Qualitative Operators for Reasoning Maps: Evaluating Multi-Criteria Options with Networks of Reasons

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Qualitative Operators for Reasoning Maps: Evaluating Multi-Criteria Options with Networks of Reasons

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Abstract:

Cognitive/causal maps have been widely used as a powerful way of capturing decision-makers' perceptions about a problem, representing it as a causes-effects discourse. Several ways of making causal inferences from this type of model have been proposed in the Operational Research and Artificial Intelligence literatures, but none, as far as we are aware, has attempted to use a causal map structure to perform a multi-criteria evaluation of decision alternatives. Recently, we have proposed a new multi-criteria method, denominated as a Reasoning Map, which permits the use of decision-makers' reasoning, structured as a network of means-and-ends (a particular type of causal map) to perform such an evaluation. In this manner, the model resembles the way that people talk and think about decisions in practice. The method also pays explicit attention to the cognitive limitations of decision-makers in providing preference information. Thus it employs qualitative assessment of preferences, utilises aggregation operators for qualitative data and provides also qualitative outputs. In this paper we discuss and evaluate possible ways of aggregating qualitative performance information in Reasoning Maps.

Key words: cognitive maps, multi-criteria analysis, qualitative decision analysis, ordinal operators.

> "**rea·son** (n.)
>
> 1. The basis or motive for an action, decision, or conviction.
> 2. A declaration made to explain or justify action, decision, or conviction.
> 3. An underlying fact or cause that provides logical sense for a premise or occurrence.
> 4. The capacity for logical, rational, and analytic thought; intelligence."
>
> *The American Heritage Dictionary of the English Language, Fourth Edition*.

1 Introduction

Causal/cognitive maps (CMs) have been widely employed in Operational Research (OR) for supporting decision making (Mingers and Rosenhead, 2004). They are a powerful way of displaying links between causes and effects as a network, thus can be used to help decision-makers in identifying possible actions that lead to greater positive effects (Eden, 2004; Montibeller and Belton, 2006).

A significant strength of CMs is that the modelling is close to natural language, which

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reflects the way decision-makers are used to talking and thinking about decisions. This feature facilitates their use in practice and may help in building confidence in the recommendations derived from analysis of the maps (Eden, 1988).

However, causal maps permit only limited forms of causal inference – the analysis of the effects that a given cause would generate. Montibeller and Belton (2006) provide an overview of approaches that have been suggested as means of extending their capacity for causal inference on the basis of constructed causal maps. One of these approaches, coming from the field of Artificial Intelligence (AI), is that of fuzzy cognitive maps (FCMs), which extend CMs in two important directions: *i*) identifying which causes generate stronger effects (Kosko, 1986); or *ii*) assessing the dynamic effects if a given cause happened (Kosko, 1992). Another approach stemming from the AI community considers causal maps as Qualitative Probabilistic Networks, in which each link denotes a probabilistic dependence and impacts are propagated across the map (Wellman, 1994). In the OR literature, it has been proposed that one could use a Bayesian network to draw inferences in causal maps (Nadkarni and Shenoy, 2001; 2004); and also a procedure for deriving an influence diagram from a CM (Buede and Ferrell, 1993), which could then be analysed using standard decision analysis tools (see also Costa and Buede, 2000). Nevertheless, although increasing the power of causal inference, none of these approaches were designed for performing a multi-criteria analysis of decision options using the causal map structure as the model for evaluation.

When decision-makers are talking and reflecting about a decision that they face, they tend to adopt a reasoning discourse, talking about the means available for them to achieve their desired ends (Buss, 1978). This discourse involves complex chains of arguments, interlinked in intricate ways. An evaluation of options that is based on such a reasoning process may therefore be helpful. It is this type of reasoning process that the recently developed Reasoning Maps method (Montibeller et al., 2007a) intends to represent.

A Reasoning Map utilises a causal map structure to capture and represent a decision maker's reasoning about the decision faced. Within that structure, it employs user-defined qualitative scales to reflect the performance of decision options and strengths of influence, aggregating these variables using user-selected qualitative operators. Reasoning Maps can be seen as providing an approach to multicriteria modelling which lies between, on the one hand, the extremely flexible representation of decision strategies as a rule based system – as typically embodied in knowledge-based experts systems (Hayes-Roth, 1985) and characterised by the dominance-based rough set (DRSA) approach to multicriteria analysis developed by Greco et al. (2005) – and, on the other hand, the prescribed format of, for example, a multi-attribute value function.

The decision-rule approach has the advantage of being natural language based and parameter free, but as a consequence it can be difficult and time-consuming to surface relevant knowledge, as well as to derive a complete/valid representation of the preference aggregation procedure. The appropriate rule structure is often derived by inference from example cases, but may be directly elicited. On the other hand, structured models (like multi-attribute value functions) have the advantage of predefined aggregation procedures, but can be based on strong assumptions (such as conditions of preference independence) and may call for the elicitation of model parameters (such as weights and thresholds) which are not intuitively meaningful to the decision maker (Larichev, 1992).

A Reasoning Map shares the advantage of the natural language base with the decision rule approach and combines this with the flexibility to select from a range of potential aggregation procedures. In doing so the method pays explicit attention to the cognitive limitations of decision-makers in providing preference information. Thus, as already indicated, it employs qualitative assessment of preferences and utilises aggregation operators for qualitative data; it also provides qualitative outputs.

The main aim of this paper is to present and evaluate the operators which could be employed by Reasoning Maps and to suggest some directions for further research in this area. Qualitative operators have attracted much interest from AI (e.g. Yager, 1995), from researchers in fuzzy methods (e.g. Godo and Torra, 2000) and from those working on the integration of AI and Multi-Criteria Decision Analysis (e.g., Greco et al., 2005). Thus we believe that the paper may be of interest for researchers interested in conceptual and practical links between Artificial Intelligence and Multi-Criteria Decision Analysis (MCDA: see Belton and Stewart, 2002; Figueira et al., 2005).

The next section presents a brief outline of the Reasoning Map approach, which sets the scene for the following section, in which we discuss different operators for aggregating data in this type of modelling. Conclusions and directions for further research are presented at the end of the paper.

2 Reasoning Maps

In this section we present briefly the Reasoning Map method. A detailed description of the approach is presented in Montibeller et al. (2007a) and examples of its use in supporting real-world decision-making processes can be found in Montibeller et al.

(2007a; 2007b).

2.1 The Means-Ends Network

A reasoning map is a means-ends network, which represents decision-makers' discourse about a decision they have to make. This discourse is about the means available to achieve their desired ends (and also about the negative outcomes of some of these means). Each node in the map represents a concept, an idea, and the edges represent the perceived (positive or negative) influence from a given means concept to a given end concept. This system of coding is in line with that utilised in causal maps (Bryson et al., 2004; Eden, 2004) and influence diagrams as employed in System Dynamics (see Diffenbach, 1982; Wolstenholme, 1999). Note, however, that unlike these two forms of representation, Reasoning Maps do not permit cycles.

For example, if a decision maker says that an "increase in Research & Development (R&D) investment would lead to a rise in the number of new products being developed" this would be represented as:

increase in R&D investment $\xrightarrow{+}$ rise number new products being developed.

If she then says that "this increase in R&D investment would also decrease the amount of dividends being paid to stockholders", this could be mapped as:

increase in R&D investment $\xrightarrow{-}$ amount of dividends paid to stockholders.

Therefore for this basic example, an increase in R&D investment has both positive and negative outcomes, denoting the multicriteria nature of this strategy.

Formally, a reasoning map is an acyclic diagraph $\mathbf{G} = (\mathbf{C}, \mathbf{D})$, with q nodes, a node set $\mathbf{C} = \{1, 2, \ldots, q\}$ and an edge set $\mathbf{D} = \{-1, 0, +1\}$. A positive value on an edge $d_{ij} \in \mathbf{D}$ represents a positive perceived influence of a means concept C_i on an end concept C_j; a negative value represents a negative influence; and a zero value represents no influence.

Aiming to provide a cognitively valid decision method (Larichev, 1992), all elicited information, aggregation of data, and model outputs in a Reasoning Map are defined in qualitative terms, without any conversion to numbers. The evaluation process uses only qualitative (crisp) assessments, via an ordinal scale where the number of levels as well as a linguistic term describing each level (such as 'strong', 'weak', etc.) are defined by the decision-maker. To give a formal definition of this scale, let $\wp$ be a partially ordered finite set with m elements $\wp = \{p_1, p_2, \ldots, p_m\}$, where each value p_i is a qualitative label.

There are two main advantages of using purely qualitative operators. Firstly, behavioural research (e.g., Budescu and Wallsten, 1985; Huizingh and Vrolijk, 1997) has shown that human beings employ ordinally consistent preferences, as long as m is kept small (around 7±2). Secondly, there are arguments that qualitative decision analysis − which does not rely on fuzzification or any other means of quantifying qualitative preference measurement − increases the transparency of the model and, therefore, may enhance decision-makers' confidence on its outputs (Larichev, 1992; Moshkovich et al., 2005).

Following Kosko (1986), Reasoning Maps also represent the strength of each link in the map, $e_{ij,}$ using this qualitative scale $\wp$. This variable measures the strength of perceived influence of the means concept C_i (with associated variable V_i) over the end concept C_j, (with associated variable V_j). It is important to acknowledge that the notions of causality and influence, as represented by the links is causal maps, are open to multiple interpretations (see Wellman, 1994; Marchant, 1999) and care must be taken to clarify the intended meaning. In a Reasoning Map, the strength of perceived influence seeks to capture, in qualitative terms, the decision-makers' perception of the extent to which an increase in the performance of an action on a means concept may lead to an increase in its performance on an end concept. However, it should be recognised that, no matter how theoretically well-defined the model is, it is always difficult, in practice, to make sure that decision-makers use the pre-defined theoretical framework consistently throughout the process (see Montibeller et al., 2007a).

In Figure 1 we present an example of a 9-concept Reasoning Map (each node denotes a concept) with the associated variables d_{ij} and e_{ij} for each link. There are three types of concepts in this map: attribute concepts, where a given decision alternative *a* is evaluated (only out-arrows, e.g., concepts 1 to 3 in Figure 1); consequence concepts, which represent the intermediate consequences of adopting a given alternative (both in-arrows and out-arrows, e.g., concepts 4 to 7 in the same figure); and value concepts, which represent the decision-maker's ultimate ends (only in-arrows, e.g., concepts 8 and 9 in the same figure).

2.2 Evaluating Decision Alternatives

Decision alternatives are evaluated by the attribute concepts in a Reasoning Map. Each i-th attribute concept has an associated variable $V^A_i(a)$ which measures the performance using the same ordinal scale $\wp$ (concepts 1 to 3 in Figure 1, for example). The attributes should be preferentially independent (ordinal independence).

For consequence and value concepts, the performance of decision alternatives has a positive and a negative component (as suggested by Zhang *et al*., 1989). Thus each of these i-th concepts has an associated variable $V_i(a) = [V^-_i(a), V^+_i(a)]$, as exemplified in Figure 1 (concepts 4 to 9).

As in cognitive maps (Axelrod, 1976) and fuzzy cognitive maps (Kosko, 1986) two parameters are required for drawing causal inferences in Reasoning Maps (for details, see Montibeller and Belton, 2006), namely, partial effects and total effects.

A partial effect is calculated for each link in the map, and a total effect for each of its concepts (except for attribute concepts). A partial effect is a qualitative function of the decision alternative's performance and the strength of the respective link

For links leaving attribute concepts (e.g., links e_{16}, e_{14}, e_{24}, e_{25}, e_{35} and e_{39} in Figure 1), the *partial effect* of the i-th attribute on the *t* concept, for an alternative *a*, is given by:

$$V^+_{it}(a) = \mathbf{PE}[e_{it}, V^A_i(a)] \text{ if the perceived influence } d_{it} \text{ is positive or} \quad [1]$$

$$V^-_{it}(a) = \mathbf{PE}[e_{it}, V^A_i(a)] \text{ if the perceived influence } d_{it} \text{ is negative} \quad [2]$$

for i =1, 2, …, n.

For example, assume that the performance of alternative *a* on the attribute concepts are the ones shown in Figure 2 and that *Minimum* is chosen as the PE operator (see Section 3.1 for a discussion on this choice). In this case, the partial effects arriving in concept 4 are (from Eq. [1]):

$V^+_{14}(a) = \mathbf{PE}[e_{14}, V^A_1(a)]$ = Min ['moderate, 'strong'] = 'moderate';

$V^+_{24}(a) = \mathbf{PE}[e_{24}, V^A_2(a)]$ = Min ['weak', 'moderate'] = 'weak'.

For the other links in the map, partial effects are split into positive and negative effects. Formally, the negative and positive values of the *partial effect* (V^+_{it} and V^-_{it}) of the i-th concept on the *t* concept, for an alternative *a*, are given by:

If the sign of influence d_{it} is negative, then:

$$V^+_{it}(a) = \mathbf{PE}[e_{it}, V^-_i(a)] \text{ and } V^-_{it}(a) = \mathbf{PE}[e_{it}, V^+_i(a)]. \quad [3]$$

If the sign of influence d_{it} is positive, then:

$$V^+_{it}(a) = \mathbf{PE}[e_{it}, V^+_i(a)] \text{ and } V^-_{it}(a) = \mathbf{PE}[e_{it}, V^-_i(a)] \quad [4]$$

for i =1, 2, …, n.

Total effects (TE) for a given alternative *a* are calculated for each t-th consequence and value concept in the map (for example, concepts 4 to 9 in Figure 1), aggregating the relevant partial effects. Again we split positive ($V^+_t(a)$) and negative performances ($V^-_t(a)$):

$$V^-_t(a) = \mathbf{TE}[V^-_{1t}(a), V^-_{2t}(a), \ldots, V^-_{nt}(a)] \quad [5]$$

$V^{+}_{t}(a) = \mathbf{TE}[V^{+}_{1t}(a), V^{+}_{2t}(a),\ldots, V^{+}_{nt}(a)]$. [6]

For example, in Figure 2, given the two positive partial effects arriving in concept 4 calculated above; and assuming that the TE operator chosen was Maximum (see section 3.2 for a discussion on this choice), the total effect is (from Eq. [6]):

$V^{+}_{4}(a) = \mathbf{TE}[V^{+}_{14}(a), V^{+}_{24}(a)]$ = Max['moderate', 'weak'] = 'moderate'.

Following this procedure, and using these PE and TE operators in Eqs. [1] to [6], it is possible to calculate the total effect for each concept as shown in Figure 3 (with PE as Minimum and TE as Maximum). Notice that the alternative *a* has a positive moderate effect on value concept 8 ($V_8(a)$ = [+m]); and both a negative weak and a positive moderate effect on value concept 9 ($V_9(a)$ = [-w,+m]). If other alternatives were available, this Reasoning Map could be used to compare their impacts on these two value concepts.

Contrary to causal maps and fuzzy cognitive maps, which have pre-defined partial and total effects operators, in Reasoning Maps we suggest some flexibility in the choice of operators (**PE** in equations [1]-[4]; **TE** in equations [5] and [6]). The discussion on, and an evaluation of, some of the possible operators is presented in the next section.

3 Qualitative Operators for Reasoning Maps

As already indicated, Reasoning Maps keep the modelling purely qualitative, without any quantification or fuzzification. Therefore, it can use operators for partial and total effects that work with qualitative, ordinal information both as input and as output.

Several types of operators have been proposed with that purpose: *Maximum* or *Minimum* (Kosko, 1986); *Weighted-Maximum* or W*eighted-Minimum* (particular cases of the Sugeno integral, see Godo and Torra, 2000); *Median* or M*ode* (traditional ways of aggregating ordinal data); the *Ordinal Weighted Average* (OWA) operators proposed by Yager (1995) (for example, *Linear Aggregation*, referred to by him as "normative aggregation", and the *Max-Min Weighted Average*).

There are also operators with quantitative weights, like the *Weighted Median* suggested by Yager (1998), the *Linguistic Ordered Weighted Average* proposed by Herrera and Herrera-Viedma (2000), and the *Ordinal Weighted* M*ean* described in Godo and Torra (2000). However, these ones are excluded from our discussion, as they require quantitative information. For a full review of ordinal operators in general, see Domingo-Ferrer and Torra (2003).

We now present possible operators for the aggregation of partial and total effects, discussing each of the operators and the pros and cons of adopting it.

3.1 Partial Effects

One of the possible operators for the partial effect in Reasoning Maps (**PE** in Equations [1]-[4]) is the use of the operator *Min*, as proposed by Kosko (1986) for his fuzzy cognitive map. The rationale for selecting this operator for a Reasoning Map is that the strength of perceived influence e_{it} works as a cap on the means-end transmission: a strong performance is brought upwards to the superior concepts if the influence is strong, but not if it the influence is weak.

An important advantage of this operator is that it does not require any parameter elicitation, which is particularly important for large maps or when the time for eliciting information from decision makers is limited (for example, if a Reasoning Map is being used in a decision conferencing mode; for details about this type of intervention, see Phillips, 2007). Also, it is quite simple to explain, which may increase decision makers' confidence in the recommendations of the model (see a discussion about the advantages of simple decision models in Edwards et al., 1988). Conversely, the main drawback of the *Min* operator is that it does not allow further modelling of decision-maker's preferences. Furthermore, it leads to a degradation of outcomes by compressing the performances of decision alternatives (for example on a given means concept, a *strong* alternative and a *moderate* one would have the same partial effect if the strength of influence is *weak*).

An alternative approach is to use a decision table, as exemplified in Table 1, where all the possible combinations of partial effects (from 'very weak' to 'very strong') are combined with the strength of influence (again from 'very weak' to 'very strong'). The decision-makers then provide the total effect for each combination, for example: a very strong performance on a means concept which has a weak strength of influence towards an end concept generates, in their view, a moderate partial effect (highlighted in bold). As suggested by Greco et al. (2005) decision tables are an easier way to elicit preferences of decision makers and the partial effect can be fully specified by the user. This type of operator may be feasible if the same table could be employed for the whole map, otherwise the burden of elicitation would be extremely heavy.

In the example described in Section 2.2, the decision Table 1 (DecTable) would provide the following results for partial effects 1-4 and 2-4 (highlighted by a circle in the table):

$V^{+}_{14}(a)$ = **PE**[e_{14}, $V^{A}_{1}(a)$] = DecTable ['moderate, 'strong'] = 'strong';

$V^{+}_{24}(a)$ = **PE**[e_{24}, $V^{A}_{2}(a)$] = DecTable ['weak', 'moderate'] = 'weak'.

3.2 Total Effects

There are several qualitative operators that can be employed for calculating total effects. They differ in several aspects, in particular: *i*) how easily they can be understood by decision-makers, *ii*) how sophisticated is the preference modelling they allow; *iii*) the elicitation burden they impose on decision-makers; *iv*) the nature of the aggregation they perform (i.e., whether or not the resultant level of total effect is influenced by the value of each and every partial effect); and *v*) their discriminatory power.

We now present and discuss these operators in light of the above considerations. The use of each operator is illustrated with the following problem:

Suppose a set of 5 ordered labels (m = 5):

$\wp$ = {'very weak (vw)', 'weak (w)', 'moderate (m)', 'strong (s)', 'very strong (vs)'}.

Then let $V_y(a)$ be the *original set* of *n* positive partial effects of an alternative *a*, arriving at node *y* of a Reasoning Map (n = 5) as shown in Figure 4:

$V_y(a)$ = {$V_{1y}(a)$ = 's'; $V_{2y}(a)$ = 's'; $V_{3y}(a)$ = 'm', $V_{4y}(a)$ = 'vs', $V_{5y}(a)$ = 'w'}.

Ordering the elements of $V_y(a)$ from the strongest to the weakest influence, creates the *ordered set* of partial effects, $V_y^o(a)$:

$V_y^o(a)$ = {$V_{1y}^o(a)$ = 'vs' ; $V_{2y}^o(a)$ = 's'; $V_{3y}^o(a)$ = 's', $V_{4y}^o(a)$ = 'm', $V_{5y}^o(a)$ = 'w'}.

There are three main types of operator: those for which the outcome depends on the position of elements in an ordered set of partial effects; those for which it depends on the frequency of occurrence of elements; and those for which it depends on the position of elements in the original set of partial effects. Each type is discussed in the sections below.

3.3 Outcome Controlled by Position in the Ordered Set

Maximum

Max [$V_y^o(a)$] : gives $V_{1y}^o(a)$ (i.e. the 1st element in the ordered set)

In the Example: Max [$V_y^o(a)$] = $V_{1y}^o(a)$ = 'vs'.

The Max operator reflects an optimistic decision attitude. Its main advantages are that it does not require extra preference elicitation and is easily understood by decision-makers. Conversely, its main drawbacks are that: it does not allow further modelling of preferences; it is non-aggregative, as only one partial effect (the strongest one) is taken into account; and it may provide indistinguishable results when comparing decision alternatives (for example, an alternative with a single *strong* partial effect and the others *weak*, would score the same as another alternative which has all partial effects *strong*).

Minimum

Min $[V_y^o(a)]$: gives $V_{ny}^o(a)$ (i.e. the last element in the ordered set)

In the Example: Min $[V_y^o(a)] = V_{5y}^o(a)$ = 'w'.

The Min represents a pessimistic decision attitude. Its main disadvantages and advantages are exactly the same as the Max operator, just described above.

Median

Median $[V_y^o(a)]$: if n is odd, gives $V_{[(n+1)/2]y}^o(a)$; (i.e. the middle element in the ordered set)

if n is even, gives either $V_{(n/2)y}^o(a)$ or $V_{[(n/2)+1]y}^o(a)$

Obs.: As the partial effects are on an ordinal scale, when n is even the decision maker should decide for the *optimistic* ($V_{(n/2)y}^o(a)$) or the *pessimistic* ($V_{[(n/2)+1]y}^o(a)$) assessment from the mid-point (n/2).

In the Example: Median $[V_y^o(a)] = V_{[(5+1)/2]y}^o(a) = V_{3y}^o(a)$ = 's'.

The *Median* produces an "average" in an ordinal sense. Its main advantages are that it is a simple, easy to understand concept and it does not require any extra parameters (unless the number of partial effects is even, then the decision-maker has to decide if she is optimistic or pessimistic for the mid-point). Similarly to Min and Max, the main disadvantages of the *Median* are that it does not allow further preference modelling and it is not aggregative.

Max-Min Weighted Average

MMW $[V_y^o(a)]$: gives *Max* [*Min*[α, $V_{1y}^o(a)$], $V_{ny}^o(a)$] ;

where α is degree of optimism/pessimism (where α measured on the scale $\wp$, with higher values representing a more optimistic outlook).

In the Example: if α = 'm', then:

MMW $[V_y^o(a)]$ = *Max* [*Min*[α, $V_{1y}^o(a)$], $V_{5y}^o(a)$] = *Max* [*Min*['m','vs'], 'w'] = Max ['m','w'] = 'm'.

This operator was proposed by Yager (1995), and it is the ordinal equivalent of the Hurwicz cardinal operator: αMax(.) + (1-α)Min(.). Therefore, if α = 1, then Max; if α = 0, then Min. For instance, if α = 'vs' (very optimistic), then in the example *MMW* [V_y^o(a)] = *Max* [V_y^o(a)] = 'vs'. If α = 'vw' (very pessimistic), then MMW [V_y^o(a)] = *Min* [V_y^o(a)] = 'w'.

One advantage of this operator is that it allows some preference modelling (the degree of optimism/pessimism), which can be elicited for the whole map, thus reducing the elicitation burden (however, if needed, α can be elicited for each node). It is partially aggregative, as it employs two partial effects for calculating its final result. Its disadvantages are that it can lead to indistinguishable results, and it produces α itself if $V_{1y}^o(a) < \alpha < V_{ny}^o(a)$. In addition, it is not as easily understandable by decision-makers as the previous ones.

Linear Aggregation

LA[V_y^o(a)] = Max[Min[w_1,V_{1y}^o(a)] , Min[w_2, V_{2y}^o(a)] , … , Min[w_n, V_{ny}^o(a)]];

where the w's are a function of the number of elements (n) and the number of labels (m).

This operator was also proposed by Yager (1995), and was denoted as "normative aggregation". A set of weights should be calculated first, using a function proposed by him (which takes as inputs the number of labels and the number of partial effects). Notice that the weights are not associated with a given partial effect V_{jy}(a), but only with a given position in the set of *ordered* partial effects V_{jy}^o(a).

In the Example: For 5 elements and 5 labels the weights are: w_1 = 'vw', w_2 = 'w', w_3 = 'm', w_4 = 's', w_5 = 'vs' (for details on how to calculate the weights, see Yager, 1995). Then:

LA [V_y^o(a)] = *Max*[*Min*[w_1,V_{1y}^o(a)] , *Min*[w_2,V_{2y}^o(a)] , *Min*[w_3,V_{3y}^o(a)] , *Min*[w_4,V_{4y}^o(a)] , *Min*[w_5,V_{5y}^o(a)]];

LA [V_y^o(a)] = *Max*[*Min*['vw','vs'], *Min*['w','s'], *Min*['m','s'], *Min*['s','m'], *Min*['vs','w']];

LA [V_y^o(a)] = *Max*['vw','w','m','m','w'] = 'm'.

This operator is the ordinal equivalent of a cardinal weighted-sum of k criteria, with w_k

= 1/n. Its main advantages are that it is fully aggregative, as it employs all partial effects for its result, and it does not require the elicitation of any extra parameter. The main disadvantages are that it does not incorporate any extra preferential information and is a quite difficult concept to be understood by decision makers.

3.4 Outcome Controlled by Frequency

Mode

Mode $[V_y(a)]$ = gives the most frequent partial effect

In the Example: *Mode* $[V_y(a)] = \{V_{2y}(a) , V_{3y}(a)\}$ = 's'.

This operator provides the most frequent partial effect. While being a quite simple concept, and not requiring any extra preference elicitation, the mode may provide unstable results, as the change in a single partial effect may change drastically the response of this function (Domingo-Ferrer and Torra, 2003). Therefore this operator is not recommendable when there are high levels of uncertainty about performances or strengths of influence.

3.5 Outcome Controlled by Position in the Original Set

Weighted Minimum

$WMin\ [V_y(a)] = Min[Min[w_1,V_{1y}(a)] , Min[w_2,V_{2y}(a)] , \ldots , Min[w_n,V_{ny}(a)]];$

where w's are elicited from the decision maker.

This is a special case of the Sugeno Integral, concerning ordinal data (Domingo-Ferrer and Torra, 2003). A weight should be elicited for each partial effect. Notice that in this operator, contrary to LA, each weight is associated *a priori* with a given partial effect $V_{jy}(a)$.

In the Example: Assuming that the decision maker defined w_1 = 's', w_2 = 'w', w_3 = 's', w_4 = 'w', w_5 = 'm', then:

$WMin\ [V_y(a)] = Min[Min[w_1,V_{1y}(a)] , Min[w_2,V_{2y}(a)] , Min[w_3,V_{3y}(a)] , Min[w_4,V_{4y}(a)] , Min[w_5,V_{5y}(a)]];$

WMin $[V_y(a)]$ = Min[Min['s','s'] , Min['w','s'] , Min['s','m'] , Min['w','vs'] , Min['m','w']];

WMin $[V_y(a)]$ = Min['s','w','m','w','w'] = 'w'.

This operator reflects a pessimistic weighted average decision attitude. Its main advantages are that it is an aggregative operator (as it considers all the partial effects for computing its result) and is relatively easy to be understood. It imposes, however, a huge elicitation burden. Besides, the role of weights could be misunderstood, as the partial performances of decision alternatives have already been modulated by the partial effect operator – so it would be like considering weights twice.

Weighted Maximum

$WMax\,[V_y(a)] = Max[Min[w_1,V_{1y}(a)]\,,\, Min[w_2,V_{2y}(a)]\,,\, \ldots\,,\, Min[w_n,V_{ny}(a)]];$

where w's are elicited from the decision maker.

This operator is similar to the previous one, but takes the Maximum of the minima between weight and partial effect. Again here each weight is associated *a priori* with a given partial effect.

In the Example: Assuming the same weights as the previous operator:

$WMax[V_y(a)] = Max[Min[w_1,V_{1y}(a)]\,,\, Min[w_2,V_{2y}(a)]\,,\, Min[w_3,V_{3y}(a)]\,,$
$Min[w_4,V_{4y}(a)]\,,\, Min[w_5,V_{5y}(a)]\,];$

$WMax\,[V_y(a)]$ = *Max*[*Min*['s','s'] , Min['w','s'] , Min['s','m'] , Min['w','vs'] , Min['m','w']];

$WMax\,[V_y(a)]$ = Max['s','w','m','w','w'] = 's'.

This operator has the same advantages and disadvantages as the previous one, but reflects an optimistic weighted average decision attitude.

In Table 2 we present a summary of these operators in terms of factors that may be important in their selection for performing the aggregation of total effects, namely: the type of decision attitude it conveyed; the nature of preference parameter(s) required; whether the position in the set of partial effect matters or not; and a general comment on advantages and disadvantages. We indicate whether the operator is available in the set of Excel macros we developed to analyse Reasoning Maps (for more details see Montibeller et al., 2007a).

We suggest that an appropriate operator for performing the total effect aggregation should be selected for each intervention, bearing in mind the decision maker's preferences and factors just discussed in this section. Clearly some of these factors are conflicting: for example, increasing the sophistication of modelling is achieved at the expense of increasing the elicitation burden; and so the choice of operator will be context dependent.

Initial attempts to employ the Reasoning Maps method in supporting real-world decision making, described in Montibeller et al. (2007a; 2007b), have shown that: i) it was relatively easy to elicit preferences and strengths of influence using an ordinal scale, but care had to be taken to make sure that the decision-makers fully understood the latter parameter; ii) decision-makers were able to appreciate the different results which stemmed from using different total effect operators, but the lack of a visual interactive software prevented a full exploration of such operators; iii) the degree of specification of the model (for example attributes to be employed for the qualitative appraisal of the performance of options, the definition of variables associated with concepts of the map, etc.) has a heavy impact on the time to conduct the analysis. As a whole, the two cases demonstrated that employing the decision method was feasible, but time consuming, with the decision-makers perceiving it as a useful tool in supporting their decision.

4 Conclusions and Directions for Further Research

In this paper we discussed several possible operators for aggregating qualitative data in a Reasoning Map. This recently developed decision method proposes a way of performing a qualitative appraisal of decision alternatives using a particular type of causal map to structure the evaluation. In this way, it permits the evaluation of options along complex chains of reasoning statements: from the means available to the ends that decision-makers want to achieve.

Most of the advantages (and limitations) of qualitative decision analysis (see Moshkovich et al., 2005) are shared by the Reasoning Map approach. The control imposed with regard to the cognitive complexity of eliciting preference information and interpreting output on the performance of options (for details see Larichev, 1992) – which lead to the use of qualitative (ordinal) scales for preference elicitation, as well as for data aggregation and output of results – can increase the accessibility and attraction of the method to managers, who utilise the verbal medium as their main communication tool (Mintzberg, 1973). For the same reason, the approach may also help in building confidence in its results. However, Reasoning Maps do not provide a quantitative evaluation of options and because the method employs ordinal data, it may result in a limited degree of discrimination, thus not providing a full rank of alternatives (for a full discussion on its limitations see Montibeller et al., 2007a). Therefore the method seems to be more useful for complex problems, where there is need for problem structuring (Rosenhead and Mingers, 2001) and decision-makers are mainly interested in understanding the impact of their actions in intricate chains of reasoning.

The investigation of ordinal operators for Reasoning Maps opens several avenues for future research, among them we suggest:

- *A more detailed study on the mathematical properties of the model*. While every operator presented in Table 1 has the basic properties needed for an aggregation function (commutative – the initial order of arguments does not matter; monotonic – as the partial values increase the overall value should not decrease; and idempotent – if all partial values are the same, the overall value should be equal them; see also Yager, 1995) a more detailed analysis of these properties in the network itself would be welcomed.
- *Full axiomatization of the method*. Linked with the item above, there is the need of a full axiomatization of the method, which can help in checking the properties and also in comparing it with existing methodologies.
- *Explore the use of Decision Rules and other operators*. The Decision Rules approach (Greco et al., 2005) is attracting growing interest in the MCDA community. We believe that there are potential synergies between this approach and Reasoning Maps. As exemplified for the partial effect operator, instead of choosing a predefined operator one could use decision rules for eliciting preferences in a Reasoning Map; this could make the method more flexible and able to accommodate complex preference structures. Another possible focus of research would be to investigate different ordinal operators that could deal with more complex inter-linkage of influences (e.g., when both partial effects A and B both have to be at a certain level in order to generate a total effect C).
- *Develop supporting software*. As stated elsewhere (Montibeller et al., 2007a) there is a strong need for developing a software that could implement the method. Such software could help to: *i*) analyse further the properties of the method (e.g., impact on the model's outputs when using different operators, varying the number of qualitative labels, or visually interacting in diverse ways with decision-makers); *ii*) implement different ordinal operators (including Decision Rules); iii) evaluate in more detail the use in practice of the Reasoning Maps method. There are open issues on how to define the "best" aggregation operator for a given decision-maker and the "right" number of qualitative levels for a particular problem/map/decision-maker; appropriate software would help enormously in conducting research on this front.
- *Further applications*. More applications of the method are clearly needed, not only to further develop the method itself but also in order to: *i*) assess for which types of

problems the method is more suitable (e.g., it may be more suitable when the problem has a more qualitative nature, hard data is difficult to obtain, or the decision-makers feel more comfortable expressing their preferences and analysing results in qualitative terms); *ii*) compare the use of Reasoning Maps with traditional MCDA methods (such as Multi-Attribute Value Analysis, see Belton and Stewart, 2002) in practice.

Concluding the paper, there are potentially strong links between MCDA and AI, which may provide interesting opportunities for future research. In particular, the approaches of eliciting and processing qualitative information by AI, and the focus on the multi-criteria evaluation of decision alternatives by MCDA, seem complementary rather than competing. The Reasoning Maps method is one of the approaches which attempts to make these links, as it utilises cognitive/causal maps and ordinal operators, two areas that have been extensively explored by AI, to perform a qualitative multi-criteria evaluation of decision options. We recognise that the method is yet in its infancy, with several research questions still open – in particular the use of ordinal operators for aggregating qualitative data – but we hope the discussion presented in this paper could be of some interest to the OR community, given the positive results obtained in applying it to some complex real world decisions (described in Montibeller et al., 2007a, 2007b) and a growing interest on linking AI and MCDA.

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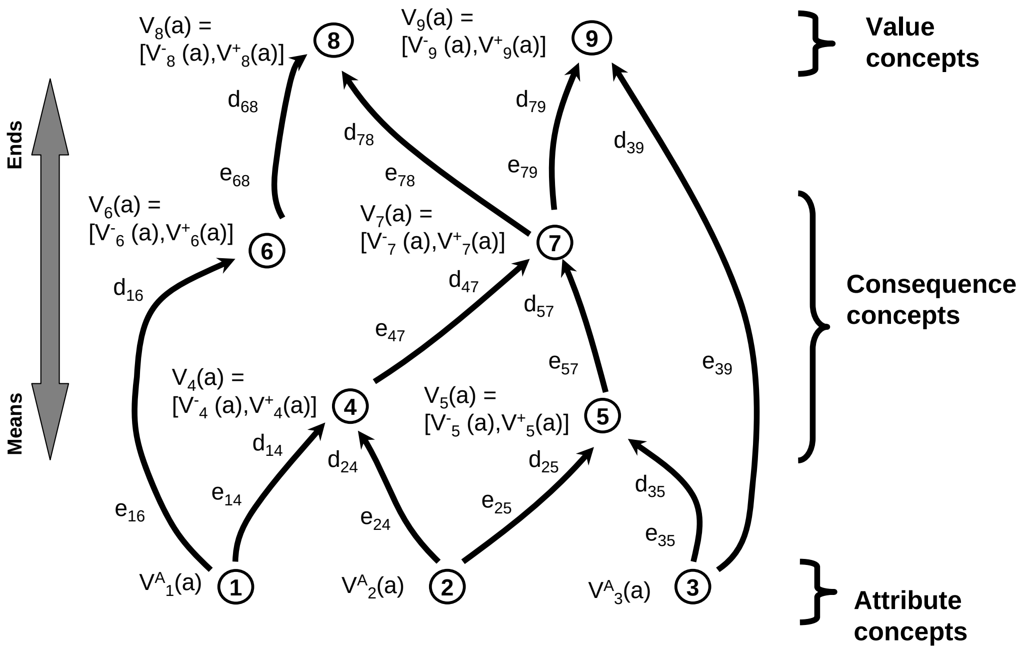

Figure 1. An Example – Variables of a Reasoning Map.

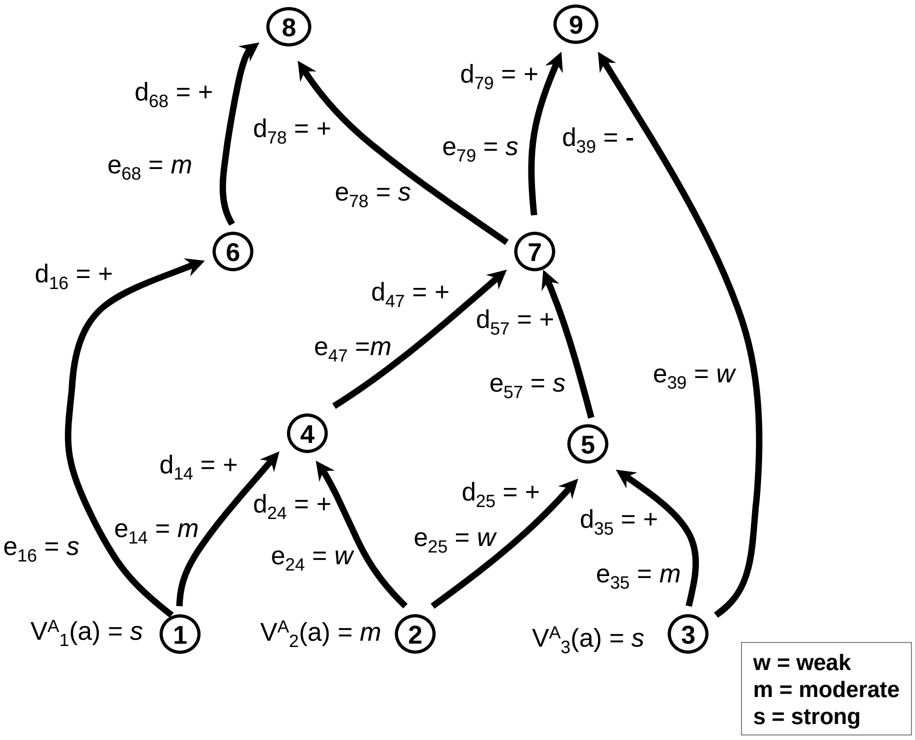

Figure 2. An Example - Evaluating a decision alternative in a Reasoning Map.

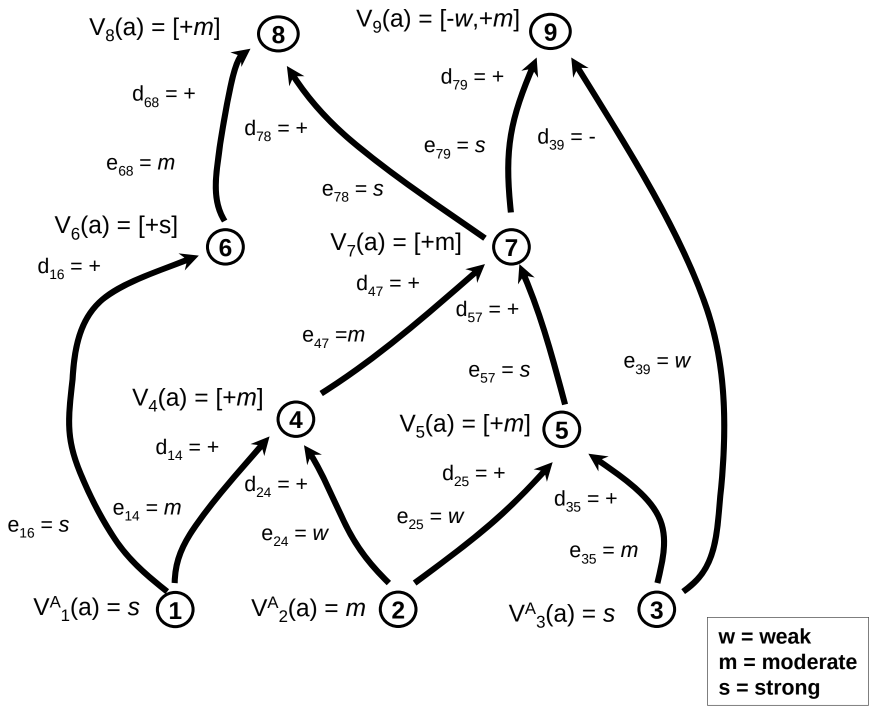

Figure 3. An Example – Total effects of alternative *a* in a Reasoning Map.

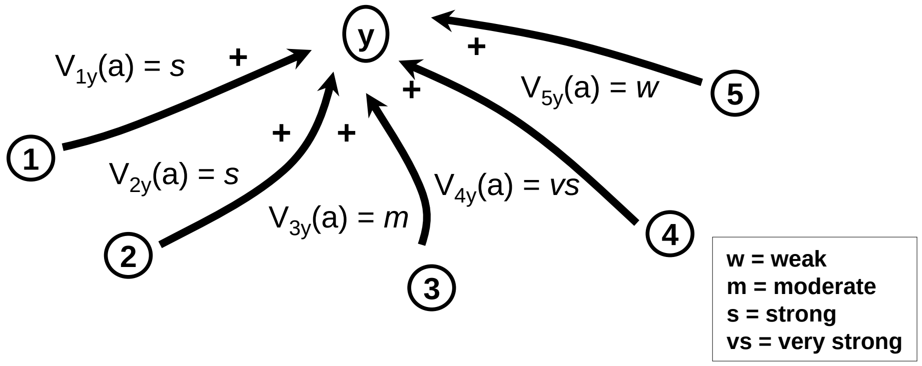

Figure 4. Aggregating five partial performances in a Reasoning Map.

		Strength of Influence				
		very weak	⇩ **weak**	**moderate**	**strong**	**very strong**
Performance on means concept means concept	**very weak**	very weak	very weak	very weak	very weak	very weak
	weak	very weak	weak	weak	moderate	moderate
	moderate	very weak	weak	moderate	moderate	strong
	strong	very weak	moderate	strong	strong	strong
	⇨ **very strong**	very weak	**moderate**	strong	strong	very strong

Table 1. A Decision Table for Calculating the Partial Effect – An example.

Operator	Decision Attitude	Type Parameter	Position matters?	Advantages	Disadvantages	Available ?
Maximum	optimistic	none	no	▪ no extra parameter ▪ easy concept	▪ no extra preferential information ▪ non-aggregative ▪ indistinguishable results	yes
Minimum	pessimistic	none	no	▪ no extra parameter ▪ easy concept	▪ no extra preferential information ▪ non-aggregative ▪ indistinguishable results	yes
Median	“average”	optimistic or pessimistic mid-point (when n is even)	no	▪ single parameter for the whole map ▪ relatively easy concept	▪ non-aggregative ▪ no extra preferential information	yes
Mode	most frequent	none	no	▪ easy concept ▪ no extra parameter	▪ non-aggregative ▪ sensitive response to a variations of a single partial effect	no
Max-Min Weighted Average	weighted average of optimistic and pessimistic effects	degree of optimism	no	▪ extra preferential information ▪ single parameter for the whole map ▪ partially aggregative	▪ indistinguishable results (response is quite sensitive to degree of optimism) ▪ not a very easy concept	yes
Linear Aggregation	weighted average	none	no	▪ aggregative ▪ no extra parameter	▪ no extra preferential information ▪ difficult concept	yes
Weighted Minimum	pessimistic weighted average	a weight for each link	yes	▪ aggregative ▪ relatively easy concept	▪ high burden of assessment ▪ role of weights may be misunderstood	no
Weighted Maximum	optimistic weighted average	a weight for each link	yes	▪ aggregative ▪ relatively easy concept	▪ high burden of assessment ▪ role of weights may be misunderstood	no

Table 2. Comparing total effect operators for Reasoning Maps.