ANALYZING THE CASE FOR GOVERNMENT INTERVENTION IN A REPRESENTATIVE DOMOCRACY*

Timothy Besley
London School of Economics and Political Science

and

Stephen Coate
University of Pennsylvania

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The Suntory Centre
Suntory and Toyota International Centres for
Economics and Related Disciplines
London School of Economics and Political Science
Houghton Street
London WC2A 2AE
Tel.: 020-7955 6698

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Abstract

The welfare economic method for analyzing the case for government intervention is often critized for ignoring the political determination of policies. The standard method of accounting for this critique studies the case for intervention under the constraint that the level of the instrument in question will be politically determined. We critize this method for its implicit assumption that new interventions will not affect the level of existing policy instruments. We argue that this assumption is particularly misleading in suggesting that political economy concerns must dampen the case for intervention.

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Contact address: Professor Timothy Besley, London School of Economics and Political Science, Houghton Street, London WC2A 2AE. Email: t.besley@le.ac.uk

1. Introduction

In what ways should the government intervene in the economy? Welfare economics has developed a powerful method for analyzing this question which has generated a set of standard prescriptions for government intervention. These include the provision of public goods and the regulation of externalities and natural monopolies.[1] Not only are these prescriptions influential in class-rooms, they have underpinned the views of generations of policy economists. Its influence notwithstanding, the welfare economic approach has its critics. Perhaps the most important are Buchanan and his followers in the public choice tradition. They argue that the approach is flawed because it ignores policy determination via a political process (see, for example, Buchanan (1962)). Thus, any political ramifications of government intervention are not taken into account. We call this the *public choice critique* of welfare economics.

Many economists now accept the basic thrust of the critique, and it is commonplace to acknowledge that political determination of policies may enter a caveat for the welfare economic model.[2] To understand the force of the critique in any specific context requires developing a theory of policy determination via the political process. When analyzing the case for a specific intervention, the typical approach (reviewed below) is to specify a model of the determination of

[1]These are traditional prescriptions. Considerations of imperfect information provide a siginificant addition to the possibilities (see, for example, Greenwald and Stiglitz (1986)).

[2]For example, Stiglitz (1994) in discussing the significance of his work with Greenwald, notes that "the Greenwald-Stiglitz theorems should not primarily be taken as a basis of a prescription for government intervention. One of the reasons that they do not provide a basis for prescription is that doing so would require a more detailed and formal model of the government." (p.32)

the new policy instrument in question while (implicitly) holding existing policies fixed. The intervention is then recommended if social welfare with the new policy at its equilibrium level exceeds social welfare without the policy. This approach typically chastens a welfare economic perspective which assumes that the analyst can select the level of the new policy. Political determination of the new policy is a constraint on policy choice that makes intervention less attractive, leading to more conservative policy advice.

Here, we argue that this approach does not do justice to the public choice critique. In a representative democracy, there can be no presumption that the introduction of a new policy instrument will leave existing policy instruments unchanged. Representatives are chosen to decide on multiple policy issues and granting them the power to choose a new instrument may dramatically alter the nature of political competition. Once the possibility of changes in other instruments is recognized, accounting for the public choice critique requires a significantly more involved analysis. Moreover, there is no reason to believe that taking it into account reduces the case for intervention relative to the welfare economic approach.

We develop our argument using a two-dimensional version of the "citizen-candidate" model of political competition introduced by Osborne and Slivinski (1996) and Besley and Coate (1997a).[3] We suppose that the government has access to a negative income tax for income redistribution purposes. We then consider the case for allowing the government to provide a public good in the face

[3]Osborne and Slivinski (1996) developed a one-dimensional version of this model under the assumption that citizens vote sincerely. Besley and Coate (1997a) develop the general case under the assumption of strategic voting.

of a free-rider problem. The model allows us to address the political determination of the new instrument (the public good) and also how the use of the income tax can change with state provision of the public good. We demonstrate the pitfalls of assuming that the income tax rate, and hence redistributive policy, is unchanged after state provision of the public good is permitted.

The remainder of the paper is organized as follows. The next section provides the relevant background material. It reviews the welfare economic approach to assessing the case for government intervention, the public choice critique and the way in which public economists have tried to take it into account. Section three outlines our economic and political model and section four characterizes political equilibrium with and without state provision of the public good. Section five compares the case for intervention under the welfare economic approach and the standard method of accounting for the public choice critique. It then critically evaluates the latter in light of the findings in section four. Section six contains some additional discussion and section seven concludes.

2. Background

2.1. The Welfare Economic Approach

We are concerned with whether the government should intervene in some particular manner. For example, whether it should provide a good, such as health insurance, to its citizens or whether it should tax or subsidize some activity. At the heart of the welfare economic approach is the idea of society's utility possibility frontier or *frontier* for short. This gives the set of utility allocations that can be attained given a particular array of policy instruments. Underlying the frontier is a model of the private economy describing how citizens respond to different policy choices. To evaluate the role of government, the frontier without the instrument being available to the government (the status quo) is contrasted with what can be attained when it is used. There are three possible cases to consider. They are illustrated in Figure 1 for a two person economy. The solid line denotes the status quo frontier, while the dashed line represents the new frontier.

In Case I, which is illustrated in panel (i), the frontier is unchanged by giving government access to the new instrument. Any utility allocation that can be achieved with the instrument can be achieved without it. Case II is the opposite extreme where the frontier with the new instrument lies everywhere outside the status quo frontier. Any utility allocation that can be achieved without the instrument can be Pareto dominated with it. This is illustrated in panel (ii). In Case III the new frontier coincides with the status quo frontier over some part of its range, but is not equal to it. Thus, either the new frontier is longer than the status quo frontier as in panel (iii), or it partially lies outside it as in panel (iv). Elongation occurs when it is not possible to Pareto dominate utility allocations on the status quo frontier with the new instrument, but it is possible to give some citizens a strictly higher utility level than they could have achieved with the initial set of instruments. A partial shift occurs when there are some utility allocations that can be Pareto dominated by giving the government access to the instrument, but not all can be. These three cases exhaust the possibilities; the new frontier can never be inside the status quo frontier since the government can always choose not to use the new instrument.[4]

To proceed further requires a social objective. Most common is to postulate the existence of a social welfare function which ranks alternative utility allocations and is increasing in each citizen's utility level. Intervention of a particular form is then recommended if and only if it can increase social welfare.[5] Intervention is not desirable in Case I where the frontier is unmoved. In Case II, where the frontier is shifted out, intervention is desirable. Whether intervention is recommended when the frontier is elongated or partially shifted out by the instrument (Case III) depends upon the precise form of the social welfare function. The key issue is whether the socially optimal utility allocation lies on the elongated part of the frontier or the part that is shifted out. Figure 2 illustrates two possibilities, one in which intervention is recommended (panel (i)) and one in which it is not (panel (ii)).

The discussion has not yet been specific about the policy instruments available in the status quo. The traditional (*first best*) view assumes that lump-sum taxes and transfers are feasible. Under the conditions of the Second Fundamental Welfare Theorem, the status quo frontier respects only technological feasibility constraints and there is no case for additional government intervention. Relaxing

[4]The approach assumes that the government commits to a vector of policies ex ante. In a dynamic model where the government makes policy choices in each period without commitment, the introduction of a new instrument (such as a capital tax) might reduce the set of utility allocations that can be achieved.

[5]An alternative approach is to recommend intervention if and only if it permits a Pareto improvement. This approach recommends against intervention in Case I and for it in Case II. In Case III, intervention is recommended when the new frontier lies to the right of the status quo utility allocation.

the conditions of this Theorem generates the classic market failure arguments for government intervention. In this first best world, Case III does not play much of a role — lump sum taxes and transfers ensure that losers from wealth enhancing interventions can be compensated to create Pareto improvements.

Modern *second best* theory dispenses with the assumption of lump-sum taxation, usually on the grounds that government does not have sufficient information to implement such tax schemes. Diamond and Mirrlees (1971) developed the now standard case where the government uses linear income and commodity taxes to finance its activities and make compensations. Mirrlees (1971) introduced the additional possibility of non-linear income taxes. Case III now plays a prominent role in the analysis since correcting traditional market failures is unlikely to generate Pareto improvements if redistributive instruments are limited. Second best analysis also supports interventions that would not be supported in a first best world. For example, public provision of private goods may permit greater redistribution and/or enhance the efficiency of redistributive efforts when lump sum taxes and transfers are not available.

2.2. The Public Choice Critique

The canonical public choice critique is illustrated in Public Economics textbooks (see, for example, Stiglitz (1986) Figure 5.1) using a diagram such as Figure 3. Suppose that the frontier shifts out when the government has the power to undertake a particular intervention. If the political process pins down the levels of the policy instruments, then it will determine which allocation from the new and old frontier will be chosen. Suppose then that we begin at a point like S

in the picture and that the political process will guide the economy to point P after the instrument is introduced. This new point is not Pareto dominant, nor is it dominant relative to the social welfare contour illustrated in the diagram. Thus studying the political process can undermine the welfare economic case for intervention.[6] Whether we desire intervention in practice therefore depends on understanding the political process.[7]

[6]It is sometimes argued in the public choice literature that the key problem with the welfare economic approach is that it fails to appreciate the importance of "political failure" in the case of government intervention. If, as suggested in Besley and Coate (1997b), this is defined as a situation where the political process produces policy choices that can be Pareto dominated by feasible alternatives, this is not correct. The caveats to the welfare economic approach identified in Figure 3 reflect the redistributive consequences of intervention rather than the possibility of inefficient policy choices. Both the status quo and the post-intervention utility allocations lie on the relevant Pareto frontiers. While political failure as defined in Besley and Coate (1997b) would add an extra dimension to the analysis, identifying whether the post-intervention policy choice is inside the frontier is neither necessary nor sufficient for deciding whether a government intervention is desirable. The key issue is how the post-intervention utility allocation compares with the status quo, not whether it lies on the new utility possibility frontier.

[7]One reaction to the public choice critique is that if the *levels* of policy instruments are being politically determined, then decisions over which instruments ought to be used will be determined endogenously too. Policy analysis is then effectively emasculated in favor of a *que sera sera* view of policy intervention with a role for public economists only as predictors of which interventions are undertaken. On this view, normative analyses are only interesting to the extent that their *prescriptions* are *predictions* about what interventions occur. Buchanan preserves a role for normative analysis by distinguishing between a constitutional stage and a policy-making stage (see, for example, Buchanan (1987)). At the former stage, citizens agree on both a *fiscal* and a *procedural constitution*. The former delineates the appropriate roles of government and the latter specifies the rules of the political process. At the policy-making stage, citizens work

The canonical presentation does not appear to note that similar logic can lead the welfare economic approach to recommend *against* a worthwhile intervention. The fact that the new instrument does not permit an increase in maximal social welfare does not rule out the possibility that social welfare will be higher at the new political equilibrium. Figure 4 illustrates this point. Again, point S represents the status quo utility allocation, while point P denotes that arising when the new instrument is added. Despite the fact that intervention does not permit an increase in maximal social welfare, social welfare is higher after the intervention.

2.3. Accounting for the Critique: The Standard Method

An analysis of government intervention that takes account of the public choice critique requires a theory of policy determination to predict the equilibrium policy levels. In terms of Figure 3, the analyst must determine the points S and P. The analyst can then compare welfare with and without intervention according to the chosen social criterion.

In the existing literature, the standard way of accomplishing this is to assume that the level of the new policy instrument will be determined by the median voter. The choice of other policies is not modelled, with the implicit assumption

within the constitution to determine policy. Normative analysis of government's role is then useful for designing the fiscal constitution. While the distinction between the constitutional and policy-making stages is somewhat artificial, it is a useful vehicle for maintaining a role for normative inquiry about the economic role of government when the political process is taken seriously. From a policy analysis perspective, such analysis seems appropriate when the analyst can influence only broad directions of policy, with exact implementation left up to the political process.

being that either the government has no policy instruments in the status quo or that these are held fixed. Welfare at the median voter's optimum is then compared with welfare without the intervention.[8] We refer to this as the *standard method* of accounting for the public choice critique. It is the approach taken, for example, in Buchanan and Vanberg (1988) who consider the implications of allowing the government to intervene to regulate an externality, and Faulhauber (1996) who considers the implications of price regulation of a monopolist producing two services. In both papers, taking account of political determination of the new instrument serves to substantially weaken the case for intervention when aggregate surplus is the welfare criterion.

This standard method of accounting for the public choice critique really only represents a modest caveat to the welfare economic approach. The analyst need worry only about the political determination of the new instrument. Moreover, it would be straightforward to incorporate additional political institutions, such

[8] A similar approach is often taken in studies analyzing how political determination of policies can affect the optimal *form* of intervention. Two different methods of intervention are compared, with the level of each being determined by the political process. All other policy instruments are implicitly held fixed. Rodrik (1986) is a study in this vein. He shows that when the social objective is aggregate surplus the standard ranking of tariffs and production subsidies in an international trade context can be reversed when the levels of these instruments are determined by rent-seeking. Also related is Finkelshtain and Kislev (1997) who compare price and quantity regulations under the assumption that the levels of these regulations will be determined by lobbying. Krusell, Quadrini and Rios-Rull (1996) compare consumption and income taxation in a dynamic general equilibrium model with majority voting. They show that the standard welfare ranking of consumption and income taxes can be overturned when the level of taxation is endogenous.

as interest groups, to get a more complete analysis. Hence, a profitable marriage between welfare economics and political economy would seem possible with a richer understanding of the case for government intervention. Unfortunately, while focusing on the political determination of the instrument in question is important, it is not the end of the story.

3. The Model

3.1. The Economic Environment

Consider a polity consisting of N citizens indexed by $i = 1, ..., N$. There are three goods: a private good x, labor ℓ and a public good g. Each citizen i is endowed with L units of labor time and can transform this into units of the private good at rate a_i (which we refer to as his *ability*). It costs c units of the private good to produce one unit of the public good.

Each citizen i has preferences over his own consumption of the private good x_i, labor supply ℓ_i, and the public good. These preferences can be represented by the additively separable utility function $x_i - \phi(\ell_i) + b(g, \theta_i)$. The function $\phi(\cdot)$ represents the disutility of labor and the function $b(\cdot)$ represents the "willingness to pay" for the public good. We assume that $\phi(\cdot)$ is a smooth, increasing and strictly convex function such that $\phi'(0) = 0$ and $\phi'(L) > a^R$. These latter two conditions guarantee interior solutions to the labor supply problem. The function $b(\cdot)$ is assumed to be smooth, increasing and strictly concave in g, and to satisfy $b(0, \theta) = 0$. The parameter θ_i represents individual i's preference for the public good, with higher values of θ_i implying a larger marginal benefit from the public good. Thus, $b(\cdot)$ satisfies the cross partial condition $b_{12} > 0$.

Each citizen in the polity is characterized by a *class* (his ability) and a *type* (his public good preference). There are just two ability groups, rich and poor, so that for all i, $a_i \in \left\{a^P, a^R\right\}$, with $a^P < a^R$. Let N^k be the size of class k. We assume throughout that the poor are in the majority, i.e. $N^P > N^R$.

There are H $(< N)$ different types, so that for all i, $\theta_i \in \{\lambda_1, ..., \lambda_H\}$, where $0 < \lambda_1 < ... < \lambda_H$. Let N_h^k be the number of individuals in class $k \in \{P, R\}$ who are of type h. In addition, let $N_h = N_h^P + N_h^R$ $(h = 1, ..., H)$ be the number of individuals of type h. We assume the existence of a *median type* μ such that $\sum_{h=1}^{\mu-1} N_h < N/2 \leq \sum_{h=\mu}^{H} N_h$. We also make the important assumption that $b_1(0, \lambda_H) < c$, which implies that the marginal valuation of the public good is smaller than the unit provision cost for even the citizen who values the public good most highly.

We consider two policy regimes denoted by $\rho \in \{M, S\}$. In regime M (*market provision*), the policy maker simply chooses a negative income tax scheme comprising a tax rate t and an income guarantee T. In regime S (*state provision*), the policy maker also chooses a level of the public good G. For notational simplicity, we will think of the policy maker as choosing (t, T, G) in either regime, but in the market regime subject to the "constitutional" constraint that $G = 0$. In either regime, we suppose that the constitution requires that $t \in [0, 1]$.

The policy triple (t, T, G) is chosen at the beginning of the period anticipating market behavior. It must be *feasible* in the sense that (i) it satisfies the constitutional restrictions, (ii) taxes are not so high that some citizens find it impossible to pay them, and (iii) it produces a balanced budget. A necessary and sufficient condition for (ii) to be satisfied is that a poor citizen could pay his taxes when working at full capacity; i.e., that $T + (1-t)\, a^P L \geq 0$.

Given (t, T, G), each citizen i chooses how much income to earn y_i, and how much of the public good to purchase g_i. Citizens behave non-cooperatively and move simultaneously in choosing (y_i, g_i). Thus, a set of decisions $\{(y_i^*, g_i^*)\}_{i=1}^N$ form a *market equilibrium* given (t, T, G) if for all i, (y_i^*, g_i^*) solves the problem

$$\max_{(y,g)} T + (1-t)\, y - cg - \phi\,(y/a_i) + b\left(G + \textstyle\sum_{j \neq i} g_j^* + g, \theta_i\right)$$
$$\textit{subject to}$$
$$g \geq 0, T + (1-t)\, y - cg \geq 0 \;\&\; y \in [0, L/a_i]\,.$$

Under our assumptions we have:

Lemma 1. *For any (t, T, G) with $t \in [0,1]$ and $T + (1-t)\, a^P L \geq 0$, there exists a unique private sector equilibrium with $g_i^* = 0$ and $y_i^* = y\,(t, T, a_i)$ for all i, where*

$$y\,(t, T, a_i) = \arg\max_y \{T + (1-t)\, y - \phi\,(y/a_i) : T + (1-t)\, y \geq 0\}\,.$$

There is thus a severe free-rider problem in the provision of the public good resulting in no private provision in the absence of government action. This stems from our earlier assumption that the marginal valuation of the public good is smaller than the unit provision cost for every citizen.

It follows from Lemma 1 that the policy triple (t, T, G) generates revenue $t \cdot \sum_{i=1}^N y\,(t, T, a_i) - NT$. Thus, budget balance requires that

$$NT + cG = t \cdot \sum_{i=1}^{N} y\,(t, T, a_i)\,.$$

We let Z_ρ denote the set of feasible policy triples in regime ρ; i.e., those satisfying this budget constraint as well as the other two feasibility conditions. We also let

$$V^i\,(t, T, G) = T + (1-t)\, y\,(t, T, a_i) - \phi\,(y\,(t, T, a_i)\,/a_i) + b\,(G, \theta_i)$$

denote citizen i's equilibrium indirect utility when the policy triple is (t, T, G).

3.2. The Political Process

The model of policy making is based on Besley and Coate (1997a). At the beginning of the period, a member of the community is selected to make policy choices, with an election determining the choice of citizen to do this. All citizens are able to run in these elections and each must choose whether or not to declare themselves as a candidate. Running for office costs an amount δ. All individuals in the society then vote over the set of self-declared candidates. The candidate with the most votes wins (there is plurality rule). In the event of ties, the winning candidate is chosen randomly with each tying candidate having an equal chance of being selected. If only one individual runs for office then he is automatically selected. If no citizen runs, taxes and expenditures are zero and laissez-faire prevails.

The political process is modelled as a three stage game. Stage one sees each citizen deciding whether or not to become a candidate. In stage two, citizens vote over the set of self-declared candidates. At stage three, the winning candidate chooses policy. In voting, citizens anticipate candidates' policy choices and vote accordingly. Potential candidates anticipate voting behavior. We analyze the three stages in reverse order.

3.2.1. Policy selection

If citizen i is elected in policy regime $\rho \in \{M, S\}$, then he selects the policy triple:

$$(t_i^\rho, T_i^\rho, G_i^\rho) = \arg\max\{V^i(t, T, G) : (t, T, G) \in Z_\rho\}.$$

Let v_{ji}^ρ denote citizen j's utility when citizen i is elected in policy regime ρ; that is,

$v^{\rho}_{ji} = V^j(t^{\rho}_i, T^{\rho}_i, G^{\rho}_i)$. Then, associated with any citizen i being the policy maker in either of our policy regimes, there will be a utility imputation $v^{\rho}_i = (v^{\rho}_{1i}, ..., v^{\rho}_{Ni})$.

The convenience of our assumed additively separable preferences is the possibility that public goods and income tax choices are separable. However, this ceases to be true, even with these preferences, in cases where private consumption is zero for some citizens, which might arise if public goods are very valuable. Hence, it is convenient to assume a bound on the marginal value of public goods to prevent any citizen financing a very high level of public goods at the expense of private consumption.

To develop the required condition, first define the function $y^*(t, a)$ implicitly from the equality $(1-t)a = \phi'(y/a)$. This is the first order condition for the unconstrained optimal earnings of a citizen with ability a. Thus, as long as $T + (1-t)y^*\left(t, a^k\right) \geq 0$ for $k \in \{P, R\}$, we have that $y(t, T, a^k) = y^*\left(t, a^k\right)$. Next, define $\overline{y}^*(t)$ to be the mean unconstrained earnings.[9] Finally, let $\widetilde{t}_R = \arg\max\left\{t \cdot y^*\left(t, a^R\right) : t \in [0,1]\right\}$ be the tax rate that maximizes the tax revenue raised from the rich at their unconstrained earnings.[10] The required upper bound on the marginal value of the public good, which will be retained throughout the analysis, is given in:

Assumption 1: For all $t \in \left[0, \widetilde{t}_R\right]$

$$b_1\left(\frac{N\left(t\overline{y}^*(t) + (1-t)\,y^*\left(t, a^P\right)\right)}{c}, \lambda_H\right) < \frac{(1-t)\,c}{N - N^R \cdot t}$$

[9] That is, $\overline{y}^*(t) = [N^P y^*(t, a^P) + N^R y^*(t, a^R)]/N$.

[10] If there is more than one maximizer, we choose the lowest such tax rate. Since $y^*\left(1, a^R\right) = 0$, it is clear that $\widetilde{t}_R < 1$.

Thus, if total spending on public goods exceeds $N\left(t\overline{y}^*(t) + (1-t)\,y^*\left(t, a^P\right)\right)$, we know that every citizen's valuation of an additional unit is less than $\frac{(1-t)c}{N-N^{R}\cdot t}$. This can be used to justify the desired separability in policy choices. The key result is:

Lemma 2. *If Assumption 1 holds, then*

$$t_i^{\rho} =^{\arg\max}_{t\in[0,1]} \left\{t\overline{y}^*(t) + (1-t)\,y^*(t, a_i) - \phi\left(\frac{y^*(t,a_i)}{a_i}\right)\right\} \text{ for } \rho \in \{M, S\},$$

$$G_i^S =^{\arg\max}_{G} \left\{b(G, \theta_i) - \frac{c}{N}G : G \geq 0\right\},$$

$$T_i^M = t_i^M \cdot \overline{y}^*\left(t_i^M\right) \text{ and } T_i^S = t_i^S \cdot \overline{y}^*\left(t_i^S\right) - \frac{c}{N}G_i^S.$$

This simple characterization of each citizen's optimal policy vector completes the analysis of the policy selection stage.

3.2.2. Voting

We now consider citizens' voting behavior in regime ρ given a set of candidates $C \subset \{1, ..., N\}$. Let $\alpha_j \in C \cup \{0\}$ denote citizen j's decision, where $\alpha_j = i$ denotes j voting for candidate i, and $\alpha_j = 0$ denotes abstention. A vector of voting decisions is $\alpha = (\alpha_1, ..., \alpha_N)$. Given C and α, let $P^i(C, \alpha)$ denote the probability that candidate $i \in C$ wins. Given our assumptions, $P^i(C, \alpha)$ equals 1 if i has the most votes or is the only candidate. It equals $1/M$, if there are M tying candidates of which i is one. It is zero otherwise.

Individuals vote to maximize their expected utility given the voting decisions of others. A *voting equilibrium* in regime ρ is a vector of decisions α^* with three properties. For each citizen j, (i) α_j^* is a best response to α_{-j}^*, i.e.,

$$\alpha_j^* \in \arg\max\left\{\sum_{i\in C} P^i\left(C,\ \left(\alpha_j, \alpha_{-j}^*\right)\right) v_{ij}^\rho \mid \alpha_j \in C \cup \{0\}\right\},$$

(ii) α_j^* is not a weakly-dominated voting strategy and (iii) $\alpha_j^* = 0$ if $v_{ij}^\rho = v_{kj}^\rho$ for all $i, k \in C$. Ruling out weakly dominated strategies is standard in the voting literature, implying that citizens do not vote for their least preferred candidate. Voting is then sincere in two candidate elections. Our third condition, which we call *abstinence of indifferent voters,* is a mild requirement which serves to sharpen some of the results.

3.2.3. Entry

The decision to run for office is motivated by the possibility of moving policy in a preferred direction. Let $\sigma_j \in \{0, 1\}$ be each citizen's entry decision, where $\sigma_j = 1$ denotes entry. A profile of entry decisions is denoted by $\sigma = (\sigma_1, ..., \sigma_N)$. The set of candidates under entry profile σ is $C(\sigma) = \{i \mid \sigma_i = 1\}$.

Each citizen's expected payoff from any given profile of entry decisions depends upon anticipated voting behavior. We let $\alpha(C)$ be the voting behavior citizens anticipate when the candidate set is C. Then, if citizen i is a candidate, he is anticipated to win with probability $P^i(C(\sigma), \alpha(C(\sigma)))$. Any citizen j's expected payoff is the probability that each candidate i wins in the race times his payoff from i's preferred policy. A profile of entry decisions σ^* is an *equilibrium of the entry game* under regime ρ given $\alpha(\cdot)$, if for each citizen i

$$\sigma_i^* \in^{\arg\max}_{\sigma_i \in \{0,1\}} \sum_{j\in C\left(\sigma_{-i}^*, \sigma_i\right)} P^j\left(C\left(\sigma_{-i}^*, \sigma_i\right), \alpha\left(C\left(\sigma_{-i}^*, \sigma_i\right)\right)\right) v_{ji}^\rho + P^0\left(C\left(\sigma_{-i}^*, \sigma_i\right)\right) v_{j0}^\rho - \delta\sigma_i,$$

where $P^0(C)$ denotes the probability that the default outcome is selected, which equals one if $C = \emptyset$ and zero otherwise.

3.2.4. Political Equilibrium

Combining the three stages, a *political equilibrium* in regime ρ is a vector of entry decisions σ and a function describing voting behavior $\alpha(\cdot)$ such that (i) σ is an equilibrium of the entry game in regime ρ given $\alpha(\cdot)$ and (ii) for all non-empty candidate sets C, $\alpha(C)$ is a voting equilibrium in regime ρ. Below, we will present conditions which guarantee that a political equilibrium exists in both regimes.

Any political equilibrium $\{\alpha(\cdot), \sigma\}$ in regime ρ defines a probability distribution over the set of feasible policy choices Z_ρ. We denote this probability distribution by $\pi(\alpha(\cdot), \sigma; \rho)$. Each citizen's expected utility in this equilibrium, ignoring entry costs, is then

$$U^i(\alpha(\cdot), \sigma; \rho) = \sum_{(t,T,G)\in\Delta(\pi(\alpha(\cdot),\sigma;\rho))} V^i(t, T, G)\, \pi(\alpha(\cdot), \sigma; \rho)(t, T, G),$$

where $\Delta(\pi)$ denotes the (finite) support of the probability distribution π.

4. Political Equilibrium Under the Two Regimes

In this section, we characterize political equilibrium under market and state provision. This characterization will then be used in the next section to evaluate the standard method of accounting for the public choice critique.

4.1. Political Equilibrium under Market Provision

Political equilibrium under market provision is straightforward to characterize. We begin by considering each citizen's policy choice. For each class $k \in \{P, R\}$, define

$$W^k(t) = t\overline{y}^*(t) + (1-t)\,y^*\left(t, a^k\right) - \phi\left(y^*\left(t, a^k\right)/a^k\right)$$

and

$$t_k = \arg\max\left\{W^k(t) : t \in [0,1]\right\}.$$

By Lemma 2, if citizen i is the policy maker and is of class k, then he will choose $\left(t_i^M, T_i^M\right) = (t_k, t_k \cdot \overline{y}^*(t_k))$. It is clear that $t_R = 0$ and that t_P satisfies the first order condition

$$\overline{y}^*(t) - y^*\left(t, a^P\right) = -t \cdot \frac{d\overline{y}^*(t)}{dt}.$$

It follows that if citizen j is of class k' and the policy maker i is of class k then $v_{ji}^M = W^{k'}(t_k)$.

Turning to voting behavior, observe that our assumption that voters do not use weakly dominated strategies guarantees voting along class lines. Thus, in any election with both rich and poor candidates, all the rich vote for the rich candidates and all the poor vote for the poor candidates. Since there can be no benefit to running against a candidate of the same class, we should expect at most one candidate from each class to enter. However, if a rich candidate challenges a poor candidate, he is bound to lose. This explains

Proposition 1. *$\{\alpha(\cdot), \sigma\}$ is a political equilibrium under market provision for sufficiently small δ if and only if it involves a single poor citizen running uncontested.*

Thus, when redistribution is the only issue, the poor majority get their preferred tax system. This is implemented by a single candidate since, as we have pointed out, there is no gain to opposing such a candidate – it would be pointless for another poor citizen to enter and futile for a rich citizen to do so. Note that an individual's type has no bearing on the political equilibrium as it does not affect their preference for redistribution.

4.2. Political Equilibrium under State Provision

The key difference between state and market provision stems from the fact that we now have a two dimensional policy space. The additional dimension complicates matters considerably. Indeed, we will show that generally there are a number of possible types of political equilibria.

We begin with a few preliminary observations about policy choices and voting behavior. Let $B(G, \lambda_h) = b(G, \lambda_h) - \frac{c}{N} \cdot G$ be the "surplus" enjoyed by a citizen of type h when the good is financed by a uniform head tax. As shown in Lemma 2, if the citizen who wins the election is of type h then he selects the public good level

$$G_h = \arg\max \{B(G, \lambda_h) : G \geq 0\}.$$

Lemma 2 also implies that if citizen i is of class k and type h, then the policy vector he selects as policy maker is $\left(t_i^S, T_i^S, G_i^S\right) = \left(t_k, t_k \overline{y}^* (t_k) - \frac{c}{N} G_h, G_h\right)$. A citizen

j of class k' and type f therefore receives the payoff $v_{ji}^{S} = W^{k'}(t_k) + B(G_h, \lambda_f)$ from having citizen i in power.

To explore voters' preferences, it is useful to define $\Delta_P = W^P(t_P) - W^P(0)$ and $\Delta_R = W^R(0) - W^R(t_P)$ as the gains from having the tax rate set by a citizen of your own class. These gains will depend on the degree of inequality in individuals' abilities and the relative sizes of the two groups. A useful result about these gains is:

Lemma 3. *The redistributive gain to the rich of having a rich citizen in power exceeds the redistributive gain to the poor of having a poor citizen in power, i.e.,* $\Delta_P < \Delta_R$.

This reflects the facts that the poor are a majority and that the tax-transfer system creates deadweight loss.

We consider voter preferences over pairs of candidates. There are three possible configurations to consider: two poor candidates, two rich candidates and one of each class. With two candidates of the same class, whose types are e and f, a citizen of type h prefers the type e candidate if $B(G_e, \lambda_h) > B(G_f, \lambda_h)$. For any pair of types e and f such that $G_e \neq G_f$, let $\lambda(e, f)$ solve $B(G_e, \lambda) = B(G_f, \lambda)$. Then our assumptions about preferences imply that if $G_e < (>)\, G_f$, then all those citizens with λ_h such that $\lambda_h < (>)\, \lambda(e, f)$ prefer the type e candidate.[11]

The case of a rich candidate of type e and a poor candidate of type f is a little more complicated. In this case, a rich citizen of type h prefers the rich candidate if $\Delta_R > B(G_f, \lambda_h) - B(G_e, \lambda_h)$, while a poor person of type h prefers

[11]This follows from studying the properties of the function $z(\lambda) = B(G_e, \lambda) - B(G_f, \lambda)$, remembering that $b_{12} > 0$.

the rich candidate if $\Delta_P < B(G_e, \lambda) - B(G_f, \lambda)$. For any pair of types (e, f) let $(\chi^k(e, f))_{k \in \{P,R\}}$ solve the pair of inequalities

$$\Delta_R = B\left(G_f, \chi^R\right) - B\left(G_e, \chi^R\right),$$

and,

$$\Delta_P = B\left(G_e, \chi^P\right) - B\left(G_f, \chi^P\right),$$

assuming such solutions exist. Now for $k \in \{P, R\}$ define $\lambda^k(e, f)$ as follows

$$\lambda^k(e, f) = \begin{cases} 0 & \text{if } \chi^k(e, f) \text{ does not exist and } G_e > G_f \\ \chi^k(e, f) & \text{if } \chi^k(e, f) \text{ exists} \\ +\infty & \text{if } \chi^k(e, f) \text{ does not exist and } G_e < G_f. \end{cases}$$

Our assumptions imply that if $G_e < (>)\, G_f$, then all rich citizens of types h such that $\lambda_h < (>)\, \lambda^R(e, f)$ and all poor citizens of types h such that $\lambda_h < (>)\, \lambda^P(e, f)$, prefer the rich candidate.

Figure 5 illustrates how citizens split between a rich and a poor candidate in the case in which the rich candidate has weaker public good preferences. Since the vertical axis measures ability and the horizontal axis measures public good preferences, each citizen can be represented as a point in the Figure. Rich citizens are arrayed along the horizontal line emanating from the point $(0, a^R)$, while poor citizens are distributed along the line emanating from $(0, a^P)$. The citizens supporting the rich candidate are the rich with types to the left of $\lambda^R(e, f)$ and the poor with types to the left of $\lambda^P(e, f)$.

Our understanding of political equilibria is completed by studying the entry game. This is the task to which we now turn, dividing the discussion according to the number of candidates who compete in equilibrium.

4.2.1. One-candidate equilibria

We begin by considering the possibility of one candidate equilibria, which was the only possibility under market provision. Necessary conditions for a one candidate equilibrium are readily determined. Observe that if there is a one candidate equilibrium, then the candidate must be poor — a single rich individual would always be vulnerable to entry by a poor person of the same type. He must also deliver the public good level preferred by the median type. Otherwise, he would lose to another poor candidate choosing the median type's optimum. We report this fact as:

Proposition 2. *Suppose that Assumption 1 is satisfied and that there exists both rich and poor citizens of the median type. Then if $\{\alpha(\cdot), \sigma\}$ is a one-candidate political equilibrium under state provision for sufficiently small δ, the single candidate is poor and chooses the public good level preferred by the median type.*

This possibility may seem like a natural one to focus on since it has a median voter flavor — redistributive policy is exactly as in the case of market provision (reflecting the fact that a majority of the population is poor) and the public good level is that desired by the median voter. However, the statement in Proposition 2 provides only necessary conditions for a one candidate equilibrium and says nothing about whether such equilibria exist. Moreover, even if a one candidate equilibrium does exist, there is no guarantee that it is unique. There may be other plausible outcomes that may give very different policy outcomes. We discuss each of these issues in turn.

The potential for non-existence arises from the possibility that a rich candidate with public good preferences to the left or the right of the median might be able to enter and defeat a median poor candidate. More formally, let $\zeta(e, f)$ denote the net support that a type e rich candidate obtains against a type f poor person; i.e.,

$$\zeta(e,f) = \begin{cases} \sum_{h\in I_-(\lambda^R)} N_h^R + \sum_{h\in I_-(\lambda^P)} N_h^P - \left[\sum_{h\in I_+(\lambda^R)} N_h^R + \sum_{h\in I_+(\lambda^P)} N_h^P\right] & \text{if } G_e < G_f \\ \sum_{h\in I_+(\lambda^R)} N_h^R + \sum_{h\in I_+(\lambda^P)} N_h^P - \left[\sum_{h\in I_-(\lambda^R)} N_h^R + \sum_{h\in I_-(\lambda^P)} N_h^P\right] & \text{if } G_e > G_f, \end{cases}$$

where $I_-(\lambda) = \{h : \lambda_h < \lambda\}$ and $I_+(\lambda) = \{h : \lambda_h > \lambda\}$ and $\lambda^R = \lambda^R(e, f)$ and $\lambda^P = \lambda^P(e, f)$. Furthermore, let

$$e^* \in \arg\max\left\{\zeta(e,\mu) : e \in \{1, ..., H\} \textit{ and } N_e^R > 0\right\}$$

be the type of rich candidate who achieves maximal net support against a median poor candidate. Then, it follows from Corollary 1 of Besley and Coate (1997a) that, if Assumption 1 is satisfied and both rich and poor citizens of the median type exist, a one candidate political equilibrium under state provision exists for δ sufficiently small if and only if $\zeta(e^*, \mu) < 0$.

For this condition not to be satisfied, it is necessary that citizens do not always vote along class lines. Specifically, there must exist some types of poor citizens such that $B(G_h, \lambda_h) - B(G_\mu, \lambda_h) > \Delta_P$. Such citizens could in principle be induced to vote for a rich candidate of the right type against a poor candidate delivering the median type's ideal public good level. If such citizens exist, then the non-existence of a one-candidate equilibrium is a real possibility because, as shown by Lemma 3, the rich have a greater incentive to vote along class lines.

To illustrate, consider an example with three types of public good preferences in which type 2 is the median type. A poor citizen of the median type is potentially vulnerable to entry by a rich citizen of either type 1 or 3. We will consider the first case — the analysis of the second is symmetric. All rich voters will support the entrant if

$$B(G_2, \lambda_3) - B(G_1, \lambda_3) < \Delta_R.$$

This says that a rich citizen whose type is furthest from the entrant will still prefer to vote for the candidate of his own class. The only poor supporters that this rich candidate could attract are those of type 1. He will get their support if

$$B(G_1, \lambda_1) - B(G_2, \lambda_1) > \Delta_P.$$

Thus, under these two conditions we have that

$$\zeta(1, 2) = N^R + N_1^P - [N_2^P + N_3^P].$$

It follows that no one-candidate equilibrium exists if $N^R + N_1^P \geq N_2^P + N_3^P$.

When one candidate equilibria do exist, the issue is one of uniqueness. The following Proposition describes two sets of circumstances in which one candidate equilibria both exist and are unique.

Proposition 3. *Suppose that Assumption 1 is satisfied. Then, for sufficiently small δ, one candidate equilibria both exist and are the only type of political equilibria under state provision if (i) all poor citizens are of the same type and all rich citizens are of the same type or (ii) $b_1(0, \lambda_H) < \frac{c}{N}$.*

The reasoning behind these conditions is simple. Under (i), the coalitions on either side of the redistributive issue are not affected by the introduction of the public good. Accordingly, the nature of political competition is unchanged. Under (ii), all citizens agree on the optimal level of the public good (namely, zero) and hence political competition is simply over the level of redistribution. While there can be no formal proof of the necessity of the conditions in this Proposition, there are strong reasons to believe that these sufficient conditions give a rather precise sense of the assumptions needed to ensure that all political equilibria involve one candidate. This will become clear below as we develop the conditions for existence of equilibria with more than one candidate.

4.2.2. Two-candidate equilibria

Two candidate equilibria are an important class of political equilibria under state provision. They typically exist in conjunction with one candidate equilibria and, more importantly, they exist in conditions where one candidate equilibria fail to exist. To study such equilibria, we make the following two assumptions:

Assumption 2: $\sum_{h=1}^{\mu-1} N_h = \sum_{h=\mu+1}^{H} N_h$.

This simply says that the median group exactly splits the population in two.

Assumption 3: For all $h, h' \in \{1, ..., H\}$, $N_h^P + N_{h'}^R < \frac{N}{3} + 1$.

This is basically a requirement of dispersion in public good preference types.

Political equilibria with two candidates are basically of two forms. They can involve two candidates of the same class or one of each class. A characterization of two candidate equilibria for citizens of the same class is given in:

Proposition 4. *Suppose that Assumptions 1 - 3 are satisfied. Then a political equilibrium under state provision exists involving type e and f candidates* ($G_e < G_f$) *of the same class for sufficiently small* δ *if and only if* $\lambda(e, f) = \lambda_\mu$.

The condition that $\lambda(e, f) = \lambda_\mu$ implies that the median group is indifferent between type e and f candidates of the same class. Types 1 through $\mu - 1$ therefore prefer the type e candidate, while types $\mu + 1$ through H prefer the type f candidate. It follows that a race between a type e and f candidate of the same class will result in a tie. This gives each candidate an incentive to run. The anticipated voting behavior supporting this equilibrium is such that all entrants expect to lose. No citizen switches his vote to a preferred entrant for fear of simply causing his original candidate to lose.

The most striking example of an equilibrium of this form is one where two rich candidates are competing. Political competition here is entirely between candidates with differing views about public good levels, with consensus between them on a policy of no redistribution via the income tax. This is true even though a majority of the citizens wish such redistribution to take place! In a sense this example shows how fragile political equilibria can be to the addition of a policy dimension. We can move from a one candidate equilibrium implementing the poor's redistributive policy to a situation where there are only rich candidates. This could arise even with rather modest dispersion in the public goods preferences.[12]

[12]This gives a sense in which the conditions in Proposition 3 for a one-candidate equilibria to be the only type of political equilibria under state provision look close to necessary. One could always introduce a small, but sufficient, dispersion in public good preferences for a two candidate equilibrium of the form described in Proposition 4 to exist.

In the case of equilibria with candidates of different classes, we have

Proposition 5. *Suppose that Assumptions 1 and 3 are satisfied. Then, a political equilibrium exists with a rich citizen of type e running against a poor citizen of type f for sufficiently small δ if and only if $\zeta(e, f) = 0$.*

The same basic flavor of the other two candidate equilibria is preserved with the two candidates attracting an equal number of votes. Now, however, the two candidates differ in their desired level of redistribution.

The two kinds of two-candidate equilibria described in Propositions 4 and 5 can coexist. In such cases, we find the possibility of very different cleavages in political competition for given tastes and technologies. In one case, there is consensus about redistribution where all candidates represent the same class, and in the other there could be a fair degree of consensus about public goods with candidates having very different views about redistribution.[13] Which kind of two candidate equilibrium we might expect to see cannot be argued on purely theoretical grounds. Particular readers may have notions of plausibility that they wish to apply. However, for our purposes, the main point to note is that the addition of an extra dimension to political competition can dramatically change the nature of policy in the other dimension (in this case income redistribution).

4.2.3. Three-candidate equilibria

In principle, three candidate equilibria can arise in one of two main forms. We could find equilibria with candidates of the same class or those with a mixture of

[13]There cannot be convergence in the public goods dimension in this model because the poor are in a majority.

classes. The first of these possibilities can be ruled out. We state this as:

Proposition 6. *Suppose that Assumptions 1 and 2 are satisfied. Then there is no political equilibrium under state provision involving three candidates of the same class.*

The basic reasoning is fairly straightforward. Equilibria in which three candidates of the same class are winning are not possible: since voters are risk averse with respect to public goods, at least one voter would choose to break the tie by switching his vote to his second most preferred candidate. Equilibria in which two of the candidates are tying and the other is losing are impossible because the loser would not affect the outcome by withdrawing. Equilibria with one candidate winning and two losers are ruled out because at least one loser would not change the outcome by withdrawing.

Three candidate equilibria involving candidates of different classes are a different matter. To illustrate, let us return to the three type example that we used above to demonstrate the possible non-existence of a one-candidate equilibrium. Consider the case in which a type 1 rich citizen would defeat a type 2 poor candidate. We will see that a three candidate equilibrium with candidates of different classes is likely to exist for small enough δ. The proposed equilibrium has two rich candidates and one poor candidate. The poor candidate and one of the rich candidates are of type 1, while the other rich candidate is of type 2.

Note first that if the rich candidate of type 1 is expected to win, the two losing candidates have no incentive to exit. If the losing type 2 rich candidate exits, then, since voting is sincere in two candidate races, the type 1 poor candidate will win. This is clearly worse for the type 2 rich candidate. If the poor candidate exits, then the median rich candidate will win. Again, this is worse for the type 1 poor candidate. Obviously, the rich candidate of type 1 has no incentive to exit since he is getting his optimal policy. Furthermore, if the rich candidate of type 1 is expected to continue to win if additional candidates join the race, then no other citizen has an incentive to enter.

Is the presumption that the rich candidate of type 1 wins and will continue to do so in the face of entry is consistent with rational voting behavior? It turns out that there exist voting equilibria to support this whenever a large enough group of voters do not regard the type 1 rich candidate as the worst of the three candidates, a sufficient condition being $N_2^P + N_3^P + 1 < N_1^P + N^R$. But, as we showed above, when a type 1 rich citizen would defeat a type 2 poor candidate it must be the case that $N_1^P + N^R > N_2^P + N_3^P$.

This example can be generalized. Defining e^*, as above, to be the type of rich candidate who maximizes the net support against the median poor candidate (denoted $\zeta(e, \mu)$), we have:

Proposition 7. *Suppose that Assumption 1 is satisfied, that there are rich and poor citizens of the median type, and that $\zeta(e^*, \mu) \geq 2$. Then, assuming there are poor citizens of type e^*, a three candidate political equilibrium exists under state provision in which a rich candidate of type e^* wins for sufficiently small δ.*

Recall that a one candidate equilibrium exists under state provision if and only if $\zeta(e^*, \mu) < 0$. This proposition therefore comes close to establishing that, under Assumption 1, a political equilibrium exists under state provision for sufficiently small δ. If a one candidate equilibrium does not exist, then a three candidate equilibrium of the above type is likely to exist. The most striking feature of the equilibrium described in this proposition is that it produces a single winning candidate who produces neither the level of redistribution nor the public good level preferred by the majority.[14]

The equilibria in Proposition 7 involve two rich candidates running against a poor candidate, with one of the rich candidates winning. It is also possible to derive conditions under which three candidate equilibria involving two poor candidates and a single rich candidate exist, with a poor candidate winning. Conditions can also be found for three candidate equilibria with two or three candidates tying. We will not proceed further with these equilibria here, nor with any equilibria involving four or more candidates. The results developed so far suffice to illustrate the problems with the standard method of accounting for the public choice critique.

4.3. Summary

Under market provision, there is a unique political equilibrium in which the tax rate is that preferred by the poor. Under state provision, the political equilibria

[14]Arguably, the voting behavior associated with this equilibrium is rather unnatural, since some individuals must vote insincerely even though they are not affecting the outcome. Specifically, the rich winner must receive some support from voters for whom he is the second choice. This reflects how permissive our notion of voting equilibrium is, once one gets beyond two candidates. To cut down the size of the set of political equilibria, one can impose further restrictions on voting behavior. One such restriction is that citizens vote sincerely (i.e., for their preferred candidate) unless their vote is affecting the outcome. This refinement will eliminate the three candidate equilibrium described in Proposition 7. However, the two candidate equilibria described in Propositions 4 and 5 remain. Thus, imposing this refinement does not remove the possibility that intervention leads to a change in the level of redistribution.

that we have identified fall into two categories. First, there are one candidate equilibria in which the level of redistribution remains that preferred by the poor and the public good level is that favored by an individual with median tastes. The other kind of political equilibria comprise those with multiple candidates who differ in their views about either redistribution or the level of the public good. While such equilibria can exist alongside one candidate equilibria, they are the only possibility if there is no Condorcet winner in the set of citizens' preferred policy outcomes.

5. The Case for Intervention

This section considers whether the state should intervene to provide the public good under the welfare economic method and under the standard method of accounting for the critique. We demonstrate the conservative implications of the latter relative to the welfare economic method. We then critically evaluate the standard method in light of the results of the previous section. We show that the standard method is misleading, particularly in its conservatism.

5.1. The Welfare Economic Approach

The first step in the welfare analysis is to understand which of the three cases defined in section 2 applies. In fact, each arises under different assumptions about citizens' marginal valuations of the public good. Case I arises when even the type with the highest public good preferences is unwilling to pay $\frac{c}{N}$ for the first unit of public good.

Proposition 8. *(Case I) If Assumption 1 is satisfied and if* $b_1(0, \lambda_H) < \frac{c}{N}$, *then for every* $(t', T', G') \in Z_S$ *there exists* $(t, T, 0) \in Z_M$ *such that* $V^i(t, T, 0) \geq V^i(t', T', G')$ *for each citizen* i.

In this situation, the frontier is unchanged by the possibility of public provision and the welfare economic approach recommends against intervention.

If the type with the lowest public good preferences is willing to pay $\frac{c}{N}$ for the first unit of public good then Case II applies — the frontier with state provision is to the right of that with market provision.

Proposition 9. *(Case II) If* $b_1(0, \lambda_1) > c/N$, *then for all* $(t', T', 0) \in Z_M$, *there exists* $(t, T, G) \in Z_S$ *such that* $V^i(t, T, G) > V^i(t', T', 0)$ *for each citizen* i.

The stated condition guarantees that starting from any initial Pareto efficient tax system, all citizens can be made better off by a policy change that provides one unit of the public good and finances it by lowering the guarantee by $\frac{c}{N}$. The result implies that the welfare economic approach always recommends state intervention to provide the public good when $b_1(0, \lambda_1) > c/N$.

Case III arises in the intermediate case in which the type with the highest public good preferences is willing to pay more than $\frac{c}{N}$, but the type with the lowest preferences is not.

Proposition 10. *(Case III) Suppose that Assumption 1 is satisfied, that* $b_1(0, \lambda_1) < \frac{c}{N} < b_1(0, \lambda_H)$ *and that there exists citizens of type* 1 *and type* H. *Then, (i) there exists* $(t', T', 0) \in Z_M$ *with the property that there exists no* $(t, T, G) \in Z_S$ *which can Pareto dominate it, and (ii) there exists some* $(t', T', G') \in Z_S$ *and some citizen* i *such that* $V^i(t', T', G') > V^i(t, T, 0)$ *for all* $(t, T, 0) \in Z_M$.

Part (i) of the Proposition implies that the frontier with state provision coincides with that under market provision over some part of its range. There are some citizens who value the public good at less than its per capita cost, and the coarse tax-transfer system makes it impossible to compensate them for its introduction. Part (ii) implies that the state provision frontier is not equal to the market provision frontier. There are some citizens who can be given a strictly higher level of utility under state provision than they could obtain under market provision.[15] Again, the coarse tax-transfer system makes it impossible to compensate high valuation citizens for not having the public good.

In the case where $b_1(0,\lambda_1) < \frac{c}{N} < b_1(0,\lambda_H)$ the welfare economic case for intervention depends on the particular social welfare function. For concreteness, we suppose throughout that the social welfare function is Utilitarian — a similar analysis could be carried out more generally. The optimal policy vector under state provision is then given by:

$$(t^*, T^*, G^*) = \arg\max\{\sum_{h=1}^{H} V^i(t;T,G) : (t,T,G) \in Z_S\},$$

and state intervention will be recommended if and only if $G^* > 0$. Since citizens have a constant marginal utility of income and redistributive taxation distorts

[15]Part (ii) of the Proposition shows that state provision elongates the frontier. A partial shift is possible when type 1 citizens are of a different class than type H citizens. Suppose, for example, that there are two types of public good preferences, λ_1 and λ_2, with $b_1(0,\lambda_2) > c/N > b_1(0,\lambda_1)$, and all poor citizens are type 2s and all rich citizens are type 1s. Then, starting from some policy vector $(t,T,0)$ with $t > 0$, it may be possible to make both rich and poor better off by lowering the tax rate and financing the provision of a small amount of the public good with a lower guarantee. The public good may be a more efficient redistributive mechanism than the tax system (particularly if $b_1(0,\lambda_1)$ is only a little smaller than $\frac{c}{N}$).

citizens' labor supply decisions, the optimal income tax rate (t^*) is zero and public goods are financed using a head tax. The socially optimal public goods level then maximizes public goods' surplus: $\sum_{h=1}^{H} N_h B(G, \lambda_h)$, with intervention being desirable if and only if $\sum_{h=1}^{H} N_h b_1(0, \lambda_h) > c$.

5.2. The Standard Method

Applying the standard method of accounting for the public choice critique, the analyst would assume that the level of redistribution in the economy (as measured by the tax rate t) would remain as in the status quo and that the level of the public good would be determined by the median voter. Since a constant tax rate implies that the public good is financed by a uniform head tax, the median voter desires an amount G_μ. Thus, the question would be whether social welfare is higher under the triple $(t_P, t_P \cdot \overline{y}^* (t_P) - \frac{c}{N} G_\mu, G_\mu)$ than under $(t_P, t_P \cdot \overline{y}^* (t_P), 0)$.[16]

To facilitate comparison with the welfare economic approach, it is useful to consider the implications of this rule in the three cases of the previous sub-section. In Case I, in which the Pareto frontier does not move ($b_1(0, \lambda_H) < \frac{c}{N}$), the median voter chooses a zero level of the public good ($G_\mu = 0$). Hence, the standard method agrees with the welfare economic approach that there is no case for state provision.

[16] The papers that use this approach do not explicitly model the other instruments. For example, in the case of public goods, the textbook analysis considers a world in which there are two goods, a private good and a public good. Citizens have exogenous endowments of the private good and different preferences for the public good. The public good is assumed to be financed by a uniform head tax. This is consistent with our model, when citizen i's "endowment" is defined as $t_P \overline{y}^*(t_P) + (1 - t_P) y^*(t_P, a_i)$.

In Case II, where the Pareto frontier shifts out ($b_1(0, \lambda_1) > \frac{c}{N}$), every citizen is better off with state provision if the lowest type is better off with G_μ than with 0 (i.e., $B(G_\mu, \lambda_1) \geq 0$). If this condition is not satisfied, the case for intervention depends on the specific social welfare function. In the Utilitarian case the welfare economic prescription is confirmed if and only if the *sum* of citizens' net willingnesses to pay for the public good is positive at the median type's optimal public good level, i.e., $\sum_{h=1}^{H} N_h B(G_\mu, \lambda_h) > 0$.[17] This is not implied by $b_1(0, \lambda_1) > c/N$, making it possible for the standard method to reject intervention when it would be advocated by a welfare economic approach.

In Case III, when the frontier is elongated ($b_1(0, \lambda_1) < \frac{c}{N} < b_1(0, \lambda_H)$), the case for intervention under the standard method will also depend on the specific social welfare function. Notice, however, that in the Utilitarian case if intervention is desirable under the standard method, then it must also be desirable under the welfare economic method. This is because $\sum_{h=1}^{H} N_h B(G_\mu, \lambda_h) > 0$ implies that $\sum_{h=1}^{H} N_h b_1(0, \lambda_h) > c$.

To summarize, in Case I the standard method agrees with the welfare economic approach that intervention is undesirable. In Case II, it does not necessarily agree that intervention is desirable, while in Case III, with a Utilitarian social welfare function, it also offers a weaker case for intervention. Thus, the standard method of accounting for the public choice critique is more conservative for a Utilitarian social welfare function.[18] This reflects the fact that the welfare economic method

[17]This is consonant with the sum of surplus - median voter analyses of Buchanan and Vanberg (1988) and Faulhaber (1996).

[18]Under certain types of social welfare functions, an exception to the conclusion that the standard method dampens the case for intervention arises. This possibility arises when $t^* < t_P$,

assumes complete freedom in selecting the level of public goods, while the standard method assumes that the median taste for public goods determines the level.

5.3. A Critique of the Standard Method

We now exploit the richer framework of our model to critically evaluate the standard method. Let $\{\alpha^{\rho}(\cdot), \sigma^{\rho}\}$ denote the political equilibrium anticipated in regime ρ. Then the utility allocation without intervention will be $\{U^i(\alpha^M(\cdot), \sigma^M; M)\}_{i=1}^N$, while that with intervention will be $\{U^i(\alpha^S(\cdot), \sigma^S; S)\}_{i=1}^N$. Accounting for the public choice critique requires a comparison of social welfare at the state provision utility allocation with that arising at the market provision utility allocation. The standard method can be misleading if the policy choice in the state provision political equilibrium is not $(t_P, t_P \cdot \overline{y}^*(t_P) - \frac{c}{N}G_\mu, G_\mu)$.

In Case I, in which the Pareto frontier does not move, Proposition 3 implies that all political equilibria are one candidate equilibria. The standard method therefore correctly predicts the state provision equilibrium. Since the median voter chooses a zero level of the public good ($G_\mu = 0$), incorporating political determination of policies does not change the case for intervention.[19]

so that the social welfare optimum involves less redistribution to the poor than the status quo political equilibrium. Then, if it is the case that provision of the public good redistributes to the rich in the sense that $\sum_h N_h^R B_1(0, \lambda_h) > 0 > \sum_h N_h^P B_1(0, \lambda_h)$, it may be the case that social welfare is higher under $(t_P, t_P \cdot \overline{y}^*(t_P) - \frac{c}{N}G_\mu, G_\mu)$ than $(t_P, t_P \cdot \overline{y}^*(t_P), 0)$ even when $G^* = 0$. While it would be preferable to simply cut the tax rate, given that this is not politically feasible, public provision at the median level maybe the second best way of benefiting the rich. This possibility cannot arise under a Utilitarian social welfare function, since the social value of any given citizen's utility increment is independent of that citizen's utility level.

[19] A caveat to this conclusion is necessary in a world in which there exists political equilibria

In Cases II and III, Proposition 3 tells us that the standard method must correctly predict the state provision equilibrium when all the poor are of the same type and all the rich are of the same type. Without this condition, however, the standard method can end up recommending either too much or too little policy intervention. We begin with a case where it fails to be conservative enough and advocates an undesirable intervention.

Suppose that the actual political equilibrium has two poor candidates with divergent public goods' preferences running against each other as in Proposition 4. Such equilibria can easily exist alongside the standard one-candidate equilibrium. The public good possibilities are then either side of the median each arising with probability one half. Denote the two candidate types delivering these policies by (e, f). For a Utilitarian social welfare function, the standard method recommends intervention when it is *not* desirable whenever

$$\sum_{h=1}^{H} N_h B(G_\mu, \lambda_h) > 0 > \sum_{h=1}^{H} N_h \frac{1}{2}[B(G_e, \lambda_h) + B(G_f, \lambda_h)].$$

The concavity of $B(\cdot, \lambda_h)$ makes this quite possible.

The standard model can also be too conservative. Suppose that the political equilibrium with state provision is one where a rich candidate e and a poor candidate f compete. Recall from the discussion in section 5.1 that a Utilitarian objective implies that reductions in redistributive taxation are welfare improving. In this equilibrium, the rich's redistributive preference arises with probability one half with a citizen of class k and type h obtaining a payoff of

that generate different policy choices prior to intervention. Even though intervention would not change the set of equilibrium utility allocations, it could act like a sunspot which moves society to a different equilibrium.

$\frac{1}{2}[W^k(0) + B(G_e, \lambda_h) + W^k(t_P) + B(G_f, \lambda_h)]$. Suppose now that we wrongly supposed that the outcome with state provision of the public good would be a tax rate t_P and a public goods level G_μ, then we would recommend against welfare improving state intervention if

$$\frac{1}{2}[N^R \Delta_R - N^P \Delta_P] + \sum_{h=1}^{H} N_h \frac{1}{2}[B(G_e, \lambda_h) + B(G_f, \lambda_h)] > 0 > \sum_{h=1}^{H} N_h B(G_\mu, \lambda_h). \tag{5.1}$$

The first inequality gives an expression for the gain from state provision in the "true" political equilibrium. It includes the term $N^R \Delta_R - N^P \Delta_P$ which measures the welfare gain from a less distortionary tax system. The standard approach ignores this and focuses exclusively on the public goods' surplus evaluated at the median's preferred level. It thereby fails to identify a possible source of welfare improvements. In this instance this results in a prescription that is too conservative.[20]

This example can also be used to illustrate a stronger claim. As we observed in the last section, with a Utilitarian social welfare function, the standard method is more conservative than the welfare economic approach. But this is not a necessary consequence of accounting for political determination. There are cases where Figure 4 applies and political economy considerations justify interventions the welfare economic method would reject. Suppose that we are in a situation where

[20]It is important to see the nature of the general argument and what is specific to the example. If we had specified the rich to be the majority and a social welfare function that favored redistribution, then one could easily construct examples where state provision was favored on account of it *increasing* redistributive taxation.

a Utilitarian welfare economist would recommend against state provision of the public good, i.e. if

$$\sum_{h=1}^{H} N_h \cdot b_1(0, \lambda_h) \leq c. \tag{5.2}$$

In spite of this, a political economy analysis that anticipates a two candidate equilibrium with a rich and a poor citizen competing can recommend intervention. This is true when the first inequality in (5.1) holds, which is quite compatible with (5.2) also holding.

Key to understanding this result is the fact that fully accounting for the public choice critique allows the level of redistribution to be affected by state provision of the public good. This corresponds to a kind of second-best reasoning. Beginning from a situation in which the income tax rate is not set optimally, introducing state provision can shift the income tax closer to its socially optimal level. This is additional to any conventional direct effects on public good provision. While state provision of a public good is a rather blunt instrument for bringing about an income tax change, it is the only feasible way of doing so given that political equilibrium determines the policy outcome. This reasoning has a Machiavellian ring to it. However, it is a natural consequence of bringing political economy concerns into the model.

6. Discussion

A key theme of our analysis is that introducing an additional policy instrument can lead to changes in the levels of existing instruments with significant redistributive consequences. We have demonstrated this in a model in which the two instruments in question are separable — citizens' willingness to pay for the public good is independent of the income tax rate. In a world in which policies are related through non-separabilities in preferences, the same phenomenon can arise for a different reason.[21] To illustrate this, consider an environment with public provision of a private good studied, for example, by Epple and Romano (1996), Fernandez and Rogerson (1997), and Gouveia (1996). Suppose that there are two homogeneous groups — rich and poor, with a single publicly provided private good, such as health care, financed by a proportional income tax. Suppose further that the constitution bans the private purchase of health care, meaning that citizens cannot "top-up" the publicly provided quantity. Consider the policy question of whether the constitutional ban should be lifted and the government should be granted the discretion to decide whether or not to impose the ban.

Allowing the government the power to relax the ban will lead to a rightward shift in the Pareto frontier. If the ban is relaxed, holding constant the rate of taxation, those who choose to top up in the private sector are better off, while those who choose not to are unaffected.[22] However, a version of Figure 3 can easily be generated. Suppose that the rich are in a majority so that a rich citizen always makes policy choices. In the status-quo, when topping up is banned, the rich desire a positive amount of publicly provided health care. However, the income tax finance of this will mean that the rich will pay a higher share of the cost than

[21]We thank Raquel Fernandez for drawing our attention to this possibility and suggesting the example to follow.

[22]This assumes that the publicly provided private good is produced at constant cost so that there are no pecuniary externalities.

the poor. Thus, the public program will disproportionately benefit the poor. If the government is given the right to lift the ban, rich citizens will want to exercise this right and their demand for state funded education will be diminished, even eliminated. This could well make the poor worse off and the result would be like a move from S to P in Figure 3. Thus, the introduction of the new instrument (the ability to relax the ban) leads to a dramatic change in the level of another policy, namely, publicly provided health care.

It should be clear that, while this result has a similar flavor, its logic is completely different from that developed in this paper. The key assumption is that the level of the new policy (i.e., whether or not the ban is in place) alters the demand for the other policy (publicly provided health care). There is no change in the political equilibrium: the rich control policy throughout. In our analysis, the level of the public good does not alter citizens' preferences over the tax rate. However, introducing the public good, changes the class of citizen who controls policy. Hence, our effects come solely through shifts in political equilibria. These shifts are made possible by the fact that the rich and the poor have heterogeneous preferences for the public good (see Proposition 3).

It is important to understand whether the shifts in political equilibria behind our results are a robust phenomenon or an artifact of our particular model of representative democracy. This is not easy to resolve since models of representative democracy which produce predictions in a two-dimensional policy environments are so thin on the ground. A natural comparison might be with a probabilistic voting model in which two Downsian parties propose platforms in an environment in which they are uncertain about voter characteristics.[23] If they exist, equilibrium policy platforms typically maximize a specific underlying function of citizens' utilities, analogous to a social welfare function. Precisely how equilibrium platforms are changed by the addition of a new instrument would need to be worked out, but it does not seem particularly likely that such changes will have significant redistributive consequences.

In support of our model, however, political scientists have frequently discussed how new policy issues can lead to major realignments of political coalitions. Our results can be interpreted along these lines by considering the set of voters who support a particular candidate as a kind of coalition. The two candidate equilibria where one or more rich individual stands for office are examples where the introduction of a second policy (the public good) splits the coalition of poor voters favoring redistributive taxation. The non-existence of a one candidate equilibrium can similarly be interpreted as a case where the coalition of poor supporting one candidate is unstable when a rich candidate enters.

When viewed as an attempt to begin developing a model that might satisfactorily account for the public choice critique, a sobering feature of our analysis is the existence of multiple equilibria. Given the importance of correctly anticipating the political equilibrium in evaluating the case for intervention, this suggests the difficulty involved in establishing the case for government intervention at a purely theoretical level. Knowledge of history, culture and institutions may be needed to provide an accurate prediction of the political consequences of intro-

[23]See Coughlin (1991) for a review of such models and Ledyard (1984) for the microfoundations.

ducing a new form of intervention. However, the evident difficulties in modeling political equilibrium cannot validate persisting with a welfare economic approach that ignores the political process altogether. It also cannot justify using a method which simply ignores the potential equilibrium effects on other policies.

7. Concluding Remarks

This paper has reconsidered the question of when economists can recommend particular forms of government intervention. We have put forward a framework that encompasses existing insights as well as suggesting some new ones. Our principal analytical contribution is to place the issue in a tractable multi-dimensional setting. We show why it is not legitimate to hold other aspects of policy making fixed when trying to assess the impact of a particular government intervention. Even if these policies are not related through preferences, interactions between them cannot be ignored. Multi-dimensional political competition can change political equilibria to focus around new cleavages as new policy issues come on the agenda. This greatly increases the possible ways in which political concerns can interfere with the welfare economic approach to policy analysis.

The paper questions a basic presumption in the public choice literature. For example, Buchanan suggests that "Public choice theory, in its redress of the imbalance in the institutional comparisons informed by and inspired by welfare economics, has shifted the pendulum "right-ward".... A comparison of market and government alternatives, both examined "warts and all", and without the "benevolent despot" blinders on, will necessarily produce a private-public sector mix less dominated by the public sector than the mix that might have been generated by prevailing ideas in, say, 1950 or 1960." Buchanan (1989) page 26. We have shown that this need not be the case and an analyst who viewed the issues in a political economy context could recommend an intervention to change the political equilibrium in a favorable way. In this sense, our analysis turns the standard public choice critique of welfare economics on its head.

References

[1] Besley, Timothy and Stephen Coate, [1997a], "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 112(1), 85-114.

[2] Besley, Timothy and Stephen Coate, [1997b], "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," *American Economic Review*, forthcoming.

[3] Buchanan, James M., [1962], "Politics, Policy, and the Pigouvian Margins," *Economica*, 17-28.

[4] Buchanan, James M., [1987], "Constitutional Economics," in *New Palgrave Dictionary of Economics*, (London: Macmillan and Co).

[5] Buchanan, James M., [1989], "The Achievements and Limits of Public Choice in Diagnosing Government Failure and in Offering Bases for Constructive Reform," in Chapter 2 in *Explorations into Constitutional Economics*, College Station: Texas A & M University Press.

[6] Buchanan, James M. and Viktor J. Vanberg, [1988], "The Politicization of Market Failure," *Public Choice*, 57(2), 101-13.

[7] Coughlin, Peter J., [1991], *Probabilistic Voting Theory*, Cambridge: Cambridge University Press.

[8] Diamond, Peter and James A. Mirrlees, [1971], "Optimal Taxation and Public Production I: Production Efficiency and II: Tax Rules," *American Economic Review*, 61, 8-27 and 261-78.

[9] Epple, Dennis and Richard Romano, [1996], "Public Provision of Private Goods," *Journal of Political Economy*, 104(1), 57-84.

[10] Faulhauber, Gerald, [1996], "Voting on Prices: The Political Economy of Regulation," typescript, Wharton School, University of Pennsylvania.

[11] Fernandez, Raquel and Richard Rogerson, [1997], "Education Finance Reform and Investment in Human Capital: Lessons from California," New York University, C.V. Starr Center for Applied Economics, RR#97-21.

[12] Finkelshtain, Isreal and Yoav Kislev, [1997], "Prices versus Quantities: The Political Perspective," *Journal of Political Economy*, 105(1), 83-100.

[13] Gouveia, Miguel, [1996], "The Public Sector and Health Care," *International Tax and Public Finance*, 3(3), 329-350.

[14] Greenwald, Bruce and Joseph E. Stiglitz, [1986], "Externalities in Economies with Imperfect Information," *Quarterly Journal of Economics*, 101, 229-264.

[15] Krusell, Per, Vincenzo Quadrini and Jose-Victor Rios-Rull, [1996], "Are Consumption Taxes Really Better Than Income Taxes?" *Journal of Monetary Economics*, 37(3), 475-503.

[16] Ledyard, John O., [1984], "The Pure Theory of Large Two-Candidate Elections," *Public Choice*, 44, 7-41.

[17] Mill, John Stuart, [1848], *Principles of Political Economy*, Book V, Chapter 1, Harmondsworth: Penguin Books.

[18] Mirrlees, James A., [1971], "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 175-208.

[19] Osborne, Martin J. and Al Slivinski, [1996], "A Model of Political Competition with Citizen Candidates," *Quarterly Journal of Economics*, 111(1), 65-96.

[20] Rodrik, Dani, [1986], "Tariffs, Subsidies and Welfare with Endogenous Policy," *Journal of International Economics*, 21(3/4), 285-99.

[21] Stiglitz, Joseph E., [1986], *Economics of the Public Sector*, New York, NY: W.W. Norton and Company.

[22] Stiglitz, Joseph E., [1994], *Whither Socialism*, Cambridge, MA: MIT Press.

8. Appendix: Proofs of Results

Proof of Lemma 1: Our assumption that $b_1(0, \lambda_H) < c$, means that for each citizen i, $b_1\left(G + \sum_{j\neq i} g_j^*, \theta_i\right) < c$, which implies that $g_i^* = 0$. It follows that y_i^* solves the problem

$$\max T + (1-t)\, y - \phi\,(y/a_i)$$
$$\textit{subject to}$$
$$T + (1-t)\, y \geq 0 \;\&\; y \in [0, L/a_i]\,.$$

Our assumptions about $\phi(\cdot)$ imply that the constraint $y \in [0, L/a_i]$ is non-binding, which yields the result. ■

Proof of Lemma 2: It is obvious that $\left(t_i^M, T_i^M\right)$ has the claimed form. Hence, we concentrate on demonstrating the result for $\left(t_i^S, T_i^S, G_i^S\right)$. By definition $\left(t_i^S, T_i^S, G_i^S\right)$ solves

$$\max_{(t,T,G)} V^i\,(t, T, G)$$
$$\textit{subject to}$$
$$T + (1-t)a^P L \geq 0,$$
$$NT + cG = t[N^P \cdot y(t, T, a^P) + N^R \cdot y(t, T, a^R)],$$
$$G \geq 0 \textit{ and } t \in [0, 1].$$

We refer to this as Problem 1. Now define Problem 2 as follows

$$\max_{(t,T,G)} \{T + (1-t)\, y^*\,(t, a_i) - \phi\,(y^*\,(t, a_i)\,/a_i) + b\,(G, \theta_i)\}$$
$$\textit{subject to}$$
$$NT + cG = Nt \cdot \overline{y}^*(t),$$
$$G \geq 0 \textit{ and } t \in [0, 1].$$

Let (t^*, T^*, G^*) solve Problem 2. Then it is apparent that (t^*, T^*, G^*) satisfies the conditions of the Lemma. Thus, to prove our result, it is enough to show that (t^*, T^*, G^*) solves Problem 1.

We begin by showing that (t^*, T^*, G^*) is feasible for Problem 1. Note first that

$$T^* + (1 - t^*)y^*(t^*, a^P) \geq 0.$$

To see this, suppose not. Then the government budget constraint of Problem 2 implies that

$$G^* > N\left[t^* \cdot \overline{y}^*(t) + (1 - t^*)\, y^*\left(t^*, a^P\right)\right]/c.$$

But since $b_1(G^*, \theta_i) = \frac{c}{N} > \frac{(1-t^*)c}{(N - N^R t^*)}$, this contradicts Assumption 1 if $t^* \leq \tilde{t}_R$. It is clear that this is true if citizen i is rich, since $t^* = 0$. Substituting the budget constraint into the objective function and rearranging, we see that if citizen i is poor, t^* solves

$$\max_t \left\{t\frac{N^R}{N}y^*\left(t, a^R\right) + \left[y^*(t, a^P)\left(1 - \frac{N^R}{N}t\right) - \phi\left(y^*\left(t, a^P\right)/a^P\right)\right]\right\}.$$

Since the term on the left hand side is maximized at $\tilde{t}_R$ and the term in parentheses is always decreasing in t, the optimum cannot exceed $\tilde{t}_R$. We therefore conclude that $T^* + (1 - t^*)y^*(t^*, a^P) \geq 0$.

It follows that the first constraint in Problem 1 is satisfied and that for all citizens j, $y(t^*, T^*, a_j) = y^*(t^*, a_j)$. This together with the government budget constraint in Problem 2 implies that the government budget constraint in Problem 1 is satisfied. This establishes feasibility.

It follows that if (t^*, T^*, G^*) does not solve Problem 1, then $V^i\left(t_i^S, T_i^S, G_i^S\right) > V^i(t^*, T^*, G^*)$. Since (t^*, T^*, G^*) solves Problem 2, this implies either that (i) $y(t_i^S, T_i^S, a^k) \neq y^*\left(t_i^S, a^k\right)$ for both $k \in \{P, R\}$, or (ii) $y\left(t_i^S, T_i^S, a^P\right) \neq y^*\left(t_i^S, a^P\right)$. Thus the optimum must involve either both classes or the poor having zero consumption. Case (i) amounts to the assertion that

$$T_i^S + \left(1 - t_i^S\right) y^* \left(t_i^S, a^R\right) < 0,$$

and case (ii) to the assertion that

$$T_i^S + \left(1 - t_i^S\right) y^* \left(t_i^S, a^R\right) \geq 0 > T_i^S + \left(1 - t_i^S\right) y^* \left(t_i^S, a^P\right).$$

To complete the proof, we must rule out both of these cases. We do this by establishing two claims. These are more general than we need here, but will be used in the proof of Proposition 9 below.

Claim 1: Let $(t', T', G') \in Z_S$ be any policy vector such that

$$T' + (1 - t') y^* \left(t', a^R\right) < 0,$$

Then, under Assumption 1, there exists a policy vector $(t, T, G) \in Z_S$ such that $T + (1 - t) y^* \left(t, a^P\right) \geq 0$ and $V^i(t, T, G) > V^i(T', t', G')$ for all $i = 1, ..., N$.

Proof: Note first that if $T' + (1 - t') y^* \left(t', a^R\right) < 0$, then both rich and poor citizens have zero consumption, earning just enough to pay their taxes. Thus,

$$y(t', T', G') = y(t', T', G') = -\frac{T'}{(1 - t')}$$

and government revenue is $-NT'/(1-t')$. The government budget constraint therefore implies that $T' = -(1-t')cG'/N$. Citizen i's utility level is then $b(G',\theta_i) - \phi\left(\frac{cG'}{Na_i}\right)$.

There are two possibilities: (a) $G' < Ny^*\left(0, a^P\right)/c$ and (b) $G' \geq Ny^*\left(0, a^P\right)/c$. Consider the policy vector $(0, -\frac{c}{N}G', G')$. In case (a), we have that

$$V^i(0, -\frac{c}{N}G', G') = b(G',\theta_i) - \frac{c}{N}G' + y^*(0,a_i) - \phi(y^*(0,a_i)/a_i) \;\; for\ all\ i = 1, ..., N.$$

Since each citizen i could have chosen to earn $\frac{c}{N}G'$ but choose to earn strictly more, his utility must exceed $b(G',\theta_i) - \phi\left(\frac{cG'}{Na_i}\right) = V^i(T', t', G')$. Thus, we have established the Claim.

In case (b), we have that $V^i(0, -\frac{c}{N}G', G') = V^i(T', t', G')$ if i is poor and $V^i(0, -\frac{c}{N}G', G') \geq V^i(T', t', G')$ if i is rich. Now for $\varepsilon \in [0, G']$, consider the package $\left(0, -\frac{c}{N}(G'-\varepsilon), G'-\varepsilon\right)$. This is clearly feasible. Letting $\xi_i(\varepsilon)$ denote citizen i's utility under the policy vector $(0, -\frac{c}{N}(G'-\varepsilon), G'-\varepsilon)$, we have that

$$\xi_i(\varepsilon) = b(G'-\varepsilon, \theta_i) - \phi\left(\frac{c}{Na_i}(G'-\varepsilon)\right) \;\; if\ y^*(0,a_i) < \frac{c}{N}(G'-\varepsilon),$$

and

$$\xi_i(\varepsilon) = -\frac{c}{N}(G'-\varepsilon) + y^*(0,a_i) + b(G'-\varepsilon,\theta_i) - \phi(y^*(0,a_i)/a_i) \;\; if\ y^*(0,a_i) \geq \frac{c}{N}(G'-\varepsilon),$$

In the former case, differentiating with respect to ε yields

$$\xi_i'(\varepsilon) = -b_1(G'-\varepsilon,\theta_i) + \phi'\left(\frac{c}{Na_i}(G'-\varepsilon)\right)\frac{c}{Na_i},$$

while in the latter case,

$$\xi_i'(\varepsilon) = -b_1(G'-\varepsilon,\theta_i) + \frac{c}{N}.$$

Since $\phi' > a_i$ when $y^*(0,a_i) < \frac{c}{Na_i}(G'-\varepsilon)$, this implies that

$$\xi_i'(\varepsilon) \geq \frac{c}{N} - b_1(G'-\varepsilon,\theta_i).$$

Now note that if $\varepsilon \in [0, G' - Ny^*(0,a^P)/c]$, Assumption 1 implies that $b_1(G'-\varepsilon,\theta_i) > \frac{c}{N}$. This means that each citizen's utility is strictly increasing in ε on $[0, G' - Ny^*(0,a^P)/c]$. Thus, for policy vectors $\left(0, -\frac{c}{N}(G'-\varepsilon), G'-\varepsilon\right)$ where ε is slightly larger than $G' - Ny^*(0,a^P)/c$, we know that

$$V^i\left(0, -\frac{c}{N}(G'-\varepsilon), G'-\varepsilon\right) > V^i(0, -\frac{c}{N}G', G') = V^i(T',t',G') \text{ for all } i.$$

This proves Claim 1. ■

Claim 2: Let $(t',T',G') \in Z_S$ be any policy vector such that

$$T' + (1-t')\,y^*\left(t',a^P\right) < 0 \leq T' + (1-t')\,y^*\left(t',a^R\right).$$

Then, under Assumption 1, there exists a policy vector $(t,T,G) \in Z_S$ such that $T + (1-t)\,y^*\left(t,a^P\right) \geq 0$ and $V^i(t,T,G) > V^i(T',t',G')$ for all $i = 1,...,N$.

Proof: When $T' + (1-t')\,y^*\left(t',a^P\right) < 0 \leq T' + (1-t')\,y^*\left(t',a^R\right)$, the poor have zero consumption and hence $y\left(t',T',a^P\right) = -T'/(1-t')$. For the rich, $y\left(t',T',a^R\right) = y^*(t',a^R)$. The government budget constraint implies that $-T' = -T(t',G')$ where

$$-T(t,G) = \frac{(1-t)\left(cG - N^R t y^*\left(t,a^R\right)\right)}{N - N^R t}.$$

Thus, if citizen i is poor,

$$V^i(T',t',G') = b(G',\theta_i) - \phi\left(-T(t',G')/(1-t')\,a^P\right)$$

while if he is rich,

$$V^i(T',t',G') = b(G',\theta_i) + T(t',G') + (1-t')\,y^*\left(t',a^R\right) - \phi\left(\frac{y^*\left(t',a^R\right)}{a^R}\right).$$

Suppose first that $t' \leq \tilde{t}_R$ and consider the policy vector $(t', T(t', G'-\varepsilon), G'-\varepsilon)$, for $\varepsilon \in [0,\bar{\varepsilon}]$, where $\bar{\varepsilon}$ is implicitly defined by

$$(1-t')y^*\left(t',a^P\right) + T(t',G'-\varepsilon) = 0.$$

If citizen i is poor, then his utility under the new policy vector, written as a function of ε is

$$\xi_i(\varepsilon) = b(G'-\varepsilon,\theta_i) - \phi\left(\frac{-T(t',G'-\varepsilon)}{(1-t')\,a^P}\right).$$

If citizen i is rich,

$$\xi_i(\varepsilon) = T(t',G'-\varepsilon) + b(G'-\varepsilon,\theta_i) + (1-t')\,y^*\left(t',a^R\right) - \phi\left(\frac{y^*\left(t',a^R\right)}{a^R}\right).$$

Thus, if citizen i is poor

$$\xi_i'(\varepsilon) = -b_1(G'-\varepsilon,\theta_i) + \phi'\left(\frac{-T(t',G'-\varepsilon)}{(1-t')\,a^P}\right)\frac{c}{a^P(N-N^Rt')},$$

while if he is rich

$$\xi_i'(\varepsilon) = -b_1(G'-\varepsilon,\theta_i) + \frac{c(1-t')}{(N-N^Rt')}$$

Since $\phi'\left(\frac{-T(t',G'-\varepsilon)}{(1-t')a^P}\right) > (1-t')\,a^P$, we have that for each citizen i

$$\xi_i'(\varepsilon) < \frac{c(1-t')}{(N-N^Rt')} - b_1(G'-\varepsilon,\theta_i).$$

Note also that the definition of $\overline{\varepsilon}$ implies that

$$G' - \varepsilon > N\left(t'\overline{y}^*(t') + (1-t')\,y^*\left(t', a^P\right)\right)/c.$$

Hence, Assumption 1 implies that for all citizens i $\xi_i'(\varepsilon) > 0$ for all $\varepsilon \in [0, \overline{\varepsilon}]$ since $t' \leq \widetilde{t}_R$. This implies that

$$V^i\left(t', T\left(t', G' - \overline{\varepsilon}\right), G' - \overline{\varepsilon}\right) > V^i(T', t', G') \; for \; all \; i$$

which, since $(1-t')y^*\left(t', a^P\right) + T\left(t', G' - \varepsilon\right) = 0$, establishes the Claim.

Now suppose that $t' > \widetilde{t}_R$. For all $t \in [0,1]$ let

$$T(t, \varepsilon) = (1-t)\,(\frac{T'}{(1-t')} + \varepsilon),$$

where ε is small and non-negative. In addition, let

$$\eta(t, \varepsilon) = N^P\left(-(\frac{T'}{(1-t')} + \varepsilon)\right) + N^R\left(-T(t, \varepsilon)\right) + N^R t y^*\left(t, a^R\right).$$

Since $T(t', 0) = T'$, it should be clear that $\eta(t', 0) = cG'$. It is also true that

$$\eta(0,0) < cG' < \eta\left(\widetilde{t}_R, 0\right).$$

By continuity, therefore, for sufficiently small ε there exists a $\widehat{t}(\varepsilon) \in (0, t_R)$ such that $\eta\left(\widehat{t}(\varepsilon), \varepsilon\right) = cG'$. Now consider the policy vector with tax rate $\widehat{t}(\varepsilon)$, guarantee $T\left(\widehat{t}(\varepsilon), \varepsilon\right)$ and public goods level $\widehat{G}(\varepsilon)$ determined from the budget constraint.

There are two cases: (a)

$$y^*\left(\widehat{t}(0), a^P\right) > -\frac{T(\widehat{t}(0), 0)}{1 - \widehat{t}(0)} = -\frac{T'}{(1-t')}$$

and (b) $y^*\left(\widehat{t}(0), a^P\right) \leq -\frac{T'}{(1-t')}$. In case (a) under the policy vector $(\widehat{t}(0), T\left(\widehat{t}(0), 0\right), \widehat{G}($ the poor choose to pay more taxes than under (T', t', G'), while the rich pay the same amount. The revenue raised under the policy vector $(\widehat{t}(0), T\left(\widehat{t}(0), 0\right), \widehat{G}(0))$ therefore exceeds cG' and hence $\widehat{G}(0) > G'$. The rich are strictly better off because they pay the same total amount of taxes but now enjoy a higher level of the public good and a lower tax rate. The poor are also better off because they receive a higher level of public good and, by revealed preference, prefer to consume

$$T(\widehat{t}(0), 0) + (1 - \widehat{t}(0))y^{*}\left(\widehat{t}(0), a^{P}\right)$$

and to work $y^{*}\left(\widehat{t}(0), a^{P}\right)/a^{P}$, than to consume 0 and to work $-T'/(1-t')a^{P}$. The policy vector $(\widehat{t}(0), T\left(\widehat{t}(0), 0\right), \widehat{G}(0))$ thus Pareto dominates (T', t', G') and serves to establish the Claim.

In case (b) under the policy vector $(\widehat{t}(0), T\left(\widehat{t}(0), 0\right), \widehat{G}(0))$ both rich and poor pay the same amount of taxes as they did under (T', t', G'). It follows that $\widehat{G}(0)) = G'$. The poor are thus equally well off under $(\widehat{t}(0), T\left(\widehat{t}(0), 0\right), \widehat{G}(0))$, while the rich are strictly better off because they face a lower tax rate. Since the poor's utility is increasing in ε and the rich's utility is decreasing in ε, it follows by continuity that, for sufficiently small ε, both the rich and the poor will be better off under the policy vector $(\widehat{t}(\varepsilon), T\left(\widehat{t}(\varepsilon), \varepsilon\right), \widehat{G}(\varepsilon))$ than under (T', t', G'). If

$$T(\widehat{t}(\varepsilon), \varepsilon) + (1 - \widehat{t}(\varepsilon))y^{*}\left(\widehat{t}(\varepsilon), a^{P}\right) \geq 0,$$

then the policy vector $(\widehat{t}(\varepsilon), T\left(\widehat{t}(\varepsilon), \varepsilon\right), \widehat{G}(\varepsilon))$ serves to establish the Claim. If not, then noting that $\widehat{t}(\varepsilon) < \widetilde{t}_{R}$, we may use our earlier argument to demonstrate the existence of a policy vector with the desired properties. This proves Claim 2. ■

These Claims establish that neither case (i) nor case (ii) are possible under Assumption 1. The result now follows. ■

Proof of Proposition 1: (Sufficiency) Let citizen i be poor and let σ be a vector of entry decisions with $\sigma_i = 1$ and $\sigma_j = 0$, for all $j \neq i$. Let $\alpha(\cdot)$ be any voting function with the property that $\alpha(C)$ is a voting equilibrium for all candidate sets C. Then we claim that $\{\alpha(\cdot), \sigma\}$ is a political equilibrium for δ sufficiently small.

To prove this, we need to show that σ is an equilibrium of the entry game given $\alpha(\cdot)$. We first check that citizen i has an incentive to run. His equilibrium payoff is $W^P(t_P) - \delta$. If he withdrew, there would be no candidates and the default outcome of zero taxation would be implemented. Thus, his payoff would be $W^P(0)$. Since $W^P(t_P) > W^P(0)$, citizen i has an incentive to run for δ sufficiently small. We now verify that no other citizen has an incentive to enter. It is clear that no other poor citizen has an incentive to enter since they are already obtaining their most preferred policy outcome. Consider then a rich citizen j. His equilibrium payoff is $W^R(t_P)$. If he entered, his payoff would be

$$P^i(\{i,j\}, \alpha(\{i,j\}))W^R(t_P) + (1 - P^i(\{i,j\}, \alpha(\{i,j\})))W^R(0) - \delta.$$

But $\alpha(\{i,j\})$ is a voting equilibrium and the weak dominance requirement implies that voters vote sincerely in two candidate races. Thus, since the poor are the majority, $P^i(\{i,j\}, \alpha(\{i,j\})) = 1$. It follows that j has no incentive to enter.

(Necessity) Now let $\{\alpha(\cdot), \sigma\}$ be a political equilibrium under market provision for sufficiently small δ. We must show that $\#C(\sigma) = 1$ and that the single candidate is poor. Suppose first that $\#C(\sigma) = k > 1$. Let $C_R(\sigma)$ be the set of rich candidates and $C_P(\sigma)$ be the set of poor candidates. In addition, let $\mathcal{W}(C(\sigma))$ be the set of winning candidates. There are three possibilities: (i) $\mathcal{W}(C(\sigma)) \subset C_R(\sigma)$ (ii) $\mathcal{W}(C(\sigma)) \subset C_P(\sigma)$ and (iii) $\mathcal{W}(C(\sigma)) \cap C_R(\sigma) \neq \emptyset$ and $\mathcal{W}(C(\sigma)) \cap C_P(\sigma) \neq \emptyset$. In case (i), it must be the case that $C_P(\sigma) = \emptyset$ since withdrawing cannot produce a worse outcome for the poor. But this means that $\#C_R(\sigma) = 1$, since no rich candidate has an incentive to run against other rich candidates. This contradicts the hypothesis that $\#C(\sigma) > 1$. A similar argument rules out case (ii). In case (iii), the result must be a tie between one or more rich candidates and one or more poor candidates. Since each citizen's vote is decisive in such a tie, the best response requirement of voting equilibrium implies that all rich citizens must be voting for a single rich candidate and all poor voters must be voting for a single poor candidate. But since the poor outnumber the rich, the result cannot be a tie. We conclude, therefore, that $\#C(\sigma) = 1$.

Now suppose that the single candidate, citizen j, were rich. Consider some poor citizen i. His equilibrium payoff is $W^P(t_P)$. If he entered, his payoff would be

$$P^i(\{i,j\}, \alpha(\{i,j\}))W^P(t_P) + (1 - P^i(\{i,j\}, \alpha(\{i,j\})))W^P(0) - \delta.$$

But $\alpha(\{i,j\})$ is a voting equilibrium and the weak dominance requirement implies that voters vote sincerely in two candidate races. Thus, since the poor are the majority, $P^i(\{i,j\}, \alpha(\{i,j\})) = 1$. It follows that j has an incentive to enter for sufficiently small δ, which contradicts the fact that σ is an equilibrium of the entry game. ■

Proof of Lemma 3: Note first that the inequality $\Delta_P < \Delta_R$ is equivalent to the inequality $W^P(t_P) + W^R(t_P) < W^P(0) + W^R(0)$. Now let

$$\beta(t) = y^*\left(t, a^P\right) + y^*\left(t, a^R\right) - \left[\phi\left(y^*\left(t, a^P\right)/a^P\right) + \phi\left(y^*\left(t, a^R\right)/a^R\right)\right].$$

It is clear that $\beta(0) = W^P(0) + W^R(0)$ and since $N^P > N^R$, $\beta\left(t^P\right) > W^P(t_P) + W^R(t_P)$. Hence, it suffices to show that $\beta(0) > \beta(t_P)$. Differentiating $\beta(t)$ and using the fact that $(1-t)a^k = \phi'\left(y^*\left(t, a^k\right)/a^k\right)$for $k \in \{P, R\}$ yields

$$\beta'(t) = t \cdot \left[\frac{\partial y^*\left(t, a^P\right)}{\partial t} + \frac{\partial y^*\left(t, a^R\right)}{\partial t}\right] < 0,$$

as required. ■

Proof of Proposition 2: Let $\{\alpha(\cdot), \sigma\}$ be a political equilibrium under state provision for sufficiently small δ with the property that $\#C(\sigma) = 1$. Let the single candidate, citizen j, be of class k and type e. We show first that $G_e = G_\mu$. Suppose not and consider some citizen i of class k and type μ. Citizen i's equilibrium payoff is given by $W^k(t_k) + B(G_e, \lambda_\mu)$. If citizen i entered, he would obtain a payoff

$$W^k(t_k) + P^i(\{i,j\}, \alpha(\{i,j\}))B(G_\mu, \lambda_\mu) + (1 - P^i(\{i,j\}, \alpha(\{i,j\})))B(G_e, \lambda_\mu) - \delta$$

But $\alpha(\{i,j\})$ is a voting equilibrium and the weak dominance requirement implies that voters vote sincerely in two candidate races. Thus, since the median group prefers citizen i, $P^i(\{i,j\}, \alpha(\{i,j\})) = 1$. Since $G_e \neq G_\mu$, it follows that j has an incentive to enter for sufficiently small δ, which contradicts the fact that σ is an equilibrium of the entry game.

The argument to show that citizen j must be poor is similar. If he were rich, then a poor citizen of the median type would have an incentive to enter for sufficiently small δ. ■

Proof of Proposition 3: Note that in case (i), all poor citizens are the same and all rich citizens are the same. Thus, there are two groups with identical and opposing preferences. The logic of Proposition 1 therefore continues to apply. In case (ii), while citizens have different types, they all agree that the optimal level of public goods is zero. Accordingly, the only issue is redistribution and the logic of Proposition 1 continues to apply. ■

Proof of Proposition 4: **(Necessity)** Suppose that a political equilibrium exists with a type e running against a type f of the same class ($e \neq f$). Each must win with positive probability to be willing to run. This requires that they have the same number of votes as each other, which, since indifferent voters abstain, requires that the condition in the Proposition be satisfied.

(Sufficiency) Suppose that the condition of the Proposition is satisfied. Take two citizens of the same class of types e and f and label them i and j. Let Ω_i be the set of citizens with types such that $\lambda_h < \lambda(e, f)$; Ω_j be the set of citizens with types with $\lambda_h > \lambda(e, f)$ and Ω_0 be the set of citizens of types such that $\lambda_h = \lambda(e, f)$. Then $(\Omega_i, \Omega_j, \Omega_0)$ is a sincere partition (see Besley and Coate (1997a)). The condition of the Proposition together with Assumption 2 implies that $\#\Omega_i = \#\Omega_j$ and that Ω_0 contains all citizens indifferent between i and j. Assumption 3 then implies that $\#\Omega_0 < (N/3) + 1$. Since $G_e < G_f$, we have that $\frac{1}{2}\left(v_{ii}^S - v_{ij}^S\right) \geq \delta$ and $\frac{1}{2}\left(v_{jj}^S - v_{ji}^S\right) \geq \delta$ for δ sufficiently small. Proposition 3 of Besley and Coate (1997a) then implies that there exists a political equilibrium involving i and j running against each other. ■

Proof of Proposition 5: This is similar to the proof of Proposition 4 and hence is omitted. ■

Proof of Proposition 6: Suppose, to the contrary, that there existed such a political equilibrium. Label the three candidates 1, 2, and 3, and let their public good types be $h(1)$, $h(2)$ and $h(3)$, with $h(1) \leq h(2) \leq h(3)$. There are three possibilities: (i) the three candidates are tying, (ii) two of the candidates are tying and (iii) only one is winning. We now proceed to rule each of these possibilities out.

(i) If all three candidates are tying, then each citizen is decisive. The best response requirement of voting equilibrium then implies that each citizen must be voting for his preferred candidate (see Besley and Coate (1997a)). Moreover, for each citizen j who is voting for candidate k

$$\frac{1}{3}\left[\sum_{i=1}^{3} B\left(G_{h(i)}, \theta_j\right)\right] \geq^{max}_{i \neq k} B\left(G_{h(i)}, \theta_j\right);$$

that is, each citizen must prefer the lottery over the three candidates to the certain win of his second best candidate. If all the candidates are not associated with the same level of public goods, then since $B(\cdot, \theta_j)$ is strictly concave,

$$\frac{1}{3}\left[\sum_{i=1}^{3} B\left(G_{h(i)}, \theta_j\right)\right] < B\left(\overline{G}, \theta_j\right)$$

where $\overline{G} = \frac{\sum_{i=1}^{3} G_{h(i)}}{3}$. It follows that, if $\overline{G} \geq G_{h(2)}$, citizens of type $h(1)$ prefer the public goods level $G_{h(2)}$ to the lottery over the three levels, while if $\overline{G} < G_{h(2)}$, citizens of type $h(3)$, must prefer the public goods level $G_{h(2)}$. Thus, since type

$h(i)$ citizens must be voting for candidate i, the above inequality cannot hold. Hence, it must be the case that all candidates are associated with the same level of public goods (i.e., $G_{h(1)} = G_{h(2)} = G_{h(3)}$). But in this case, each candidate would be better off withdrawing.

(ii) If two candidates are tying, then since each voter is decisive, each voter must be voting for their preferred candidate of the two winners. Since the tying candidates must receive equal support, either the candidates must be associated with the same public goods level or the median type is indifferent between the two candidates' public good levels. In the former case, it is clear that the losing candidate has no incentive to remain in the race. In the latter case, if the losing candidate were to exit, the median group (by AIV) would abstain. By Assumption 2, then the remaining candidates would still tie and the probability distribution over policy would be unchanged. Hence, it is always worthwhile for the losing candidate to exit.

(iii) The single winner must be candidate 2. If it were candidate 1 (3), then candidate 3 (1) would have an incentive to drop out (see Proposition 5 of Besley and Coate (1997a)). The entry conditions also imply that $G_{h(1)} < G_{h(2)} < G_{h(3)}$ and that candidates 1 and 3 would at least tie with candidate 2 in a two-way race. Thus

$$B\left(G_{h(1)}, \lambda_\mu\right) \geq B\left(G_{h(2)}, \lambda_\mu\right)$$

and

$$B\left(G_{h(3)}, \lambda_\mu\right) \geq B\left(G_{h(2)}, \lambda_\mu\right).$$

But the first of these inequalities imply $h(2) > \mu$ and the second implies $h(2) < \mu$

— a contradiction. ■

Proof of Proposition 7: Assume that $e^* < \mu$ (the argument for $e^* > \mu$ is analogous). Select a rich candidate of type e^*, a poor candidate of type e^* and a rich citizen of type μ. Label them i, j, k respectively and consider the vector of entry decisions σ such that only these three citizens enter. We will construct a voting function $\alpha(\cdot)$ such that $\{\alpha(\cdot), \sigma\}$ is a political equilibrium.

If the candidate set is $\{i, j\}$, then all of the rich vote for i and all of the poor vote for j. If the candidate set is $\{i, k\}$, all citizens with types such that $\lambda_h < (>)\, \lambda(e^*, \mu)$ vote for $i\,(k)$, with the remainder abstaining. If the candidate set is $\{j, k\}$, all poor citizens with types such that $\lambda_h < (>)\, \lambda^P(\mu, e^*)$ vote for $j\,(k)$, all rich citizens with types such that $\lambda_h < (>)\, \lambda^R(\mu, e^*)$ vote for $j\,(k)$, and the remainder abstain. For the candidate set $\{i, j, k\}$, all rich citizens and all poor citizens with types such that $\lambda_h < \lambda(e^*, \mu)$ vote for i. All poor citizens with types such that $\lambda_h > \lambda^P(\mu, e^*)$ vote for k and all poor citizens with types such that $\lambda_h \in \left[\lambda(e^*, \mu), \lambda^P(\mu, e^*)\right]$ vote for j. We suppose that for the candidate set $\{i, j, k, g\}$ for all $g \neq \{i, j, k\}$, citizens vote as for the set $\{i, j, k\}$. For all remaining candidate sets, we let any voting equilibrium be selected.

Notice that the voting function that we have constructed always selects a voting equilibrium. Voting in the two-candidate cases described above is sincere and in the three candidate case no citizen is voting for his least preferred choice. Thus, voting decisions are not weakly dominated. To check that each citizen's voting decision is a best response in the three candidate case, note that the number of voters for i is $\sum_{h \in I_-(\lambda(e^*,\mu))} N_h^P + N^R$ while the number of voters for the other candidates is no greater than $N^P - \sum_{h \in I_-(\lambda(e^*,\mu))} N_h^P$. But we know from the fact that $\zeta(e^*,\mu) \geq 2$, that

$$\sum_{h \in I_-(\lambda^P(e^*,\mu))} N_h^P + \sum_{h \in I_-(\lambda^R(e^*,\mu))} N_h^R > \sum_{h \in I_+(\lambda^P(e^*,\mu))} N_h^P + \sum_{h \in I_+(\lambda^R(e^*,\mu))} N_h^R + 1$$

Since $\lambda^P(e^*,\mu) < \lambda(e^*,\mu)$, this implies that

$$\sum_{h \in I_-(\lambda^P(e^*,\mu))} N_h^P + N^R > N^P - \sum_{h \in I_-(\lambda(e^*,\mu))} N_h^P + 1$$

Thus i is winning by more than one vote and nobody can affect the outcome by switching his vote.

To complete the proof, we need to check that σ is an equilibrium of the entry game given $\alpha(\cdot)$ for sufficiently small δ. It is clear that i has an incentive to run, because he gets his preferred outcome and neither of the other candidates would deliver this outcome. Citizen j (k) remains in the race to prevent k (j) from winning. No other citizen wishes to enter given the assumed voting behavior. ■

Proof of Proposition 8: Let $(t', T', G') \in Z_S$ be a policy vector such that $G' > 0$. There are three possibilities: case (a) in which

$$(1-t')y^*(t', a^R) + T' < 0;$$

case (b) in which

$$(1-t')y^*(t', a^P) + T' < 0 \leq (1-t')y^*(t', a^R) + T';$$

and case (c) in which

$$0 \leq (1-t')y^*(t', a^P) + T'.$$

In case (a), Claim 1 of Lemma 2 tells us that there exists a policy vector $(t, T, G) \in Z_S$, satisfying the inequality which characterizes case (c) which Pareto dominates (t', T', G'). Similarly, in case (b) Claim 2 of Lemma 2 tells us that there exists a policy vector $(t, T, G) \in Z_S$, satisfying the inequality which characterizes case (c) which Pareto dominates (t', T', G'). Thus, it suffices to establish the result for case (c).

Consider for $\varepsilon \in [0, G']$ the policy vector $(t', T' + \varepsilon \frac{c}{N}, G' - \varepsilon)$. This is clearly feasible. Letting $\xi_i(\varepsilon)$ denote citizen i's utility under this policy vector, we have

$$\xi_i(\varepsilon) = T' + \varepsilon \frac{c}{N} + b(G' - \varepsilon, \theta_i) + (1 - t') y^*(t', a_i) - \phi\left(\frac{y^*(t', a_i)}{a_i}\right).$$

Thus,

$$\xi_i'(\varepsilon) = \frac{c}{N} - b(G' - \varepsilon, \theta_i) \geq \frac{c}{N} - b(G' - \varepsilon, \lambda_H) > 0.$$

It follows that

$$V^i(t', T' + G'\frac{c}{N}, 0) > V^i(t', T', G') \; for \; all \; i.$$

This completes the proof. ∎

Proof of Proposition 9: Let $(t', T', 0) \in Z_M$. We may assume that $t' < 1$, since if $t' = 1$, $T' = 0$ and $V^i(0,0,0) > V^i(1,0,0)$ for all citizens i. It then follows that $T' = t' \cdot \overline{y}^*(t') > 0$. Now consider the policy vector $(t', T' - \frac{c}{N}G, G)$. Then, for sufficiently small G,

$$V^i(t', T' - \frac{c}{N}G, G) = T' - \frac{c}{N}G + (1 - t')y^*(t', a_i) - \phi(\frac{y^*(t', a_i)}{a_i}) + b(G, \theta_i).$$

Differentiating the right hand side and evaluating at $G = 0$, we have that

$$\frac{dV^i(t', T', 0)}{dG} = b_1(0, \theta_i) - \frac{c}{N} \geq b_1(0, \lambda_1) - \frac{c}{N} > 0.$$

The first inequality follows from $b_{12} > 0$ and the second is the condition stated in the Proposition. It follows that, for sufficiently small G,

$$V^i(t', T' - \frac{c}{N}G, G) > V^i(t', T', 0) \ for \ all \ i.$$

The result follows. ■

Proof of Proposition 10: (i) Consider a citizen i of class k and type 1. By Lemma 2, this citizen's (uniquely) optimal policy vector in the set Z_S is $(t_k, t_k \cdot \overline{y}^*(t_k), 0)$. It follows that there exists no $(t, T, G) \in Z_S$ which can Pareto dominate $(t_k, t_k \cdot \overline{y}^*(t_k), 0)$. (ii) Consider a citizen i of class k and type H. By Lemma 2, this citizen's optimal policy vector in the set Z_S is $(t_k, t_k \cdot \overline{y}^*(t_k) - \frac{c}{N}G_H, G_H)$ where $G_H > 0$. Any $(t, T, 0) \in Z_M$ provides this citizen with a lower level of utility. ■

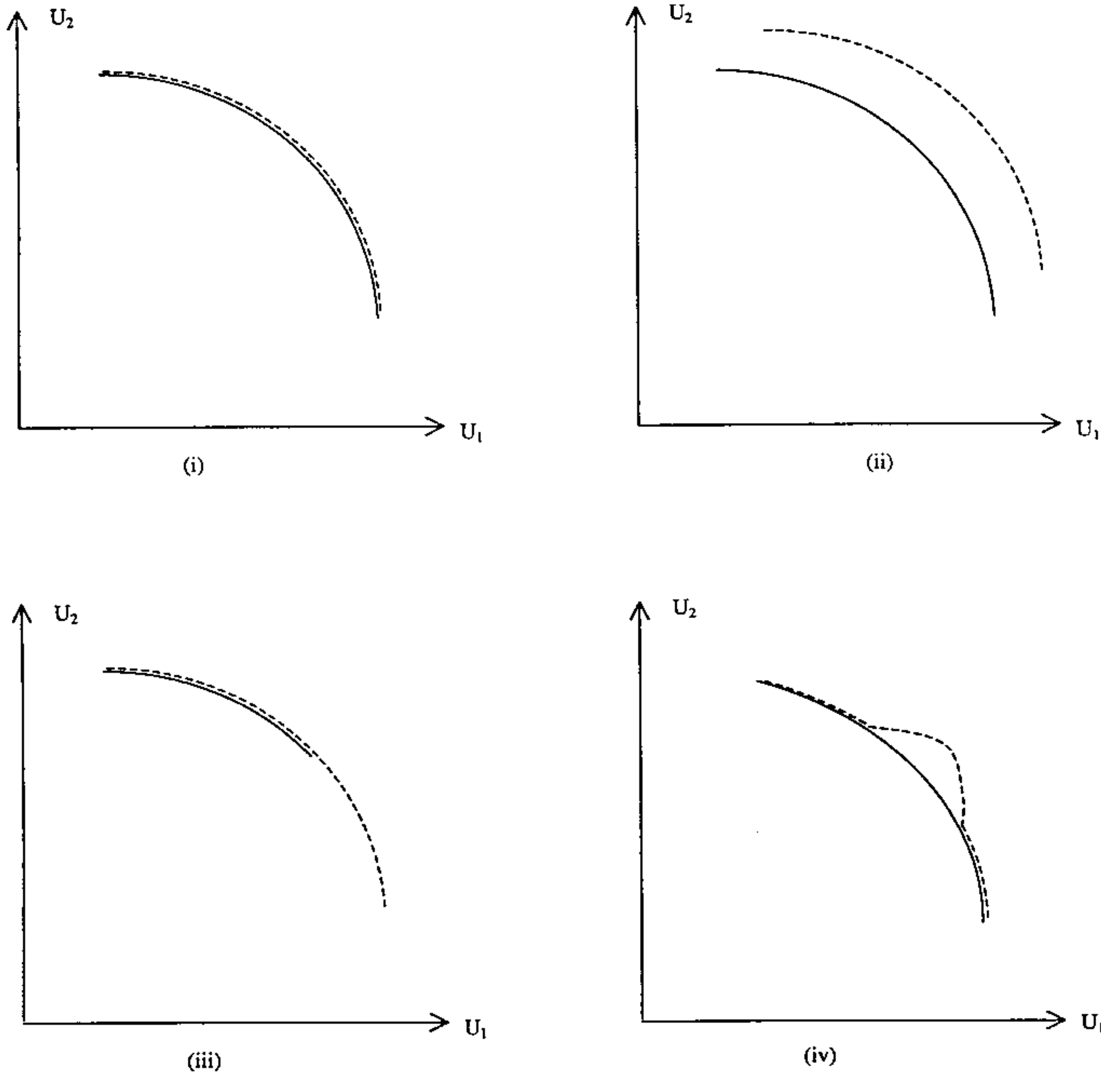

Figure 1

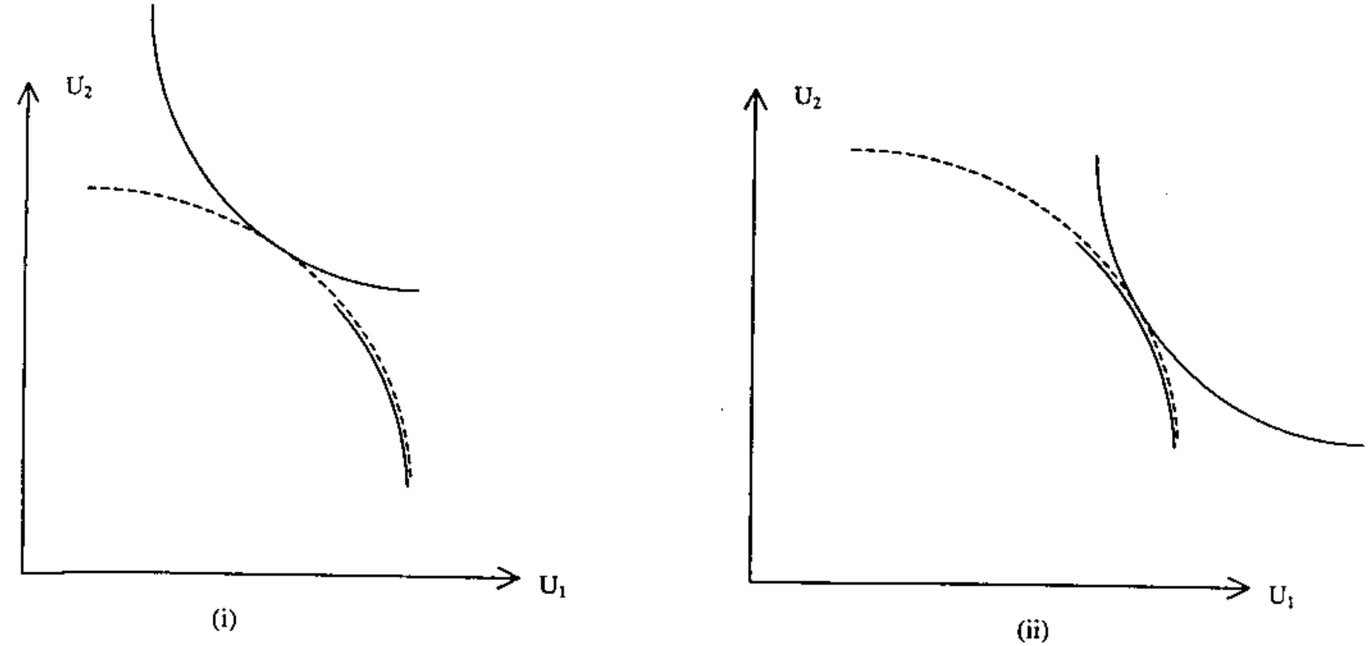

Figure 2

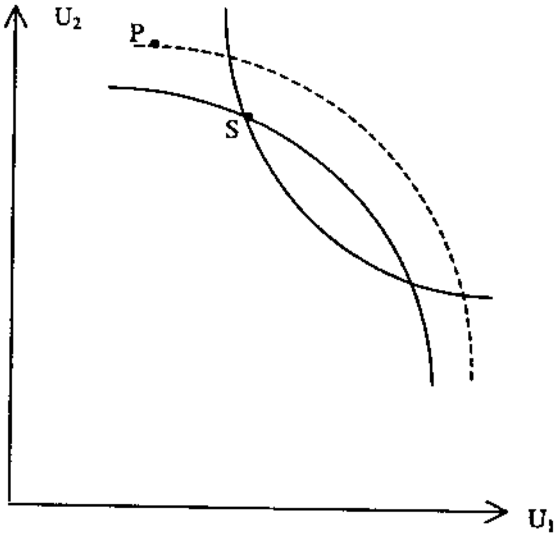

Figure 3

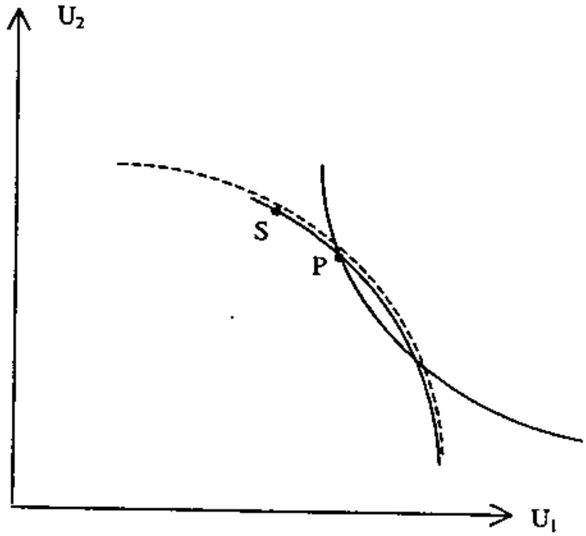

Figure 4

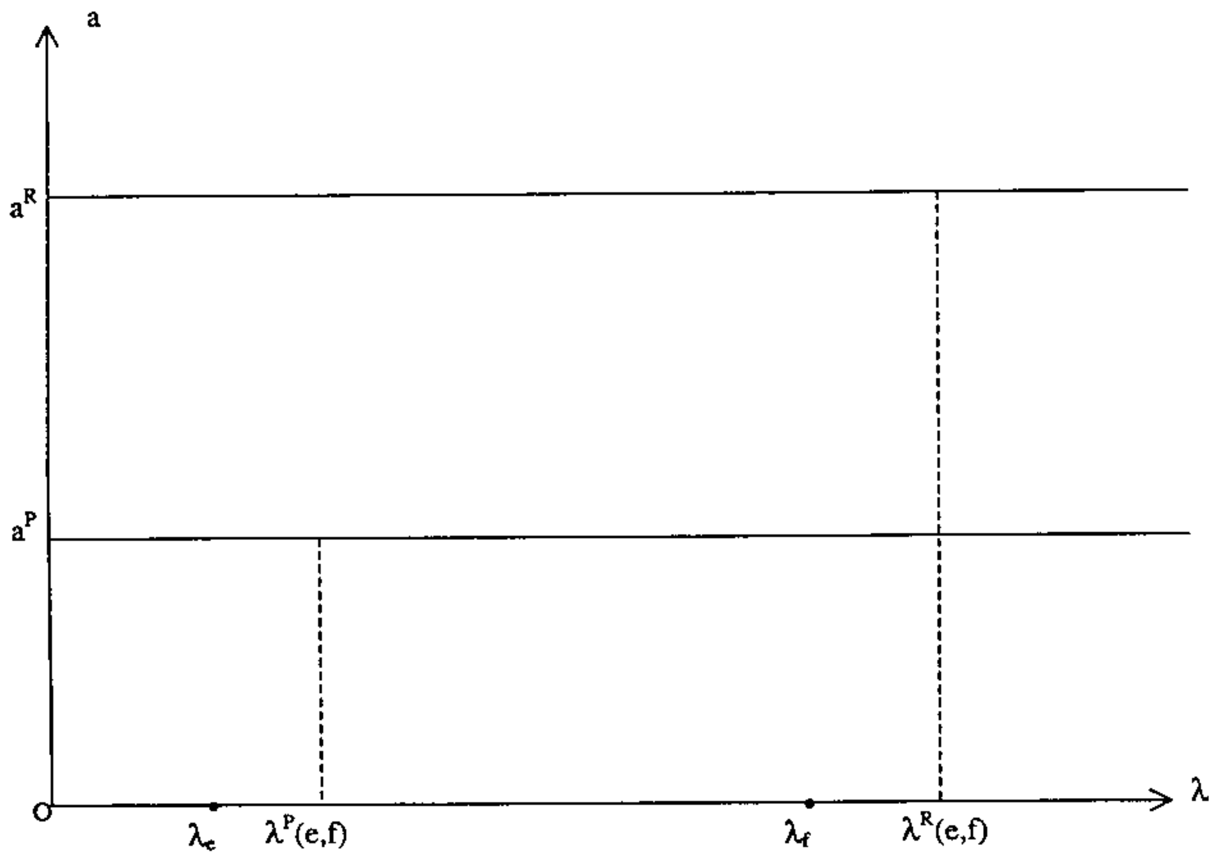

Figure 5