

# Choice of Corporate Risk Management Tools under Moral Hazard\*

Jan Bena<sup>†</sup>  
CERGE-EI<sup>‡</sup>, LSE

June 2006

## Abstract

This paper examines the choice of tools for managing a firm's operational risks: cash reserves, insurance contracts, and financial assets under an optimal financing contract that solves moral hazard between insiders and outside investors. Risk management is valuable as it reduces the costs of raising external financing, increases debt capacity, lessens underinvestment, and improves welfare. I show that insurance is superior as it facilitates the outside financing relationship but leads to inefficient excessive continuation if used without coverage limits. When insurance against an operational risk is not available, the firm uses financial assets instead or resorts to holding cash reserves.

*Keywords:* Risk management, Corporate insurance, Moral hazard, Optimal contracting

*JEL classification:* D82, G22, G31

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\*I am grateful to Ron Anderson, Antoine Faure-Grimaud, David Webb, Margaret Bray, Andrea Prat, Sergey Slobodyan, Jan Hanousek, and seminar participants at the International Symposium on Insurance and Finance in Bergen; the LSE, London; the Czech Economic Society Meeting in Prague; and CERGE-EI, Prague for useful comments and suggestions. I have benefited from my stay at the Financial Markets Group, the London School of Economics and Political Science, London.

<sup>†</sup>Please address correspondence to: Jan Bena, The Financial Markets Group, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK, Email: j.bena@lse.ac.uk, Tel: +44 (0) 20 7955 6270, Fax: +44 (0) 20 7852 3580.

<sup>‡</sup>A joint workplace of the Center for Economic Research and Graduate Education, Charles University, Prague, and the Economics Institute of the Academy of Sciences of the Czech Republic. Address: CERGE-EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic.

# 1 Introduction

Risk management is a very important area of corporate decision making as it entails assessing and adjusting corporations' exposure to various sources of risk by the use of financial derivatives, insurance contracts, and other activities. Financial executives and CEOs report risk management as one of their most important concerns.<sup>1</sup> The literature on risk management generally assumes that managers act in the best interests of firms' shareholders and describes the optimal risk shifting between agents. In contrast, this paper investigates the role risk management plays in the relationship between corporate insiders and outside investors with conflicting interests under moral hazard in a world of universal risk neutrality. In other words, I derive the reason why corporate risk management is pursued from agency-based considerations. In my setup, risk management is valuable as it reduces the costs of raising external financing, increases a firm's debt capacity, lessens underinvestment, and improves welfare.

I analyze how insurance contracts are employed to manage a firm's operational risk, and I compare situations when insurance is available and when it is not in an optimal contracting framework.<sup>2</sup> In my setting, the insurance contract eliminates information asymmetry between a corporate insider and an outside investor and hence allows for better contracting. A better contracting environment leads to lower costs of raising funds and the insurance contract facilitates the outside financing relationship. I also show that insurance contracts are not uniformly beneficial. Besides an efficiency increase due to increased debt capacity, insurance has a negative effect as well; it ex post induces the excessive continuation of the firm, which is ex ante inefficient. I investigate in detail costs and benefits from using insurance on a firm's debt capacity, the optimality of the investment decision, and

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<sup>1</sup>See, for example, Bodnar et al. (1998) or Rawls and Smithson (1990).

<sup>2</sup>See von Thadden (1995) for a similar optimal contracting framework.

insiders' payoff to provide conditions under which insurance is beneficial. To avoid inefficient excessive continuation, the insider seeks insurance contracts with upper limits on the extent of insurance coverage. If setting such limits is impossible or if no insurance contract for a particular operational risk is available—when it is difficult to write formal insurance contracts because the underlying shock cannot be well-described *ex ante* or objectively measured *ex post*—the insider substitutes insurance by investing in financial assets or resorts to holding cash. Finally, I investigate costs and benefits from using financial assets and compare these with costs and benefits from using insurance contracts and holding cash reserves.

This paper is related to the literature on the determinants of hedging and the literature on corporate demand for liquidity. The hedging literature investigates how violating the assumptions of Modigliani and Miller (1958) makes risk management activities beneficial. Stulz (1984) argues that firms do hedge because managers who are in charge of corporate decision making are risk averse. Smith and Stulz (1985) show that firms should manage risks if they face a convex corporate tax schedule or deadweight costs of financial distress. DeMarzo and Duffie (1995) add that hedging is used because it improves the informativeness of corporate earnings as a signal of management ability. Finally, Brown and Toft (2002) analyze whether a firm facing both hedgeable and unhedgeable risk should use standard forward and options contracts or more exotic derivative structures. From this literature, my model is close to that used by Froot et al. (1993). According to their model, firms who face convex costs of external financing do perform risk management operations if their investment decisions depend on the amount of internal resources available.<sup>3</sup> In this case, the optimal risk management policy equalizes the marginal payoff from

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<sup>3</sup>Rochet and Villeneuve (2004) have provided theoretical treatments of precautionary corporate liquidity holdings. Anderson and Carverhill (2005), and Mello and Parsons (2000) study a demand for liquidity and hedging decision of a financially constrained firm in a dynamic setting. A similar framework is used by Boyle and Guthrie (2003), who concentrate on how liquidity of a financially constrained firm affects the timing of corporate investment decisions.

total resources available for investing across all states of the world. Similarly to Froot et al., insurance is useful in my model because it reduces the costs of external financing. In contrast to their paper, I endogenize the costs of external financing by introducing moral hazard into the borrowing/lending relationship. The moral hazard is solved and the borrowing/lending decision (and hence the investment decision) is determined by the optimal financing contract between the insider and the outside investor.

My model builds upon the analysis by Holmström and Tirole (1998), which establishes a link between the literature on corporate demand for liquidity and corporate risk management. Holmström and Tirole show that firms do hold cash as a buffer to cover stochastic liquidity shocks to their cash flows; in other words, cash reserves are used as a rough risk management tool. I take the extended version of their risk-management-by-holding-cash result as a benchmark and further enrich the analysis by introducing insurance contracts and financial assets as on-hand tools available for changing the risk exposure of a firm.

I characterize basic elements of the model in the next section. In section 3, I solve the optimal contracting problem between the insider and the outside investor when to hold cash reserves is the only risk management option. Section 4 defines insurance and solves for an optimal contract with insurance. I discuss costs and benefits from using insurance contracts in section 5, discuss financial assets in section 6, and conclude in section 7.

## 2 Model

There are three dates  $t = 0, \frac{1}{2}, 1$ ; two periods (the first between dates 0 and  $\frac{1}{2}$  and the second between dates  $\frac{1}{2}$  and 1); and two agents. Both agents are risk-neutral with an additively separable utility function over undiscounted consumption streams:  $u(c_0, c_{\frac{1}{2}}, c_1) = c_0 + c_{\frac{1}{2}} + c_1$ . There is one universal good (called "cash")

used for consumption and investment, which is storable at a zero interest rate. One agent (manager, insider, or she) has access to a stochastic constant-returns-to-scale production technology, which I call from now on a "project." The manager has endowment of cash  $A > 0$  at date 0, and no endowments at dates  $\frac{1}{2}$  and 1. The manager raises additional cash from the other agent (investor, outsider, or he) who is assumed to have enough cash at all dates. The parties sign an optimal financial contract that allows the manager to raise additional funding to invest in the project at date 0.

The project bears an operational risk at date  $\frac{1}{2}$  and a fundamental risk at date 1. Due to the fundamental risk at date 1, the project does not always deliver a positive payoff: if investment of size  $I > 0$  is made at date 0 and the project succeeds, the payoff at date 1 is  $RI > 0$ ; whereas, if the project fails, the payoff at date 1 is 0.<sup>4</sup> Date 1 project's payoffs are verifiable. The fundamental risk is subject to effort moral hazard. In the second period, the manager privately chooses probability  $p$  of project's success (of payoff  $RI$ ). She can exert either high or low effort. If she exerts high effort, the probability of success is  $p_H$ ; if she exerts low effort, the probability of success is  $p_L$ , where  $0 < p_L < p_H < 1$ . If the manager shirks, she enjoys private benefits  $BI > 0$ . The amount of cash invested  $I$  is a continuous variable and is subject only to financing constraints.

The operational risk captures the idea that the manager might need more cash before the project is completed than was planned initially at date 0. At intermediate date  $\frac{1}{2}$ , an additional uncertain amount of cash has to be spent to cover such unexpected cash need (a liquidity shock). If required additional cash is paid, the project continues and the final payoff is realized at date 1 as described above.<sup>5</sup> If additional cash is not paid the project terminates at date  $\frac{1}{2}$  yielding nothing

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<sup>4</sup>For simplicity I consistently define all "cash" variables on a per-unit-of-investment basis; therefore, cash payoff at date 1,  $RI$ , is  $R \cdot I$  where  $R > 0$  can be understood as the gross return per unit of investment.

<sup>5</sup>By covering the liquidity shock, the initial size of the project  $I$  remains unchanged.

and investment  $I$  is lost. The liquidity shock attains two values  $\tilde{\rho}I = \{\underline{\rho}I, \bar{\rho}I\}$  (referred to as the low shock and the high shock), where  $0 < \underline{\rho} < \bar{\rho}$ . The project is hit by low liquidity shock  $\underline{\rho}I$  with probability  $\alpha$  and by high liquidity shock  $\bar{\rho}I$  with probability  $1 - \alpha$ . Liquidity shock can be interpreted as a shock to a firm's cash flows. The high shock is more harmful as it makes the firm more difficult to sustain. The structure of the project and risks involved are common knowledge, but once realized the size of the liquidity shock is revealed only to the manager and is unobservable for the investor. The manager can unilaterally quit the contract with the investor and consume all cash left in the project (if any) at date  $\frac{1}{2}$ , which creates the second source of moral hazard.

Finally, I assume that the project cannot be liquidated partially at date  $\frac{1}{2}$  (nondivisibility) and the manager cannot pay out more cash than she has at any given date (limited liability). The timeline of the project is depicted in the figure below.

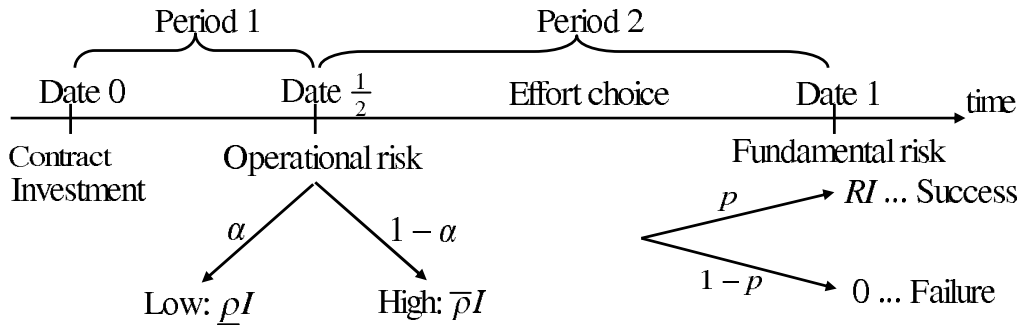


Figure 1: Project's timeline

The following assumptions apply:

**A1** Given the realization of high liquidity shock  $\bar{\rho}I$ , the project has a negative continuation value at date  $\frac{1}{2}$  even if high effort is exerted,  $p_H R - \bar{\rho} < 0$ .<sup>6</sup>

**A2** Given the realization of low liquidity shock  $\underline{\rho}I$ , the project has a positive

<sup>6</sup>Since  $p_H > p_L$ , if low effort is exerted  $p_L R - \bar{\rho} < 0$  as well.

continuation value at date  $\frac{1}{2}$  if high effort is exerted,  $p_H R - \underline{\rho} > 0$ , but a negative continuation value if low effort is exerted,  $p_L R - \underline{\rho} < 0$ .

### 3 Contracting without Insurance

#### 3.1 First-best

Assumption **A1** implies that if the project is hit by the high liquidity shock it should be abandoned at date  $\frac{1}{2}$ , and assumption **A2** implies that high effort should be exerted if the low shock is realized. With these optimal rules, the expected first-best value of the project is  $V_{FB} = A + [\alpha(p_H R - \underline{\rho}) - 1] I_{FB}$ .

**Lemma 1.** *Optimal first-best investment is infinite,  $I_{FB}^* \rightarrow \infty$ , if  $\alpha(p_H R - \underline{\rho}) > 1$ ; zero,  $I_{FB}^* = 0$ , if  $\alpha(p_H R - \underline{\rho}) < 1$ ; and any positive real number,  $I_{FB}^* \in [0, \infty)$ , if  $\alpha(p_H R - \underline{\rho}) = 1$ .*

**A3** For the remainder of the paper I assume that the expected first-best return per unit of investment is positive,  $\alpha(p_H R - \underline{\rho}) > 1$ , and the project should be undertaken with infinite investment in the first-best world.

#### 3.2 Optimal Financing Contract under Moral Hazard

This section establishes the benchmark for the analysis. Similarly to Holmström and Tirole (1998), I show that a pile of cash held from date 0 to date  $\frac{1}{2}$  allows the manager to sustain the liquidity shock and hence serves as a risk management tool.

To raise financing the manager offers a contract to the investor at date 0. The market for external financing is competitive and the investor accepts the contract if he breaks even. The manager and the investor cannot commit to a two-period contract and the parties will renegotiate the initial contract if it is beneficial for

them to do so at date  $\frac{1}{2}$ . The optimal financing contract is characterized in the following proposition:<sup>7</sup>

**Proposition 1.** *The optimal contract  $\mathcal{C}^* = \{T_R^*, T_0^*, I^*, L^*\}$  has the following properties:*

a) *date 1 transfer from the manager to the investor following the project's failure is  $T_0^* = 0$ ;*

b) *date 1 transfer from the manager to the investor following the project's success is  $T_R^* = \min\{T_R^{\frac{1}{2}}, T_R^1\}$ , where*

$$\begin{cases} T_R^{\frac{1}{2}} \equiv R - \frac{\rho}{p_H} < R \text{ and} \\ T_R^1 \equiv R - \frac{B}{p_H - p_L} < R; \end{cases}$$

c) *total loan provided by the investor satisfies  $L^* = I^* - A + \rho I^* = \alpha p_H T_R^* I^* > 0$ ;*

d) *investment is  $I^* = A \frac{1}{1 + \rho - \alpha p_H T_R^*} > 0$ ;*

e) *manager's expected payoff is  $P^* = A \left(1 + \frac{\alpha(p_H R - \rho) - 1}{1 + \rho - \alpha p_H T_R^*}\right) > A$ ;*

f) *investor's expected profit is  $\Pi^* = 0$ ;*

*For the contract to exist, the following conditions on  $\rho$  have to be satisfied: if  $T_R^* = T_R^{\frac{1}{2}}$ , the existence condition is  $0 < \underline{\rho}_{\min}^{\frac{1}{2}} < \rho < \underline{\rho}_{\max}$ ; and if  $T_R^* = T_R^1$ , the existence condition is  $0 < \underline{\rho}_{\min}^1 < \rho < \underline{\rho}_{\max}$ ; where  $\underline{\rho}_{\min}^{\frac{1}{2}} \equiv \frac{\alpha p_H R - 1}{1 + \alpha}$ ,  $\underline{\rho}_{\min}^1 \equiv \alpha p_H T_R^1 - 1$ , and  $\underline{\rho}_{\max} \equiv \frac{\alpha p_H R - 1}{\alpha}$ .*

There are two moral hazards the optimal contract has to solve. First, the manager can shirk in the second period, which limits transfer  $T_R^1$  the investor can receive at date 1 by the amount needed to induce the manager to exert high effort,  $\frac{B}{p_H - p_L}$ . Second, the manager can consume cash at date  $\frac{1}{2}$ , which limits transfer  $T_R^{\frac{1}{2}}$  the investor can receive by the amount needed to induce the manager not to do it when the low shock arrives and to continue the project,  $\frac{\rho}{p_H}$ . Since both incentive constraints have to be satisfied at the same time, transfer  $T_R^*$  is determined by the incentive compatible condition that is binding first. The focus of this paper is on the case not analyzed by Holmström and Tirole (1998), when the cash stealing moral

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<sup>7</sup>Proof of Proposition 1 is provided in the Appendix.



hazard is more severe than the effort moral hazard,  $T_R^{\frac{1}{2}} < T_R^1 \iff B < \frac{p_H - p_L}{p_H} \underline{\rho}$ .

As the size of the liquidity shock is unobservable for the investor and he is unable to elicit truth-telling, he does not provide any additional contingent financing at date  $\frac{1}{2}$ . Intuitively, if the manager sends a message that the liquidity shock is high, it is optimal to shut the project down and not to provide any additional cash. If the investor is ready to provide additional financing at date  $\frac{1}{2}$  following a low shock message, the manager will always report the shock to be low. If the true liquidity shock is low, the project is efficiently continued, but if the true shock is high, the managers consumes all additional cash provided and the project is abandoned. Also, to induce the manager to tell the truth by adjusting her date 1 payoff is not feasible as well. This is because the date  $\frac{1}{2}$  cash stealing moral hazard is followed by the effort moral hazard in the second period. The incentive constraint that elicits high effort in the second period is binding and fixes the payoffs for the investor and the manager at date 1. Hence, the manager's date 1 payoff cannot be altered to elicit the truth about the size of the liquidity shock at date  $\frac{1}{2}$  as it would conflict with the high effort choice.

The cash stealing moral hazard is solved by the optimal contract as follows: at date 0, when the contract is signed, part of total loan  $L^*$  is invested into the project,  $I^* - A$ , and the rest is held as cash reserves from date 0 to date  $\frac{1}{2}$  to cover the coming liquidity shock. The size of the cash holding is equal to the amount needed to cover low liquidity shock  $\underline{\rho}I^*$ . For this implementation to work, one has to assume either that investment  $I^*$  is verifiable or that the investor is able to commit to not providing any additional cash at date  $\frac{1}{2}$ . These assumptions are necessary to incentivize the manager not to invest the whole loan at date 0 (her payoff is increasing in investment) and put aside cash to cover the low liquidity shock. The second possible implementation is as follows: the investor provides his loan in two noncontingent instalments  $L^* = L_0^* + L_{\frac{1}{2}}^*$ , where  $L_0^* = I^* - A$  is the loan

at date 0 and  $L_{\frac{1}{2}}^* = \underline{\rho}I^*$  is the loan at date  $\frac{1}{2}$ . The manager uses  $L_0^*$  for investment and  $L_{\frac{1}{2}}^*$  is used to cover the low liquidity shock. In both cases the investor pays amount  $\underline{\rho}I^*$  no matter what liquidity shock arrives, and this translates into the costs of external financing as cash is inefficiently consumed by the manager when the high shock is realized and the project is terminated.

When the project is hit by the high liquidity shock (probability  $1 - \alpha$ ) or when the project ends up with failure (probability  $1 - p_H$ ), the payoff to the investor is zero. When the project is hit by the low liquidity shock and results in success (probability  $\alpha p_H$ ) the investor receives per-unit-of-investment transfer  $T_R^*$ . As a result, the investor is willing to provide loan  $L^*$  equal only to the expected pledgeable income,  $\alpha p_H T_R^* I^*$ .

The level of investment  $I^*$  is finite and therefore lower than in the first-best case. Investment is increasing in the manager's initial cash endowment  $A$ , with probability that the low liquidity shock appears  $\alpha$ , with probability that the project succeeds (given high effort)  $p_H$ , and with project's payoff following success  $R$ . On the other hand, an increase in low liquidity shock  $\underline{\rho}$  or an increase in private benefits  $B$  increases the cost of external financing and optimal investment  $I^*$  decreases.

## 4 Contracting with Insurance

### 4.1 Insurance

I introduce operational risk insurance by enabling the manager to invest cash into an insurance contract whose payoff is perfectly negatively correlated with the project's liquidity shock  $\tilde{\rho}$ . If the manager pays  $\lambda H$  in insurance premium at date 0, the insurance coverage at date  $\frac{1}{2}$  is  $\underline{\rho}H$  if the low liquidity shock is realized and  $\bar{\rho}H$  if the high liquidity shock is realized ( $H$  is the number of cash units of insurance purchased and  $\lambda$  stands for the unit price of insurance). To be consistent with the

perfect competitiveness in the market for outside financing, I assume that insurance is available at a fair price:  $\lambda H \equiv \alpha \underline{\rho} H + (1 - \alpha) \bar{\rho} H$ . Insurance is a very powerful instrument in contracting between the manager and the investor as it allows them to overcome the problem of the unobservable liquidity shock at date  $\frac{1}{2}$ .

Real world insurance contracts that coincide with this model are, for example, corporate property and casualty insurance or business interruption insurance. Chesler and Anglim (2001; 1) describe the business interruption insurance as follows: “The purpose of business interruption coverage is to protect the insured against a loss of income if it suffers a loss that causes it to suspend operations. ...[B]usiness interruption coverage can put the insured in the position it would be in if no loss had occurred.”

In this model, the investor is by assumption separated from the insurer; in other words, the investor does not write insurance contracts and the insurance company is not a lender. This is because the main interest of this paper is to investigate how the plain use of the insurance contract with standard properties that mimic real world affects the contracting of a loan between the manager and the investor. I do not aim at designing the optimal insurance coverage agreed among the manager, the investor, and an insurance company. At the same time, the assumption that the investor is separated from the insurer is supported by extensive anecdotal evidence suggesting that mergers of banks and insurance companies and the, so-called, "bancassurance" model is a failure.<sup>8</sup> Most probably this is because insurance and lending involve different monitoring technologies which result in necessary specialization and separation. The investor specializes in assessing a project's fundamental value and provides a loan based on the expected repayment, whereas the insurer specializes in assessing the size of damages to the business' operations triggered

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<sup>8</sup>See for example the article in McKinsey Quarterly 2003 No. 2 by Lars et al. stating: “And though many large bancassurance deals took place in the period, they receive the thumbs-down: market weren't convinced that uniting two activities as different as insurance and banking creates value.”(p. 2)

by random events and adjusts the payoff accordingly. In order to write insurance contracts, the insurer has to be able to ex ante define and well describe risks covered by the contract and to objectively measure the impact of events on the firm's operations ex post. This is complex as many operational risks are business specific and require special expertise.<sup>9</sup> In other words, not only are the investor and the insurer specialized in observing distinct features of the project, the timing is also different; the investor does the monitoring ex ante, before the investment is made, whereas the insurer monitors ex post, after the shock hits the firm. Consistently, I assume that it is prohibitively costly for the investor to observe date  $\frac{1}{2}$  liquidity shock and for the insurer to observe terminal date 1 project's payoffs.<sup>10</sup> Finally, I assume that the payoff from the insurance contract directly offsets the liquidity shock and hence the manager cannot consume the payoff from insurance before it is used to cover the liquidity shock. This is a standard provision of property and casualty or business interruption insurance contracts.

## 4.2 First-best with Insurance

Within the first-best framework with the possibility of insurance, I first discuss the case when the liquidity shock is fully insured and hence eliminated.

**Lemma 2.** *In the case of full insurance,  $H = I_{FB,i}$ ,<sup>11</sup> the expected first-best value of the project is  $V_{FB,i} = A + (p_H R - 1 - \lambda)I_{FB,i}$  and the expected first-best per-unit-of-investment return is lower relative to the first-best case with cash reserves.*

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<sup>9</sup>See for example the article in McKinsey Quarterly December 2005 by Markus et al. arguing: "Knowledge and talent management are critical to [Property and casualty insurer's] success, particularly because risk data for niche products are limited. This deficiency makes statistical analysis difficult and means that companies must base their underwriting more on informed judgement."(p. 4)

<sup>10</sup>The alternative explanation of separation of raising external financing and providing insurance is as follows: Insurance serves as a monitoring device for dispersed investors who due to a free-riding problem have no incentive to monitor risks involved in the business. In this case, the insider pays the insurance premium to the insurer to do the monitoring for the investors.

<sup>11</sup>Subscript  $i$  indicates the case with full insurance.

*Proof.* The expected first-best value of the project under full insurance,  $H = I_{FB,i}$ , is  $V_{FB,i} = A - I_{FB,i} - \lambda H + \alpha [p_H R I_{FB,i} - \underline{\rho}(I_{FB,i} - H)] + (1 - \alpha) [p_H R - \bar{\rho}(I_{FB,i} - H)]$ , which simplifies to  $V_{FB,i} = A + (p_H R - 1 - \lambda)I_{FB,i}$ . Full insurance gives higher expected per-unit-of-investment returns relative to the first-best no-insurance case iff  $p_H R - 1 - \lambda \geq \alpha(p_H R - \underline{\rho}) - 1$ , which is, under fair price of insurance, equivalent to  $\bar{\rho} \leq p_H R$ . The last inequality violates **A1**.  $\square$

Similarly, it is easy to show that in the first-best world partial insurance,  $H < I_{FB,i}$ , is worse than no-insurance as well.

**Lemma 3.** *In the case of partial insurance,  $H < I_{FB,i}$ , the expected first-best per-unit-of-investment return is lower relative to the first-best case with cash reserves.*<sup>12</sup>

The results of Lemmas 2 and 3 are driven by the fact that in the first-best world without moral hazard, the risk neutrality of agents precludes any positive effect insurance might have. At the same time, full insurance leads to the elimination of the high liquidity shock, the project is not terminated when the high shock is realized, and full insurance hence induces the excessive continuation of the project relative to the no insurance case. The efficiency from not covering the high liquidity shock and stopping the project when it arrives is lost. In the case of partial insurance, efficiency decreases as well since the part of insurance premium paid to cover the high liquidity shock is a waste.

### 4.3 Optimal Financing Contract with Full Insurance under Moral Hazard

This section shows that in contrast to the first-best world full insurance has a positive effect when the manager is separated from the investor and the project

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<sup>12</sup>Proof of Lemma 3 is provided in the Appendix.

is subject to moral hazard. The following assumption ensures that full insurance could be beneficial:

**A4** In expectation formed at date 0, the project has a positive value if high effort is exerted and the expected value of the liquidity shock is paid:  $\alpha(p_H R - 1 - \underline{\rho}) + (1 - \alpha)(p_H R - 1 - \bar{\rho}) > 0$ .<sup>13</sup>

If **A4** is satisfied, the expected first-best value of the project under full insurance  $V_{FB,i}$  is positive and increasing in investment. Similarly to the case without insurance, the manager offers the contract to the investor at date 0, the market for external financing is competitive, and the investor accepts the contract if he breaks even. The optimal financing contract is characterized in Proposition 2.<sup>14</sup>

**Proposition 2.** *The optimal contract  $\mathcal{C}_i^* = \{T_{R,i}^*, T_{0,i}^*, I_i^*, L_i^*\}$  has the following properties:*

- a) date 1 transfer from the manager to the investor following the project's failure is  $T_{0,i}^* = 0$ ;
- b) date 1 transfer from the manager to the investor following the project's success is  $T_{R,i}^* \equiv R - \frac{B}{p_H - p_L} < R$ ;
- c) total loan provided by the investor satisfies  $L_i^* = (1 + \lambda)I_i^* - A = p_H T_{R,i}^* I_i^* > 0$ ;
- d) investment is  $I_i^* = A \frac{1}{1 + \lambda - p_H T_{R,i}^*} > 0$ ;
- e) manager's expected payoff is  $P_i^* = A \left(1 + \frac{p_H R - (1 + \lambda)}{1 + \lambda - p_H T_{R,i}^*}\right) > A$ ;
- f) investor's expected profit is  $\Pi_i^* = 0$ ;

For the contract to exist, the following conditions on  $\lambda$  have to be satisfied:  $0 < \lambda_{\min} < \lambda < \lambda_{\max}$ , where  $\lambda_{\min} \equiv p_H T_{R,i}^* - 1$  and  $\lambda_{\max} \equiv p_H R - 1$ .<sup>15</sup>

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<sup>13</sup>Note that **A1** implies that  $p_H R - 1 - \bar{\rho} < 0$ , and therefore one needs  $p_H R - 1 - \underline{\rho} > 0$  in order for **A4** to be satisfied. **A1** and **A2** also imply that in expectation formed at date 0, the project has a negative value if low effort is exerted:  $\alpha(p_L R - 1 - \underline{\rho}) + (1 - \alpha)(p_L R - 1 - \bar{\rho}) < 0$ .

<sup>14</sup>Proof of Proposition 2 is provided in the Appendix.

<sup>15</sup>The equilibrium exists for reasonable parameter values since  $0 < \lambda_{\min} \Leftrightarrow \frac{p_H R - 1}{p_H} > \frac{B}{p_H - p_L}$ , which is consistent with the no-insurance case, and  $\lambda_{\min} < \lambda_{\max}$  for any  $B > 0$ . See the proof of this Proposition for more details.

At date 0, when the contract is signed, part of total loan  $L_i^*$  is invested into the project,  $I_i^* - A$ , and the rest is used to purchase insurance coverage for the coming liquidity shock,  $\lambda I_i^*$ . Since the liquidity shock is fully eliminated by insurance, there is no need for the investor to provide cash to the manager at date  $\frac{1}{2}$  and for the manager to hold cash from date 0 to  $\frac{1}{2}$ . The inefficiency caused by the unobservability of the liquidity shock and the possibility of the manager to consume cash reserves when the high liquidity shock arrives is eliminated by insurance. This reduces the cost of raising cash from the outside investor, increases the debt capacity of the project,  $L_i^*$ , and hence increases investment  $I_i^*$ . On the other hand, the project is not abandoned if the high liquidity shock arrives which causes a new inefficiency—the project is continued excessively (relative to the no insurance case) with an adverse effect on its debt capacity and investment.

As in the case with cash reserves the manager can shirk in the second period, which limits transfer  $T_{R,i}^*$  the investor can receive by the amount needed to induce the manager to exert high effort,  $\frac{B}{p_H - p_L}$ . The cash-stealing incentive constraint is eliminated by insurance. The investor is willing to provide loan  $L_i^*$  equal to the expected pledgeable income,  $p_H T_{R,i}^* I_i^*$ . The level of investment  $I_i^*$  is finite (lower than  $I_{FB,i}^*$ ), increasing in the manager's initial cash endowment,  $A$ , increasing in the probability that the project succeeds (given high effort),  $p_H$ , and increasing in project's payoff following success,  $R$ . On the other hand, an increase in cost of insurance  $\lambda$  or an increase in private benefits  $B$  decreases investment.

## 5 Costs and Benefits of Insurance

### 5.1 Effect of Full Insurance on Investment, Debt Capacity, and Insider's Payoff

This section shows under what conditions the full insurance contract increases investment, the debt capacity of the project, and the manager's payoff. The effect of full insurance is not obvious as the efficiency increase from the elimination of the liquidity shock trades off against the inefficient excessive project's continuation relative to the no insurance case.

Consider the case without insurance when the date 1 incentive constraint on managerial effort is binding,  $T_R^* = T_R^1 = T_{R,i}^*$ . Investment under full insurance is higher than investment in the no-insurance case iff

$$I_i^* > I^* \Leftrightarrow (1 - \alpha)(\bar{\rho} - \underline{\rho}) < (1 - \alpha)p_H T_{R,i}^* \Leftrightarrow \bar{\rho} - \underline{\rho} < p_H \left( R - \frac{B}{p_H - p_L} \right). \quad (1)$$

The left-hand-side of (1) stands for the expected cost of excessive continuation associated with full insurance (the additional cash spent on the insurance premium to inefficiently cover the high liquidity shock) whereas the right-hand-side stands for the expected additional pledgeable income the investor receives when the project is fully insured. If the value of the option to close the project following the high liquidity shock is high ( $\bar{\rho} - \underline{\rho}$  is high) and/or if the effort moral hazard is large ( $\frac{B}{p_H - p_L}$  is large), condition (1) is not satisfied and the total investment in the project is higher under no insurance.

The debt capacity of the project under full insurance is higher than the debt capacity in the no-insurance case iff

$$L_i^* > L^* \Leftrightarrow \frac{p_H T_{R,i}^*}{1 + \lambda} > \frac{\alpha p_H T_R^1}{1 + \underline{\rho}} \Leftrightarrow \bar{\rho} - \underline{\rho} < \frac{1 + \underline{\rho}}{\alpha}. \quad (2)$$



The debt capacity of the project under full insurance is higher if the expected return to the investor per unit of total investment under insurance  $\frac{p_H T_{R,i}^*}{1+\lambda}$  is higher than the expected return per unit of total investment under no insurance  $\frac{\alpha p_H T_R^*}{1+\underline{\rho}}$ . If the low liquidity shock is high and/or the cost of insurance is low, the debt capacity is higher under insurance.

Finally, the manager's payoff under full insurance is higher than the manager's payoff in the no-insurance case iff

$$P_i^* > P^* \Leftrightarrow \frac{p_H (R - T_{R,i}^*)}{1 + \lambda} > \alpha \frac{p_H (R - T_R^1)}{1 + \underline{\rho}} + \frac{1 - \alpha}{1 + \underline{\rho}} \left( \underline{\rho} - \frac{p_H T_{R,i}^*}{1 + \lambda} \right). \quad (3)$$

The left-hand-side of inequality (3) stands for the expected manager's payoff per unit of total investment when the project is fully insured. The first term on the right-hand-side of (3) is the expected manager's payoff per unit of total investment when the project is not insured and the low shock arrives. The second term stands for the difference in the expected manager's payoff per unit of total investment between the no-insurance and the full insurance case. When the high liquidity shock arrives and the project is not insured, the manager consumes cash  $\underline{\rho}$  she brings to date  $\frac{1}{2}$ , whereas she consumes the equivalent of this cash given by fraction  $\frac{p_H T_{R,i}^*}{1+\lambda}$  when the project is insured.

The situation when the date  $\frac{1}{2}$  non-stealing incentive constraint is binding in the case without insurance,  $T_R^* = T_R^{\frac{1}{2}} < T_{R,i}^*$ , gives similar results. The investment under full insurance is higher than the investment in the no-insurance case iff

$$I_i^* > I^* \Leftrightarrow \alpha p_H T_R^{\frac{1}{2}} < p_H T_{R,i}^* - (1 - \alpha)(\bar{\rho} - \underline{\rho}). \quad (4)$$

The left-hand-side of (4) stands for the investor's total expected payoff if the project is not insured. The right-hand-side is the investor's total expected payoff if the project is fully insured decreased by the cost of excessive continuation associated

with this insurance (insurance premium paid inefficiently). The debt capacity of the project under full insurance is higher than the debt capacity in the no-insurance case iff

$$L_i^* > L^* \Leftrightarrow \frac{p_H T_{R,i}^*}{1 + \lambda} > \frac{\alpha p_H T_R^{\frac{1}{2}}}{1 + \underline{\rho}}. \quad (5)$$

The debt capacity of the project under full insurance is higher if the expected return to the investor per unit of total investment under full insurance  $\frac{p_H T_{R,i}^*}{1 + \lambda}$  is higher than the expected return per unit of investment under no insurance  $\frac{\alpha p_H T_R^{\frac{1}{2}}}{1 + \underline{\rho}}$ . Finally, the manager's payoff under full insurance is higher than the manager's payoff in the no-insurance case iff

$$P_i^* > P^* \Leftrightarrow \frac{p_H R - (1 + \lambda)}{1 + \lambda - p_H T_{R,i}^*} > \frac{\alpha(p_H R - \underline{\rho}) - 1}{1 + \underline{\rho} - \alpha p_H T_R^{\frac{1}{2}}}. \quad (6)$$

Overall, full insurance tends to dominate holding cash reserves in managing operational risk as the low shock increases (the inefficiency caused by the consumption of the cash reserves by the manager when the high shock arrives in the no-insurance case increases) or as the high shock decreases (the inefficiency caused by the excessive project's continuation when the high shock arrives under the insurance decreases).

Also, the manager never prefers to combine holding a partial cash reserve (holding cash from date 0 to  $\frac{1}{2}$  in an amount lower than the size of the low liquidity shock) with covering the rest by purchasing partial insurance against both shocks,  $H < I^*$ . This is because, if the manager combines a partial cash reserve with partial insurance such that the low shock is just covered, the amount of cash she can consume when the high shock is realized is higher than if cash reserves only are used. Hence, in this case, cash holding in the amount equal to the size of the low liquidity shock dominates as it leads to lower inefficient consumption and higher debt capacity than the combination of partial cash holding and partial insurance. Similarly,

if the manager combines partial cash reserves with partial insurance,  $H < I^*$ , such that the high shock is just covered, the project is excessively continued as in the full insurance case, but, in addition, the manager can inefficiently consume cash when the low shock is realized. In this case, the combination of partial insurance and a partial cash holding is dominated by full insurance,  $H = I^*$ .

## 5.2 Low-shock Insurance

Since the insurance against the high liquidity shock leads to inefficiency, the next step is to analyze the case when only insurance against the low liquidity shock is available. Such insurance contract specifies that if the manager pays premium  $\gamma H$  at date 0, the insurance payoff at date  $\frac{1}{2}$  is  $\underline{\rho}H$  if the low liquidity shock is realized and zero if the high liquidity shock is realized. All assumptions about insurance remain unchanged including its availability at a fair price:  $\gamma H \equiv \alpha \underline{\rho}H$  ( $H$  is again the number of cash units of insurance purchased and  $\gamma$  stands for the unit price of the low-shock insurance).

The optimal financing contract between the manager and the investor under the low-shock insurance is characterized in Proposition 3.<sup>16</sup>

**Proposition 3.** *The optimal contract  $\mathcal{C}_{\underline{i}}^* = \{T_{R,\underline{i}}^*, T_{0,\underline{i}}^*, I_{\underline{i}}^*, L_{\underline{i}}^*\}$  has the following properties:*

a) *date 1 transfer from the manager to the investor following the project's failure is  $T_{0,\underline{i}}^* = 0$ ;*

b) *date 1 transfer from the manager to the investor following the project's success is  $T_{R,\underline{i}}^* \equiv R - \frac{B}{p_H - p_L} < R$ ;*

c) *total loan provided by the investor satisfies  $L_{\underline{i}}^* = (1 + \gamma)I_{\underline{i}}^* - A = \alpha p_H T_{R,\underline{i}}^* I_{\underline{i}}^* > 0$ ;*

d) *investment is  $I_{\underline{i}}^* = A \frac{1}{1 + \gamma - \alpha p_H T_{R,\underline{i}}^*} > 0$ ;*

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<sup>16</sup>Proof of Proposition 3 is analogous to the one for Proposition 2 and is omitted. Subscript  $\underline{i}$  indicates the case with the low-shock insurance.

e) manager's expected payoff is  $P_{\underline{i}}^* = A \left( 1 + \frac{\alpha p_H R - (1+\gamma)}{1+\gamma - \alpha p_H T_{R,\underline{i}}^*} \right) > A$ ;

f) investor's expected profit is  $\Pi_{\underline{i}}^* = 0$ ;

g) operational risk is fully insured  $H_{\underline{i}}^* = I_{\underline{i}}^*$ ;

For the contract to exist, the following conditions on  $\underline{\rho}$  have to be satisfied:  $0 <$

$\underline{\rho}_{\min,\underline{i}} < \underline{\rho} < \underline{\rho}_{\max,\underline{i}}$ , where  $\underline{\rho}_{\min,\underline{i}} \equiv \frac{\alpha p_H T_{R,\underline{i}}^* - 1}{\alpha}$  and  $\underline{\rho}_{\max,\underline{i}} \equiv \frac{\alpha p_H R - 1}{\alpha}$ .<sup>17</sup>

As the investor knows that the low liquidity shock is covered by insurance, the manager cannot credibly claim any additional funds at date  $\frac{1}{2}$  and the project is efficiently terminated if the high liquidity shock is realized. At the same time, since the manager carries no cash from date 0 to  $\frac{1}{2}$ , the inefficient consumption of cash when the high liquidity shock is realized is prevented too. As a result, the low-shock insurance efficiently solves the moral hazard resulting from the unobservability of the liquidity shock and the only remaining friction between the parties is the nonverifiability of effort in the second period (fundamental moral hazard). The low-shock insurance case is equivalent to the investor observing the size of the liquidity shock directly.

The following lemma confirms that the low-shock insurance is strictly better than both no insurance and full insurance:

**Lemma 4.** *The optimal financing contract under the low-shock insurance induces higher investment, higher debt capacity, and higher payoff to the manager relative to both the no insurance and the full insurance case:  $I_{\underline{i}}^* > I^*$  and  $I_{\underline{i}}^* > I_i^*$ ;  $L_{\underline{i}}^* > L^*$  and  $L_{\underline{i}}^* > L_i^*$ ;  $P_{\underline{i}}^* > P^*$  and  $P_{\underline{i}}^* > P_i^*$ .<sup>18</sup>*

According to Lemma 4, the low-shock insurance, if available at a fair price, is the best and the most efficient tool to manage the operational risk. More importantly, if the manager can choose between all possibilities analyzed so far, she selects low-shock insurance to maximize her payoff. This is efficient since the level of investment

<sup>17</sup>The equilibrium exists for reasonable parameter values since  $\underline{\rho}_{\min,\underline{i}} > 0 \Leftrightarrow \frac{\alpha p_H R - 1}{\alpha p_H} > \frac{B}{p_H - p_L}$ , which is consistent with the no-insurance case, and  $\underline{\rho}_{\min,\underline{i}} < \underline{\rho}_{\max,\underline{i}}$  for any  $B > 0$ .

<sup>18</sup>Proof of Lemma 4 is provided in the Appendix.

is the highest and the value of the project is the highest. In other words, to prevent the inefficient continuation of a firm with a negative value, the manager optimally chooses to set ex ante a limit above which liquidity shocks are not covered by insurance. Reasons for less than full hedging or insurance coverage analyzed so far in the literature come from transaction costs, asymmetric information, or incentives problems between hedging/insurance markets and a firm. I provide a new rationale why firms should not offload all risks even if it is possible to do so at fair costs. From the outside investor's point of view, the low-shock insurance is beneficial since it serves as a commitment device for the manager not to consume cash and not to lie about the size of the liquidity shock at date  $\frac{1}{2}$ .

## 6 Contracting with Financial Assets

### 6.1 Financial Assets

As an alternative to insurance, I allow the insider to manage the operational risk by investing in a financial asset. The financial asset is modeled to serve as an imperfect hedge only, and the analysis in this section can also be viewed as the one in which the key assumption behind the superiority of the low-shock insurance is relaxed. As argued earlier, writing insurance contracts requires expertise and is feasible only if the underlying shock can be described in the contract and the impact of the event on the value of the firm can be ex post verified by the insurer. In contrast to insurance, the asset's payoffs are not perfectly negatively correlated with the operational risk (the size of the liquidity shock) and is used as a risk management tool when it is impossible to write a formal insurance contract. The aim is to determine what asset types are used and to provide a cost/benefit analysis across all risk management tools—cash reserves, insurance contracts, and financial assets—in a single tractable framework.

At date 0 the manager invests cash into the financial asset to receive the following payoff structure at date  $\frac{1}{2}$ : If the low liquidity shock is realized, the asset gives payoff 0 with probability  $m$  and payoff 1 with probability  $1 - m$ . In contrast, if the high liquidity shock is realized, the asset gives payoff 1 with probability  $m$  and 0 with probability  $1 - m$ . Consistently with the previous analysis, the asset is priced fairly and is available in a zero net supply,  $\delta H \equiv \alpha(1 - m)H + (1 - \alpha)mH$ . The amount of cash units of the asset (or the face value of the asset) purchased is denoted as  $H$ . Typically, price is  $0 < \delta < 1$  as  $\alpha \in (0, 1)$  and  $m \in (0, 1)$ . Finally, I assume that asset's payoffs at date  $\frac{1}{2}$  are unobservable for the investor. The asset's payoff structure is depicted in the following figure.

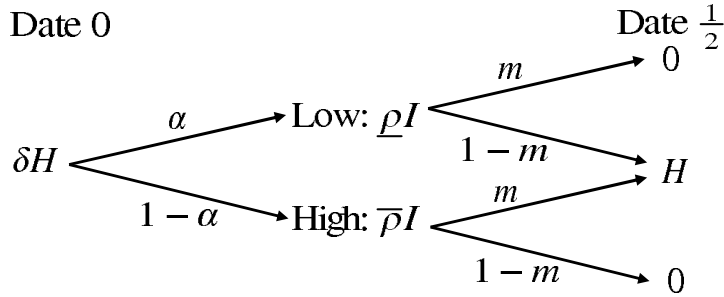


Figure 2: Financial asset's payoff structure

## 6.2 First-best with Financial Assets

Within the first-best framework I compare the use of the financial asset with the benchmark case when the cash reserves only are used. When using the financial asset, the manager buys just enough of it to be able to sustain the low liquidity shock when the asset's payoff is positive,  $H = \underline{\rho}I_A$ .<sup>19</sup>

**Lemma 5.** *When the financial asset only is used as a risk management tool, the manager buys  $H = \underline{\rho}I_{FB,A}$  units of the asset. The expected first-best value of the project is  $V_{FB,A} = A + [\alpha(1 - m)(p_H R - \underline{\rho}) - 1] I_{FB,A}$  and the expected first-best*

<sup>19</sup>Subscript  $A$  indicates the case with the financial asset.

*per-unit-of-investment return is lower relative to the first-best case with cash for any  $m \in (0, 1)$ .*<sup>20</sup>

The financial asset allows the manager to sustain the low liquidity shock only with probability  $1 - m$  when the asset gives a positive payoff. With probability  $m$  the asset's payoff is zero, the low liquidity shock cannot be covered, and the project is inefficiently terminated. Holding cash reserves, therefore, dominates investing in the asset in the first-best world whenever  $m \in (0, 1)$  and the financial asset is equivalent to the case with cash if  $m = 0$ .

### 6.3 Optimal Financing Contract with Financial Assets

Similarly to the full insurance case, there is no benefit from using the financial asset in the first-best world. This section shows that the financial asset has a positive effect when the manager is separated from the investor and the project is subject to moral hazard. The following assumption ensures that the financial asset could be beneficial.

**A5** There exists  $\hat{m} > 0$  defined by  $\alpha(1 - \hat{m})(p_H R - \underline{\rho}) \equiv 1$  such that for  $m \in (0, \hat{m})$  the expected first-best per-unit-of-investment return of the project with investment in the financial asset is positive.

According to **A5** for  $m \in (0, \hat{m})$  the expected first-best value of the project under risk management using the financial asset,  $V_{FB,A}$ , is positive and increasing in investment. As in the previous cases, the manager offers the contract to the investor at date 0, the market for external financing is competitive, and the investor accepts the contract if he breaks even. The optimal financing contract is characterized in Proposition 4.<sup>21</sup>

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<sup>20</sup>Proof of Lemma 5 is analogous to the one for Lemma 2 and is omitted.

<sup>21</sup>Proof of Proposition 4 is similar to the one for Proposition 2 and is omitted.

**Proposition 4.** *The optimal contract  $C_A^* = \{T_{R,A}^*, T_{0,A}^*, I_A^*, L_A^*\}$  has the following properties:*

a) *date 1 transfer from the manager to the investor following the project's failure is  $T_{0,A}^* = 0$ ;*

b) *date 1 transfer from the manager to the investor following the project's success is*

$$T_{R,A}^* = \min\{T_{R,A}^{\frac{1}{2}}, T_{R,A}^1\} \text{ where } \begin{cases} T_{R,A}^{\frac{1}{2}} \equiv R - \frac{\rho}{p_H} < R \text{ and} \\ T_{R,A}^1 \equiv R - \frac{B}{p_H - p_L} < R; \end{cases}$$

c) *total loan provided by the investor satisfies  $L_A^* = (1 + \delta\rho)I_A^* - A = \alpha(1 - m)p_H T_{R,A}^* I_A^* > 0$ ;*

d) *investment is  $I_A^* = A \frac{1}{1 + \delta\rho - \alpha(1-m)p_H T_{R,A}^*} > 0$ ;*

e) *manager's expected payoff is  $P_A^* = A \left(1 + \frac{\alpha(1-m)(p_H R - \rho) - 1}{1 + \delta\rho - \alpha(1-m)p_H T_{R,A}^*}\right) > A$ ;*

f) *investor's expected profit is  $\Pi_A^* = 0$ ;*

g) *the face value of the financial asset purchased is  $H^* = \rho I_A^*$ ;*

*For the contract to exist, the following conditions on  $m$  have to be satisfied:  $0 <$*

$$m_{\min} < m < m_{\max}, \text{ where } m_{\min} \equiv \frac{\alpha(p_H T_{R,A}^* - \rho) - 1}{\alpha(p_H T_{R,A}^* - \rho) + (1 - \alpha)\rho} \text{ and } m_{\max} \equiv 1 - \frac{1}{\alpha(p_H R - \rho)} = \hat{m}.$$

At date 0, when the contract is signed, part of total loan  $L_A^*$  is invested in the project,  $I_A^* - A$ , and the rest is used to purchase the asset,  $\delta\rho I_A^*$ . In contrast to the full insurance case, the liquidity shock is not eliminated by investing in the financial asset and hence both incentive constraints are a part of the contract as in the cash reserves case.

The costs and benefits from using the financial asset can be best described by comparing this case to the insurance and the cash reserves case. If  $m = 0$  the financial asset case is analogous to the low-shock insurance ( $I_A^* = I_{\underline{i}}^*$ ,  $L_A^* = L_{\underline{i}}^*$ , and  $P_A^* = P_{\underline{i}}^*$ ) and hence provides the best possible protection against the liquidity shock. In contrast, if  $m = 1$ , the financial asset case is the worst as it gives a positive payoff only when the high shock is realized. If  $0 < m < 1$ , the financial asset has an advantage relative to full insurance as it does not induce the inefficient



continuation of the project when the high shock is realized. On the other hand, it has a disadvantage relative to full insurance as it allows the manager to sustain the low liquidity shock only with probability  $1 - m$ . With probability  $m$  the asset's payoff is zero, the low liquidity shock is not covered, and the project is inefficiently terminated. Parameter  $m$  measures some basis risk that cannot be hedged using the asset. The lower the  $m$ , the better the hedge of the liquidity shock that can be achieved using the financial asset. Clearly, there have to be threshold values of  $m$  such that for lower values the financial asset dominates full insurance and the opposite is true for higher values of  $m$ . If  $T_{R,A}^* = T_{R,A}^1 = T_{R,i}^*$ ; the investment is higher when the financial asset is used relative to the full insurance case,  $I_A^* > I_i^*$ , if and only if  $m < m_{I,i}^1$ ; the debt capacity is higher,  $L_A^* > L_i^*$ , if and only if  $m < m_{L,i}^1$ ; and the expected manager's payoff is higher,  $P_A^* > P_i^*$ , if and only if  $m < m_{P,i}^1$ . Similarly, if  $T_{R,A}^* = T_{R,A}^{\frac{1}{2}} < T_{R,i}^*$ ; the investment is higher when the financial asset is used relative to the full insurance case,  $I_A^* > I_i^*$ , if and only if  $m < m_{I,i}^{\frac{1}{2}}$ ; the debt capacity is higher,  $L_A^* > L_i^*$ , if and only if  $m < m_{L,i}^{\frac{1}{2}}$ ; and the expected manager's payoff is higher,  $P_A^* > P_i^*$ , if and only if  $m < m_{P,i}^{\frac{1}{2}}$ .<sup>22</sup> In comparison to the cash reserves case, the financial asset has again a disadvantage as it allows the manager to sustain the low liquidity shock only with probability  $1 - m$ . The benefit of the financial asset relative to the cash reserves case is that the expected inefficient consumption of cash when the high liquidity shock arrives is lower; cash is inefficiently consumed only with probability  $m$  when the asset gives a positive payoff. In other words, to transfer one unit of cash from date 0 to date  $\frac{1}{2}$  is cheaper using the financial asset relative to holding a cash reserve. In order to have one unit of cash at date  $\frac{1}{2}$ , the manager has to put aside one unit of cash at date 0 and the amount invested into the project decreases by the same amount. In contrast, when purchasing the financial asset the manager pays price  $\delta < 1$  to receive one unit

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<sup>22</sup>Analytical expressions for thresholds  $m_{I,i}^1$ ,  $m_{L,i}^1$ ,  $m_{P,i}^1$ ,  $m_{I,i}^{\frac{1}{2}}$ ,  $m_{L,i}^{\frac{1}{2}}$ , and  $m_{P,i}^{\frac{1}{2}}$  are provided in section 8.5 of the Appendix.

of cash at date  $\frac{1}{2}$ . Overall, the inefficient termination of the project when the low shock is realized trades off against the financial asset's lower cost. The investment in the project when the financial asset is used is higher relative to the cash reserves case,  $I_A^* > I^*$ , if and only if  $m < m_I$ , and the debt capacity is higher,  $L_A^* > L^*$ , if and only if  $m < m_L$ . Similar inequalities hold for the manager's expected payoff: If  $T_{R,A}^* = T_{R,A}^{\frac{1}{2}} = T_R^{\frac{1}{2}}$ , then  $P_A^* > P^*$  if and only if  $m < m_P^{\frac{1}{2}}$ ; and if  $T_{R,A}^* = T_{R,A}^1 = T_R^1$ , then  $P_A^* > P^*$  if and only if  $m < m_P^1$ .<sup>23</sup>

The comparison indicates that risk management using a full insurance contract or cash reserves can be dominated by the use of the financial asset if the asset's payoff is positive when the low shock arrives with a high enough probability and if the asset's payoff is zero when the high shock arrives with a high enough probability. The opposite is true if the asset's payoff is positive when the high shock arrives with a high enough probability and if the asset's payoff is zero when the low shock arrives with a high enough probability. In other words, the payoff from the asset has to be sufficiently negatively correlated with the size of the liquidity shock.

## 7 Conclusion

I analyze three possible ways how firms manage their operational risks—building up cash reserves, using insurance contracts, and investing in financial assets. I investigate the effect from employing these tools on the ability of the firm to raise funding from outside investors in an optimal contracting framework. Risk management activities are pursued in my setup as they alleviate moral hazard between insiders and outside investors, and hence decrease the cost of external financing, increase the debt capacity of the firm, and allow the firm to invest more.

The best way to manage operational risk is to use insurance contracts provided

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<sup>23</sup>Analytical expressions for thresholds  $m_I$ ,  $m_L$ ,  $m_P^{\frac{1}{2}}$ , and  $m_P^1$  are provided in section 8.5 of the Appendix.

that the insurer observes shocks affecting the firm, adjusts insurance payoff accordingly, and behaves competitively. I provide a new rationale for having limits on insurance coverage because the insurance of risks of all sizes is ex ante inefficient as it ex post leads to the excessive continuation of the firm. I show that the insurance coverage limits are set such that the firm with a negative continuation value caused by a pending large shock to the firm's operations is liquidated relying on limited liability. If insurance against an operational risk is not available or if insurance contracts with coverage limits do not exist, the firm uses financial assets or resorts to holding cash reserves. I provide a detailed analysis of the costs and benefits of these risk management tools, and I state conditions under which one is preferred to the others. Insurance with no coverage limits dominates holding cash when the lowest value of a shock with which the firm can be hit is high or when the highest value of a shock with which the firm can be hit is low. Finally, I show that accumulating cash reserves and the use of insurance contracts is always dominated by investing in financial assets if the correlation between assets' payoffs and the shocks is sufficiently aligned.

## 8 Appendix

### 8.1 Proof of Proposition 1

#### 8.1.1 Contract Structure

The contract is contingent on date 1 verifiable payoffs. Assumption **A2** implies that the contract has to implement high effort in the second period. The realization of the liquidity shock at date  $\frac{1}{2}$  is not observable for the investor and the contract can be contingent only on messages sent by the manager,  $\{\underline{\hat{\rho}}, \widehat{\rho}\}$ , where  $\underline{\hat{\rho}}$  is understood as the low liquidity shock message and  $\widehat{\rho}$  as the high liquidity shock message. The

contract is an 8-tuple

$$\mathcal{C} = \{T(\underline{\hat{\rho}}, R), T(\underline{\hat{\rho}}, 0), T(\widehat{\bar{\rho}}, R), T(\widehat{\bar{\rho}}, 0), t(\underline{\hat{\rho}}), t(\widehat{\bar{\rho}}), I, L\}, \quad (7)$$

where  $T(\underline{\hat{\rho}}, R)$  is date 1 per-unit-of-investment cash transfer from the manager to the investor following message  $\underline{\hat{\rho}}$  and outcome  $R$  (transfers  $T(\underline{\hat{\rho}}, 0)$ ,  $T(\widehat{\bar{\rho}}, R)$ ,  $T(\widehat{\bar{\rho}}, 0)$  are defined analogously);  $t(\underline{\hat{\rho}})$  is date  $\frac{1}{2}$  per-unit-of-investment cash transfer from the investor to the manager following message  $\underline{\hat{\rho}}$  (transfer  $t(\widehat{\bar{\rho}})$  is defined analogously);  $I > 0$  is total cash invested by the manager into the project at date 0; and  $L$  is total cash transfer (amount lent) from the investor to the manager,  $L \geq I - A$ .

### 8.1.2 Truth-telling

Let's first analyze the situation at date  $\frac{1}{2}$  following the arrival of high liquidity shock  $\bar{\rho}$ . Only the manager knows the true value of the liquidity shock. Assumption **A1** implies that no matter what amount of cash the manager brought to date  $\frac{1}{2}$  from date 0, she is better off by consuming this cash (as well as consuming any additional cash she receives, if any, from the investor at this date) and not continue the project.<sup>24</sup> If she sends message  $\widehat{\bar{\rho}}$ , the investor does not provide any additional cash,  $t(\widehat{\bar{\rho}}) \leq 0$ , and since the manager has no incentive to return the cash,  $t(\widehat{\bar{\rho}}) \geq 0$ , there is no cash transfer following the high liquidity shock message,  $t(\widehat{\bar{\rho}}) = 0$ . If the manager sends message  $\underline{\hat{\rho}}$  and the transfer from the investor is positive,  $t(\underline{\hat{\rho}}) > 0$ , the manager consumes transfer  $t(\underline{\hat{\rho}})$ . To prevent the manager from lying about the size of the shock, the transfer that follows the low liquidity shock message has to be nonpositive,  $t(\underline{\hat{\rho}}) \leq 0$ , and since the manager has no incentive to return the cash to the investor, there is no cash transfer,  $t(\underline{\hat{\rho}}) = 0$ , following the low liquidity shock message as well. Therefore, date  $\frac{1}{2}$  transfers are equal zero,  $t(\underline{\hat{\rho}}) = t(\widehat{\bar{\rho}}) = 0$ , and

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<sup>24</sup>Following the high liquidity shock, the project has a negative continuation value and neither the manager nor the investor can commit ex ante to pursue the project that loses money.

are consistent with not lying when the high liquidity shock arrives.

If low liquidity shock  $\underline{\rho}$  arrives at date  $\frac{1}{2}$ , the project is worth continuing if high effort is exerted in the second period. (Assume that the contract is such that high effort is implemented and the manager has just enough cash to cover the low liquidity shock. These assumptions are verified in the next subsection.) If the manager sends message  $\widehat{\rho}$ , the investor does not provide any additional cash,  $t(\widehat{\rho}) = 0$ , and the manager continues the project iff the following condition is satisfied:

$$p_H \left[ R - T(\widehat{\rho}, R) \right] - (1 - p_H)T(\widehat{\rho}, 0) \geq \underline{\rho}. \quad (8)$$

On the left-hand-side there is the payoff from continuing the project, and on the right-hand-side there is the payoff from consuming cash immediately. If the manager sends message  $\underline{\rho}$ , the investor does not provide any additional cash,  $t(\underline{\rho}) = 0$ , as he knows that the manager has enough cash to cover the low shock and the manager continues the project iff the analogous condition to (8) is satisfied:

$$p_H \left[ R - T(\underline{\rho}, R) \right] - (1 - p_H)T(\underline{\rho}, 0) \geq \underline{\rho}. \quad (9)$$

Assuming that the contract is such that the project is continued when the low liquidity shock arrives (again, this is verified in the next subsection), the manager does not lie about the size of the liquidity shock iff

$$p_H \left[ R - T(\underline{\rho}, R) \right] - (1 - p_H)T(\underline{\rho}, 0) \geq p_H \left[ R - T(\widehat{\rho}, R) \right] - (1 - p_H)T(\widehat{\rho}, 0),$$

which can be rewritten as

$$p_H \left[ T(\widehat{\rho}, R) - T(\underline{\rho}, R) \right] - (1 - p_H) \left[ T(\widehat{\rho}, 0) - T(\underline{\rho}, 0) \right] \geq 0. \quad (10)$$

Conditions (8), (9), and (10) imply that without loss of generality one can set

$T_R \equiv T(\widehat{\rho}, R) = T(\underline{\rho}, R)$  and  $T_0 \equiv T(\widehat{\rho}, 0) = T(\underline{\rho}, 0)$ , and rewrite contract (7) in the simplest form:

$$C' = \{T_R, T_0, I, L\}. \quad (11)$$

### 8.1.3 Manager's Problem

According to assumptions **A1** and **A2**, the contract has to induce high effort and make the manager abandon the project when the high liquidity shock is realized. The expected payoff for the manager is

$$P = A + L - I + \alpha [p_H(R - T_R) - (1 - p_H)T_0 - \underline{\rho}] I, \quad (12)$$

and the expected profit for the investor is

$$\Pi = -L + \alpha [p_H T_R + (1 - p_H)T_0] I. \quad (13)$$

The optimal contract can be found as a solution to the following problem:

$\max_{C'} \{P\}$  subject to:

$$P \geq A \quad (PC^{manager})$$

$$\Pi \geq 0 \quad (PC^{investor})$$

$$p_H(R - T_R) - (1 - p_H)T_0 \geq p_L(R - T_R) - (1 - p_L)T_0 + B \quad (IC^{date1})$$

$$p_H(R - T_R) - (1 - p_H)T_0 \geq \underline{\rho} \quad (IC^{date\frac{1}{2}})$$

$$A + L - I \geq 0 \quad (14)$$

$$A + L - I \geq \underline{\rho} I \quad (15)$$

$$A + L - I \geq \underline{\rho} I - (RI - T_R I) \quad (16)$$

$$A + L - I \geq \underline{\rho} I + T_0 I \quad (17)$$

The manager maximizes her payoff so that both parties want to participate in the project, constraints  $(PC^{manager})$  and  $(PC^{investor})$ ; high effort is exerted in the second period, constraint  $(IC^{date1})$ ; and the manager prefers to continue the project following realization of the low liquidity shock, constraint  $(IC^{date\frac{1}{2}})$ . Assume that the manager's payoff  $P$  is increasing in the amount of cash invested into the project, and it is optimal for her to invest as much cash as possible at date 0. (This assumption is verified at the end of this paragraph.) If the manager invested all cash and the project fails at date 1 (its payoff is zero), the manager has no cash to repay the investor, and the optimal contract has to set  $T_0^* = 0$ . For the same reason, if the manager invested all cash and the project succeeds at date 1 (its payoff is  $R$ ), transfer following success  $T_R^*$  is not greater than  $R$ . Using  $T_0^* = 0$ , one can rewrite incentive constraint  $(IC^{date1})$  as  $T_R^* \leq T_R^1 \equiv R - \frac{B}{p_H - p_L}$  and incentive constraint  $(IC^{date\frac{1}{2}})$  as  $T_R^* \leq T_R^{\frac{1}{2}} \equiv R - \frac{\rho}{p_H}$ . The manager's payoff  $P$  is decreasing in  $T_R$  whereas the investor's payoff  $\Pi$  is increasing in  $T_R$ . Hence, depending on parameter values, at least one of the two incentive constraints,  $(IC^{date1})$  or  $(IC^{date\frac{1}{2}})$ , is binding. Since both incentive constraints have to be satisfied at the same time, the optimal contract sets  $T_R^* = \min\{T_R^{\frac{1}{2}}, T_R^1\}$ . Also, the manager's payoff  $P$  is increasing in  $L$  whereas the investor's payoff  $\Pi$  is decreasing in  $L$ . Since the market for loans is competitive the investor's participation constraint is binding as well. Binding participation constraint  $(PC^{investor})$  implies that the optimal loan is equal to the expected pledgeable payoff for the investor:

$$L^* = \alpha p_H T_R^* I^*. \quad (18)$$

Substituting (18) into the manager's expected payoff (12) and simplifying gives

$$P^* = A + [\alpha(p_H R - \underline{\rho}) - 1] I^*. \quad (19)$$

According to assumption **A3**,  $\alpha(p_H R - \underline{\rho}) > 1$ , and the manager's payoff is increasing in investment  $I^*$ .

Inequalities (14) to (17) are feasibility constraints imposed by the structure of the model. Condition (14) states that the loan provided by the investor has to cover the need for external financing given the investment decision at date 0. Condition (15) says that given loan  $L$ , investment  $I$ , and the manager's cash  $A$ , the insider is able to withstand the low liquidity shock at date  $\frac{1}{2}$ . Inequality (16) is the limited liability condition that imposes an upper bound on possible repayment  $T_R$  to the investor if the project succeeds at date 1. Similarly, (17) is the limited liability condition that imposes an upper bound on possible repayment  $T_0$  to the investor if the project fails at date 1. Note that because  $T_0^* = 0$  constraint (17) is identical to (15) and can be removed from the problem. Similarly, constraint (14) can be removed since if (15) is satisfied, (14) is satisfied as well. Rearranging constraint (16) one gets  $L \geq I - A + \underline{\rho}I - (R - T_R)I$ , and since  $T_R^* \leq R$ , constraint (16) is satisfied when (15) is satisfied and (15) is the only relevant constraint. Since the manager's payoff is increasing in investment, she keeps the least amount of cash possible from date 0 to date  $\frac{1}{2}$  and constraint (15) is binding:

$$L^* = I^* - A + \underline{\rho}I^*. \quad (20)$$

The optimal investment level can be solved from equations (18) and (20):

$$I^* = A \frac{1}{1 + \underline{\rho} - \alpha p_H T_R^*}. \quad (21)$$

#### 8.1.4 Existence

Assumption **A3** imposes an upper bound on the size of the small liquidity shock  $\underline{\rho} < \underline{\rho}_{\max} \equiv \frac{\alpha p_H R - 1}{\alpha}$ . To have positive investment in equilibrium, according to (21)  $1 + \underline{\rho} - \alpha p_H T_R^* > 0$ . If  $T_R^* = T_R^{\frac{1}{2}}$ , investment is positive iff  $\underline{\rho} > \underline{\rho}_{\min}^{\frac{1}{2}} \equiv \frac{\alpha p_H R - 1}{1 + \alpha}$  and



the sufficient condition for the equilibrium to exist is

$$0 < \underline{\rho}^{\frac{1}{2}} \equiv \frac{\alpha p_H R - 1}{1 + \alpha} < \underline{\rho} < \underline{\rho}_{\max} \equiv \frac{\alpha p_H R - 1}{\alpha}. \quad (22)$$

Assumption **A3** implies that  $\alpha p_H R - 1 > 0$  and according to conditions in (22) the equilibrium always exists since  $0 < \underline{\rho}^{\frac{1}{2}} < \underline{\rho}_{\max}$  is always satisfied. If  $T_R^* = T_R^1$ , investment is positive iff  $\underline{\rho} > \underline{\rho}_{\min}^1 \equiv \alpha p_H \left( R - \frac{B}{p_H - p_L} \right) - 1$  and the sufficient condition for the equilibrium to exist is

$$0 < \underline{\rho}_{\min}^1 \equiv \alpha p_H \left( R - \frac{B}{p_H - p_L} \right) - 1 < \underline{\rho} < \underline{\rho}_{\max} \equiv \frac{\alpha p_H R - 1}{\alpha}. \quad (23)$$

According to conditions (23), equilibrium exists iff  $0 < \underline{\rho}_{\min}^1 < \underline{\rho}_{\max}$ , which can be rewritten as

$$\frac{\alpha p_H R - 1}{\alpha p_H} > \frac{B}{p_H - p_L} > \frac{\alpha p_H R - 1}{\alpha p_H} - \frac{\alpha p_H R - 1}{\alpha^2}.$$

Since assumption **A3** holds,  $\alpha p_H R - 1 > 0$ , and the equilibrium exists for some values of  $B$ .

## 8.2 Proof of Lemma 3

Define  $\bar{H}$  as the level of insurance that solves equation  $p_H R I_{FB,i} - \bar{\rho}(I_{FB,i} - \bar{H}) = 0$ . If  $0 < H < \bar{H}$ , it is ex post optimal to liquidate the project if the high liquidity shock arrives and the value of the project is  $V_{FB,i} = A - I_{FB,i} - \lambda H + \alpha [p_H R I_{FB,i} - \underline{\rho}(I_{FB,i} - H)]$ , which under a fair insurance price simplifies to  $V_{FB,i} = A - (1 - \alpha)\bar{\rho}H + [\alpha(p_H R - \underline{\rho}) - 1] I_{FB,i}$ . Since  $(1 - \alpha)\bar{\rho}H > 0$ , the value of the project is lower relative to the first-best case without insurance.

If  $H > \bar{H} > 0$ , it is ex post optimal to continue the project if the high liquidity shock arrives and the value of the project is  $V_{FB,i} = A - I_{FB,i} - \lambda H +$

$\alpha [p_H R I_{FB,i} - \underline{\rho}(I_{FB,i} - H)] + (1 - \alpha) [p_H R - \bar{\rho}(I_{FB,i} - H)]$ , which under a fair insurance price simplifies to  $V_{FB,i} = A + (1 - \alpha)(p_H R - \bar{\rho})I_{FB,i} + [\alpha(p_H R - \underline{\rho}) - 1] I_{FB,i}$ . Assumption **A1** implies that  $(1 - \alpha)(p_H R - \bar{\rho})I_{FB,i} < 0$  and the value of the project is lower relative to the first-best case without insurance.

## 8.3 Proof of Proposition 2

### 8.3.1 Manager's Problem

The proof is analogical to the case without insurance but there is no liquidity shock at date  $\frac{1}{2}$ . According to assumptions **A1**, **A2**, and **A4**, the contract has to induce high effort. The payoff for the manager is

$$P_i = A + L_i - (1 + \lambda)I_i + [p_H(R - T_{R,i}) - (1 - p_H)T_{0,i}] I_i, \quad (24)$$

and the profit for the investor is

$$\Pi_i = -L_i + [p_H T_{R,i} + (1 - p_H)T_{0,i}] I_i. \quad (25)$$

The optimal contract can be found as a solution to the following problem:

$$\max_{\mathcal{C}'} \{P_i\} \text{ subject to:} \quad (26)$$

$$P_i \geq A \quad (PC_i^{manager})$$

$$\Pi_i \geq 0 \quad (PC_i^{investor})$$

$$p_H(R - T_{R,i}) - (1 - p_H)T_{0,i} \geq p_L(R - T_{R,i}) - (1 - p_L)T_{0,i} + B \quad (IC_i^{date1})$$

$$A + L_i - I_i - \lambda I_i \geq 0 \quad (27)$$

$$A + L_i - I_i - \lambda I_i \geq T_{R,i} I_i - R I_i \quad (28)$$

$$A + L_i - I_i - \lambda I_i \geq T_{0,i} I_i \quad (29)$$

The manager maximizes her payoff so that both parties want to participate in the project, constraints  $(PC_i^{manager})$  and  $(PC_i^{investor})$ ; and high effort is exerted in the second period, constraint  $(IC_i^{date1})$ . Assume that the manager's payoff  $P_i$  is increasing in the amount of cash invested into the project, and it is optimal for her to invest as much cash as possible at date 0. (This assumption is verified at the end of this paragraph.) If the manager invested all cash and the project fails at date 1 (its payoff is zero), the manager has no cash to repay the investor, and the optimal contract has to set  $T_{0,i}^* = 0$ . For the same reason, if the manager invested all cash and the project succeeds at date 1 (its payoff is  $R$ ), transfer  $T_{R,i}^*$  is not greater than  $R$ . Using  $T_{0,i}^* = 0$ , one can rewrite incentive constraint  $(IC_i^{date1})$  as  $T_{R,i}^* \leq R - \frac{B}{p_H - p_L}$ . The manager's payoff  $P_i$  is decreasing in  $T_{R,i}$  whereas the investor's payoff  $\Pi_i$  is increasing in  $T_{R,i}$ . Hence, incentive constraint  $(IC_i^{date1})$  is binding and  $T_{R,i}^* = R - \frac{B}{p_H - p_L}$ . Also, the manager's payoff  $P_i$  is increasing in  $L_i$  whereas the investor's payoff  $\Pi_i$  is decreasing in  $L_i$ . Since the market for loans is competitive the investor's participation constraint is binding as well. Binding participation constraint  $(PC_i^{investor})$  implies that the optimal loan is equal to the expected pledgeable payoff for the investor:

$$L_i^* = p_H T_{R,i}^* I_i^*. \quad (30)$$

Substituting (30) into the manager's expected payoff (24) and simplifying gives

$$P_i^* = A + (p_H R - 1 - \lambda) I_i^*. \quad (31)$$

According to assumption **A4**,  $p_H R - 1 - \lambda > 0$ , and the manager's payoff is increasing in investment  $I_i^*$ .

Inequalities (27) to (29) are feasibility constraints imposed by the structure of the model. Condition (27) states that the loan provided by the investor has to cover

the need for external financing given investment decision at date 0 and insurance premium  $\lambda I_i$ . Inequality (28) is the limited liability condition that imposes an upper bound on possible repayment  $T_{R,i}$  to the investor if the project succeeds at date 1. Similarly, (29) is the limited liability condition that imposes an upper bound on possible repayment  $T_0$  to the investor if the project fails at date 1. Because  $T_0^* = 0$ , constraint (29) is identical to (27) and can be removed from the problem. Rearranging constraint (28) one gets  $L_i \geq (1 + \lambda)I_i - A - (R - T_{R,i})I_i$  and since  $T_{R,i}^* \leq R$  constraint (28) is satisfied when (27) is satisfied and (27) is the only relevant constraint to the problem. Since the manager's payoff is increasing in investment, she invests all cash at date 0 and constraint (27) is binding:

$$L_i^* = (1 + \lambda)I_i^* - A. \quad (32)$$

The optimal investment level can be solved from equations (30) and (32):

$$I_i^* = A \frac{1}{1 + \lambda - p_H T_{R,i}^*}. \quad (33)$$

### 8.3.2 Existence

Assumption **A4** imposes an upper bound on the price of insurance  $\lambda < \lambda_{\max} \equiv p_H R - 1$ . To have a positive investment in equilibrium, according to (33),  $1 + \lambda - p_H T_{R,i}^* > 0$ . This gives the lower bound on the price of insurance  $\lambda > \lambda_{\min} \equiv p_H \left( R - \frac{B}{p_H - p_L} \right) - 1$ . Equilibrium exists iff  $0 < \lambda_{\min} < \lambda_{\max}$ , which can be rewritten as  $0 < \frac{B}{p_H - p_L} < \frac{p_H R - 1}{p_H}$ . Since assumption **A4** holds,  $p_H R - 1 > 0$ , and the equilibrium exists for some positive values of  $B$ .

## 8.4 Proof of Lemma 4

The investment under the low-shock insurance is bigger than under the no insurance:  $I_{\underline{i}}^* > I^*$  iff  $1 + \underline{\rho} - \alpha p_H T_R^* > 1 + \alpha \underline{\rho} - \alpha p_H T_{R,\underline{i}}^*$ . If  $T_R^* = T_R^1 = T_{R,\underline{i}}^*$ , the inequality simplifies to  $\underline{\rho} > \alpha \underline{\rho}$ , which always holds; if  $T_R^* = T_R^{\frac{1}{2}} < T_{R,\underline{i}}^*$ , the inequality is a fortiori satisfied. The debt capacity under the low-shock insurance is bigger than under the no insurance:  $L_{\underline{i}}^* > L^*$  iff  $\alpha p_H T_{R,\underline{i}}^* I_{\underline{i}}^* > \alpha p_H T_R^* I^*$ . If  $T_R^* = T_R^1 = T_{R,\underline{i}}^*$ , the inequality simplifies to  $I_{\underline{i}}^* > I^*$ , which holds as was shown above; if  $T_R^* = T_R^{\frac{1}{2}} < T_{R,\underline{i}}^*$ , the inequality is a fortiori satisfied. The manager's payoff under the low-shock insurance is bigger than under the no insurance:  $P_{\underline{i}}^* > P^*$  iff  $\frac{\alpha p_H R - (1 + \gamma)}{1 + \gamma - \alpha p_H T_{R,\underline{i}}^*} > \frac{\alpha(p_H R - \underline{\rho}) - 1}{1 + \underline{\rho} - \alpha p_H T_R^*}$ . This inequality simplifies to  $1 + \underline{\rho} - \alpha p_H T_R^* > 1 + \alpha \underline{\rho} - \alpha p_H T_{R,\underline{i}}^*$ , which is satisfied as was shown when comparing the investment levels.

The investment under the low-shock insurance is bigger than under the full insurance:  $I_{\underline{i}}^* > I_i^*$  iff  $1 + \lambda - p_H T_{R,i}^* > 1 + \gamma - \alpha p_H T_{R,\underline{i}}^*$ . Since  $T_{R,i}^* = T_{R,\underline{i}}^*$ , the inequality simplifies to  $p_H \frac{B}{p_H - p_L} > p_H R - \bar{\rho}$ . This inequality is satisfied as the left-hand-side is positive for  $B > 0$  whereas the right-hand-side is negative by assumption **A1**. The debt capacity under the low-shock insurance is bigger than under full insurance:  $L_{\underline{i}}^* > L_i^*$  iff  $\alpha p_H T_{R,\underline{i}}^* I_{\underline{i}}^* > p_H T_{R,i}^* I_i^*$ , which simplifies to  $\alpha(1 + \lambda) > 1 + \gamma$ . Substituting for fair insurance prices  $\lambda$  and  $\gamma$  one gets  $\bar{\rho} - \underline{\rho} > \frac{1}{\alpha}$ . The last inequality is satisfied because assumptions **A1** and **A4** hold. **A1** implies that  $\bar{\rho} > \bar{\rho}_{lowest} \equiv p_H R$ . Substituting  $\bar{\rho}_{lowest}$  into **A4** leads to  $\bar{\rho} - \underline{\rho} > \frac{1}{\alpha} + \frac{\bar{\rho} - \bar{\rho}_{lowest}}{\alpha}$ . Since  $\bar{\rho} > \bar{\rho}_{lowest}$ , inequality  $\bar{\rho} - \underline{\rho} > \frac{1}{\alpha}$  has to be satisfied if **A1** and **A4** hold. The manager's payoff under the low-shock insurance is bigger than under full insurance:  $P_{\underline{i}}^* > P_i^*$  iff  $\frac{\alpha p_H R - (1 + \gamma)}{1 + \gamma - \alpha p_H T_{R,\underline{i}}^*} > \frac{p_H R - (1 + \lambda)}{1 + \lambda - p_H T_{R,i}^*}$ , which simplifies to  $\alpha(1 + \lambda) > 1 + \gamma$ . As shown above, this inequality is satisfied.

## 8.5 Financial Asset Thresholds

Threshold values of  $m$  such that for lower values the financial asset dominates full insurance and the opposite is true for higher values of  $m$  are as follows.

If  $T_{R,A}^* = T_{R,A}^1 = T_{R,i}^*$  then

$$m_{I,i}^1 \equiv \frac{(1-\alpha)(\bar{p}-p_H T_{R,A}^1)}{\alpha(p_H T_{R,A}^1 - \rho) + (1-\alpha)\rho},$$

$$m_{L,i}^1 \equiv \frac{(1-\alpha)[\alpha(\bar{p}-\rho)-1]}{(1-\alpha)[\alpha(\bar{p}-\rho)+\rho] + \alpha}, \text{ and}$$

$$m_{P,i}^1 \equiv \frac{p_H B(1-\alpha)[\alpha(\bar{p}-\rho)-1]}{(p_H - p_L)(1-\alpha)\rho[p_H R - \bar{p} - 1 + \alpha(\bar{p}-\rho)] + \alpha p_H B[1 - (1-\alpha)\rho + \bar{p}(1-\alpha)]};$$

if  $T_{R,A}^* = T_{R,A}^{\frac{1}{2}} < T_{R,i}^*$  then

$$m_{I,i}^{\frac{1}{2}} \equiv \frac{(1-\alpha)\bar{p} - p_H(T_{R,i}^* - \alpha T_{R,A}^{\frac{1}{2}})}{\alpha p_H T_{R,A}^{\frac{1}{2}} + \rho(1-2\alpha)},$$

$$m_{L,i}^{\frac{1}{2}} \equiv \frac{\alpha T_{R,A}^{\frac{1}{2}}[\alpha(\bar{p}-\rho) - \bar{p} - 1] + T_{R,i}^*(1+\alpha\rho)}{\alpha T_{R,A}^{\frac{1}{2}}[\alpha(\bar{p}-\rho) - \bar{p} - 1] + T_{R,i}^*(2\alpha-1)}, \text{ and}$$

$$m_{P,i}^{\frac{1}{2}} \equiv \frac{p_H B[\alpha(p_H R - \rho) - 1] - (p_H - p_L)\alpha\rho[p_H R - \bar{p} - 1 + \alpha(\bar{p}-\rho)]}{\alpha p_H B(p_H R - \rho) - (p_H - p_L)(2\alpha-1)\rho[p_H R - \bar{p} - 1 + \alpha(\bar{p}-\rho)]}.$$

Threshold values of  $m$  such that for lower values the financial asset dominates holding cash reserves and the opposite is true for higher values of  $m$  are as follows.

$$m_I \equiv \frac{(1-\alpha)\rho}{\alpha(p_H T_{R,A}^* - \rho) + (1-\alpha)\rho},$$

$$m_L \equiv \frac{(1-\alpha)\rho}{1 + (1-\alpha)2\rho},$$

$$m_{P}^{\frac{1}{2}} \equiv \frac{(1-\alpha)[\alpha(p_H R - \rho) - 1]}{2\alpha[(1-\alpha)(p_H R - \rho) + 1] - 1}, \text{ and}$$

$$m_P^1 \equiv \frac{(1-\alpha)\rho[\alpha(p_H R - \rho) - 1]}{(1-\alpha)\rho[2\alpha(p_H R - \rho) - 1] + \alpha p_H \frac{B}{p_H - p_L}}.$$

## 9 References

- Anderson, R., Carverhill, A., 2005, A Model of Corporate Liquidity, CEPR Discussion Paper No. 4994
- Bodnar, G., Hyat, G., Marston, R., 1998, 1998 Wharton Survey of Financial Risk Management by US Non-Financial Firms, *Financial Management*, Vol. 27, pp. 70-91

- Boyle, G., Guthrie, G., 2003, Investment, Uncertainty, and Liquidity, *Journal of Finance*, Vol. 58, No. 5, pp. 2143-2166
- Brown, G., Toft, K., 2002, How Firms Should Hedge, *Review of Financial Studies*, Vol. 15, No. 4, pp. 1283-1324
- Chacko, G., Tufano, P., Verter, G., 2001, Cephalon, Inc. Taking Risk Management Theory Seriously, *Journal of Financial Economics*, Vol. 60, pp. 449-485
- Chesler, R. D., Anglim, A. J., 2001, Essentials Of Business Interruption Insurance Law, A commentary reprinted from the December 2001 issue of Mealey's Business Interruption Insurance
- DeMarzo, P., Duffie, D., 1995, Corporate Incentives for Hedging and Hedge Accounting, *Review of Financial Studies*, Vol. 8, No. 3, pp. 743-771
- Froot, K., 2001, The Market for Catastrophe Risk: A Clinical Examination, *Journal of Financial Economics*, Vol. 60, pp. 529-571
- Froot, K., Scharfstein, D., Stein, J., 1993, Risk Management: Coordinating Corporate Investment and Financing Policies, *Journal of Finance*, Vol. 48, No. 5, pp. 1629-1658
- Froot, K., Stein, J., 1998, Risk Management, capital budgeting, and capital structure policy for financial institutions: an integrated approach, *Journal of Financial Economics*, Vol. 47, pp. 55-82
- Graham, J., Rogers, D., 2002, Do Firms Hedge in Response to Tax Incentives?, *Journal of Finance*, Vol. 57, No. 2, pp. 815-839
- Holmström, B., Tirole, J., 1998, Private and Public Supply of Liquidity, *Journal of Political Economy*, Vol. 106, No. 1, pp. 1-39

- Hunter, W., Smith, S., 2002, Risk management in the global economy: A review essay, *Journal of Banking and Finance*, Vol. 26, pp. 205-221
- Mauer, D., Triantis, A., 1994, Interactions of Corporate Financing and investment Decisions: A Dynamic Framework, *Journal of Finance*, Vol. 49, No. 4, pp. 1253-1277
- Mayers, D., Smith, C., 1987, Corporate Insurance and the Underinvestment Problem, *Journal of Risk and Insurance*, Vol. 54, No. 1, pp. 45-54
- Mello, A., Parsons, J., 2000, Hedging and Liquidity, *Review of Financial Studies*, Vol. 13, No. 1, pp. 127-153
- Mian, S., 1996, Evidence on Corporate Hedging Policy, *Journal of Financial and Quantitative Analysis*, Vol. 31, No. 3, pp. 419-439
- Modigliani, F., Miller, M., 1958, The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review*, Vol. 48, No. 3, pp. 261-297
- Nance, D., Smith, C., Smithson, Ch., 1993, On the Determinants of the Corporate Hedging, *Journal of Finance*, Vol. 48, No. 1, pp. 267-284
- Rawls, S., Smithson, C., 1990, Strategic Risk Management, *Journal of Applied Corporate Finance*, Vol. 2, No. 4, pp. 6-18
- Rochet, J., Ch., Villeneuve, S., 2004, Liquidity Risk and Corporate Demand for Hedging and Insurance, working paper, GREMAQ, Toulouse University
- Smith, C., Stulz, R., 1985, The Determinants of Firms' Hedging Policies, *Journal of Financial and Quantitative Analysis*, Vol. 20, No. 4, pp. 391-405
- Stulz, R., 1984, Optimal Hedging Policies, *Journal of Financial and Quantitative Analysis*, Vol. 19, pp. 127-140



Stulz, R., 1996, Rethinking Risk Management, *Journal of Applied Corporate Finance*, Vol. 9., No. 3, pp. 8-24

Tufano, P., 1996, Who Manages Risk? An Empirical Examination of Risk Management Practices in the Gold Mining Industry, *Journal of Finance*, Vol. 51, No. 4, pp. 1097-1137

von Thadden, ELU, 1995, Long-Term Contracts, Sort-Term Investment and Monitoring, *Review of Economic Studies*, Vol. 62, No. 4, pp. 557-575