Interval Effects:

Superadditivity and Subadditivity in Intertemporal Choice

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Abstract

Time discounting is influenced both by the delay to outcomes, and by the interval separating them. In contrast to the effects of delay, interval effects have received relatively little research attention. Previous research has shown that for intervals of moderate length, the rate of discounting decreases as intervals get longer (subadditive discounting). In this paper we show that, in addition, for short intervals the rate of discounting increases as intervals get longer, implying a U-shaped relationship between discounting and interval length. Superadditive discounting is shown in two studies. In Experiment 1, we show that short intervals are more likely to give rise to superadditive than subadditive discounting. In Experiment 2, we show that discounting for short intervals is lower than that for intervals of moderate length, but that discounting for moderate-length intervals is greater than that for long ones. In the discussion we place these findings in a broader context.

When given a choice between two possible times at which an outcome can occur, people usually prefer to receive it earlier if it is a good outcome, and later if it is a bad one. This phenomenon of *time discounting* has been the subject of much research, which has revealed how the strength of the preference for earlier outcomes over later ones is influenced by a host of factors, including the nature of the outcomes, how they are described, and their timing. The ultimate goal of research into time discounting has been to develop a *behavioral discount function* that characterizes how the value of an outcome changes as a function of when it will occur.

We will discuss discount functions by referring to the relative value of outcomes that occur at two different times, as in $100 to be received in either 12 or 18 months. We use d to denote the *delay* to the earlier outcome and t to denote the *interval* separating the outcomes. In the example d = 12 months and t = 6 months. The discount function, denoted F, gives the value of an outcome at the end of the interval, $d+t$, as a fraction of its value at the beginning of the interval, d (Laibson, 2002):

$$F(d,t) = \frac{v(x,d+t)}{v(x,d)}, \tag{1}$$

where $v(x,d)$ and $v(x,d+t)$ indicate the value of x when delayed by d and $d+t$ units of time, respectively. The discount factor, denoted δ, gives this fraction as an average per unit of time:

$$\delta(d,t) = F(d,t)^{1/t}. \tag{2}$$

It is this standardized δ that enables us to compare the discounting that occurs over intervals of different length.

Using the above terminology, *exponential* discounting, often regarded as the normatively correct discount function (e.g., Strotz, 1955; see discussion in Read, 2004), is defined as:

$$F(d,t) = \frac{(1+r)^d}{(1+r)^{d+t}} = \left(\frac{1}{1+r}\right)^t; \delta(d,t) = \left(\frac{1}{1+r}\right)^{\frac{t}{t}} = \frac{1}{1+r}.$$

where $r>0$. Exponential discounting is defined by two psychological properties: *delay independence* (δ is not influenced by d) and *interval independence* (δ is not influenced by t). As an illustration of these, delay independence means the *proportional* loss in the value of $100 will be the same over the intervals 0→12 months and 12→24 months, while interval independence means the *total* loss in value will be the same over the successive subintervals 0→12 months and 12→24 months as over the undivided interval 0→24 months.

The delay independence assumption has long been recognized as unrealistic (Samuelson, 1937; Strotz, 1955), so it is rarely incorporated into descriptive models, which are based on *hyperbolic* discounting (Ainslie, 1975; Laibson, 1997; Loewenstein & Prelec, 1992; O'Donoghue & Rabin, 1999). The defining property of hyperbolic discounting is that δ increases with *d* (the *delay* effect) which entails that, for instance, someone who is indifferent between \$100 now and \$150 in 12 months (δ=0.67) will prefer \$150 in 24 months to \$100 in 12 months (implying δ>0.67)[1]. The following function, proposed by Mazur (1987), is the most widely cited hyperbolic discount function:

$$F(d,t)=\frac{1+kd}{1+k(d+t)};\delta(d,t)=\left(\frac{1+kd}{1+k(d+t)}\right)^{\frac{1}{t}}.$$

Where $k>0$ is a discounting parameter.

Conventional hyperbolic discounting models, such as Mazur's, incorporate interval independence. To see this, compare the effect of discounting over two successive intervals, first from d→d+t, and then from d+t→d+t+s, with the effect of discounting over the undivided interval d→d+t+s (as shown in Figure 1). The effect of discounting over the shorter intervals *t* and *s* will be identical to that of discounting over the undivided interval *t+s*:

$$F(d,t)\times F(d+t,s)=\frac{1+kd}{1+k(d+t)}\times\frac{1+k(d+t)}{1+k(d+t+s)}=F(d,t+s).$$

This is *additive* discounting (Read, 2001).

Insert Figure 1 about here

The empirical adequacy of the interval independence assumption has been challenged by a number of recent studies. A frequently reported finding is that δ is lower (i.e., there is more discounting) when an interval is divided into shorter subintervals (e.g., Baron, 2000; Pender, 1996; Read, 2001; Read & Roelofsma, 2003). This pattern is called *subadditive* discounting: The decision weight assigned to a time interval increases when it is divided into subintervals and separate judgments are made for each subinterval. Or, in symbols:

$$F(d,t)\times F(d+t,s)<F(d,t+s).$$

For instance, someone who is indifferent between \$100 now and \$150 in 12 months and between \$150 in 12 months and \$200 in 24 months (a loss in value of \$100 over a subdivided 24 month delay) will usually prefer \$200 in 24 months to \$100 now (implying a loss in value of less than \$100 over an undivided 24 month delay).

The term *subadditive* discounting was chosen in part because of the resemblance of this phenomenon to subadditive *probability* weighting (Starmer & Sugden, 1993), in which a given probability receives more weight if it is subdivided into a set of independent constituents. Subadditivity is not, however, the only characteristic of probability weighting, and it might not be the only interval effect either. It is well established that differences in outcome probabilities are given less than proportional weight if they do not exceed a significance threshold (Tversky, 1969; Kahneman & Tversky, 1979). For instance, a 51% chance of something happening is treated the same as a 52% chance. Both Rubinstein (1988) and Leland (1994) have used such threshold effects to explain anomalies in risky choice and, recently, have extended this analysis to intertemporal choice (Rubinstein, 2001, 2003; Leland, 2002). One implication of their perspective is that small differences between delays may be given less than proportional weight. Therefore, when an interval is divided into short, sub-threshold intervals we might observe *superadditive* discounting, where δ is higher (less discounting) for the divided interval than for the undivided one.

More formally, this suggests that the effect of the interval t, in the discount function $F(d,t)$, has two regions separated by a significance threshold t_ε. Below this threshold, F decreases at an increasing rate (yielding superadditivity), and above the threshold it decreases at an increasing rate (subadditivity). An example of such a function is shown in Figure 2.1. It has an inverted S-shape, concave over the range of sub-threshold intervals, and convex thereafter, with an inflection point at t_ε. Figure 2.2 shows the corresponding values of δ for this function, which has an interval of superadditivity (when δ is decreasing in interval length) followed by subadditivity (when δ is increasing in interval length).

Insert Figure 2 about here

Although the purpose of this paper is not to advocate a particular discount function, Figure 2 depicts a variant of a generalized logistic function (Johansson, 1973) that captures the delay effect and both interval effects:

$$F(d,t) = \frac{1}{1+b\left[(d+t)^a - d^a\right]^c};\delta(d,t) = \left(\frac{1}{1+b\left[(d+t)^a - d^a\right]^c}\right)^{\frac{1}{t}}, \tag{3}$$

where $0<a<1$, $b>0$, and $c>1$. Each parameter specification captures one aspect of how intervals are evaluated[2]. The specification for a captures the delay effect: The impact of the interval t is reduced the later it begins. That is, the same 3-week interval will lead to more discounting when it begins in 1 week than when it begins in 15 weeks. The specification for b captures the threshold level, with lower values of b corresponding to higher thresholds[3]. Finally, the specification for c gives rise to the characteristic inverted S-shaped F-function, yielding both superadditivity and subadditivity. In short, therefore, Equation 3 summarizes the major results we expected to obtain in our experiments. First, we expected the delay effect (hyperbolic discounting), with more discounting over early intervals than over late ones. Second, we expected superadditivity for comparisons between short and medium-length intervals, and subadditivity for comparisons between medium-length and long intervals.

Below, we report two experiments that show evidence for both subadditivity and superadditivity. Experiment 1 was a small-scale choice study in which we investigated short to medium-length intervals. We examined whether, among choice patterns revealing interval effects, superadditive patterns were more frequent than subadditive ones. Experiment 2 was a choice-titration study in which we took exact measures of δ over short, medium, and long intervals to test the hypothesis of superadditivity and subadditivity.

Experiment 1

Method

The purpose of Experiment 1 was to determine if choices between options separated by small to medium differences in delay would reveal patterns consistent with superadditivity -- choices of larger-later (LL) options for small differences in delay, and choices of smaller-sooner (SS) options for larger differences. The option pairs used in the current study are given in Table 1, together with the compound interest rate for each option pair.

Insert Table 1 about here

A random order of the option pairs was determined and a Latin square of six different orders was designed on the basis of that random order. Six different questionnaires were prepared in accordance with the Latin Square. The questionnaires presented each option pair on a separate page, in the following format:

	Option 1	**Option 2**
You receive	£500	£525
When	1 week from today	2 weeks from today
Your choice:	❒	❒

The respondents, 120 students from the London School of Economics, checked the box corresponding to the option they preferred.

We hypothesized that the short intervals of Experiment 1 would generally be below the threshold t_ε and so any interval effects observed, would be more likely to take the form of superadditivity than subadditivity.

Results

Classification of choice patterns and frequency of their occurrence. Given all possible dyadic choices between four options, there are 64 possible choice patterns. We can divide these, following the method of Roelofsma and Read (2000), into 24 transitive patterns (T-patterns) and 40 intransitive ones (I-patterns). An I-pattern is one that contains *at least* one preference cycle. To illustrate the difference, consider the following four preference relationships between A, B, and C:

B$\succ$A, C$\succ$B, and C$\succ$A (T-pattern, willing to wait)

B$\succ$A, C$\succ$B, but A$\succ$C (I-pattern, superadditivity)

A$\succ$B, B$\succ$C, and A$\succ$C (T-pattern, wants it now)

A$\succ$B, B$\succ$C, but C$\succ$A (I-pattern, subadditivity).

It is only in I-patterns that we can *observe* subadditivity and superadditivity, although the underlying preferences may exist without being observed[4]. Therefore, our attention is focused on I-patterns. In total, out of the 120 questionnaires, fully 84 were T-patterns, primarily because most respondents restricted their choice to SS or LL alone.

The I-patterns can be divided into those that exhibit (i) superadditivity, (ii) both superadditivity and a delay effect, (iii) subadditivity, (iv) both subadditivity and a delay effect, and (v) an anomalous pattern. A *delay effect* occurs when, for an interval of given length, SS is preferred to LL for an early interval, but LL is preferred to SS for a later interval. An example would be when A$\succ$B, but C$\succ$B. *Subadditivity* occurs when, as in the example above, for an interval of given length that can be divided into subintervals, SS is preferred to LL for all its component subintervals, but LL is preferred to SS for the undivided interval. *Superadditivity* occurs when, as in the example above, LL is preferred to SS for each subinterval of an interval, but SS is preferred to LL for the undivided interval. The *anomalous* I-patterns are those that violate the delay effect, i.e., when, for an interval of given length, LL is preferred to SS for an early interval, but SS is preferred to LL for a later interval. Table 2 depicts all the (theoretically) consistent I-patterns (those in categories i to iv) observed in this study, and their frequency of occurrence.

Insert Table 2 about here

The occurrence rates of superadditivity and subadditivity were 12 and 5, respectively, so superadditive patterns were more than twice as frequent as subadditive ones. However, we should take into account that the base rate is different for superadditivity, which is revealed by 9 choice patterns, and subadditivity, which is revealed by only 7 choice patterns. Moreover, we should take into account that choice behavior has error associated with it, so that a participant may have ‘accidentally’ ended up in the superadditive category, the subadditive category, or indeed the anomalous category. We therefore relate the frequency with which each of these categories is observed to the frequency with which it can be expected to be observed by chance alone. Specifically, for each category, we calculate:

$$Expected\ \ frequency = \frac{Number\ of\ \ choice\ \ patterns\ in\ category}{Total\ \ number\ of\ \ choice\ \ patterns} \times Number\ of\ \ participants$$

Table 3 provides the number of choice patterns, the expected and observed frequency, and the χ^2-deviation between the expected and observed frequency for each category. As can be seen, anomalous I-patterns occurred less often than expected, subadditivity occurred as often as expected, while superadditivity occurred more often than expected. This result was significant, $\chi^2(2)=5.48$, $p=.06$.

Insert Table 3 about here

Experiment 1 was not definitive. The major result is that for short to medium-length intervals, violations of interval independence more commonly take the form of superadditivity than subadditivity. We did not take measures of the rate at which outcomes are discounted over intervals of varying length, nor did we test for both superadditivity and subadditivity over a wider range of interval lengths. In Experiment 2 we took direct measures of δ over short, medium-length, and long intervals.

Experiment 2

Method

Repeated choices were made between pairs of options, SS and LL, with the outcome of one option, x_{SS} or x_{LL}, being adjusted following each choice until an indifference point, meaning a point at which the two options were equal in value, was reached. The titration procedure is described in the Appendix. At the indifference point, the discounted value of SS equals the discounted value of LL and so, applying Eqs. (1) and (2), the discount factor is obtained as follows:

$$\delta(d,t)=\left(\frac{x_{SS}}{x_{LL}}\right)^{\frac{1}{t}}. \qquad (4)$$

Design and hypotheses. We tested nine option pairs according to the design in Figure 3. There were three interval lengths: *Short* intervals of one week, *medium* intervals of three weeks, and a *long* interval of 17 weeks. Each medium interval was spanned by three short intervals. There was a medium interval at the beginning and end of the long interval. The discount factors, as obtained with Eq. (4), are denoted $\delta_j(d,t)$, where d takes on values e (early) or l (late), t takes on values s (short), m (medium), or l (long), and the subscript j indicates the order of occurrence of the three short intervals spanning a medium interval.

Insert Figure 3 about here

Our first hypothesis was that there would be a delay effect, meaning that holding interval length constant, earlier intervals would show lower values of δ than would later intervals:

H1: $\delta(e,s)<\delta(l,s)$;

$\delta(e,m)<\delta(l,m)$.

Hypothesis 2 was our major focus. We predicted that δ would be higher for short intervals than for medium ones (superadditivity) and lower for medium intervals than for long ones (subadditivity):

H2: $\delta(\cdot,s)>\delta(\cdot,m)<\delta(\cdot,l)$.

We also tested *strong* superadditivity and subadditivity, which would occur if the two interval effects outweigh the delay effect. We describe these hypotheses with reference to Figure 1, which shows a discounting interval $d \rightarrow d+t+s$, both undivided and divided into two subintervals, $d \rightarrow d+t$ and $d+t \rightarrow d+t+s$. By the delay effect alone, δ is higher over the later interval than over the earlier one: $\delta(d+t,s)>\delta(d,t)$. Therefore, if two intervals of different length begin at the same time, the delay effect implies that δ will be higher for the longer one: $\delta(d,t+s)>\delta(d,t)$. The superadditivity hypothesis, however, predicts that, when the shorter of the two intervals is below the threshold, δ will be higher for the shorter one: $\delta(d,t)>\delta(d,t+s)$. The *strong* superadditivity hypothesis is that superadditivity outweighs the delay effect, meaning that, for two intervals of different length that begin at the same time, the shorter interval will yield a higher delta than the longer one. In relation to Figure 3:

H2a: $\delta_1(e,s)>\delta(e,m)$;

$\delta_1(l,s)>\delta(l,m)$.

By an analogous argument, if two intervals of different length end at the same time, the delay effect implies that δ will be higher for the shorter interval: $\delta(d+t,s)>\delta(d,t+s)$. The subadditivity hypothesis, however, predicts that, when the two intervals are above the threshold, δ will be higher for the longer one: $\delta(d,t+s)>\delta(d+t,s)$. The *strong* subadditivity hypothesis is that subadditivity outweighs the delay effect, meaning that, for two intervals of different length that end at the same time, the longer interval will yield a higher delta than the shorter one. In relation to Figure 3:

H2b: $\delta(\cdot,l) > \delta(l,m)$.

Participants and procedure. The participants were 53 students from the London School of Economics who were paid £5. Experimental sessions were run in a computer room with several participants at a time. The participants were seated at a desk with an IBM-compatible computer. An instruction sheet informed them they could choose between (hypothetical) amounts of money available at different times. The instruction sheet illustrated how the situations would be presented on the computer screen. The participants were asked to attend closely to the amounts of money as well as the amounts of time, because both would frequently change throughout the experimental session. The participants were also instructed how to respond: The left and right arrow keys were used to make choices, the down arrow key confirmed choices, and the up arrow key allowed them to correct mistakes.

Before the experimental session, participants completed a series of practice trials starting with £400 in 26 weeks and £500 in 52 weeks. This was followed by 18 series of experimental trials, comprising 2 replications of the 9 option pairs.

Results

Admission of participants. Our analyses are conducted on the results from 42 participants. The remaining results were not used for two reasons. Firstly, four participants chose SS on the first trial for at least one interval. Our titration task was designed to elicit a choice of LL on the first trial for any reasonable level of discounting (demanding simple interest of less than 100% *per week*). Secondly, seven participants did not display *weak monotonicity*, meaning that, on at least one occasion, they demanded more compensation for a shorter interval than for a longer one when the shorter interval was a subset of the longer one. To illustrate, weak monotonicity would be violated by a participant who was indifferent between £500 and £750 over the medium interval 1→4 weeks, but was indifferent between £500 and £800 over the short interval 1→2 weeks. The survival rate of 79% of the sample is typical of those reported in other studies (e.g., 81% in Ahlbrecht & Weber, 1997, 72% in Benzion, Rapoport, & Yagil, 1989, and 84% in Shelley, 1993).

Insert Figure 4 about here

The delay effect. Figure 4.1 displays the geometric means of δ for early and late, short and medium intervals. In support of **H1**, when interval length was held constant δ was higher for late intervals than for early ones. This was confirmed by an ANOVA with log(δ) as the dependent measure[5] and interval onset (early versus late) and interval length (short versus medium) as within-participant factors, which revealed that the main effect of interval onset was significant, $F(1,41)= 7.28$, $p=.01$, $\eta^2=.15$.

In addition to the delay effect, superadditivity manifested itself, in that δ was higher for short intervals than for medium ones. The main effect of interval length was significant, $F(1,41)=6.39$, $p=.02$, $\eta^2=.14$.

Superadditivity and subadditivity. Figure 4.2 depicts the geometric means of δ for short, medium, and long intervals. In support of **H2**, δ was higher for short intervals than for medium ones (superadditivity) and lower for medium intervals than for long ones (subadditivity), $\delta(\cdot,s)> \delta(\cdot,m)<\delta(\cdot,l)$, confirming the U-shape relationship between interval length and discount factors. In an ANOVA with interval length (short versus medium versus long) as a within-participant factor, the effect of interval length was significant, $F(2,82)= 8.43$, $p=.00$, $\eta^2=0.17$. Moreover, the quadratic contrast, which provides a direct test of **H2**, was also significant $F(1,41)=11.23$, $p=.00$, $\eta^2=.22$.

Strong superadditivity. In support of **H2a**, δ was higher for a short interval than for a medium one when both began equally early, $\overline{\delta_1(e,s)}=.92$ and $\overline{\delta(e,m)}=.90$, or equally late, $\overline{\delta_1(l,s)}=.96$ and $\overline{\delta(l,m)}=.94$, confirming strong superadditivity. In an ANOVA with interval length (short versus medium) and interval onset (early versus late) as within-participant factors, the main effect of interval length was marginally significant, $F(1,41)=3.58$, $p=.07$, $\eta^2=.08$.

While outweighed by superadditivity, the delay effect *did* manifest itself, in that δ was higher for late intervals than for early ones of the same length. The main effect of interval onset was significant, $F(1,41)=6.51$, $p=.01$, $\eta^2=.14$.

Strong subadditivity. In support of **H2b**, δ was lower for a medium interval than for a long one when both ended equally late, $\overline{\delta(l,m)}=.94$ and $\overline{\delta(\cdot,l)}=.96$, confirming strong subadditivity. In an ANOVA with interval length (medium versus long) as within-participant factor, this effect was significant, $F(1,41)=4.11$, $p=.05$, $\eta^2=.09$.

Discussion

Models of the discounting process usually build on the assumption that people choose between delayed outcomes by first computing the discounted value of each option, and then choosing the one with the highest value. Interval effects pose a challenge to this view, suggesting that people take into account not only the delay to outcomes, but also the interval between them. Consequently, the value put on outcomes depends not only on their temporal proximity to the present (delays), but also on their temporal proximity to one another (intervals). Superadditive intertemporal choice suggests that people underweight short (i.e., sub-threshold) intervals relative to moderate length ones, while subadditive discounting suggests that they overweight moderate length intervals relative to longer ones. Combined, the two interval effects suggest that there are two psychologically relevant interval lengths in the discounting process: A threshold toward which the decision weight assigned to an interval is marginally increasing (superadditive discounting) and a time horizon toward which the decision weight assigned to an interval is marginally decreasing (subadditive discounting). The threshold is the point below which people tend to overlook that one outcome occurs further ahead into the future than another. The time horizon is the point beyond which they tend to stop looking further ahead into the future (cf. Rachlin, Siegel, & Cross, 1994).

The pattern of superadditivity followed by subadditivity bears resemblance to findings from research on the pain of waiting (Heuter & Swart, 1998), showing that people ignore, or at least assign less than proportional weight to, waiting times that do not exceed a significance threshold, but that, once beyond this threshold, there is a rapid increase in intolerance, which reaches a ‘resignation’ plateau as the waiting time is prolonged. Undoubtedly, the critical points in the weighting of time depend on context, with factors like expectations and outcome characteristics playing important moderating roles. For instance, we might expect that the threshold will be greater for larger outcomes than for smaller ones. This is consistent with our intuitions about threshold effects while waiting for different outcomes: It might take 20 minutes for us to become restive when sitting in a fine restaurant, 4 minutes at a fast food restaurant, 2 minutes at a bus stop, but months or even years when waiting for the delivery of a luxury yacht (a hypothesis we have not personally tested). While the behavioral discount function developed in this paper captures all known violations of delay and interval independence in time discounting, more work is needed to capture such violations of *outcome* independence.

The results in this paper, along with many early ones, argue for a fundamental rethinking of intertemporal choice. When people are asked how they make choices (intertemporal or otherwise) one of the first things they say is that they make *comparisons* between dimensions of choice. It is surprising, then, that the comparative nature of decision making has not been incorporated into most models of intertemporal choice. Interval effects are the product of such a comparison process, since the interval is itself a comparison between two delays. This comparison process is reflected in the discount function suggested above (Eq. 3). But this function is still conceptualized within the discounting paradigm. We suggest that a more radical move is needed, one that views intertemporal choice as the result of weighting differences between delays directly against differences between outcomes. In such a model, the very notion of discounting would disappear.

Appendix: Choice titration procedure

(NB. As explained in the text, the few participants who, despite the simple interest of 100% per week, chose SS on the first trial were discarded from the analyses. To simplify exposition, therefore, we restrict our description of the titration method to the case of those who first chose LL.)

The starting point was chosen to elicit a choice of LL over SS by any reasonable participant. The initial smaller-sooner amount (x_{SS}) was always £500, while the larger-later amount (x_{LL}) was x_{SS} plus £500 times the number of weeks separating the outcomes. For example, the first trial for the early, medium-length interval was:

Amount	£500	£2,000	[500+(3×500)]
When received	in 1 week	in 4 weeks	

For the short intervals of 1 week, x_{LL} was £1,000 [500+(1×500)]; for the long interval of 17 weeks, x_{LL} was £9,000 [500+(17×500)].

From this point on, we used a 'split the difference' procedure to get close to the indifference point. The rules for the procedure can be summarized as follows:

1. Before the first choice of SS, a choice of LL meant that x_{LL} was reduced by half the difference between value it took on the previous trial and £500. To illustrate, after the choice of LL for the example above, the following adjustment would be made:

Amount	£500	£1,250	[2,000-(2,000-500)/2]
When received	In 1 week	In 4 weeks	

2. When SS was chosen, x_{LL} was increased by half the difference between its value on the previous trial, and value it had when LL was last chosen. To illustrate, if SS was chosen for the example,

Amount	£500	£1,625	[1,250+(2,000-1,250)/2]
When received	In 1 week	In 4 weeks	

3. After the first choice of SS, a choice of LL meant that meant that x_{LL} was reduced by half the difference between the value it took on the previous trial and value it had when SS was last chosen. To illustrate, a choice of LL would now lead to:

Amount	£500	£1,435	[1,625-(1,625-1,250)/2, rounded down]
When received	In 1 week	In 4 weeks	

As shown in the last example, x_{LL} was rounded to the nearest multiple of £5, with a random upward or downward adjustment of £2.50 if there were two nearest. This split the difference procedure continued until the difference being split was less than £5. If this point was reached before SS was ever chosen, the value of x_{LL} would therefore be less than £505 and the procedure was ended. If SS had been chosen at least once, however, we then switched to a 'unit change' procedure, where x_{LL} was adjusted by £1 at a time until the difference between the value x_{LL} took the last time LL was chosen, and the value it took the last time SS was chosen, was less than £1. The indifference point was estimated as the midpoint between the most recent value of x_{LL} that was chosen and rejected.

Endnotes

[1] The empirical support for the delay effect is ambiguous. It is seen in some studies (Green, Fristoe, & Myerson, 1994; Keren & Roelofsma, 1995; Kirby & Herrnstein, 1995), but not, or not reliably, in others (Ahlbrecht & Weber, 1997; Baron, 2000; Holcomb & Nelson, 1992; Read, 2001; Read & Roelofsma, 2003).

[2] The function is quite general, and specific parameter combinations can give rise to a wide range of discounting phenomena. When a=1, there is no delay effect, i.e., $F(d,t)=F(0,t)$. When b=0, there is no discounting, i.e., $F(d,t)=1$. When $0<c\leq 1$, there is subadditive discounting only, i.e., $F(d,t) \times F(d+t,s) < F(d,t+s)$. Notably, when both a=1 and $0<c\leq 1$, our function reduces to Read's (2001) formulation.

[3] The inflection point (where the F-function changes from concave to convex) is given by:

$$t_{\varepsilon} = \left[\left(\frac{1}{b} \cdot \frac{c-1}{c+1} \right)^{\frac{1}{c}} + d^{a} \right]^{\frac{1}{a}} - d.$$

Although t_{ε} is a function of a, b, c, and d, it is primarily affected by b. Note that, because the above expression is always positive, and increasing in d whenever $0<a<1$, $b>0$ and $c>1$, it follows that the threshold point is higher the later an interval begins.

[4] For instance, someone who has a weekly δ of less than .97 over the first week and the second week, when they are evaluated individually, but a weekly δ of .96 when the two weeks are combined, would nonetheless always choose LL in this experiment.

[5] We use log(δ) in our ANOVAs because it does not distort the functional relationship between δs. For instance, δ=.8 denotes twice the patience of δ=.4, while δ=.4 denotes twice the patience of δ=.2. When using logs, this functional relationship is retained, because log(.8)-log(.4)=log(.4)-log(.2).

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Table 1. Stimuli of choice study: Option pairs and compound interest rates.

Option pairs		Compound interest rates (per week)
LL	**SS**	
A = £500 in 1 week	**B** = £525 in 2 weeks	5.0%
B = £525 in 2 weeks	**C** = £550 in 3 weeks	4.8%
C = £550 in 3 weeks	**D** = £575 in 4 weeks	4.6%
A = £500 in 1 week	**C** = £550 in 3 weeks	4.9%
B = £525 in 2 weeks	**D** = £575 in 4 weeks	4.7%
A = £500 in 1 week	**D** = £575 in 4 weeks	4.8%

Table 2. Results of choice study: Classification of consistent I-patterns.

Delay effect	Superadditivity					Subadditivity				
	Option	Option B	Option C	Option D	*N*	Option	Option B	Option C	Option D	*N*
Yes	A	LL	SS	LL		A	SS	LL	LL	
	B		LL	LL		B		SS	LL	
	C			LL	0	C			LL	0
	A	LL	SS	SS		A	SS	SS	LL	
	B		LL	LL		B		SS	SS	
	C			LL	0	C			LL	0
	A	SS	LL	SS		A	SS	SS	LL	
	B		LL	LL		B		SS	LL	
	C			LL	2	C			SS	2
	A	SS	SS	LL		A	SS	SS	SS	
	B		LL	SS		B		SS	LL	
	C			LL	0	C			SS	0
	A	SS	SS	SS						
	B		LL	SS						
	C			LL	5					
	A	SS	LL	SS						
	B		SS	LL						
	C			LL	0					
No	A	LL	LL	SS		A	SS	LL	LL	
	B		LL	LL		B		SS	LL	
	C			LL	2	C			SS	2
	A	LL	SS	LL		A	SS	LL	SS	
	B		LL	SS		B		SS	LL	
	C			LL	1	C			SS	0
	A	LL	SS	SS		A	SS	SS	LL	
	B		LL	SS		B		SS	SS	
	C			LL	2	C			SS	1
				Total	12				**Total**	5

A delay effect occurs when there is a shift from SS to LL when moving down on one or both diagonals. Interval effects can be detected by looking at 'triangles' of responses - two or three items on any diagonal, and a pivot item on the same row as the first of these items and the same column as the last (e.g., the diagonal items BC and CD, and the third item BD). Intransitivity occurs when all diagonal items differ from the pivot item.

Table 3. Results of choice study: Interval effects (I-patterns).

Category statistics	Choice patterns			Total
	Superadditivity	Subadditivity	Anomalous	
No. of choice patterns	9	7	24	40
Expected frequency	6.75	5.25	18	30
Observed frequency	12	5	13	30
χ^2-deviation	4.08	0.01	1.39	5.48

$$\chi^2 - deviation = \frac{(Observed\ frequency - Expected\ frequency)^2}{Expected\ frequency}.$$

Figure 1. A discounting interval, both undivided and divided into two subintervals.

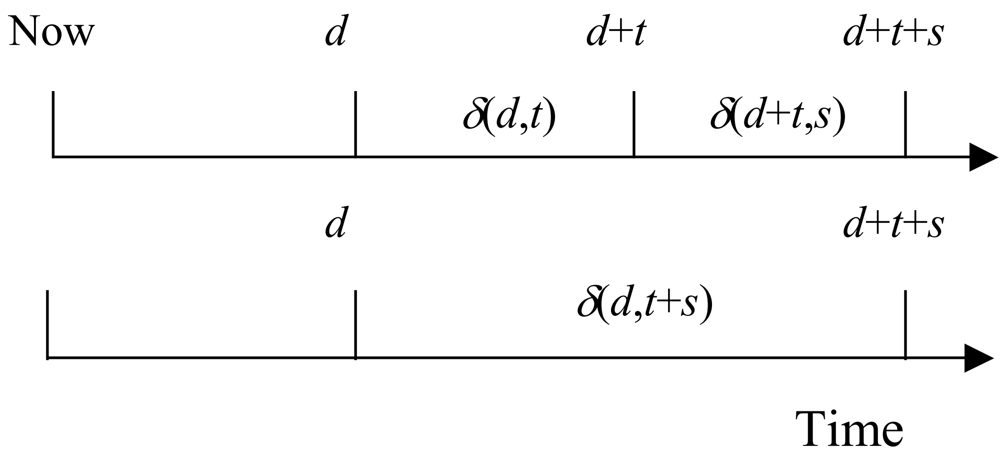

Figure 2.1. Discount functions *F* over intervals of different length.

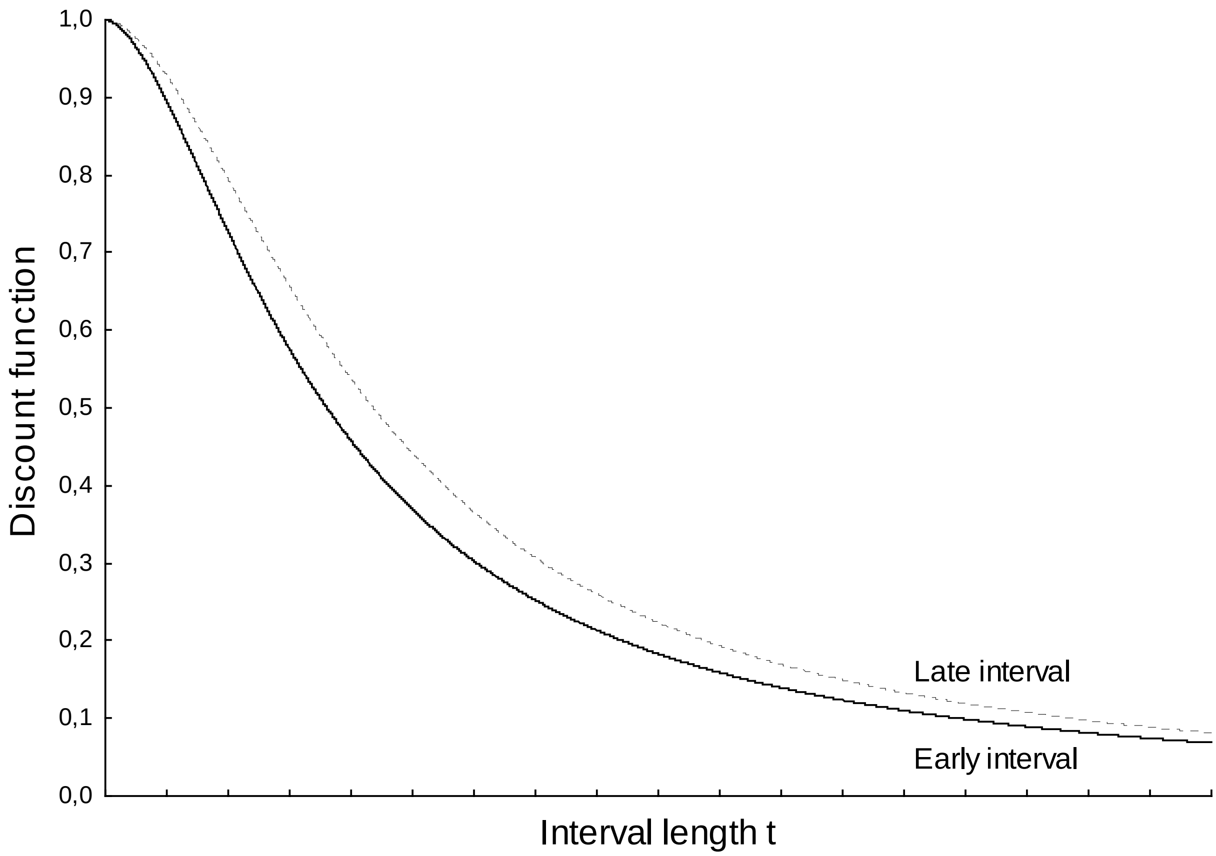

Figure 2.2. Discount factors δ over intervals of different length.

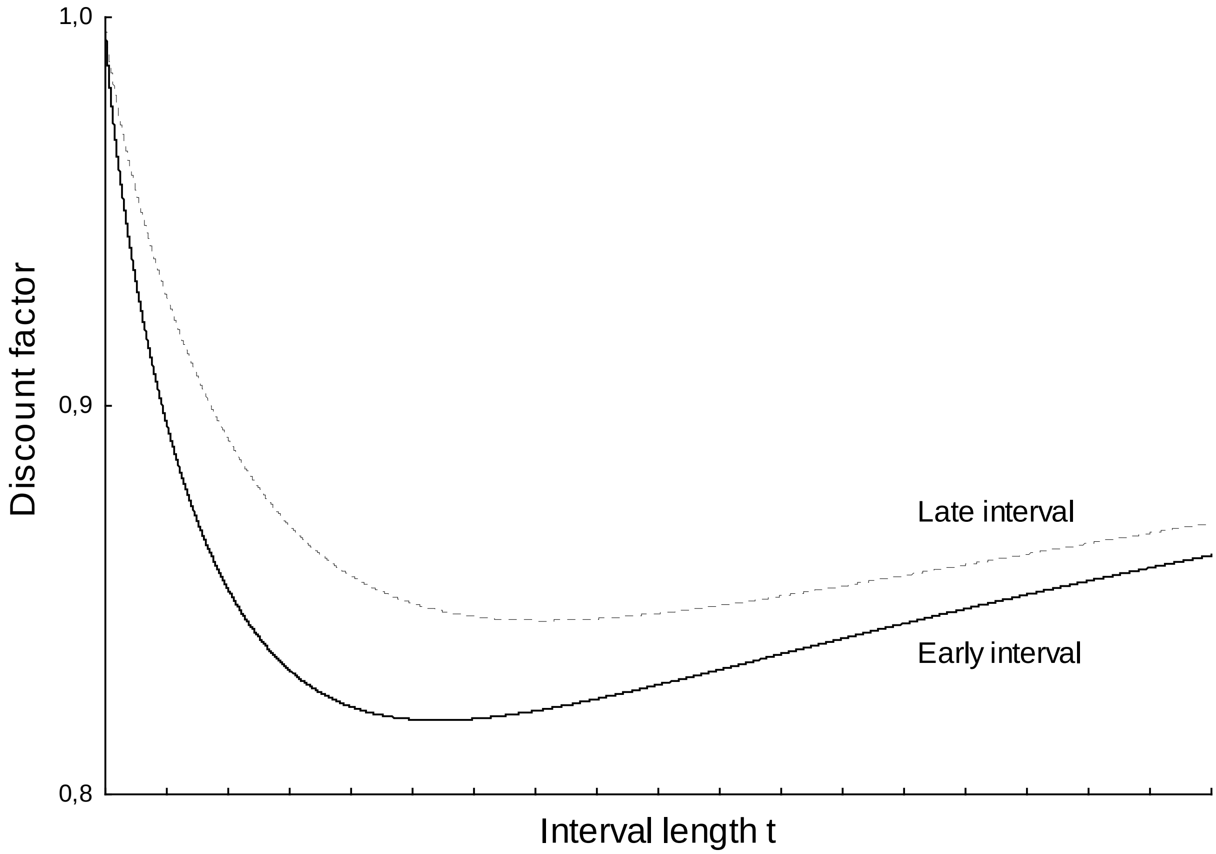

Figure 3. Design of choice-titration study. The arrows indicate that the individual δs are combined into an overall δ through computation of a geometric mean, e.g., $\delta(e,s)=[\delta_1(e,s) \times \delta_2(e,s) \times \delta_3(e,s)]^{1/3}$.

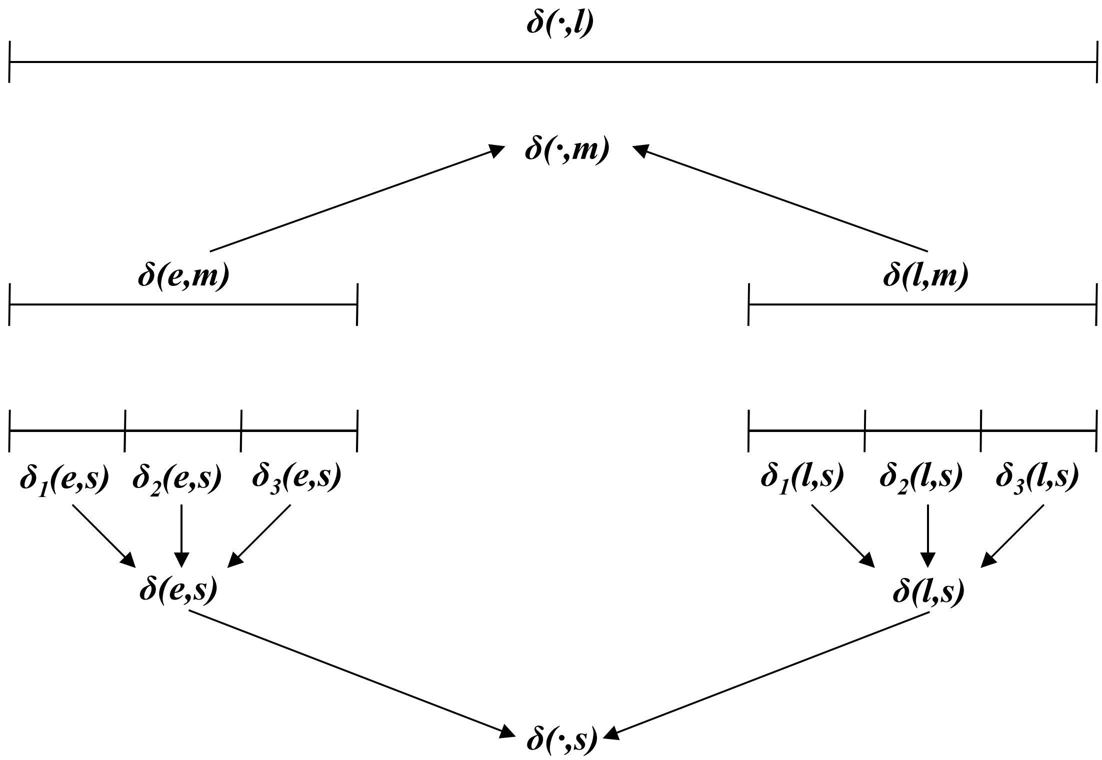

Figure 4.1. Results of choice-titration study: Superadditivity and the delay effect. This figure compares δ over early and late, small and medium-length intervals, testing superadditivity, $\delta(e,s)>\delta(e,m)$ and $\delta(l,s)>\delta(l,m)$, and the delay effect, $\delta(l,s)>\delta(e,s)$ and $\delta(l,m)>\delta(e,m)$.

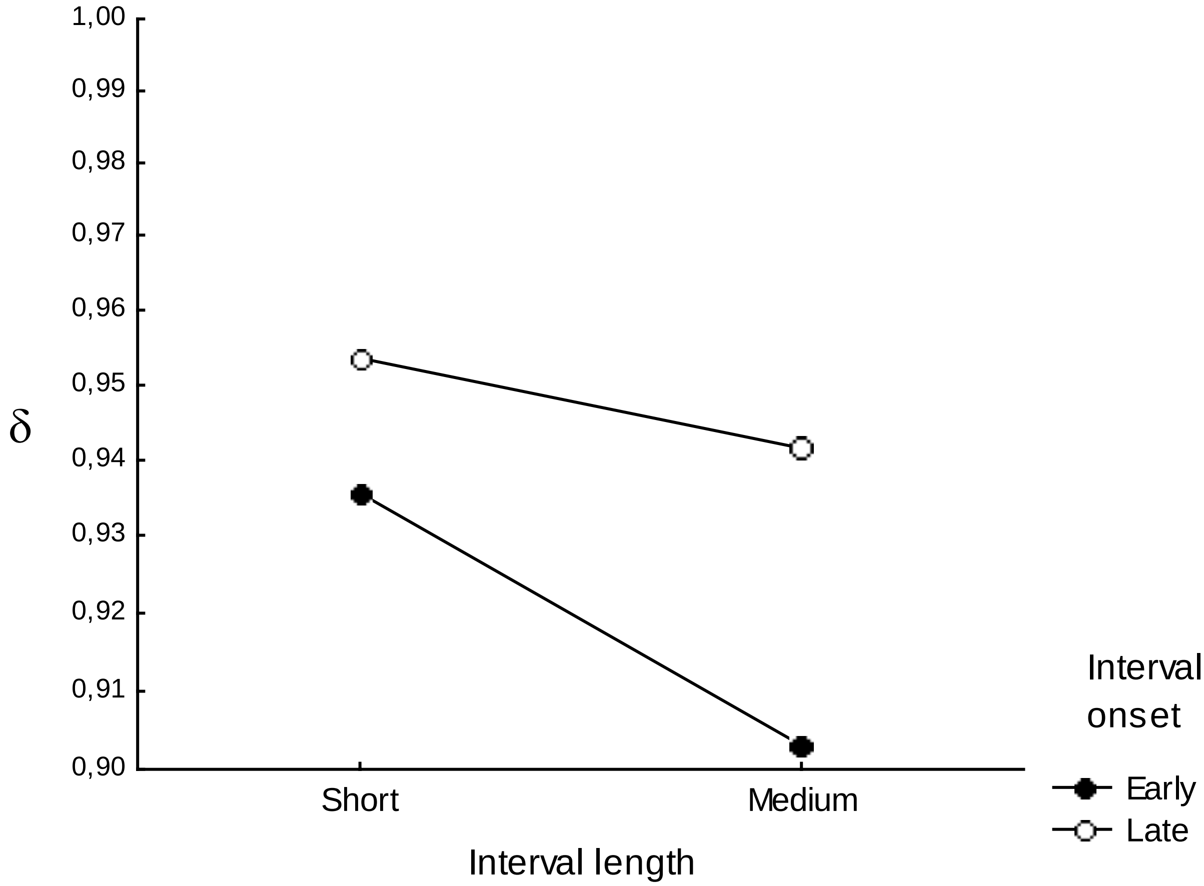

Figure 4.2. Results of choice-titration study: Superadditivity and subadditivity. This figure compares δ over small, medium-length, and long intervals, testing superadditivity and subadditivity, $\delta(\cdot,s) > \delta(\cdot,m) < \delta(\cdot,l)$.