Convergence, Endogenous Growth, and Productivity Disturbances

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Nontechnical Summary

For policy-makers and non-specialist observers, interest in income convergence across countries lies in a simple question, Are the poor catching up with the rich? Concern over this is the same as that over the dynamics of income distributions and inequality across people and families within societies. Observers need not have a stake in a set of precise economic hypotheses in order to be interested in the answer to this question. The issues here are basic and fundamental, and cut across broad fields of economics.

Therefore, in empirical studies of macroeconomic growth, it would appear that researchers are doubly blessed. Convergence is of interest in two important ways: first, in the sense just described, and second in its ability, according to some economists, to provide insight on whether growth is better characterized as exogenous or endogenous. Just as the social payoffs to understanding this are potentially large, so too the input in the number of empirical papers written.

The research reported below raises some simple theoretical warnings on reading too much into the link between convergence and the endogeneity of the growth process. Previous research on stochastic growth models had already shown that cross-country convergence can occur, even with increasing returns to scale in the aggregate production function. We strengthen that conclusion by showing that absent auxiliary, purely statistical and non-economic assumptions on cross-country interaction—assumptions up to now left only implicit—the notion of convergence cannot be sensibly addressed in a representative-economy model. Making explicit those assumptions, however, then calls into question some widely-accepted empirical findings. (We also provide a particularly simple, and we think insightful, extension and proof of a previous result on how convergence can occur in the presence of increasing returns.)

In our view the way out of these difficulties is two-fold. On the theoretical end, the researcher needs to be explicit about the nature of cross-country interaction—whether that is through merchandise trade,

exchange of ideas, communication, or coalition-formation into blocs. On the empirical end, we favor eschewing regression-based analyses, and going directly to explicit models of distribution dynamics. Clearly, this paper does not complete the program on either, but only points to their necessity.

To summarize then, the lesson we take from these theoretical manipulations is that the link is a tenuous one between convergence empirics and theoretical notions of exogenous and endogenous growth. We do *not*, however, conclude that this means convergence empirics are uninteresting. Instead, for reasons given in the first paragraph above, we think these empirics are revealing of much that is useful. These empirics call for a theoretical modelling that is explicit about the relations between individual cross-section units, and thus mesh well with recent theoretical work on social and economic interaction.

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ABSTRACT

Kelly (1992) has recently shown that evidence on convergence cannot be taken as evidence against endogenous growth in general. This study uses a well-known class of stochastic growth models to show other difficulties with traditional empirical studies of convergence. Key parameters typically cannot be estimated consistently in cross-section regressions. When the parameters are assumed known, implications for convergence are unavailable except under restrictive and economically unmotivated assumptions. Those same assumptions that relate key parameters to cross-country convergence render cross-section regressions impossible to estimate consistently.

Keywords: cross-country dependence, cross-country regression, increasing returns, stochastic growth, time-series regression

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1. Introduction

Rightly or wrongly, many macroeconomists continue to associate economic convergence with a predictive success of exogenous growth models, and in particular, a success over endogenous growth structures. Drawing so stark a distinction between different growth models is no doubt a caricature, but it serves useful clarifying purposes. Perhaps most important, it suggests that evidence for convergence is evidence that ongoing growth is impossible—without some device such as ad hoc, exogenously-specified (i.e., unmodeled) technical change. This is because convergence ought to mean that those economies that are ahead tend to slow down; those behind tend to catch up.

In an interesting recent contribution Kelly (1992) has shown that convergence is consistent with a large and important class (Long and Plosser, 1983) of endogenous growth models that display ongoing, persistent growth. That work explicitly breaks the incorrect line of reasoning described in the previous paragraph. It shows that the empirical evidence on convergence does *not* have the implications that many had attributed to it. Taken to a natural extreme, that work might be thought to cast doubt altogether on the informativeness of empirical studies of convergence.

In our view, such a conclusion is unwarranted. Empirical studies of convergence are informative because they address basic questions like, Are the rich getting richer and the poor, poorer? Are the poor catching up with the rich? Such issues have long been of concern to economists working in public finance and income distribution. These issues bear intellectual interest independent of the exact nature of technical change and economic growth. Research on growth and convergence empirics has shed new insight on these classical questions, and has moreover provided methodological payoffs for research in other areas. Tools for studying convergence empirics shed light on many different problems, including the evolving size distribution of firms in an industry; regional dynamics; and the dynamic behavior of wage disparities.

However, the analysis in Kelly (1992) does raise important new issues for interpreting convergence in economic growth. The results there—on convergence and persistent growth simultaneously occurring—turn crucially on a clever stochastic specification of a work-horse endogenous growth model. Such a setup is ideal for further developing explicit testable implications. In this paper, we exploit the framework to do exactly that: We clarify how a large class of exogenous and endogenous growth models differ in their dynamic, aggregate behavior (answer: not that much); we interpret earlier, well-known convergence studies (lessons more subtle than often imagined, relying on implicit, unwarranted assumptions); and we draw some general lessons for subsequent studies on convergence and growth. In the process, we also clarify the mechanism behind the convergence result in Kelly (1992).

The remainder of this paper is organized as follows. Section 2 sets down the basic model, simplified to emphasize only the key issues. Section 3 discusses the model's empirical implications, and relates them to well-known published studies of growth and convergence. Section 4 presents again the convergence result from Kelly (1992), only providing more intuition, and removing an ambiguity in the original argument. Finally, Section 5 concludes.

2. The basic model

We take the simplest model; this will suffice to consider all the relevant issues. Assume, therefore, a growth model having only a single sector. This is as in the most stripped-down cases in Kelly (1992), which in turn derive from the well-known model of Long and Plosser (1983).

However, in contrast to the models in Kelly (1992), we make explicit the different economies imagined in the model. For discussing convergence and cross-section dynamics, it is essential to clarify what happens when to different economies.[1] Thus, consider economies indexed by j, evolving mutually autonomously, and each

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[1] Quah, in a series of papers (Quah, 1993a–b, 1995, 1996a–d), has emphasized this in criticizing the usual “convergence hypothesis” tests. More on this below.

in equilibrium characterized by the planning problem: at time t_0, solve

$$\sup_{\{(C_j(t),K_j(t+1)):t\geq t_0\}} E_{t_0}\sum_{t=t_0}^{\infty}(1+\rho)^{-t}\log C_j(t), \quad \rho>0 \tag{1}$$

$$\text{s.t. } Y_j(t)=\theta(t)K_j(t)^{\alpha(t)} \tag{2}$$

$$C_j(t)\leq Y_j(t)-K_j(t+1),\ t\geq t_0,$$

$$K_j(t_0)>0 \text{ given.}$$

Subscripts denote an economy's label; parentheses enclose index in time. Assume that $\{\,(\theta(t),\alpha(t)):\ \text{integer } t\,\}$ is a jointly stationary vector process with all entries (a.s.) positive.

At time t agents in every economy observe the same history

$$\mathcal{F}(t)=\{\,\theta(s),\alpha(s),Y_j(s),K_j(s):\ s<t,\ \text{all } j\,\};$$

expectations conditioned on this history will be denoted $E_t=E\,(\,\cdot\,|\mathcal{F}(t))$, a notation already used in (1). Thus, by our timing assumption, $E_t\theta(t)\neq\theta(t)$ and $E_t\alpha(t)\neq\alpha(t)$ in general. Not allowing agents to see at time t the productivity disturbances $\theta(t)$ and $\alpha(t)$ is not crucial for the discussion, but we follow Kelly (1992) in this.[2] Similarly, the specification follows Kelly (1992) in assuming full depreciation, although this could be relaxed using the analysis in Stokey and Lucas (1989, pp. 10-11).

The important feature here—one whose implications will manifest in some of the discussion below—is that (θ,α) is invariant in j. Put another way, these disturbances are *common across economies*. While not emphasized in Kelly (1992)

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[2] For technical reasons, we might require (θ,α) to have compact support, but such considerations won't be essential in the discussion to follow. Also, assuming (θ,α) has first-order Markov structure would fit the problem more neatly into Stokey-Lucas (1989) notation, but again this won't be essential.

or the many empirical studies on these issues, convergence—in the way that these researchers intend—is impossible to discuss without being explicit about this.

The other feature to note is that "capital share" $\alpha(t)$, while identical across economies, is not a constant, but instead permitted to be a nondegenerate stochastic process: the convergence results in Kelly (1992) rely critically on this. We write "capital share" in quotes for the simple reason that with externalities in accumulating K, the exponent α will of course exceed capital's share of factor payments in a decentralized competitive equilibrium.

Finally, we repeat that except for the commonality in (θ, α) the economies j are assumed to evolve autonomously of each other—allowing interaction, say due to factor mobility across economies, generates dynamics that differ from those under this autonomy assumption.[3]

At time t the planner chooses consumption and investment rules that are $\mathcal{F}(t)$-measurable functions; thus, at time t, values for $C_j(t)$ and $K_j(t+1)$ are determined and known. Assume that $\{\,\alpha(t) : \text{ integer } t\,\}$ is such that the value function for problem (1) is bounded (a.s.). Then optimal investment implies the capital stock:

$$K_j(t+1) = \left((E_t\alpha(t))(1+\rho)^{-1}\right) Y_j(t), \tag{3}$$

so that from equation (2) output behaves as:

$$Y_j(t+1) = \theta(t+1) \left[\left((E_t\alpha(t))(1+\rho)^{-1}\right) Y_j(t)\right]^{\alpha(t+1)}.$$

Similarly, again from (2) the capital stock evolves as:

$$K_j(t+1) = \left((E_t\alpha(t))(1+\rho)^{-1}\right) \theta(t) K_j(t)^{\alpha(t)}.$$

Taking logs and defining $y_j \stackrel{\text{def}}{=} \log Y_j$ and $k_j \stackrel{\text{def}}{=} \log K_j$ give the first-order stochastic

[3] The implications of such factor mobility are explored in, e.g., Barro et al (1995) and Lucas (1993), and with an emphasis on empirics in Quah (1996a).

difference equations:

$$\begin{aligned} y_j(t+1) &= \log\theta(t+1) + \alpha(t+1)\log(E_t\alpha(t)) - \alpha(t+1)\log(1+\rho) \\ &\qquad + \alpha(t+1)y_j(t) \\ &= \eta(t+1) + \alpha(t+1)y_j(t) \end{aligned} \tag{4}$$

and

$$\begin{aligned} k_j(t+1) &= \log\theta(t) + \log(E_t\alpha(t)) - \log(1+\rho) + \alpha(t)k_j(t) \\ &= \epsilon(t) + \alpha(t)k_j(t) \end{aligned} \tag{5}$$

where we have defined η and ϵ thus:

$$\begin{aligned} \eta(t) &= \log\theta(t) + \alpha(t)\log(E_{t-1}\alpha(t-1)) - \alpha(t)\log(1+\rho) \\ \epsilon(t) &= \log\theta(t) + \log(E_t\alpha(t)) - \log(1+\rho). \end{aligned}$$

Note that η and ϵ are both stationary and common across economies: these properties are immediate from the corresponding ones in (θ, α).

The coefficients on lagged y and k in equations (4) and (5) come directly from the technology assumption (2)—not the investment and consumption functions, given indirectly in (3). Thus, whether the planner optimally selects investment and consumption, or whether there are accumulation externalities as in Romer (1986), the lag coefficients in (4) and (5) are *the same*: it is only the equation 'residuals' η and ϵ that change.[4]

In this (standard) model, therefore, the dynamics of output and capital in a single or representative economy do *not* distinguish efficient growth from inefficient growth with externalities. Those dynamics are the same under both kinds of growth, and it is (roughly speaking) only levels that change. This message, while not central, carries over to tests of the convergence hypothesis that we discuss more below.

[4] It is easy to see that if the decentralized competitive equilibrium ignores an aggregate capital externality, only equation (3) changes, with no impact on lagged y and k coefficients in equations (4) and (5).

3. Some important special cases and convergence empirics

To clarify the issues at stake, we consider in this section some important specializations of the model. The discussion focuses on empirical issues—as in, e.g., Bernard and Durlauf (1996), Canova and Marcet (1995), Carlino and Mills (1993), Friedman (1992), and Quah (1993a–b, 1995, 1996a–b, d)—rather than the theoretical convergence concerns in Kelly (1992). We return to the latter in the next section.

First, take the standard case where $\alpha(t) = \alpha$ for all t (a.s.). Then

$$\eta(t) = \log\theta(t) + \alpha\log((1+\rho)^{-1}\alpha),$$

so that, up to a shift in mean, η inherits all its stochastic properties directly from θ. Also, (4) becomes

$$\begin{aligned} y_j(t+1) &= \alpha y_j(t) + \eta(t+1) \qquad \text{for } t \geq t_0, \\ \text{with } y_j(t_0) &= \log\theta(t_0) + \alpha k_j(t_0). \end{aligned} \tag{6}$$

Iterating, we get

$$y_j(t) = \alpha^{t-t_0} y_j(t_0) + \sum_{s=0}^{t-1-t_0} \alpha^s \eta(t-s). \tag{7}$$

If $\alpha = 1$ then y_j for each j is an integrated (order 1) process. If, further, θ is iid through time, then equation (6) says that y_j (for each j) is a random walk with drift $E\eta = E\log\theta + \alpha\log((1+\rho)^{-1}\alpha)$ that could be positive or negative.

Most economists would take this case, $\alpha(t) = \alpha = 1$, to be *the* quintessential case of no convergence, although as will become apparent, such a conclusion does not follow. For one, studying equation (6), if the drift $E\eta$ is negative, then y_j diverges to $-\infty$ (a.s.) so that Y_j converges to 0 (again, a.s.). In a different setting (discussed further below) it is exactly this device—that divergence in logs to $-\infty$ implies convergence in levels to 0—that gives the results in Kelly (1992). But, here, this is trivial and uninteresting, and we can always assume $E\eta$ to be nonnegative since it depends only on exogenously-determined parameters. Assume this then, regardless of the value of α. When $\alpha < 1$ standard time-series reasoning applied to equation (6) says that y_j is eventually stationary (unless $k_j(t_0)$ had been chosen from the appropriate distribution whereupon y_j is immediately stationary, without qualification). When $\alpha = 1$, under our assumption on $E\eta$, then y_j is an integrated order 1 sequence with nonnegative drift, and thus neither y_j nor Y_j converges.[5]

We can now relate this to cross-section analyses of the convergence hypothesis (e.g., Barro and Sala-i-Martin (1992) and others). In equation (7) subtract $y_j(t_0)$ from both sides and then divide by $t - t_0$; this gives:

$$\begin{aligned}\frac{[y_j(t) - y_j(t_0)]}{t - t_0} &= \left[(t - t_0)^{-1}(\alpha^{t-t_0} - 1)\right] y_j(t_0) \\ &\qquad + (t - t_0)^{-1} \sum_{s=0}^{t-1-t_0} \alpha^s \eta(t - s), \\ &= \gamma(t - t_0) y_j(t_0) \\ &\qquad + (t - t_0)^{-1} \sum_{s=0}^{t-1-t_0} \alpha^s \eta(t - s), \qquad (8)\end{aligned}$$

defining the function γ in the obvious way. But this is just the usual convergence equation. The left-hand side of (8) is an average growth rate; the right hand side shows the abeyant dependence on initial conditions through

$$\gamma(t - t_0) = (t - t_0)^{-1}(\alpha^{t-t_0} - 1).$$

Off $t - t_0 = 0$, the function $\gamma(t - t_0)$ is negative whenever α is less than 1.

Studies of cross-country convergence typically focus instead on β, defined implicitly by:

$$\gamma(T) = (\exp(-\beta T) - 1)/T$$

[5] More precisely, neither y_j nor Y_j converges in distribution to a well-defined random variable.

(e.g., Barro and Sala-i-Martin, 1992). Inverting this relation, we see that:

$$\begin{aligned}\beta &= -(t-t_0)^{-1}\log(1+(t-t_0)\gamma(t-t_0)) \\ &= -(t-t_0)^{-1}\log(1+(\alpha^{t-t_0}-1)) = -\log\alpha.\end{aligned}$$

Thus, β the rate of β-convergence is positive precisely when α is less than 1, is zero when α equals 1, and varies negatively and monotonically in α.

This clarifies the relation—implied by the standard model—between cross-section and time-series tests of convergence (e.g., Barro and Sala-i-Martin (1992), Bernard and Durlauf (1996), Carlino and Mills (1993)). Both kinds of tests examine the same coefficient, but under different transformations. Where these cross-section and time-series tests differ, of course, is also clear. If η is indeed common across all economies then no cross-section regression on equation (8) can hope to estimate either β or α consistently. Thus, researchers working under the assumption that productivity disturbances are common across economies *must* disbelieve the usual cross-country convergence findings. If, however, in equation (6) $\eta(t+1)$ turns out to be uncorrelated with $y_j(t)$ then a straightforward time-series regression—carried out separately for each economy j even—will consistently recover α, regardless of the commonality of η across economies.

Writing the problem as above, it also becomes apparent that a convergence regression with $\beta = 0$ is exactly the situation where $\alpha = 1$, i.e., the case of a unit root in a time series.[6] A large literature on this, beginning with the important work of Nelson and Plosser (1982) has developed, with its principal finding that $\alpha = 1$ is an approximation not lightly discarded. Why is the cross-section on convergence *concluding the opposite* taken to be so much more compelling?

It must be that researchers believe that cross-section evidence tells something about the behavior of poor and rich economies relative to each other—and not just about the univariate properties of output in a single economy. However, while

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[6] Quah (1996b) develops further this comparison between cross-section convergence regressions and unit root processes.

recovering β or equivalently α says whether y_j—taken by itself, independently of other economies—might or might not converge to a stationary distribution, it typically says *nothing* about whether incomes in different economies are converging towards each other! Why is this?

If η is common across economies, then—as we have already argued above—cross-section regressions are difficult to interpret. However, such a situation does happen to be one where knowledge of β or α is informative, for from equation (7), we have

$$\lim_{t_0 \to -\infty} y_j(t) = \lim_{t_0 \to -\infty} \alpha^{|t_0|} y_j(t_0) + \sum_{s=1}^{\infty} \alpha^s \eta(t-s), \tag{9}$$

and (assuming $y_j(t_0)$ bounded) the restriction $\alpha < 1$ implies that the first term on the right hand side vanishes, whereas $\alpha = 1$ implies that it does not. The second term on the right of (9) does not depend on j, and thus is identical for all economies. Therefore, here, the value of α—or equivalently β even though it cannot be estimated from cross-section convergence regressions—does allow inferring whether rich and poor economies are converging towards each other.[7] However, when β has any hope of being consistently estimated by cross-section regressions—the η's are not perfectly correlated across j—the second term is no longer the same across economies, and β's value says nothing about the behavior of rich and poor economies relative to one another.

The results here relate to those of Bernard and Durlauf (1996). Those authors show that cross-section regression tests can reject the null of no-convergence too often (when the data are generated by a specific new-growth model). At the

[7] As a side issue of interest, note that in this case, $\beta = 0$ or $\alpha = 1$ does not imply growing inequality relative to the leading economy: instead, inequalities simply persist, neither growing nor shrinking. This thus provides a simple counterexample to the common claim (e.g., Parente and Prescott, 1993) that $\alpha = 1$ cannot be correct as cross-section dispersions of y in reality do not appear to be growing. More intricate examples with the same message can be easily constructed: just let η be distributed with a strong common component across economies.

same time, they argue that time-series regression tests can be sensitive to initial conditions. Our finding, by contrast, is that under the assumption of interest—η common, so that cross-country convergence is meaningfully related to $\alpha < 1$—the cross-section regression cannot even be consistently estimated, much less provide hypothesis tests. And, while our theoretical findings suggest a preference for time-series regression analyses, we agree with Bernard and Durlauf (1996) on the incipient sensitivity to initial conditions.

If we turn to when η does vary across economies, then most natural is to assume that η_j is iid across j. If $\alpha < 1$, then as $t \to \infty$, the cross-section distribution of y becomes identical to the invariant or steady-state distribution of any single one of the y_j's (by the Glivenko-Cantelli theorem). Further, that invariant distribution will typically be nondegenerate, so that what we mean by convergence of rich and poor economies is subtle. The situation is exactly that where a Galton's fallacy argument applies to interpreting cross-section convergence regressions—e.g., Friedman (1992), Quah (1993b)—and a finding of β-convergence then does not mean the gap between rich and poor is falling.

When the invariant cross-section distribution is nondegenerate, individual economies will, typically, still be transiting across different parts of the invariant distribution. There will therefore be time intervals when already rich economies are growing richer while already poor ones are growing poorer, even though there is "convergence" in the sense that $\alpha < 1$ or $\beta > 0$. Also, notice that when the invariant distribution is unique, then all initial distributions must converge to it, even initial distributions with dispersions smaller than the invariant one's (see, e.g., Fig. 1 in Quah, 1996b). Put differently, $\alpha < 1$ or equivalently $\beta > 0$ is consistent with a cross-section distribution that shows, over short and medium runs, *increasing* dispersion between rich and poor.

To summarize, then, this reasoning has suggested that everything depends on what we assume about the η's. But these are exogenous disturbances: no economic reasoning in the model establishes their properties. Empirically studying (cross-economy) convergence is thus delicate and subtle, even in models as explicit and simple as that here. Part of the difficulty is that in the simplest, most common models used for analyzing economic growth, all cross-economy interaction occurs only through unmodelled exogenous disturbances. In that case, interpretable economic assumptions on technology and externalities place almost no restrictions on how rich and poor economies behave relative to one another. But it is the latter observations that drive theoretical research (e.g., as argued persuasively by Romer, 1994). It is the behavior of the rich and poor relative to each other that are the high-stakes motivation in understanding economic growth. If one were interested only in representations like equation (6), then studying univariate time series representations (e.g., Carlino and Mills, 1993) might already be best. However, under conditions where cross-economy regressions might be sensible, estimates of and theoretical restrictions on α or β say almost nothing about the behavior of rich and poor economies relative to one another.[8]

We take as two-fold the concrete implications thus far. First, consistently estimating α or β is not straightforward. Second, even if α and β were known, any implications for the poor catching up with the rich are unavailable except under overly restrictive auxiliary, non-economic assumptions. Moreover, in the standard model used here, those same assumptions would make impossible consistent estimation of the cross-section convergence regression. Therefore, to address empirical convergence issues, we prefer bypassing altogether these α and β parameters and the cross-section convergence regression. Instead, we would use the models of explicit distribution dynamics developed in Quah (1993a–b, 1995, 1996a–b, d). The empirical results there provide a richness of characterization in terms of polarization and stratification, that is unavailable in the standard regression framework.

[8] Canova and Marcet (1995) and den Haan (1995) have investigated alternative specifications for the stochastic disturbances than we have done here. They too have emphasized the difficulty in interpreting the empirical findings from standard regression analyses. And, for understanding the theoretical significance of convergence, they too have emphasized the overly important role played by arbitrary, exogenous specification of the stochastic structure.

4. Convergence properties

Return now to the model that Kelly (1992) used to argue for convergence with stochastic, endogenous growth. The convergence problem studied here is univariate: what happens in the economy under consideration?[9]

Iterate equation (4), the general version of equation (6), to obtain:

$$y_j(t) = \left[\prod_{s=1}^{t-t_0} \alpha(t_0+s)\right] y_j(t_0) + \sum_{s=0}^{t-1-t_0} \left[\prod_{r=0}^{s-1} \alpha(t-r)\right] \eta(t-s); \tag{10}$$

this is just the analogue of equation (7).

Equation (10) makes transparent the convergence result in Kelly (1992). Since η is common, cross-economy differences manifest only through the first summand on the right hand side of (10). But notice that if $\alpha(t)$ is strictly positive, with $\mathrm{Var}(\alpha(t)) > 0$, and $E\alpha(t) = 1$, then the product limit in this first summand behaves as

$$\prod_{s=1}^{t} \alpha(t_0+s) \overset{a.s.}{\to} 0 \qquad \text{as } t \to \infty, \tag{11}$$

so that all economies *do* converge towards each other, even with "stochastic constant returns to scale" $E\alpha(t) = 1$ or "stochastic increasing returns" $E\alpha(t) > 1$.

Why is (11) correct? It follows, of course, from the argument in Proposition 1 of Kelly (1992)—except we will see that (11) remains true even when α is not serially independent. (The statements in Kelly (1992) are only for iid α, so we obtain here a trivial improvement on those results.) We repeat the argument so

[9] As argued above, this discussion can be extended to convergence between pairs or within groups of countries only with implicit assumptions that neither are economically motivated nor play a crucial role in the economics of the problem. Nevertheless, we feel that the univariate convergence issue remains an interesting scientific question, and so we complete the study here.

that we can add some comments after it. Fix t_0 and define

$$\zeta(t) = \log\left(\prod_{s=1}^{t} \alpha(t_0 + s)\right) = \sum_{s=1}^{t} \log \alpha(t_0 + s),$$

or, equivalently,

$$\zeta(0) = 0$$
$$\zeta(t) = \zeta(t-1) + \log \alpha(t_0 + t), \qquad t \geq 1.$$

When $\mathrm{Var}(\alpha(t)) > 0$, Jensen's inequality together with $E\alpha(t) = 1$ gives

$$E \log \alpha(t_0 + t) < 0,$$

so that ζ is an integrated process with negative drift. Thus, ζ diverges to $-\infty$ (a.s.), from which (11) immediately follows. This holds even when α is serially dependent, provided only that α is stationary and ergodic. (Such arguments are routine in econometric proofs of test consistency in unit root models.)

Moreover, this convergence result works even when $E\alpha(t) \neq 1$: when α is positive (a.s.) all that is required is $E \log \alpha(t) < 0$, and that can happen even when $E\alpha(t) > 1$, and certainly when $E\alpha(t) < 1$.

There is a little more intuition to develop here: Why, when $\alpha(t) > 0$ (a.s.), with $E\alpha(t) = 1$, and $\mathrm{Var}(\alpha(t)) > 0$ does $\prod_{s=1}^{t} \alpha(t_0 + s) \stackrel{a.s.}{\rightarrow} 0$ whereas the limit is 1 when $\mathrm{Var}(\alpha(t))$ vanishes? Take a simple two-point distribution on $\{1 - \nu, 1 + \nu\}$ with equal probability in α's stationary measure on each point. Positivity in α (a.s.) gives $|\nu| < 1$; positive variance gives $\nu \neq 0$. In the stationary distribution, there are approximately equal numbers of occurrences in either value, $1 - \nu$ or $1 + \nu$. Pair them up, and notice that $(1 - \nu) \times (1 + \nu)$ equals $1 - \nu^2$ which is strictly between 0 and 1. Now multiply many such pairs, and the product goes to 0. When $\mathrm{Var}(\alpha(t)) = 0$ in this example, we have $\nu = 0$ as well, and the product pairs $(1 - \nu) \times (1 + \nu)$ then always just equal 1, and the limiting product remains there.

5. Conclusion

This note has done two things. First, in Section 3, we have exploited the structure of stochastic growth models (Kelly, 1992; Long and Plosser, 1983) to clarify why usual characterizations of convergence can be misleading: key parameters are typically not identified, but even when they are known, implications for convergence are unavailable except under restrictive and economically unmotivated assumptions. Moreover, in the standard model we have used for the discussion in Section 3, the same assumptions that relate cross-section convergence meaningfully to the key parameters render those parameters impossible to estimate consistently. Second, in Section 4, we have provided an alternative and easy way to understand the theoretical convergence findings of Kelly (1992).

The results in Kelly (1992) have provoked some of our thinking on the meaning of empirical findings on convergence. What becomes clear is that to understand economic convergence or divergence what we need are results on the dynamics of the entire cross section of economies, not theorems on the dynamics of a "representative" economy. Interestingly, it is instead the last on which most of the empirical convergence literature has focused.[10] Then, extending that intuition to the behavior of the entire cross section typically takes place only under assumptions that have no economic ideas attached (e.g., either the commonality or the independence of η's in Section 3).

There is a perverse circularity in trying to obtain convergence results for endogenous growth models. The initial motivation for those growth models, as articulated by Lucas (1988) and Romer (1986) was precisely to understand the large and persistent gap between poor and rich. If that gap weren't as large and

[10] Quah (1996c) is a counter-example: there, the author studies the economic reasons underlying bloc-formation and polarization across the cross section of countries, and then relates the predictions of the model to empirical findings. Ben-David (1994) is a comparable exercise, although using different motivations. Galor (1996) and Quah (1996d) link a range of theoretical ideas to these empirical issues.

as persistent, no endogenous growth models would have been needed in the first place. If one interprets findings that convergence has occurred as saying that Lucas's and Romer's *measurements* were mistaken right from the start, why do we still want to construct and understand endogenous growth models? What exactly is at stake then?

Our answer to this is that the usual cross-section convergence findings are misleading, for reasons given above. Empirical analyses better designed for investigating the dynamics between rich and poor (e.g., Quah 1993a–b, 1995, 1996a–b, d) show Lucas's and Romer's stylized facts to be the important and economically significant characterizations. An endogenous growth model to "explain" convergence is a fine technical exercise, but seems to have its priorities all wrong.

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