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The Ownership of Ratings*

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Abstract

A prevalent feature in rating markets is the possibility for the client to hide the outcome of the rating process, after learning that outcome. This paper identifies the optimal contracting arrangement and the circumstances under which simple ownership contracts over ratings implement this optimal solution. We place ourselves in a setting where the decision to obtain a rating is endogenous and where the cost of such a piece of information is a strategic variable (a price) chosen by a rating agency. We then show that clients hiding their ratings can only be an equilibrium outcome if they are sufficiently uncertain of their quality at the time of hiring a certification intermediary and if the decision to get a rating is not observable. For some distribution functions of clients' qualities, a competitive rating market is a necessary condition for this result to obtain. Competition between rating intermediaries will unambiguously lead to less information being revealed in equilibrium.

Key Words: Certification, Corporate Governance.

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1 Introduction

On January 31st, 2003, Standard & Poor's assigned its first Corporate Governance rating (CGS) to a US company, the Federal Mortgage Association, Fannie Mae. Those ratings are described by S&P as *independent assessments* resulting from *an interactive process that does not follow a "check the box" approach* and likely, contain information not widely available otherwise. Importantly, the rating is made public free of extra charge at the company's discretion and, according to S&P, a "majority" of firms do not reveal their ratings. S&P also commits not to reveal whether a particular company has even approached them for a rating: "*assessments can be provided to companies on a confidential basis*".

Although this business model is relevant to academics and practitioners involved in the important debate on corporate governance, and more specifically on the issue of whether a "market" solution will produce the right kind of information to investors, S&P's contractual offer is also of some interest to contract theorists at large. There are in fact many instances where ratings are provided on terms similar to the ones described in the previous example. Students taking a test to measure their academic abilities or their language skills pay some upfront fee and, once they get their results, have the option to reveal them or not to potential universities.[1] On page 30 of his 1995 book, Oliver Hart argues that "*the owner of an asset has residual control rights over that asset: the right to decide all usages of the asset in any way not inconsistent with a prior contract, custom or law*". For an informational asset like a rating, an important dimension of control over this asset will regard its disclosure. Defining an ownership contract over ratings as giving full disclosure rights to its owner, our research question is to identify the circumstances under which such a simple (ownership) contract can emerge as the optimal possible arrangement.

To answer that question, we take a mechanism design approach and consider successively different market structures of the rating market. We identify the set of renegotiation-proof contracts and show how and when a simple ownership contract emerges as the optimal arrangement. Firms hiding a rating can only emerge as an equilibrium outcome if firms are sufficiently uncertain of their quality at the time of hiring the rating intermediary and if a fraction of firms does not ask for a rating. In fact, for some distribution functions of firms' qualities, a competitive market is a necessary condition for this result to obtain. Importantly, this result also implies that some imprecision in the rating technology *alone* does not justify that rating agencies might offer to their clients the

[1] For instance, "*GMAC® recognizes a responsibility to safeguard the information in its files from unauthorized or inappropriate disclosure. GMAT® scores and personally identifiable examinee information will be released only at your specific request, except as otherwise set forth in the privacy policy contained in the GMAT®.*"

option to hide a rating in equilibrium. Indeed, the option to hide a rating will turn out to be an ineffective insurance device against the possibility of errors in the rating technology. Related results also show that a monopolist in the rating market will actually engage in excessive production of ratings. Doing so increases the monopolist's ability to extract rents from firms as not having a rating is then a more negative signal. Bertrand duopolists, unable to engage in cross-subsidization over firm's types, will offer ratings for a fewer (and better) set of types. Therefore competition between rating intermediaries will lead to less information being revealed in equilibrium, in particular at the lower end of the distribution of firms' quality. This, in the perspective of corporate governance or more generally of the working of certification markets, suggests that competition might not be encouraged in rating markets, if the social value of this information is high.

The existing literature does not suggest that ownership contracts of the type documented above could be optimal. In an important paper, Lizzeri (1999) shows two results. First, a monopolist intermediary will commit to never reveal any rating and will optimally make a simple announcement to the effect that a given firm has hired its services. If market participants expect every firm to hire such an intermediary, the absence of this announcement is an out of equilibrium outcome. He shows that the only possible out of equilibrium beliefs must be that market participants (investors, consumers...) then take that firm to be of the lowest possible quality. Firms are then indifferent between going to the intermediary, and being charged a fee equal to the average value of a firm (or product...) in this market, or being mistaken for the worst kind. The reason why then the monopolist commits itself to conceal the exact value of a given firm is that firms worth less than average would pay less than the average value if some information was revealed. As a second result, Lizzeri shows that competition may lead to full information revelation. In a related set-up but with risk-averse buyers or competitive sellers, Peyrache and Quesada (2004) show that the equilibrium will entail partial disclosure of information. To a large extent, the divergence of those results with ours stems from our emphasis on renegotiation-proof contracts.

The literature on voluntary-disclosure of information is generally not supportive of the view that once the firm has the ownership of its rating, it may actually choose to conceal it. Grossman and Hart (1980) and Grossman (1981) show that whenever a seller is perfectly informed and can certify at no cost the quality of the good sold, the only equilibria will involve unravelling and will result in all the information being disclosed. Milgrom (1981) contains an example with a similar result where in addition, revealing information is costly. The underlying force behind such a result is that the market interprets withheld information as information that is unfavorable about the firm's value. The unravelling argument has been the focus of many subsequent articles such as

Milgrom and Roberts (1986), Farrell (1986) or Okuno-Fujiwara, Postlewaite and Suzumura (1990). The accounting literature built extensively on this idea and analyzed under what circumstances a firm (or a manager) will choose to withhold information. Verrecchia (1983) shows how the existence of disclosure-related costs provides an explanation for why a manager exercises discretion in disclosing information. Dye (1985) also amends the "disclosure" principle by assuming that the firm is informed only with some exogenous probability and the market (the investors) does not observe the nature of the information a firm (a manager) possesses. The rational behind these contributions is the idea that if uninformed agents doubt of the motives of informed ones, there might be less of an adverse selection effect when information is withheld, leading therefore to partial disclosure in equilibrium. Shavell (1994) offers a related result in a set up where firms can first choose to acquire information regarding their quality and, in a second step, remain silent or disclose it. Contrary to Verrechia (1983) and Dye (1985) but following Shavell (1994), we consider endogenous information acquisition by the firm (the manager). However, we differ from this last contribution in that, instead of being an exogenous constant, this cost of getting the information (the price of a rating) is also a strategic variable determined by a third party (the rating agency) which, ultimately, depends on the market structure.

Our analysis is a mechanism design exercise that involves both screening and signalling elements and as such is related to Rochet and Stole (2002) to the extent that firms' reservation utilities are endogenous; and it also shares some features of the general set-up analysed by Segal and Whinston (1999, 2003) as ours is a case of contracting with (informational) externalities. Our final section deals with competition in contracts in such a context.

The paper is organized as follows. A brief second section introduces the model. In Section 3 we take the behavior of rating intermediaries as given and we look for the conditions under which the option of concealing the rating may be of some value. Knowing those conditions, in Section 4 we investigate whether concealing the rating can be part of an equilibrium. Section 5 analyses in what circumstances a simple ownership contract can implement the optimal contract. Section 6 discusses the main results and concludes. Proofs that are not in the main text are provided in the Appendix.

2 The model

We first describe the base model and then provide an important preliminary result implied by the inability of the parties *not* to renegotiate the initial contract. To fix ideas, we sketch our model in a set-up capturing the S&P example of the introduction.

2.1 The Set-up

Consider a firm, a certification intermediary (two in the competitive case) and a number ($n \geq 2$) of competitive, risk neutral, investors.[2] A firm's governance comes in various qualities which result in an incremental value of v for its investors.[3] We assume that v is distributed on some bounded support, according to a cdf $F_v(\cdot)$ with density $f_v(\cdot)$. In most of the paper we will assume that the distribution of v is uniform on $[0, 1]$.[4] Initially, investors regard any firm as average, and take its value to be $\frac{1}{2}$. Suppose that, at the time of hiring the intermediary, the firm gets a signal $\mu \in M$ about v. We assume that $\mu \sim U[v - \theta, v + \theta]$ for some $\theta \in [0, \frac{1}{2})$, so $M = [-\theta, 1 + \theta]$. This implies in particular that if we consider two different signals $\mu_1 < \mu_2$, the distribution of v conditional on μ_2 first order stochastically dominates the distribution conditional on μ_1.

The intermediary possesses a certification technology which provides an informative signal, σ, about the true value of v at a cost $c > 0$. We suppose that $\sigma \sim U[v - \omega, v + \omega]$.[5] Except in one occasion, we will concentrate on the case where the signal is perfectly revealing, i.e. $\omega = 0$. We consider this signal to be hard information. That is, it can either be concealed or truthfully revealed to the market. This assumption of hard information is a modeling device to concentrate on intermediaries with strong reputational concerns that optimally choose not to manipulate their information.[6] Finally, we assume that this signal, referred to from now on as a rating, is observed both by the certification intermediary and the firm.[7]

[2] The risk neutrality assumption implies that there is no social value of information. One may then wonder why it is of any interest to study what information will be revealed in equilibrium in such a setting. It is easy to incorporate in our model a loss function that would justify the need to provide accurate information to investors. We take the short cut that ultimately the more information is revealed by the rating intermediary, the better it is for society.

[3] A number of recent studies supports the simple proposition that greater shareholder governance translates into greater shareholder value. See for example Gompers, Ishii and Metrick (2003) or Arcot and Bruno (2006).

[4] None of the results contained in Section 3 depend on the uniform distribution assumption. Some of those in Section 4 do, but we discuss there and in Section 6 the extent to which this may be so.

[5] Here we take as given that the space of ratings coincides with the space of firm's types. Dziuda and Vogel (2006) offers an analysis of the design of the optimal rating grid.

[6] Our setting is thus different from the literature that studies the reputational incentives of financial information providers in the context of cheap talk games, as in Mariano (2004) or Sette (2004). There, the rating agency can fully manipulate its report. One might conjecture that when information is soft, the possibility to hide a bad rating might reduce firms' incentives to bribe rating agencies to manipulate such soft information. If that was correct, this could be another rationale for this contracting feature, complementary to ours.

[7] In practice, obtaining a rating is largely the outcome of a long exchange of information between the auditor and the auditee. Theoretically, one could design ex post incentive compatible mechanisms that would induce revelation of the rating by the intermediary. Intuitively, the surplus created by the production of a rating is maximized when all the information is revealed between the intermediary and the firm. If the rating intermediary has all the bargaining power, it can extract all this surplus when this information is revealed to the firm. Moreover, once v is revealed, μ

The intermediary and the firm can enter into a transaction by which in exchange of a fee, the intermediary performs an audit that results in a rating. Contracts between the intermediary and the firm are signed after the latter has received its informative signal on v. We do not put any restriction on the class of contracts that can be offered to the firm. The intermediary then offers a contract $\{p_0, p(\sigma), d(\sigma)\}_\mu$ that can stipulate both an up-front fee p_0, a fee contingent on what rating is ultimately obtained $p(\sigma)$ and a disclosure rule $d(\sigma) \in [0, 1]$ where $d(\sigma)$ is the probability that a rating of σ is publicly revealed. This contract offer could in principle be contingent (in an incentive compatible way) on any report of the firm on the signal it received, μ. The contract also specifies whether the hiring decision is to be made public or kept secret. Based on the information they obtain, investors update their beliefs regarding the quality of the corporate governance and, given that they are risk neutral, pay the expected quality.
As usual, the information structure at the time of renegotiation is crucial in determining the set of renegotiation-proof contracts. We assume, in what follows, that renegotiation can occur once the intermediary has generated a rating. That is, the intermediary knows, at the time of renegotiation, how good the quality of the corporate governance of the firm is. By assuming that the intermediary makes a take-it-or-leave-it offer to the firm, we endow the former with full bargaining power. However, the firm still has the choice to refuse the contract offered during the renegotiation phase in which case the original contract prevails.
The timing of the game can be summarized as follows.

1. Intermediaries post (simultaneously in the competitive case) contracts $\{p_0, p(\sigma), d(\sigma)\}_\mu$ and choose whether contracting is going to remain secret or public.

2. Nature chooses v and the firm gets an informative signal μ on its true value v.

3. The firm and the market participants observe the offer of the intermediaries.

4. The firm decides whether to approach one intermediary to obtain a rating.

5. If the firm pays p_0, to an intermediary, this intermediary and the firm learn the intermediary's rating and the game moves to stage 6. Otherwise the game moves to stage 8.

6. The intermediary can make an offer to renegotiate to a new contract $\{p'(\sigma), d'(\sigma)\}$.

7. The firm accepts or refuses the renegotiation offer and the intermediary executes the corresponding contract. The rating is then revealed or not to the market according to the contract in force.

has no residual informative value.

8. Market participants update their beliefs about v using Bayes' rule whenever possible. Firm's value is then equal to the updated expected value of v, given all the information provided.

9. Market participants bid for the firm and pay its expected value.

We first aim at identifying the set of renegotiation-proof contracts in this set-up. Ultimately, our goal is to understand both the circumstances under which a simple ownership contract emerges as the optimal arrangement and the reasons why a non-negligible proportion of firms, once given the option, prefers to hide the rating. We answer these two questions sequentially. In the following section, we start by identifying some necessary conditions for the option to be valuable. Before doing so, let us provide an important preliminary result.

2.2 Renegotiation-proofness

We first derive the willingness to pay of a firm for any contract offered by one intermediary when $\omega = 0$, that is when $\sigma = v$.[8] Denote by ϕ the information that market participants have when no rating is revealed. The absence of a rating does not imply that market participants keep their prior beliefs, since they may learn something from the fact that no rating is revealed. In general, denote by $E_v[v|\tilde{\phi}]$ the updated value that market participants place on a firm without a rating. Given an offer $\{p_0, p(v), d(v)\}_\mu$, a firm of type μ will hire an intermediary if and only if:

$$E_v \left[(d(v)v + (1 - d(v))E_v[v|\phi_1] - p(v) - p_0) \,|\mu \right] \geq E_v[v|\phi_0] \tag{1}$$

where ϕ_1 is the market participants' information if they observe that the firm has hired an intermediary and $d(v) = 0$ while ϕ_0 is their information when they observe that a firm did not hire the intermediary. In the case of secret contracting, $\phi_1 = \phi_0$ as market participants cannot tell whether the absence of a rating (the only thing they see) is due to the firm not hiring the intermediary or the intermediary hiding the rating of a rated firm.

We have taken the view that the firm and the intermediary(ies) cannot commit not to renegotiate over the first contract (signed under asymmetric information). This boils down to assuming that both parties are able to fully realize gains from trade available to them right before disclosing any information. This implies that in order to characterize the optimal contract, we can restrict attention to initial offers that are renegotiation-proof. That is, whatever the contract signed initially, we require that there is no contract at stage 6 that would be preferred by both the intermediary and the firm. In particular, this implies that the decision to reveal or not the rating must be *ex*

[8] The case $\omega > 0$ is covered in the Appendix.

post efficient. For any firm with true value v that has hired the intermediary, the requirement that a renegotiation-proof contract will have to satisfy is the following:

Lemma 1 *When $\omega = 0$, a contract is renegotiation-proof if and only if it has the property that:*

$$d(v) = \begin{cases} 1 & \text{if} \quad v \geq E_v[v|\phi_1] \\ 0 & \text{if} \quad v < E_v[v|\phi_1]. \end{cases} \tag{2}$$

Proof. When $\omega = 0$, renegotiation happens under complete information and so must result in an ex post efficient disclosure policy. For that to be so, we need disclosure to happen if and only if: $v \geq E_v[v|\phi_1]$. Indeed, the firm is considered to be worth $E_v[v|\phi_1]$ when the rating is not revealed to the market, while v if it is revealed. ■

The goal is to characterize the optimal contract that satisfies renegotiation-proofness and then use our results to analyze how this optimal contract can be implemented through an appropriate allocation of ownership over the rating. The Appendix provides an easy generalization to the case of imperfect rating, where $d(\sigma) = 1$ if and only if $E_v[v|\sigma] \geq E_v[v|\phi_1]$. A direct implication of Lemma 1 and first order stochastic dominance is that the willingness to pay for a rating increases in the signal μ in any renegotiation-proof equilibrium.

3 The Value of the No-Disclosure Option

We now identify a few necessary conditions for the no-disclosure option to have value. To do so, we first define a particular class of equilibria:

Definition 1 *A threshold equilibrium is defined as an equilibrium in which all types μ above a certain threshold $\hat{\mu}$ hire one intermediary, while all those below do not.*

We will show later that an implication of Lemma 1 - and the fact that firms with more positive signals have a higher willingness to get a rating - is that in both market structures we will consider, the only possible equilibria have to be threshold equilibria. In this section, we assume that this is so and derive a few implications of the threshold structure. But before concentrating on this class of equilibria, we can narrow down our search for the circumstances under which hiding a rating may be part of an equilibrium by noticing that:

Proposition 1 *If intermediary i reveals that a given firm has hired its services (public contracting), then whatever the outcome of the rating, it is revealed: $\forall\, i$, $\forall\, \sigma$, $d_i(\sigma) = 1$.*

Proof. Start with the case $\omega = 0$, so that $\sigma = v$. Denote by $g(\cdot|\mu)$ the conditional density of the firm's value, given a signal μ. Call $M_i \subseteq M$ the set of types who hire intermediary i and $V_i = \{v \in [0,1] : \exists \mu \in M_i : g(v|\mu) > 0\}$, the set of ratings that intermediary i can encounter in equilibrium. Suppose that there exists some subset $\bar{V}_i \subset V_i$ of ratings that are not revealed (i.e., $d_i(v) = 0$ for $v \in \bar{V}_i$). Under public contracting, $E_v[v|\phi_1] = E_v[v|\mu \in M_i \text{ and } v \in \bar{V}_i]$. Now, define $\bar{v}_i := \sup \bar{V}_i$. For a type $\bar{v}_i$ it is optimal to renegotiate the disclosure policy and set $d(\bar{v}_i) = 1$, since, by definition, $\bar{v}_i > E_v[v|\phi_1]$. This argument unravels and all ratings are revealed.

Now when $\omega > 0$, we can similarly establish that a firm that would obtain the highest attainable rating $\overline{\sigma}_i$ supposed to be hidden would actually prefer to see this rating revealed instead. The same unravelling argument applies. ■

The proof of Proposition 1 does not assume any particular form for the contract offered by intermediary i, nor does it depend on the exact market structure. It only uses the simple intuition that once a firm is seen to have hired an intermediary but that no rating is revealed, it must be bad news. An important implication is then that there cannot be an equilibrium where firms hire intermediaries but do not reveal ratings unless those hiring contracts are kept secret. We explore this possibility in the next section, successively considering settings where the firm is perfectly and imperfectly informed of its own type. Before, note that a setting where all firms ask for a rating can easily be reinterpreted as a particular form of public contracting, since in equilibrium investors know that the firm has hired an intermediary. Closely related is then the following result.

Corollary 1 *In an equilibrium where all types of firms hire an intermediary, then all ratings are disclosed.*

For completeness, such an equilibrium could only be supported by the out-of-equilibrium beliefs that a firm without a rating must be of the worst possible type (i.e. $\mu = -\theta$).

In the remainder of this section, we specialize our analysis to the case of a fully informed firm: $\theta = 0$. We establish that in any threshold equilibrium the value of the no disclosure option in this case is zero. To do so, we need to focus on the case of secret contracting as we have already shown that if the hiring decision is public, the option has no value.

As the firm knows exactly its value v, the willingness to pay of a type v for a contract with a disclosure rule $d(\sigma)$ is:

$$E_\sigma\{d(\sigma)E_v[v|\sigma] + (1 - d(\sigma))E_v[v|\phi]|v\} - E_v[v|\phi] = E_\sigma\{d(\sigma)(E_v[v|\sigma] - E_v[v|\phi])|v\}$$

where $\phi_0 = \phi_1 = \phi$ as the hiring decision is secret. Moreover, we know from Lemma 1 how $d(\sigma)$ must be designed in any renegotiation-proof equilibrium. As firms are informed about v they know exactly their willingness to pay. In particular, when $\omega = 0$, a firm for which $v < E_v[v|\phi]$ in a given putative equilibrium is willing to pay 0 to hire the intermediary. From this, it follows that

Proposition 2 *For all $\omega \geq 0$, in any threshold equilibrium, the option to hide the rating has no value if the firm is perfectly informed about its type ($\theta = 0$).*

Proof. Suppose first that $\omega = 0$, so $\sigma = v$. As firms are fully informed of their type, they know at the time of hiring an intermediary whether $d(v)$ will be 0 or 1. If they know that their rating will not be disclosed, their willingness to pay is 0. Therefore, all firms with $v < E_v[v|\phi]$ in any putative equilibrium are pooled with the same willingness to pay of 0. Note that no intermediary can make positive profit with these types. If they produce a rating, their rating will be hidden. The same outcome would be achieved by not producing any rating for those types. That would save c, more than their willingness to pay of 0. So, only firms with types $v \geq E_v[v|\phi]$ and for which $d(v) = 1$ hire the intermediary and no rating is ever hidden.

The proof when $\omega > 0$ is slightly more involved and is relegated to the Appendix. ■

Whenever the firm is fully informed of its type at the time of hiring the intermediary, there is no difference between public and secret contracting. In both cases, a firm without a rating can only be a firm that has not hired the intermediary.

The first part of Proposition 2 (when $\omega = 0$) amounts to saying that in the absence of any uncertainty, an option contract is of no value. Either the firm, who knows v, wants this value to be certified, and if so will want the rating to be revealed, or it does not apply for a rating. What is more surprising is that in a threshold equilibrium not any sort of uncertainty justifies that the option to hide a rating should be given to firms. In particular, as long as $\theta = 0$, a noisy rating technology does not create a need for this option. This is counterintuitive in that one might think that firms could be worried that the rating technology wrongly assigns them too low a rating (firms should not be worried about positive errors and will not wish to hide exceedingly good ratings). In particular, a firm could fear that too low a rating might result in the market undervaluing it. But in a threshold equilibrium where firms are perfectly informed of their true value at the time of hiring an intermediary, the market can *never* put a value on a firm lower than the threshold value. To see this, notice that the disclosure of a rating actually reveals two pieces of news: of course, it reveals the value of σ, but it also tells the market that this particular firm did apply for a rating. In a threshold equilibrium, this means that $v \geq \hat{v}$ where $\hat{v}$ denotes a candidate equilibrium threshold. It could well be that the rating σ is below $\hat{v}$, but still market participants will not put a value on a rated firm below $\hat{v}$. Alternatively, one could say that any rating that suggests that a firm is worth less than $\hat{v}$ is disregarded, on the account that the rating technology is noisy (while the firm's information is perfect): such a rating must be a "mistake". Now if the option to hide a rating is useless to a $\hat{v}$ type of firm, any firm with a higher true value will also have no use of this option. Indeed, the value put by the market on a firm that hides a rating is independent of its true value while the expected value obtained by revealing a rating is weakly increasing in its true value (as better firms are more likely to get better ratings). Notice that firms strictly above the threshold firm might be disappointed by the rating they obtain. Still, revealing such a disappointing rating will result in being valued strictly more than $\hat{v}$. Not revealing any rating is telling the market that the firm is worth at most $\hat{v}$.

What this discussion also highlights is that the threshold structure is key for the result to obtain. The next section will first establish that only threshold equilibria can exist.

4 Hiding the Rating as an Equilibrium Outcome

Whether or not offering the no-disclosure option emerges as an equilibrium outcome depends on the set of types who hire an intermediary and therefore on the prices offered by those. Therefore, it is important to distinguish between different market structures. We consider two possible cases: a monopoly intermediary and two Bertrand competitors. We now concentrate on $\omega = 0$.

The possibility of a noisy signal may induce firms to make two sorts of mistakes when deciding to hire a rating intermediary. First, they may decide on the basis of their signal μ to hire an intermediary, but upon learning their exact value v, they wished they had not applied for a rating. It is for this set of excessively "optimistic" firms that the option to hide a rating may be attractive. This will be all the more so, if a second type of mistake takes place in this market: firms who on the basis of their signal decide not to hire a rating intermediary, while if they knew their true value they would have been better off getting a rating and revealing it. The larger the number of those excessively "pessimistic" firms, the higher the value the market puts on a firm without a rating. The rest of the paper will show that the interplay of these two forces will be key in determining the value of the option.

As the objective of this section is to identify when ratings can be profitably hidden, we focus now on the case of secret contracting, with firms imperfectly informed of their true quality at the time of hiring an intermediary ($\theta > 0$). Moreover, to simplify notation, let us call $v_\phi \equiv E_v[v|\phi]$ the value that the market gives to a firm when no rating is revealed.

4.1 Monopoly Intermediary

Let us first show that we can place ourself in the framework of Section 3, namely that the equilibrium is of threshold type.

Lemma 2 *For any distribution of types and signals, the monopolistic intermediary can extract the entire surplus from all types of firms. Any equilibrium belongs to the threshold class.*

Proof. Suppose that the monopolist could charge to each type μ a price equal to its willingness to pay (the case of symmetric information). Then, since the willingness to pay is increasing in μ, if the monopolist finds it profitable to take on type $\hat{\mu}$, it will also find it profitable to take all types above $\hat{\mu}$. Thus, in this case the equilibrium would certainly have a threshold structure.

In our case, the monopolist is not informed about μ, only the firm is. However, we show that the monopolist can offer a simple incentive compatible mechanism that extracts all the firm's surplus and is ex post renegotiation-proof. Take a simple menu of contracts contingent on the firm's announcement about its type $\widetilde{\mu}$, and possibly the rating v: $\{p_0(\widetilde{\mu}), d(v, \widetilde{\mu}), p(v, \widetilde{\mu})\}$ where $p(v, \widetilde{\mu})$ is an additional charge to be paid by the firm if a rating v is revealed (there is no need to consider more general contracts that could also levy an additional charge when no rating is revealed, possibly contingent on that rating). To be incentive compatible, we need:

$$\forall(\tilde{\mu}, \mu),\ p_0(\mu) + E_v\left[\left(d(v,\mu)\left(-p(v,\mu)+v\right) + \left(1-d(v,\mu)\right)v_\phi\right)|\mu\right] \geq$$

$$p_0(\tilde{\mu}) + E_v\left[\left(d(v,\tilde{\mu})\left(-p(v,\tilde{\mu})+v\right) + \left(1-d(v,\tilde{\mu})\right)v_\phi\right)|\mu\right]$$

Consider the following pricing structure: Denote by $\hat{\mu}$ the lowest value of the signal μ for which a firm applies for a rating. Take $p_0 = \epsilon$, with $\epsilon > 0$, very small, and $d(v) = 0$, $p(v) = 0$ if $v \leq v_\phi$ while $d(v) = 1$ and $p(v) = v - v_\phi - \frac{\epsilon}{\Pr(v > v_\phi|\hat{\mu})}$ otherwise. This contract is renegotiation-proof. Since the offer is independent of the announcement, it is trivially incentive compatible. With this offer, the utility of a type μ who accepts the contract is

$$v_\phi + \epsilon\left(\frac{\Pr(v > v_\phi|\mu)}{\Pr(v > v_\phi|\hat{\mu})} - 1\right).$$

The rent of any type μ who hires the intermediary can be made arbitrarily close to 0 by approaching ϵ to 0. Thus, the monopolist extracts all the rents. Moreover, first order stochastic dominance implies that $\Pr(v > v_\phi|\mu)$ increases with μ, so $\frac{\Pr(v>v_\phi|\mu)}{\Pr(v>v_\phi|\hat{\mu})} > 1$ if and only if $\mu > \hat{\mu}$. That is types below $\hat{\mu}$ stay out of the market and types above accept the offer. The equilibrium must be a threshold equilibrium.[9] ■

The willingness to pay of a firm of type μ, according to Lemma 1 when the threshold is $\hat{\mu}$ is

$$\begin{aligned} p(\mu, \hat{\mu}) &= \Pr(v \geq v_\phi(\hat{\mu})|\mu) E_v[v|v \geq v_\phi(\hat{\mu}), \mu] + \Pr(v < v_\phi) v_\phi(\hat{\mu}) - v_\phi(\hat{\mu}), \\ &= \Pr(v \geq v_\phi(\hat{\mu})|\mu)\left(E_v[v|v \geq v_\phi(\hat{\mu}), \mu] - v_\phi(\hat{\mu})\right). \end{aligned}$$

Since the intermediary can extract the entire surplus, the profit is

$$\Pi(\hat{\mu}^*) = \max_{\hat{\mu}} \Pr(\mu \geq \hat{\mu})\left(E_\mu\left[p(\mu, \hat{\mu})|\mu \geq \hat{\mu}\right] - c\right).$$

[9] Whenever $\omega > 0$, replace v by $E_v[v|\sigma]$ in the analysis above and a similar proof goes through.

We now proceed to show that for the case where firms' values v and signals μ are uniformly distributed, there is no circumstances under which the option to hide a rating is of some value when the intermediary is a monopolist.

Proposition 3 *For all $\theta \geq 0$, the unique renegotiation-proof equilibrium profit is $1/2 - c$ and, in all renegotiation-proof equilibria, all types of firms ask for a rating. The monopolist can reach this profit level with a price structure so that $p_0 + p(v) = v$ and $d(v) = 1$ $\forall v$. Such equilibrium is supported by the out of equilibrium beliefs that a firm without rating is worth zero.*

Proof. See Appendix. ■

Since the intermediary takes on all types, it is irrelevant whether contracts are secret or public. It is worth emphasizing that there is some cross-subsidization between types. Indeed, when a firm gets a signal μ, the monopolist's expected profit on that type is $E_v[v|\mu] - c$. When this is negative, the intermediary expects to lose money on those types. It still pays for the intermediary to take them on, as not doing so will admittedly save $c - E_v[v|\mu] > 0$ but will reduce the willingness to pay for its services of all firms with higher values as it would increase v_ϕ. The situation is in fact quite similar to a discriminating monopolist who starts contracting with the consumers with the highest valuation and who is surely willing to do so with all consumers with a valuation exceeding the cost of delivering the good or service (as again, the monopolist extracts here all the surplus). But there is an extra effect in our set-up: as the monopolist walks down the demand curve, the demand curve is shifted upwards. This effect explains why the monopolist is willing to contract with types with an ex ante valuation lower than c.

Importantly, contracting with all types has the impact of destroying the value of the no disclosure option, as explained in Corollary 1. It is correct that the monopolist's profit per type is higher when there is a possibility that some ratings are not revealed, compared to a situation where all ratings are revealed, in a *given* threshold equilibrium. If there was some "rating-rationing" (some low types being not rated), the value of a firm without a rating would be less than the same value if all rated companies had their ratings disclosed. Indeed if some ratings were not revealed, this would lower v_ϕ as only types with a true value *below* v_ϕ can prefer no revelation. Lowering the value of not having a rating has the advantage ex ante of increasing firms' willingness to pay. But this can only happen if some types are not rated as otherwise the usual unravelling argument implies that $d(v) = 1$ for all v. What our analysis shows is that the monopolist can use one of two possible ways to reduce the value the market puts on a firm without a rating, v_ϕ. One is indeed not to reveal what a rated firm is worth if this is low enough, provided that not all types of firms obtain a rating. And one is to reduce the threshold above which firms get a rating. Those two ways become exclusive when this threshold is brought down to the lowest type. It happens to be the case that for the uniform distribution, the former effect is less strong than the latter and that indeed the monopolist prefers taking on all types. This is specific to this distribution but this arbitrage between making a loss on low types, increasing the willingness to pay of all types above and reducing the attractiveness of the no disclosure option remains general.

This result contrasts with Lizzeri (1999) where all companies ask for a rating and the intermediary only publicly reveals whether it has been hired by the firm.[10] The difference comes from our emphasis on renegotiation-proofness. Indeed, in equilibrium, a firm who hires the intermediary is worth $1/2$ if no information is revealed. Therefore, a firm of type $v > 1/2$ that has hired the intermediary is willing to pay an extra fee (at most $v - 1/2$) for the intermediary to reveal its type instead of remaining valued as an average firm in the absence of such an announcement. If the firm and the intermediary can renegotiate the initial contract, they will find a way to certify this information to the market as doing so creates some additional surplus.

This result is also related to, but again quite different from, Shavell (1994). Shavell considers a set-up where first firms, upon observing the realisation of a cost of becoming informed, can decide to learn their true value. If they do so, this gives them the option to reveal this information. For low cost realisations, it pays for them to acquire this option. There is no unravelling in that, upon observing no information, the market does not know whether this is due to a high cost of information acquisition or to a low value realisation. The obvious difference with our analysis is that in our model, first firms have some information about their types (albeit possibly noisy) and second the cost of revealing something to the market (a rating) is endogenous, chosen by the rating agency. It emerges that in equilibrium the rating agency charges different prices for conducting a rating to firms with different (expected) qualities. In particular, this cost of producing a rating increases with firm's quality instead of being constant as assumed in Shavell. Contrary to his paper, we do not examine here the incentives for firms to improve the quality of their initial information. We could allow θ to be somehow endogenous where firms could possibly reduce it at some cost $k(\theta)$. Notice though that as firms are getting no surplus in our set-up, their incentives for doing so would be minimized.

Our result is that all firms get a rating and all ratings are revealed. In this sense, the rating market could not work better from the viewpoint of information revelation, taking as given the distribution

[10]Following our notation, the equilibrium contract in Lizzeri (1999) is $p_0 = 1/2$ and, $d(v) = 0 \ \forall v$. It is noticeable that the monopolist's profit in his analysis is the same as in ours (if, like him, we assume $c = 0$). In other words, this contract is still optimal once contingent mechanisms are allowed.

of firms' quality. Thus, if one is sufficiently confident that market forces will lead to a monopolistic structure of the rating market, then one should see no need for regulatory intervention. This conclusion should however be toned down: firms have no incentives to improve their quality, as their equilibrium payoff is independent of v.

4.2 Competing Intermediaries

We now turn our attention to the opposite case of Bertrand competition: two intermediaries who can produce a perfectly accurate rating at a cost c compete on the rating market. It is enough, for our purpose of exhibiting cases where ratings are hidden in equilibrium, to focus on the case of uniform distributions and $c \leq \frac{1}{4}$.[11] Following the analysis of Section 3, let us solely consider the case of secret contracting.

Unsurprisingly, an equilibrium where intermediaries capture all the surplus as described in Proposition 3 cannot survive in a competitive framework. Indeed, the standard Bertrand-like reasoning applies and one of the intermediaries always has incentives to undercut her competitor to attract all types willing to obtain a rating. This is proved in the following lemma:

Lemma 3 *When two identical intermediaries, $i = 1, 2$, compete in contracts, equilibrium prices are such that intermediaries make zero profit on each type μ asking for a rating. That is, $E[p^i|\mu] = c$ for all μ and for $i = 1, 2$. The equilibrium is a threshold equilibrium.*

Proof. Suppose that intermediaries have proposed 2 different contracts. The firm decides whether to ask for a rating and if so from which intermediary. Suppose that the market observes no rating. With secret contracting, the absence of a rating may be due to 3 reasons: (a) the firm didn't ask for a rating, (b) the firm was rated by intermediary 1 but did not reveal it or (c) the firm was rated by intermediary 2 but did not reveal it. The three cases are indistinguishable for the market. Thus, in any case, in the absence of a rating the market is willing to pay v_ϕ, independently of the true firm's value realisation and of which intermediary it hired. If market participants observe a rating v, then they are willing to pay v regardless of the identity of the intermediary that gives the information. Then, from Lemma 1, the information that will be revealed after asking for a rating is again the same whatever the identity of the intermediary and the contract initially offered. Namely, the rating will be disclosed whenever $v > v_\phi$. Therefore, firms get valued in the same way whether they go to intermediary i or j. Given this homogeneity in the service provided by intermediaries, intermediary i cannot make a strictly positive profit on any subset of types, without intermediary j having an incentive to undercut it. Therefore the only equilibrium involves $E[p^i|\mu] = c$, $\forall \mu, i$.

Being aware of the pricing strategy that emerges in equilibrium, we then get that any type μ expects to pay c in equilibrium. Since the willingness to pay is increasing in μ, if type $\hat{\mu}$ accepts one offer, so do all types

[11] Considering cases where $\frac{1}{4} < c < \frac{1}{2}$ would add cumbersome computations without changing our main message.

$\mu \geq \hat{\mu}$ and the equilibrium belongs to the threshold class. ■

As we have established that any equilibrium must be a threshold equilibrium, we concentrate on cases in which $\theta > 0$. For the no-disclosure option to be truly valuable, we need to exhibit an equilibrium where there exists some $\mu \geq \hat{\mu}$ so that the set of attainable values of v contains some range for which $v < v_\phi(\hat{\mu})$. This last condition is itself endogenous. Given the threshold structure we have that

$$v_\phi(\hat{\mu}) = E_v[v|(\mu, v) \in \phi(\hat{\mu})], \tag{3}$$

where $\phi(\hat{\mu}) = \{(\mu, v) \in [-\theta, 1+\theta] \times [0,1] : \mu < \hat{\mu} \text{ or } (\mu \geq \hat{\mu} \text{ and } v < v_\phi(\hat{\mu}))\}$ is the set that represents the information held by the market when no information is revealed.

Consider a putative symmetric equilibrium where both intermediaries offer the same contract. Suppose that firms with $\mu \geq \hat{\mu}$ ask for certification while others do not. A firm that asks and obtains a rating v can either reveal it and get v, or withhold it and get v_ϕ, where v_ϕ is the type which ex post is just indifferent between revealing or not. We are now equipped to provide the following result.

Proposition 4 *The option of hiding the rating has a strictly positive value if and only if $\theta > c$. That is, when $\theta \leq c$, the only equilibrium entails full revelation for all types who ask for a rating to any intermediary. On the contrary, when $\theta > c$, some ratings are hidden in equilibrium. The equilibrium is such that there exists $\hat{\mu}$ and $v_\phi(\hat{\mu})$ such that*
(i) a firm hires any one of the intermediaries if and only if $\mu \geq \hat{\mu}$ where $\hat{\mu}$ is such that

$$\Pr(v \geq v_\phi(\hat{\mu})|\hat{\mu}) \left(E_v[v|v \geq v_\phi(\hat{\mu}),\, \hat{\mu}] - v_\phi(\hat{\mu})\right) = c. \tag{4}$$

(ii) a rating is disclosed if and only if $v \geq v_\phi(\hat{\mu})$ where $v_\phi(\hat{\mu})$ is implicitly given by (3).

Proof. See Appendix. ■

Some intuition for the role of $\theta - c$ can be gathered from looking at one of the equilibrium possibilities. Suppose the equilibrium could be such that $\hat{\mu}$ falls in an intermediate range of the signal's distribution (precisely, between θ and $1-\theta$). Then, the lowest rating that can be attained in equilibrium would be $\hat{\mu} - \theta$. If this is larger than v_ϕ, there will be no point in hiding the lowest possible rating and therefore all ratings will be revealed. Knowing that, the threshold type's expected utility of hiring one of the two intermediaries is just its expected rating, $\hat{\mu}$, net of the fee charged by an intermediary, c. This threshold type must be indifferent between applying for a rating and staying out of the rating market, i.e. $\hat{\mu} - c = v_\phi$. Therefore, no rating will be hidden if $\hat{\mu} - \theta \geq v_\phi = \hat{\mu} - c$ or $\theta \leq c$.

This illustrates that if the quality of the firms' information is relatively high (θ small) in comparison to the cost of producing an accurate rating (c high), the option of hiding the rating has no value: in equilibrium all ratings are revealed. However, this equilibrium ceases to exist when the firm is poorly informed about its type (θ large). In such a case, a firm is no longer guaranteed that the lowest possible rating it can get is necessarily higher than the market participants' expectation in the absence of a rating. An intermediary now wants to deviate to offering secret contracts that include the no-disclosure option. By choosing this contract, the firm is insured against bad ratings. Upon receiving μ, the larger is θ, i.e. the more uncertain the firm is about its true value at the time of hiring the intermediary, the worse the rating can be. Firms are then keener on having this ex post insurance, to prevent them from greater disappointments. Interestingly, there is a second effect that reinforces the attractiveness of the no disclosure option when θ is large. In a threshold equilibrium, more noise in the firm's signal also implies that, holding constant the true distribution of types, the average true quality of firms with signals lower than any threshold is higher than if firms' signals were more precise (v_ϕ increases with θ). The market knows that and puts a higher value on a firm that does not have a rating when firms' signals are more noisy. The insurance device provided by the no disclosure option is more likely to be exercised, and it offers a higher return when exercised.

The unravelling result (all ratings revealed) breaks down when we introduce secret contracting and firms have noisy information about their types. Note that this happens when the rating industry is more valuable: the cost of producing a rating is low and it considerably improves on the information otherwise available to firms (θ is large). In that respect, it is interesting to remark that most S&P's clients are firms located in countries where corporate governance quality is most uncertain (in particular, outside the US and the UK). One can hypothesize that in countries like the US or the UK, θ is small so that there is no point in offering such contracts there.

To conclude this section, note that an important difference between a monopolistic market structure for the rating market and a competitive one is that for the same parameter values, competition between intermediaries reduces the amount of information that is revealed to market participants in equilibrium. Fewer firms get a rating, and among those some may withhold their ratings.

5 Allocation of Ownership

In the previous section we have determined the shape of the equilibrium contract under different market structures. Observe that, in both settings, either the price paid to obtain a rating (in the monopoly setting) or the disclosure rule (in the competing setting) were contingent on the realization of v. In practice, one might be concerned about the costs of writing such contracts or about their enforcement. It is therefore a question of interest to ask whether incomplete contracts that do not specify contingent mechanisms can implement the same equilibria that we obtained in the previous section.

Of particular interest, both in the literature and in practice as exemplified by the S&P contract, are simple ownership contracts. Following Hart (1995), the party endowed with the ownership over the rating is given the right to decide all usages of this information in any way not inconsistent with a prior contract. This section shows that in our set-up some form of ownership can always implement the optimal contract. It also shows that in case the contracting environment is *limited* to non contingent contracts, the unique equilibrium of such a modified contractual game in the competitive case is to offer ownership contracts that replicate the outcome obtained with fully contingent contracts.

Before pursuing, let us define two ownership contracts of the following kind. A contract is said to establish *full ownership* over the rating if the contractual arrangement is given by (p_0, O), where O is a discrete variable, $O \in \{I, F\}$ that identifies the party who is given ownership (I for intermediary, F for firm). Such contracts assign property rights over the rating in exchange of a price p_0. In contrast, a contract establishes *partial ownership* for the intermediary over the rating if the contractual arrangement is given by (p_0, I, d) In this case, the intermediary has committed to disclose the information with probability d. However, she still has partial ownership in the sense that she can renegotiate this disclosure policy ex post.[12]

5.1 The Monopolist Case

Consider the possibility to offer a contract where the monopolist keeps the *full* ownership of the rating: the monopolist has the right to hide or to reveal the rating. Of course, this does not exclude some ex post renegotiation over disclosure.

Proposition 5 *The optimal contract under a monopoly intermediary is implementable with a simple ownership contract,* $(p_0, O) = (0, I)$*, where the intermediary offers a fixed price* $p_0 = 0$ *ex ante*

[12] We do not need to specify a partial ownership contract for the firm since, having the right to decide how to use the rating, there cannot be any renegotiation.

and retains ownership over the rating. The firm and the intermediary renegotiate over its revelation with the monopolist extracting all the surplus at this stage.

Proof. The monopolist, having the right to reveal or not the rating, can extract $v - v_\phi$ by making a take-it-or-leave offer to the firm when $v > v_\phi$ for revealing the rating. If the monopolist offers $p_0 = 0$, all types ask for a rating and the intermediary gets all the surplus at the renegotiation stage. This results in the monopolist getting an expected profit of $1/2 - c$ as before. ■

By keeping ownership over the rating, the intermediary is endowed with the right to resell it to the firm once its true value is obtained. As we have seen, the fact that μ is private information plays no role, and the equilibrium in this situation remains the one described in Proposition 3. Note that the flexibility in the disclosure rule and pricing strategy that was lost by contractual incompleteness is recovered at the renegotiation stage.

There is a second implication of the last result: in a world where contingent contracts are impossible, giving the monopolist the ownership of the rating is also *uniquely* optimal.

It is straightforward that the monopolist cannot implement the profit maximizing complete contract by transferring ownership to the firm (with an offer (p_0, F)), since in this case, the only way to attract all types of firm is by setting $p_0 = 0$. This results in all information being revealed ex post, but negative profit to the intermediary. Similarly, a contract that allocates partial ownership to the monopolist cannot maximize its profit. Such a contract reveals with some ex ante, non contingent probability, d, the rating obtained by the firm. With probability $1 - d$ we are back to the renegotiation game as under full intermediary ownership. The monopolist can then as before, credibly threaten the firm to leave it with $\min\{v, v_\phi\}$. With probability d the firm can at least guarantee itself v ex post. Because $\min\{v, v_\phi\} \leq v$, partial ownership can only increase the bargaining position of the firm ex post. Notice that it does not follow immediately that this reduces the monopolist's expected profits as this ex post loss might be recouped by charging a higher price ex ante. But as this price p_0 cannot be contingent on μ, and as we are looking for an incomplete contract that replicates the profit maximizing complete contract where all types hire the monopolist, it must be that p_0 does not exceed the valuation to pay of the lowest type, i.e. 0. Then, it is never optimal for the monopolist to leave ex post rents to the firm and it is best to choose $d = 0$, that is, a full ownership contract.

5.2 The Competitive Case

It is straightforward to check that a simple ownership contract implements the competitive equilibrium obtained under complete contracts: consider the possibility for intermediaries to sale the ownership of the rating to firms for an ex ante price of c, i.e., a full ownership contract of the form $(p_0, O) = (c, F)$. Firms will choose to disclose their rating if and only if $v \geq v_\phi$. Knowing this and the price charged to them, firms above some threshold $\hat{\mu}$ will apply for a rating. This threshold must be the same as in Section 4 because there too firms had to pay an expected amount of c. Whether the option to hide the rating is exercised depends as before on the comparison of $\max\{\hat{\mu} - \theta, 0\}$ and v_ϕ. Can this be an equilibrium?

Suppose one intermediary offers this contract. Because the set of possible contractual offers of the competing intermediary is now reduced (compared to the case where contingent contracts can also be offered), it must remain that the best reply to such an offer is again to charge c and to offer $d(v) = 0$ if and only if $v < v_\phi$. This is easily achieved by a simple ownership contract and therefore those offers form an equilibrium.

In a world where contingent contracts are ruled out, it could be that there are now some additional equilibria. This would happen if there were some best replies available to intermediaries when contingent contracts were feasible that would now be no longer available. A closer look at the exact contractual possibilities is now needed to rule out this possibility. In fact, we establish the following.

Proposition 6 *Under incomplete contracting and Bertrand competition, transferring ownership over ratings to firms is the unique equilibrium when $\theta > c$.*

Proof. Existence has been established in the text. The uniqueness result comes from applying Lemma 3 to the new set-up. Given that firms have the possibility to offer simple ownership contracts that generate zero profits to intermediaries, Lemma 3 is still valid here: intermediaries must make zero profit on each type μ or they would be undercut. When $\theta > c$, only full ownership to firms can ensure to a given type μ an optimal revelation of the rating with no further negotiation. Ex post renegotiation will result in firms paying additional amounts that will vary depending on what rating they obtain. To guarantee that a given intermediary makes zero profit on each possible type, we would need an initial charge that would vary with μ, as different μ would have different expected ex post transfers. This is not possible with non contingent contracts so that the only contract compatible with zero profit for all types must prevent ex post renegotiation.

So at least, one of the intermediaries will transfer ownership. Assume that intermediary i makes an offer (c, F) while intermediary j offers a contract (p_0, I). Then, a firm that goes to intermediary j expects renegotiation to happen if $v > v_\phi$, in which case, the intermediary will ask for a payment of $v - v_\phi$. Thus, if $p_0 > 0$, intermediary j gets no clients, and intermediary i could deviate to a contract $(c + \epsilon, F)$, that will increase profits for ϵ small. If $p_0 = 0$, then the only clients that intermediary j could get are those for which $\Pr(v > v_\phi|\mu)(E_v[v|v > v_\phi, \mu] - v_\phi) < c$, so intermediary j makes loses. Thus, the only equilibrium is such that both intermediaries offer (c, F). ■

When there is competition among intermediaries, transferring ownership gives the flexibility in the disclosure rule that is lost due to contract incompleteness. In the monopoly case, this flexibility was recovered through ex post renegotiation.

6 Discussion

A central result of the extant literature on disclosure of information is that whenever the latter can be certified at no cost then, under very general conditions, there will be full disclosure in equilibrium. Key to this result is the assumption that it is common knowledge that the agent who applies for a rating knows the relevant information. When receivers account for the noise in the firms' initial signals, the unravelling argument sometimes does not apply. We have shown that this might be an equilibrium outcome when intermediaries compete with each other, the information of the firm is initially sufficiently imprecise and the decision of hiring an intermediary is not observable by the market. To conclude, we discuss some extensions of our model.

Alternative distribution functions. A fair question to ask is how our results depend on the specific assumptions made about the different distributions. In the competitive case, the equilibrium price is equal to the marginal cost, c, and, consequently there are always some types who prefer to stay out of the rating market. Under secret contracting and poor information held by firms, some rated firms will then prefer to be pooled with those types who stayed out rather than to reveal their low true values. As apparent in the Appendix, some steps in the proof of Proposition 4 do not depend on the shape of the distribution (in particular Lemma 4). In particular, if v is distributed on some interval $[0, V]$, where $V > 0$, then if $\hat{\mu} \in [\theta, V - \theta]$ and the distribution of v conditionally on μ is symmetric around μ, it remains true that $\theta > c$ is a sufficient condition for the no-disclosure option to be valuable.

In the monopoly setting, things are less clear. Our result that the option has no value relies on the fact that the monopoly intermediary does not want to leave some types out of the rating market. This certainly depends on the shape of the distribution. Under alternative distribution functions, the intermediary may decide to leave some types out and the option of hiding the rating could become valuable. However, for any distribution of values and signals that satisfies the first order stochastic dominance assumption we can show the following result.

Proposition 7 *Suppose that the profit of the monopoly intermediary as a function of the threshold $\hat{\mu}$ above which all types ask for a rating, $\Pi(\hat{\mu})$, is single-peaked. Denote by v_ϕ^M (resp. v_ϕ^B) the value of a firm without a rating when the market structure of the rating industry is monopolistic (resp. Bertrand competition). Then, necessarily $v_\phi^B > v_\phi^M$. The mass of firms that reveal a rating is smaller under Bertrand competition than in the monopolistic case.*

Proof. See Appendix ■

Under Bertrand competition, rating intermediaries must make zero profit on each type μ they contract with. Therefore no cross-subsidies are possible and there is no way a firm with a willingness to pay lower than c will apply for a rating. Consider the lowest type $\hat{\mu}^B$ that obtains a rating in any competitive equilibrium, possibly with an offer that includes the no-disclosure option. A monopolist rating intermediary has an incentive to accept this type as well because if the monopolist simply replicated the competitive contracting offer, the valuation to pay of this threshold type is enough to cover c. In addition, the monopolist would get some additional profit by being able to charge to all firms with a willingness to pay above c a larger price. Thus, the monopolist is in fact willing to go further and take on types on which it makes a second order loss, i.e. some types $\mu' < \hat{\mu}^B$ because this still increases the valuation to pay of all types above, a first order gain. Therefore, it must then be that the lowest type that gets a rating in a monopolistic market is lower than in a competitive market. This also implies that the lowest rating that can be produced is lower in the monopolistic structure. So if some ratings are disclosed under competition, the same ratings (and some others) will also be disclosed by the monopolist, resulting in $v_\phi^B > v_\phi^M$.
Competition reduces information production. To some extent, what our results exhibit is a tendency for a monopolistic rating intermediary to "over-produce" information. Doing so costs more than what it is worth to the firm, but it allows the monopolist to extract more rents from all firms. In that respect, our analysis shows that the market for ratings is quite different from other markets where monopolists are deemed to under-produce either in symmetric information set-ups or when asymmetrically informed about for instance consumers' types (as in the case of a discriminating monopolist).

Welfare analysis. Our modelling approach based on risk neutral agents did not confer any social value to information. We implicitly assumed in our discussions that more information revealed was a good thing. Given the cost c of performing a rating, unless the value of information is large, there should be in fact an optimal level of rating disclosures. The comparison of the two market structures in such a model would not be trivial as two effects can be distinguished. First, we have seen that a monopolistic structure will be conducive to a larger set of firms applying for a rating than under Bertrand competition. Following our previous discussion, the monopolistic structure will involve a welfare loss by generating too many ratings. But second, Bertrand competition might generate a larger set of ratings produced but unrevealed. This is also inefficient as the cost of producing the rating is paid but the rating is then hidden, generating no new information. One should also stress that competition suppresses information about rather bad types of firms (i.e. those with sufficiently small signal μ or value v).

Mandatory Disclosure. A natural benchmark for our analysis is a setting where firms are free to ask for a rating but, once the rating is provided, full disclosure by the rating agency is mandatory.[13] In the case of uniform distributions, this certainly does not affect the analysis under a monopoly market structure since, absent such type of regulatory constraint, all ratings are already revealed. Similarly, this does not affect either the analysis whenever, in a competitive market structure, the option has no value. However, this would have an impact on the equilibrium under competing intermediaries once considering that $\theta > c$. The result, in this setting is the following.

Proposition 8 *Suppose that v is distributed according to some cdf $F_v(\cdot)$ on some support $[0, V]$ with mean $E[v]$. Assume that $c < V - E[v]$ and μ is uniform on $[v - \theta, v + \theta]$, in such a way that the distribution of v conditional on μ_2, $G(v|\mu_2)$, first order stochastically dominates $G(v|\mu_1)$ if $\mu_2 \geq \mu_1$. Mandatory disclosure of ratings under Bertrand competition weakly increases the lowest threshold above which all types of firms apply for a rating in any equilibrium.*

Proof. See Appendix ∎

Allowing rating agencies to offer the option of hiding the rating has a positive impact on the firm's willingness to pay for a rating. However, the effect on the amount of information that is revealed in equilibrium is ambiguous since some information will be withheld. Note also that, given that in our analysis information has no direct social value, social welfare is higher when the option cannot be offered. Indeed, such regulation avoids cases where the cost c of providing a rating is incurred by the rating agency but the rating is ultimately not disclosed. In the competitive case, ultimately firms collectively bear the cost of producing the rating. Without an explicit social benefit of producing a rating, in the current set-up firms will prefer, before knowing their type, that the rating market is shut down. This also implies that they would prefer a mandatory disclosure rule as this is a way to limit the number of ratings produced in equilibrium.

[13] Even if the decision to get a rating is not observable, one can imagine that the rating agency can be audited and, if caught withholding ratings, be forced to pay a large fine.

Lack of information about the rating agency. Suppose there is a proportion of firms who are unaware of the existence of intermediaries who evaluate corporate governance. Those "ignorant" firms will not ask for a rating, no matter how good their signal is. If this is the case, the value of v_ϕ is less responsive to the strategic decisions made by firms informed about the workings of the rating market. This possibility will reduce (but not eliminate) the forces behind the unravelling effect and is likely to increase the value of the no disclosure option. Still, the probability that a rating will not be revealed is higher under competition.

7 Appendix

Proof of Proposition 2 when $\omega > 0$

Suppose that the firm, still perfectly informed on v (i.e. $\theta = 0$), is now uncertain about the rating it will get because the intermediary's technology is imprecise, i.e. $\omega > 0$. Previous proofs have to be amended to account for the fact that ratings are noisy. In particular, Lemma 1 might not hold any longer because now renegotiation takes place under asymmetric information. We start by extending this result.

It cannot be that $d(\sigma) < 1$ when $E_v[v|\sigma, v \geq \hat{v}] > v_\phi$, otherwise the rating agency could simply offer to reveal the rating for sure, against an additional fee of $(1 - d(\sigma))\left[E_v[v|\sigma, v \geq \hat{v}] - v_\phi\right]$. Similarly, it cannot be that $d(\sigma) > 0$ when $E_v[v|\sigma, v \geq \hat{v}] < v_\phi$. This implies that the firm's willingness to pay is $\Pr(E_v[v|\sigma, v \geq \hat{v}] \geq v_\phi]|v) E_\sigma[E_v[v|\sigma, v \geq \hat{v}] - v_\phi]|v]$, increasing in v.

Consider the case of secret contracting and $\theta = 0$. Importantly, the threshold structure implies that a firm with a rating must have a value $v \geq \hat{v}$. This is true even if $\sigma < \hat{v}$ so that the revelation of a rating cannot result in an updated value less than $\hat{v}$, i.e., $E_v[v|\sigma, v \geq \hat{v}] \geq \hat{v}$, for any σ. Suppose now that there is a set of ratings Σ_n that are not revealed in equilibrium: $d(\sigma) = 0$ if $\sigma \in \Sigma_n$. Then, $v_\phi = E_v[v|v < \hat{v} \text{ } or \text{ } (\sigma \in \Sigma_n, v \geq \hat{v})]$. Call $\bar{\sigma}_n = \sup \Sigma_n$. It must be that $E_v[v|\bar{\sigma}_n, v \geq \hat{v}] > E_v[v|\phi] = v_\phi = E_v[v|v < \hat{v} \text{ } or \text{ } (\sigma \in \Sigma_n, v \geq \hat{v})]$. Renegotiation-proofness implies that $\bar{\sigma}_n$ must be revealed. This argument unravels and thus all ratings are revealed.

Proof of Proposition 3

Call $F_\mu(\cdot)$ the (unconditional) cdf of the signal μ and $f_\mu(\cdot)$ its density. The profit of a monopolist who takes all types above $\hat{\mu}$ writes as

$$\Pi(\hat{\mu}) = \int_{\hat{\mu}}^{1+\theta} \Pr(v \geq v_\phi(\hat{\mu})|\mu)[E_v(v|v \geq v_\phi(\hat{\mu}), \mu) - v_\phi(\hat{\mu})] f_\mu(\mu) d\mu - (1 - F_\mu(\hat{\mu}))c.$$

It is equal to $\frac{1}{2} - c$ whenever $\hat{\mu} = -\theta$ and to 0 whenever $\hat{\mu} = 1 + \theta$.

Call $h(\cdot|v)$ the conditional density of the signal μ for given value v. Denoting by $\Pr(\phi(\hat{\mu})) = \Pr((\mu, v) \in \phi(\hat{\mu}))$ and using the fact that

$$\Pr(\phi(\hat{\mu}))v_\phi(\hat{\mu}) + (1 - \Pr(\phi(\hat{\mu})))E_v[v|(\mu, v) \notin \phi(\hat{\mu})] = E_v[v] = \frac{1}{2},$$

and

$$\int_{\hat{\mu}}^{\mu_{\max}} \Pr(v \geq v_\phi(\hat{\mu})|\mu)[E_v[v|v \geq v_\phi(\hat{\mu}), \mu] - v_\phi(\hat{\mu})] f_\mu(\mu) d\mu = (1 - \Pr(\phi(\hat{\mu}))) \left[E_v[v|(\mu, v) \notin \phi(\hat{\mu})] - v_\phi(\hat{\mu})\right],$$

we can rewrite the profit of the monopolist who takes all types above $\hat{\mu}$ as

$$\Pi(\hat{\mu}) = \frac{1}{2} - v_\phi(\hat{\mu}) - (1 - F_\mu(\hat{\mu}))c. \tag{5}$$

For the uniform case we have that simple Bayesian updating gives the density of μ,

$$f_\mu(\mu) = \int_0^1 h(\mu|v) f_v(v) dv = \begin{cases} \frac{\mu+\theta}{2\theta} & if \quad \mu \in [-\theta, \theta) \\ 1 & if \quad \mu \in [\theta, 1-\theta] \\ \frac{1+\theta-\mu}{2\theta} & if \quad \mu \in (1-\theta, 1+\theta] \end{cases}$$

and

$$g(v|\mu) = \frac{h(\mu|v) f_v(v)}{f_\mu(\mu)} = \begin{cases} \frac{1}{1+\theta-\mu} & if \quad \mu > 1-\theta, & v \in [\mu-\theta, 1], \\ \frac{1}{2\theta} & if \quad \mu \in [\theta, 1-\theta], & v \in [\mu-\theta, \mu+\theta], \\ \frac{1}{\mu+\theta} & if \quad \mu < \theta, & v \in [0, \mu+\theta]. \end{cases}$$

Since the monopolist can always ensure a profit level of $\frac{1}{2} - c$ with a contract $p_0 = 0$, $p(v) = v$ and $d(v) = 1$ that attracts all types of firm, it only remains to show that that the profit with contracts that exclude some types does not exceed $1/2 - c$. To do so, we have to consider different cases according to whether we can find a solution where some ratings could be hidden and according to the range of $\hat{\mu}$.

Case 1: The option has no value. That is,

$$v_\phi(\hat{\mu}) = E_v[v|\mu \leq \hat{\mu}] \leq \max\{\hat{\mu} - \theta, 0\}.$$

Since $v_\phi \geq 0$, it must be that $\hat{\mu} \geq \theta$.

1.a) Suppose that $\theta \leq \hat{\mu} \leq 1 - \theta$. This implies that $v_\phi(\hat{\mu}) = \frac{\hat{\mu}}{2} + \frac{\theta^2}{6\hat{\mu}}$ and $F_\mu(\hat{\mu}) = \hat{\mu}$. Thus, equation (5) writes

$$\begin{aligned} \Pi_{1.a}(\hat{\mu}) &= \frac{1}{2} - c - \left(\frac{\hat{\mu}}{2} + \frac{\theta^2}{6\hat{\mu}}\right) + \hat{\mu}c, \\ &= \frac{1}{2} - c - \hat{\mu}\left(\frac{1}{2} - c\right) - \frac{\theta^2}{6\hat{\mu}} \leq \frac{1}{2} - c, \end{aligned}$$

because $c \leq 1/2$ and $\hat{\mu} > \theta \geq 0$. Thus, no $\hat{\mu} \in [\theta, 1-\theta]$ for which the option has no value can do better than $\hat{\mu} = -\theta$.

1.b) Suppose $\hat{\mu} > 1-\theta$. This implies that

$$v_\phi(\hat{\mu}) = \frac{-2-\hat{\mu}^3+3\theta+3\hat{\mu}^2\theta+\theta^3+3\hat{\mu}(1-\theta^2)}{3(2\hat{\mu}(1+\theta)-\hat{\mu}^2-(1-\theta)^2)}$$

and $F_\mu(\hat{\mu}) = 1 - \frac{(1+\theta-\hat{\mu})^2}{4\theta}$. Thus, equation (5) writes

$$\begin{aligned}
\Pi_{1.b}(\hat{\mu}) &= \frac{1}{2} - c - \frac{-2-\hat{\mu}^3+3\theta+3\hat{\mu}^2\theta+\theta^3+3\hat{\mu}(1-\theta^2)}{3(2\hat{\mu}(1+\theta)-\hat{\mu}^2-(1-\theta)^2)} + \left(1 - \frac{(1+\theta-\hat{\mu})^2}{4\theta}\right)c, \\
&\leq \frac{1}{2} - c - \frac{-2-\hat{\mu}^3+3\theta+3\hat{\mu}^2\theta+\theta^3+3\hat{\mu}(1-\theta^2)}{3(2\hat{\mu}(1+\theta)-\hat{\mu}^2-(1-\theta)^2)} + \left(1 - \frac{(1+\theta-\hat{\mu})^2}{4\theta}\right)\frac{1}{2},
\end{aligned}$$

because $c \leq 1/2$. Rearranging terms we have that,

$$\begin{aligned}
\Pi_{1.b}(\hat{\mu}) &\leq \frac{1}{2} - c - \frac{(1+\theta-\hat{\mu})^3(3(\hat{\mu}-(1-\theta))+2\theta)}{24\theta(2\hat{\mu}(1+\theta)-\hat{\mu}^2-(1-\theta)^2)}, \\
&\leq \frac{1}{2} - c,
\end{aligned}$$

because for $\hat{\mu} \in [1-\theta, 1+\theta]$ we have, $(1+\theta-\hat{\mu})^3 \geq 0$, $3(\hat{\mu}-(1-\theta))+2\theta > 0$ and $2\hat{\mu}(1+\theta)-\hat{\mu}^2-(1-\theta)^2 > 0$.

Case 2: The option has value. That is,

$$v_\phi(\hat{\mu}) > \max\{\hat{\mu}-\theta, 0\}.$$

2.a) Suppose that $\hat{\mu} < \theta$. Recall that

$$v_\phi(\hat{\mu}) = \frac{\int_{-\theta}^{\hat{\mu}}\int_0^{\mu+\theta} v dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} v dv d\mu}{\int_{-\theta}^{\hat{\mu}}\int_0^{\mu+\theta} dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} dv d\mu}. \tag{6}$$

Solving (6) for $\hat{\mu}$ this gives

$$\hat{\mu}(v_\phi) = v_\phi - \theta + (6\theta)^{\frac{1}{3}} v_\phi^{\frac{2}{3}}. \tag{7}$$

Thus, $\hat{\mu} < \theta$ implies that

$$v_\phi + (6\theta)^{\frac{1}{3}} v_\phi^{\frac{2}{3}} < 2\theta,$$

Denote by $s(v_\phi) \equiv v_\phi + (6\theta)^{\frac{1}{3}} v_\phi^{\frac{2}{3}} - 2\theta$. We solve the equation $s(v_\phi) = 0$, and find three roots, only one of them real and equal to

$$\bar{v}_\phi \equiv \frac{(2(1+\sqrt{5}))^{\frac{2}{3}} - 2^{\frac{4}{3}}}{(1+\sqrt{5}))^{\frac{1}{3}}}\theta,$$

We check that $\bar{v}_\phi \in [0, 1)$ for $\theta \in [0, 1/2]$, and that $s(v_\phi) < 0$ iff $v_\phi < \bar{v}_\phi$.

The profit function (5) writes

$$\begin{aligned}\Pi_{2.a}(v_\phi) &= \frac{1}{2}-c-v_\phi+\frac{(\hat{\mu}(v_\phi)+\theta)^2}{4\theta}c,\\ &= \frac{1}{2}-c-v_\phi+\frac{\left(v_\phi+(6\theta)^{\frac{1}{3}}v_\phi^{\frac{2}{3}}\right)^2}{4\theta}c.\end{aligned}$$

Hence, $\Pi_{2.a}(v_\phi)$ is decreasing for v_ϕ close to 0 and convex for any $v_\phi \in [0,\bar{v}_\phi]$, so its maximum must be at a corner. Replacing at $\bar{v}_\phi$, we get that

$$\Pi_{2.a}(\bar{v}_\phi) < \frac{1}{2}-c$$

for any $c < 1/2$. Thus, the maximum profit if for $v_\phi = 0$, which implies $\hat{\mu} = -\theta$.

2.b) Suppose that $\hat{\mu} \in [\theta, 1-\theta]$. In this case,

$$v_\phi(\hat{\mu}) = \frac{\int_{-\theta}^{\theta}\int_0^{\mu+\theta} v dv d\mu + \int_\theta^{\hat{\mu}}\int_{\mu-\theta}^{\mu+\theta} v dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} v dv d\mu}{\int_{-\theta}^{\theta}\int_0^{\mu+\theta} dv d\mu + \int_\theta^{\hat{\mu}}\int_{\mu-\theta}^{\mu+\theta} dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} dv d\mu}. \tag{8}$$

Solving (8) for $\hat{\mu}$ gives again equation (7). The profit function (5) writes

$$\begin{aligned}\Pi_{2.b}(v_\phi) &= \frac{1}{2}-c-v_\phi+\hat{\mu}(v_\phi)c,\\ &= \frac{1}{2}-c-v_\phi+\left(v_\phi-\theta+(6\theta)^{\frac{1}{3}}v_\phi^{\frac{2}{3}}\right)c.\end{aligned}$$

Hence, $\Pi_{2.b}(v_\phi)$ is concave for any v_ϕ and has a maximum at

$$v_\phi^* = \frac{16}{9}\left(\frac{c}{1-c}\right)^3\theta.$$

So,

$$\Pi_{2.b}(v_\phi) \leq \Pi_{2.b}(v_\phi^*) = \frac{1}{2}-c-\frac{\theta c}{9(1-c)^2}\left(9-18c+c^2\right) < \frac{1}{2}-c$$

for any $c \in [0, 1/2]$.

2.c) Suppose that $\hat{\mu} > 1-\theta$. In this case,

$$v_\phi(\hat{\mu}) = \frac{\int_{-\theta}^{\theta}\int_0^{\mu+\theta} v dv d\mu + \int_\theta^{1-\theta}\int_{\mu-\theta}^{\mu+\theta} v dv d\mu + \int_{1-\theta}^{\hat{\mu}}\int_{\mu-\theta}^{1} v dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} v dv d\mu}{\int_{-\theta}^{\theta}\int_0^{\mu+\theta} dv d\mu + \int_\theta^{1-\theta}\int_{\mu-\theta}^{\mu+\theta} dv d\mu + \int_{1-\theta}^{\hat{\mu}}\int_{\mu-\theta}^{1} dv d\mu + \int_{\hat{\mu}}^{v_\phi+\theta}\int_{\mu-\theta}^{v_\phi} dv d\mu}. \tag{9}$$

Solving (9) for $\hat{\mu}$ gives

$$\hat{\mu}(v_\phi) = \frac{2-3v_\phi+v_\phi^3-3\theta(1-2v_\phi-v_\phi^2)}{3(1-v_\phi)^2}. \tag{10}$$

Using (10), we have that the condition $\hat{\mu} > 1-\theta$ is equivalent to

$$\frac{1}{2} \geq \theta > \frac{(1-v_\phi)^3}{6v_\phi^2}. \tag{11}$$

Since the option has value it must be that $\hat{\mu} - \theta < v_\phi$. Using (10) this condition is equivalent to

$$2(1 - v_\phi)^3 - 6\theta(1 - 2v_\phi) < 0.$$

This is impossible if $v_\phi \geq 1/2$. Thus, $v_\phi < 1/2$ and

$$\theta > \frac{(1 - v_\phi)^3}{3(1 - 2v_\phi)} > 0. \tag{12}$$

Combining (11) and (12) we obtain that

$$\frac{(1 + \sqrt{5})^{2/3} - 2^{2/3}}{(2(1 + \sqrt{5}))^{1/3}} < v_\phi < \frac{1}{2}.$$

The profit is

$$\Pi_{2.c}(v_\phi) = \frac{1}{2} - c - v_\phi + \left(1 - \frac{(1 + \theta - \hat{\mu}(v_\phi))^2}{4\theta}\right) c.$$

So for any $c \in [0, 1/2]$

$$\Pi_{2.c}(v_\phi) \leq \frac{1}{2} - c - v_\phi + \left(1 - \frac{(1 + \theta - \hat{\mu}(v_\phi))^2}{4\theta}\right) \frac{1}{2}. \tag{13}$$

Using (10), we obtain that

$$\begin{aligned} -v_\phi + \left(1 - \frac{(1 + \theta - \hat{\mu}(v_\phi))^2}{4\theta}\right) \frac{1}{2} \quad &= \quad \frac{(1 - 2v_\phi)\left(3(1 - v_\phi)^4 - 3\theta(1 - 2v_\phi) - (1 - v_\phi)^3\right)}{6(1 - v_\phi)^4} - \frac{1 - v_\phi}{72\theta}, \\ &< \quad \frac{(1 - 2v_\phi)(1 - 3v_\phi)}{6(1 - v_\phi)} - \frac{(1 - v_\phi)^2}{36} < 0, \end{aligned}$$

where the inequalities come from the fact that $\frac{1}{2} > \theta > \frac{(1-v_\phi)^3}{3(1-2v_\phi)}$ and $\frac{(1+\sqrt{5})^{2/3}-2^{2/3}}{(2(1+\sqrt{5}))^{1/3}} < v_\phi < \frac{1}{2}$. Thus, for any feasible θ and v_ϕ, $\Pi_{2.c}(v_\phi) < \frac{1}{2} - c$.

Proof of Proposition 4

We know that the firm that is indifferent between asking for a rating or staying out of the certification market is given by

$$\Pr(v \geq v_\phi(\hat{\mu})|\hat{\mu}) E_v[v|v \geq v_\phi(\hat{\mu}),\, \hat{\mu}] + (1 - \Pr(v \geq v_\phi(\hat{\mu})|\hat{\mu}))\, v_\phi(\hat{\mu}) - c = v_\phi(\hat{\mu}).$$

Rearranging terms, this gives equation (4). In particular, this implies that $\hat{\mu} > -\theta$ in any equilibrium if $c > 0$. Thus, the mass of types who stay out is strictly positive, which means that $v_\phi > 0$.

Now, the worst rating that could be obtained by a firm endowed with a signal $\hat{\mu}$ is $\max\{\hat{\mu} - \theta, 0\}$. Therefore, the option will have some value if and only if

$$v_\phi(\hat{\mu}) > \max\{\hat{\mu} - \theta, 0\}. \tag{14}$$

We first establish:

Lemma 4 *Whenever $\theta \leq \hat{\mu} \leq 1-\theta$, competing intermediaries offer the no-disclosure option (i.e. $\exists v$, so that $d(v) = 0$) if $\theta > c$.*

Proof: We show that it is impossible to have $\theta > c$ and the option not offered. Suppose by contradiction that the option is not offered. Then the indifferent type is defined by

$$\begin{aligned} v_\phi &= E[\max\{v, v_\phi\}|\hat{\mu}] - c \\ &= E[v|\hat{\mu}] - c \\ &= \hat{\mu} - c \end{aligned}$$

and we need for the option not to be offered that

$$\hat{\mu} - \theta \geq v_\phi = \hat{\mu} - c$$

so $c \geq \theta$, a contradiction.

Lemma 5 *Whenever $\theta \leq \hat{\mu} \leq 1-\theta$, competing intermediaries offer the no-disclosure option (i.e. $\exists v$, so that $d(v) = 0$) only if $\theta > c$.*

We show that it is impossible to have the option offered and $\theta \leq c$. When $\hat{\mu} \geq \theta$, the no-disclosure option has some value if $v_\phi > \hat{\mu} - \theta$. Using the uniform distribution and under the assumption that the option is valuable, equation (4) can be written as

$$\left(1 - \frac{v_\phi - (\hat{\mu} - \theta)}{2\theta}\right)\left(\frac{v_\phi + (\hat{\mu} + \theta)}{2} - v_\phi\right) = c,$$

or equivalently

$$\hat{\mu} + \theta - 2\sqrt{\theta c} = v_\phi.$$

Using this, we can check that condition (14) holds only if $\theta > c$.

We have now established that in the case of the uniform distribution and when $\theta \leq \hat{\mu} \leq 1-\theta$, the no-disclosure option is offered if and only if $\theta \geq c$. We now turn our attention to the case where $\hat{\mu} < \theta$.

Lemma 6 *Suppose $\hat{\mu} < \theta$, then there must always exist some v so that $d(v) = 0$.*

Proof: Suppose this is not true. Then we would need

$$0 \geq \hat{\mu} - \theta > v_\phi(\hat{\mu})$$

but $v_\phi(\hat{\mu}) \geq 0$. A contradiction.

So in an equilibrium where the option is not exercised, it must be that $\hat{\mu} > \theta$. Therefore, the only possibility for having an equilibrium where $\hat{\mu} < \theta$ is that the option is offered. We start looking for one where $\hat{\mu} < \theta \leq c$.

Lemma 7 *There is no equilibrium where $\hat{\mu} < \theta \leq c$.*

Proof: Suppose this is not true. As $\hat{\mu} < \theta$ we know that the option must be exercised sometimes for such an equilibrium to exist. The condition for the option to be exercised is simply that $v_\phi > 0$, which is true. In the case of uniform distribution, Equation (4) when $\hat{\mu} < \theta$ and $v_\phi > 0$ becomes

$$\frac{\hat{\mu} + \theta - v_\phi}{\hat{\mu} + \theta} \left(\frac{\hat{\mu} + \theta + v_\phi}{2} - v_\phi \right) = c,$$

or equivalently,

$$\hat{\mu} + \theta - \sqrt{2c(\hat{\mu} + \theta)} = v_\phi.$$

Now, $\hat{\mu} < \theta \leq c$ implies that $\hat{\mu} + \theta - \sqrt{2c(\hat{\mu} + \theta)} < 0$, i.e. that $v_\phi < 0$, a contradiction.

Therefore, when $\hat{\mu} < \theta$, the only possible equilibrium candidates have that either $c \leq \hat{\mu} < \theta$ or $\hat{\mu} \leq c < \theta$ and the option is offered. In any case, $c < \theta$ when the option is offered.

Lemma 8 *Whenever $\hat{\mu} > 1 - \theta$, competing intermediaries offer the no disclosure option only if $\theta > c$.*

Suppose that $\hat{\mu} > 1 - \theta$. Condition (14) is again $v_\phi(\hat{\mu}) > \hat{\mu} - \theta$. Using the uniform distribution and under the assumption that (14) holds, equation (4) can be written as

$$\frac{1 - v_\phi(\hat{\mu})}{1 + \theta - \hat{\mu}} \left(\frac{v_\phi(\hat{\mu}) + 1}{2} - v_\phi \right) = c,$$

or equivalently

$$v_\phi(\hat{\mu}) = 1 - \sqrt{2c(1 + \theta - \hat{\mu})}.$$

Again, (14) implies that $\theta > c$.

Lemma 9 *If $c < 1/4$, there is no equilibrium with $\hat{\mu} > 1 - \theta$ in which the option has no value.*

When $\hat{\mu} > 1 - \theta$, under the uniform distribution $E_v[v|\hat{\mu}] = \frac{\hat{\mu} - \theta + 1}{2}$, so the indifferent type is defined by

$$\frac{\hat{\mu} - \theta + 1}{2} - c = E_v[v|\mu \leq \hat{\mu}]. \tag{15}$$

We know that for any $\hat{\mu} < 1 + \theta$, $E_v[v|\mu \leq \hat{\mu}] < E_v[v|\mu \leq 1 + \theta] = 1/2$. Combining this with (15) and the fact that $c \leq 1/4$ we obtain that

$$\hat{\mu} < \frac{1}{2} + \theta. \tag{16}$$

Now, suppose that the option is not offered (i.e., it has no value). It must be that $E_v[v|\mu \leq \hat{\mu}] \leq \hat{\mu} - \theta$, or, using (15),

$$\frac{\hat{\mu} - \theta + 1}{2} - c \leq \hat{\mu} - \theta,$$

which is equivalent to

$$1 - (\hat{\mu} - \theta) \leq 2c \leq \frac{1}{2},$$

because $c \leq 1/4$. Hence, we must have that

$$\hat{\mu} \geq \frac{1}{2} + \theta, \tag{17}$$

which contradicts (16). Thus, if the option has no value the case $\hat{\mu} > 1 - \theta$ can never happen for $c \leq 1/4$.

We have therefore established that the option is offered if and only if $\theta > c$.

Proof of Proposition 7

The monopolist's ability to extract all surplus does not depend on the types' distribution so that in a monopolistic market structure, the equilibria will have a threshold structure, as under Bertrand competition. Denote by $\hat{\mu}$ the threshold in any possible equilibrium. By definition we can express $v_\phi(\hat{\mu})$ as follows,

$$\Pr(\phi(\hat{\mu}))v_\phi(\hat{\mu}) = \int_{\mu_{\min}}^{\hat{\mu}} \int_0^1 vg(v|\mu)f_\mu(\mu)dvd\mu + \int_{\hat{\mu}}^{\mu_{\max}} \int_0^{v_\phi} vg(v|\mu)f_\mu(\mu)dvd\mu,$$

where

$$\Pr(\phi(\hat{\mu})) = \int_{\mu_{\min}}^{\hat{\mu}} \int_0^1 g(v|\mu)f_\mu(\mu)dvd\mu + \int_{\hat{\mu}}^{\mu_{\max}} \int_0^{v_\phi} g(v|\mu)f_\mu(\mu)dvd\mu.$$

This implies, in particular that

$$\frac{dv_\phi}{d\hat{\mu}} = \frac{f_\mu(\hat{\mu})}{\Pr(\phi(\hat{\mu}))} \int_{v_\phi}^1 (v - v_\phi)g(v|\hat{\mu})dv \geq 0.$$

Therefore, it is enough to show that $\hat{\mu}^B > \hat{\mu}^M$. The profit of the monopolist is given by:

$$\Pi(\hat{\mu}) = \int_{\hat{\mu}}^{\mu_{\max}} [p(\mu, \hat{\mu}) - c]f_\mu(\mu)d\mu,$$

with

$$p(\mu, \hat{\mu}) = \int_{v_\phi(\hat{\mu})}^1 (v - v_\phi(\hat{\mu}))g(v|\mu)dv.$$

Deriving this function with respect to $\hat{\mu}$ we get

$$\frac{\partial \Pi}{\partial \hat{\mu}} = -[p(\hat{\mu}, \hat{\mu}) - c]f_\mu(\hat{\mu}) + \int_{\hat{\mu}}^{\mu_{\max}} \frac{\partial p(\mu, \hat{\mu})}{\partial \hat{\mu}} f_\mu(\mu)d\mu.$$

Under Bertrand competition, the lowest type who asks for a rating is $\hat{\mu}^B$ such that $p(\hat{\mu}^B, \hat{\mu}^B) = c$. Evaluating the monopolist's first order condition at $\hat{\mu}^B$, we get

$$\frac{\partial \Pi}{\partial \hat{\mu}}(\hat{\mu}^B) = \int_{\hat{\mu}^B}^{\mu_{\max}} \frac{\partial p(\mu, \hat{\mu}^B)}{\partial \hat{\mu}} f_\mu(\mu) d\mu = -\frac{dv_\phi}{d\hat{\mu}}(\hat{\mu}^B) \int_{\hat{\mu}^B}^{\mu_{\max}} \int_{v_\phi(\hat{\mu}^B)}^{1} g(v|\mu) f_\mu(\mu) dv d\mu < 0$$

since v_ϕ increases with $\hat{\mu}$. Since $\Pi(\hat{\mu})$ is single-peaked, the maximum profit is found for a $\hat{\mu}^M < \hat{\mu}^B$. A monopolist intermediary attracts more firms seeking for a rating and, therefore, lowers the lowest rating that is revealed. More information is then revealed in the monopoly case.

Proof of Proposition 8

Whenever the equilibrium is such that the no-disclosure option is not offered (in either market structures), the mandatory rules does not affect the equilibrium. Consider now an equilibrium where some ratings are not disclosed. We show that in such a case, it is impossible that the number of firms increase following the introduction of the mandatory rule. Denote by $\hat{\mu}$ the threshold equilibrium absent the mandatory rule and $\hat{\mu}^m$ in presence of such a rule. We show that $\hat{\mu} > \hat{\mu}^m$ is impossible.

Assume that the no-disclosure option is offered absent the mandatory rule. Denote by $q(\tilde{\mu}) = E_v[v|\mu \leq \tilde{\mu}]$ and $r(\tilde{\mu}) = E_v[v|\tilde{\mu}] - c$. Also denote by $R(\tilde{\mu}) = E_v[\max\{v, v_\phi\}|\tilde{\mu}] - c \geq r(\tilde{\mu})$ for all $(v_\phi, \tilde{\mu})$. Finally implicitly define $Q(\tilde{\mu}) = E_v[v|\mu < \tilde{\mu}$ or $\mu \geq \tilde{\mu}$ and $v < Q(\tilde{\mu})]$. The following lemma provides some comparison between $Q(\tilde{\mu})$ and $q(\tilde{\mu})$ for all $\tilde{\mu}$

Lemma 10 *For any $\tilde{\mu}$, we have $Q(\tilde{\mu}) \leq q(\tilde{\mu})$*

Proof. First, if $\Pr(\mu \geq \tilde{\mu}$ and $v < Q(\tilde{\mu})) = 0$, then $Q(\tilde{\mu}) = q(\tilde{\mu})$.
Second, suppose that $\Pr(\mu \geq \tilde{\mu}$ and $v < Q(\tilde{\mu})) > 0$.
$Q(\tilde{\mu})$ is such that

$$\int_{\mu<\tilde{\mu}} \int_0^V (v - Q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu = 0$$

and $Q(\tilde{\mu})$ is such that

$$\int_{\mu<\tilde{\mu}} \int_0^V (v - Q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu + \int_{\mu\geq\tilde{\mu}} \int_0^{Q(\tilde{\mu})} (v - Q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu = 0.$$

Thus

$$\begin{aligned}\int_{\mu<\tilde{\mu}} \int_0^V (v - q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu \quad &= \quad \int_{\mu<\tilde{\mu}} \int_0^V (v - Q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu \\ &\quad + \int_{\mu\geq\tilde{\mu}} \int_0^{Q(\tilde{\mu})} (v - Q(\tilde{\mu}))\, g(v|\mu) f_\mu(\mu) dv d\mu,\end{aligned}$$

which implies that

$$(Q(\tilde{\mu}) - q(\tilde{\mu})) \int_{\mu<\tilde{\mu}} \int_0^V g(v|\mu) f_\mu(\mu) dv d\mu = -\int_{\mu\geq\tilde{\mu}} \int_0^{Q(\tilde{\mu})} (Q(\tilde{\mu}) - v)\, g(v|\mu) f_\mu(\mu) dv d\mu < 0.$$

■

In the absence of mandatory disclosure, the lowest type that hires any of the intermediaries has a net gain of doing so $R(\hat{\mu}) - Q(\hat{\mu}) > r(\hat{\mu}) - q(\hat{\mu})$ and moreover, $R(\hat{\mu}) - Q(\hat{\mu}) = 0$.

- Suppose first that the distribution $F_{\mu}(\mu)$ is log-concave. Then as shown by Bagnoli and Bergstrom (2005) the function $r(\hat{\mu}) - q(\hat{\mu})$ is increasing in $\hat{\mu}$. Therefore if $\hat{\mu} > \hat{\mu}^m$,

$$c > E_v[v|\hat{\mu}^m] - E_v[v|\mu < \hat{\mu}^m]$$

and the type $\hat{\mu}^m$ cannot be the threshold type under mandatory disclosure, a contradiction.

- Suppose now that the distribution $F_{\mu}(\mu)$ is not log-concave and that there is now a possibility that $r(\hat{\mu}) - q(\hat{\mu})$ is not everywhere increasing in $\hat{\mu}$. We have that $r(-\theta) = -c < q(-\theta) = 0$. We also have that $r(V + \theta) = V - c > q(V + \theta) = E[v] > 0$. Now, when the mandatory rule is removed, so that firms can hide ratings, the value of obtaining a rating is now $R(\hat{\mu}) = E[\max\{v, v_{\phi}\}|\hat{\mu}] - c \geq r(\hat{\mu})$ for all $(v_{\phi}, \hat{\mu})$. Moreover, the value of not having a rating is now $Q(\hat{\mu}) = E[v|\mu < \hat{\mu}$ or $\mu \geq \hat{\mu}$ and $v < Q(\hat{\mu})] \leq q(\hat{\mu})$ for all $\hat{\mu}$. There could be now multiple values of $\hat{\mu}$ for which $R(\hat{\mu}) - Q(\hat{\mu}) = 0$ or under mandatory disclosure, $r(\hat{\mu}) - q(\hat{\mu}) = 0$. However, at the lowest value of $\hat{\mu}$ for which $r(\hat{\mu}) - q(\hat{\mu})$ reaches 0, it does so from below. Since, for any $\hat{\mu}$, the function $R(\hat{\mu}) - Q(\hat{\mu}) > r(\hat{\mu}) - q(\hat{\mu})$ and that $R(-\theta) - Q(-\theta) = -c$, then it entails that the lowest value $\hat{\mu}_{min}$ for which $R(\hat{\mu}) - Q(\hat{\mu}) = 0$ is lower than the lowest value $\hat{\mu}'_{min}$ for which $r(\hat{\mu}) - q(\hat{\mu}) = 0$.

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