IMPERFECT COMMON KNOWLEDGE IN FIRST GENERATION MODELS OF CURRENCY CRISES

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Abstract

First generation models assume that the level of reserves of a Central Bank in a fixed exchange rate regime is common knowledge among consumers, and therefore the timing of the attack on the currency, in an economy with persistent deficit, can be correctly anticipated. In these models, the collapse of the peg leads to no discrete change in the exchange rate. We relax the assumption of perfect information and introduce uncertainty about the willingness of a Central Bank to defend the peg. In this new setting, there is a unique equilibrium at which the fixed exchange rate is abandoned. In our model, the lack of common knowledge will lead to a discrete devaluation of the local currency once the peg finally collapses.

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1 Introduction

Traditionally, exchange rates have been explained by macroeconomic fundamentals. However, there seem to be significant deviations from them. The need to understand these unexplained movements have drawn attention on the basic assumptions of the traditional models, such as the homogeneity of market consumers or the irrelevance of private information.

First generation models of currency crises assume that consumers are perfectly informed about macroeconomic fundamentals. The original model is due to Krugman (1979). He presents an economy in which the level of the Central Bank's foreign reserves is common knowledge among consumers. In this setting, market participants not only know the level of reserves, but also, that the other agents know it too. There is perfect transmission of information and speculators can precisely coordinate the attack on the currency. The model concludes that the attack is therefore accurately predictable and implies zero devaluation. However, during currency crises, large discrete devaluations are normally observed.

In the standard first generation model, there is no private information. This is a very strong assumption that supposes that all information about fundamentals is available for all market agents and that they all receive and share the same information. This may seem unrealistic. A natural generalisation of the problem would try to incorporate in the model not only public information.

Guimarães (2004) considers a first generation model in which a speculative attack is not an instantaneous event, but it requires some time to force the Central Bank to abandon the peg. He introduces uncertainty in this setting by assuming that agents do not know whether they would be able to escape the devaluation or not. He argues that there is a unique equilibrium in which agents delay their decisions to attack the currency due to these market frictions. This will lead to a discrete devaluation of the currency.

A different approach is presented in Pastine (2002). He incorporates a forward-looking rational policy maker that dislikes speculative attacks and it is capable of choosing the timing of the move to a floating exchange rate regime. Pastine (2002) shows that the monetary authority has an incentive to introduce uncertainty into the speculators' decisions in order to avoid predictable attacks on the currency. In this model, the standard perfectly predictable attack is replaced by an extended period of speculation which gradually places increasing pressure on the policy maker.

Broner (2001, 2003) has been one of the leading pioneers to highlight the actual problems of the first generation models of currency crises. In Broner (2003), he relaxes the assumption of perfect information including a new type of investors: the uninformed consumers. He considers an economy in which there are agents who are perfectly informed about the level of the Central Bank's reserves, because they have access to private information (informed consumers), and market participants who can only reach public information (uninformed consumers). The ratio between informed and uninformed consumers is fixed exogenously and determines the resulting equilibria. He concludes that, if the fraction of informed consumers is high enough ($\alpha > \alpha^*$), market agents face a situation similar to the lack of private information, that is, there exits a unique equilibrium of zero devaluation. However, when the private information is high (for smaller fractions of informed consumers, $\alpha < \alpha^*$), the model arrives to a large set of equilibria, characterised by possible discrete devaluations, which differ on the informed consumers' propensity to attack the currency.

In Broner's model, it is still common knowledge among informed consumers the threshold level of the Central Bank's reserves and therefore, the exact time when the peg is attacked. Another generalisation of the first generation models could aim to include private information through uncertainty on the arbitrageurs' beliefs about other market agents. In a competitive environment, rational agents may not be willing to reveal what they know to other participants.

Abreu and Brunnermeier (2003) consider an economy, in which arbitrageurs become sequentially aware that the market price has departed from fundamentals, and they do not know whether they have learnt this information early or late relative to the other rational arbitrageurs. They assumed this information structure to argue the existence and persistence of asset bubbles despite the presence of rational agents in the economy.

In this paper, we incorporate Abreu and Brunnermeier's information structure to introduce uncertainty about the willingness of the Central Bank to defend the peg in first generation models of currency crises. The application of Abreu and Brunnermeier's dynamic model to currency crises presents several advantages over the original setting. It provides a reasonable explanation for the assumption that the asset's market price remains constant until the time when the bubble bursts. In our setting, the market price (the exchange rate) does remain invariable until the peg collapses. The Central Bank is committed to maintain the exchange rate fixed and it will devote its foreign reserves to defend the peg. Hence, the cumulative selling pressure will not affect the fixed exchange rate until it exceeds a certain threshold and the Central Bank can no longer maintain the peg. Also, in Abreu and Brunnermeier's model it is necessary to introduce a final condition which guarantees the burst of the asset bubble even if no arbitrageur sells his shares. In a first generation model, in which a government runs a persistent budget deficit, the Central Bank will be ultimately forced to abandon the peg when the foreign reserves are exhausted (even if no speculator attacks the currency).

We consider an economy with a fixed exchange rate regime and a persistent deficit that reduces the Central Bank's reserves. In this setting, we suppose a continuum of agents who, one by one, cannot affect the exchange rate. They can choose between local and foreign currency. We assume that holding local currency generates higher returns, although there is a risk of devaluation. Rational agents take their investment decisions by evaluating the trade off between these higher profits and the fear of capital losses. At some random time $t = \widetilde{t}$, arbitrageurs become sequentially aware that the shadow price has exceeded the peg, and they have to decide between cancelling and maintaining their positions. Market agents notice the mispricing in random order and they do not know if other agents are also aware of it. They would prefer to hold local currency for as long as possible, since it produces higher profits, but they would not want to wait for too long because of the capital losses caused by devaluation.

We suppose that rational agents have financial constraints which limit their individual maximum positions and their impact on the exchange rate. To force the exchange rate off the peg, it will be necessary to coordinate the attack on the currency. Rational consumers face a synchronization problem and at the same time a competition dilemma: only a fraction of all participants can leave the market before the peg collapses, because as soon as a large enough number of agents sell out local currency, the Central Bank will be forced to abandon the fixed exchange rate regime and those consumers who, at that time, still hold local currency will suffer a capital loss.

We prove that there exists a unique equilibrium in which, for moderate heterogeneity among agents, the peg is abandoned after enough consumers have hold overvalued currency for some period of time τ^* since they noticed that the fixed exchange rate lies below the shadow price. Rational agents will hold a maximum long position in local currency until they fear the attack is imminent. At that time, they will sell out causing the fall of the peg. For extreme levels of dispersion of opinion among market participants, the fixed exchange rate regime collapses when the Central Bank's foreign reserves are completely exhausted. In either case, the abandonment of the peg implies a discrete devaluation of the currency. In the former, we will prove that arbitrageurs optimally hold overvalued currency for some time $\tau^* > 0$ ("After becoming aware of the bubble, they [arbitrageurs]... optimally choose to ride the bubble over some interval.", in Abreu and Brunnermeier's setting). Hence, the attack will take place strictly after the shadow price exceeds the peg causing a jump in the exchange rate. In the latter, the Central Bank will defend the peg until the reserves are exhausted. This will necessary occur some time after the shadow price exceeds the peg, which will imply a discrete change in the exchange rate. In this paper we derive a first generation model of currency crises with imperfect information, which explains the discrete jump in the exchange rate generally observed in the transition from a fixed to a floating exchange rate regime.

In independent work, Rochon[1] (2004) applies Abreu and Brunnermeier's structure to currency crises. She considers an economy with a fixed exchange rate regime in which a negative shock triggers a gradual depletion of the Central Bank's foreign reserves. In her setting, agents are uncertain about the magnitude of the reserves and about other speculators' information. In Rochon (2004), the peg is abandoned whenever the accumulated exogenous outflow plus the selling pressure exceed the initial level of reserves. This leads to a discrete devaluation. The similarities between Rochon (2004) and our work lie in the application of Abreu and Brunnermeier's information structure to currency crises. However, our paper differs from Rochon (2004) in several ways. First, we consider a different setting; we assume a first generation model, as described in Krugman (1979), in which a government runs a persistent deficit which will gradually reduce the Central Bank's foreign reserves. Rochon (2004) defines a general model in which the Central Bank is committed to defend the fixed exchange rate regime but it is not forced to finance an expansionary monetary policy (although in the final section she also imposes this condition in order to determine, endogenously, the rate of devaluation). In her economy, an adverse exogenous shock generates an outflow of reserves, which is responsible for the gradual exhaustion of the Central Bank's foreign reserves (instead of a persistent budget deficit). Second, we suppose that rational agents have imperfect information about the shadow price, while in Rochon (2004) the key variable is the level of reserves. From there on, the derivation of the unique equilibrium and the timing of the attack are clearly different.

The paper is organised in the following order. Section 2 analyses the Krugman

[1] I am thankful to Fernando Broner for bringing this paper to my attention at the International Economics seminar (London School of Economics, December 2004)

model. We explain Abreu and Brunnermeier's model in Section 3. Section 4 illustrates the generalisation of the traditional first generation model of currency crises. We relax the assumption of perfect information and define the new setting. Section 5 studies the resulting unique equilibrium. We show that the exchange rate collapses at a final time, when the Central Bank's foreign reserves are depleted, if extreme levels of dispersion of opinion are considered, and at an earlier moment in time, given moderate heterogeneity among consumers. We derive the timing of the attack and analyse the determinants of the period of time τ^* during which rational agents optimally hold overvalued domestic currency. Finally, we present our conclusions in Section 6.

2 The Krugman Model

The original model of speculative attacks on fixed exchange rates is due to Krugman (1979). He considered an economy with a fixed exchange rate regime where the government runs a budget deficit that will gradually reduce the Central Bank's reserves. The model concludes that the peg will be abandoned before the reserves are completely exhausted. At that time, there will be a speculative attack that eliminates the lasting foreign exchange reserves and leads to the abandonment of the fixed exchange rate.

In this model, the Central Bank faces two different tasks. First, it must satisfy the financial needs of the government and second, it has to maintain a fixed exchange rate. The Central Bank finances the deficit by issuing government debt and defends the peg through direct intervention in the foreign exchange rate market. In this economy, the asset side of the Central Bank's balance sheet at time t is made up of domestic credit (D_t) and the value in domestic currency of the international reserves (R_t). The balance sheet's liability side consists of the domestic currency in circulation, the money supply (M_t^s). Hence:

$$M_t^s = D_t + R_t$$

The budget deficit grows at a constant rate μ ($\mu > 0$):

$$\mu = \frac{\dot{D}_t}{D_t}$$

Also assume that the purchasing parity holds:

$$S_t = \frac{P_t}{P_t^*}$$

where, at time t, P_t is the domestic price level, S_t the exchange rate of domestic currency for foreign, and P_t^* is the foreign price level. We can fixed $P_t^* = 1$, and therefore the exchange rate can be identified with the domestic price level ($P_t = S_t$).

In this model it is supposed that money is only created through the deficit. As long as the Central Bank is committed to defend the peg, it will print money to finance the deficit. This will tend to raise the money supply, and hence affect the domestic prices and the exchange rate. Domestic prices will begin to increase bringing about an incipient depreciation of the currency. To maintain the exchange rate fixed, the authorities will reduce the foreign reserves to purchase the domestic currency and foreign reserves will fall as domestic assets continually rise. Ultimately, if the budget is in deficit, pegging the rate becomes impossible, no matter how large the initial reserves were.

However, the model concludes that the attack comes *before* the stock of foreign reserves would have been exhausted in the absence of speculation. Why? In Krugman's model, consumers can correctly anticipate the exhaustion of the reserves, they can only choose between domestic and foreign money and it is also supposed that foreigners do not hold domestic money. Then, the assumption of perfect foresight implies that speculators, anticipating an abandonment of the peg, will attack the exchange rate to acquire the Central Bank's reserves and to avoid a capital loss.

To determine the timing of the crisis, we introduce the following definition.

Definition 1 *The shadow floating exchange rate or* ***shadow price*** *at time t (S_t) is the exchange rate that would prevail at time t if the Central Bank held no foreign reserves, allowed the currency to float but continued to allow the domestic credit to grow over time.*

In Appendix A we derive an expression for the shadow floating exchange rate. To simplify the analysis, it is convenient to express all magnitudes in logarithms[2]. We present logarithmic versions of the previous equations and describe the monetary equilibrium by the Cagan equation. Then, the logarithm of shadow price is

[2] We use the standard notation in which an upper case letter represents a variable in levels and a lower case one its logarithm: $s_t = ln(S_t)$. From now on, exchange rates will be expressed in logarithms. To simplify the reasoning we will still refer to them as fixed exchange rate and shadow price, where, to be precise, it should say fixed *log*-exchange rate and *log*-shadow price.

given by:

$$s_t = \gamma + \mu \times t$$

where γ and μ are constants and μ is the rate of growth of the budget deficit. The time of the attack on the currency T is defined as the date on which the shadow price reaches the peg ($s_t = \overline{s}$):

$$T = \frac{\overline{s} - \gamma}{\mu}$$

In the Krugman model, the level of the Central Bank's foreign reserves is common knowledge among consumers. Thus, the timing of the attack is accurately predictable and the transition from a fixed to a floating exchange rate regime occurs without discrete jumps in the exchange rate.

3 The Abreu And Brunnermeier Model

Abreu and Brunnermeier (2003) present a model in which an asset bubble can survive despite the presence of rational arbitrageurs. They consider an information structure where rational arbitrageurs become sequentially aware that an asset's market price has departed from fundamentals and they do not know if other arbitrageurs have already noticed the mispricing. The model concludes that if the arbitrageurs' opinions are sufficiently dispersed, the asset bubble bursts for exogenous reasons when it reaches its maximum size. And in the case of moderate levels of dispersion of opinion, Abreu and Brunnermeier (2003) prove that endogenous selling pressure advances the bubble collapse. They demonstrate that these equilibria are unique. Also, the model shows how news events can have a disproportionate impact on market prices, since they allow agents to synchronise their exit strategies.

This model considers two types of agents: behavioral traders (influenced by fads, fashions, over-confidence...) and rational arbitrageurs. Initially the stock price p_t grows at the risk-free interest rate r ($p_t = e^{rt}$) and rational arbitrageurs are fully invested in the market. At $t = 0$, the price starts growing at a faster rate g ($g > r$). Behavioral traders believe that the stock price p_t will grow at a rate g in perpetuity. Hence, whenever the stock price falls below $p_t = e^{gt}$, they are willing to buy any quantity of shares (up to their aggregate absorption capacity κ). Then, at some random time t_0 (exponentially distributed on $[0, \infty)$), rational arbitrageurs become (in random order) sequentially aware that the price is too high. However, the price continues to grow at a rate $g > r$ and hence, only a fraction $(1 - \beta(\cdot))$ of the price is explained by the fundamentals, where $\beta(\cdot)$ represents the "bubble component". Rational agents understand that the market will eventually collapse but still prefer to ride the bubble as it generates higher returns.

In Abreu and Brunnermeier's model, the bubble collapses as soon as the cumulative selling pressure exceeds some threshold κ (the absorption capacity of the behavioral traders) or ultimately at $t = t_0 + \overline{\tau}$ when it reaches its maximum size ($\overline{\beta}$). It is assumed that arbitrageurs, one by one, have limited impact on the price, because of the financial constraints they face. Consequently, large movements in prices require a coordinated attack. They consider that in each instant t, from $t = t_0$ until $t = t_0 + \eta$, a mass of $1/\eta$ arbitrageurs becomes aware of the mispricing, where η can be understood as a measure of the dispersion of opinion among agents concerning the timing of the bubble. Since t_0 is random, they do not know how many other rational arbitrageurs have noticed the mispricing, because they will only become aware of the selling pressure when the bubble finally bursts. Rational arbitrageurs face temporal miscoordination. Then, an arbitrageur who becomes aware of this mispricing at time t_i has the following posterior cumulative distribution for the date (t_0) on which the price departed from its fundamental value, with support $[t_i - \eta, t_i]$:

$$\Phi(t_0|t_i) = \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}$$

Therefore, from $t = t_0 + \eta\kappa$ onwards, the mispricing is known to enough arbitrageurs to correct it. Nevertheless, they do not attempt to, since as soon as they coordinate, the selling pressure will burst the bubble. However, there is also a competitive component in the model: only a fraction κ of the arbitrageurs will be able to sell out before the bubble collapses (because it bursts the moment the selling pressure surpasses κ). Thus, arbitrageurs have also an incentive to leave the market.

In this setting, each arbitrageur can sell all or part of her stock of shares until a certain limit due to some financial constraints. It is possible to buy back shares and to exit and re-enter the market multiple times. The strategy of an agent who became aware of the bubble at time t_i is defined as the selling pressure at time t: $\sigma(\cdot, t_i) = [0, t_i + \overline{\tau}] \mapsto [0, 1]$. The action space is normalised to be the continuum between $[0, 1]$, where 0 indicates a maximum long position and 1 a maximum short position. Then, the aggregate selling pressure of all agents at time $t \geq t_0$ is given by $s(t, t_0) = \int_{t_0}^{min\{t,\ t_0+\eta\}} \sigma(t, t_i) dt_i$ and therefore the bursting time can be expressed as:

$$T^*(t_0) = \inf\{t | s(t, t_0) \geq \kappa \text{ or } t = t_0 + \overline{\tau}\}$$

Given this information structure, arbitrageur t_i's beliefs about the date on which the bubble bursts are described by:

$$\Pi(t_0|t_i) = \int_{T^*(t_0)<t} d\Phi(t_0|t_i)$$

In their analysis, Abreu and Brunnermeier focus on trigger-strategies in which an agent, who sells out at t, continues to attack the bubble at all times thereafter. In this case, solving the optimisation problem of the arbitrageur who notices the bubble at time t_i and sells out at time t yields the following condition:

Lemma 1 (Abreu & Brunnermeier) (sell out condition). *If arbitrageur t_i's subjective hazard rate is smaller than the 'cost-benefit ratio', i.e.*

$$h(t|t_i) < \frac{g-r}{\beta(t - T^{*-1}(t))}$$

trader t_i will choose to hold the maximum long position at t. Conversely, if $h(t|t_i) > \frac{g-r}{\beta(t-T^{*-1}(t))}$ she will trade to the maximum short position.

where $h(t|t_i) = \frac{\pi(t|t_i)}{1-\Pi(t|t_i)}$ is the hazard rate that the bubble will burst at time t, $\Pi(t|t_i)$ represents the arbitrageur t_i's beliefs about the bursting date and $\pi(t|t_i)$ denotes the associated conditional density. They conclude that an arbitrageur who becomes aware of the mispricing at time t_i will hold a maximum long position until his subjective hazard rate becomes larger than the cost-benefit ratio. That is, arbitrageur t_i will ride the bubble until his subjective probability that the bubble will burst in the next trading round is high enough. At that time, arbitrageur t_i will trade to the maximum short position to get out of the market.

They consider two different scenarios. When arbitrageurs' opinions are sufficiently dispersed, Abreu and Brunnermeier (2003) prove that the selling pressure does not affect the time when the bubble collapses, because each arbitrageur optimally rides the bubble for so long that, at the end of the horizon ($t = t_0 + \overline{\tau}$), there is not enough pressure to burst the bubble (less than κ will have sold out). They show that there is a unique equilibrium at which the bubble bursts for exogenous reasons at $t = t_0 + \overline{\tau}$. A different conclusion is reached when a moderate level of heterogeneity is assumed. In this case, they demonstrate that there is a unique and symmetric equilibrium in which each arbitrageur sells her shares τ^* periods after becoming aware of the mispricing. The bubble bursts at $t = t_0 + \eta\kappa + \tau^*$ ($< t_0 + \overline{\tau}$, given small values for η).

The model assumes an information structure based on the lack of common knowledge (when an arbitrageur becomes aware of the mispricing, he does not know if others know) and derives that these equilibria are unique. However, in typical applications, the symmetry information game has multiple equilibria. Abreu and Brunnermeier (2003) argue that the fact that arbitrageurs are competitive (since at most a fraction of them can leave the market prior to the crash) leads to a unique equilibrium even under symmetric information.

4 Imperfect Common Knowledge

This section presents a first generation model of currency crises in which the traditional assumption of perfect information is relaxed. We introduce the new setting and we derive the sell out condition that determines the moment when a rational agent fears the abandonment of the peg and prefers to attack the currency.

Consider an economy similar to the one described in Krugman (1979) and summarised in Section 2. In this setting, the level of the Central Bank's foreign reserves is common knowledge among consumers and therefore, the peg is attacked whenever it leads to no discrete change in the price level, i.e., as soon as the shadow price reaches the fixed exchange rate ($s_t = \overline{s}$). In our model we incorporate the information structure presented in Abreu and Brunnermeier (2003) (and reviewed in Section 3) to introduce uncertainty about the willingness of the Central Bank to defend the peg. We consider that the level of reserves is no longer common knowledge among consumers and in this paper we analyse the consequences of this uncertainty on the abandonment of the fixed exchange rate.

4.1 The Model

The following process is assumed in our setting. The Central Bank establishes the fixed exchange rate at a certain level $\overline{s}$ as depicted in Figure 1. We denote (the logarithm of) the shadow price[3] by s_t. We assume that $\widetilde{t}$ is exponentially distributed on $[0, \infty)$ with cumulative distribution function $\mathrm{F}(\widetilde{t}) = 1 - e^{-\lambda \widetilde{t}}$ ($\lambda > 0$, $\widetilde{t} \geq 0$). Prior to $t = \widetilde{t}$, the peg lies above the shadow price s_t ($\overline{s} > s_t$) and the fixed exchange rate cannot collapse, since speculators would only attack the peg if it profitable for them. If before $t = \widetilde{t}$ the peg is abandoned, the currency would immediately revaluate to reach the shadow price (the local currency would worth more while arbitrageurs hold short positions in domestic currency). Hence, anticipating this capital loss, rational agents will not attack the currency and

[3] In Appendix A we prove that the logarithm of the shadow price is a linear function of time ($s_t = \gamma + \mu \times t$), or equivalently, that the shadow price grows exponentially.

therefore no speculative attack will occur before $t = \widetilde{t}$. From $t = \widetilde{t}$ onwards, the shadow price exceeds the fixed exchange rate ($\overline{s} < s_t$) and the peg might be attacked.

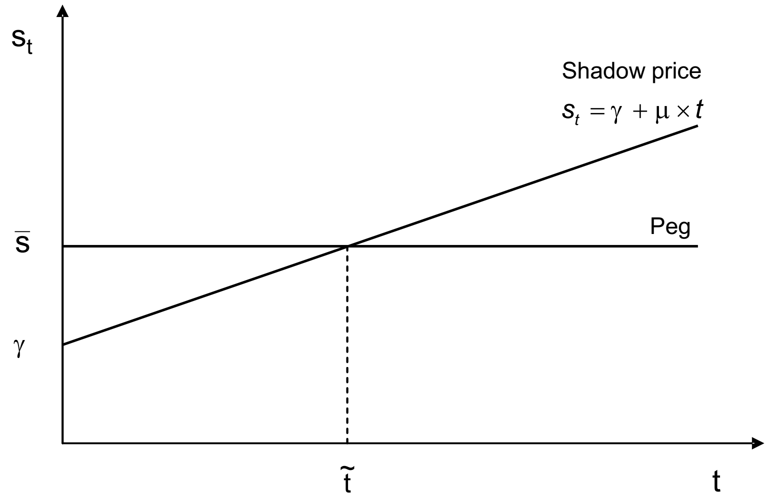

Figure 1: Fixed exchange rate ($\overline{s}$) and shadow price (s_t) - We represent a fixed exchange rate regime in which the Central Bank finances a persistent deficit and maintains the exchange rate fixed at a certain level ($\overline{s}$). We plot the random time $t = \widetilde{t}$, when (the logarithm of) the shadow price (s_t) reaches the peg ($\overline{s}$).

Agents and Actions

In our model, there is only one type of agent: the rational arbitrageurs. We assume a continuum of speculators, with mass equal to one, who one by one cannot affect the exchange rate because of some financial constraints which limit their maximum market positions. In currency crises, however, it may seem more realistic to consider that only a few relevant institutions actively participate in currency markets and that information might be clustered. This assumption would not modify the intuition of our results. In Brunnermeier and Morgan (2004), they prove that the equilibrium delay in such games always exceeds equilibrium delay in the game with a continuum of agents and no information clustering (i.e. in the Abreu and Brunnermeier model). Hence, defining a finite number of speculators and allowing for information clustering increases the optimal waiting time τ^*, and therefore it delays longer the attack on the fixed exchange rate regime, causing a larger discrete devaluation of the home currency.

Let us denote by agent i the rational agent who, at time t_i, receives a signal indicating that the shadow price exceeds the fixed exchange rate. Agent i may take one of two actions. He can hold local currency or buy foreign currency. Investing in domestic currency generates a return equal to r while the foreign investment yields r^*. We impose the following condition:

Assumption 1 $r > r^*$

We consider that the local currency pays an interest rate r higher than the foreign currency ($r > r^*$) to guarantee that, initially, agents invest in domestic currency. Hence, in our economy, rational agents originally prefer to invest in domestic currency because of the higher profits, but they understand that the exchange rate will be attacked and the peg eventually abandoned. Therefore, the only decision is when to exit. Selling too early leads to less profitable outcomes, but if they wait too long and they do not leave the market before the fixed exchange rate collapses, they will incur in capital losses associated with the devaluation.

An individual agent is limited in the amount of currency he can buy or sell. As in Abreu and Brunnermeier (2003), we can normalise the action space to lie between $[0,1]$ and define the strategy of rational agent i by his selling pressure at time t: $\sigma(\cdot, t_i) = [0, t_i + \overline{\tau}] \mapsto [0,1]$. A selling pressure equal to zero ($\sigma(t, t_i) = 0$) indicates a maximum long position in local currency and a value equal to one ($\sigma(t, t_i) = 1$) implies that rational agent i has sold out all his holdings of domestic currency (maximum pressure). Let $s(t, \widetilde{t})$ denote the aggregate selling pressure of all market participants at time $t \geq \widetilde{t}$.

Collapse of the Peg

The fixed exchange rate can collapse for one of two reasons. It is abandoned at $t = \widetilde{t} + \tau^* + \eta\kappa$ when the aggregate selling pressure exceeds a certain threshold κ ($s(t, \widetilde{t}) \geq \kappa$) or ultimately at a final time when all foreign reserves are exhausted, say at $t = \widetilde{t} + \overline{\tau}$. Let us denote this collapsing date by $T^*(\widetilde{t}) = \widetilde{t} + \zeta$, where $\zeta = \tau^* + \eta\kappa$ if the abandonment of the peg is caused by arbitrageurs' selling pressure (endogenous collapse) and $\zeta = \overline{\tau}$ if it is due to the exhaustion of reserves (exogenous collapse). Since we have assumed that rational agents have no market power, they will need to coordinate to force the abandonment of the peg. However, only a proportion $\kappa < 1$ of agents can exit the market before the peg is abandoned. Therefore, rational agents face both, cooperation and competition.

Information Structure

To simplify the analysis, we assume that at the random time $t = \widetilde{t}$ when the shadow price reaches the peg, rational agents begin to notice this mispricing. They become aware sequentially and in random order and they do not know if they have noticed it early or late compared to other agents. They cannot know if they are the first or the last to know. Specifically, at each instant (between $\widetilde{t}$ and $\widetilde{t} + \eta$), a new mass $1/\eta$ of rational agents receives a signal indicating that the shadow price exceeds the fixed exchange rate, where η is a measure of the dispersion of opinion among them. The timing of agent i's signal is uniformly distributed on $[\widetilde{t}, \widetilde{t}+\eta]$, but since $\widetilde{t}$ is exponentially distributed each agent does not know how many other market participants have received the signal before him. Agent i only knows that at $t = t_i + \eta$ all other arbitrageurs received their signals. Conditioning on $\widetilde{t} \in [t_i - \eta, t_i]$, agent i's posterior cumulative distribution function for the date $\widetilde{t}$ on which the shadow price reached the peg is $\Phi(\widetilde{t}|t_i) = \frac{e^{\lambda\eta} - e^{\lambda(t_i - \widetilde{t})}}{e^{\lambda\eta}-1}$. Then, agent i's posterior cumulative distribution function over the collapsing date $T^*(\widetilde{t})$ is

$$\Pi(T^*(\widetilde{t})|t_i) = \frac{e^{\lambda\eta} - e^{\lambda(t_i+\zeta-T^*(\widetilde{t}))}}{e^{\lambda\eta}-1}$$

given that $T^*(\widetilde{t}) \in [t_i + \zeta - \eta, t_i + \zeta]$.

Further Assumptions

We consider the following statements to simplify the analysis and the specification of our setting:

Assumption 2 *In equilibrium, an agent holds either a maximum long position or a maximum short position in local currency:* $\sigma(t, t_i) \in \{0, 1\} \;\forall\; t, t_i$.

We consider that a rational agent prefers to invest his whole budget in local currency, since it generates higher returns, until a certain time when he fears that the attack on the currency is imminent and decides to cancel his position by selling all his stock of domestic currency. Hence, his selling pressure is initially equal to 0 (when he is fully invested in domestic currency) and equal to 1 once he sells out. The information structure considered in our model and Assumption 2 imply the following result:

Corollary 1 *If agent i holds a maximum short position at time t in local currency $(\sigma(t, t_i) = 1)$, then at time t any agent j $(\forall t_j \leq t_i)$ has already sold out his stock of domestic currency $(\sigma(t, t_j) = 1,\ \forall t_j \leq t_i)$.*

We assume that once a rational agent sells his stock of domestic currency, any agent that became aware of the mispricing before him, is already out of the market.

Assumption 3 *No re-entry.*

To simplify the analysis we suppose that once a rational agent gets out of the market, he will not enter again. Intuitively, an agent sells out when he believes that the attack is close. Then, even if he does not observe the attack during some period of time after leaving the market, he still will not know when the fixed exchange rate will collapse, but certainly it will happen sooner than he thought when he exited the market. Therefore, if he does not change his beliefs, he will not have an incentive to re-enter the market.

The Sell Out Condition

In our economy, a rational agent can choose between buying domestic or foreign currency. Initially, market participants are fully invested in local currency because of the higher returns $(r > r^*)$, but there is a risk of devaluation. Hence, each agent will sell exactly at the moment when the fear of the devaluation of the home currency offsets the excess of return derived from investing in local currency. Ideally, he would like to sell *just* before the exchange rate is abandoned and the domestic currency suffers devaluation or equivalently just before the appreciation of the foreign currency. In Appendix B we define the size of the expected appreciation of the foreign currency perceived by agent i as $A_f^i(t-t_i) = \left|E\left[\frac{\frac{1}{S_t}-\frac{1}{\overline{S}}}{\frac{1}{\overline{S}}}\middle|t_i\right]\right| = 1 - E[\frac{\overline{S}}{S_t}|t_i] \geq 0$ (if $E[\frac{\overline{S}}{S_t}|t_i] \leq 1$), and we present the optimisation problem which yields the following sell out condition:

Lemma 2 (sell out condition). *Rational agent i prefers to hold a maximum long position in local currency at time t if his hazard rate is smaller than the 'greed-to-fear ratio', i.e., if*

$$h(t|t_i) < \frac{r-r^*}{1-E[\frac{\overline{S}}{S_t}|t_i]} = \frac{r-r^*}{A_f^i(t-t_i)}$$

He trades to a maximum short position in local currency, if $h(t|t_i) > \dfrac{r-r^*}{A_f^i(t-t_i)}$.

In our model, a rational agent who notices the mispricing at time $t = t_i$ compares his subjective hazard rate ($h(t|t_i)$) with the 'greed-to-fear ratio' ($\frac{r-r^*}{A_f^i(t-t_i)}$) and trades to a maximum short position as soon as he observes that the probability of devaluation given that the peg still holds is larger that the 'greed-to-fear ratio'.

5 Equilibrium

The exchange rate collapses as soon as the cumulative selling pressure exceeds a threshold κ or at a final date $t = \widetilde{t} + \overline{\tau}$ when all foreign reserves are exhausted. This statement implies that no fixed exchange rate regime in an economy with persistent deficit can survive in the long term. We will focus our analysis in the first scenario, in which the peg collapses for endogenous reasons.

5.1 Endogenous Collapse Of The Peg

We have seen that rational agents become aware of the mispricing in random order during an interval $[\widetilde{t},\ \widetilde{t}+\eta]$, where $\widetilde{t}$ is exponentially distributed and represents the time when the shadow price reaches the peg. To simplify the analysis we have supposed that $\widetilde{t}$ is also the moment when the first agent notices the mispricing. η is a measure of the heterogeneity of the rational agents (a larger η corresponds to a wider dispersion of opinion among market participants). Since all arbitrageurs are ex-ante identical, we restrict our attention to symmetric equilibria. Then, for moderate values of the parameter η, we will show that there exists a unique symmetric equilibrium in which the peg falls when the aggregate selling pressure surpasses a certain threshold (κ).

Consider the following backward reasoning. If at the final date $t=\widetilde{t}+\overline{\tau}$ the selling pressure has not exceed the threshold κ, we have assumed that the exchange rate collapses because the Central Bank's foreign reserves are exhausted, and that this final condition is common knowledge among all market participants[4]. This induces a rational agent i to sell out at τ_1 periods after he observes the mispricing ($t_i+\tau_1<\widetilde{t}+\overline{\tau}$) in order to avoid a capital loss. Therefore, the currency will come under attack at $t=\widetilde{t}+\tau_1+\eta\kappa<\widetilde{t}+\overline{\tau}$ (since we consider small values of the

[4]Specifically, $\overline{\tau}$ is common knowledge among arbitrageurs but not $t=t_i+\overline{\tau}$, which will depend on the random time when each agent notices the mispricing. It is supposed that once a rational agent finds out that the shadow price has exceeded the peg, he knows that the Central Bank's foreign reserves will last *at most* $\overline{\tau}$ periods and he also knows that all other arbitrageurs, who are aware of the mispricing, will know it too. But this agent does not know if he has learnt this information early or late compared to the other market participants, and hence he does not know if the attack on the currency will happen before this final date.

parameters η and κ). But if the peg is abandoned at $t = \widetilde{t} + \tau_1 + \eta\kappa$, rational agents will sell out earlier, let us say $\tau_2 < \tau_1$ periods after they notice that the shadow price exceeds the peg. But given the new timing, agents will choose to sell even earlier (τ_3) and so on. As the selling date advances, the cost of devaluation of the domestic currency (or the appreciation of the foreign currency) diminishes and therefore, the benefit from holding local currency rises, that is, the 'greed-to-fear ratio' increases as the selling date advances. In our setting, at each instant, a rational agent compares his hazard rate to his 'greed-to-fear ratio'. He will prefer to sell local currency until the time ($t = t_i + \tau^*$) when the 'greed-to-fear ratio' equals the probability of the peg collapsing given that it still holds (the equality defines the switching condition). This guarantees that rational agents will have an incentive to hold "overvalued" local currency for some period of time after they become aware of the mispricing, or equivalently, that the decreasing sequence of periods converges to $\tau^* > 0$. This result is depicted in Figure 2.

We can derive an expression for τ^* from previous results. We have assumed that, in equilibrium, a rational agent holds either a maximum long position or a maximum short position in local currency, depending on the relation between the probability of the peg collapsing and the profits derived from holding local currency. Hence, we can obtain τ^* from the time ($t = t_i + \tau^*$) when agent i will switch from maximum holding to maximum selling. This is given by:

$$\tau^* = \frac{1}{\mu} \ln \left[\left(1 - \frac{r - r^*}{h} \right)^{-1} \right] - \tau'$$

where $r - r^*$ is the excess of return, $h = h(t|t_i)$ denotes the hazard rate (which we will prove that remains constant over time), μ is a positive constant corresponding to the slope of the linear logarithm of the shadow price (s_t) which represents the

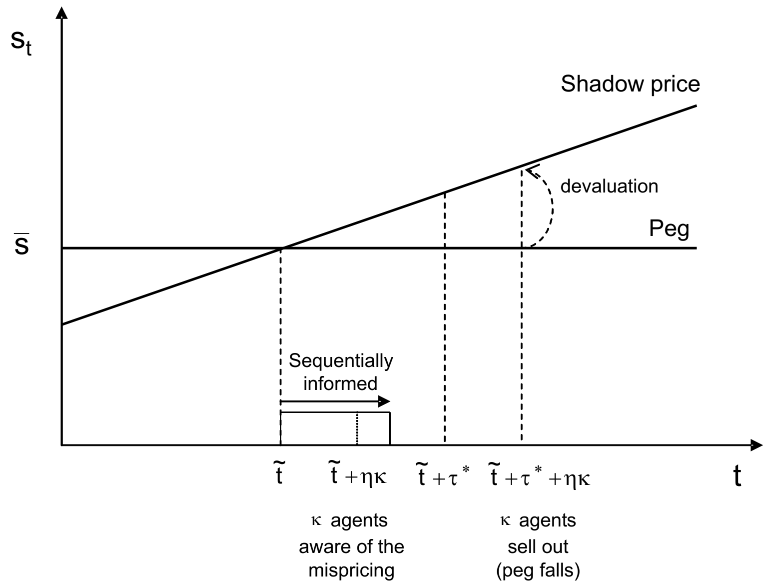

Figure 2: Collapse of the peg (moderate levels of dispersion of opinion) - We plot (the logarithm of) the fixed exchange rate ($\overline{s}$), (the logarithm of) the shadow price (s_t) and relevant moments in time. At $t = \widetilde{t}$, the shadow price exceeds the peg and rational agents become sequentially aware of it. At $t = \widetilde{t} + \eta\kappa$, enough agents have noticed the mispricing but they prefer to wait τ^* periods before selling out. At $t = \widetilde{t} + \tau^* + \eta\kappa$, the selling pressure surpasses the threshold κ and the peg is finally abandoned. There is a discrete devaluation of the exchange rate.

rate of growth of the budget deficit and τ' is indicative of the difference between the date t_i at which the agent receives the signal about the mispricing and the time when he believes the foreign currency begins appreciating.

We can summarise this result in Proposition 1:

Proposition 1 *There exists a* ***unique symmetric equilibrium*** *at which each rational agent sells out τ^* periods after becoming aware of the mispricing, where:*

$$\tau^* = \frac{1}{\mu} \ln \left[\left(1 - \frac{r - r^*}{h} \right)^{-1} \right] - \tau'$$

Thus, the fixed exchange rate will be abandoned at $t = \widetilde{t} + \tau^* + \eta\kappa$.

Proof

Rational agent i prefers to invest in local currency for as long as possible, since this strategy generates higher returns than buying foreign currency ($r > r^*$). But, at a certain moment in time ($t = t_i$), he learns that the shadow price exceeds the peg and that there exists a risk of devaluation. We have argued that he still prefers to hold domestic currency (for some period of time τ^*) until he fears that the attack on the currency is imminent and he decides to sell out. This occurs at $t = t_i + \tau^*$. Rational agent i sells whenever his 'greed-to-fear ratio' equals his hazard rate, i.e. when $h(t|\widetilde{t}) = \frac{r-r^*}{A_f^i(t-t_i)}$. From this switching condition we will derive the optimal waiting time τ^*, i.e., the period of time when a rational agent knows that he is holding overvalued currency.

We will organise the proof in three steps. In the first one, we demonstrate that the 'greed-to-fear ratio' is decreasing in time. Step 2 shows why the hazard rate is constant in time. Finally, in Step 3 we derive the expression for τ^* from the time when the hazard rate equals the 'greed-to-fear ratio' and rational agent i changes from a maximum long position to a maximum short position in local currency.

Step 1 *The 'greed-to-fear ratio' decreases in time.*

Proof

The 'greed-to-fear ratio' is defined as:

$$\frac{r - r^*}{A_f^i(t - t_i)}$$

where $r - r^*$ is the excess of return derived from investing in domestic currency and $A_f^i(t - t_i)$ denotes the size of the expected appreciation of the foreign currency feared by agent i:

$$A_f^i(t - t_i) = \left| E\left[\frac{\frac{1}{S_t} - \frac{1}{\overline{S}}}{\frac{1}{\overline{S}}} \middle| t_i \right] \right| = 1 - E\left[\frac{\overline{S}}{S_t} \middle| t_i \right]$$

and

$$E\left[\frac{\overline{S}}{S_t} \middle| t_i \right] = \int_{t_i - \eta}^{t_i} e^{-\mu(t-\widetilde{t})} \phi(\widetilde{t}|t_i) d\widetilde{t} = k e^{-\mu(t-t_i)}$$

where $k = \frac{\lambda}{\lambda-\mu} \frac{e^{(\lambda-\mu)\eta}-1}{e^{\lambda\eta}-1}$, $k \in [0,1]$ for $\lambda \geq \mu$. Hence, $A_f^i(t - t_i)$ is an strictly increasing and continuous function of the time elapsed since agent i received his signal indicating that the shadow price exceeded the peg.

Assumption 1 establishes that the excess of return $(r - r^*)$ is positive and constant in time. Therefore the 'greed-to-fear ratio', $\frac{r-r^*}{A_f^i(t-t_i)}$, decreases in time. Intuitively, the further in time, the larger the possible appreciation of the foreign currency once the peg collapses, and therefore the smaller the benefits (in relative terms) that a rational agent obtains from holding local currency. Thus, the 'greed-to-fear ratio' will decrease in time.

□

Step 2 *The hazard rate is constant in time.*

Proof

The hazard rate is defined as: $h(T^*(\widetilde{t})|t_i) = \frac{\pi(T^*(\widetilde{t})|t_i)}{1-\Pi(T^*(\widetilde{t})|t_i)}$, where $\pi(T^*(\widetilde{t})|t_i)$ is the conditional density function and $\Pi(T^*(\widetilde{t})|t_i)$, the conditional cumulative distribution function of the date on which the peg collapses. The hazard rate represents, at each time, the probability that the peg is abandoned, given that it has survived until $t = t_i$. We have considered that the timing of agent i's signal is uniformly distributed on $[\widetilde{t}, \widetilde{t} + \eta]$ and that $\widetilde{t}$ is exponentially distributed. Then, a rational agent, who becomes aware that the shadow price exceeds the peg at $t = t_i$, has a posterior density function of the date on which the peg is abandoned, with support $[t_i + \tau^* + \eta\kappa - \eta, t_i + \tau^* + \eta\kappa]$, given by: $\pi(T^*(\widetilde{t})|t_i) = \frac{\lambda e^{\lambda(t_i+\zeta-T^*(\widetilde{t}))}}{e^{\lambda\eta}-1} = \frac{\lambda e^{\lambda(t_i+\tau^*+\eta\kappa-T^*(\widetilde{t}))}}{e^{\lambda\eta}-1}$, where $\zeta = \tau^* + \eta\kappa$ if the peg collapses for endogenous reasons. This is depicted in Figure 3.

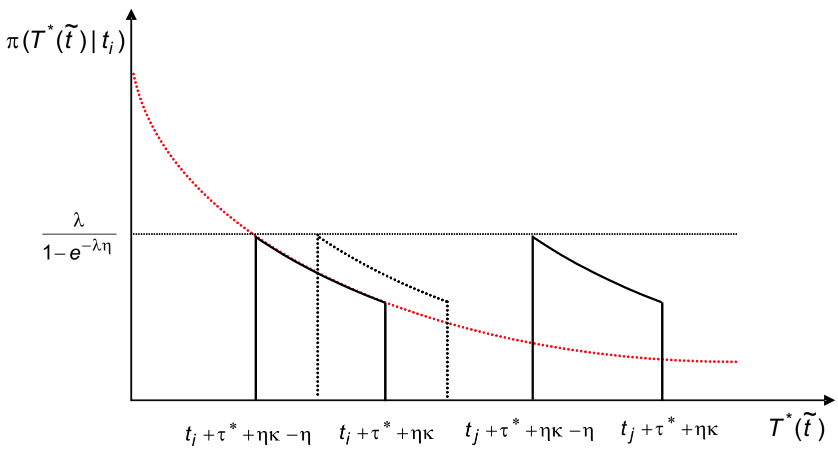

Figure 3: Posterior density function for rational agent i and j

At time $t = t_i$, rational agent i only knows if the peg has collapsed or not. But if the fixed rate regime has not been attacked, agent i cannot know when it will happen, since he does not know if any other agent became aware of the mispricing before him. At any other time $t = t_j > t_i$, agent j faces an equivalent scenario (shifted from t_i to time t_j, but with no additional information), i.e., agents cannot learn from the process (if the peg has not collapsed, they cannot know when the attack will take place). Therefore, the hazard rate, over the collapsing dates $t = t_i + \tau^*$, is constant in time and it is given by the following expression:

$$h(t_i + \tau^* | t_i) = \frac{\lambda}{1 - e^{-\lambda\eta\kappa}} \equiv h$$

$\square$

Step 3 *The optimal* τ^* *is given by:* $\tau^* = \frac{1}{\mu} \ln \left[\left(1 - \frac{r-r^*}{h}\right)^{-1} \right] - \tau'$

We have obtained that the 'greed-to-fear ratio' is a decreasing function of time, while the hazard rate is constant. Therefore, we can derive τ^* from the time $t = t_i + \tau^*$ when rational agent i fears that the collapse of the peg is imminent and decides to sell out the local currency. We have proved that he will hold local currency during τ^* periods after becoming aware of the mispricing, i.e., agent i will hold a long position in local currency until $t = t_i + \tau^*$, when his 'greed-to-fear ratio' equals his hazard rate:

$$\underbrace{\frac{r - r^*}{A_f^i(t - t_i)}}_{\substack{Greed\ to\ Fear \\ Ratio}} \Bigg|_{t=t_i+\tau^*} = \underbrace{\frac{\lambda}{1 - e^{-\lambda\eta\kappa}}}_{Hazard\ Rate} \quad \Rightarrow \quad \frac{r - r^*}{1 - ke^{-\mu(t_i+\tau^*-t_i)}} = h \quad \Rightarrow$$

$$\Rightarrow \quad \tau^* = \frac{1}{\mu} \ln \left[\left(1 - \frac{r - r^*}{h}\right)^{-1} \right] - \tau' \quad \Rightarrow \quad \tau^* = \frac{1}{\mu} \ln \left[\frac{h}{h - (r - r^*)} \right] - \tau'$$

where $\tau' = \frac{1}{\mu} \ln \frac{1}{k} \in [0, \infty)$. The optimal waiting time τ^* has two components. The first one defines the trade-off between the excess of return derived from investing in domestic currency and the capital loss bore by rational agent i if the fixed exchange rate is abandoned before he sells out. The second component τ' is due to the information structure and measures the period of time elapsed between the date $t = t_i$ at which agent i receives the signal and the time when he believes that the foreign currency starts appreciating. To simplify the reasoning suppose that there is no delay, i.e., assume that at $t = t_i$ rational agent i receives the signal about the mispricing and believes that the shadow price has "just" reached the peg. In this case, $\tau' = 0$ and each arbitrageur waits $\tau^* = \frac{1}{\mu} \ln \left[\left(1 - \frac{r-r^*}{h}\right)^{-1} \right]$ after receiving the signal. Then, he exits the market.

Figure 4 represents the decreasing 'greed-to-fear ratio', the hazard rate, τ^* and the time of the attack. It is interesting to note that if the hazard rate is larger than the excess of return ($h > r - r^*$), rational agents wait a finite positive period of time ($\tau^* > 0$) before selling out. At some point in time, they believe that the attack on the fixed exchange rate is imminent and they cancel their positions in home currency. There is an endogenous collapse of the peg. However, if $h = r - r^*$ arbitrageurs sell overvalued currency after waiting for an infinite time ($\tau^* = \infty$) to elapse since receiving the signal. Finally, if $h < r - r^*$, agents never sell, i.e., if the probability that the peg collapses (given that it still holds) is lower than the excess of return derived from investing in domestic currency, then arbitrageurs do not fear a devaluation and hence they never leave the market. In this case ($h \leq r - r^*$) the fixed exchange rate collapses for exogenous reasons when the Central Bank's foreign reserves are exhausted.

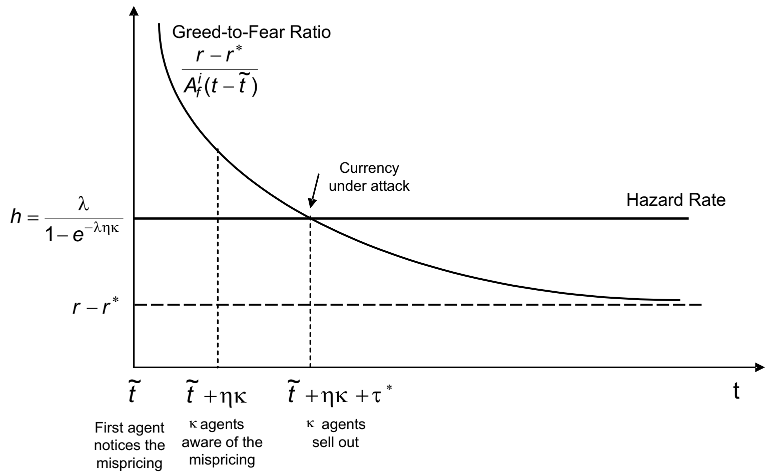

Figure 4: TIMING OF THE ATTACK - A rational agent holds a maximum long position in local currency until he believes that the attack is imminent, and he prefers to sell out, that is, when his 'greed-to-fear ratio' equals his hazard rate. At $t = \tilde{t}+\tau^*+\eta\kappa$ enough agents have sold out and the currency comes under attack.

□

5.2 Determinants of τ^*

- *Excess of return* - The period of time τ^* during which rational agents optimally hold overvalued local currency depends directly on the excess of return derived from investing in domestic currency. Increasing the spread between returns $(r - r^*)$ delays the attack on the peg, since a more attractive local currency will induce market participants to hold overvalued currency for a longer time. The size of the delay depends on:

$$\frac{\partial \tau^*}{\partial (r - r^*)} = \frac{1}{\mu} \cdot \frac{1}{h - (r - r^*)}$$

This suggests that rising returns will have a small impact on the delay of the attack if the probability of the peg collapsing (given that it still holds) and the slope of the (logarithm of the) shadow price are high.

- *Hazard rate* - τ^* is a decreasing function of the hazard rate ($\frac{\partial \tau^*}{\partial h} < 0$). The hazard rate represents the probability that the fixed exchange rate is abandoned given that the peg is still in place. Assuming the information structure presented in this paper, the hazard rate remains constant over time[5] and equal to:

$$h = \frac{\lambda}{1 - e^{-\lambda \eta \kappa}}$$

  where λ characterises the exponential distribution of $\widetilde{t}$ (the time when the shadow price exceeds the peg), η is a measure of the dispersion of opinion among agents and κ defines the threshold level of cumulative selling pressure that triggers the attack on the currency.

  A lower heterogeneity among market participants and a smaller threshold increase the hazard rate and advance the currency crises. In the limit as η tends to zero, we converge to the Krugman setting in which the fundamentals are common knowledge and the peg is attacked as soon as the shadow price reaches the fixed exchange rate. In this case, $h \to \infty$ and $\tau^* = 0$. Also, a higher κ delays the crisis. If the Central Bank is determined to commit a larger proportion of its foreign reserves to defend the fixed exchange rate, the peg will survive longer.

- *Slope of the shadow price* - The optimal time τ^* depends inversely on the slope of (the logarithm of) the shadow price μ ($\frac{\partial \tau^*}{\partial \mu} < 0$). μ can be seen as the speed of depletion of the Central Bank's foreign reserves. This suggests that a steeper (logarithm of the) shadow price implies a faster rate of exhaustion

[5]This is proved in Subsection 5.1.

of reserves and ultimately that the Central Bank's reserves will be exhausted earlier. Hence, a higher slope, reduces τ^* and advances the attack on the fixed exchange rate.

μ also represents the rate of growth of the government's budget deficit. Then, the faster the level of government's expenditure, the shorter speculators will be willing to hold the overvalued local currency, hence, advancing the attack on the currency.

Our analysis suggests that in a first generation model in which a government runs a persistent deficit, which grows at a constant rate μ, rising the spread between returns to make the local currency more attractive, committing more foreign reserves to defend the peg and inducing dispersion of opinion among market participants will delay the attack on the currency. However, since the expansionary monetary policy makes a fixed exchange rate ultimately unsustainable, these policy instruments would only increase the size of the devaluation whenever it occurs. The only effective means would be to reduce the rate of growth of the budget deficit (μ). This would delay the speculative attack on the fixed exchange rate and diminish the size of the devaluation when the peg finally collapses.

6 Conclusion

In this paper we relax the traditional assumption of perfect information in first generation models of currency crises. We consider an economy similar to the one described in Krugman (1979) and, to introduce uncertainty about the willingness of a Central Bank to defend the peg, we incorporate the information structure presented in Abreu and Brunnermeier (2003).

At a random time $t = \widetilde{t}$, the shadow price exceeds the fixed exchange rate and sequentially, but in random order, rational agents become aware of this mispricing. They understand that the currency will be attacked and the peg eventually abandoned, but still prefer to hold local currency during some period of time (τ^*) after they notice the mispricing, since we have assumed that holding domestic currency generates higher returns. We derive an expression for τ^*. The optimal period of time τ^* is the same for all market participants and it is independent from the time when each agent notices the mispricing. Increasing the excess of return $(r - r^*)$ obtained from investing in domestic currency, reducing the hazard rate or diminishing the level of persistent deficit, would increase the period of time when rational agents prefer to hold overvalued local currency and therefore it would delay the attack on the currency.

In our model, rational agents sequentially know that the peg lies below the shadow price but they do not know if other agents have already noticed it, i.e., macroeconomic fundamentals are no longer common knowledge among market participants. Therefore, rational agents in this economy face a synchronisation problem. However they do not have incentives to coordinate, since as soon as they do, the peg collapses. There is also a competitive component in our model: only a fraction κ of the rational agents can leave the market before the fixed exchange rate is abandoned. Consequently, in our setting, the currency does not come under attack the moment the shadow price exceeds the peg, as in Krugman's model, but at a later time when a large enough mass of rational agents has waited τ^* periods and decides to leave the market because of fear of an imminent attack.

We conclude that there is a unique equilibrium in which the fixed exchange rate collapses when the selling pressure surpasses a certain threshold (κ) or ultimately at a final date $t = \widetilde{t} + \overline{\tau}$, when the Central Bank's foreign reserves are exhausted, if dispersion of opinion among rational agents is extremely large. This equilibrium is unique, depends on the heterogeneity among agents and leads to discrete devaluations. This result differs from the zero-devaluation equilibrium in Krugman's model and also from other settings (Broner (2003)) in which lack of perfect information brings multiple equilibria.

A The shadow price

In this section, we will derive an expression for the shadow price in a first generation model of currency crises. To simplify the calculations, it is convenient to express magnitudes in logarithms. We will use upper case letters to represent a variable in levels and lower case letters to indicate its logarithm.

We consider an economy with a fixed exchange rate regime ($\overline{s} = ln(\overline{S})$), in which a government runs a persistent deficit. In particular, we assume that it grows at a positive constant rate μ:

$$\mu = \frac{\dot{D}_t}{D_t} = \frac{1}{D_t}\frac{d(D_t)}{dt} = \dot{d}_t \quad \Rightarrow \quad \dot{d}_t = \mu \tag{1}$$

where $d_t = ln(D_t)$ is the logarithm of the domestic credit.

The Central Bank has two main tasks: to finance the government's deficit by issuing debt and to maintain the exchange rate fixed through open market operations. In our economy there are no private banks. Then, from the Central Bank's balance sheet the money supply at time t, M_t^s, is made up of domestic credit (D_t) and the value in domestic currency of the international reserves (R_t):

$$M_t^s = D_t + R_t \tag{2}$$

We assume that the purchasing parity holds. Then, the exchange rate S_t is defined as:

$$S_t = \frac{P_t}{P_t^*} \quad \Rightarrow \quad s_t = p_t - p_t^*$$

We can take the foreign price as the numeraire ($P_t^* = 1 \;\Rightarrow\; p_t^* = 0$). Then,

$$s_t = p_t \tag{3}$$

The monetary equilibrium is described by the Cagan equation:

$$m_t^s - p_t = -\delta \times \dot{p}_t$$

By equation 3, the Cagan equation can then be written as:

$$m_t^s - s_t = -\delta \times \dot{s}_t \tag{4}$$

The shadow price is the exchange rate that would prevail in the market if the peg is abandoned. The Central Bank will defend the peg until reserves reach a minimum level. To simplify the analysis, assume that the Central Bank abandons the fixed exchange rate when the reserves are exhausted, i.e., when $R_t = 0$. Then, the money supply (equation 2) is given by:

$$M_t^s = D_t \quad \Rightarrow \quad m_t^s = d_t \tag{5}$$

Equations 1 and 5 imply:

$$\dot{m_t^s} = \dot{d}_t = \mu \quad \Rightarrow \quad m_t^s = m_0^s + \mu \times t$$

Hence, substituting this result in equation 4 we obtain:

$$m_0^s + \mu \times t - s_t = -\delta \times \dot{s}_t \quad \Rightarrow \quad m_0^s + \mu \times t - s_t + \delta \times \dot{s}_t = 0$$

To solve this differential equation, we can try a linear solution:

$$s_t = constant + \mu \times t$$

Then,

$$m_0^s + \mu \times t - constant - \mu \times t + \delta \times \mu = 0 \quad \Rightarrow \quad constant = m_0^s + \delta \times \mu$$

Therefore, (the logarithm of) the shadow price is given by:

$$s_t = \underbrace{m_0^s + \delta \times \mu}_{\gamma} + \mu \times t = \gamma + \mu \times t \quad \Rightarrow \quad s_t = \gamma + \mu \times t \tag{6}$$

where γ and μ are constants and μ represents the rate of growth of the budget deficit.

We have proved that (the logarithm of) the shadow is a linear function of time.

B Sell Out Condition

In this section we derive the sell out condition stated in Lemma 2. In our economy, a rational agent can choose between either holding local or foreign currency. Investing in domestic currency generates a return equal to r, while the foreign currency yields r^*. Assumption 1 ($r > r^*$) makes the domestic investment more attractive. Therefore, market participants are initially fully invested in domestic currency. At some random time $t = \widetilde{t}$ the shadow price reaches the fixed exchange rate and from then onwards the peg might be attacked.

We want to determine the optimal selling date for the arbitrageur who becomes aware of the mispricing at time $t = t_i$. Each rational agent's payoff from selling out depends on the price at which he can sell the domestic currency. At time $t \geq \widetilde{t}$ the peg may or may not hold. The price (in local currency) of an asset which yields a constant rate r is e^{rt}. The payoff function is denominated in foreign currency, hence the price of domestic currency if the peg still holds is $p_t = e^{rt}\frac{1}{\overline{S}}$, where $\overline{S}$ is the fixed exchange rate. However, if the peg has been abandoned, the price will be: $E[p_t|t_i] = E[e^{rt}\frac{1}{S_t}|t_i]$, which can be expressed as a fraction of the pre-crisis price as: $E[p_t|t_i] = e^{rt}\frac{1}{\overline{S}}E[\frac{\overline{S}}{S_t}|t_i]$, and $E[\frac{\overline{S}}{S_t}|t_i]$ can be understood as a rate of variation in the exchange rate. Then, the rational agent i's payoff[6] from selling out at time t is given by:

[6] We assume there are no transaction costs to simplify the specification of the payoff function. We could easily incorporate transaction costs in our setting. For example, let us define the transaction cost at time t equal to ce^{r^*t} (as in Abreu and Brunnermeier (2003)). This convenient formulation guarantees that the optimal solution is independent of the size of the transaction costs.

$$\int_{t_i}^{t} e^{-r^*s}\ e^{rs}\ E\left[\frac{1}{S_s}\bigg|t_i\right]\ \pi(s|t_i)\ ds + e^{-r^*t}\ e^{rt}\ \frac{1}{\overline{S}}\ (1-\Pi(t|t_i))$$

where $\Pi(t|t_i)$ represents agent i's conditional cumulative distribution function of the date on which the peg collapses and $\pi(t|t_i)$ indicates the associated conditional density.

Differentiating the payoff function with respect to t yields:

$$\underbrace{\frac{\pi(t|t_i)}{1-\Pi(t|t_i)}}_{h(t|t_i)} = \frac{r-r^*}{1-E[\frac{\overline{S}}{S_t}|t_i]} \quad \Rightarrow \quad h(t|t_i) = \frac{r-r^*}{A^i_f(t-t_i)}$$

where $A^i_f(t-t_i)$ is the size of the expected appreciation of the foreign currency feared by agent i once the fixed exchange rate is abandoned. $A^i_f(t-t_i)$ is a strictly increasing and continuous function of $t-t_i$, the time elapsed since agent i becomes aware of the mispricing.

Therefore, arbitrageur i maximises his payoff to selling out at time t when his hazard rate equals the 'greed-to-fear ratio'. Thus, rational agent i holds:

- a maximum long position in local currency, if $h(t|t_i) < \dfrac{r-r^*}{A^i_f(t-t_i)}$, or
- a maximum short position in local currency, if $h(t|t_i) > \dfrac{r-r^*}{A^i_f(t-t_i)}$.

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