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By

Bob Nobay
Ivan Paya
David A. Peel

DISCUSSION PAPER NO 601

DISCUSSION PAPER SERIES

November 2007

Bob Nobay is Senior Research Associate at the Financial Markets Group, London School of Economics and Political Science. David Peel is a Professor at the School of Management, Lancaster University, England and Ivan Paya is a Senior Lecturer at the School of Management, Lancaster University. Any opinions expressed here are those of the authors and not necessarily those of the FMG.
Inflation Dynamics in the US – A Nonlinear Perspective

Bob Nobay\textsuperscript{a} Ivan Paya\textsuperscript{b} David A. Peel\textsuperscript{c}

\textsuperscript{a}London School of Economics, Financial Markets Group, Houghton Street, London WC2 2AE, UK (Corresponding author; e-mail: a.nobay@lse.ac.uk)

\textsuperscript{b}Lancaster University Management School, Lancaster, LA1 4YX, UK (e-mail: i.paya@lancaster.ac.uk)

\textsuperscript{c}Lancaster University Management School, Lancaster, LA1 4YX, UK (e-mail: d.peel@lancaster.ac.uk)
Abstract

A stylized fact of US inflation dynamics is one of extreme persistence and possible unit root behavior. If so, the implications for macroeconomics and monetary policy are somewhat unpalatable. Our econometric analysis proposes a parsimonious representation of the inflation process, the nonlinear ESTAR, rather than the IMA process with time-varying parameters as in Stock and Watson (2007). The empirical results confirm a number of the key features such as regime changes and implicit Federal Reserve inflation targets. We address the issue of whether the source of the Great Moderation can be ascribed to good luck rather than good policy.

Keywords: Unit Root, Inflation persistence, nonlinear ESTAR.

JEL classification: C15, C22, E31
1 Introduction

A stylized fact of the dynamics of US inflation, as first highlighted in the pioneering contribution of Nelson and Schwert (1977), clearly indicate that it is a very persistent process. In fact, Barsky (1987), Ball and Cecchetti (1990), and Brunner and Hess (1993) suggested that U.S. inflation contains a unit root. Moreover, the unit root property appears to be shared for a wide array of economies examined in O’Reilly and Whelan (2005) and Cecchetti et al. (2007). More recently, in influential contributions, Stock and Watson (2007) and Cogley and Sargent (2007) have parsimoniously modeled inflation as an unobserved component trend-cycle model with stochastic volatility, a model that in its reduced form also exhibits a unit root. Stock and Watson show that the estimate of the moving average coefficient in their implied IMA(1,1) model for the mean of inflation has declined sharply since the early 1980’s. They attribute this to large changes in the variance of the error in the permanent stochastic trend component relative to the variance of the error in the transitory component of their model so that the magnitude of the MA coefficient varies inversely with the ratio of the permanent to the transitory disturbance variance.

The unit root feature of inflation is now reflected in theoretical models of the inflationary process. Woodford (2006) allows for the unit root feature by assuming that the inflation target follows a random walk. Cogley and Sbordone (2006) and Sbordone (2007) reformulate the New Keynesian supply curve, since the standard formulation is based on the assumption that
inflation is stationary.\textsuperscript{1} There are, however, severe economic and statistical problems with the assumption of a unit root in the inflation process. For instance, the assumption would imply, \textit{ceteris paribus}, that the nominal exchange rate, via purchasing power parity, is an I(2) process. Moreover, asset arbitrage would require nominal asset returns in general to exhibit I(1) behavior, and this is dramatically at odds with empirical findings. Further, the assumption of a random walk in the inflation target in theoretical models implies that the target will ultimately take negative values which is also economically absurd. Cogley and Sargent (2002) are mindful of the problem - they impose parameter restrictions to ensure that inflation is always stationary, since otherwise, it would imply infinite asymptotic variance of inflation, which can be ruled out as theoretically absurd, given the central banks’ loss function which includes inflation variance.

How robust, though, is the stylized fact that inflation follows a unit root process? Within the linear framework adopted in the extant literature, an alternative avenue is to consider whether inflation is fractionally integrated (see, e.g., Hassler and Wolters, 1995; Baillie et al., 1996; Baum et al., 1999; and Baillie et al., 2002).\textsuperscript{2} The fractionally integrated model has the property that although inflation is still very persistent, and could ultimately exhibit

\textsuperscript{1}As is well-recognised, and discussed robustly in Cochrane (2007), there are related issues of indeterminacy in this literature.

\textsuperscript{2}The ARFIMA(p,d,q) class of processes take the form

\[ x_t = (1 - L)^{-d} u_t \]

where \( x_t \) is a stationary ARMA(p,q) process, and \( d \) is a non integer. See, e.g., Granger and Joyeaux (1980) for discussion of the properties of fractional processes.
infinite variance, it is still mean reverting so that inflation does not exhibit a unit root. A major shortcoming of this literature, however, is that they do not allow for possible structural breaks in the series to reflect regime changes as reflected in the analyses of the US Great Moderation. Regime changes are known to spuriously induce the fractional property (see Diebold and Inoue, 2001; Franses et al., 1999; and Granger and Hyung, 1999). Consequently it is reasonable, from a linear perspective, to assume that the empirical evidence supports the extant view that the inflation series exhibits unit root behavior.

The focus of this paper is to consider an alternative parameterization of the inflation process. We borrow from the recent literature on exchange rate dynamics which mimic the findings in inflation analysis. In the empirical exchange rate literature, a commonplace finding is that real exchange rates can be described by either a unit root or a fractional processes (see Diebold et al., 1991; Cheung and Lai, 1993). More recently, and drawing on the theoretical analyses following Dumas (1992), it has been shown that the dynamics of real exchange rate adjustment, given transactions costs or the sunk costs of international arbitrage, induce nonlinear adjustment of the real exchange rate to purchasing power parity (PPP). Whilst globally mean reverting this nonlinear process has the property of exhibiting near unit root behavior for small deviations from PPP. Essentially, small deviations from PPP are left uncorrected if they are not large enough to cover transactions costs or the “sunk costs of international arbitrage”. Empirical work shows that the Exponential Smooth Autoregressive (ESTAR) model provides a parsimonious fit to PPP data (see Michael et al., 1997; and Paya and Peel, 2006). Of particular interest are the resultant implied dynamics of real exchange rates, as
derived from the nonlinear impulse response functions for the ESTAR models. They show that whilst the speed of adjustment for small shocks around equilibrium is highly persistent and relatively slow, larger shocks mean-revert much faster than the “glacial rates” previously reported for linear models. In this respect the nonlinear models provide a solution to the PPP puzzle outlined in Rogoff (1996).

A natural counterpart in monetary policy analysis is that the central bank pursues an implicit or explicit inflation target and that adjustment to this target is nonlinear. One model of the policy maker that implies this reduced form behavior of the inflation rate is the opportunistic approach to disinflation is set out by Orphanides and Wilcox (2002) and Aksoy et al. (2006). The key feature of their model, as stated by Aksoy et al. (2006), is that “a central bank controls inflation aggressively when inflation is far from its target, but concentrates on output stabilization when inflation is close to its target, allowing supply shocks and unforeseen fluctuations in aggregate demand to move inflation within a certain band”. In this regard it is relevant that Martin and Milas (2007) estimate threshold Taylor rules for the period

3A recent paper which focuses on this issue is Peter Ireland (2005). He draws inferences about the behaviour of the Federal Reserve’s implicit inflation target within a New Keynesian model.

4Gregoriou and Kontonikas (2006a) show that deviations of inflation in several targeting countries, not including the US, appear stationary on the basis of the Kapetanios et al. (2003) test. However, in Gregoriou and Kontonikas (2006b) they model the first difference of the deviations of inflation rates from target as ESTAR process which is inconsistent. Byers and Peel (2000) model inflation dynamics in three hyperinflations with a more complex ESTAR process exploiting the possible multiple equilibria property of the general ESTAR model.
They suggest the response of interest rates to inflation is zero when inflation is in the “band”. They also point out that the Opportunistic Approach to inflation has similarities with “constrained discretion” as advocated by Bernanke and Mishkin (1997) and Bernanke (2003).

We conjecture that inflation behaves as a near unit root process for inflation rates close to the implicit target of the policy maker but is mean reverting for large deviations. In this respect, the nature of the implied inflation adjustment process is similar to that suggested to explain deviations from purchasing power parity.

One simple ESTAR process that captures the PPP dynamics and also the inflation adjustment mechanism postulated above can be represented as follows:

\[ y_t = \alpha + e^{-\gamma(y_{t-1}-\alpha)^2} \sum_{i=1}^{p} \beta_i (y_{t-i} - \alpha) + u_t \]  

(1)

where \( y_t \) is the inflation rate, \( \alpha \) is a constant, \( \Phi(p) = \sum_{i=1}^{p} \beta_i \), \( u_t \) is a random disturbance term, and the transition function is \( G(.; \gamma) = e^{-\gamma(y_{t-1}-\alpha)^2} \), with \( \gamma > 0 \). Within this framework, the equilibrium or implicit inflation target is given by \( \alpha \). The ESTAR transition function is symmetric about \( y_{t-1} - \alpha \).

The parameter \( \gamma \) is the transition speed of the function \( G(.) \) towards 0 (or 1) as the absolute deviation grows larger or smaller. Particular emphasis is reserved for the unit root case, \( \Phi(p) = 1 \). In this case, \( y_t \) behaves as a random walk process when it is near the implicit target \( \alpha \). When the deviations from equilibrium are larger, the magnitude of such deviations along with the magnitude of \( \gamma \) imply that \( G(.) \) is less than one so that \( y_t \) is mean reverting.
This ESTAR model provides an explanation of why PPP deviations or inflation deviations analyzed from a linear perspective might appear to be described by either a non-stationary integrated I(1) process, or alternatively, described by fractional processes. Pippenger and Goering (1993) show that the Dickey Fuller tests have low power against data simulated from an ESTAR model. Michael et al. (1997) illustrate that data that is generated from an ESTAR process can appear to exhibit the fractional property. That this would be the case was an early conjecture by Acosta and Granger (1995).

The remainder of the paper is structured as follows. In the next section we discuss and carry out a sequence of econometric tests to discriminate between the linear unit root IMA(1,1) model of Stock and Watson and the ESTAR model outlined above. Our results establish that the ESTAR model provides a parsimonious explanation of US inflation. In section 3 we undertake an analysis of the impulse response functions from our ESTAR models. We take into account the distinctive features of nonlinear models which lead to impulse response functions that are history dependent and depend on the sign and size of current and future shocks as well. The economic implications are discussed further in section 4. Our results allow us to consider further the findings and interpretations of Mishkin (2007), Nelson (2005), Romer and Romer (2002), Sargent (1999) and Stock and Watson amongst others, in regards to monetary policy characterizations of the postwar US economy. Concluding comments are offered in Section 5.
2 Nonlinear Model

2.1 Linearity Testing

We examine quarterly US inflation measured by the log difference of PCE chain type index or GDP price index over the period 1947.Q1 to 2004.Q4. The data is available from the Federal Reserve Economic Database (FRED) and are seasonally adjusted.\(^5\) We divide the sample into two main sub-periods for detailed analysis. These periods are 1947.Q1 to 1982.Q4, and 1983.Q1 to 2004.Q4, respectively. The second period corresponds to a dramatic reduction in the volatility of inflation following the Volcker deflation and is regarded as a different policy regime as demonstrated in the estimates of Taylor Rules (see, e.g., Clarida et al., 2000; Dolado et al., 2004; and Martin and Costas, 2007). There is more debate about the precise beginning and ending of the first regime but the results are robust for the first sample and marginally more significant for the PCE index. Cogley and Sargent (2007) note colleagues in the Federal Reserve pay more attention to this measure of inflation for policy purposes. Consequently we report analysis of the PCE index.

Within the framework we consider, the key empirical issue is that of discriminating between alternative representations, so as to chose the most parsimonious statistical representation of inflation.

We begin by applying a set of specific linearity tests. Escribano and Jorda (EJ hereafter) (1999) extended the familiar nonlinearity test procedure for-

\(^5\)This data was kindly made available to us by Timothy Cogley can be found at http://research.stlouisfed.org/fred2/. The series have FRED mnemonics PCECTPI and GDPCTPI respectively.
mulated by Terasvirta (1994) and reviewed further in van Dijk et al. (2002). They proposed a new specification strategy to discriminate between the ESTAR and logistic STAR (LSTAR) models. Their specification strategy is shown to be consistent and to generate higher correct selection frequencies than that of Terasvirta (1994). The test is implemented following a series of steps. The linear AR process for \( y_t \) is initially specified using certain model selection criterion (Akaike, Schwartz). The linearity test is then specified using the lag length \( (p) \) of the linear process and a Taylor expansion of \( y_t \) for the cases of an ESTAR and a LSTAR:

\[
y_t = \delta_0 + \delta_1 x_t + \lambda_1 x_t z_{t-d} + \lambda_2 x_t z_{t-d}^2 + \lambda_3 x_t z_{t-d}^3 + \lambda_4 x_t z_{t-d}^4 + v_t \tag{2}
\]

where \( x_t = (y_{t-1}, \ldots, y_{t-p})' \) with \( p \) determined in the first step, and \( z_{t-d} \) is the transition variable, in our case equals to \( y_{t-d} \), where \( d \) is the delay parameter. The null hypothesis in this test \( (H_0^1) \) is that \( y_t \) follows a stationary linear process so that \( H_0^1: \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0 \). The computation of the test is carried out utilizing the \( F \) version of the test. If linearity is rejected, we follow the EJ procedure to discriminate between the ESTAR and LSTAR nonlinear models. The null hypothesis of nonlinear ESTAR corresponds to \( H_0^E: \lambda_2 = \lambda_4 = 0 \) in (2) and its \( F \)-statistic \( (F_E) \) is computed. For the null of an LSTAR, \( H_0^L: \lambda_1 = \lambda_3 = 0 \) in (2) with its corresponding \( F \)-statistic.

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\(^6\)Logistic LSTAR models embody asymmetric adjustment to deviations from equilibrium whilst the adjustment is symmetric in the ESTAR models.

\(^7\)The \( \chi^2 \) version of the test yielded similar results. The delay parameter \( d \) can be determined by searching over a certain range of values (e.g., \( d \in [1, 8] \)) and choose the one that minimizes the p-value of the test for \( H_0^1 \). In our case, we choose \( d = 1 \) as is the one that has a clear economic interpretation.
If the minimum $p$-value corresponds to $F_L$, we select LSTAR, if it corresponds to $F_E$, we select ESTAR.

In our case, for the null of a linear stationary process ($H_{10}$) in the US inflation series we obtain $p$-values of 0.006 and 0.66 for the first and second period, respectively. In the first period, the minimum $p$-value corresponds to the $F_E$ test and consequently it is possible to reject the null of linear stationary process in favor of a nonlinear stationary ESTAR model in the first period.

An alternative linearity testing procedure would be, given theoretical priors, to have a linear unit root inflation as the null hypothesis. Stock and Watson (2005) fit a stochastic volatility process to the inflation series. In particular, they assume an unobserved component model for inflation $y_t$ with the following state-space representation:

\[
\begin{align*}
y_t &= \tau_t + \varepsilon_{yt} \\
\tau_t &= \tau_{t-1} + \varepsilon_{\tau t}
\end{align*}
\]

where the innovations are conditionally normal martingale differences with the following variances

\[
\begin{align*}
h_{yt} &= h_{yt-1} e^{\sigma_y \eta_{yt}} \\
h_{\tau t} &= h_{\tau t-1} e^{\sigma_{\tau} \eta_{\tau t}}
\end{align*}
\]

where $\eta_{yt}, \eta_{\tau t}$ are i.i.d. Gaussian shocks with mean zero and mutually independent. The model implies an integrated I(1) process for inflation. Consequently we also undertake the tests of Kapetanios et al. (2003) (KSS
hereafter) and Kilic (2003) where the null of a linear unit root process is tested against the alternative of a globally stationary nonlinear ESTAR model.

Under the null hypothesis, using a first order Taylor approximation of the nonlinear model KSS obtain the following auxiliary regression

$$\Delta y_t^* = \sum_{j=1}^{p} \Delta y_{t-j}^* + \delta y_{t-1}^3 + error$$

(3)

Testing for $\delta = 0$ against $\delta < 0$ corresponds to testing the null hypothesis, and the $t - statistic$ is given by

$$t_{NL}(\hat{\delta}) = \frac{\hat{\delta}}{s.e(\hat{\delta})}$$

(4)

where $s.e(\hat{\delta})$ denotes the estimator standard error. The asymptotic distribution of (4) is not standard since, under the null, the underlying process is nonstationary. KSS show that their test has greater power than the ADF and also that of Enders and Granger (1998) to discriminate against ESTAR. We obtain values for the KSS test of -5.77 and -4.79 for the two sub-periods. These values are highly significant using the conventional critical values provided in KSS, and therefore suggesting we can reject the null of a unit root in inflation in favor of the ESTAR process.

In order to make certain that the implementation of the KSS test is robust within our framework we carry out a Monte Carlo exercise. In particular, we generate the true DGP as the unobserved component trend-cycle model with stochastic volatility (IMAV) of Stock and Watson calibrated with the

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*KSS examine the properties of their test under three different assumptions of stochastic processes with nonzero mean and/or linear deterministic trend. In the cases where $y_t^*$ exhibits significant constant or trend, $y_t^*$ should be viewed as the de-meaned and/or detrended variable.*
values in our sub-samples. We use the same sample size as the actual data which is 144 observations for the first period and 88 for the second one, and simulate 9,999 data samples for each sub period. We then apply the KSS test to this simulated data for each sub-period in order to obtain the new ninety five percent critical values. These are -4.95 and -4.59, which are below our actual values obtained for the real data. Consequently the KSS test points to a clear rejection of the null of a linear unit root in favor of an ESTAR process.\footnote{An alternative test of the unit root test null against a nonlinear ESTAR alternative is developed by Kiliç (2003). This test uses a grid search over the space of values for the parameters $\gamma$ and $c$ to obtain the largest possible t-value for $\phi$ in the following regression}

The third linearity test we perform is the one developed in Harvey and Leybourne (2007) (HL hereafter). They test the null hypothesis of a linear process, which could be either stationary or non-stationary, since their statistic is consistent against either form. Their methodology is based on a Taylor approximation of a nonlinear stationary or nonstationary series which yields the following regression equation

$$\Delta y_t^* = \phi y_{t-1}^3 (1 - \exp(-\gamma (z_t - c)^2)) + \text{error}$$

where $z_t$ is the transition variable, in this case $(\Delta y_{t-1}^*)$. The null hypothesis is $H_0 : \phi = 0$ (unit root case) and the alternative $H_1 : \phi < 0$. The Kiliç (2003) test has potential advantages over the KSS test. First, it computes the test statistic even when the threshold parameter needs to be estimated in addition to the transition parameter. Second, Kilic claims that it has more power. For the same reasons as in the case of the KSS test above, we undertake the same Monte Carlo experiment and obtain new 95% critical values of -3.45, and -3.47 respectively. The values obtained with our actual data in the two periods were -4.91 and -4.24 giving further support to the alternative of an ESTAR.
\[ y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-1}^2 + \alpha_3 y_{t-1}^3 + \alpha_4 \Delta y_{t-1} + \alpha_5 (\Delta y_{t-1})^2 + \]
\[ + \alpha_6 (\Delta y_{t-1})^3 + \varepsilon_t \]  

(5)

The null hypothesis of linearity is \( H_{0L} : \alpha_2 = \alpha_3 = \alpha_5 = \alpha_6 = 0 \). The alternative hypothesis (nonlinearity) is that at least one of those \( \alpha \)'s is different from zero. The statistic is then

\[ W_T^* = \exp(-b |DF_T|^{-1}) \frac{RSS_1 - RSS_0}{RSS_0/T} \]

(6)

where \( |DF_T| \) is the absolute value of the ADF statistic, and the value of \( b \) is provided in HL such that, for a given significance level, the critical value of \( W_T^* \) coincides with that from a \( \chi^2 \) distribution.\(^\text{10}\) The values we obtain for the first and second periods are 23.73 and 7.44, respectively. Linearity is clearly rejected in the first period but not in the second one.\(^\text{11}\)

A second step of the test is to determine the stationarity or nonstationarity of the processes using the Harris, McCabe and Leybourne (2003) test statistic. In our case stationarity could not be rejected. Given the existence of a discrepancy between the KSS and the HL tests for the second period we check the power of both statistics under the alternative of an ESTAR process with a range of parameter values similar to the ones obtained in the

\(^{10}\)Actually, HL provides the coefficients of the seventh-order polynomial of \( b \) in \( \alpha \) (significance level) such that it is possible to compute \( b \) for any desired significance level \( \alpha (= 0.99, 0.95, 0.90, \ldots) \).

\(^{11}\)As our prior for the alternative model is an ESTAR we included a fourth power in (5) for the test in the second period using the same rational than Escribano and Jorda. However, the test still rejects the null hypothesis with a p-value of 0.28. Using only three powers in (5) yields a p-value of 0.82.
estimation provided in the next section. The KSS test appears to be more powerful in this case as, according to table 3 in KSS and table 3 in HL, the power of the KSS and HL tests is 0.98 and 0.25, respectively.

Overall, our battery of tests clearly suggest that a linear process, either stationary or non stationary, can be rejected in favour of a nonlinear ESTAR process in the first period. For the second period a non-stationary linear process can be clearly rejected on the basis of the KSS test in favour of the ESTAR process.\(^\text{12}\)

### 2.2 Nonlinear Estimates: the ESTAR model

In Tables 1a and 1b we present the results of the estimation of ESTAR models using non-linear least squares for the main sub-periods, as justified above, and a few other periods for comparison of parameter stability. In the estimation of ESTAR model, the transition parameter, \(\gamma\), is estimated by scaling it by the variance of the transition variable. This scaling is suggested for two reasons. One is to avoid problems in the convergence of the algorithm. Second, it makes it easier to compare speeds of adjustment (see Terasvirta,1994).

When the ESTAR transition parameter is estimated as zero we obtain a unit root process. Consequently the critical significance values are non-standard. Accordingly the critical values for the normalized speed of adjustment coefficient have been obtained through Monte Carlo simulation. We generate 9,999 series as the DGP series for each sub-period from the IMAV model of Stock and Watson calibrated with values in each sub-sample. We

\(^{12}\)These results are also in contrast to those found in Pivetta and Reis (2007) where they could not reject the unit root using a modified version of the Cogley and Sargent (2002) model where stationarity restrictions had been removed.
then estimate ESTAR processes on the simulated data so as to obtain the
distribution of the $t$-statistic of the $\gamma$ parameter at various significance levels.

The ESTAR model in the first period is jointly estimated with a GARCH(1,1)
process.\(^{13}\) In the second period this is unnecessary as there is no evidence of
residual mispecification.\(^{14}\) The estimated coefficients in Table 1a are signif-
icient and inflation appears parsimoniously explained by an ESTAR process
with two autoregressive lags. Even though we discuss the economic interpre-
tation of these results in section 4, it is worth mentioning that the second
period displays significantly lower target inflation, $\alpha$, and significantly larger
speed of adjustment of inflation towards $\alpha$ than the first period. Figures 1a
and 2a plot the actual inflation series, the fitted series and the residuals for
the two sub-samples reported in Table 1a. It is evident from these figures
that the variance of the residuals varies at the beginning and at the end of
the first sub-sample, the size of the residuals is larger in the first period and
that inflation moves around a lower level in the second period. For compar-
ison purposes, Figures 1b and 2b plot the actual, fitted, and residual series
obtained from the IMA(1,1) model.

An alternative approach is to fit the ESTAR process for the whole period
allowing the intercept and the speed of adjustment to change by introduc-
tion of a dummy variable ($d82$). This takes the value of zero up to the
fourth quarter of 1982 and unity afterwards. To obtain critical values for

\(^{13}\)The estimated GARCH(1,1) takes the following form: $\sigma^2_t = k + \varphi \varepsilon^2_{t-1} + \phi \sigma^2_{t-1}$.

\(^{14}\)The diagnostic residuals in each estimation reported in Table 1b were satisfactory
except for the period 1980.1-1995.2 where there was remaining autocorrelation at lag 4,
on the basis of the test of Eitrheim and Terasvirta (1996). Standard errors for this case
are computed using the Newey-West procedure.
the dummy variable coefficients we employ the wild bootstrap which allows for heteroskedasticity of any form or changing over the longer sample period (see, e.g., Wu, 1986; Mammen, 1993; and Davidson and Flachaire, 2001). The result displayed in Table 2, for the sample period where the dummies are most significant, is consistent with the results reported in Table 1 confirming the significant differences in the implicit inflation target and the speed of response to shocks in the two periods.

\begin{equation}
\begin{aligned}
\epsilon_i &= 1 \quad \text{with probability } p = 0.5 \\
\epsilon_i &= -1 \quad \text{with probability } p = 0.5
\end{aligned}
\end{equation}

The $\epsilon_i$ are mutually independent drawings from a distribution independent of the original data. The distribution has the properties that $E\epsilon_i = 0$, $E(\epsilon_i^2) = 1$, $E(\epsilon_i^3) = 0$, and $E(\epsilon_i^4) = 1$. As a consequence any heteroskedasticity and non-normality due to the fourth moment in the estimated residuals, $\hat{\epsilon}_t$, is preserved in the created residuals, $u^b_t$. We then simulate the ESTAR model in Table 2, 10,000 times with the coefficients on the dummy variables set to zero, using residuals. $u^b_t$, $i = 1, 2, \ldots, 10,000$, using the actual initial values of $y_{t-1}, y_{t-2}$ as starting values. We then estimate the ESTAR model with the dummy variables included to obtain the critical values. Analysis by Goncalves and Kilian (2002) is suggestive, in a slightly different context, that the wild bootstrap will perform as well as the conventional bootstrap, which is based on re-sampling of residuals with replacement, even when the errors are homoskedastic. The converse is not true.
3 Nonlinear Impulse Response Functions

In this section we examine the speed of mean reversion of the nonlinear model of inflation. To calculate the half-lives of inflation deviations \((y_t - a)\) within the nonlinear framework we need to obtain the Generalized Impulse Response Function (GIRF) for nonlinear models introduced by Koop et al. (1996). They differ from the linear response functions in that they depend on initial conditions, on the size and sign of the current shock, and on the future shocks as well. The GIRF is defined as the average difference between two realizations of the stochastic process \(\{y_{t+h}\}\) which start with identical histories up to time \(t - 1\) (initial conditions) but one realization is “hit” by a shock at time \(t\) while for the other one is not

\[
GIRF_h(h, \delta, \omega_{t-1}) = E(y_{t+h}|u_t = \delta, \omega_{t-1}) - E(y_{t+h}|u_t = 0, \omega_{t-1}) \quad (7)
\]

where \(h = 1, 2, \ldots\) denotes horizon, \(u_t = \delta\) is an arbitrary shock occurring at time \(t\), and \(\omega_{t-1}\) defines the history set of \(y_t\). The value of (7) has to be approximated using stochastic simulation since it is not possible to obtain an analytic expression for the conditional expectation involved in (7) for horizons larger than one (see Gallant et al., 1993; and Koop et al., 1996).\(^{16}\)

For each history, we construct 5,000 replications of the sample paths \(\hat{y}_0^*, \ldots, \hat{y}_h^*\) based on \(u_t = \delta\) and \(u_t = 0\) by randomly drawn residuals as noise for \(h \geq 1\). The difference of these paths is averaged across the 5,000 replications and it is stored. In order to obtain the final value for (7) we average across all histories. In the case of nonlinear models, monotonicity in the impulse

\(^{16}\)See Murray and Papell (2002) and Killian and Zha (2002) for a comprehensive analysis of impulse responses and estimating procedures.
response need not hold and shock absorption becomes slower as the shock becomes smaller. Hence, we calculate the $x$-life of shocks for $(1 - x) = 0.50$, and 0.75 where $(1 - x)$ corresponds to the fraction of the initial effect $u_t$ that has been absorbed.

For a particular value of inflation at time $t$, the series is hit with a shock of size $\delta$. The shock size is usually determined in terms of the residual standard deviation ($\hat{\sigma}_u$) of the model, such that $\delta = k\hat{\sigma}_u$. In this way, one can compare shocks absorption for a given value of $k$ but for models with different standard errors. Moreover, it is also possible to convert it to a common measure in terms of the level of the dependent variable. In our case, the residual standard deviation in the first period is $\hat{\sigma}_{1,u} = 0.0046$ which corresponds roughly to an additive 2% per annum shock on the level of inflation at quarter $t$. In the first sub-sample the largest change in inflation on a given quarter took place in the early fifties and was equal to 0.024($\simeq 5\hat{\sigma}_{1,u}$), or roughly 10% in annual terms. However, in the second period $\hat{\sigma}_{2,u} = 0.0024$ which corresponds to a 1% per annum shock to inflation level in a particular quarter. The largest change in inflation in the second sub-sample equals 0.007($\simeq 3\hat{\sigma}_{2,u}$) and took place in the eighties. We therefore consider the following set of values for $k = 1, 3, 5$. The particular choice of $k$’s allows us to compare and contrast the persistence of small, and large shocks within and across periods.

Table 3 shows the results for the GIRFs in both sub-samples. Two points are worth mentioning. First, the inflation series displays a clear nonlinear pattern. In particular large shocks tend to be absorbed much faster than small shocks. In Figures 3 and 4 we display the GIRFs for both sub-periods, and it is visually evident that larger shocks revert quicker than small shocks.
Second, inflation was significantly more persistent in the first period than in the second period. However, the magnitude of shocks is higher in the first period. Figures 5 and 6 display the GIRFs for both sub-periods along with the impulse response from the IMA models. Not surprisingly, impulse responses for the IMA models do not die out after the second period implying a much more persistent inflation series.

Assuming these shocks are exogenous to the policy maker, one might wonder what would have happened if the “model” in the second period had been hit by shocks of the same size of the first period? To address this issue, we carry out a counterfactual exercise of subjecting the second period model to first period shocks. In Table 3 column four, we display in brackets the result of deriving the impulse responses for the second period model with the first period residuals. The answer appears to be that inflation would have been much less persistent.

As a further check we also undertake the following experiment. Residuals in the first period have a standard deviation around twice as high as the residuals in the second period. Consequently we simulate the second period impulse responses with shocks twice as large as the benchmark and compared results. That is, we simulated the impulse responses in the second-period with shocks of $k = 2, 6, 10$ to compare with shocks of $k = 1, 3, 5$. The absorption of shocks are slightly slower than using the residuals from first period –the results however are qualitatively the same.
4 Policy Implications

Mishkin (2007) amongst others reminds us that in interpreting stylized facts about changes in inflation dynamics, we must be cautious in interpretation based on reduced-form relationships as they are about correlations and not necessarily about true structural relationships. Given this caveat the reduced form ESTAR models for the two main sample periods and the associated impulse response functions suggest that economic policy was conducted in a distinctively different manner in the two periods. In particular our estimates support the view that the policy maker had a significantly different equilibrium or implicit inflation target in the two periods – approximately 4.89% per annum in the first period and 2.79% in the second.

The speed of response to shocks appears to be significantly different in the two periods. In the first period we estimate that fifty percent of a 2% shock would be dissipated within five quarters whilst in the second period this dissipation rate would take less than 2 quarters. On the other hand, in the first period we estimate a 10% shock would take 3 quarters for 50% dissipation whilst in the second period 50% would be fully dissipated within the quarter. Consequently the ESTAR model estimates suggest that inflation is now much less persistent than in the first period.

This is also the conclusion of Stock and Watson based on their reduced form model. However, their model attributes the decrease in the persistence of inflation to a reduction in the variance of the permanent component of inflation relative to the transitory component. One interpretation, from the perspective of the ESTAR model, is that the variance of shocks was greater in the first period than in the second. Furthermore policy makers responded
less aggressively to shocks in the first period. In particular their response to shocks of small magnitude was more benign, than was the case in the second period –or would have been the case in the second period if shocks had been of a similar magnitude to those in the first period. In this respect the ESTAR estimates are consistent with first, the good-luck hypothesis, that is that shocks were smaller in the second period (Stock and Watson, 2003; Ahmed, Levin, and Wilson, 2004), and second, improved policymaking in the sense that the Fed had a lower implicit inflation target and responded more rapidly to inflation shocks. This change in policy makers preferences between the two periods suggests that the more favorable inflation scenario can persist in the future, that is lower inflation is not simply due to good luck.

The opportunistic approach to disinflation set out by Orphanides and Wilcox (2002) and Aksoy et al. (2006) provides a general framework that allows inflation to move within a band and can motivate the ESTAR model in both periods. However to explain why the target in the second period appears to have been lower on average and the speed of response to shocks faster we have to look elsewhere.

Nelson (2005) (also see Romer and Romer, 2002) argues for the monetary policy neglect hypothesis whereby policy makers took a non-monetary view of the inflation process. We can interpret this as implying both a slower response to inflation shocks and possibly a higher equilibrium or implicit inflation target.\textsuperscript{17}

\textsuperscript{17}Nelson stresses that a satisfactory explanation must be consistent with the estimated monetary policy reaction function. However although we agree with this observation recent empirical evidence suggests such policy responses are nonlinear rather than linear as he
Orphanides (2003) and Sargent (1999) provide different reasons to Nelson and Mishkin as to why the implicit inflation target might have been higher in the first period and Sargent also provides a rationale as to why the response to shocks might have been slower. Orphanides suggests that the policy makers were too optimistic about the economy’s productive potential so that ex post they appeared to follow excessively expansionary monetary policy. Sargent (1999) suggests that policy makers acted on the basis that there was a long run trade-off between inflation and real output. Moreover, their perception was that this trade-off had worsened so that increasing inflation rates were required to obtain a given real output gain. This led to higher levels of inflation (and hence higher average inflation in the period) before the policy maker was forced to deflate.

With similar implications Mishkin (2007) argues that since the late 1970s, the Federal Reserve has increased their commitment to price stability, in both words and actions, and has pursued more-aggressive monetary policy to control inflation. He also argues that such policies have helped anchor inflation expectations so that any given shock to inflation will now have a much less persistent effect on actual inflation. The impact of a shock on inflation dynamics is, of course, not independent of the policy response, that is, the coefficient on inflation in the Taylor rule. However it is also clear that the dynamic response of inflation to a shock, for a given policy response, is also not independent of expectations of inflation in any structural model of the inflationary process. From the perspective of anchoring inflation expectations as stressed in Mishkin it is informative to note that the Federal Reserve of Cleveland’s daily ten year ahead series inflation expectations derived from

assumes in his informative paper.
real and nominal bonds (TIPS) between 2005 and 2007, whether in raw or
adjusted form, appears to exhibits a unit root.\textsuperscript{18} This would suggest that
long term inflation expectations are not anchored. However visual inspection
of the series clearly suggest otherwise - inflation expectations remaining in
a relatively narrow range over the period (see figures 7-8) as opposed to the
drift in a unit root process. In fact, as displayed in Table 4, an ESTAR
model parsimoniously captures the dynamics of these expectations series il-
lustrating their mean reverting property. This then is consistent with long
run inflation expectations being anchored.

5 Conclusions

There is, by now, a vast literature that has focused on regime changes in the
conduct of monetary policy in the US. In particular, the issue of whether the
Great Moderation is a result of dramatic changes in monetary policy (changes
in coefficients) or a reflection of the covariance structure of disturbances. In
this paper we have sought to examine a particularly unpalatable feature of
inflation dynamics in the US, namely it’s unit root property. We undertake
a comprehensive array of statistical tests to show that ESTAR models par-
simoniously capture the dynamic behavior of US inflation in the post-war

\textsuperscript{18}We consider two daily series for inflation expectations (TIPS1, TIPS2) obtained from
the Federal Reserve of Cleveland over the period 01/01/2005 - 07/16/2007. The table
below displays the unit root and stationarity tests for each of the two series where an
asterisk denotes rejection of the null.

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF</th>
<th>PP</th>
<th>KPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIPS1</td>
<td>-2.19</td>
<td>-2.24</td>
<td>0.54*</td>
</tr>
<tr>
<td>TIPS2</td>
<td>-2.66</td>
<td>-2.79</td>
<td>0.85*</td>
</tr>
</tbody>
</table>
period. Our results show that whilst inflation is a near unit root process when close to target or equilibrium, it is globally mean reverting. This property is, a priori, surely more appealing from an economic perspective than the unit root alternative. Moreover, the implied dynamics, as derived from the impulse response functions, indicate distinctive speeds of adjustment between the generally accepted policy regimes. Overall, the results deliver adjustment speeds that are much faster and plausible than is implied in the extant literature.

The model estimates imply that inflation persistence is less and the implicit inflation target or equilibrium inflation rate lower after 1982 than in the earlier period. These appear consistent with monetary policies been followed in each of the two distinct periods within the general framework of the opportunistic policy maker. The model estimates and derived impulse response functions are consistent with the hypothesis that policy makers in the second period were fortunate to face shocks of lower variance than in the first period but also responded more aggressively to these shocks in the context of a lower inflation target. Hence, rather than the usual characterization of good policy/good luck in the literature, our results support the view that monetary policy was, in the second period, better in the sense of targeting lower inflation, but also benefited from good luck.
REFERENCES


Enders, Walter and Granger, Clive W.J. “Unit Root Tests and


**Gregoriou Andros and Kontonikas, Alexandros.** “Modeling the Non-linear Behaviour of Inflation Deviations From the Target.” Mimeo Uni-
versity of Glasgow, 2006b.


Mammen, Enno. “Bootstrap and Wild Bootstrap for High Dimensional


**Paya, Ivan and Peel, David A.** “A New Analysis of the Determinants


Stock, James H. and Watson, Mark W. “Why Has U.S. Inflation Become Harder to Forecast?” Journal of Money, Credit and Banking, 2007,


Table 1a. Results for estimated ESTAR model

Estimated model: \( y_t = a + B(L)y_{t-1}e^{-\gamma(y_t-1-a)^2} + \epsilon_t \)

PCE inflation 1947Q1-1982Q4

<table>
<thead>
<tr>
<th>( a )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \gamma )</th>
<th>( s )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>0.72</td>
<td>1 - ( \beta_1 )</td>
<td>0.064</td>
<td>0.0047</td>
<td>0.66</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(0.08)</td>
<td>(0.024)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[0.048\]

GARCH: \( \varphi = 0.15 \) \( \phi = 0.78 \)

Diagnostics: \( JB = 0.01 \) \( Q(1) = 0.76 \)

\( Q(4) = 0.65 \) \( A(1) = 0.89 \) \( A(4) = 0.43 \)

PCE inflation 1983Q1-2004Q4

<table>
<thead>
<tr>
<th>( a )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \gamma )</th>
<th>( s )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0069</td>
<td>0.73</td>
<td>1 - ( \beta_2 )</td>
<td>0.188</td>
<td>0.0025</td>
<td>0.35</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(0.11)</td>
<td>(0.065)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[0.012\]

Diagnostics: \( JB = 0.96 \) \( Q(1) = 0.46 \)

\( Q(4) = 0.22 \) \( A(1) = 0.39 \) \( A(4) = 0.58 \)

Notes: Figures in brackets are the Newey-West standard errors.

\( s \) denotes standard error of the regression \( Q(l) \), \( A(l) \) and \( JB \) are the \( p \)-values of the Eitrheim and Terasvirta (1996) LM test for autocorrelation in nonlinear series for \( l \) number of lags; the LM test for ARCH effects up to \( l \) lags, and the normality Jarque-Bera test, respectively. Figures in square brackets represent the \( p \)-value of the \( \gamma \) parameter obtained through Monte Carlo simulation.
Table 1b. ESTAR estimates for different regime policies

<table>
<thead>
<tr>
<th>Period</th>
<th>$\alpha$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\gamma$</th>
<th>$s$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960.1-1982.4</td>
<td>0.013</td>
<td>0.84</td>
<td>1 - $\beta_2$</td>
<td>0.045</td>
<td>0.0030</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.11)</td>
<td>[0.10]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1966.1-1979.2</td>
<td>0.016</td>
<td>1</td>
<td></td>
<td>0.060</td>
<td>0.0033</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td></td>
<td>[0.12]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980.1-1995.2*</td>
<td>0.014</td>
<td>0.70</td>
<td>1 - $\beta_2$</td>
<td>0.048</td>
<td>0.0030</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.13)</td>
<td>[0.08]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983.1-2004.4</td>
<td>0.0069</td>
<td>0.75</td>
<td>1 - $\beta_2$</td>
<td>0.187</td>
<td>0.0026</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.11)</td>
<td>[0.02]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987.1-2004.4</td>
<td>0.007</td>
<td>0.75</td>
<td>1 - $\beta_2$</td>
<td>0.137</td>
<td>0.0025</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.11)</td>
<td>[0.08]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: An asterisk denotes significant autocorrelation and Newey-West standard errors

Square brackets denote p-values using Monte Carlo simulation under the unit root null
Table 2. Results for estimated ESTAR model

Estimated model:  
\[ y_t = a + a^*d82 + [\beta_1(y_{t-1} - a - a^*d82) + \beta_2(y_{t-2} - a - a^*d82)]e^{(-\gamma - \gamma^*d82)(y_{t-1} - a - a^*d82)^2} \]

<table>
<thead>
<tr>
<th>US PCE inflation 1953Q1-2004Q4</th>
<th>( a )</th>
<th>( a^* )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \gamma )</th>
<th>( \gamma^* )</th>
<th>( s )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>-0.006</td>
<td>0.74</td>
<td>1 - ( \beta_1 )</td>
<td>0.028</td>
<td>0.75</td>
<td>0.003</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.0017)</td>
<td>(0.08)</td>
<td>(0.011)</td>
<td>(0.25)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.10]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.09]</td>
<td>[0.00]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Diagnostics:  
\( Q(1) = 0.61 \)  \( Q(4) = 0.25 \)  
\( A(1) = 0.16 \)  \( A(4) = 0.003 \)  \( JB = 0.01 \)

Notes: Figures in square brackets represent the \( p \)-value of the \( t \) statistics obtained through wild Bootstrap simulation.
Table 3. Generalized Impulse Response Function

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>50%</td>
<td>5</td>
<td>2(2)</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>12</td>
<td>5(4)</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>50%</td>
<td>5</td>
<td>0(0)</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>12</td>
<td>4(1)</td>
</tr>
<tr>
<td>$k = 5$</td>
<td>50%</td>
<td>3</td>
<td>0(0)</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>10</td>
<td>3(0)</td>
</tr>
</tbody>
</table>
Table 4. Results for estimated ESTAR model

Extended model:  \( y_t = a + B(L)y_{t-1}e^{-\gamma(y_{t-1}-a)^2} + \epsilon_t \)

<table>
<thead>
<tr>
<th></th>
<th>TIPS1</th>
<th></th>
<th>TIPS2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Result</td>
<td>Result</td>
<td>Result</td>
</tr>
<tr>
<td>(a)</td>
<td>0.0062</td>
<td>0.0057</td>
<td>0.0057</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>1</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.009</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>(s)</td>
<td>0.021</td>
<td>0.049</td>
<td>0.049</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>ARCH</td>
<td>(\varphi = 0.11)</td>
<td>(\varphi = 0.14)</td>
<td></td>
</tr>
<tr>
<td>Diagnostics:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(JB)</td>
<td>0.015</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(Q(1))</td>
<td>0.50</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>(Q(4))</td>
<td>0.22</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>(A(1))</td>
<td>0.90</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>(A(4))</td>
<td>0.93</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Notes: Figures in brackets are the Newey-West standard errors. 
\(s\) denotes standard error of the regression \(Q(l), A(l)\) and \(JB\) are the \(p\)-values of the Eitrheim and Terasvirta (1996) LM test for autocorrelation in nonlinear series for \(l\) number of lags; the LM test for ARCH effects up to \(l\) lags, and the normality Jarque-Bera test, respectively. Figures in square brackets represent the \(p\)-value of the \(\gamma\) parameter obtained through Monte Carlo simulation.
Figure 1a. Actual, fitted inflation, and residual series using model (1) for the period 1947-1982.
Figure 1b. Actual, fitted inflation, and residual series using IMA(1,1) model for the period 1947-1982.
Figure 2a. Actual and fitted inflation series along with residual using model (1) for the period 1983-2004
Figure 2b. Actual, fitted inflation, and residual series using model (1) for the period 1983-2004
Figure 3. GIRFs First period. Solid line: 5% shock, Dotted line: 3% shock, Triangle Line: 1% shock
Figure 4. GIRFs Second period. Solid line: 5% shock, Dotted line: 3% shock, Triangle Line: 1% shock
Figure 5. Impulse Response Functions First Period. Solid lines are GIRFs from ESTAR model, and stars lines are from IMA models.
Figure 6. Impulse Response Functions Second Period. Solid lines are GIRFs from ESTAR model, and stars lines are from IMA model.
Figure 7. Expected inflation (series Tips1) obtained from Federal Reserve of Cleveland.
Figure 8. Expected inflation (series tips2) obtained from the Federal Reserve of Cleveland.