083 | 0.0044 | 0.0056 |
| Net equity val | 0.0908 | 0.0921 | 0.0898 | 0.0887 | 0.0846 | 0.0957 | 0.0873 | 0.0923 | 0.0907 | 0.0813 | 0.1002 | 0.0900 | 0.0918 | 0.0908 | 0.0907 |
| Debt val | 0.0614 | 0.0642 | 0.0582 | 0.0600 | 0.0569 | 0.0569 | 0.0646 | 0.0634 | 0.0602 | 0.0360 | 0.0663 | 0.0613 | 0.0616 | 0.0614 | 0.0614 |
| Net firm val | 0.1521 | 0.1563 | 0.1480 | 0.1487 | 0.1415 | 0.1525 | 0.1519 | 0.1557 | 0.1509 | 0.1173 | 0.1665 | 0.1512 | 0.1534 | 0.1521 | 0.1521 |
| Leverage | 0.3920 | 0.4032 | 0.3750 | 0.3799 | 0.3615 | 0.3618 | 0.4106 | 0.4006 | 0.3827 | 0.2510 | 0.3983 | 0.3943 | 0.3812 | 0.3920 | 0.3891 |
| Equity volatility | 0.3166 | 0.2824 | 0.3444 | 0.3244 | 0.3599 | 0.3353 | 0.2948 | 0.3176 | 0.3102 | 0.2800 | 0.2971 | 0.3208 | 0.3003 | 0.3167 | 0.3126 |

# TABLE IV

## Maximizing Equity versus Maximizing the Firm Value

| Tax rate $\tau$ | 30% | | 30% | | 0% | | 0% | |
|---|---|---|---|---|---|---|---|---|
| Bankruptcy cost $\alpha$ | 30% | | 5% | | 30% | | 5% | |
| Value being maximized | Equity | Firm | Equity | Firm | Equity | Firm | Equity | Firm |
| Measures at $\rho = 0.10$ | | | | | | | | |
| Liquidity | 0.00261 | 0.00267 | 0.00161 | 0.00194 | 0.00723 | 0.00774 | -0.00131 | -0.00107 |
| Net equity val | 0.01168 | 0.00793 | 0.01332 | 0.01093 | 0.02251 | 0.01985 | 0.02498 | 0.02240 |
| Debt val | 0.05619 | 0.06652 | 0.06009 | 0.06662 | 0.06080 | 0.06662 | 0.06188 | 0.06663 |
| Net firm val | 0.06787 | 0.07445 | 0.07340 | 0.07755 | 0.08331 | 0.08647 | 0.08686 | 0.08903 |
| Leverage | 0.79724 | 0.86252 | 0.80106 | 0.83803 | 0.67154 | 0.70712 | 0.72334 | 0.75754 |
| Perp. cr. spread | 0.01118 | 0.00013 | 0.00657 | 0.00004 | 0.00579 | 0.00004 | 0.00464 | 0.00003 |
| Measures at $\rho = 0.15$ | | | | | | | | |
| Liquidity | 0.01361 | 0.01402 | 0.01039 | 0.01162 | 0.02495 | 0.02641 | 0.01701 | 0.01757 |
| Net equity val | 0.05328 | 0.05062 | 0.05479 | 0.05316 | 0.08130 | 0.07947 | 0.08436 | 0.08266 |
| Debt val | 0.06090 | 0.06652 | 0.06307 | 0.06662 | 0.06353 | 0.06662 | 0.06388 | 0.06664 |
| Net firm val | 0.11418 | 0.11713 | 0.11786 | 0.11978 | 0.14483 | 0.14609 | 0.14824 | 0.14930 |
| Leverage | 0.47658 | 0.50719 | 0.49180 | 0.50700 | 0.37420 | 0.38618 | 0.38658 | 0.39937 |
| Perp. cr. spread | 0.00568 | 0.00013 | 0.00342 | 0.00004 | 0.00296 | 0.00004 | 0.00261 | 0.00002 |
| Measures at $\rho = 0.20$ | | | | | | | | |
| Liquidity | 0.00444 | 0.00445 | 0.00254 | 0.00289 | 0.01194 | 0.01291 | 0.00678 | 0.00701 |
| Net equity val | 0.09075 | 0.08818 | 0.09228 | 0.09070 | 0.13448 | 0.13271 | 0.13758 | 0.13596 |
| Debt val | 0.06138 | 0.06652 | 0.06338 | 0.06662 | 0.06380 | 0.06662 | 0.06412 | 0.06665 |
| Net firm val | 0.15213 | 0.15470 | 0.15566 | 0.15732 | 0.19827 | 0.19933 | 0.20169 | 0.20261 |
| Leverage | 0.39203 | 0.41797 | 0.40060 | 0.41583 | 0.30349 | 0.31389 | 0.30755 | 0.31794 |
| Perp. cr. spread | 0.00517 | 0.00013 | 0.00312 | 0.00004 | 0.00270 | 0.00004 | 0.00238 | 0.00002 |

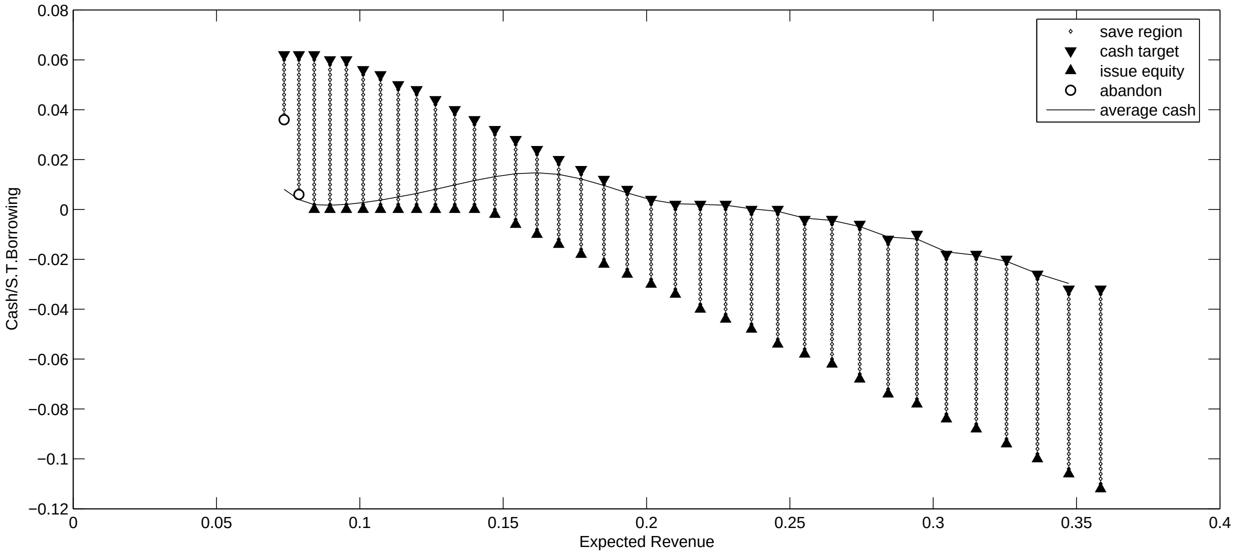
# TABLE V

# Analyzing Two Debt Covenants

| | Benchmark (No covenant) | No S.T. borrowing | No dividend if $\rho_t < f + q - L$ $L = 0.01$ | $L = 0.02$ |
|---|---|---|---|---|
| Measures at $\rho = 0.10$ | | | | |
| Liquidity | 0.002611 | 0.002849 | 0.000000 | 0.000000 |
| Net equity val | 0.011681 | 0.011722 | 0.010777 | 0.010779 |
| Debt val | 0.056193 | 0.056200 | 0.058564 | 0.058564 |
| Net firm val | 0.067874 | 0.067921 | 0.069341 | 0.069342 |
| Leverage | 0.797240 | 0.794120 | 0.844570 | 0.844560 |
| Perp. cr. spread | 0.011183 | 0.011175 | 0.008302 | 0.008302 |
| Equit volatility | 1.538630 | 1.508820 | 2.061170 | 2.060960 |
| Measures at $\rho = 0.15$ | | | | |
| Liquidity | 0.013605 | 0.014243 | 0.008310 | 0.012239 |
| Net equity val | 0.053281 | 0.053277 | 0.052917 | 0.053083 |
| Debt val | 0.060900 | 0.060909 | 0.062254 | 0.062269 |
| Net firm val | 0.114180 | 0.114186 | 0.115170 | 0.115352 |
| Leverage | 0.476580 | 0.474260 | 0.504160 | 0.488040 |
| Perp. cr. spread | 0.005682 | 0.005672 | 0.004253 | 0.004237 |
| Equity volatility | 0.396230 | 0.391850 | 0.440210 | 0.407390 |
| Measures at $\rho = 0.20$ | | | | |
| Liquidity | 0.004438 | 0.006416 | 0.005624 | 0.005635 |
| Net equity val | 0.090750 | 0.090720 | 0.090605 | 0.090606 |
| Debt val | 0.061379 | 0.061388 | 0.062636 | 0.062636 |
| Net firm val | 0.152129 | 0.152108 | 0.153241 | 0.153243 |
| Leverage | 0.392030 | 0.387250 | 0.394270 | 0.394240 |
| Perp. cr. spread | 0.005169 | 0.005160 | 0.003861 | 0.003861 |
| Equity volatility | 0.316640 | 0.309990 | 0.313230 | 0.313190 |

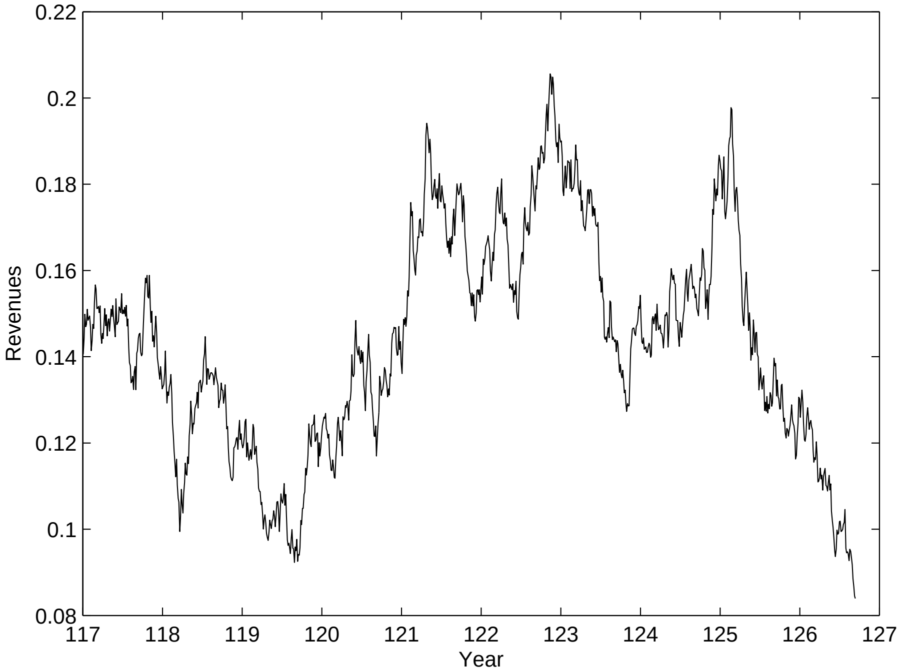
## Figure 1:

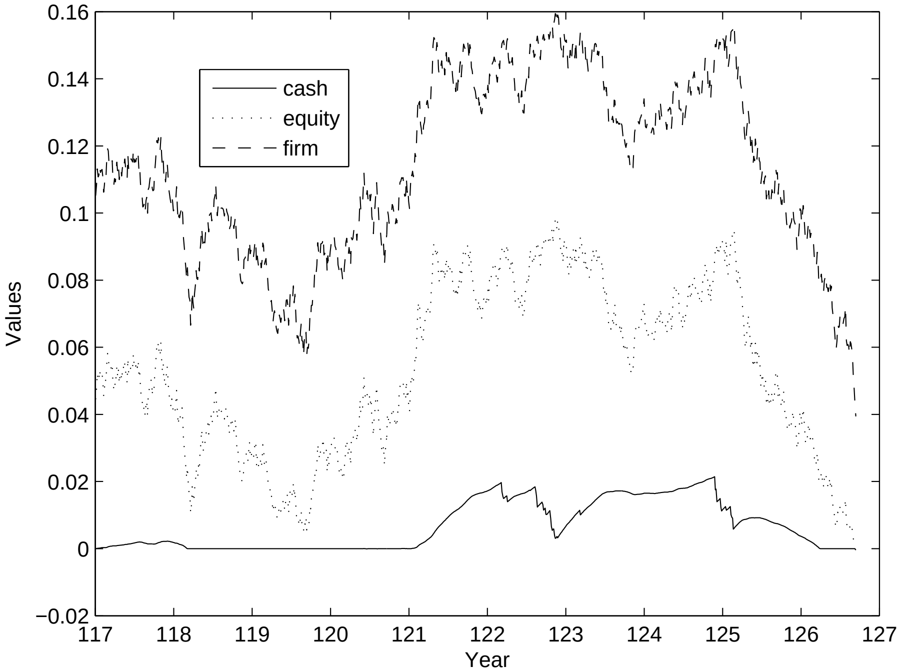
This figure depicts the regions of 'save' (i.e. do retain earnings, and do not pay dividends), 'pay dividends' and 'issue equity', arising from the solution to the PDEs of Section 1. The 'save' region $\mathcal{S}$ is depicted by diamonds. The firm pays dividends for $(\rho, C)$ not in the 'save' region, so long as $C$ is positive. The target liquid asset holding is the upper boundary of the $\mathcal{S}$ region and is depicted by downward triangles. The region in which the firm will issue new equity is indicated by upward triangles. These also indicate the lower limit of liquidity, or equivalently minus the limit of the short term borrowing facility. The points where shareholders would pay a liquidating dividend and abandon the firm to creditors are indicated with circles. The realized liquid asset holding, averaged over 300 simulations of the firm history, as a function of $\rho$, is depicted by the solid line.



## Figure 2:

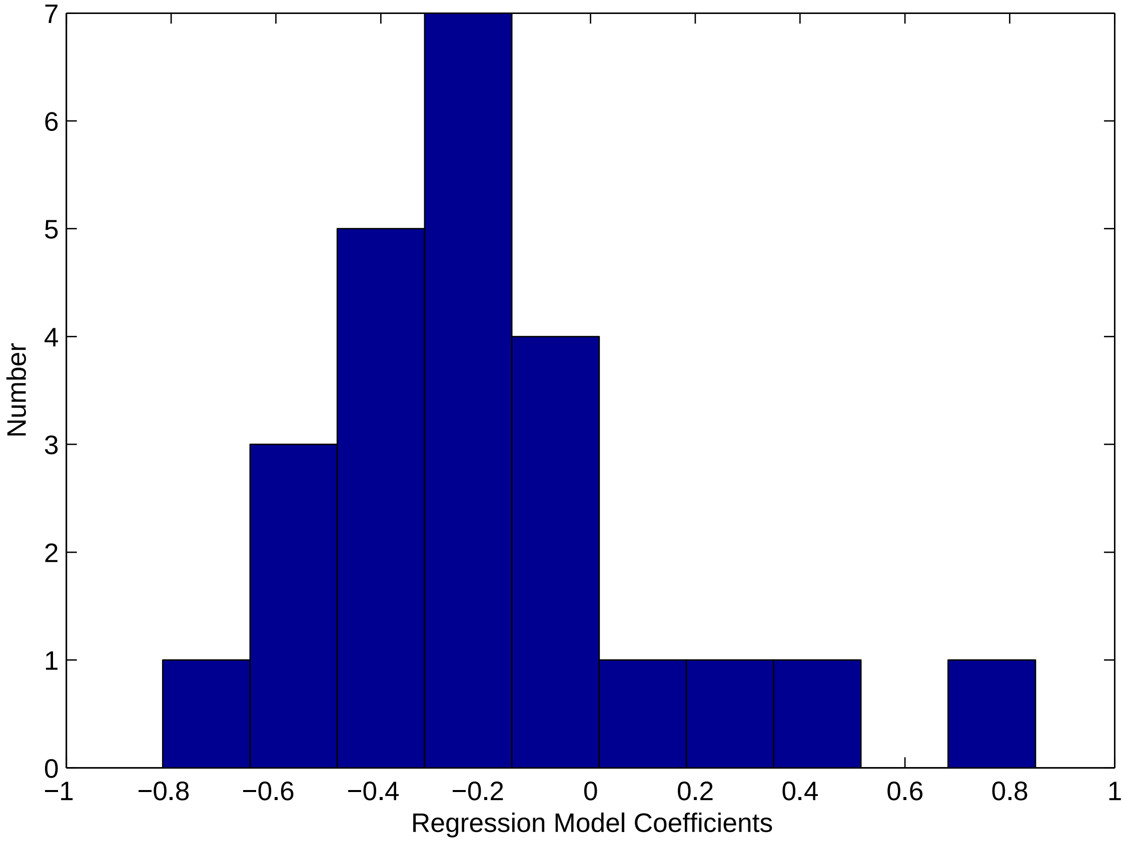
Simulated time series of the expected revenues $\rho_t$, (top graph), and the liquidity reserve $C_t$, equity value $J_t$, and total firm value $J_t + P_t$ (all on the bottom graph, in increasing order).



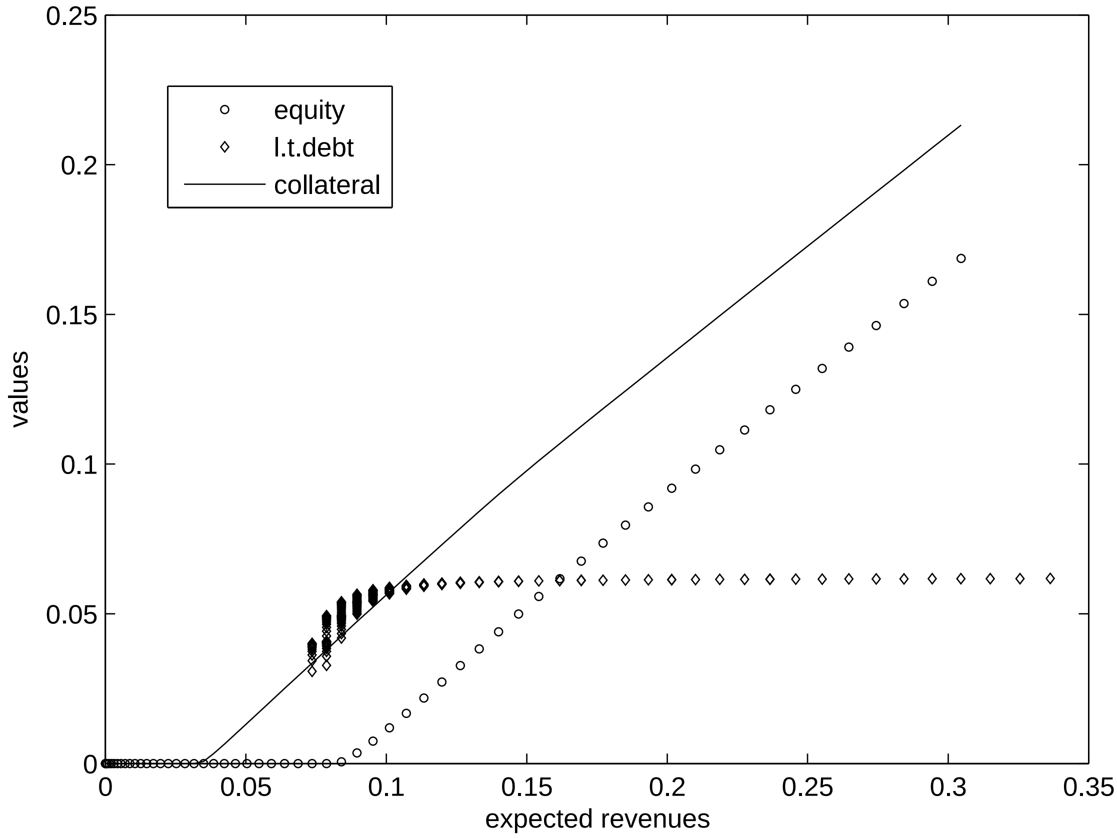
## Figure 3:

This is the distribution of coefficients from the regression of changes of the cash/asset ratio on its one-year lagged value as in Opler *et.al.* Figure 3. This is based on 121 years of simulated history for the benchmark model broken into sub samples of 5 years each. As in the Opler *et.al.* study the coefficients are generally negative reflecting mean reversion. However, large positive coefficients are not uncommon, reflecting a reaction to a substantial change of firm profitability which induces a persistent change in cash holdings. The median coefficient is -0.205 as compared to -0.242 found by Opler *et.al.*.



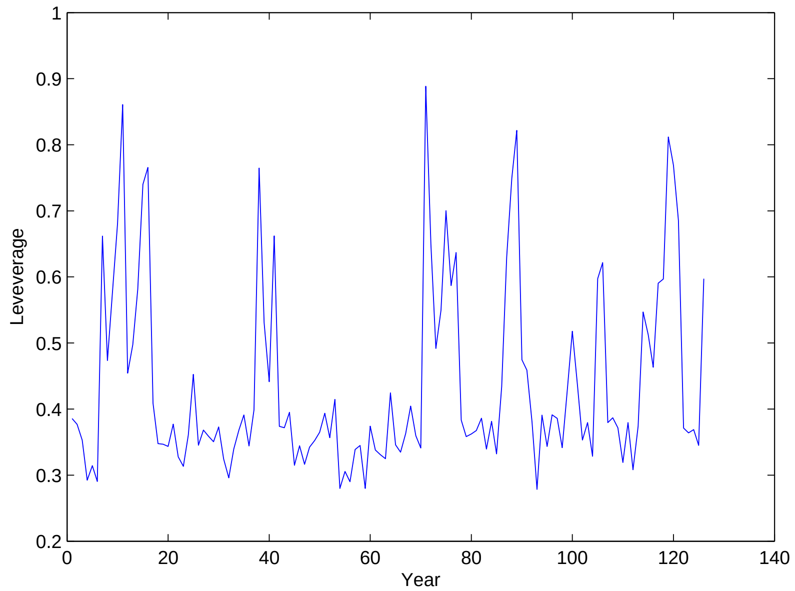
## Figure 4:

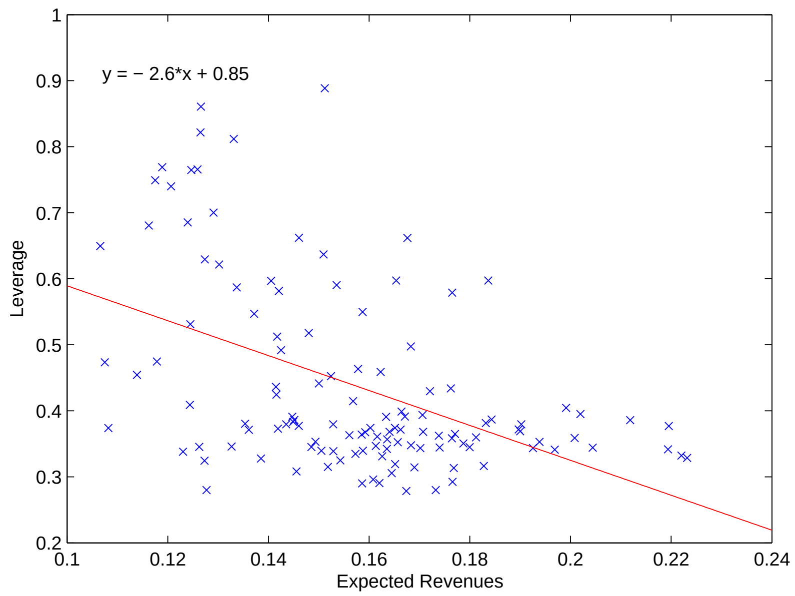
Here the circles represent the value of the equity in the benchmark example, when the liquidity reserve is zero. The solid line is the value of collateral, i.e., $1 - \alpha$ times unlevered firm value. The diamonds the debt value in the benchmark case, for all levels of liquidity.



## Figure 5:

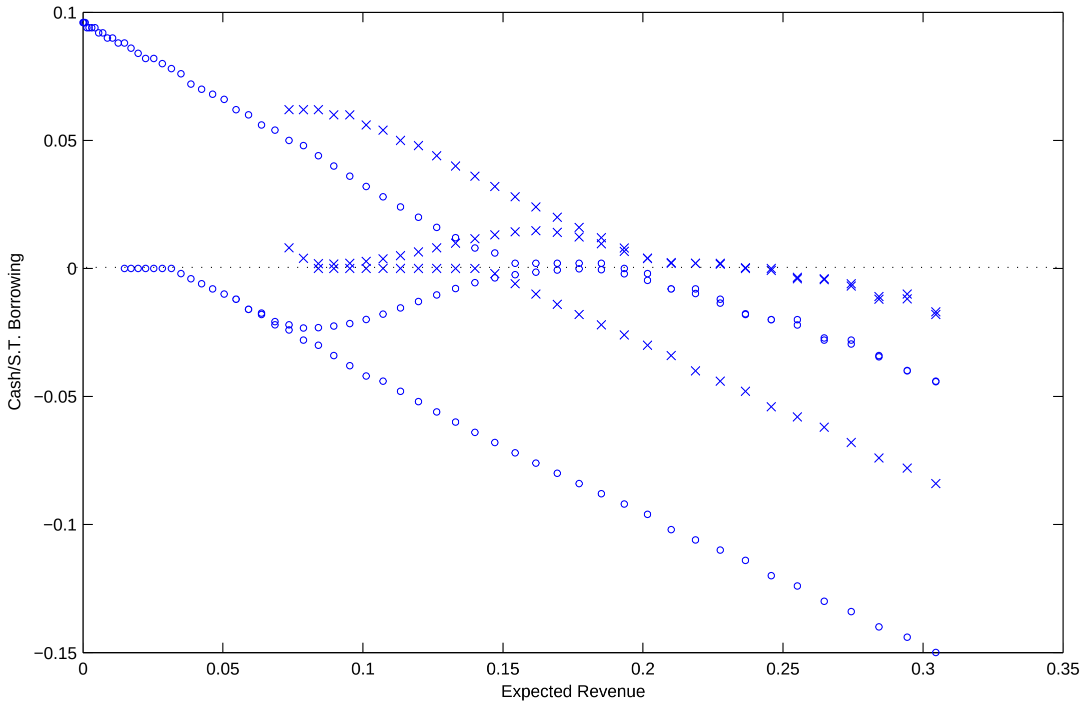
The top panel is the annual time series of the leverage defined as the book value of total debt to the value of the firm (book value of debt plus market value of equity) derived from the simulation of the benchmark model under the optimal policy. The lower panel is the plot of leverage against the previous year's expected revenues. The estimated slope of the regression of leverage on lagged expected revenues is $-2.6$ and is significant at the 1% level.



## Figure 6:

This figure depicts optimal policy for two different levels of long-term debt. The 'x' correspond to solution for the case $q = 0.004$. From top to bottom the lines give the cash target, the average cash and the debt-capacity/equity issue boundary and reproduce the results of Figure 1. This is the "high debt" case. The lines in circles give the comparable results for the "low debt" case, which is calculated for $q = 0.0$ i.e., no long-term debt.



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