Beyond Discounting: the Tradeoff Model of Intertemporal Choice

Daniel Read
Marc Scholten

Working Paper LSEOR 06.88 ISBN: 07530 2080 7

First published in Great Britain in 2006
by the Operational Research Group, Department of Management
London School of Economics and Political Science

Typeset, printed and bound by:
The London School of Economics and Political Science
Houghton Street
London WC2A 2AE

Working Paper No: LSEOR 06.88 ISBN: 07530 2080 7

Running Head: THE TRADEOFF MODEL OF INTERTEMPORAL CHOICE

Beyond Discounting:
The Tradeoff Model of Intertemporal Choice

Marc Scholten
Instituto Superior de Psicologia Aplicada,
Lisbon, Portugal
and
Daniel Read
Durham Business School,
and London School of Economics and Political Science
United Kingdom

Abstract

Research on intertemporal judgments and choices between a smaller-sooner and a larger-later outcome has revealed many anomalies to the discounted-utility model. Attempts to account for these anomalies within the discounting paradigm have resulted in convoluted and psychologically opaque models. We therefore develop a new model of intertemporal choice, the tradeoff model, in which choice results from a tradeoff between the perceived time difference (interval) and the perceived outcome difference (compensation). This model is both more parsimonious and more intuitive than any rival discounting model of comparable scope. Moreover, it accurately describes archival data as well as data from a new experiment.

Most choices involve tradeoffs between outcomes occurring at different times. Examples include deciding whether to party or work, eat or diet, and spend or invest. These are *intertemporal* choices. Much research has been devoted to observing these choices, and finding ways to model them. Many of the preference patterns revealed by this research are 'anomalies' to a normative standard derived from economics. In turn, these anomalies have been addressed by developing variants of the normative model. Most of these variants are, like the normative model itself, *delay-discounting models*: A value is assigned to each outcome, this value is discounted as a function of the delay to the outcome, and the option with the highest discounted value is chosen. Very influential delay-discounting models are Samuelson's (1937) discounted-utility model, commonly seen as the normative standard for intertemporal choices, and the model of Loewenstein and Prelec (1992), which accounts for many preference patterns that are anomalous to the discounted-utility model.

Delay-discounting models belong to the broad class of *alternative-based choice* models, in which an overall value is assigned to each option and the option with the highest value is chosen. Another class is that of *attribute-based choice* models, in which options are compared along their attributes and the option favored by the comparisons is chosen (Payne, Bettman & Johnson, 1988).[1] The formal modeling of attribute-based choice processes has a long tradition in psychology (e.g., González-Vallejo, 2002; Restle, 1961; Tversky, 1969), but it has never been undertaken for intertemporal choice.

In this paper, we develop and apply an attribute-based model of intertemporal choice, which we call the *tradeoff model*. The thrust of this model is that intertemporal tradeoffs are not made implicitly, by computing and comparing discounted values, but explicitly, by weighing perceived time differences against perceived outcome differences.[2] We show that the tradeoff model is both more parsimonious and more intuitive than any rival discounting model of comparable scope.

We begin with the basics of discounting theory. We then describe the anomalous preference patterns that have been observed and how they have been explained within the discounting paradigm. Subsequently, we develop the tradeoff model and apply it to archival data as well as data from a new experiment. We end with a discussion of the relation between the tradeoff model and other models of choice based on tradeoffs between attribute differences, the relation between the tradeoff model and bilinear choice models (among which discounting models), and the scope of the tradeoff model within the domain of intertemporal choice.

Basics of Discounting Theory

We focus on choices between smaller-sooner (*SS*) and larger-later (*LL*) outcomes, such as that between \$100 in 1 month and \$250 in 13 months. The outcomes are designated as x_S and x_L (\$100 and \$250), and their respective delays as t_S and t_L (1 and 13 months). According to discounting models, intertemporal choices are governed by the discounted values of the two options. In *delay*-discounting models, these are given as

$$V(x_S, t_S) = d(t_S)v(x_S)$$
$$V(x_L, t_L) = d(t_L)v(x_L),$$

where $V(x,t)$ is the value of x given that it will be received after a wait of t, $v(x)$ is the value x will have when it is received, and $d(t)$ is a *discount factor* decreasing in t. Indifference arises when *SS* and *LL* have equal discounted values, i.e.,

$$d(t_S)v(x_S) = d(t_L)v(x_L).$$

The indifference point allows us to derive the *discount fraction*, which is a measure of the discounting over the interval $t_S \rightarrow t_L$:

$$F_{t_S \rightarrow t_L} = \frac{d(t_L)}{d(t_S)} = \frac{v(x_S)}{v(x_L)}.$$

The discount fraction can be interpreted as the value of outcome x after delay t_L relative to its value after t_S. Thus, a higher value of $F_{t_S \rightarrow t_L}$ indicates *less* discounting. In turn, we can derive a *one-period discount fraction*, which is a measure of the *average* discounting over the interval:

$$\delta_{t_S \rightarrow t_L} = \left(F_{t_S \rightarrow t_L}\right)^{1/(t_L - t_S)},$$

where t_S and t_L are specified in an appropriate unit, usually in years. A higher value of $\delta_{t_S \rightarrow t_L}$ indicates less discounting *per unit of time*.

Because the discount function d and the value function v are unknown, researchers often define their expectations about intertemporal preferences as deviations from a null model that assumes constant discounting of outcomes per unit of time. This model, usually called the *exponential-discounting model* (e.g., Keller & Strazzera, 2002), suggests that, for any option pair, indifference between *SS* and *LL* arises when $\delta^{t_S} x_S = \delta^{t_L} x_L$.[3] Under the null hypothesis, δ is constant across option pairs. Research has shown, however, that δ varies with many aspects of the option pairs.[4] Within the discounting paradigm, this variation has been accommodated in two ways, through modifications of the discount function $d(t) = \delta^t$ or the value function $v(x) = x$.

Preference Patterns and Discounting Theory

In this section, we describe the anomalous preference patterns addressed by the tradeoff model and show how attempts to account for these patterns within the discounting paradigm have proved unsatisfactory.

Delay effect. A preference pattern that has been the focus of many investigations is the 'delay effect' (Thaler, 1981), which is that a later outcome is discounted less per unit of time than an earlier one (i.e., δ increases with *t*) or, equivalently, that there is less discounting over a later interval than over an earlier one of the same length (i.e., δ increases when t_S and t_L increase by the same additive constant *a*).[5] For instance, someone who is indifferent between \$100 now and \$110 in 1 month will prefer \$110 in 12 months to \$100 in 11 months: There is more discounting over the first month than over the twelfth.

The usual way to account for the delay effect is by modifying the discount function. Actually, there is considerable, perhaps remarkable (e.g., Rubinstein, 2003), agreement among psychologists and economists that the notion of exponential discounting should be replaced by some form of *hyperbolic discounting* (Ainslie, 1975, 1991), in which δ increases with the delay to an outcome, or *quasi-hyperbolic discounting* (Laibson, 1997), also called *present-biased preferences* (O'Donoghue & Rabin, 1999), in which δ is lower when the interval separating the outcomes begins now than when it begins later (an instance of the delay effect that is called the *immediacy effect*; see Read, Loewenstein, & Kalyanaraman, 1999). A general formulation of hyperbolic discounting has been given by Loewenstein and Prelec (1992):

$$d(t) = \left(\frac{1}{1+\alpha t} \right)^{\beta / \alpha},$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$ is the departure from exponential discounting. Given this discount function, the discount fraction for the interval $t_S \to t_L$ is

$$F_{t_S \to t_L} = \left(\frac{1+\alpha t_S}{1+\alpha t_L} \right)^{\beta / \alpha} = \left(\frac{1+\alpha t_S}{1+\alpha t_S + \alpha(t_L - t_S)} \right)^{\beta / \alpha}.$$

Thus, there is more discounting over a longer interval than over a shorter one, because $F_{t_S \to t_L}$ decreases with $t_S \to t_L$, *and* there is less discounting over a later interval than over an earlier one of the same length, because $F_{t_S \to t_L}$ increases when t_S and t_L increase by the same amount of time. Because, in the latter case, t_L - t_S does not change, there is also less discounting per unit of time, i.e., $\delta_{t_S \to t_L}$ increases.

Because the delay effect is the only preference pattern in which δ varies strictly with the delay to an outcome, it is also the only preference pattern that can be easily accommodated by modifying the discount function. The two preference patterns discussed next, in which δ also varies with characteristics of the outcome itself, require more extensive modifications of discounting theory.

Magnitude effect. The 'magnitude effect' (Prelec & Loewenstein, 1991) is that a larger outcome is discounted less than a smaller one of the same sign (i.e., δ increases with the magnitude of x) or, equivalently, there is less discounting over an interval when the magnitude of the outcomes increases by the same factor (i.e., δ increases when the magnitude of x_S and x_L increases by the same multiplicative constant m). For instance, someone who is indifferent between \$100 now and \$110 in 1 month will prefer \$1,100 in one month to \$1,000 now. The magnitude effect has been addressed in two different ways.

The magnitude effect has often been ascribed to the discount function (e.g., Green, Myerson, & McFadden, 1997; Kirby, 1997; Kirby & Maraković, 1996). This solution is unsatisfactory from a formal standpoint, because it sacrifices the separability of delay and outcome in a psychophysical model: The discounted value of a delayed outcome is now given as

$$V(x,t) = d(x,t)v(x).$$

More importantly, however, the solution is unsatisfactory from a psychological perspective, because the question "Why is a larger outcome discounted less than a smaller one?" receives the answer "Because it is larger." Loewenstein (1988), opposing such a development, argued that postulating different discount curves for different *types* of consumption (e.g., smoking, drinking, eating), "would collapse the concept of discounting to a tautology." (p. 212) This argument seems even more valid for different *amounts* of consumption of the same thing.

Loewenstein and Prelec (1992) ascribe the magnitude effect to the value function rather than the discount function. Their value function has three properties that are familiar from prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1991): *Reference dependence* (outcomes are evaluated as gains and losses relative to a neutral reference point), *diminishing sensitivity* (the marginal impact of an outcome decreases as the magnitude of the outcome increases), and *loss aversion* (losses loom larger than gains). The dashed value function in Figure 1 has these properties. The magnitude effect is accommodated by a fourth property: The contrast between the more curved regions near the reference point and the less curved regions away from the reference point is greater than in prospect theory. The solid value function in Figure 1 has this property as well.

Insert Figure 1 about here

A formal definition of the fourth property is *increasing proportional sensitivity* (Prelec & Loewenstein, 1991): Augmenting the outcomes by the same multiplicative constant augments the value of the larger outcome *relative to* the value of the smaller one, or

$\frac{v(mx_L)}{v(mx_S)} > \frac{v(x_L)}{v(x_S)}$ iff $m > 1$.[6]

For instance, the percentage by which the value of \$110 exceeds that of \$100 is greater than the percentage by which the value of \$11 exceeds that of \$10.[7] An equivalent definition is that the *elasticity* of the value function increases with outcome magnitude (Loewenstein & Prelec, 1992).

While the elasticity property accommodates the magnitude effect, it has not been invoked as an explanatory device by other theories of judgment and choice, so that its *epistemic* appeal seems rather restricted.

Sign effect. The 'sign effect' (Thaler, 1981) is that a loss is discounted less than a gain of the same magnitude (i.e., δ is higher when $x > 0$ than when $x < 0$).[8] For instance, someone who is indifferent between gaining \$100 now and gaining \$110 in one month will rather lose \$100 now than lose \$110 in one month.

The sign effect is accommodated by a fifth property of Loewenstein and Prelec's (1992) value function: The section above the reference point is more curved than the section below it (see Figure 1). A formal definition of this property is *loss amplification* (Prelec & Loewenstein, 1991): Changing the outcomes from gains to losses has the same effect as increasing their magnitude by the same multiplicative constant, or

$\frac{v(-x_L)}{v(-x_S)} = \frac{v(mx_L)}{v(mx_S)} > \frac{v(x_L)}{v(x_S)}$ iff $x_S, x_L > 0$ and $m > 1$.

An equivalent definition is that the value function is more elastic for losses than for gains (Loewenstein & Prelec, 1992).

Loss amplification, or a greater elasticity for losses than for gains, identifies an *interaction* between loss aversion and diminishing sensitivity, in that sensitivity to losses diminishes less than sensitivity to gains. However, such an interaction has not been invoked as an explanatory device by other theories of judgment and choice. Rather, loss aversion and diminishing sensitivity are usually viewed as *independent* properties of the value function. The tradeoff model developed in this paper accounts for the sign effect on the basis of loss aversion alone, not in interaction with diminishing sensitivity.[9]

Rescheduling effect. In many studies, the sign effect is either confounded with or eclipsed by another preference pattern, which is that a gain is discounted more, and a loss is discounted less, when it is postponed from t_S to t_L than when it is 'preponed' over the same interval.[10] We call this effect, originally described by Loewenstein (1988), the 'rescheduling effect.'

Imagine one is entitled to receive \$100 immediately but is given the opportunity to receive it in 1 month along with some compensation for postponing it. Suppose one demands \$10 in compensation. Now imagine that one is entitled to receive \$110 in 1 month but is given the opportunity to receive the money right now and pay a compensation for preponing it. How much would one be willing to pay in compensation? In the first scenario, the equivalent of \$110 in 1 month is \$100 now, so one *should* be willing to pay \$10 in the second scenario as well. However, the rescheduling effect is that one would offer less than \$10 in compensation. Thus, δ is lower when a gain is postponed than when it is preponed over the same interval.

Loewenstein and Prelec (1992) ascribe the rescheduling effect to reference dependence and loss aversion. On the one hand, when a gain is postponed, the smaller-sooner gain is not evaluated as a gain; rather, *foregoing* the smaller-sooner gain is evaluated as a *loss*. The larger-later gain, which must compensate for this loss, is evaluated as a gain. Thus, the compensation demanded for postponing the gain has *two* components: A compensation for the delay (the discounting component) *and* a compensation for the loss (the loss-aversion component). The greater the discounting and the greater the loss aversion, the larger the compensation demanded. Therefore, when a gain is postponed, both components work in the same direction, toward a lower δ.

On the other hand, when a gain is *preponed*, foregoing the larger-later gain is evaluated as a loss, whereas the smaller-sooner gain is evaluated as a gain. In this case, discounting and loss aversion work in opposite directions: The greater the discounting, the larger the compensation offered; the greater the loss aversion, the *smaller* the compensation offered. Thus, discounting decreases δ, whereas loss aversion increases it. Overall, δ will be lower when a gain is postponed than when it is preponed over the same interval.

In Loewenstein and Prelec's (1992) model, the sign effect is ascribed to the asymmetric *elasticity* of the value function, whereas the rescheduling effect, which is basically a sign effect for compensations, is ascribed to the asymmetric *steepness* of the value function: Compensations to be paid loom larger than compensations to be received. This need to explain very similar phenomena with very different devices poses a threat to the parsimony of Loewenstein and Prelec's model, as well as other comprehensive discounting models of intertemporal choice. Indeed, the tradeoff model developed in this paper accounts for *both* phenomena (the sign effect and the rescheduling effect) on the basis of loss aversion.

Although the four preference patterns discussed above can be addressed by modifying delay-discounting models, other patterns cannot. These patterns, which are called 'interval effects' (Scholten & Read, 2006), require a more drastic revision of the discounting paradigm.

Interval effects. Imagine we divide an interval, e.g., $t_S \rightarrow t_L$, into a series of shorter, contiguous intervals, e.g., $t_S \rightarrow t_M$ and $t_M \rightarrow t_L$. For instance, we divide one day into morning and afternoon. Now imagine two procedures. In the first, we obtain separate measures of δ for the morning and the afternoon and then combine them into a single measure. In the second, we obtain a single measure of δ directly for the whole day. Delay-discounting models suggest that the two procedures should yield the same result. Expressed in terms of discount fractions,

$$F_{t_S \rightarrow t_M \rightarrow t_L} = F_{t_S \rightarrow t_M} \cdot F_{t_M \rightarrow t_L} = \frac{d(t_M)}{d(t_S)} \cdot \frac{d(t_L)}{d(t_M)} = \frac{d(t_L)}{d(t_S)} = F_{t_S \rightarrow t_L} \; .$$

Even when allowing for outcome dependence of the discount factors, i.e., $d(x,t)$, the different procedures should yield the same result. Recent studies, however, have shown that this does *not* occur (Baron, 2000; Read, 2001; Read & Roelofsma, 2003; Roelofsma & Read, 2000; Scholten & Read, 2006).

The overall conclusion that can be drawn from these studies is that there is *usually* more discounting when an interval is divided into a series of shorter intervals than when it is left undivided (*subadditivity*), but that there is *less* discounting when an interval is divided into a series of *very* short intervals (*superadditivity*). More formally, consider the average discounting over a series of intervals:

$$\delta^{(n)} = \sqrt[n]{\prod_{i=1}^{n} \delta_i} \; ,$$

where n is the number of intervals into which an interval is divided. If $m > n$, subadditivity occurs when $\delta^{(m)} < \delta^{(n)}$, whereas superaddivity occurs when $\delta^{(m)} > \delta^{(n)}$. Equivalently, if we aggregate short intervals into longer ones, these longer intervals into still longer ones, and so forth, $\delta^{(n)}$ will decrease and then increase (see Scholten & Read, 2006, Figure 2).

Interval effects suggest that the discounting over an interval is not only a function of the delay *to* the outcomes, but also of the delay *between* them (see Footnote 5). Recently, interval effects have been accommodated by a generalization of Loewenstein and Prelec's (1992) delay-discounting model. In Scholten and Read's (2006) *interval-discounting model*, the discounted values of *SS* and *LL* are given as

$$V(x_S, t_S) = D(0, t_S)v(x_S)$$
$$V(x_L, t_L) = D(0, t_S)D(t_S, t_L)v(x_L),$$

so that indifference arises when

$$v(x_S) = D(t_S, t_L)v(x_L),$$

where *v* is a value function with the five properties discussed earlier, and *D* is an interval-discount function. Specifically, *D* is a generalization of Loewenstein and Prelec's (1992) delay-discount function:

$$D(t_S, t_L) = \left(\frac{1}{1 + \alpha (t_S^{\tau} - t_L^{\tau})^{\vartheta}} \right)^{\beta / \alpha},$$

where $\beta > 0$ is the degree of discounting and $\alpha > 0$, $\vartheta > 1$, and $0 < \tau < 1$ are departures from exponential discounting, indicating subadditivity, superadditivity, and diminishing sensitivity to delays, respectively.

The interval-discounting model achieves a gain in scope, but this comes with a loss in psychological plausibility. The model basically proposes a combination of alternative-based and attribute-based choice: A discounted value is assigned to each option and the option with the highest value is chosen (alternative-based choice), but, to obtain the discounted values, the options are directly compared along the time attribute (attribute-based choice). Although the interval-discounting model is not unique in combining both choice modes (e.g., Mellers & Biagini, 1994; Shafir, Osherson, & Smith, 1993), it *is* unique in proposing that the options are compared along one attribute (time) but *not* along the other (outcome). The question arises why the outcome attribute should be treated any differently from the time attribute: Why not directly compare the options along the outcome attribute as well? This question is the point of departure for the development of the tradeoff model.

Development of the Tradeoff Model

The tradeoff model addresses intertemporal judgments and choices that involve a comparison between a smaller-sooner (*SS*) and a larger-later (*LL*) outcome. It focuses on situations in which people treat such options 'dispassionately but intuitively.' That is, they are neither overridden by powerful emotions (e.g., Loewenstein, 1996), nor guided by formal reasoning (e.g., Kahneman, 2003).

The thrust of the tradeoff model is that judgments and choices are made by weighing the perceived *interval* separating the outcomes against the perceived *compensation* for obtaining a gain later rather than sooner or incurring a loss sooner rather than later. This weighing process may be more or less cursory, depending on the options under consideration and the task at hand (e.g., matching or choice). The more cursory assessments are procedural, whereas the more careful assessments are psychophysical. In this paper, we emphasize the psychophysics, but we briefly discuss the procedures as well.

We use $f(t_S, t_L) > 0$ to denote the advantage of the smaller-sooner gain or the larger-later loss along the time attribute and $g(x_S, x_L) > 0$ to denote the advantage of the larger-later gain or the smaller-sooner loss along the outcome attribute. Indifference between *SS* and *LL* arises when

$$f(t_S, t_L) = g(x_S, x_L)\,. \tag{1}$$

Preference for the larger-later gain or the smaller-sooner loss arises when $g(x_S, x_L) > f(t_S, t_L)$. We will systematically develop this model, starting with its most elementary specification.

Intra-attribute subtractivity

In the simplest formulation of the tradeoff model, the advantage of one option along the time attribute is weighted against the advantage of the other option along the outcome attribute by a single tradeoff parameter:

$$\kappa(t_L - t_S) = \begin{cases} x_L - x_S & \text{if } x_S, x_L > 0 \\ x_S - x_L & \text{if } x_S, x_L < 0, \end{cases} \tag{2}$$

where $\kappa > 0$. Thus, an advantage is given by the absolute difference between the options along an attribute (*intra-attribute subtractivity*; cf. Tversky & Krantz, 1970). The tradeoff parameter κ reflects the units in which the delays and the outcomes are measured (e.g., when changing days into weeks, κ increases sevenfold) as well as the rate at which time differences are traded off against outcome differences (e.g., when doubling the compensation demanded or offered for each additional week, κ is also doubled).

The one-parameter model in Equation 2 accounts for two robust preference patterns. The first, a shared implication of normative and descriptive models, is that increasing both delays by the same multiplicative constant changes indifference into preference for *SS* (gains) or *LL* (losses). The second pattern is the *magnitude effect*: Increasing the magnitude of both outcomes by the same multiplicative constant changes indifference into preference for *LL* (gains) or *SS* (losses). According to the tradeoff model, the multiplicative constant increases the difference between the options along the time or outcome attribute, thus increasing the impact of that attribute.

Reference dependence and loss aversion

Like prospect theory, the tradeoff model assumes *reference dependence* and *constant* *loss aversion* (see Tversky & Kahneman, 1991): Each outcome is evaluated as a gain or a loss relative to a neutral reference point and changing an outcome from a gain to a loss increases its perceived magnitude by a multiplicative constant. The implications of loss aversion depend on what the reference point is.

Sign effect. If the reference point is current wealth, gains and losses coincide with the actual amounts to be received or paid (Kahneman & Tversky, 1979). In this case, indifference between *SS* and *LL* arises when

$$\kappa(t_L - t_S) = \begin{cases} x_L - x_S & \text{if } x_S, x_L > 0 \\ \Lambda(x_S - x_L) & \text{if } x_S, x_L < 0, \end{cases} \tag{3}$$

where $\Lambda > 1$ is the loss-aversion parameter. Loss aversion gives rise to the *sign effect*: Changing both outcomes from gains to losses changes indifference into preference for *SS*, because Λ increases the perceived difference between the options along the outcome attribute, thus increasing the impact of that attribute.

Rescheduling effect. The above derivation of the sign effect applies only if both outcomes are evaluated as deviations from current wealth. However, the reference point may be affected by the formulation of the options and by the expectations of the decision maker (Kahneman & Tversky, 1979). Specifically, one of the options may be designated as an *entitlement* (a right to receive x at t) or a *commitment* (a responsibility to pay x at t), and the other as an option that includes a compensation for rescheduling x. In this case, it is natural to treat the entitlement or commitment as the *default*. As a result of adaptation to the default, the reference point may shift away from current wealth. By *how much* the reference point will shift, however, is unclear.

Loewenstein and Prelec (1992) assumed *complete* adaptation to an entitlement or a commitment. When specifying, for instance, the minimum to be received at t_L to forego a receipt at t_S, the outcomes are evaluated as a gain of x_L at t_L and a loss of x_S at t_S (see also Shelley, 1993). Thus, receiving x_L is evaluated as a gain relative to current wealth, whereas foregoing x_S is evaluated as a loss relative to a reference point that has shifted away to current wealth *plus* the entitlement.

There is the theoretical possibility, however, that adaptation varies anywhere between one extreme case of no adaptation and the other extreme case of complete adaptation. Indeed, the possibility of *incomplete* adaptation has also been considered by Loewenstein and others (Hoch & Loewenstein, 1991; Strahilevitz & Loewenstein, 1998). Incomplete adaptation implies that foregoing x_S would not be as bad as a total loss of x_S. Figure 2 depicts the implications of rescheduling an entitlement if there is no adaptation, if there is complete adaptation, and if there is, as we propose, incomplete adaptation.[11]

Insert Figure 2 about here

In Figure 2, x_S denotes an entitlement and $\hat{x}_L$ denotes the outcome that compensates for rescheduling the entitlement. The top left panel shows no adaptation, the case described by Equation 3. Both x_S and $\hat{x}_L$ are evaluated as deviations from current wealth, the origin of the value function. The top right panel shows complete adaptation. Receiving x_S is evaluated as a neutral outcome, the origin of the dashed value function, whereas foregoing x_S is evaluated as a loss. Loss aversion implies that the relief from not losing x_S is greater than the pleasure of gaining x_S, so that, when holding t_L and t_S constant, $\hat{x}_L$ is larger than in the case of no adaptation. The bottom panel shows incomplete adaptation. Receiving x_S is evaluated as a gain of x_S - R_S, while foregoing x_S is evaluated as a loss of R_S. This case is less extreme than that of no adaptation ($R_S = 0$) and complete adaptation ($R_S = x_S$), so that $\hat{x}_L$ is less extreme as well.

Applying the above reasoning to all four rescheduling scenarios, we arrive at the following specification of the tradeoff model:

$$\kappa(t_L - t_S) = \begin{cases} \hat{x}_L - [(x_S - R_S) - \Lambda(0 - R_S)] & \text{if } R_S, x_S, \hat{x}_L > 0 \quad \text{(postponing a gain)} \\ \Lambda\hat{x}_S - [\Lambda(x_L - R_L) - (0 - R_L)] & \text{if } R_L, \hat{x}_S, x_L < 0 \quad \text{(preponing a loss)} \\ [(x_L - R_L) - \Lambda(0 - R_L)] - \hat{x}_S & \text{if } R_L, \hat{x}_S, x_L > 0 \quad \text{(preponing a gain)} \\ [\Lambda(x_S - R_S) - (0 - R_S)] - \Lambda\hat{x}_L & \text{if } R_S, x_S, \hat{x}_L < 0 \quad \text{(postponing a loss).} \end{cases}$$

Because the above reference-point model assumes a linear value function, it can be given as

$$\kappa(t_L - t_S) = \begin{cases} (\hat{x}_L - x_S) - (\Lambda - 1)R_S & \text{if } R_S, x_S, \hat{x}_L > 0 \quad \text{(postponing a gain)} \quad (4.1) \\ \Lambda(\hat{x}_S - x_L) + (\Lambda - 1)R_L & \text{if } R_L, \hat{x}_S, x_L < 0 \quad \text{(preponing a loss)} \quad (4.2) \\ (x_L - \hat{x}_S) + (\Lambda - 1)R_L & \text{if } R_L, \hat{x}_S, x_L > 0 \quad \text{(preponing a gain)} \quad (4.3) \\ \Lambda(x_S - \hat{x}_L) - (\Lambda - 1)R_S & \text{if } R_S, x_S, \hat{x}_L < 0 \quad \text{(postponing a loss).} \quad (4.4) \end{cases}$$

Thus, the compensation that one wants to receive for postponing a gain or preponing a loss, or that one is willing to pay for preponing a gain or postponing a loss, can be broken down into a component that reflects the weighing of the compensation against the interval over which the rescheduling occurs and a component that reflects the reluctance to accept the compensation, which results from adaptation. Equation 4 reduces to Equation 3 when there is no adaptation, i.e., when $R_S = R_L = 0$.

To derive the rescheduling effect for gains, we solve Equation 4.1 for $\hat{x}_L$:

$\hat{x}_L = \kappa(t_L - t_S) + x_S + (\Lambda - 1)R_S$.

This is the minimum that one wants to receive at t_L in exchange for x_S at t_S. Substituting $\hat{x}_L$ for x_L in Equation 4.3 and solving for $\hat{x}_S$,

$\hat{x}_S = x_S + (\Lambda - 1)(R_S + R_L)$.

This is the minimum that one wants to receive at t_S in exchange for $\hat{x}_L$ at t_L. It is evident that $\hat{x}_S > x_S$ as long as there is loss aversion *and* adaptation. In a similar fashion, the rescheduling effect for losses can be derived from Equations 4.2 and 4.4.

Conclusion. The tradeoff model offers a parsimonious and psychologically plausible explanation of the sign effect and the rescheduling effect: Loss aversion is the common cause of *both*, while the degree of adaptation to a rescheduled outcome determines the degree to which the sign effect is outweighed by the rescheduling effect. Although this explanation of the rescheduling effect is conceptually simple, the formal specification of the tradeoff model becomes quite complex, as evident from Equation 4. Therefore, we will discuss the remaining properties of our model without the complicating factor of rescheduling.

Diminishing sensitivity

Equation 3 implies that the impact of an attribute will not change when the attribute amounts change by the same additive constant. For instance, the difference between 1 and 2 months (or \$1 and \$2) would have the same impact as that between 11 and 12 months (or \$11 and \$12). This is at odds with the principle of *diminishing sensitivity* (Tversky & Kahneman, 1991), which has its roots in the Weber-Fechner law. The principle states that sensitivity to a given difference between attribute amounts decreases as the distance of the attribute amounts from the reference point increases. The tradeoff model incorporates diminishing sensitivity to both delays and outcomes. We remove the assumption that delays and outcomes are treated objectively by the decision maker, and propose that the tradeoff is between *perceived* intervals and *perceived* compensations:

$$\kappa(w(t_L) - w(t_S)) = \begin{cases} v(x_L) - v(x_S) & \text{if } x_S, x_L > 0 \\ v(x_S) - v(x_L) & \text{if } x_S, x_L < 0, \end{cases} \tag{5}$$

where w is a delay-perception function and v is a value function. Subjective delays are positive deviations from the present, i.e., $w(t) \geq 0$ for $t \geq 0$, they increase with objective delays, i.e., $w'(t) > 0$, but at a decreasing rate, i.e., $w''(t) < 0$. Correspondingly, subjective outcomes are either positive or negative deviations from current wealth, i.e., $v(x) \geq 0$ for $x \geq 0$

and $v(x) < 0$ for $x < 0$, their subjective magnitude increases with their objective magnitude, i.e., $v'(x) > 0$, but at a decreasing rate, i.e., $v''(x) < 0$ for $x \geq 0$ and $v''(x) > 0$ for $x < 0$.[12] In addition, a loss is perceived to be greater than a gain of equal magnitude, i.e., $v(-x) = -\Lambda v(x)$ for $x \geq 0$.

Diminishing sensitivity accounts for two preference patterns. The first, a shared implication of normative and descriptive models, is that increasing the magnitude of the outcomes by the same additive constant changes indifference into preference for *SS* (gains) or *LL* (losses). The second pattern is the *delay effect*: Increasing the delays by the same additive constant changes indifference into preference for *LL* (gains) or *SS* (losses). According to the tradeoff model, the additive constant decreases the perceived difference between the options along the time or outcome attribute, thus decreasing the impact of that attribute.

Diminishing sensitivity also attenuates the extremely large effects generated by intra-attribute subtractivity. The linear model in Equation 3 implies, for instance, that someone who is indifferent between \$1 sooner and \$11 later should also be indifferent between \$100 sooner and \$110 later, because $x_L - x_S = \$10$ in both cases. This is an extremely large magnitude effect, because the person is sensitive only to *absolute* differences between the options, and not at all to *proportional* differences. In case of any sensitivity to proportional differences, the person would prefer \$100 sooner to \$110 later. To restore indifference, the later outcome would have to be larger than \$110, i.e., $x_L - x_S$ increases as x_S increases. Because of the magnitude effect, however, it would not have to be as large as \$1,100, i.e., $(x_L - x_S) / x_S$ decreases as x_S increases. The combination of diminishing sensitivity and intra-attribute subtractivity, as described by the nonlinear model in Equation 5, yields this pattern.

To see the generality of the result that diminishing sensitivity attenuates the effects generated by intra-attribute subtractivity, let x and y be the sooner outcomes (in the above example, $x = 1$ and $y = 100$), let a be the amount added to both x and y (in the example, $a = 10$), and let e be the *extra* amount added to y. Holding t_S and t_L constant, indifference among each pair of options implies that

$$v(x + a) - v(x) = v(y + a + e) - v(y).$$

Rearrangement of the terms yields

$$v(y) - v(x) = v(y + a + e) - v(x + a). \quad (6)$$

The concavity of the value function v implies that

$$v(y) - v(x) > v(y + a) - v(x + a).$$

In Equation 6, then, it must be true that $e > 0$. How large e should be to restore indifference depends on the specific shape of the value function v, to be discussed when we apply our model to actual data.

Augmenting and diminishing relative sensitivity

In a general sense, any formal model of intertemporal choice proposes a weighing of delays and outcomes within some theoretical structure. For clarity of exposition, we draw a distinction between two types of weighing functions in the tradeoff model. On the one hand, the *intra-attribute weighing functions*: These are the delay-perception function *w*, which weighs delays against one another, and the value function *v*, which weighs outcomes against one another. On the other hand, there is an *inter-attribute weighing function*, which weighs perceived intervals against perceived compensations. In Equation 5, this is a scalar function, which multiplies perceived intervals by the tradeoff parameter κ. However, as discussed earlier, intertemporal preferences are usually subadditive in intervals. This suggests that, in general, the weight of perceived intervals is marginally decreasing relative to the weight of perceived compensations (*diminishing relative sensitivity*). We introduce a tradeoff function to capture this:

$$Q(w(t_L) - w(t_S)) = \begin{cases} v(x_L) - v(x_S) & \text{if } x_S, x_L > 0 \\ v(x_S) - v(x_L) & \text{if } x_S, x_L < 0. \end{cases} \tag{7}$$

The tradeoff function Q, which includes the tradeoff parameter κ, increases with perceived intervals, i.e., $Q'(\cdot) > 0$, but at a decreasing rate, i.e., $Q''(\cdot) < 0$. We emphasize that Q, like κ, is strictly comparative: Equation 7 does not imply that the weight of perceived intervals is *itself* marginally decreasing, but only that it is marginally decreasing *relative to* the weight of perceived compensations.

Although, as discussed earlier, subadditivity is the general rule, it can reverse into superadditivity for very short intervals. To capture this, the tradeoff function Q is marginally *increasing* over short perceived intervals, i.e., $Q''(\cdot) > 0$, but marginally decreasing over longer ones, i.e., $Q''(\cdot) < 0$. The marginally increasing weight of short perceived intervals (*augmenting relative sensitivity*) reflects a more general tendency of people to underweight small perceived intra-attribute differences, as originally suggested by Tversky's (1969) additive-difference model.

This completes the development of the tradeoff model. Table 1 draws a comparison between the tradeoff model and Scholten and Read's (2006) interval-discounting model, the only discounting model of comparable scope. The tradeoff model is more parsimonious than the interval-discounting model, because it replaces the (psychologically implausible) proposal of interval discounting with the (more intuitive) principle of intra-attribute subtractivity, and can thereby remove increasing proportional sensitivity and loss amplification as explanatory devices. The tradeoff model also has a broader scope than the interval-discounting model, in that it allows for incomplete adaptation to rescheduled outcomes.

Insert Table 1 about here

Applications of the Tradeoff Model

Indifference data

According to the tradeoff model, indifference between *SS* and *LL* arises when $f(t_S, t_L) = g(x_S, x_L)$. This raises two issues: How to determine the indifference point and how to estimate and evaluate the tradeoff model.

The indifference point is usually determined by fixing the two delays and one of the outcomes, and finding the magnitude of the other (variable) outcome that yields indifference between *SS* and *LL*. In *matching*, the magnitude of the variable outcome is specified by the participants. In *choice-based matching*, its magnitude is adjusted in response to each of a series of choices until the largest amount that yields preference for one option is sufficiently close to the smallest amount that yields preference for the other option. The midpoint between those two amounts is usually taken to be the indifference point. We analyze data from both types of studies.

To estimate the tradeoff model, we minimize the sum of squared deviations between the observed and predicted values of the dependent variable (which is some function of the two delays and the two outcomes at the indifference point) across all option pairs:

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 ,$$

where y_i and $\hat{y}_i$ are the observed and predicted value, respectively, for option pair *i* and *n* is the number of option pairs.[13] We thus maximize

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \overline{y})^2} ,$$

where $\overline{y}$ is the average value of y_i across the *n* option pairs. This average is the best ordinary least-squares estimate of a *constant* value *y*. Thus, R^2 compares the predictive accuracy of the tradeoff model with that of a null model, according to which $y_i = y$ for all *i*. For this evaluation of the tradeoff model to be meaningful, the null model has to be meaningful, which ultimately depends on the choice of *y*.

Discounting models have frequently been estimated and evaluated on the variable outcomes that yield indifference between *SS* and *LL* (e.g., Green et al., 2005; Green et al., 1997; Kirby, 1997; Murphy et al., 2001; Rachlin, Raineri, & Cross, 1991; Scholten & Read, 2006; Simpson & Vuchinich, 2000). However, when using outcomes as the dependent variable, the null model predicts a constant outcome across option pairs, which is an overly naïve standard of comparison. First, it fails to predict that larger outcomes are discounted by a greater amount than smaller ones (e.g., if \$10 is discounted by \$5 over a period of time, \$100 will be discounted by more than \$5 over that period), as predicted by *any* discounting model.[14] Second, it fails to predict that outcomes are discounted by a greater amount over longer delays than over shorter ones (e.g., if \$10 is discounted by \$5 over 1 period of time, that outcome will be discounted by more than \$5 over 2 periods), as contradicted *only* by a model of "dichotomous time preferences" (Loewenstein & Prelec, 1992, p. 580), "in which the present outcome has unit weight, and all future events are discounted by a common constant." Thus, when using outcomes as dependent variable, the null model is too easy to defeat: *Any* alternative model can come out favorably by predicting very general discounting effects.

To remove these general discounting effects from model estimation and evaluation, the dependent variable should be some indicator of the *proportion* by which outcomes are discounted *per unit of time*. Such indicators can be derived from the exponential-discounting model. According to one formulation of this model, indifference between *SS* and *LL* arises when $x_S/(1+\rho)^{t_S} = x_L/(1+\rho)^{t_L}$, where $\rho > 0$ is a one-period discount *coefficient*. Solving for ρ,

$$\rho = \left(\frac{x_L}{x_S}\right)^{\frac{1}{t_L - t_S}} - 1. \tag{8}$$

An equivalent formulation of this model is $\delta^{t_S} x_S = \delta^{t_L} x_L$, where $0 < \delta < 1$ is a one-period discount *fraction*. Solving for δ,

$$\delta = \left(\frac{x_S}{x_L}\right)^{\frac{1}{t_L - t_S}}. \tag{9}$$

The proportion by which the outcomes are discounted per unit of time is $\rho / (1 + \rho) = 1 - \delta$. This is the discount *rate*, which, according to the null model, is constant across outcome pairs. The alternative model than predicts that ρ or δ will vary. The predictions of the tradeoff model can be obtained by solving Equation 1 for the variable outcome and substituting it into

Equation 8 or 9.

We apply our model to aggregate values of ρ or δ. In the literature, indifference data have been aggregated across participants by taking arithmetic means of ρ (e.g., Benzion et al., 1989), geometric means of ρ (e.g., Chapman, 1996), arithmetic means of δ (e.g., Read & Roelofsma, 2003), or geometric means of δ (e.g., Scholten & Read, 2006). We favor the last procedure, because it preserves the ratio information in the data: Taking the geometric mean of δ is the same as taking the geometric mean of the variable outcome and computing δ from there.[15] However, our first analysis is conducted on the arithmetic means of ρ reported in a highly influential paper by Benzion et al. (1989). The purpose of this analysis is to examine the performance of the flexible reference-point model in Equation 4 and to explore whether and how the degree of adaptation to the rescheduled outcome varies with its magnitude and sign.

Indifference data from matching

In the matching study undertaken by Benzion et al. (1989), participants specified, for 64 option pairs, the magnitude of the variable outcome that yielded indifference between *SS* and *LL*. Each question was embedded in a scenario that designated the fixed outcome as an entitlement or commitment and the variable outcome as a compensation for rescheduling the fixed outcome. The 64 option pairs resulted from a 2 (postponing versus preponing) × 2 (outcome sign) × 4 (outcome magnitude) × 4 (delay to the later outcome) design. The sooner outcome was always available immediately, so that the effect of the delay *to* the later outcome (the delay effect) was confounded with the effect of the interval *between* the sooner and the later outcome (subadditivity; see also Footnote 5). Therefore, we will call this the *pseudo-delay effect*. The data (Benzion et al., 1989, Table 1) revealed four major preference patterns: The rescheduling effect, the sign effect, the magnitude effect, and the pseudo-delay effect.

We examine the accuracy with which Equation 4 accounts for these data and we explore whether and how the degree of adaptation to rescheduled outcomes varies with their magnitude and sign. While complete adaptation implies a one-to-one relation between the magnitude of the outcomes and the degree of adaptation, *incomplete* adaptation opens many possibilities. For instance, it has been suggested that people adapt more readily to gains than to losses of equal magnitude (Strahilevitz & Loewenstein, 1998; Thaler & Johnson, 1990). To test for incomplete and *asymmetric* adaptation, we estimate 2 (outcome sign) × 4 (outcome magnitude) = 8 values of *R*.

To preserve the computational tractability of the flexible reference-point model in

Equation 4, we do not introduce functional forms that capture diminishing sensitivity to outcomes and diminishing (relative) sensitivity to delays (perceived intervals).[16] Instead, we allow the tradeoff parameter κ to vary freely with the magnitude of the rescheduled outcome and the delay to the later outcome. We expect the variation of κ to exhibit an orderly pattern, because it is supposed to reflect unspecified properties of well-behaved functions, $v(x)$ and $Q(w(t_L))$.[17] Moreover, we test two hypotheses about how κ varies with outcome magnitude and delay.

First, as discussed earlier, a linear $v(x)$ implies insensitivity to proportional differences between outcomes, whereas a concave $v(x)$ yields some degree of proportionality.[18] Thus, the hypothesis of *magnitude dependence* is that κ will increase with outcome magnitude.

Second, if $Q(w(t_L))$ is linear, the perceived compensation for rescheduling is proportional to t_L. For instance, when a person is indifferent between the options and t_L doubles, the perceived compensation must increase by a factor 2κ for indifference to be maintained. However, if $Q(w(t_L))$ is concave, indifference will be maintained if the perceived compensation increases by a factor of less than 2κ. The hypothesis of *delay dependence* is that κ will decrease with delay. A sufficient condition for this to occur is that the combined effect of diminishing sensitivity to delay and diminishing relative sensitivity to perceived intervals be greater than the effect of diminishing sensitivity to outcomes.

To test for magnitude and delay dependence, we estimate 4 (outcome magnitude) × 4 (delay to the later outcome) = 16 values of κ. Combining the eight values of R and the 16 values of κ with a single value of Λ, we estimated 25 free parameter values from 64 data points (one-period discount coefficients). The results are given in Table 2.

Insert Table 2 about here

There was evidence of incomplete adaptation to rescheduled outcomes: R was always greater (in absolute terms) than zero, but always smaller than the outcome being rescheduled. There was also evidence of asymmetric adaptation to gains and losses: For outcomes of \$40, \$200, and \$1,000, R was greater (in absolute terms) for gains than for losses, but for outcomes of \$5,000, this asymmetry reversed (see the top panel of Figure 3). Outcome magnitude and sign accounted for 80% and 11%, respectively, of the variation of R on a logarithmic scale, their interaction for 9%.

Insert Figure 3 about here

There was also evidence of diminishing sensitivity to outcomes and diminishing (relative) sensitivity to delays (perceived intervals): κ increased steadily and reliably with outcome magnitude (magnitude dependence), and decreased steadily, but not always reliably, with the delay to the later outcome (delay dependence). More specifically, κ increased by a nearly constant proportion as outcome magnitude increased by a constant proportion; conversely, κ decreased by a nearly constant proportion as the delay to the later outcome increased by a constant proportion (see the bottom panel of Figure 3). Outcome magnitude and delay accounted for 96% and 4%, respectively, of the variation of κ on a logarithmic scale, their interaction for virtually nothing. This highly regular variation of κ bolsters our confidence in the present estimation of the flexible reference-point model.

Finally, there was evidence of loss aversion: Λ was reliably greater than one, although its magnitude of 1.14 was modest compared with estimates obtained by applying subjectively expected value models to risky choices, e.g., $\Lambda = 2.25$ (Tversky & Kahneman, 1992), and $\Lambda = 1.43$ (Schmidt & Traub, 2002).

Figure 4 displays the observed and predicted values of ρ. The top panel shows that ρ is greater when postponing a gain than when preponing it, but smaller when postponing a loss than when preponing it (the rescheduling effect). Also, ρ is greater for gains than for losses (the sign effect). The bottom panel shows that ρ is greater for smaller outcomes than for larger ones (the magnitude effect) and greater for longer delays than for shorter ones (the pseudo-delay effect).

Insert Figure 4 about here

Because the estimated reference-point shifts indicated a tendency to adapt *less* to gains of \$5,000 than to losses of equal magnitude, Figure 5 displays observed and predicted values of ρ for those outcomes only. Comparison with the overall results in the top panel of Figure 4 reveals that the rescheduling effect is attenuated for gains of \$5,000 but not for losses of equal magnitude and that the highest value of ρ no longer occurred for the postponement of gains but rather for the preponement of losses. This pattern of results also emerged in a matching study by Shelley (1993, Figures 3 and 6), using the same delays and outcomes as Benzion et al. (1989).[19]

Insert Figure 5 about here

In sum, the results from this first application of the tradeoff model suggest that adaptation to a rescheduled outcome is incomplete (e.g., an entitlement of $200 yields a reference-point shift of less than $200), that adaptation increases with outcome magnitude (e.g., an entitlement of $200 yields a greater reference-point shift than an entitlement of $40), and that the level of adaptation to gains is *generally* higher than the level of adaptation to losses (e.g., an entitlement of $200 yields a greater reference-point shift than a commitment of $200). As outcome magnitude increases, however, adaptation to losses increases at a higher rate than adaptation to gains and may eventually reach a higher level. The issue of adaptation to a rescheduled outcome deserves greater attention, both empirically and theoretically.

The results are also indicative of diminishing sensitivity to outcomes and diminishing (relative) sensitivity to delays (perceived intervals). Because of the delay-interval confound, it was not possible to distinguish between diminishing sensitivity to delays and diminishing relative sensitivity to perceived intervals. This *will* be possible, however, in the second application of the tradeoff model. In the next section, therefore, we specify functional forms that capture these properties.

The weighing of delays

The delay-perception function w has two properties: Reference dependence (with the present serving as the reference point) and diminishing sensitivity to delays. Diminishing sensitivity has two limits. One is *constant sensitivity*, in which case subjective time runs at the same speed as objective time and w becomes a scalar function, i.e., $w(t) = t$. The other limit is *insensitivity*, in which case any delay is treated as if it had no duration at all and w becomes a zero function i.e., $w(t) = 0$ for any t. Both reference dependence and diminishing sensitivity are incorporated in the following delay-perception function:

$$w(t) = \frac{1}{\tau}\log(1+\tau t), \tag{10}$$

where $\tau > 0$ is a diminishing-sensitivity parameter. The delay-perception function becomes a scalar function, with constant sensitivity, as τ goes to zero and a zero function, with complete insensitivity, as τ goes toward infinity.

The tradeoff function Q has three properties: Augmenting relative sensitivity, diminishing relative sensitivity, and a progression of augmenting and diminishing relative sensitivity as intervals increase in perceived length. Either form of relative sensitivity has two limits: Constant sensitivity and insensitivity. Letting $T = w(t_L) - w(t_S)$,

$$Q(T) = \frac{\kappa}{\alpha}\log\left(1+\alpha\left(\frac{T}{\mathrm{B}}\right)^{\mathrm{B}}\right), \tag{11}$$

where $\alpha > 0$ reflects diminishing relative sensitivity and $B \geq 1$ reflects augmenting relative sensitivity. Figure 6 displays four variants of *Q*, corresponding to the presence or absence of augmenting or diminishing relative sensitivity.

Insert Figure 6 about here

Figure 7 displays the characteristically S-shaped tradeoff function *Q* as well as the coordinates of the inflection point. The coordinate along the abscissa depends only on α and *B*, whereas the coordinate along the ordinate depends also on the tradeoff parameter κ. As augmenting relative sensitivity vanishes, the inflection point goes to the origin, yielding a concave *Q*. As diminishing relative sensitivity vanishes, the inflection point goes to infinity, yielding a convex *Q*.

Insert Figure 7 about here

Indifference data from choice-based matching

Our second analysis is conducted on geometric means of δ, as reported by Scholten and Read (2006). The tradeoff model is specified as in Equation 7, the delay-perception function *w* as in Equation 10, and the tradeoff function *Q* as in Equation 11. However, the data provide no information about loss aversion, because outcomes were always gains, nor about diminishing sensitivity to outcomes, because the smaller-sooner gain was always of the same magnitude. The value function *v* can therefore be specified as $v(x) = x$ while the tradeoff parameter κ can be held constant.

In a choice-based matching study, participants were presented with nine pairs of delayed gains. For each option pair, they made an initial choice between a sooner gain of £500 and a later gain that offered a simple interest of 100% per week. Most participants chose *LL*. The later gain was then adjusted in response to each of a series of choices until indifference between *SS* and *LL* was reached. The nine option pairs corresponded to the nine intervals displayed in Figure 8: Six short intervals of 1 week, two medium-length intervals of 3 weeks, and one long interval of 17 weeks. There was a medium-length interval at the beginning and end of the long interval. Each medium-length interval spanned three short intervals. The subintervals at the beginning and end of the long interval will hereafter be designated as 'early' and 'late' intervals, respectively. The data revealed three major preference patterns: The delay effect, subadditivity, and superadditivity.

Insert Figure 8 about here

As in the original analysis conducted by Scholten and Read (2006), we estimated four time-weighing parameters from nine data points. However, we estimated the parameters from one-period discount fractions (δ) rather than outcomes (x_L). The results are given in Table 3. The parameter estimates of the tradeoff model confirmed the delay effect (τ), subadditivity (α), and superadditivity (B).

Insert Table 3 about here

The tradeoff model, once estimated on δ, accounts for 84.35% of the variance of δ. Table 4 shows that, had the model been estimated on outcomes (x_L), it would have accounted for only 75.45% of the variance of δ. Conversely, the tradeoff model, once estimated on x_L, accounts for 99.78% of the variance of x_L, but most of this variance, 98.69%, would *also* have been accounted for had the model been estimated on δ. Therefore, the improved accuracy with which we describe anomalies to the exponential-discounting model (i.e., systematic variations of δ) does not impair our ability to capture general discounting effects (in this study, the result that x_L increases with t_L - t_S).

Insert Table 4 about here

Figure 9 displays the observed and predicted values of δ. The top panel shows that δ first decreases but then increases with interval length (superadditivity and subadditivity), while the bottom panel shows that it increases with the delay to interval onset (the delay effect).

Insert Figure 9 about here

In sum, the tradeoff model accurately describes the co-occurrence of the delay effect, subadditivity, and superadditivity. Our next analysis examines the weighing of delays *in combination with* the weighing of outcomes. This analysis also extends the tradeoff model to states of equal *or differential* preference between *SS* and *LL*. To conduct this analysis, we need a value function that includes diminishing sensitivity to the outcomes and a tradeoff model that includes differential preference between the options. These are provided in the next two sections.

The weighing of outcomes

The value function v has the three properties that are familiar from prospect theory: Reference dependence, loss aversion, and diminishing sensitivity. Reference dependence and loss aversion were already specified in the first application of the tradeoff model. Diminishing sensitivity has two limits: Constant sensitivity, in which case the value of an outcome is equal to its *monetary* value and v becomes a scalar function, i.e., $v(x) = x$, and insensitivity, in which case any outcome is treated as if it had no value at all and v becomes a zero function, i.e., $v(x) = 0$ for any x. Reference dependence, loss aversion, and diminishing sensitivity are incorporated in the following value function:

$$v(x) = \begin{cases} \frac{1}{\gamma}\log(1+\gamma x) & \text{if } x \geq 0 \\ -\frac{\Lambda}{\gamma}\log(1+\gamma(-x)) & \text{if } x < 0, \end{cases} \tag{12}$$

where $\gamma > 0$ is a diminishing-sensitivity parameter.[20] The value function becomes a scalar function, with constant sensitivity, as γ goes to zero and a zero function, with complete insensitivity, as γ goes toward infinity.[21]

Within the theoretical structure of the tradeoff model, Equation 12 yields meaningful boundaries on the compensation demanded or offered for waiting. To illustrate, consider indifference between the following option pairs:

(I) x = \$1 sooner and $x + a$ = \$11 later,

(II) y = \$100 sooner and $y + a + e$ = \$110 + e later.

The compensation demanded is a = \$10 in I and $a + e$ = \$10 + e in II. According to Equation 12, $a + e$ cannot be smaller than \$10 (*constant compensation*) or larger than \$1,000 (*proportional compensation*). That is, $a \leq a + e \leq a \cdot y/x$. These boundaries are obtained by solving Equation 6 for $a + e$, specifying v as in Equation 12, and letting γ go to zero and infinity, respectively.

Preference data

We generalize the tradeoff model to states of indifference or preference by applying Restle's (1961) binary-choice model. The odds of choosing *LL* among a pair of gains or *SS* among a pair of losses can be given as

$$\Omega = \frac{P}{1-P} = \left(\frac{g(x_L, x_S)}{f(t_L, t_S)} \right)^{\vartheta}, \tag{13}$$

where $0 < P < 1$ is the probability of choosing *LL* among a pair of gains or *SS* among a pair of losses and $\vartheta \geq 0$ is the degree of determinism in choice behavior: The higher ϑ, the higher the odds that the option favored by the tradeoff will be chosen. For indifference data, $\Omega = 1$ for all option pairs, so that Equation 13 reduces to Equation 1.

Below, we conduct a choice-based matching study that yields preference data as well as indifference data. This allows us to cross-validate our model by estimating its parameters from the indifference data and then applying those estimates to the preference data, or vice versa.

Indifference and preference data from choice-based matching

Participants. The participants were 34 students (nine females and 25 males) from the London School of Economics and 18 students (15 females and three males) from the Instituto Superior de Psicologia Aplicada in Lisbon. The participants from London were paid £5, those from Lisbon received €7.50.

Design. The design was composed of three within-participant factors: The delay to the outcomes (standard, additively increased, or multiplicatively increased), the magnitude of the outcomes (small or large), and their sign (positive or negative). Orthogonal manipulation of these factors yielded 12 option pairs, which are displayed in Table 5.1. The presentation order of these pairs was randomized across participants.

Insert Table 5.1 about here

Procedure. Experimental sessions were run by computer, which, for each option pair, involved an adjustment procedure. On the first trial, participants chose from a pair of delayed outcomes as displayed in Table 5.1. The delays were then fixed, as was the outcome not chosen on the first trial. On subsequent trials, the outcome chosen on the first trial was adjusted. The adjustment procedure followed that described by Scholten and Read (2006, Appendix).

Preference data. For each option pair, *P* was taken to be the proportion of participants who chose *LL* (gains) or *SS* (losses) on the first trial. These choice proportions are given in Table 5.1.

Indifference data. For each option pair and each participant, the indifference-inducing outcome, x_S or x_L, was taken to be the midpoint between the values of the variable outcome on the last two trials. We then computed aggregate values of x_S and x_L across fixed and variable outcomes. These values, and those of δ, are given in Table 5.2.

Insert Table 5.2 about here

Results. The data revealed four major preference patterns: The magnitude effect, the sign effect, the delay effect, and subadditivity. Consistent with other studies using intervals of at least several months (Baron, 2000; Read, 2001; Read & Roelofsma, 2003), there was no indication of superadditivity. The tradeoff model was specified as in Equation 7, the delay-perception function w as in Equation 10, the tradeoff function Q as in Equation 11 with $B = 1$, and the value function v as in Equation 12. Thus, we estimated five parameters from 12 data points (one-period discount fractions). The results are given in Table 6. The parameter estimates of the tradeoff model confirmed the sign effect (Λ), the delay effect (τ), and subadditivity (α). The magnitude effect, generated by intra-attribute subtractivity, was attenuated by diminishing sensitivity (γ).

Insert Table 6 about here

Figure 10 displays the observed and predicted values of δ. The top panel shows that δ increases when the delays increase by the same additive constant (the delay effect) or by the same multiplicative constant (a shared implication of the delay effect and subadditivity).

Insert Figure 10 about here

The top panel also shows that δ is higher for multiplicatively increased delays than for additively increased delays. This indicates that subadditivity outweighs the delay effect. To verify this, we divide the interval 12 → 36 (demarcated by the multiplicatively increased delays and denoted *U*) into an early interval 12 → 23 (denoted *E*), an intermediate interval 23 → 29 (demarcated by the additively increased delays and denoted *M*), and a late interval 29 → 36 (denoted *L*). The delay effect implies that δ will be higher for *M* than for *U*, because the interval *E* preceding *M* is longer than the interval *L* succeeding *M*, i.e., *M* occurs toward the end of *U*.[22] Subadditivity has the opposite implication, because *M* is shorter than *U*. We verify that δ is higher for *U* than for *M*, indicating that subadditivity outweighed the delay effect.

The bottom panel of Figure 10 shows that δ is higher for large outcomes than for small ones (the magnitude effect) and that δ is higher for losses than for gains (the sign effect). In addition, the magnitude effect is greater for gains than for losses. This confirms the magnitude by sign interaction effect previously identified by Loewenstein and Prelec (1992).

We evaluated the goodness-of-fit of the tradeoff model to the one-period discount fractions, outcomes, and choice odds, after we estimated it on each of these dependent variables.[23] The results are given in Table 7.

Insert Table 7 about here

Estimating the parameters from one data set and then applying those estimates to the other obviously detracted from the goodness-of-fit, but the tradeoff model stood up well in the cross-validation. Inspection of the left and right panel of Figure 11 reveals that the detraction from goodness-of-fit was caused primarily by the *overall* levels of discounting that were predicted: Model estimation on choice odds yielded one-period discount fractions that were generally too low (i.e., too much discounting; see the left panel), whereas model estimation on one-period discount fractions or outcomes yielded choice odds that were generally too high (i.e., too little discounting; see the right panel). This seems to suggest that people are *more impulsive* in choice than in choice-based matching, which is quite intuitive.

Insert Figure 11 about here

Pruning

We earlier mentioned that the comparison of the options under consideration may be more or less cursory. The more careful assessments are the psychophysical ones just described, and the more cursory assessments are procedural ones, as described in this section.

We propose that the psychophysical model applies when there is a meaningful tradeoff between the timing and the desirability or undesirability of the outcomes. We also propose, however, that, when there is either *no* tradeoff or such a trivial one that a careful weighing of relative advantages is unnecessary, decision makers use low-effort heuristics, which we call *pruning* operations.

One pruning operation is *elimination of dominated options* (EDO): If X is better than Y along one attribute, and no worse along the other, then choose X. EDO is used when there is no tradeoff between delay and outcome, e.g., when the choice is between a smaller-later (SL) outcome and a larger-sooner (LS) one. EDO is called detection of dominance in prospect theory and corresponds to Stage I of Rubinstein's (2003) similarity model of intertemporal choice.

The other pruning operation is *elimination of nearly-dominated options* (ENO): If X is better than Y along one attribute, and only trivially worse along the other, then choose X. ENO is used when there *is* a tradeoff between delay and outcome, i.e., a choice between SS and LL, but the difference between the options along one attribute is 'negligible' and therefore pruned away. Take, for instance, the following options:

X = ($150, 2 months),

Y = ($151, 24 months).

For most people, the additional gain of $1 is 'negligible,' but the additional wait of 22 months is not. The outcome difference is therefore pruned away and the smaller-sooner outcome is chosen. ENO corresponds to one of the simplification procedures in prospect theory, called elimination of small differences between prospects, and to Stage II of Rubinstein's (2003) similarity model of intertemporal choice.

EDO and EDO are used when choice is 'obvious.' Both operations are fast and free of doubt. Indeed, the moment one starts to wonder whether an attribute difference is negligible or not, one has stopped pruning, and choice has been passed on to a more careful weighing of relative advantages.

ENO distinguishes itself from both EDO and the more careful weighing of relative advantages by a high degree of *inter-attribute sensitivity*: Whether the difference along an attribute is considered to be negligible or not depends to a high degree on the difference along the *other* attribute. Take, for instance, the following options:

X = ($151, 2 months),

Y' = ($150, 24 months),

Z' = ($150, 2 months and 1 day).

X dominates both *Y*' and *Z*'. Therefore, the choice between *X* and *Y*' is as obvious as the choice between *X* and *Z*'. Now consider the following options:

X = ($150, 2 months),

Y = ($151, 24 months),

Z = ($151, 2 months and 1 day).

The attribute differences are exactly the same as in the previous example, but this time *X* implies a tradeoff with both *Y* and *Z*. When *X* is compared with *Y*, only one of the two attribute differences is small and therefore likely to be pruned away. However, when *X* is compared with *Z*, *both* attribute differences are small, so that *neither* is likely to be pruned away. Thus, ENO will not dictate whether to choose *X* or *Z*, and choice is passed on to the tradeoff process described by the psychophysical model.

General Discussion

Research on intertemporal judgments and choices between a smaller-sooner and a larger-later outcome has revealed many preference patterns that are anomalies to the discounted-utility model, the economic standard for rational intertemporal choices. Attempts to account for these anomalies within the discounting paradigm have resulted in convoluted and psychologically opaque models. We developed a new model of intertemporal choice, the tradeoff model, in which choice results from a tradeoff between the perceived time difference (interval) and the perceived outcome difference (compensation). This model is both more parsimonious and more intuitive than any rival discounting model of comparable scope. Moreover, it accurately describes archival data as well as data from a new experiment.

In this final section, we discuss the relation between the tradeoff model and other models of choice based on tradeoffs between attribute differences, the relation between the tradeoff model and bilinear choice models, and the scope of the tradeoff model within the domain of intertemporal choice.

Tradeoffs and choice

The tradeoff model draws a distinction between more cursory, procedural assessments of the options and more careful, psychophysical assessments. When there is either no tradeoff or such a trivial one that a careful weighing of relative advantages is unnecessary, choice is based on low-effort pruning operations: Elimination of dominated and nearly-dominated options. These operations correspond to Stage I and Stage II, respectively, of Rubinstein's (2003) procedural approach to intertemporal choice (see also Leland, 2002). "If," according to Rubinstein (2003), "the two first stages were not decisive, the choice is made using a different criterion." (p. 1210) The psychophysics of the tradeoff model is *a* possible description of the 'different criterion' used in Stage III. Our description of Stage III is a natural continuation of Stages I and II, in that each of the three stages is a form of attribute-based choice.

The formal modeling of attribute-based choice processes, pioneered by Restle (1961) and Tversky (1969), has recently seen a growing interest. Recent developments include stochastic models that predict probabilities of choice from a set of two (González-Vallejo, 2002; González-Vallejo & Reid, 2006) or three options (Kivetz, Netzer, & Srinivasan, 2004a, 2004b; Scholten, 2002; Tversky & Simonson, 1993), as well as stochastic-dynamic models that also predict *latencies* of choice from a set of two (Diederich, 1997) or three options (Roe, Busemeyer, & Townsend, 2001; Usher & McClelland, 2004). All of these modeling attempts, however, address either risky choice or choice between consumer products. The tradeoff model is the first to broaden the scope of attribute-based choice modeling to intertemporal choice.

The tradeoff model is a stochastic model of choice between two options and is therefore most directly comparable to the *stochastic-difference model* of González-Vallejo (2002; González-Vallejo & Reid, 2006). As applied to the intertemporal choices addressed by the tradeoff model, the stochastic-difference model suggests that indifference between *SS* and *LL* arises when

$$H(v(x_S), v(x_L)) - H(w(t_S), w(t_L)) + \psi = \varepsilon,$$

where ψ is a decision threshold and ε is a random error. When $\psi > 0$, there is a bias in favor of *LL* (gains) or *SS* (losses); when $\psi < 0$, there is a bias against these options. The tradeoff function *H* translates the attributes into a common currency.[24] González-Vallejo (2002) proposes that the tradeoff function *H* is the proportional difference between its arguments:

$$\frac{v(x_L) - v(x_S)}{v(x_L)} - \frac{w(t_L) - w(t_S)}{w(t_L)} + \psi = \varepsilon,$$

which simplifies to

$$\frac{w(t_S)}{w(t_L)} - \frac{v(x_S)}{v(x_L)} + \psi = \varepsilon.$$

Because the *same* tradeoff function *H* applies to perceived outcomes and perceived delays, there are not many alternatives to defining it as a proportional-difference function. This, however, invites most of the problems that we have identified for Scholten and Read's (2006) interval-discounting model. Most importantly, it must invoke two elasticity properties for *v*, i.e., increasing the magnitude of the outcomes by the same multiplicative constant or changing their sign from positive to negative decreases the ratio $v(x_S)$ / $v(x_L)$, and, on top of this, an elasticity property for *w*, i.e., increasing the delays by the same multiplicative constant decreases the ratio $w(t_S) / w(t_L)$. A key distinction between González-Vallejo's (2002) and our model is that the stochastic-difference model takes the *proportional*-difference rule as the process within which *elasticity* operates, whereas the tradeoff model takes an *absolute*-difference rule as the process within which *diminishing sensitivity* operates. We have argued that our proposal is both more parsimonious and more intuitive as a description of preference patterns in intertemporal choice.

So much for tradeoffs?

Discounting models belong to the general class of bilinear choice models, in which the value of an outcome is multiplied by a weight corresponding to its delay or probability. Thus, the immediacy equivalent of a delayed outcome is:

$$x_S = v^{-1}[d(t_L)v(x_L)].$$

The certainty equivalent of a risky outcome can be described analogously (see Birnbaum & Sutton, 1992). The multiplicative operation implies that, if we plot the immediacy equivalent against the delayed outcome, we will obtain a diverging fan of lines, with a steeper slope corresponding to a shorter delay. This has generally been confirmed for delayed outcomes (Stevenson, 1986, 1992, 1993) as well as, analogously, risky ones (Anderson & Shanteau, 1970; Shanteau, 1974; Tversky, 1967a, 1967b). The tradeoff model, however, describes the immediacy equivalent as:

$$x_S = \begin{cases} v^{-1}\left[v(x_L) - Q(w(t_L))\right] & \text{if } x_S, x_L > 0 \\ v^{-1}\left[v(x_L) + Q(w(t_L))\right] & \text{if } x_S, x_L < 0. \end{cases}$$

The subtraction or addition will not produce the diverging fan unless v^{-1} includes an exponential function of its argument. The inverse of the generalized logarithmic function in Equation 12 meets this requirement. Consider, for instance, the 12 × 2 = 24 options in Table 5.2. Let t_j and x_j denote the delay and outcome, respectively, of option *j* and let $\hat{y}_j$ denote the immediacy equivalent of option *j* as estimated by the tradeoff model. Thus,

$$\hat{y}_j = \begin{cases} v^{-1}\left[v(x_j) - Q(w(t_j))\right] & \text{if } \hat{y}_j, x_j > 0 \\ v^{-1}\left[v(x_j) + Q(w(t_j))\right] & \text{if } \hat{y}_j, x_j < 0. \end{cases}$$

For each delayed outcome, we obtain the immediacy equivalent by specifying *w*, *Q*, and *v* as in Equations 10, 11, and 12, respectively, and employing the parameter estimates from Table 6. The results are displayed in Figure 12. The diverging fan of lines is obvious, both for losses (left panel) and for gains (right panel). Thus, the evidence for a 'multiplicative operation,' sometimes treated as evidence against attribute-based choice models (e.g., Stevenson, 1992), is fully compatible with the tradeoff model.

Insert Figure 12 about here

Control in intertemporal choice

The tradeoff model addresses intertemporal judgments and choices that involve a comparison between a smaller amount of money after a smaller amount of time (*SS*) and a larger amount of money after a larger amount of time (*LL*). Although this has been the focus of most experimentation on, and formal modeling of, individual intertemporal choices, it is nonetheless a *narrow* focus. For instance, we have not extended the tradeoff model to choices between sequences of outcomes (e.g., Loewenstein & Prelec, 1991, 1993), the modeling of which will require yet more psychological apparatus, possibly the operation of multiple reference points and other forms of intra-attribute sensitivity. But even the domain of intertemporal choices between *SS* and *LL* is not exhaustively described by our model. Rather, we suggest there is a continuum of choices ranging from 'uncontrolled' to 'rationally controlled,' with 'intuitively controlled' choices, as described by our model, falling in between. This is depicted in Figure 13.

Insert Figure 13 about here

Uncontrolled choices are the 'poster children' of intertemporal choices. Examples include choosing whether to have the tiramisu, to smoke another cigarette, or to engage in unprotected or forbidden sex. The person is hardly a decision maker any longer, but rather a servant of uncontrollable fears and desires (Loewenstein, 1996, 2000). These choices cannot easily be viewed as deliberate tradeoffs between *SS* and *LL*. Sometimes, *LL* cannot even be conceived as such (e.g., how is using a condom an *LL* in comparison with not using one?), and may thus not enter into the decision. Sometimes, of course, and normally as the result of a *prior* decision, people may struggle with their desires by making the tradeoff explicit to themselves (e.g., 'is the pleasure of this dessert worth the extra time I will have to spend in the gym to burn off the calories?'), but these are rare occasions, and even on these occasions the pull of the present pleasure is so great that we doubt whether any additional insight will come from modeling it as a variant of the *g*-function in the tradeoff model.

Rationally controlled choices are at the other end of the spectrum. These are made through formal reasoning. In the introduction, we described two ways in which a person could choose between delayed outcomes: Either by directly comparing the options along the time attribute and the money attribute, as described by the tradeoff model, or by first computing and then comparing discounted values. Someone who does the latter is rationally controlled. We expect, or rather *hope*, that the intertemporal choices made by a new Harvard MBA, when at work (i.e., not when cruising the night clubs or making personal investment decisions), can be formally modeled as the maximization of discounted value.

Intuitively controlled choices, addressed by the tradeoff model, fall in between these two ends of the continuum. These choices are *controlled*, because the person is not enslaved by unbridled passion, and therefore has the capacity to deliberate and to choose the option that reason commands. But the control is *intuitive*, because the person is not an agent that acts as if informed by tables of interest rates and discounted values. Rather, an explicit tradeoff is made between the timing and the desirability or undesirability of the outcomes. The questions that first come to mind are 'how much sooner?' and 'how much smaller?,' or 'how much later?' and 'how much larger?' The answers are assessments similar to those that would be made when assessing any quantity such as loudness ('how much louder?') or brightness ('how much brighter?'). The question then becomes how much weight should be given to the perceived time difference relative to the perceived outcome difference. It is this intuitive process that we have modeled in this paper.

References

Ahlbrecht, M., & Weber, M. (1997). An empirical study on intertemporal decision making under risk. *Management Science, 43,* 813-826.

Ainslie, G. W. (1975). Specious reward: A behavioral theory of impulsiveness and impulsive control. *Psychological Bulletin, 82,* 463-496.

Ainslie, G. W. (1991). Derivation of “rational” economic behavior from hyperbolic discount curves. *American Economic Review, 81,* 334-340.

al-Nowaihi, A., & Dhami, S. (2006). A note on the Loewenstein-Prelec theory of intertemporal choice. *Mathematical Social Sciences, 52,* 99-108.

Anderson, N. H., & Shanteau, J. C. (1970). Information integration in risky decision making. *Journal of Experimental Psychology, 84,* 441-451.

Baron, J. (2000). Can we use human judgments to determine the discount rate? *Risk Analysis, 20,* 861-868.

Benzion, U., Rapoport, A., & Yagil, J. (1989). Discount rates inferred from decisions: An experimental study. *Management Science, 35,* 270-284.

Birnbaum, M. H., & Sutton, S. E. (1992). Scale convergence and utility measurement. *Organizational Behavior and Human Decision Processes, 52,* 183-215.

Brandstätter, E., Gigerenzer, G., & Hertwig, R. (2006). The priority heuristic: Making choices without trade-offs. *Psychological Review, 113,* 409-432.

Chapman, G. B. (1996). Temporal discounting and utility for health and money. *Journal of Experimental Psychology: Learning, Motivation, and Cognition, 22,* 771-791.

Diederich, A. (1997). Dynamic stochastic models for decision making with time constraints. *Journal of Mathematical Psychology, 41,* 260-274.

Frederick, S., Loewenstein, G., & O’Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature, 40,* 351-401.

González-Vallejo, C. (2002). Making trade-offs: A probabilistic and context-sensitive model of choice behavior. *Psychological Review, 109,* 137-155.

González-Vallejo, C., & Reid, A. A. (2006). Quantifying persuasion effects on choice with the decision threshold of the stochastic choice model. *Organizational Behavior and Human Decision Processes, 100,* 250-267.

Green, L., Fristoe, N., & Myerson, J. (1994). Temporal discounting and preference reversals in choice between delayed outcomes. *Psychonomic Bulletin & Review, 1,* 383-389.

Green, L., Myerson, J., & Macaux, E. W. (2005). Temporal discounting when the choice is between two delayed rewards. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 31,* 1121-1133.

Green, L., Myerson, J., & McFadden, E. (1997). Rate of temporal discounting decreases with amount of reward. *Memory & Cognition, 25,* 715-723.

Hoch, S., & Loewenstein, G. (1991). Time-inconsistent preferences and consumer self-control. *Journal of Consumer Research, 17,* 492-507.

Holcomb, J. H., & Nelson, P. S. (1992). Another experimental look at individual time preference. *Rationality and Society, 4,* 199-220.

Hsee, C. K., Loewenstein, G. F., Blount, S., & Bazerman, M. H. (1999). Preference reversals between joint and separate evaluation of options: A review and theoretical analysis. *Psychological Bulletin, 125,* 576-590.

Kahneman, D. (2003). A perspective on judgment and choice. *American Psychologist, 58,* 697-720.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica, 47,* 363-391.

Keller, L. R., & Strazzera, E. (2002). Examining predictive accuracy among discounting models. *Journal of Risk and Uncertainty, 24,* 143-160.

Keren, G., & Roelofsma, P. (1995). Immediacy and certainty in intertemporal choice. *Organizational Behavior and Human Decision Processes, 63,* 287-297.
Kirby, K. N. (1997). Bidding on the future: Evidence against normative discounting of delayed rewards. *Journal of Experimental Psychology: General, 126,* 54-70.
Kirby, K. N., & Herrnstein, R. J. (1995). Preference reversals due to myopic discounting of delayed reward. *Psychological Science, 6,* 83-89.
Kirby, K. N., & Maraković, N. N. (1996). Delay-discounting probabilistic rewards: Rates decrease as amounts increase. *Psychonomic Bulletin & Review, 3,* 100-104.
Kivetz, R., Netzer, O., & Srinivasan, V. (2004a). Alternative models for capturing the compromise effect. *Journal of Marketing Research, 41,* 237-257.
Kivetz, R., Netzer, O., & Srinivasan, V. (2004b). Extending compromise effect models to complex buying situations and other context effects. *Journal of Marketing Research, 41,* 262-268.
Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics, 62,* 443-477.
LeBoeuf, R. A. (2006). Discount rates for time versus dates: The sensitivity of discounting to time-interval description. *Journal of Marketing Research, 43,* 59-72.
Leland, J. W. (2002). Similarity judgments and anomalies in intertemporal choice. *Economic Inquiry, 40,* 574-581.
Loewenstein, G. (1987). Anticipation and the valuation of delayed outcomes. *Economic Journal, 97,* 666-684.
Loewenstein, G. (1988). Frames of mind in intertemporal choice. *Management Science, 34,* 200-214.
Loewenstein, G. (1996). Out of control: Visceral influences on behavior. *Organizational Behavior and Human Decision Processes, 65,* 272-292.
Loewenstein, G. (2000). Emotions in economic theory and economic behavior. *American Economic Review, 90,* 426-432.
Loewenstein, G., & Prelec, D. (1991). Negative time preference. *American Economic Review, 81,* 347–352.
Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics, 107,* 573-597.
Loewenstein, G. & Prelec, D. (1993). Preferences for sequences of outcomes. *Psychological Review, 100,* 91-108.
Maddala, G. S. (1994). *Limited-dependent and qualitative variables in econometrics.* Cambridge, MA: Cambridge University Press.
Mellers, B. A., & Biagini, K. (1994). Similarity and choice. *Psychological Review, 101,* 505-518.
Murphy, J. G., Vuchinich, R. E., & Simpson, C. A. (2001). Delayed reward and cost discounting. *The Psychological Record, 51,* 571-588.
O'Donoghue, T., & Rabin, M. (1999). Doing it now or later. *American Economic Review, 89,* 103-124.
Payne, J. W. (1976). Task complexity and contingent processing in decision making: An information search and protocol analysis. *Organizational Behavior and Human Performance, 16,* 366-387.
Payne, J. W., Bettman, J. R., & Johnson, E. J. (1988). Adaptive strategy selection in decision making. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 14,* 534-552.
Pitt, M. A., Myung, I. J., & Zhang, S. (2002). Toward a method of selecting among computational models of cognition. *Psychological Review, 109,* 472-491.
Prelec, D., & Loewenstein, G. (1991). Decision making over time and under uncertainty: A

common approach. *Management Science, 37,* 770-786.

Rachlin, H., Raineri, A., & Cross, D. (1991). Subjective probability and delay. *Journal of the Experimental Analysis of Behavior, 55,* 233-244.

Read, D. (2001). Is time-discounting hyperbolic or subadditive? *Journal of Risk and Uncertainty, 23,* 5-32.

Read, D., Frederick, S., Orsel, B., & Rahman, J. (2005). Four score and seven years from now: The date/delay effect in temporal discounting. *Management Science, 51,* 1326-1335.

Read, D., Loewenstein, G., & Kalyanaraman, S. (1999). Mixing virtue and vice: Combining the immediacy effect and the diversification heuristic. *Journal of Behavioral Decision Making, 12,* 257-273.

Read, D., & Roelofsma, P. H. M. P. (2003). Subadditive versus hyperbolic discounting: A comparison of choice and matching. *Organizational Behavioral and Human Decision Processes, 91,* 140-153.

Restle, F. (1961). *Psychology of judgment and choice: A theoretical essay.* New York, NY: John Wiley & Sons.

Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision theory: A dynamic connectionist model of decision making. *Psychological Review, 108,* 370-392.

Roelofsma, P. H. M. P., & Read, D. (2000). Intransitive intertemporal choice. *Journal of Behavioral Decision Making, 13,* 161-177.

Rubinstein, A. (2003). “Economics and psychology”? The case of hyperbolic discounting. *International Economic Review, 44,* 1207-1216.

Russo, J. E., & Dosher, B. A. (1983). Strategies for multiattribute binary choice. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 9,* 676-696.

Samuelson, P. (1937). A note on measurement of utility. *Review of Economic Studies, 4,* 155-161.

Schmidt, U., & Traub, S. (2002). An experimental test of loss aversion. *Journal of Risk and Uncertainty, 25,* 233-249.

Scholten, M. (2002). Conflict-mediated choice. *Organizational Behavior and Human Decision Processes, 88,* 683-718.

Scholten, M., & Read, D. (2006). Discounting by intervals: A generalized model of intertemporal choice. *Management Science, 52,* 1426-1438.

Shafir, E. B., Osherson, D. N., & Smith, E. E. (1993). The advantage model: A comparative theory of evaluation and choice under risk. *Organizational Behavior and Human Decision Processes, 55,* 325-378.

Shanteau, J. C. (1974). Component processes in risky decision making. *Journal of Experimental Psychology, 103,* 680-691.

Shelley, M. K. (1993). Outcome signs, question frames, and discount rates. *Management Science, 39,* 806-815.

Simpson, C. A., & Vuchinich, R. E. (2000). Reliability of a measure of temporal discounting. *The Psychological Record, 50,* 3-16.

StatSoft (2003). Statistica (Version 6) [Computer software]. Tulsa, OK: Author.

Stevenson, M. K. (1986). A discounting model for decisions with delayed positive or negative outcomes. *Journal of Experimental Psychology: General, 115,* 131-154.

Stevenson, M. K. (1992).The impact of temporal context and risk on the judged value of future outcomes. *Organizational Behavior and Human Decision Processes, 52,* 455-491.

Stevenson, M. K. (1993). Decision making with long-term consequences: Temporal discounting for single and multiple outcomes in the future. *Journal of Experimental Psychology: General, 122,* 3-22.

Strahilevitz, M. A., & Loewenstein, G. (1998). The effect of ownership history on the valuation of objects. *Journal of Consumer Research, 25,* 276-289.
Strotz, R. H. (1955-1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies, 23,* 165-180.
Thaler, R. (1981). Some empirical evidence on dynamic inconsistency. *Economics Letters, 8,* 201-207.
Thaler, R. H., & Johnson, E. J. (1990). Gambling with the house money and trying to break even: The effects of prior outcomes on risky choice. *Management Science, 36,* 643-660.
Tversky, A. (1967a). Additivity, utility, and subjective probability. *Journal of Mathematical Psychology, 4,* 175-201.
Tversky, A. (1967b). Utility theory and additivity analysis of risky choices. *Journal of Experimental Psychology, 75,* 27-36.
Tversky, A. (1969). Intransitivity of preferences. *Psychological Review, 76,* 31-48.
Tversky, A., & Kahneman, D. (1991). Loss aversion in riskless choice: A reference-dependent model. *Quarterly Journal Economics, 107,* 1039-1061.
Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5,* 297-323.
Tversky, A., & Krantz, D. H. (1970). The dimensional representation and the metric structure of similarity data*. Journal of Mathematical Psychology, 7,* 572-596.
Tversky, A., & Simonson, I. (1993). Context-dependent preferences. *Management Science, 39,* 1179-1189.
Usher, M., & McClelland, J. L. (2004). Loss aversion and inhibition in dynamical models of multialternative choice. *Psychological Review, 111,* 757-769.
Yates, J. F., & Watts, R. A. (1975). Preferences for deferred losses. *Organizational Behavior and Human Decision Processes, 13,* 294-306.

Authors' Note

Marc Scholten, Instituto Superior de Psicologia Aplicada, Lisboa, Portugal. Daniel Read, Durham Business School, Durham, United Kingdom and London School of Economics, London, UK. The authors are grateful for the financial support received from the Fundação para a Ciência e Tecnologia (FCT), projects POCTI/PSI/42275/2001 and POCI/PSI/56030/2004.

Correspondence concerning this paper should be addressed to Marc Scholten, Instituto Superior de Psicologia Aplicada, Rua Jardim do Tabaco 34, 1149-041 Lisboa, Portugal. E-Mail: scholten@ispa.pt.

TABLE 1

Comparison of the interval-discounting model and the tradeoff model.

Interval-discounting model	Tradeoff model
Interval discounting	Intra-attribute subtractivity
Increasing proportional sensitivity	
Loss amplification	
Loss aversion	Loss aversion
Complete adaptation to rescheduled outcomes	Adaptation to rescheduled outcomes
Diminishing sensitivity	Diminishing sensitivity
Superadditive discounting (short intervals)	Augmenting relative sensitivity (short intervals)
Subadditive discounting (medium to long intervals)	Diminishing relative sensitivity (medium to long intervals)

TABLE 2

Data from Benzion et al. (1989): Magnitude of the rescheduled outcome, delay to the later outcome, goodness-of-fit to one-period discount coefficients and parameter estimates of the tradeoff model, and hypothesis tests.[a]

$100\% \times R^2 = 91.02\%$

Magnitude of rescheduled outcome	Delay to later outcome	κ	Magnitude dependence[b]		Delay dependence[c]		R		Incomplete adaptation[d]		Asymmetric adaptation[e]		Λ	Loss aversion[f]	
			$t(39)$	p	$t(39)$	p	Gains	Losses	$t(39)$	p	$t(39)$	p		$t(39)$	p
40	½	16.33	-	-	-	-	20.06	-4.95	-3.04	.00	2.28	.01	1.14	3.15	.00
40	1	10.95	-	-	-6.56	.00									
40	2	8.11	-	-	-3.51	.00									
40	4	5.61	-	-	-3.52	.00									
200	½	60.63	13.23	.00	-	-	67.86	-5.46	-5.33	.00	2.10	.02			
200	1	40.57	8.58	.00	-4.37	.00									
200	2	33.54	8.16	.00	-1.56	.06									
200	4	25.20	7.67	.00	-2.13	.02									

1,000	½	272.53	12.30	.00	-	-						
1,000	1	196.54	8.89	.00	-3.24	.00	400.32	-72.54	-4.20	.00	2.11	.02
1,000	2	150.28	7.08	.00	-1.99	.03						
1,000	4	124.67	7.61	.00	-1.26	.11						
5,000	½	919.57	7.29	.00	-	-						
5,000	1	730.30	5.96	.00	-1.55	.06	511.52	-971.29	-8.36	.00	-0.80	.79
5,000	2	588.66	5.10	.00	-1.17	.12						
5,000	4	488.64	4.86	.00	-0.90	.19						

[a]The tradeoff model was estimated on 64 data points (r) collected from 204 participants. Tests are one-tailed t-tests.
[b]Testing whether κ increases reliably when the magnitude of the rescheduled outcome increases (e.g., from $1,000 to $5,000), holding the delay to the later outcome constant.
[c]Testing whether κ decreases reliably when the delay to the later outcome increases (e.g., from 2 yr to 4 yr), holding the magnitude of the rescheduled outcome constant.
[d]Testing whether the highest level of adaptation is reliably lower than the level of complete adaptation (e.g., whether a positive reference-point shift of $400.32 is reliably smaller than a full shift of $1,000 or whether a negative reference-point shift of $971.29 is reliably smaller than a full shift of $5,000).
[e]Testing whether the level of adaptation to a gain is reliably higher than the level of adaptation to a loss (e.g., whether a positive reference-point shift of $400.32 is reliably greater than a negative shift of $72.54).
[f]Testing whether Λ is reliably greater than one.

TABLE 3

Data from Scholten and Read (2006): Goodness-of-fit to one-period discount fractions and parameter estimates of the tradeoff model.[a]

$100\% \times R^2 = 84.35\%$

Parameter	Estimate	$t(5)$	p
κ	104.76	2.08[b]	.05
α	0.51	0.86[b]	.21
B	1.68	2.79[c]	.02
τ	0.03	2.44[b]	.03

[a]The tradeoff model was estimated on nine data points (logarithmic transforms of δ) collected from 42 participants.
[b]Testing whether the estimate is reliably greater than zero (one-tailed t-test).
[c]Testing whether the estimate is reliably greater than one (one-tailed t-test).

TABLE 4

Data from Scholten and Read (2006): Fitting and cross-fitting the tradeoff model to one-period discount fractions and outcomes (100% × R^2).[a]

Tradeoff model estimated on	Tradeoff model evaluated on	
	δ	x_L
δ	84.35	98.69
x_L	75.46	99.78

[a]The tradeoff model was estimated and evaluated on logarithmic transforms of the dependent variables.

TABLE 5.1

Preference data from choice-based matching study: Delays, outcomes, and choice probabilities.

t_S[a]	t_L[a]	x_S[b]	x_L[b]	P[c]	$\hat{P}$[d]
3	9	20	40	.481	.439
3	9	-20	-40	.731	.728
3	9	200	400	.712	.629
3	9	-200	-400	.769	.853
23	29	20	40	.750	.783
23	29	-20	-40	.904	.925
23	29	200	400	.865	.887
23	29	-200	-400	.981	.964
12	36	20	40	.269	.301
12	36	-20	-40	.692	.596
12	36	200	400	.481	.483
12	36	-200	-400	.712	.762

[a]Delays in months.
[b]Outcomes in pounds or euros.
[c]Relative frequencies (N = 52).
[d]Predictions from the tradeoff model.

TABLE 5.2

Indifference data from choice-based matching study: Delays, outcomes, and one-period discount fractions.

t_S[a]	t_L[a]	x_S[b,c]	x_L[b,c]	δ[c]	$\hat{\delta}$[d]
3	9	15.94	33.11	.885	.885
3	9	-29.27	-44.45	.933	.933
3	9	184.72	296.61	.924	.921
3	9	-301.87	-430.45	.943	.948
23	29	19.19	29.00	.934	.934
23	29	-32.27	-40.62	.962	.961
23	29	191.52	261.15	.950	.954
23	29	-345.80	-400.95	.976	.970
12	36	13.55	36.46	.960	.961
12	36	-28.79	-44.26	.982	.978
12	36	176.18	331.57	.974	.974
12	36	-282.65	-445.06	.981	.983

[a]Delays in months.
[b]Outcomes in pounds or euros.
[c]Geometric means (N = 52).
[d]Predictions from the tradeoff model.

TABLE 6

Data from choice-based matching study: Goodness-of-fit to one-period discount fractions and parameter estimates of the tradeoff model.[a]

$100\% \times R^2 = 98.80\%$

Parameter	Estimate	$t(7)$	p
κ	4.58	2.11[b]	.04
α	1.24	1.39[b]	.10
τ	0.16	2.41[b]	.02
γ	0.08	6.19[b]	.00
Λ	1.52	6.83[c]	.00

[a]The tradeoff model was estimated on 12 data points (logarithmic transforms of δ) collected from 52 participants.
[b]Testing whether the estimate is reliably greater than zero (one-tailed t-test).
[c]Testing whether the estimate is reliably greater than one (one-tailed t-test).

TABLE 7

Data from choice-based matching study: Fitting and cross-fitting the tradeoff model to one-period discount fractions, outcomes, and choice odds (100% × R^2).[a]

Tradeoff model estimated on	Tradeoff model evaluated on		
	δ	x_L	Ω^c
δ	98.80	99.89	84.44
x_L	98.56	99.90	82.99
Ω^b	91.96	99.65	91.62

[a]The tradeoff model was estimated and evaluated on logarithmic transforms of the dependent variables.

[b]The tradeoff model was estimated on log choice odds by minimizing a weighted least-squares loss function (see Maddala, 1994, pp. 28-32), in which the squared deviations between observed and predicted log choice odds were weighted by

$w = \sqrt{NP(1-P)}$.

Minimizing an ordinary least-squares loss function (w = 1 for all option pairs) led to very similar results.

[c] $\vartheta = 2.81$.

Figure Captions

Figure 1. Examples of the value function from prospect theory (dashed curves) and the value function proposed by Loewenstein and Prelec (1992; solid curves). The two value functions are scaled such that they cross at the edge of the graph.

Figure 2. Postponing a gain: The valuation of outcomes in case of no adaptation (top left panel), complete adaptation (top right panel), and incomplete adaptation (bottom panel) to the postponed outcome. The greater the adaptation to that outcome, the greater the compensation demanded for postponing it.

Figure 3. Applying the tradeoff model to the data from Benzion et al. (1989): The top panel shows the estimated values of R, confirming incomplete and asymmetric adaptation; the bottom panel shows the estimated values of κ, confirming magnitude and sign dependence. Outcomes are in dollars, delays are in years.

Figure 4. Observed values of ρ (left) from Benzion et al. (1989) and predicted values (right) from the tradeoff model. The top panel pools across 4 (outcome magnitude) × 4 (delay to the later outcome) = 16 data points, showing the rescheduling effect and the sign effect. The bottom panel pools across 2 (postponing versus preponing an outcome) × 2 (outcome sign) = 4 data points, showing the magnitude effect and the pseudo-delay effect. Outcomes are in dollars, delays are in years.

Figure 5. Observed values of ρ (left) from Benzion et al. (1989) and predicted values (right) from the tradeoff model for $5,000 outcomes, pooled across four delays to the later outcome.

Figure 6. Four variants of the tradeoff function Q, showing constant relative sensitivity (linear function; $\alpha \to 0$ and $B = 1$), augmenting relative sensitivity (convex function; $\alpha \to 0$), diminishing relative sensitivity (concave function; $B = 1$), and a sequence of augmenting and diminishing relative sensitivity (S-shaped function).

Figure 7. The inflection point of the tradeoff function Q.

Figure 8. The nine intervals used by Scholten and Read (2006).

Figure 9. Observed values of δ (solid curves) from Scholten and Read (2006) and predicted values (dashed curves) from the tradeoff model. The top panel pools across the six short intervals and across the two medium-length intervals, showing superadditivity and subadditivity. The bottom panel pools across the four early intervals and across the four late intervals (in either case, three short intervals and one medium-length interval), showing the delay effect. Observed and predicted values of δ displayed on a logarithmic scale.

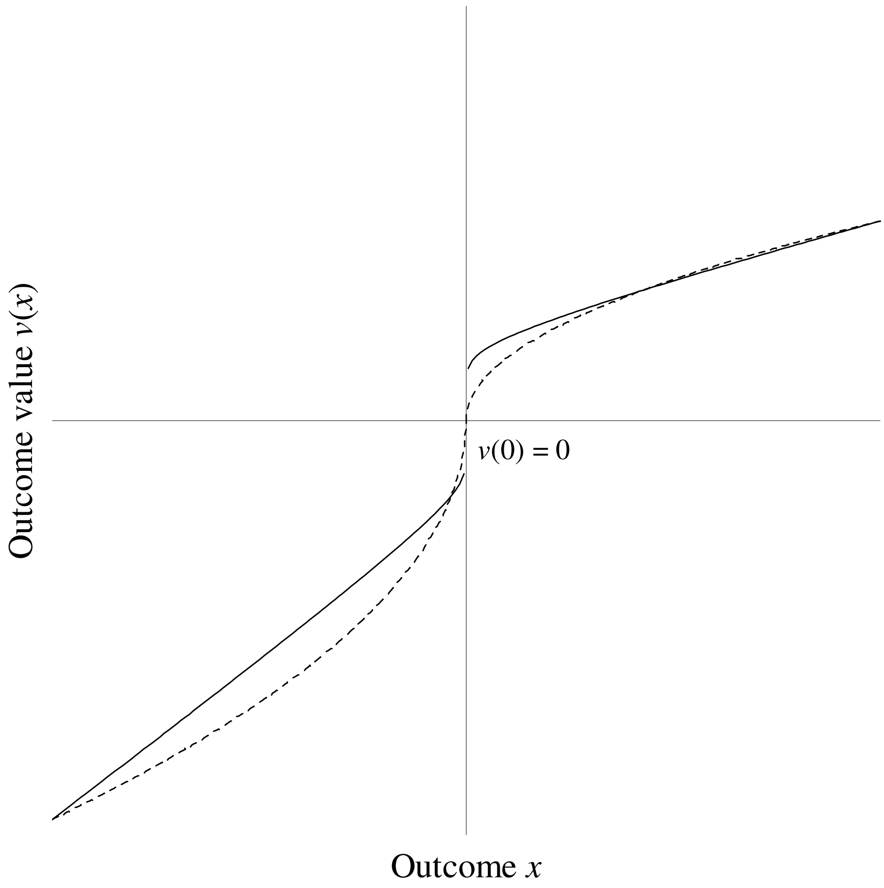
Figure 10. Observed values of δ (solid curves) from the choice-based matching study and predicted values (dashed curves) from the tradeoff model. The top panel pools across the four outcome conditions, showing the delay effect and strong subadditivity. The bottom panel pools across the three delay conditions, showing the magnitude effect, the sign effect, and their interaction. Observed and predicted values of δ displayed on a logarithmic scale.

Figure 11. Choice-based matching study: Observed and predicted values of the dependent variables, displayed on a logarithmic scale.

Figure 12. Indifference data from the choice-based matching study: Estimated immediacy equivalents of the 24 delayed outcomes. Outcomes are in pounds or euros, delays are in months.

Figure 13. Control in intertemporal choice.

FIG. 1



$\hat{x}_L$

FIG. 2

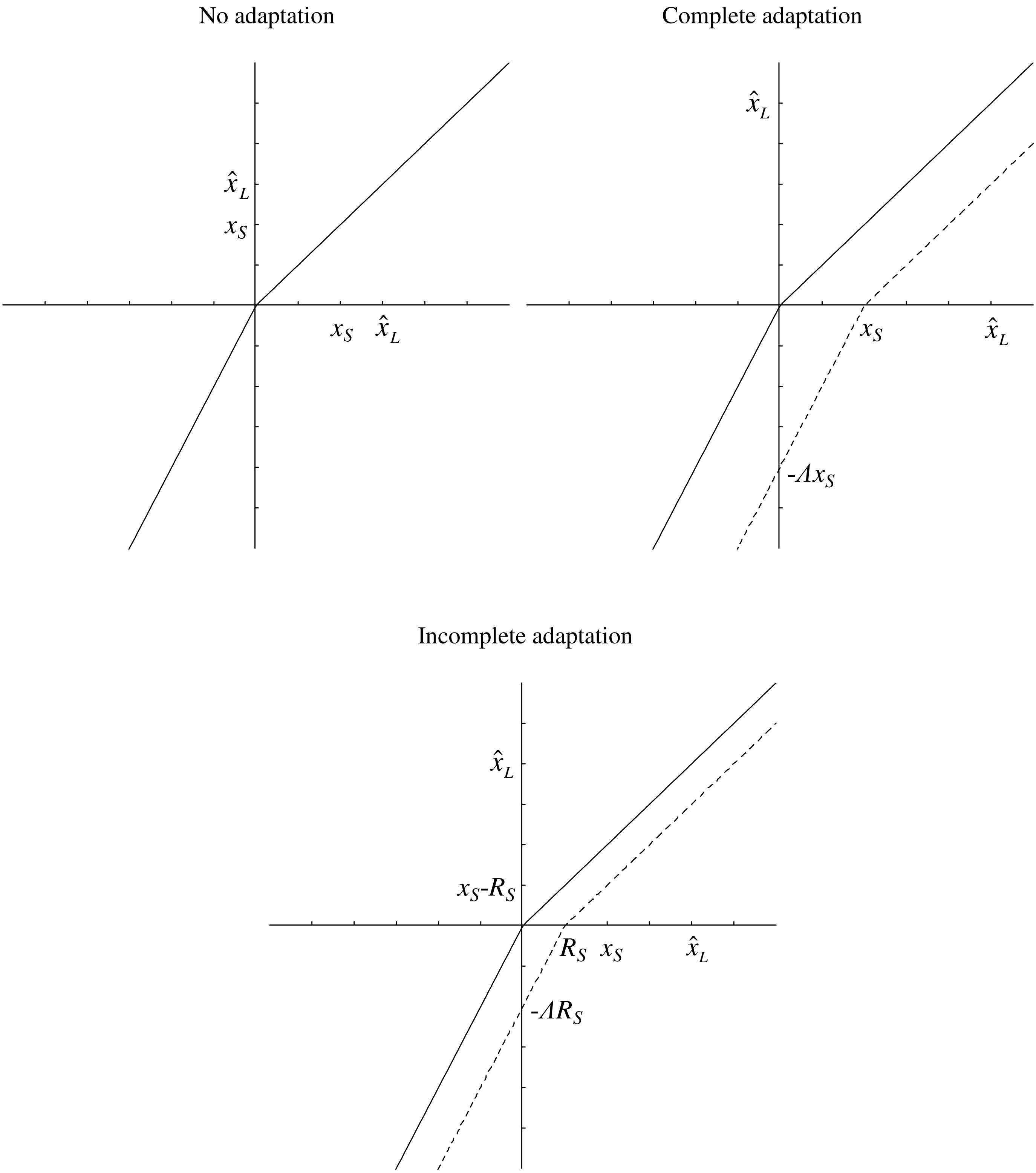

No adaptation
$\hat{x}_L$
x_S
x_S
$\hat{x}_L$
Complete adaptation
$\hat{x}_L$
x_S
$\hat{x}_L$
$-\Lambda x_S$
Incomplete adaptation
$\hat{x}_L$
x_S-R_S
R_S
x_S
$\hat{x}_L$
$-\Lambda R_S$

FIG. 3

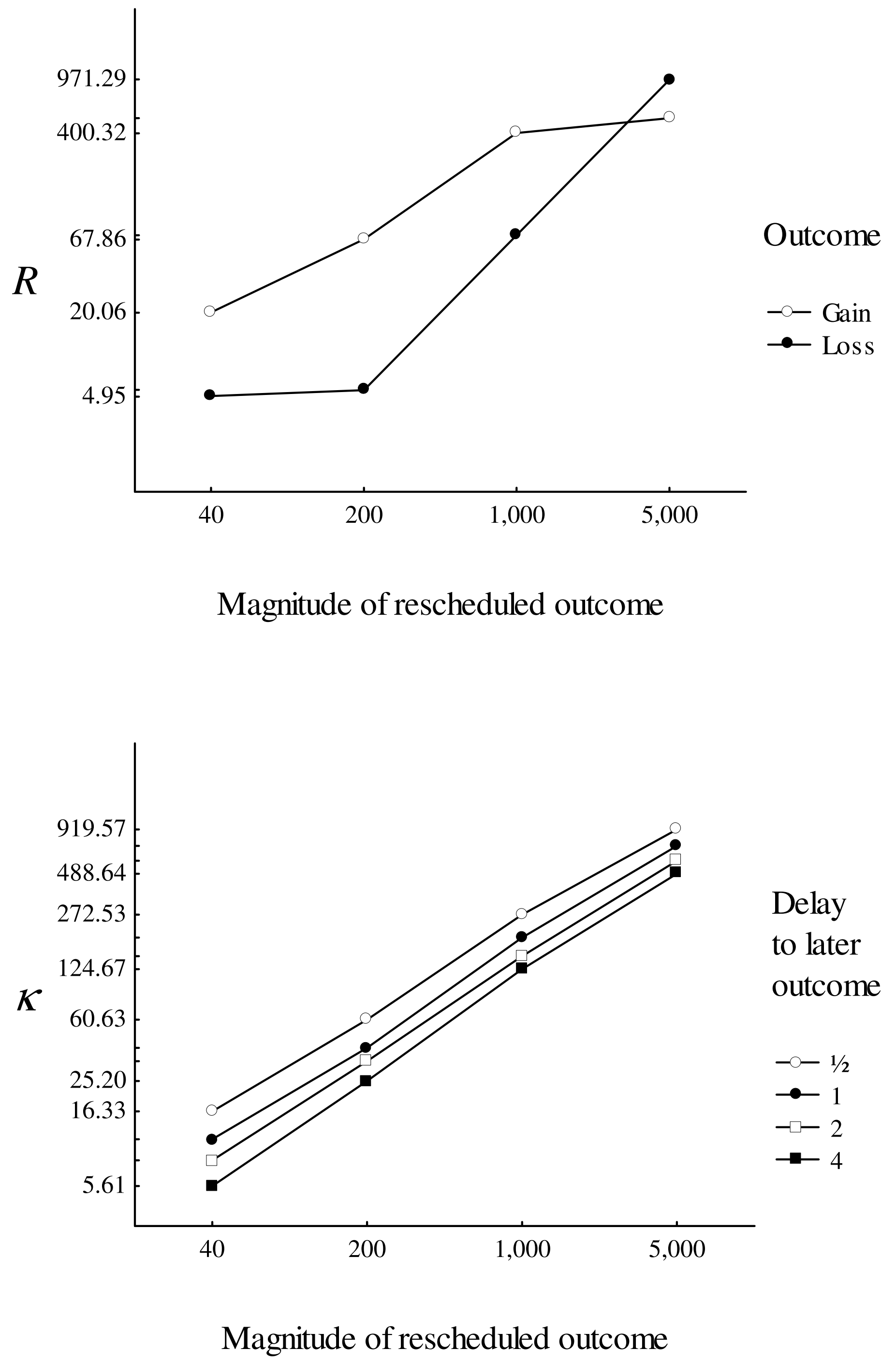
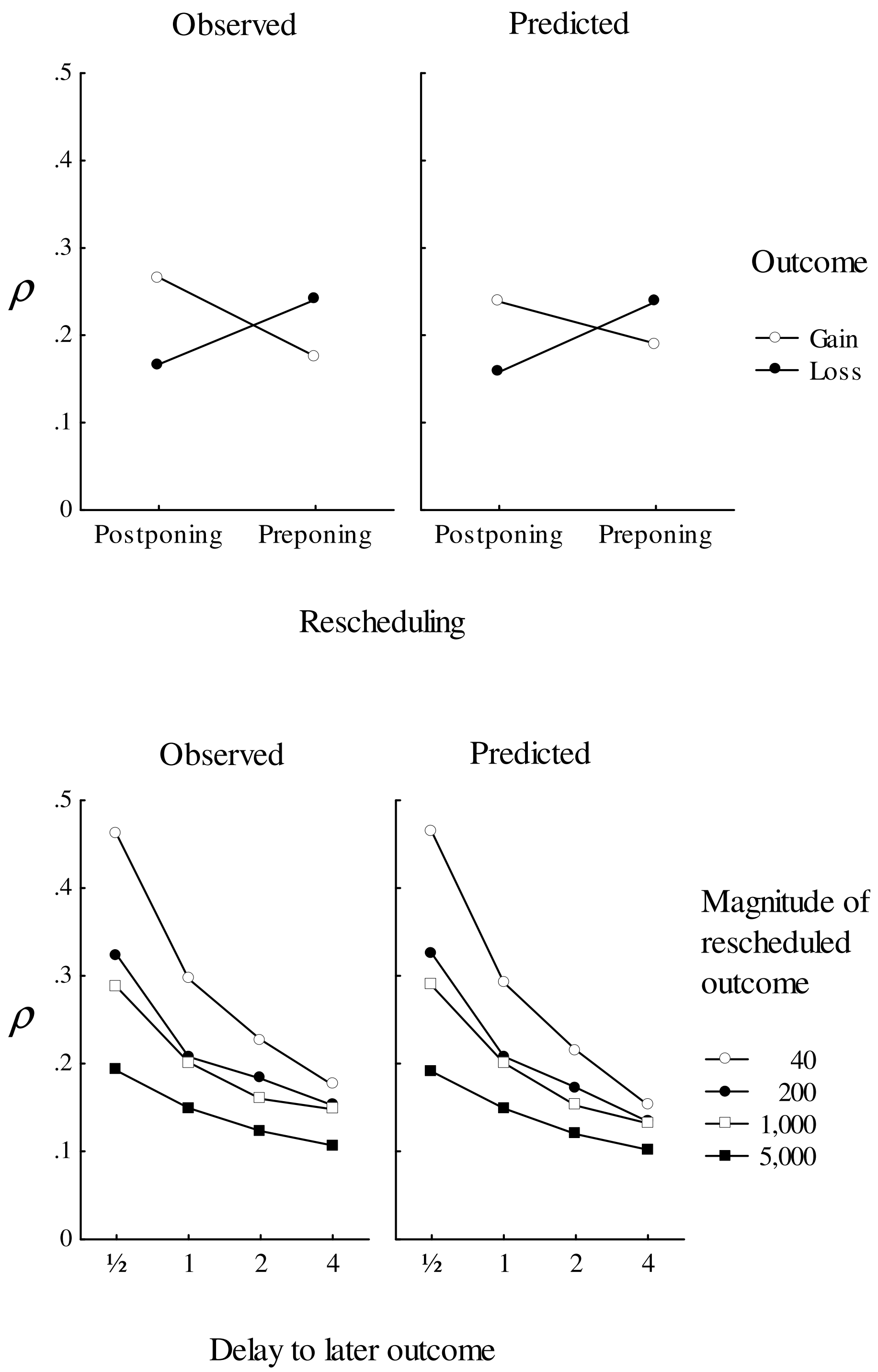

971.29
400.32
67.86
R
20.06
4.95
40
200
1,000
5,000
Magnitude of rescheduled outcome
Outcome
Gain
Loss
919.57
488.64
272.53
124.67
κ
60.63
25.20
16.33
5.61
40
200
1,000
5,000
Magnitude of rescheduled outcome
Delay
to later
outcome
½
1
2
4

FIG. 4



Observed
Predicted
.5
.4
.3
.2
.1
0
ρ
Outcome
Gain
Loss
Postponing
Preponing
Postponing
Preponing
Rescheduling
Observed
Predicted
.5
.4
.3
.2
.1
0
ρ
Magnitude of rescheduled outcome
40
200
1,000
5,000
½
1
2
4
½
1
2
4
Delay to later outcome

FIG. 5

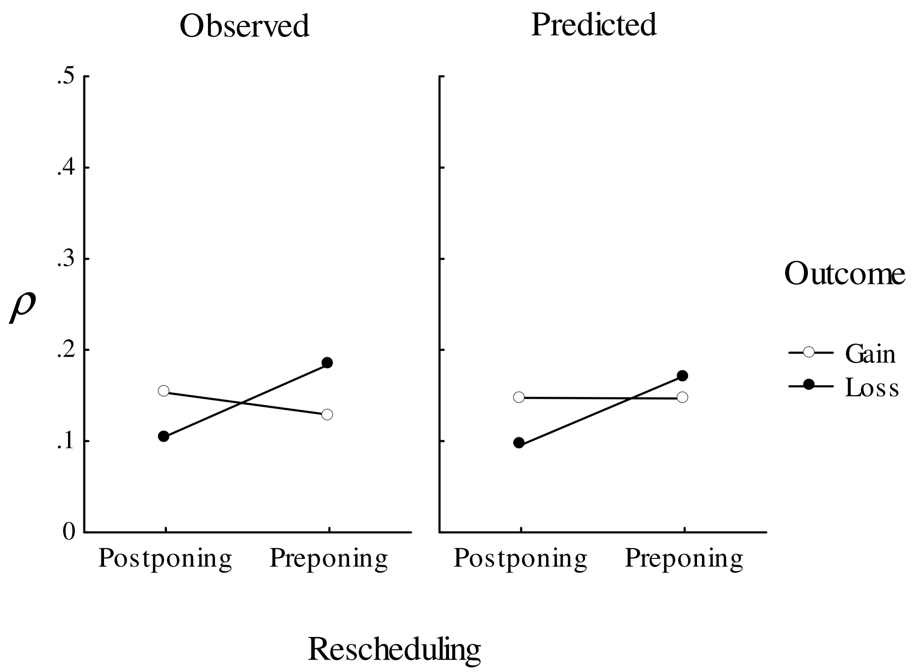

Observed
Predicted
.5
.4
.3
.2
.1
0
ρ
Postponing
Preponing
Postponing
Preponing
Rescheduling
Outcome
Gain
Loss

FIG. 6

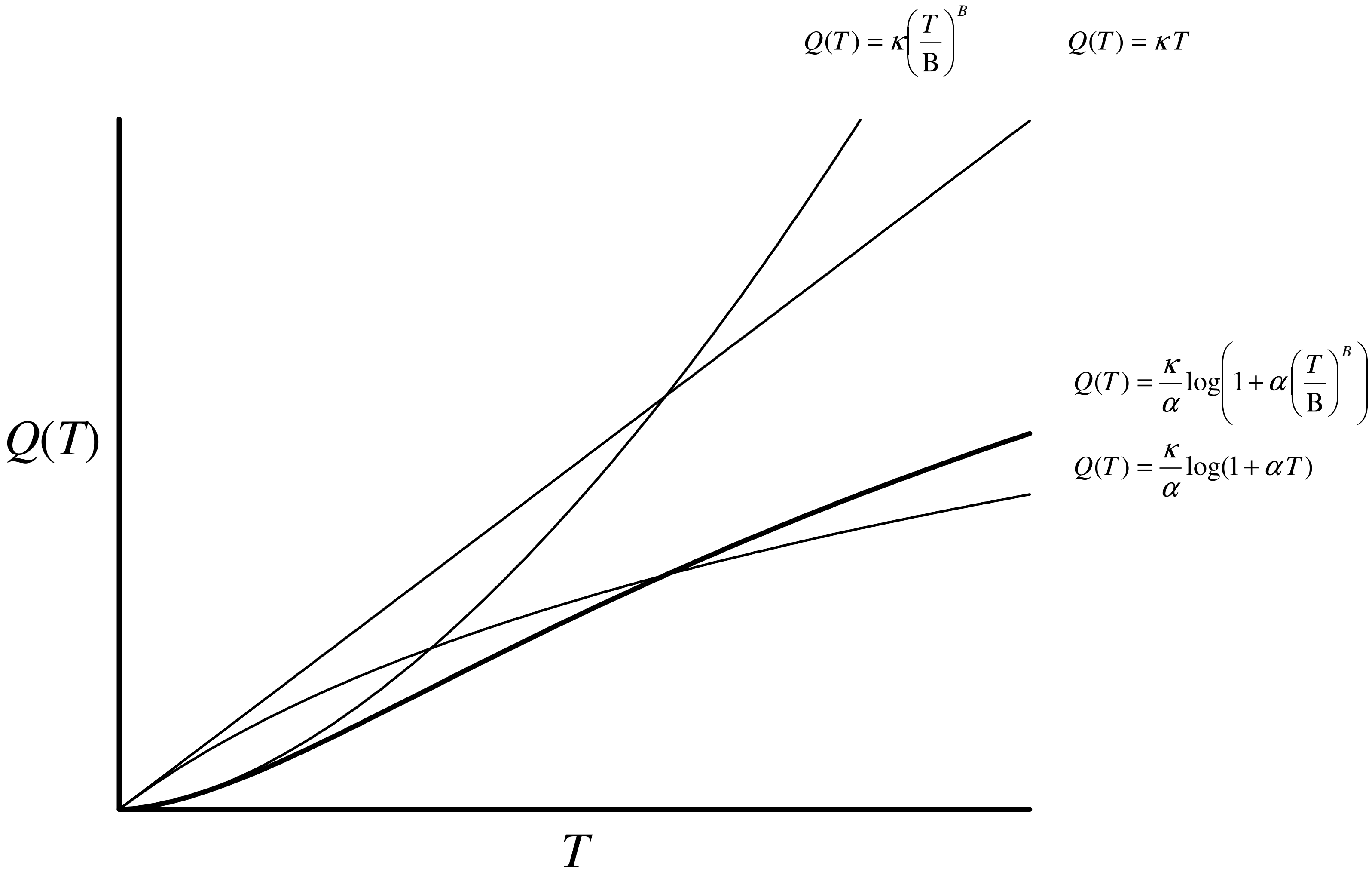

$Q(T) = \kappa\left(\frac{T}{B}\right)^{B}$
$Q(T) = \kappa T$
$Q(T) = \frac{\kappa}{\alpha}\log\left(1+\alpha\left(\frac{T}{B}\right)^{B}\right)$
$Q(T) = \frac{\kappa}{\alpha}\log(1+\alpha T)$
$Q(T)$
T

FIG.7

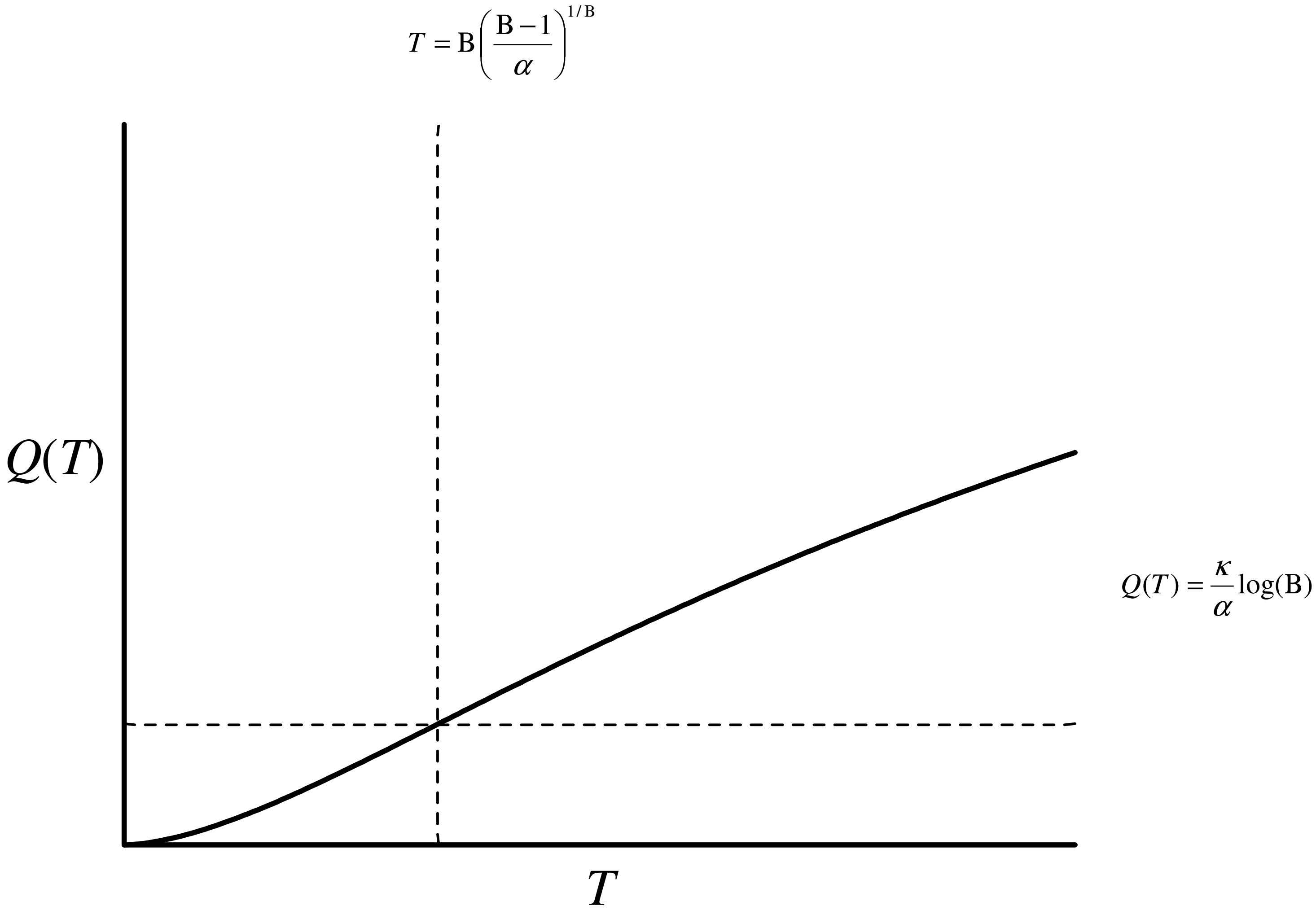


FIG. 8

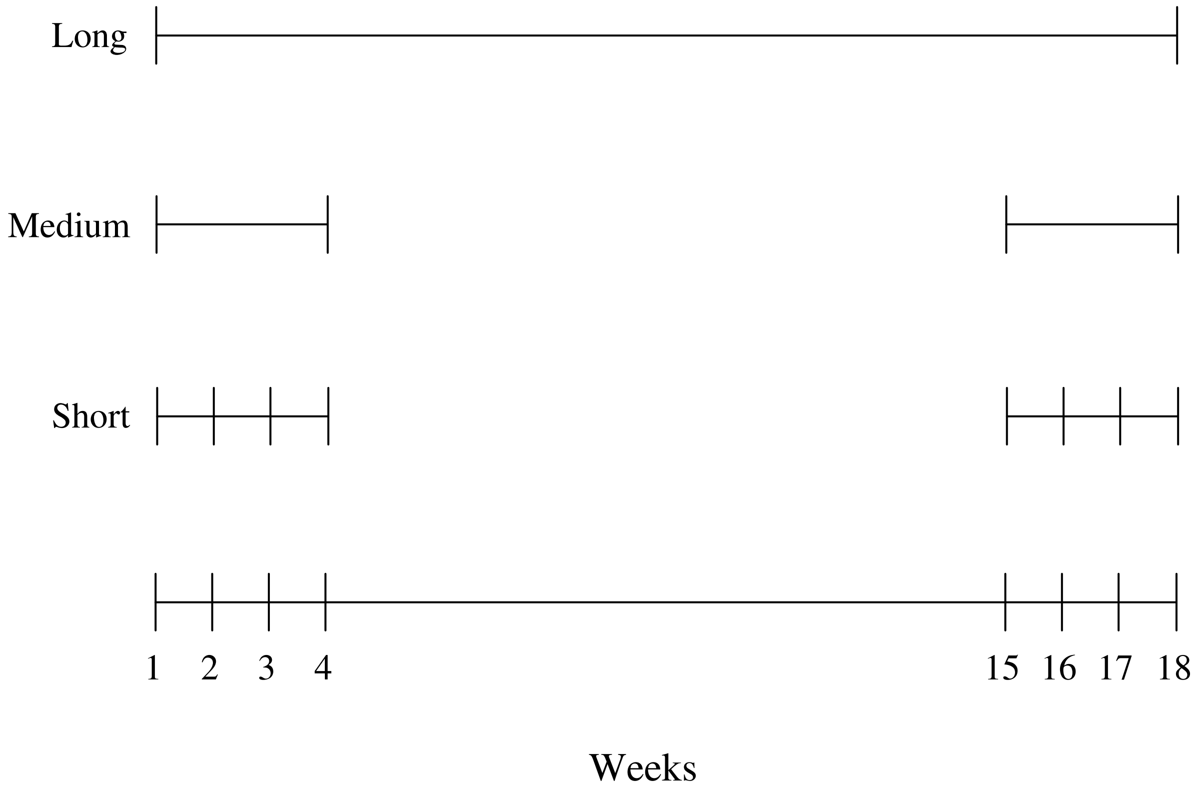


FIG. 9

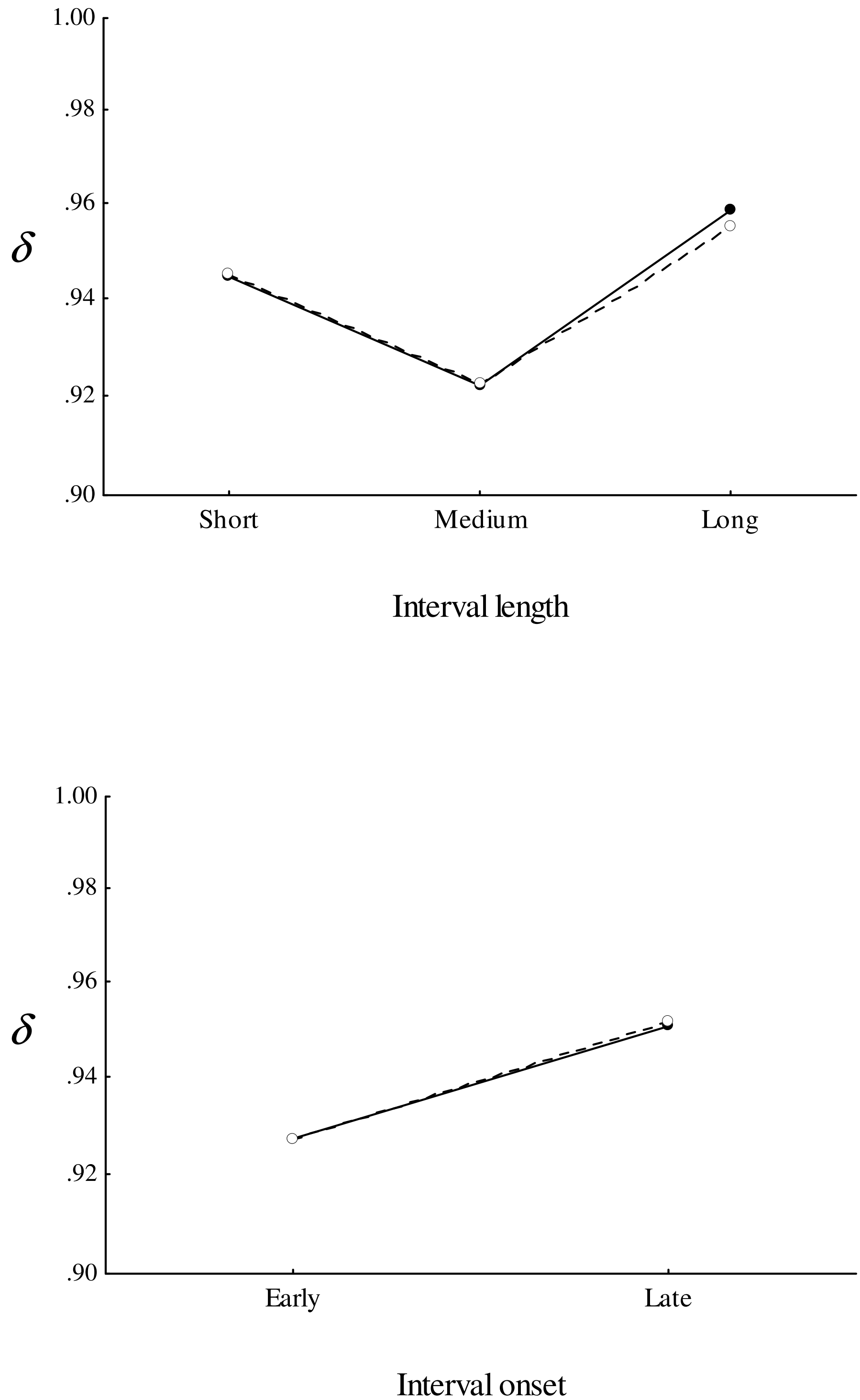

1.00
.98
.96
δ
.94
.92
.90
Short
Medium
Long
Interval length
1.00
.98
.96
δ
.94
.92
.90
Early
Late
Interval onset

FIG. 10

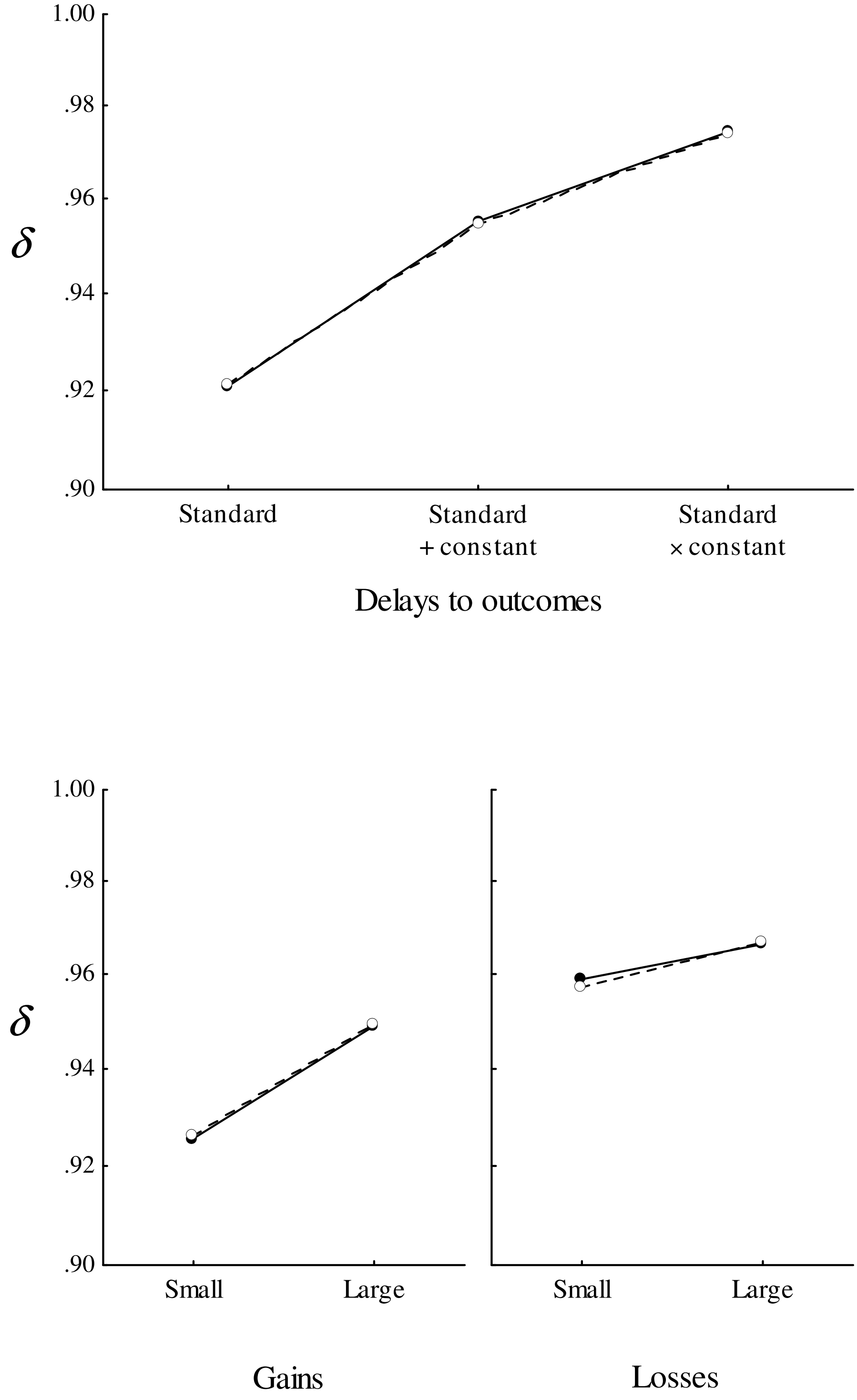
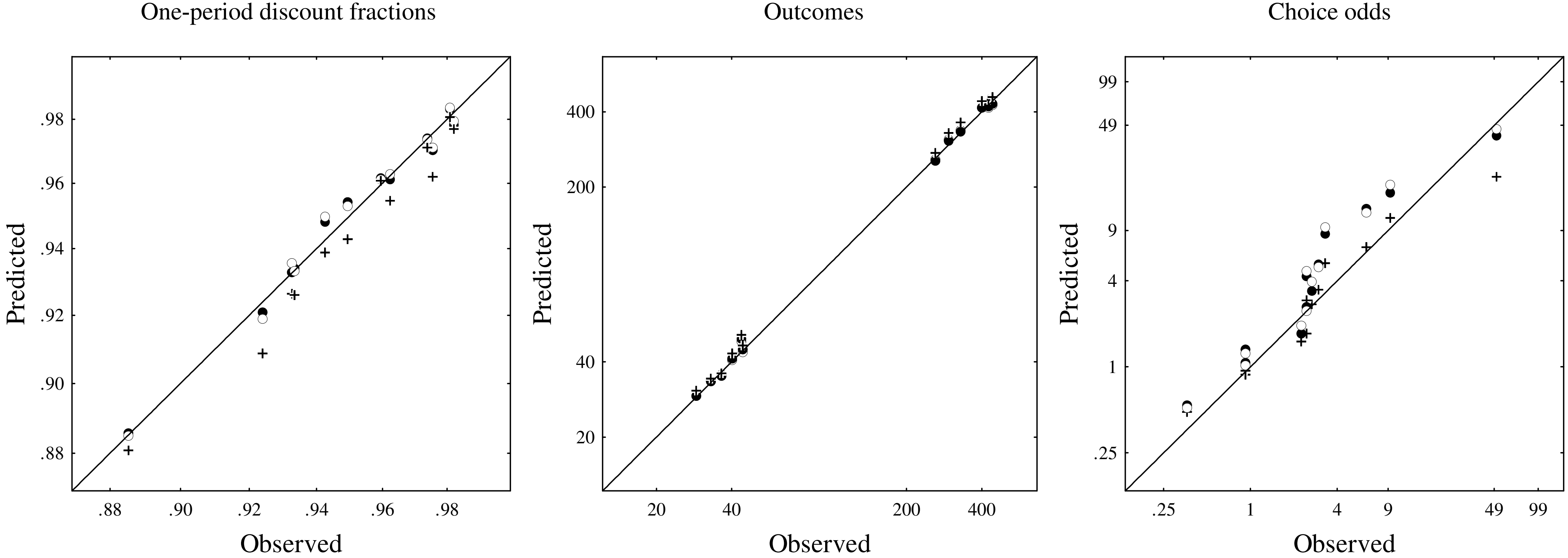

1.00
.98
.96
.94
.92
.90
δ
Standard
Standard
+ constant
Standard
× constant
Delays to outcomes
1.00
.98
.96
.94
.92
.90
δ
Small
Large
Small
Large
Gains
Losses

FIG. 11

Tradeoff model evaluated on:



Tradeoff model estimated on: One-period discount fractions; Outcomes; Choice odds.

FIG. 12

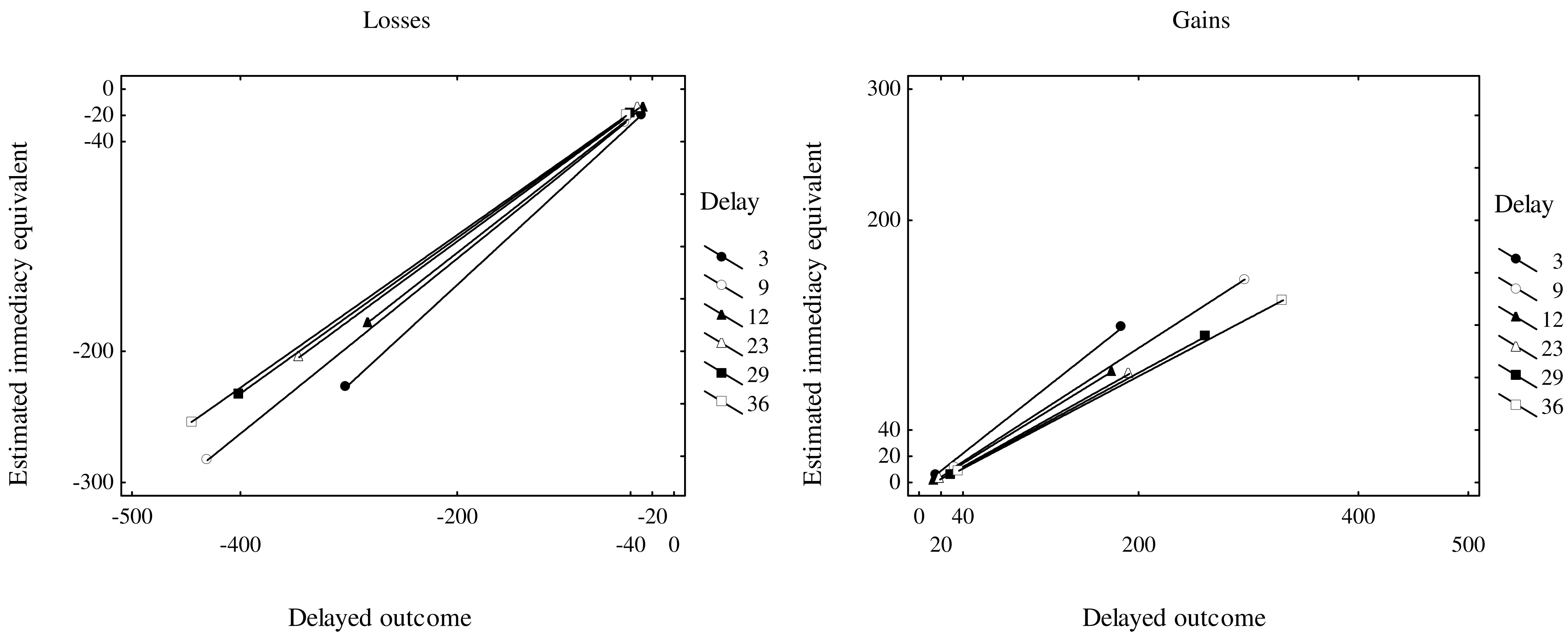
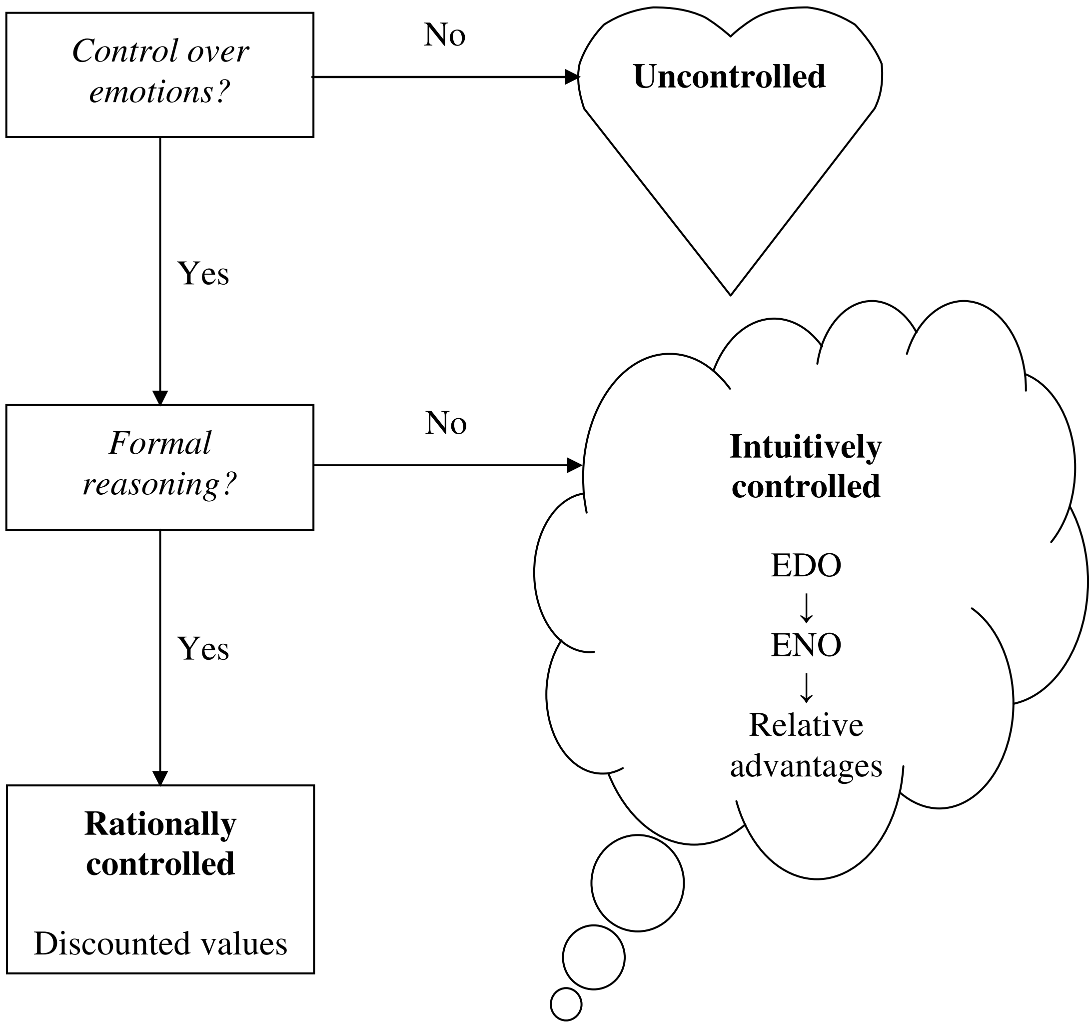

Losses
Estimated immediacy equivalent
0
-20
-40
-200
-300
-500
-400
-200
-40
-20
0
Delayed outcome
Delay
3
9
12
23
29
36
Gains
Estimated immediacy equivalent
300
200
40
20
0
0
20
40
200
400
500
Delayed outcome
Delay
3
9
12
23
29
36

FIG. 13


Control over emotions?
No
Uncontrolled
Yes
Formal reasoning?
No
Intuitively controlled
EDO
↓
ENO
↓
Relative advantages
Yes
Rationally controlled
Discounted values

Endnotes

[1]Analogous distinctions have been made between *independent* and *comparative* evaluation (Tversky, 1969), *interdimensional* and *intradimensional* processing (Payne, 1976), *holistic* and *dimensional* choice strategies (Russo & Dosher, 1983) and *separate* and *joint* evaluation (Hsee, Loewenstein, Blount, & Bazerman, 1999).
[2]We thus associate the term 'tradeoff' with attribute-based choice (González-Vallejo, 2002) rather than alternative-based choice (Brandstätter, Gigerenzer, & Hertwig, 2006).
[3]The exponential-discounting model is *not* equivalent to Samuelson's (1937) discounted-utility model. According to the discounted-utility model, indifference between *SS* and *LL* arises when $\delta^{t_S} u(c + x_S) + \delta^{t_L} u(c) = \delta^{t_S} u(c) + \delta^{t_L} u(c + x_L)$, where *c* is a constant baseline consumption level and *u* is a concave utility function over consumption levels. Thus, apart from the exponential discounting, the models are quite different with respect to the evaluation of outcomes: The exponential-discounting model effectively assumes that *u* is a *linear* utility function over *deviations from* the constant baseline consumption level.
[4]The normative status of exponential discounting arises because only constant discounting ensures that preferences will not change merely due to the passage of time (Strotz, 1955-1956). The exponential-discounting model, however, assumes exponential discounting of *outcomes*, rather than outcome *utilities*, so that variation of δ does not necessarily disprove the constant discounting as prescribed by the discounted-utility model (see also Frederick, Loewenstein, & O'Donoghue, 2002). Although the discounted-utility model may account for some variation of δ, it often fails on closer scrutiny (see Loewenstein & Prelec, 1992) and it certainly fails to account for *all* variation of δ that has proven reliable across studies.
[5]Not counting the studies in which the effect of the delay *to* outcomes is confounded with that of the interval *between* outcomes (see Read, 2001), the delay effect is seen in some (Green, Fristoe, & Myerson, 1994; Green, Myerson, & Macaux, 2005; Keren & Roelofsma, 1995; Kirby & Herrnstein, 1995; Scholten & Read, 2006), but not, or not reliably, in others (Ahlbrecht & Weber, 1997; Baron, 2000; Holcomb & Nelson, 1992; Read, 2001; Read & Roelofsma, 2003). The delay effect has been confirmed in studies where time periods were presented as delays proper (e.g., "in 1 year") and disconfirmed mostly in studies where time periods were presented as dates (e.g., "on December 1, 2007"). Recent studies show that using dates instead of delays indeed attenuates or eliminates the delay effect (LeBoeuf, 2006; Read, Frederick, Orsel, & Rahman, 2005).
[6]The paper by Loewenstein and Prelec (1992) contains a typographical error, in that their Inequality 18 reads '<' instead of '>'. However, the surrounding text is correct (see also Prelec & Loewenstein, 1991; but see al-Nowaihi & Dhami, 2006, for a different reading).
[7]Because of diminishing sensitivity, both percentages are smaller than 10.
[8]Not counting the studies in which the effect of the *sign* of an outcome is confounded with that of the *rescheduling* of an outcome (see Shelley, 1993), the sign effect is seen in some (Murphy, Vuchinich, & Simpson, 2001; Yates & Watts, 1975), but not, or not reliably, in others (Ahlbrecht & Weber, 1997; Benzion, Yagil, & Rapoport, 1989; Loewenstein, 1987; Shelley, 1993).
[9]Thaler (1981) originally ascribed the sign effect to loss aversion. However, a discounting model does not generate a sign effect on the basis of loss aversion alone, only in interaction with diminishing sensitivity. Therefore, he probably thought of loss *amplification* rather than loss *aversion*.
[10]The word 'preponing' is Indian English for 'bringing forward in time.' The former may be awkward to Anglo-American readers, but it is more concise than the latter, and it may also be

less confusing to non-native speakers.
[11]See also Hoch and Loewenstein (1991, Figure 1), Loewenstein (1988, Figures 1, 2, and 3), Shelley, 1993, Figure 1), and Strahilevitz and Loewenstein (1998, Figures 1a and 1b).
[12]The expression that "the subjective magnitude of outcomes increases with their objective magnitude" *also* applies to losses: Greater losses correspond to lower values of x and smaller losses correspond to higher values of x, so that $v'(x) > 0$.
[13]We use the Hooke-Jeeves and Quasi-Newton routine of the Statistica software (StatSoft, 2003) for model estimation.
[14]We use the term 'discounting' merely to describe empirical regularities in a data set, without intending any reference to a psychological process.
[15]This procedure thus avoids the problem, discussed by Kirby (1997), that the *arithmetic* mean of individual discount *rates* is not the same as an aggregate discount rate computed from the *arithmetic* mean of the variable outcome.
[16]Because of the delay-interval confound, the present analysis cannot distinguish between diminishing sensitivity to delays and diminishing relative sensitivity to perceived intervals.
[17]This is different for the variation of R, which we expect to confirm incomplete and asymmetric adaptation, but may otherwise not exhibit an orderly pattern. Indeed, the prime motive for this first application of the tradeoff model is to explore the variation of R.
[18]A linear value function has also been adopted in earlier examinations of the rescheduling effect (Loewenstein, 1988; Shelley, 1993; see also Hoch & Loewenstein, 1991).
[19]We could not include the data collected by Shelley (1993) in our analysis, because she did not report the discount coefficients from all 96 cells in her design. There were 32 more cells than in the design of Benzion et al. (1989), because she added a 'neutral' condition to the rescheduling factor. In this condition, Equation 4 should reduce to Equation 3, which only predicts a sign effect, i.e., lower discount coefficients for losses than for gains. However, the wording of the matching task in this condition was far from neutral: "You owe a debt of \$40 in four years to a public institute. What is the (negative) value, -\$*x*, of that debt to you now?" (Shelley, 1993, Figure 2). This can easily be 'misinterpreted' as rescheduling a commitment, especially in the context of more explicit wordings to that effect. Actually, the data from the 'neutral' condition revealed *higher* discount coefficients for losses than for gains (Shelley, 1993, Figures 3 and 6), suggesting that an undesired rescheduling effect still outweighed the sign effect.
[20]Tversky and Kahneman's (1992) value function has two diminishing-sensitivity parameters, one for gains and one for losses, thus allowing for different degrees of diminishing sensitivity to gains and losses. Equation 12 has only one diminishing-sensitivity parameter γ and captures any differential sensitivity to gains and losses with the loss-aversion parameter Λ.
[21]Formal choice models often assume a power value function (e.g., Tversky & Kahneman, 1992), rather than a logarithmic one. The power analogue of Equation 12 is:

$$v(x) = \begin{cases} \dfrac{1}{1+\gamma} x^{\frac{1}{1+\gamma}} & \text{if } x \geq 0 \\[2ex] -\dfrac{\Lambda}{1+\gamma} (-x)^{\frac{1}{1+\gamma}} & \text{if } x < 0, \end{cases}$$

which becomes a scalar function when $\gamma = 0$ and a zero function as γ goes toward infinity. The power function has a greater *complexity* than the logarithmic function, i.e., a greater capacity to fit data, but the logarithmic function seems to have a greater *generalizability* than the power function, i.e., a greater capacity to account for data to which it was not fitted (see Pitt, Myung,

& Zhang, 2002).
[22]Most models that account for the delay effect, including the tradeoff model, imply that δ will be higher for M than for U even if E is as long as L, because δ increases by a greater proportion from E to M than from M to L, i.e., $\delta_M / \delta_E > \delta_L / \delta_M$ or $\delta_M > \sqrt{\delta_E \delta_L}$.
[23]Applying a preference model to choice odds has the same problem as applying an indifference model to outcomes: An overly naïve null model. For instance, if one is equally likely to choose \$5 now or \$10 after 1 period of time, one would be more likely to choose \$95 now than \$100 after that period and more likely to choose \$5 now than \$10 after 2 periods, but the null model would assign equal probabilities throughout.
[24]González-Vallejo (2002) denotes H as d and ψ as δ, which in this paper represent a discount factor and a one-period discount fraction, respectively.