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The Majority Judgement Voting Procedure: A Critical Evaluation[1]

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ABSTRACT

We evaluate critically some of the properties of the *Majority Judgement* voting procedure recently proposed by Balinski and Laraki (2007a, 2007b) for the election of one out of two or more candidates.

Keywords Majority judgement, Median, Ordinal grades, Voting paradoxes, Voting procedures

The Majority Judgement Voting Procedure: A Critical Evaluation

1 Introduction

Arrow's (1951, 1963) Impossibility Theorem asserts that no voting procedure for selecting one out of three or more alternatives can satisfy simultaneously a small number of natural desiderata. This of course does not mean that all voting procedures are equally bad, and every once in a while one encounters in the literature a newly proposed voting system based on a novel principle.[1] As far as we know, the last two such voting systems proposed before 2007 were *approval voting*, due to Brams and Fishburn (1978, 1983); and *voting by veto*, due to Mueller (1978), extended by Moulin (1983: 138–140), and generalized by Felsenthal and Machover (1992b).

In 2007, nearly thirty years later, a new voting procedure, called *majority judgement* (MJ), was proposed by Balinski and Laraki (B&L) (2007a; 2007b).

The MJ voting procedure is designed for electing one out of n candidates ($n \geq 2$).[2] As in some sport and wine-ranking contests, each voter/judge awards each candidate/contestant a *grade*, measured on an ordinal scale. The ordinal grades employed may be expressed in numerals (e.g., $1, 2, 3, \ldots, 100$) or in words (e.g., Reject, Poor, Acceptable, Good, Very Good, Excellent – B&L (2007b) used these six grades in their 2007 French presidential election experiment).

Next, the *median grade* of each candidate – that is, the median of the grades awarded to the candidate – is determined. The winner is selected, by a process described below, from among the candidates with highest median grade.[3]

In case of a tie between the median grades, denoted by α, of two or more leading candidates, B&L propose the following iterative tie-breaking algorithm.

1. Delete one α grade from each of the tied candidates.

[1]This is quite apart from amendments to or elaborations of existing voting systems, e.g., methods for dealing with the Condorcet voting system in case a top cycle occurs (cf. Kemeny, 1959; Young, 1988; Felsenthal and Machover, 1992a).

[2]However, it can be adapted in a natural way for electing more than one candidate.

[3]If the number of voters is even and a candidate's two middle grades are different, then the median is not uniquely defined. B&L take the lower of the two middle grades as the median grade. This asymmetry of the MJ procedure creates some problems, as we will show in Examples 3.2 and 3.7 below.

2. Compute for each of the tied candidates the new value of α, the median grade.

3. If the new α grade of one of the (previously) tied candidates is higher than that of each of the other (previously) tied candidates, then this candidate is the winner. If there is still a tie between two or more candidates with the new median grade α, go to step 1.

It is not difficult to see that this algorithm must terminate with one winner, or with a dead heat between several candidates who have exactly the same grade distributions (that is: who, for each grade, have been awarded this grade by the same number of voters). In the latter case – which is extremely unlikely if the electorate is large – the winner must be chosen by lottery from among these top candidates.

This tie-breaking iterative process may get tedious due to the number of iterations needed if there are many voters – as is the usual case in public elections. In such cases B&L (2007b: 12–13, 39–40) provide the following 'simplified' rule for breaking ties.

They assign to each candidate an ordered triple – called the candidate's *majority-value* – (p, α^*, q), where:

$$\begin{aligned}\alpha &:= \text{candidate's median grade,}\\ p &:= \text{number of grades above } \alpha \text{ awarded to the candidate,}\\ q &:= \text{number of grades below } \alpha \text{ awarded to the candidate;}\end{aligned}$$

and the superscript $*$ is $+$, 0 or $-$ according as $p > q$, $p = q$ or $p < q$, respectively.

The winner must be selected from among those whose median grade α is maximal. To this end, one defines a total ordering $\succ$ among the majority-value triples with this maximal median α.

First, put:

$$(p, \alpha^+, q) \succ (r, \alpha^0, s) \succ (t, \alpha^-, u) \text{ for all } p, q, r, s, t \text{ and } u. \tag{1}$$

Next, for triples with the same middle-term superscript put:

$$(p, \alpha^+, q) \succ (r, \alpha^+, s) \text{ if } (p > r) \text{ or } (p = r \text{ and } q < s), \tag{2}$$

$$(p, \alpha^0, q) \succ (r, \alpha^0, s) \text{ if } p > r, \tag{3}$$

$$(p, \alpha^-, q) \succ (r, \alpha^-, s) \text{ if } (q < s) \text{ or } (q = s \text{ and } p > r). \tag{4}$$

Note that, by the definition of the superscript $*$, in case (2) we have $p > q$ and $r > s$; in case (3) we have $p = q$ and $r = s$; and in case (4) we have $p < q$ and $r < s$.

The winner is the candidate whose majority-value triple is maximal in this ordering. If there are several such candidates, then presumably the winner must be chosen by lottery from among them.

However, as we shall see in Examples 3.1 and 3.2, this simplified rule does not always yield the same result as the iterative algorithm. For this reason, we shall always use the latter.

The idea that a candidate with the highest median grade ought to be elected is not novel and has been around for some time. As far as we know, it was first proposed in the modern social choice literature by Bassett and Persky (1999). However, the crucial innovation of B&L (2007a) is to propose a tie-breaking algorithm, without which it would be impossible to use the MJ procedure in practice. Note that for an electorate of reasonable size ties between two or more leading candidates occur very rarely under conventional voting procedures such as plurality ('first past the post'), approval voting, Borda score, and alternative vote; and hence it is reasonable in such rare cases to break the tie by lottery. However, under a grading system it is quite likely that several candidates will share the highest median grade, and it would be unreasonable to break the tie between them by lottery, except in the rare case where *all* their grades are the same.

As we noted at the outset, Arrow's Impossibility Theorem implies that no voting procedure is perfect: every voting procedure satisfies some desiderata but not others. The evaluation of any given voting procedure depends on the number and importance of the desiderata that it satisfies, as well as on the number, seriousness, and likelihood of occurrence of the paradoxes to which it is vulnerable because it fails to satisfy other desiderata. Here 'importance' and 'seriousness' are of course a matter of opinion, and therefore the final judgement about a procedure is largely subjective.

In Section 2 we will outline several desiderata that are satisfied by the MJ procedure, and in Section 3 we will outline some problems and paradoxes afflicting the MJ procedure. However, no attempt will be made to outline necessary or sufficient conditions for the occurrence of these paradoxes under the MJ procedure, nor will we attempt to evaluate, for lack of sufficient empirical data, the likelihood of these paradoxes occurring in practice. For reasons of space we shall also not compare the MJ procedure with other voting procedures. The interested reader can find a comprehensive comparative analysis of voting procedures in Nurmi (1987).

2 Advantages of the MJ procedure

The MJ procedure has the following desirable properties.[4]

- *Voter-expressivity*: promoted by allowing voters to award (ordinal) grades to all candidates.
- *Anonymity*: all voters are treated equally.
- *Neutrality*: all candidates are treated equally.
- *Unanimity*: if all voters award candidate x a higher grade than to every other candidate, then x is elected.
- *Transitive ordering*: candidates are ranked in a transitive ordering; one candidate is necessarily ranked ahead or behind another, unless they have identical sets of grades.
- *Independence of irrelevant alternatives*: if candidate x wins, then x would still win if another candidate, y, is removed, *ceteris paribus*.
- *Monotonicity*: if candidate x wins, then x would still win if one of x's grades is increased, *ceteris paribus*.
- *Immunity to candidate cloning*: if candidate x wins, then x would still win if another candidate is added with grade distribution identical to that of x or of another candidate, *ceteris paribus*.
- The MJ procedure satisfies the *resolvability criterion*: the probability of a dead heat between two or more leading candidates – requiring lottery for choosing the final winner – approaches zero rapidly as the number of voters increases.
- Use of the median encourages voters to grade *sincerely*: in other words, to award each candidate the grade they believe is the 'right one'. (However, as we shall see in Example 3.3, in the competitive context of the MJ procedure this is true only with some reservations.)

Let us also note that the MJ procedure reduces to the Approval Voting procedure (Brams and Fishburn 1978, 1983) if there are just two available grades: say, 'approve' or 'disapprove'. When the number of grades is greater than two, some of the advantages of Approval Voting are shared or even

[4]Here and in the sequel, when we say that a candidate 'wins' we mean that s/he wins outright or is one of the dead-heaters from among whom the winner is chosen by lottery.

enhanced by the MJ procedure; but as we shall see some of the advantages are lost.

We shall now turn to list, exemplify, and discuss the main disadvantages of the MJ procedure so as to enable the reader to better evaluate this procedure and reach a decision as to whether the above advantages compensate for the disadvantages.

3 Problems afflicting the MJ procedure

In this section we shall use the following notation. Candidates will be denoted by lower-case letters, $x, y, z, \ldots$; voters will be labelled by italicized digits, *1*, *2*, *3*, ...; and grades will be denoted by upper-case letters, in increasing order: $A \prec B \prec C \prec \ldots$.

Note that although under the MJ procedure voters are not asked to rank-order the candidates explicitly, a (weak) total ordering of the candidates is implicit in the grades awarded them by a given voter. It can hardly be denied – and we shall therefore assume – that a voter who awards candidate x a higher grade than y prefers the former to the latter.[5]

An issue that we wish to mention here in passing is the problematic nature of aggregation of grades. When (cardinal or ordinal) grades awarded to a given candidate by different judges are lumped together, for example in order to determine the median grade, this presupposes that they are commensurable: so that, for example, grade G awarded by judge *1* is somehow better, denotes greater excellence and must count for more than grade F awarded by another judge, *2*. But this presupposition is not easy to justify. However, as this issue is not peculiar to social choice, we shall set it aside and assume for the sake of argument that aggregation of grades is meaningful.

Discrepancy in tie-breaking

Before turning to more important matters, we show that the simplified rule and the iterative algorithm for breaking ties between candidates with maximal median grade (both of which were presented in the Introduction) may

[5] B&L (2007a: 8720) claim that 'A measure or grade is a message that has strictly nothing to do with a utility. A judge may dislike a wine and yet give it a high grade because of its merits; he or she may also like a wine and yet, with great satisfaction, give it a low grade because of its demerits.' This may apply to judges grading wines, musicians, or sportsmen; but, it seems to us, not to voters when grading political candidates. A voter who prefers candidate x to y will, presumably, derive greater utility from x being elected than from y being elected, especially if the winning candidate is to serve as the voter's representative in some decision-making body.

lead to different outcomes. As far as we can tell, this may happen for two distinct reasons.

First, a candidate's majority-value triple, used in the simplified rule, does not provide complete information as to the candidate's distribution of grades: it tells us only *how many* grades the candidate was awarded above his or her median grade, and *how many* below the median; but it does not tell us how these (non-median) grades are distributed. For this reason, the simplified rule may produce an *apparent* dead heat, with several candidates having the same triple, whereas in fact the iterative algorithm can tell them apart, as is illustrated by the following example.

3.1 Example Consider two candidates with the following distributions of grades:

$$\begin{array}{llllllll} x: & A, & A, & B, & B, & C, & D, & D, \\ y: & A, & A, & B, & B, & C, & C, & D. \end{array}$$

Both candidates have the same triple, $(3, B^+, 2)$; so the simplified rule cannot tell them apart. But after four iterations of the algorithm (dropping one median grade at a time), we get

$$\begin{array}{llll} x: & A, & D, & D, \\ y: & A, & C, & D, \end{array}$$

which makes x the winner.

A second cause of discrepancy between the iterative algorithm and the simplified rule seems to be the asymmetry in the definition of the median, as illustrated by the following example.

3.2 Example Consider two candidates with the following distributions of grades:

$$\begin{array}{lllll} x: & A, & B, & D, & D, \\ y: & B, & B, & C, & C. \end{array}$$

Since B&L take the median to be the lower of the two middle grades, both candidates have median grade B. The triples of x and y are $(2, B^+, 1)$ and $(2, B^+, 0)$, respectively; so according to part (2) of the simplified rule, y is the winner. However, one iteration of the algorithm yields

$$\begin{array}{llll} x: & A, & D, & D, \\ y: & B, & C, & C, \end{array}$$

which makes x the winner. Note that if we were to take the *higher* of the two middle grades as the median, then x would win outright, with median grade D as against y's median grade, C.

We shall see in Example 3.7 below that the asymmetry in the definition of the median can affect the outcome even when there are no ties.

Strategic grading

As we mentioned in Section 2, use of the median grade encourages sincere grading. To see this, consider the following scenario. A student submits an examination paper to a panel of several examiners and the final grade awarded to the paper is to be the median of the grades awarded by the examiners. Under these circumstances, if an examiner were informed of the grades awarded by the other examiners, s/he would have no motive to change the grade s/he had (sincerely) awarded. This is so because if the grade awarded by the given examiner happens to coincide with the median grade – then s/he certainly has no motivation to change it. If the grade s/he (sincerely) awarded turns out to be lower than the median grade then, *ceteris paribus*, the median grade would not change if s/he were to award a *lower* grade than the original one; and awarding a *higher* grade than the original one may only increase the median grade – contrary to what s/he considers to be the right final grade. Similar considerations apply if it turns out that the grade the given examiner had awarded is higher than the median grade. So no examiner has an incentive to 'vote' insincerely.

However, the incentive to vote sincerely is not maintained if the stage of grading every single student is followed by a second stage where, similar to the situation under the MJ procedure, the student obtaining the highest median grade is to receive some prize or be appointed to some office. As illustrated by the following example, once such a competitive second stage is introduced then, in accordance with Gibbard's (1973) and Satterthwaite's (1975) theorems, there may be circumstances where voters operating under the MJ procedure with full knowledge of how the other voters are about to vote may be motivated to vote strategically: in a manner that does not reflect their sincere opinion regarding the grades some candidates really deserve.

3.3 Example Suppose three judges, *1*, *2* and *3*, grade three students, x, y and z, in a competitive examination, to be won by the student having the highest median grade. Let the grades that the judges would award if acting sincerely be as follows:

$$\begin{array}{rccc} & \mathit{1} & \mathit{2} & \mathit{3} \\ x: & D & B & B \\ y: & A & D & A \\ z: & C & A & D \end{array}$$

If all judges act sincerely, then z will win.

However, suppose now that the judges are aware of this distribution of grades. Would any of them have an incentive to change the grades they awarded to the three candidates?

Judge *3* is content with the present outcome, and has no incentive to make any change. But judges *1* and *2*, who would presumably prefer x to win, can achieve this by acting insincerely, either separately or jointly: judge 1 can get x to win by downgrading z from C to A; and judge *2* can achieve this outcome by upgrading x from B to D. Moreover, even if judge *3* suspects that one or both of the other two judges may act insincerely, s/he can do nothing to outwit them.

This simple example demonstrates that the MJ procedure is manipulable if voters, or at least some of them, can obtain information on how other voters intend to vote. Obtaining such information on large electorates is generally not very difficult because opinion polls are usually conducted and published prior to public elections, from which sophisticated voters can glean information that may allow them to see that they would be better off by voting insincerely. In this respect the MJ procedure does not seem to have an advantage over any other voting procedure – it is potentially manipulable like all the rest.

Violation of reinforcement

The *reinforcement axiom*, due to Young (1974), requires that if several disjoint electorates elect the same candidate, this candidate should also be elected, *ceteris paribus*, if the electorates are merged. The following example shows that the MJ procedure violates this desideratum.

3.4 Example Suppose there are three regions, I, II, and III, in each of which 101 voters grade each of two candidates, x and y. The following tables show the distributions of grades. The figure next to a grade is the number of voters awarding that grade.

Region I

$x:$ $21A$, $31B$, $48C$, $1D$,

$y:$ $40A$, $11B$, $48C$, $2D$;

Region II

$$\begin{array}{llllll} x: & 1A, & 46B, & 14C, & 40D, \\ y: & 1A, & 45B, & 33C, & 22D; \end{array}$$

Region III

$$\begin{array}{lllll} x: & 40B, & 20C, & 41D, \\ y: & 48B, & 3C, & 50D. \end{array}$$

If the regions are merged, then the distribution of grades will be

Merged

$$\begin{array}{lllll} x: & 22A, & 117B, & 82C, & 82D, \\ y: & 41A, & 104B, & 84C, & 74D. \end{array}$$

In all four elections the two candidates have equal median grades, so the tie-breaking algorithm needs to be used. The number of iterations required are 2, 7, 2 and 13, respectively.[6] We find that y wins in each of the three separate regions; but when the regions are merged, x wins.[7]

Indifference and abstention

Voters who are indifferent as between all candidates may decide to abstain from voting, or to express their indifference actively. Under the MJ procedure these two courses of action may have quite different effects on the outcome.

3.5 Example Suppose there are two candidates x and y, and five voters. Two of the voters are disgruntled and dislike both candidates equally. Let us assume that if these two voters stay at home, the candidates will have the following distribution of votes:

$$\begin{array}{llll} x: & C, & D, & D, \\ y: & B, & E, & E, \end{array}$$

so y will win. But now suppose the two disgruntled voters decide to express actively their equal loathing of the candidates. Then the distributions of votes will be

$$\begin{array}{llllll} x: & A, & A, & C, & D, & D, \\ y: & A, & A, & B, & E, & E, \end{array}$$

[6] If in those situations addressed in Footnote 3, in which the median is not uniquely defined, we were to choose the higher of the two middle grades instead of the lower, then the number of iterations required to break the ties would be 1, 8, 1 and 14, respectively.

[7] The same outcomes are obtained using the simplified rule presented in the Introduction.

making x the winner.[8]

The same example can be reinterpreted to illustrate an even more bizarre phenomenon. In an experiment conducted by B&L in 2007, they wished to test their proposed MJ procedure in the context of the French presidential elections. Participants were asked to grade the 12 presidential candidates and were informed that a voter failing to grade a candidate would be deemed to have awarded this candidate the lowest 'to Reject' grade (cf. B&L 2007b: Table 7). Presumably, some such convention is necessary because the MJ procedure requires all candidates to have the same number of grades.

Now let us assume that the two disgruntled voters dislike candidate x but have no view at all about y. If they abstain, then y will win. But if they decide to participate, they will award x the lowest grade, A; and they will fail to grade y, and so be deemed to award y the same grade, A. Consequently, x will win. In other words, by participating in the polls, these voters deprive the candidate of whom they have no opinion of victory, and give it to the candidate they positively loath.

A more extreme effect of abstention is a phenomenon known in the social choice literature as the *no-show paradox* (Fishburn and Brams 1983; Moulin 1988). It occurs if one or more voters can obtain a more desirable outcome if they do not participate in the election than if they do and vote sincerely. The *strong version* of this paradox occurs where the voters' most favourite candidate gets elected if they do not participate in the election, but is not elected if they vote sincerely.

A special case of the no-show paradox is the *twin paradox* (Moulin 1988): a group of voters who grade the candidates in exactly the same way obtain a better outcome if some of them do not vote and some vote sincerely. The following example shows that the MJ procedure is vulnerable to the twin paradox in its strong version.

3.6 Example Consider the following election with two candidates and seven voters:

	1	*2*	*3*	*4*	*5*	*6*	*7*
x:	A	A	A	D	E	E	E
y:	B	B	B	C	F	F	F

Here x wins. But now suppose that voters *1* and *2*, both of whom awarded the same grades as voter *3*, and who prefer candidate y, abstain from voting.

[8]This example would also work with one disgruntled voter. We choose to use two in order not to make the outcome depend on the asymmetry in the definition of the median.

Then we get:

$$\begin{array}{rccccc} & \mathit{3} & \mathit{4} & \mathit{5} & \mathit{6} & \mathit{7} \\ x: & A & D & E & E & E \\ y: & B & C & F & F & F \end{array}$$

Thus by abstaining, voters *1* and *2* cause their favourite candidate to win.

Asymmetry of the median

The asymmetry in the definition of the median can affect the outcome even when there are no ties.

3.7 Example Consider two candidates who are graded by four judges as follows:

$$\begin{array}{rcccc} & \mathit{1} & \mathit{2} & \mathit{3} & \mathit{4} \\ x: & B & C & F & G \\ y: & A & D & E & F \end{array}$$

Here y is the outright winner according to B&L. But if we were to take the higher of the two middle grades as the median, then x would have won outright. Moreover, x is arguably the better candidate, as judges *1*, *3* and *4* award x one grade higher than y, while only judge *2* awards x one grade lower than y.

This example can be interpreted somewhat differently, to illustrate *reversal asymmetry*. Suppose that in the grading shown above the ordering of the grades were reversed: instead of $A \prec B \prec \ldots \prec G$ we would have $A \succ B \succ \ldots \succ G$. In that case y would still win, because the lower of x's and y's two middle grades are now F and E respectively, and $F \prec E$. *In other words, a complete reversal of the grades has no effect on the outcome: y wins either way!*

Violation of majoritarianism

The MJ procedure is majoritarian in the rather narrow sense that when a majority of the voters awards a grade G to a candidate, that candidate's median grade is G. Also, the median, which plays a major role in the procedure, is defined (or at any rate definable) in terms of the majority of grades. But it is certainly not majoritarian in the usual sense. Consider the following extremely simple example.

3.8 Example Suppose there are three voters who award the following grades to two candidates:

$$\begin{array}{cccc} & \mathit{1} & \mathit{2} & \mathit{3} \\ x: & B & C & F \\ y: & A & D & E \end{array}$$

Here y has the higher median grade, and so wins according to the MJ procedure. But two out of the three voters grade x above y.

Of course, this is just a toy example, but it is quite easy to illustrate the same effect with a large number of voters: *under the MJ procedure it is always possible that the winning candidate is the one whom an overwhelming majority of the voters grade below all rivals.* For example, if we replace each of voters *1* and *3* by a million clones, then y will still win according to the MJ procedure, although 2 million voters grade x above y, as against a single voter who does the reverse.

Also note that if we insert additional grades between A and B and between E and F, then the difference between the grades awarded by the majority voters (*1*, *3* and their clones) to x and y can be made as large as we please in favour of x. Nevertheless the MJ procedure gets y elected.

Addendum

After submitting this paper for publication, we discovered on the internet at `http://tinyurl.com/andret` a blog – entitled 'On Balinski & Laraki's "majority judgement" median-based range-like voting scheme' – containing a critique of the MJ procedure that is broadly similar to ours. We do not know when this blog was posted on the internet; but as far as we are aware at the time of writing this addendum (February 2009), it has not appeared in print. Nevertheless, we thought we should acknowledge this blog here.

The blog's author, Warren D. Smith, is an ardent advocate of a procedure called *range voting* (RV). This procedure, which is actually used in several non-public elections, requires the voter to award each candidate a (cardinal) numerical grade out of a range of admissible grades. The winner is chosen by lot from among the candidates whose mean grade is highest.

W. D. Smith states (correctly) that, unlike the MJ procedure, the RV procedure satisfies the reinforcement axiom and is immune to the no-show paradox (provided that abstention from grading a candidate counts as awarding that candidate the lowest possible mark). However, while he shows, as we have done in Example 3.8, that the outcome under the MJ procedure may be extremely non-majoritarian – and implies that it should therefore be ‘dismissed from consideration as a voting system’ – he fails to note that the same problem afflicts the RV procedure as well.

To see this, replace in Example 3.8 the (ordinal) grades A, B, C, D, E, F by the (cardinal) grades $1, 2, 3, 6, 7, 8$, respectively. Then the mean grades of candidates x and y are 4.333 and 4.667, respectively; so y wins in this example according to the RV procedure as well – although an absolute majority of the voters grade x above y.

In the absence of empirical data we are unable to tell whether, *ceteris paribus*, a non-majoritarian candidate is more likely to be elected under MJ or under RV. However, in assessing the relative desirability of RV versus MJ, one should also remember that although RV has some advantages over MJ, it also has some disadvantages – the main one being that RV is considerably more prone to strategic voting than MJ.

4 Discussion

In assessing their MJ procedure B&L (2007b: 36) claim that ‘it satisfies almost every criterion that has been advanced across the years to test whether a method of voting is acceptable.’ This seems to us an overstatement: as was shown in the previous section, the MJ procedure is susceptible to various paradoxes, some of them quite serious.

B&L are aware of the vulnerability of the MJ procedure to most, if not all, the paradoxes exhibited in the previous section. But they either dismiss them as unimportant, or consider them as desirable properties. See, for example, their discussion at B&L (2007b: 25) of the reinforcement axiom (which they call ‘winner-consistency’) and of the no-show and related paradoxes.

Voting paradoxes fall into two broad types. The first consists of ‘what-if’ paradoxes: a hypothetical change in the input data of an election (modification of voters’ preferences or behaviour, addition or removal of a candidate or a voter, redistricting) results in a counter-intuitive or undesirable change in the outcome.

Paradoxes of the second type do not relate to a change in the election input data, but occur if some such data produce counter-intuitive or undesirable outcomes.

Vulnerability of the MJ procedure to ‘what-if’ paradoxes makes it open to manipulation by voters, as in the case of insincere grading (Example 3.3) and abstention (Examples 3.5 and 3.6); or to gerrymandering, as in the case of violation of reinforcement (Example 3.4).

Of these, the most serious in our view is the no-show paradox.[9] As argued by Nurmi (1999: 53):

> 'Vulnerability to the no-show paradox is a serious drawback in a voting system. After all, any reasonable voter would expect that by voting he is contributing to the possibility that his favorite wins. The realization that the very act of communicating his true preferences makes the outcome worse from his point of view than it would have been had he decided not to vote at all, may be demoralizing. It certainly undermines the very rationale of going to the polls.'

Let us now turn to paradoxes of the second type that afflict the MJ procedure.

The anomalies that arise due to the slight asymmetry in the definition of the median (Example 3.7) are, in our opinion, not of very great concern. Although we do not offer any quantitative theoretical or empirical estimate as to the likelihood of their occurrence, it is quite clear that if the number of voters is large, it is very improbable that the median grade of a candidate will not be uniquely defined (either initially or in the course of applying the tie-breaking algorithm). And even if this happens, it does not necessarily affect the final outcome, as can be seen from working out the four sets of data in our Example 3.4.[10]

But the paradox illustrated in our Example 3.8 is quite another matter. In our view it is the most serious problem afflicting the MJ procedure. Most social-choice theorists as well as most ordinary members of the public would be reluctant to favour a procedure that may fail to elect a candidate whom a great majority of the voters grade above all rivals, *and can do so even if there is only one rival.* B&L will need to produce very powerful counter-arguments to overcome this reluctance.

A general comment may be in order here. An underlying reason for the various voting paradoxes is that the input data of an election contain much more information than the outcome. Thus a great deal of information must get lost when the input is aggregated to produce the outcome. For example, in preferential voting procedures (in which a voter is required to order the candidates in an order of preference), when there are just three candidates there are six possible orderings (or 13, if indifference among candidates is

[9] By the way, this paradox afflicts many voting procedures (including all Condorcet-consistent voting procedures when there are more than three candidates and at least 25 voters), but all scoring (positional) procedures (such as plurality and Borda's count) are immune to it.

[10] See Footnote 6.

allowed). So the electorate is partitioned into six (or 13) classes according to preference. The relevant data are the relative size of each class in the total electorate. This information is given by five independent numbers (or 12, if indifference is allowed). But the outcome consists of just one piece of information: the name (or numerical label) of the winning candidate. Loss of information is inevitable; and it is greatly exacerbated when there are more candidates.

The MJ procedure requires an input that contains considerably *more* information than the merely preferential procedures. The latter require from a voter no more than an ordering of the candidates in order of preference. In fact, all that is used as input is the proportion of voters that have a given order of preference. The MJ procedure requires, and makes some use of, more than that: a voter is asked not merely to state whether s/he prefers x to y but for a more nuanced information. There are several different graded ways in which x can be preferred to y: for example, x is excellent and y is very bad, or x is not so bad and y is not so good, etc. (The same applies, *mutatis mutandis*, also to the RV procedure mentioned in the Addendum of Section 3. In fact, this procedure requires as input even more information than MJ, because cardinal grades are more nuanced than merely ordinal ones!)

The strengths of the MJ procedure derive largely from the fact that it makes use of such more nuanced information. But this is also a source of weakness, which is revealed most starkly when there are just two candidates.

Arrow's Theorem applies to preferential procedures and assumes that there are at least three candidates. But these procedures can of course be used when there are only two candidates. Arrow's Theorem does not hold in this exceptional case, because then preferential procedures use as input an exceptionally small amount of information, given by a single number: the proportion of voters who prefer the first candidate to the second. Because of this, no information need be lost in aggregating the input, and therefore these procedures need not display a paradox in this case; in fact, all the commonly used procedures elect the candidate preferred by a majority of the voters.

But the MJ procedure requires as input considerably more information even when there are just two candidates. (And the same applies also to the RV procedure.) Therefore some information does get lost in aggregation, and – as we have shown in Example 3.8 – the lost information may be: which of the two candidates is supported by a majority of the voters.

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