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**Job Creation and Job Destruction in the Presence of
Informal Labour Markets**

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Abstract

Recessions and policy interventions in labour markets in developing countries are characterized not only by changes in the unemployment rate, but also by changes in the proportion of formal or protected jobs. This reallocation between formal and informal jobs is large and occurs mainly because the job finding rate of formal jobs reacts substantially more than the job finding rate of informal jobs. This paper presents a search and matching model to capture this fact. I assume that firms operate the within firm margin of formality, choosing to legalize only those matches that are good enough to compensate the costs of formality. In this framework, recessions or stricter regulations in the labour market trigger two effects. As expected, they lower the incentives to post vacancies (meeting effect), but also affect the firms' hiring standards, favouring informal contracts (offer effect). This new channel sheds light on how the actions of policy makers alter the outcomes in an economy with informal jobs. For instance, attempts to protect employment by increasing firing costs will reallocate workers to informal jobs, where job separation is high. They are also likely to increase unemployment.

JEL classification: J64, H26, O17

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1 Introduction

Developing countries are characterized by the fact that between 40% to 80% of the labour force is employed in semi-illegal irregular jobs.¹ This type of employment has been called the informal employment. A less known fact is that the proportion of informal employment varies considerably both during the business cycle and when there is a change in policy. The share of informal employment in Mexico oscillated 4 percentage points every four years during the 1992-2004 period, increasing sharply in recessions. Similarly, informal employment increased in Brazil in 12 percentage points throughout the 1990's as a result of a mix of policy reforms and downturns.

Evidence on the underlying worker flows in Mexico and Brazil provides insights on how these changes are generated.² First, the job finding rate of formal jobs is strongly procyclical and highly volatile. Conversely, the job finding rate of informal jobs is relatively stable and does not present a strong cyclical pattern. Furthermore, despite significant increases in the job separation rate in informal jobs, the proportion formal employment decreases in recessions. This pattern also seems to be present in the reallocation of workers in Brazil in the 1990's.

This paper provides a framework that is able to explain these stylized facts. I construct a model that focuses on the within firm margin of informality. Firms can potentially hire either formal or informal workers. They decide their optimal level of formality according to the quality of the match and the cost of formal employment. I show that this mechanism can explain the different behaviours of the job finding rate of formal and informal jobs, as well as the changes in the share of formal employment.

The motivation for modeling this margin is rooted in two observations in the data. First, evidence suggests that firms are not entirely formal or informal. Around 25% of owners of small firms (4 or 5 employees) report the use of a combination of formal and informal contracts. Similarly, on average, a non-trivial proportion of formal firms employ informal workers. Around 30% of the jobs in medium sized firms (11-50 employees) and 8% in large firms (more than 250 employees) are informal. Secondly, firms seem to modify this degree of compliance with labour regulations over the business cycle. In periods of expansion 15% of the flows from unemployment into large firms result in an informal contract. In the middle of the 1995 crisis in Mexico this proportion doubled. This suggests that firms in developing countries do not only

¹See Hart (1972), de Soto (1989) and Schneider and Enste (2000) for extensive reviews of the causes and consequences of this type of employment. See also Djankov et al. (2002) and Schneider (2003) for detailed cross country estimates of the size of the informal economy for developed and developing countries.

²See Bosch and Maloney (2006) and Bosch et al (2006).

adjust the number of workers they desire to employ, but also the type of contractual relationships they establish with their employees.

In this model, the firm's choice of hiring standards plays a primary role in the adjustment of the labour market. Firms post "generic" vacancies and, when the worker arrives, they decide whether to establish a formal or an informal relationship. The outcome depends on the quality of the match and the trade-offs between formal and informal employment. As standard in search and matching models, a positive macroeconomic shock fosters vacancy creation. Hence, the number of meetings between firms and workers increases (*meeting effect*). Moreover, firms extend the use of formal contracts because they can take better advantage of the increase in productivity (*offer effect*). As a result, the job finding rate of formal jobs increases, since the two effects, the general macro effect and the substitution effect, reinforce each other. The effect on the job finding rate of informal jobs is ambiguous, because the two effects go in opposite directions, and therefore, it can be procyclical or countercyclical.

Furthermore, this paper provides a tool to study the effects of policy changes on the allocation of workers in developing countries. I examine the impact of three labour market interventions: hiring costs, firing costs and payroll taxes. I argue that the effect of policies do not only occur through the creation of vacancies and destruction of jobs, but also through the effects on the firm's hiring standards. For instance, protecting formal jobs by raising firing penalties reduces the job separation rate in formal jobs. However, changes in the firm's hiring decisions reallocate jobs from formal into informal employment. That is, production is shifted from "protected" jobs, where job separation is low, to "unprotected" jobs, where job separation is high. The quantitative effects of policies on unemployment will depend on how much labour is reallocated from one type of jobs to the other. This, in turn, depends on the opportunity costs of employing informal labour. I argue that in developing economies, where the wedge between formal and informal technology is low and enforcement is likely to be deficient, small regulatory changes can have a large reallocation effect.

Two alternative explanations could be put forward to rationalize the different behaviour of the worker flows and the changes in the share of formality. First, it could be argued that the cyclical patterns are due to compositional changes in the industry/occupation mix. While the manufacturing sector employs primarily formal jobs (around 80% of jobs are formal), the service sector extensively uses informal labour (only 40% of jobs are formal). Therefore, shocks affecting disproportionately the manufacturing sector during recessions could be the driving force behind changes in the proportion of formal jobs. I show that the patterns observed in the data cannot be explained by such compositional effects. Hence, in recessions, the increase in informality occurs, largely, because the share of formal employment de-

creases in all industries, occupations and population groups, not because labour is reallocated from "low informality" industries/occupations towards "high informality" industries/occupations.

The second possible explanation relates to policy regulation. Consider an economy populated by two types of firms, formal and informal. If severance payments constrain firing decisions of formal workers, then shocks for formal firms have to be fully absorbed through the hiring margin. Informal firms, on the contrary, are able to use both margins and therefore are able to adjust to the desired level of employment much quicker. This faster adjustment of informal labour would necessarily increase the proportion of formal employment during recessions, which is at odds with the data.

This paper is related to a growing literature on the existence of informal jobs in the labour market.³ Previous papers have studied two other margins along which informal jobs are created. First, a series of models focus on the worker's decision to participate in the informal labour market. They usually assume the exogenous existence of both formal and informal firms posting vacancies. Then, heterogeneous workers direct their search towards one of the two type of firms according to their moral costs of operating in the informal sector (Fugaza and Jaques, 2002 and Kolm and Larsen, 2002), to worker's education (Kolm and Larsen, 2004), or to productivity differences (Boeri and Garibaldi, 2006).⁴ Other types of models endogenize the firm's choice. Kugler (1999) assumes that firms sort themselves into formal or informal statuses according to their ex-ante productivity levels. Workers are then matched randomly into formal and informal firms. Antunes and Calvalcanti (2004) and Bosch (2004) suggest occupational choice models. Agents are allowed to decide between becoming a formal entrepreneur, an informal entrepreneur or workers in search of a job. Finally, Albrecht et al. (2006) argue that workers' productivity is the major determinant of participation in the informal sector. In a model with heterogeneous workers, they show that the appearance of informal jobs is rooted in the decision of low productivity workers to become informal self-employed.

The remainder of the paper is organized as follows. The next section presents the empirical evidence on worker flows for Mexico and Brazil. Section III presents the details of the model. Section IV outlines the main predictions of the variables of the model when the economy is subject to macroeconomic shocks or changes in regulations. Section V presents some extensions of the model that accommodate

³For early models of the informal sector in developing countries see Lewis (1954), Harris-Todaro (1970) and Fields (1990). Also see Loayza (1996) for and Satchi and Temple (2006) for growth models with informal jobs.

⁴Alternatively, Bouev (2002) suggests that workers may search randomly.

some particularities of informal employment. Section VI calibrates the model and quantifies the possible effects of policy on the unemployment rate and the share of formality. Finally, section VII concludes.

2 Empirical Evidence

There is broad agreement in the literature on the definition of an informal worker. Generally speaking, formal workers are those working in firms licensed with the government and conforming to tax and labour laws, including minimum wage directives, pension and health insurance benefits for employees, workplace standards of safety etc. The informal workers, on the contrary, are those owners of firms that are largely delinked from state institutions and obligations and employees who are not covered by formal labour protections.

The motivation for this model is drawn from the study of gross worker flows in Mexico and Brazil during the last two decades (Bosch and Maloney, 2006 and Bosch et al., 2006). Detailed labour surveys in Mexico and Brazil allow me to compute with precision not only the proportion of informal employment in the economy, but also the gross movements of workers among employment statuses (see Appendix B for details). Workers are classified into formality/informality on the basis of lack of compliance with labour legislation- in particular lack of contributions by the employer to the social security agency as the critical distinguishing characteristic⁵. Empirical papers, including Bosch and Maloney (2006) and Bosch et al. (2006) distinguish between two types of informal workers: informal salaried employees and informal self-employed. I choose to pool together these two types of employment and focus on "informality" as a whole based of the lack of protection criteria. Although these two types of employment may have different considerations, they share very similar dynamics in and out of employment. Moreover, they both constitute a very flexible "unregulated" source of labour for formal firms.⁶ Nevertheless, I explicitly consider how to integrate the self-employment decision in latter sections of the paper.

In what follows I summarize the main facts emerging from the data. Figures 1 and 2 show the two main indicators of the functioning of Mexican and Brazilian labour markets, the unemployment rate and the share of formal employment. First, unemployment rates are relatively low compared to OECD countries. Despite major macroeconomic shocks in these two countries during the 1980's and 1990's, the un-

⁵Contributions to social security programs by the employer implicitly impose a series of commitments for the firm, such as paid vacations, minimum wages, severance payments etc.

⁶Excluding self-employed workers from the sample does not qualitatively change the results.

employment rate never reaches the two digit numbers. Second, informality comprises a substantial part of total employment (between 35%-45%). Informality is mainly countercyclical, increasing especially in periods of deep recessions, such as 1995 in Mexico and 1982-83 in Brazil. Finally, whereas Mexico has not undertaken a major labour reform since the 1970's, Brazil went through a number of reforms during the 1980's and 1990's, the most significant of which was a change in the constitution in 1988. Three changes in regulations affected directly the labour market. There was a significant increase in firing costs, the numbers of maximum working hours per week was reduced from 48 to 44, and union power was enhanced. Although the effect of all these policies is difficult to estimate⁷, it is clear that from the beginning of 1990's Brazil experienced a major shift in the allocation of workers between formal and informal jobs. The share of formal employment went from over 65% of total employment to 53% in roughly 10 years. Also, average unemployment increased from 3% to 8%.

Gross worker transitions underlying the evolution of the stocks provide further insights in understanding these labour markets. Figures 3 and 4 show the job finding rate of unemployed workers in Mexico and Brazil for the two types of jobs together with the unemployment rate.⁸ Similarly, figures 5 and 6 show the separation rates.

Table 1 summarizes the cyclical properties of the series. The data unveils important patterns. The job finding rate of formal jobs is strongly procyclical. The cross correlation of the job finding rate with respect to unemployment is very high, -0.86 and -0.81 for Mexico and Brazil respectively. Conversely, the job finding rate of informal jobs does not result to have a profound cyclical pattern. It is weakly procyclical in Brazil and slightly countercyclical in Mexico. Overall, the large recessions and recoveries in these two countries during the 1980's and 1990's brought important changes in the job finding rate of formal jobs with only minor fluctuations in the job finding rate of informal jobs.

Moreover, these patterns in the job finding rate do not only correspond to the cyclical behaviour of the labour market. Long run trends also seem to be dominated by adjustments in the job finding rate of formal jobs. After the reform in Brazil, the monthly job finding rate of formal jobs decreased from 0.15 to 0.05, whereas job finding rate of informal jobs stayed on trend around 0.22.

⁷The constitutional reform in Brazil coincided with an the process of trade liberalization of the economy.

⁸The transition probabilities presented in figures 3 to 6 correspond to quarterly transitions for Mexico and monthly transitions quarterly averaged for Brazil. Hence mobility level is not entirely comparable between these two countries. Throughout the paper I refer to them as the job finding and job separation rates although technically correspond to discrete transition probabilities. The continuous transition rates for Mexico and Brazil respectively follow exactly the same cyclical pattern. See Bosch and Maloney (2006) and Bosch et al. (2006).

Table 1: Correlations and Volatility of HP detrended Worker Flows

x	Mexico		Brazil	
	Corr(u, x)	Std(x)/Std(u)	Corr(u, x)	Std(x)/Std(u)
F.emp	(0.53)	1.18	(0.57)	0.76
JFR_f	(0.86)	2.29	(0.81)	1.96
JFR_i	0.24	1.27	(0.29)	1.22
JSR_f	0.75	0.27	0.32	0.14
JSR_i	0.92	0.65	0.91	1.17

JFR _{j} =Job Finding Rate of j type of jobs, j =formal, informal

JSR _{j} =Job Separation Rate of j type of jobs, j =formal, informal

u =Unemployment rate. F.emp=Share of Formal employment.

Source: ENEU 1987-2004

Additionally, the job separation rate is countercyclical in both types of jobs. However, it is substantially higher for informal jobs. On average, around twice as high. Furthermore, the volatility of the job separation rate in informal jobs is also higher. This is especially true for Brazil where the job separation rate in formal jobs is reasonable constant over time.⁹ However, despite large increases in the job separation rate in informal jobs in recessions, the share of the formal employment decreases in downturns. This highlights that the behaviour of the job finding rate is driving the reallocation of workers between formal and informal jobs.

This paper abstracts from direct flows from occupied jobs. I do consider flows from informal jobs into formal jobs in one of the extensions of the model. In practice, direct transitions between formal and informal jobs do occur. Nevertheless, the empirical evidence from direct flows does not contradict, in essence, the modelling approach (see Bosch and Maloney, 2006). That is, the hiring behaviour of firms is the main driving force of changes in the share of formal employment. Flows from informal towards formal jobs are highly procyclical, very much like those from unemployment. Similarly, flows from formal to informal jobs are also largely procyclical but less volatile than the former. Quantitatively, this implies that changes in the share of formal employment are primarily driven by access to formal jobs rather than increasing outflows from formal jobs.

⁹The empirical evidence of the Brazilian reform is still inconclusive. Paes de Barros and Corseuil (2001) fail to find any robust findings on how job destruction rates were affected by this reform. This is consistent with the data previously presented. Figures 4 and 6 seem to suggest that the major adjustments were concentrated in the job finding rate of formal jobs and that job separation rate was not substantially altered.

2.1 Compositional effects.

It could be argued that the patterns found in share of formal employment are due to changes in the composition of the labour force or in the industry/occupational mix. For instance, manufacturing is mainly formal, between 75% to 80% of workers are formal. Conversely, services employ informal labour extensively (only 40% of jobs are formal). Therefore asymmetric shocks to these industries could be behind the patterns the share of formal employment. I investigate this hypothesis further. I use a probit model where I regress the probability of being a formal worker (conditional on employment) on a set of time dummies. This gives the unconditional evolution of the share of formal employment similar to figures 1 and 2. I also run the same regression but including, sex, age, education, industry and occupational dummies in addition to the time dummies. Figures 7 and 8 present the pattern of the time dummies of these two regressions for Mexico and Brazil. The difference between the two series captures the amount of time variation in the share of formal employment that can be explained by observables. These figures suggest that compositional changes are not the explanation to the procyclicality of the share of formal employment.

2.2 Changes in the Formalization of Firms: The within firm margin.

I argue that the observed changes in the job finding rates can be explained by modelling firms' hiring choice between formal and informal labour. There is evidence that firms conduct part of their operations informally. Kenyon (2006) shows that the percentage of sales reported by Brazilian firms ranges between 60% in micro firms to 80% in very large firms. Moreover, Maloney (2004) points at anecdotal evidence from Latin America suggesting that medium to large firms will commonly have a substantial share of their operations, including workers off the books. In Argentina, even individual workers will receive pay, partly "blanco" on the books and partly "negro" off the books and without the corresponding labour taxes paid by either worker or firm.

This is confirmed by my data. Table 2 shows the share of formal employment, as well as, the proportion of hires from unemployment that result in a formal contract by firm size in Mexico.¹⁰ The degree of formalization of a firm is invariably related to its size. Indeed, small firms are mostly informal (only 15% of employment is formal). However, table 2 suggests that the lack of compliance with labour laws is not exclusive of small firms. Large firms, which are in all likelihood formal in other dimensions,

¹⁰Unfortunately firm size is not available for Brazil.

also make use of informal contracts. On average, 8% of jobs and around 20% of the hiring in firms with more than 250 employees are informal. Similarly, not all small firms are completely informal. This fact is corroborated by data from the Mexican microenterprise survey (ENAMIN) run every two years. This survey reinterviews those who in the ENEU claim to be either self-employed or owners of a microfirm (less than 6 employees) and obtains detailed information about their operations, and most importantly for this paper, on the status of the workers they hire.¹¹ Table 3 shows, by size and year, the percentage of firms that are completely formal (i.e. all workers they employ are registered with the social security), completely informal, or they use a mix of formal and informal contracts. Two observations are of particular interest in this table. First, there is direct evidence that there is a within firm margin of informality. Between 20% to 30% of firms of 4 and 5 employees use a mix of formal and informal contracts. Second, for all firm sizes the degree formalization falls during the crisis of 1995 and recovers to the pre-crisis value by 2002. This suggests that this margin is sensitive to changes in the business cycle even for very small firms.

Table 2: Degree of Formality by Firm Size. Mexico. Average:1987-2004.

Size	Share of Formality	Share of Emp.	% of Formal Flows from <i>u.</i>	% of Total Flows from <i>u.</i>
1 to 10	15.83	42.29	10.97	47.38
11 to 15	62.36	3.55	44.99	4.17
16 to 50	76.84	9.49	61.99	10.86
51 to 100	85.26	5.29	73.00	6.13
101 to 250	89.64	3.79	78.87	4.07
More than 250	92.04	35.59	79.39	28.15

Source: ENEU 1987-2004

¹¹See data appendix for details.

Table 3: Degree of Formality of Small Firms
by Number of Employees. Mexico 1992-2002

Employees	Type	92	96	02
1	F	10.78	7.75	13.50
	I	89.22	92.25	86.50
	M			
2	F	17.63	16.06	20.39
	I	77.04	79.40	72.82
	M	5.33	4.54	6.80
3	F	28.80	18.67	30.86
	I	58.58	69.07	59.48
	M	12.62	12.27	9.67
4	F	25.60	19.76	41.61
	I	54.40	53.89	42.95
	M	20.00	26.35	15.44
5	F	41.33	32.89	42.25
	I	36.00	42.11	29.58
	M	22.67	25.00	28.17
% Formal Emp.		28.38	22.38	31.37

F=all employees formal, I=All employees informal

M=Mixed of formal and informal employees

Source: ENAMIN 1992,1996 and 2002

Further evidence on the adjustment on the degree of formalization can be obtained by looking at workers flows. Figure 9 shows the probability that a transition of a worker from unemployment results in a formal contract for large and small firms. This tries to capture propensity to hire formal workers, controlling for changes in total hiring. This ratio is strongly procyclical for both small and large firms. While one could argue that most of the changes in this ratio for small firms could be entirely driven by creation and destruction of firms, it is somewhat harder to argue that this is the case for large firms. In expansions between 20% to 15% of new hires are informal. In the 1995 crisis this ratio increased to 33%. It is this margin of informality and its adjustment what the model will exploit to explain the changes in the job finding rates and ultimately on the share of formal employment.

3 The Model

This paper introduces into a Mortensen and Pissarides (1994) type of model a within firm decision between two types of contracts; formal and informal. The intuition of the model can be summarized as follows. Entrepreneurs post vacancies in search for workers. The total number of matches between firms and workers, m , is given by the matching technology

$$m = m(u, v) \tag{1}$$

where v and u represent the number of vacancies and unemployed workers respectively. The matching technology is homogeneous of degree one and increasing and concave in both its arguments. The rate at which firms with vacant jobs find workers is given by

$$q(\theta) = m(u, v)/v \tag{2}$$

where $\theta = \frac{v}{u}$ is generally referred to as the market tightness of the economy. Similarly workers find firms at a rate

$$\theta q(\theta) = m(u, v)/u \tag{3}$$

Workers are ex ante equal, but when the worker and the firm are brought together, some match pairs result to be more productive than others. Once the productivity of the match is realized, the firm has to decide whether to write a formal or an informal contract with the worker or, alternatively, not to consummate the match at all. There is a trade off between formal and informal jobs. If a formal contract is signed, the firm incurs in a hiring cost (i.e. the firm has to train the worker). Then, the job-worker match has access to a better overall technology parameter, but they have to observe a number of labour regulations.¹² Conversely, informal jobs are less productive but avoid all labour regulations imposed by the government. They are, however, subject to monitoring and jobs may be destroyed if the government detects them. Once the match is established both formal and informal jobs are subject to idiosyncratic productivity shocks that may, endogenously, terminate matches.

¹²Initially I just consider hiring and firing cost as the only regulations. Section IV deals with payroll taxes.

3.1 Payoffs

Let V be the present discounted value (PDV) to the entrepreneur of the expected profit from posting a vacancy. Similarly, $J_f^l(z)$ and $J_i(z)$ represent the PDV for the firm of occupied formal and informal jobs respectively, where l identifies the new and ongoing formal matches.

$$rV = -pk + q(\theta) \int_{z_{\min}}^{z_{\max}} \max [J_f^n(z) - pc, J_i(z), V] dG(z) - q(\theta)V \quad (4)$$

$$rJ_f^l(z) = pz - w_f^l(z) + \lambda \int_{z_{\min}}^{z_{\max}} \max [J_f^o(s), V - pF] dG(s) - \lambda J_f^l(z) \quad , l = n, o \quad (5)$$

$$rJ_i(z) = \delta pz - w_i(z) + \lambda \int_{z_{\min}}^{z_{\max}} \max [J_i(s), V] dG(s) - \lambda J_i(z) + \phi(V - J_i(z)) - \phi p\sigma \quad (6)$$

The interpretation of equations (4) to (6) is straight forward. Vacant jobs have a current flow cost of pk and vacancies meet workers at a rate $q(\theta)$. Once the worker and the vacancy have met, the productivity of the match is drawn from a known distribution function, $G(z)$. Given the realization of the match specific productivity, the firm has three choices. First, it can formalize the relationship, in which case the firm enjoys $J_f^n(z)$ but has to pay hiring costs, pc , and it is subject to future firing costs, pF . Second, the firm can also avoid regulations by hiring the worker informally, $J_i(z)$. Third, if the realization of the productivity is too low the firm can decide to keep the vacancy open.

For occupied formal and informal jobs, the first two terms capture the instantaneous profit of the job. That is, the combination of a general productivity parameter p and the match specific productivity parameter z minus the wage in each type of job. Note that formal jobs operate with a higher general productivity parameter than informal jobs, being $\delta < 1$. I attribute this to the existence of the initial hiring cost¹³. Moreover, the introduction of firing costs in the formal jobs gives rise to two different value functions for occupied jobs. One for the newly created jobs $J_f^n(z)$, (when firing costs are not applicable in the bargaining process) and another for ongoing matches for which firing costs are considered when wages are bargained, J_f^o (see Pissarides 2000).

¹³See Gonzaga (2003), Almeida and Carneira (2005) for papers referring to the productivity differential between formal and informal jobs in Brazil.

All types of jobs are subject to idiosyncratic productivity shocks *à la* Mortensen and Pissarides (1994). When a shock arrives at a rate λ wages are renegotiated and the job-worker pair decides whether or not to continue production. In the case of formal jobs, if the job-worker pair decides to terminate the match, the firm has to pay a firing cost, pF .¹⁴ Informal jobs are also subject to a monitoring activity from the government that destroys the match at a rate ϕ . If informal matches are detected the firm incurs in a penalty of $p\sigma$.

I assume that once the nature of the relationship between the worker and the firm has been established (formal or informal) it cannot be modified. Hence, initial formal jobs are always formal, independently of the evolution of the idiosyncratic productivity shock. I relax this assumption in later sections of the paper.

The value functions for the workers, equations (7) to (9), have equally simple interpretation; U represents the PDV of an unemployed workers. While searching, the unemployed gets a value of b . They meet jobs at a rate $\theta q(\theta)$. Depending on the type of contract offered by the firm, the worker enjoys the match specific PDV of a new formal job $W_f^n(z)$, or the PDV of an informal job $W_i(z)$. Alternatively, workers can decide to keep searching. Once the workers are employed, they obtain a wage depending on the contract (formal/informal) and the productivity of the match, which is subject to changes upon the arrival of shocks.

$$rU = b + \theta q(\theta) \int_{z \min}^{z \max} \max [(W_f^n(z), W_i(z), U)] dG(z) - \theta q(\theta)U \quad (7)$$

$$rW_f^l(z) = w_f^l(z) + \lambda \int_{z \min}^{z \max} \max [W_f^o(s), U] dG(s) - \lambda W_f^l(z) \quad , l = n, o \quad (8)$$

$$rW_i(z) = w_i(z) + \lambda \int_{z \min}^{z \max} \max [W_i(s), U] dG(s) - \lambda W_i(z) + \phi(U - W_i(z)) \quad (9)$$

3.2 Bargain over wages

Following the literature, when workers and firms first meet, or when an idiosyncratic shock arrives, they bargain over the surplus of the job according to a Nash bargain solution. In this particular model three types of bargaining situations may arise. Equation (10) shows the bargaining rule for a new formal match, an ongoing formal match and an informal match respectively.

¹⁴For simplicity I have considered that both types of jobs are subject to the same distribution of shocks.

$$\begin{aligned}
J_f^n(z) - pc - V &= \frac{(1-\eta)}{\eta}(W_f^n(z) - U) \\
J_f^o(z) - V + pF &= \frac{(1-\eta)}{\eta}(W_f^o(z) - U) \\
J_i(z) - V &= \frac{(1-\eta)}{\eta}(W_i(z) - U)
\end{aligned} \tag{10}$$

where η is the share of the surplus that goes to the worker. When the worker and the firm meet for the first time and they decide to write a formal contract the gain firm is $J_f^n(z) - pc - V$. Since the match is not formed, the firm is not obliged to any firing costs to the worker in the case of disagreement. However the firm has to pay the hiring cost, pc , upon the signing of the contract. When a shock arrives to an ongoing match, the firing cost becomes operational. However, the hiring cost is already sunk. The gain for the firm of continuing the match is $J_f^o(z) - V + pF$. Obviously, these considerations do not apply for the informal jobs.

Equation (10) also highlights a well known fact of Nash bargaining. Firms and workers always agree on the type of contract, as well as when to destroy the match. From the hiring point of view, firms are willing to hire a worker formally as long as $J_f^n(z) - pc > J_i(z)$ and $J_f^n(z) - pc - V > 0$, which necessarily implies that $W_f^n(z) > W_i(z)$ and $W_f^n(z) > U$. Similarly the firm-worker match choose an informal contract if $J_f^n(z) - pc < J_i(z)$ and $J_i(z) > 0$.

3.3 Equilibrium

This section defines and characterizes the steady-state equilibrium of the model. Four conditions determine the equilibrium of the model. The first equilibrium condition states that the creation of vacancies is driven by free entry. This implies that all profit opportunities from new jobs are exploited and therefore $V = 0$.

The second and the third equilibrium conditions relate to the hiring decision of the firm. These are given by

$$J_f^n(R) - pc = J_i(R) \tag{11}$$

$$J_i(W) = 0 \tag{12}$$

I define R as the level of productivity that makes the firm indifferent between hiring the worker formally or informally, equation (11). Let W be the level of pro-

ductivity that makes the firm indifferent between hiring the worker informally or keep searching for workers, equation (12).

The fourth and final equilibrium condition of the model defines the job separation threshold of formal jobs, equation (13). Let D be the productivity level that makes an ongoing formal job unprofitable. Since firing costs do not apply to informal jobs, the job separation threshold of informal jobs is also given by W .

$$J_f^o(D) + pF = 0 \quad (13)$$

Figure 10 illustrates the firm's hiring and firing decisions. It plots the PDV for the firm of the different contracts against the level of idiosyncratic productivity of the match z . The profitability of occupied jobs depends positively on the productivity of idiosyncratic productivity of the match. This makes $J_f^n(z) - pc$, $J_f^o(z) + pF$ and $J_i(z)$ upward-sloping. However, the slope of the latter is flatter because of the overall productivity wedge, δ , and the existence of monitoring, ϕ . The fixed costs of formal jobs ensure the existence and uniqueness of R . Similarly, W and D are obtained in the intersection of $J_i(z)$ and $J_f^o(z) + pF$ with the horizontal axis respectively. Of course, it may be the case that these crossing points are not compatible with the existence of informal jobs. I restrict my analysis to the spectrum of parameters where both formal and informal jobs coexist, that is when $R > W > 0$.

I can now rewrite equations (4) to (9) using the different thresholds. For firms

$$pk = q(\theta) \left[\int_R^{z \max} (J_f^n(z) - pc) dG(z) + \int_W^R J_i(z) dG(z) \right] \quad (14)$$

$$rJ_f^l(z) = pz - w_f^l(z) + \lambda \int_D^{z \max} J_f^o(s) dG(s) - \lambda G(D)pF - \lambda J_f^l(z) \quad (15)$$

$$rJ_i(z) = \delta pz - w_i(z) + \lambda \int_W^{z \max} J_i(s) dG(s) - (\lambda + \phi) J_i(z) - \phi p\sigma \quad (16)$$

and for workers

$$rU = b + \theta q(\theta) \left[\int_R^{z \max} (W_f^n(z) - U) dG(z) + \int_W^R (W_i(z) - U) dG(z) \right] \quad (17)$$

$$rW_f^k(z) = w_f^k(z) + \lambda \int_D^{z \max} W_f^o(s) dG(s) + \lambda G(D)U - \lambda W_f^k(z) \quad (18)$$

$$rW_i(z) = w_i(z) + \lambda \int_W^{z \max} W_i(s) dG(s) + \lambda G(W)U - (\lambda + \phi) W_i(z) + \phi U \quad (19)$$

Now it is clear what the optimal hiring decision of the firm-worker pair depends on the idiosyncratic productivity of the match. When the worker and the firm meet for the first time, they jointly decide what type of contract to sign. If the idiosyncratic productivity is higher than R the contract is formal, if it is between R and W the contract is informal and if it is less than W no contract is signed. Once jobs start production, they are subject to productivity shocks. Formal jobs are destroyed if the productivity shock is lower than D , whereas informal jobs are destroyed if the productivity shock is smaller than W .

Using (14) to (19) and the bargaining solution, equation (10), I can derive the wage functions for each of the three types of matches, equations (20) to (22). Note that all three wage functions depend positively on the productivity of the match. For new formal jobs, the firing cost enters negatively as the workers have to compensate the firm for future firing costs. For ongoing matches the firing cost enters positively, since the firm has to pay the cost if the worker does not agree to continue the match. All wages depend positively on the market tightness of the economy.

$$w_i(z) = (1 - \eta)b + \eta p (\delta z + \theta k) - \eta \phi p \sigma \quad (20)$$

$$w_f^n(z) = (1 - \eta)b + \eta p (z + \theta k - \lambda F - (r + \lambda) c) \quad (21)$$

$$w_f^o(z) = (1 - \eta)b + \eta p [z + \theta k + rF] \quad (22)$$

With these wage functions and (14) to (19) it is straight forward to obtain the four equilibrium equations of the model. Substitution of the corresponding wage equation into (16) gives

$$J_i(z) = \frac{1}{r + \phi + \lambda} \left[(1 - \eta) (\delta p z - b - \phi p \sigma) - \eta p \theta k + \lambda \int_W^{z \max} J_i(s) dG(s) \right] \quad (23)$$

Evaluating $J_i(z)$ at W and subtracting it from equation (23) .

$$J_i(z) = \frac{(1 - \eta) p \delta (z - W)}{r + \phi + \lambda} \quad (24)$$

Finally substituting equation (24) back into (23) and evaluating at W , I obtain the threshold that drives the value of an occupied informal job to zero. Note that this

threshold determines the lower limit of the hiring decision and the firing of informal workers. It is also useful to get an expression for δW for comparability with the formal counterpart.

$$\begin{aligned} W &= \frac{b}{\delta p} + \frac{\phi\sigma}{\delta} + \frac{\eta\theta k}{(1-\eta)\delta} - \frac{\lambda}{r+\phi+\lambda} \int_W^{z^{\max}} (s-W) dG(s) \\ \delta W &= \frac{b}{p} + \phi\sigma + \frac{\eta\theta k}{(1-\eta)} - \frac{\lambda\delta}{r+\phi+\lambda} \int_W^{z^{\max}} (s-W) dG(s) \end{aligned} \quad (25)$$

Analogously for ongoing formal jobs

$$J_f^o(z) + pF = \frac{(1-\eta)p(z-D)}{(r+\lambda)} \quad (26)$$

$$D = \frac{b}{p} + \frac{\eta\theta k}{(1-\eta)} - \frac{\lambda}{(r+\lambda)} \int_D^{z^{\max}} (s-D) dG(s) - rF \quad (27)$$

Equations (25) and (27) constitute the two equilibrium conditions determining job separations. The job separation rate in formal jobs is given by $\lambda G(D)$. Informal workers also suffer job separations due to the exogenous government monitoring. Therefore the job separation rate is given by $\lambda G(W) + \phi$. By comparing equations (25) and (27), the first result of the model is obtained. The job separation rate is always greater for informal workers. This is due to three separate effects. The type of job specific productivity differential, the existence of firing costs and the monitoring of informal jobs. Note that, as in Mortensen and Pissarides (1994), both types of jobs are kept open even if the reservation productivity of the job is below the reservation wage of unemployed workers, $rU = b + \frac{\eta\theta pk}{(1-\eta)}$. This occurs because occupied jobs have an option value, captured by the integral term in both equations. This option value represents the potential increase in productivity for the job when a shock arrives. This option value is higher for formal jobs, since, on average, informal jobs are less productive and they have an external source of job destruction.

The third equilibrium equation determines the optimal hiring policy of the firm. Substituting equation (21) into (15) gives the value of new formal jobs

$$J_f^n(z) - pc = \frac{(1-\eta)p(z-D)}{(r+\lambda)} - (1-\eta)p(F+c) \quad (28)$$

equating (28) to (24) I obtain what is the idiosyncratic productivity of the match that makes the firm (and the worker) indifferent between a formal and an informal contract, R .

$$\begin{aligned} \frac{(1-\eta)p(R-D)}{(r+\lambda)} - (1-\eta)p(F+c) &= \frac{(1-\eta)p\delta(R-W)}{r+\phi+\lambda} \\ R &= \frac{(r+\lambda+\phi)[D+(r+\lambda)(F+c)] - (r+\lambda)\delta W}{(r+\lambda)(1-\delta) + \phi} \end{aligned} \quad (29)$$

Equation (29) states that the expected gain from the marginal formal worker must be equal to the expected gain for the marginal informal worker.

The fourth and final equilibrium equation of the model is the free entry condition. Substituting (26) and (24) into (14) I obtain

$$\begin{aligned} \frac{k}{q(\theta)} &= \int_R^{z^{\max}} \left[\frac{(1-\eta)(z-D)}{(r+\lambda)} - (1-\eta)(F+c) \right] dG(z) \\ &+ \int_W^R \frac{(1-\eta)\delta(z-W)}{(r+\phi+\lambda)} dG(z) \end{aligned} \quad (30)$$

The left hand side of this equation represents the expected cost of the vacancy, which has to be equal to the expected profit from posting the vacancy. From equation (30), it is straight forward to see the existence and uniqueness of the equilibrium in this model. As market tightness increases, the cost of the vacancy also increases, since vacancies are kept unfilled for longer periods of time. On the other hand, greater market tightness means higher wages and higher job separation rates for formal and informal jobs. This reduces the expected profitability of the vacancy, lowering the right hand side of the equation. Finally as θ changes so does R , however it is easy to show that, in equilibrium, R does not modify expected profits from the vacancy on virtue of the envelope property it satisfies, that is, the productivity level that makes the firm indifferent between hiring formal and informal workers. Hence for particular values of F , c and k there is a unique value of θ , that satisfies equation (30). Once θ is obtained, the different thresholds can be retrieved from equations (25), (27), and (29).

Figures 11 and 12 show the equilibrium in the R - θ space and D , W - θ respectively. From the hiring decision (29), there is a positive relation between R and θ (see appendix for proofs). The intuition is straight forward. Lower θ leads to lower wage demands. As a result, present and future profits for all types of jobs increase. However, future profits of informal jobs are discounted at a higher rate. This is due to the fact that duration of informal jobs is shorter. Overall, the surplus of formal jobs increases more than the surplus of informal jobs. Therefore, R decreases.

Similarly, from equation (30), in equilibrium, θ is independent of R , for the reasons stated above. Figure 12 shows the job separation conditions for both types

of jobs. These two conditions slope upwards on θ since wages are positively related to θ .

The steady state values of the stocks close the model.

$$u = \frac{n_f \lambda G(D) + n_i (\lambda G(W) + \phi)}{\theta q(\theta) [1 - G(W)]} \quad (31)$$

$$n_f = \frac{\theta q(\theta) [1 - G(R)] u}{\lambda G(D)} \quad (32)$$

$$n_i = 1 - n_f - u \quad (33)$$

$$\pi = \frac{n_f}{n_i + n_f}$$

where u is the unemployment rate and n_f and n_i the number of formal and informal jobs in the economy. The labour force is normalized to 1. Flows in and out of unemployment determine its steady state value. Flows into unemployment come from occupied formal and informal jobs. Formal jobs are destroyed at a rate $\lambda G(D)$, whereas informal jobs are destroyed at rate $\lambda G(W) + \phi$. Flows out of unemployment can be divided in two. The job finding rate of formal jobs is given by

$$JFR_f = \theta q(\theta) [1 - G(R)] \quad (34)$$

and the job finding rate of informal jobs is given by

$$JFR_i = \theta q(\theta) [G(R) - G(W)] \quad (35)$$

Finally, π captures the share of formal employment in the economy.

4 Comparative Statics

Next I examine the implications of changes in the key parameters of the model. Two sets of parameters are particularly interesting; productivity shocks and policy interventions.

4.1 Macroeconomic shocks: Changes in productivity

Figures 13 and 14 illustrate the implication of an increase in productivity, p . Note that changes in productivity are equivalent to the inverse changes in unemployment benefits, b . An increase in the general productivity parameter shifts both job separation conditions downwards. Similarly, the formal/informal hiring condition also shifts downwards, since now firms and workers have more incentives to sign formal contracts. This is because formal jobs are able to take better advantage of the increase in productivity.¹⁵ Finally, the free entry condition is shifted to the right as the expected profit from both types of jobs increases. From the graphs, there is an unambiguous increase in the market tightness, however, the impact on R , D and W is ambiguous. It is easy to show, however, that all three margins decrease, implying a higher conditional probability of signing formal contracts and lower job separation thresholds in both types of jobs (see appendix for proofs).

Proposition 1 *An increase in productivity, p , reduces the job separation rate in both types of jobs.*

This is immediate from figures 13 and 14. In response to a good productivity shock both D and W decrease. Hence, the job separation rate in both types of jobs, $\lambda G(D)$ and $\lambda G(W) + \phi$, decrease. However, the model is silent in which of these two probabilities decreases more. This depends on the assumptions about the distribution of shocks, $G(x)$.

Proposition 2 *An increase in productivity, p , unambiguously increases the job finding rate of formal jobs, but has an ambiguous effect on the job finding rate of informal jobs.*

The job finding rate of formal jobs is given by $\theta q(\theta)[1 - G(R)]$. It is useful to distinguish between two separate effects. On the one hand, the probability that a worker meets a firm, which is governed by θ , the *meeting effect*. And on the other hand, the conditional probability of signing a formal contract, $[1 - G(R)]$, the *offer effect*. A positive productivity shock increases θ and reduces R . Hence, the meeting effect and the offer effect reinforce each other, highly increasing the chance of an

¹⁵Note that, all the effects of changes in the overall parameter of productivity of the economy come from the fact that income while unemployed, b , remains constant. There are other elements that could be subject to changes in productivity such as the hiring costs, or firing costs, which in this model are assumed to be proportional to productivity. In the short run, an increase in productivity could reduce all those fixed costs, strengthening the effects highlighted here.

unemployed worker of obtaining a formal job. This is consistent with the strong procyclicality of the job finding rate of formal jobs found in Mexico and Brazil.

Similarly, the job finding rate of informal jobs is given by $\theta q(\theta)[G(R) - G(W)]$. Again it is useful to separate the two effects. Now the conditional probability of signing an informal contract is given by $[G(R) - G(W)]$. A positive productivity shock increases the meeting rate, but lowers R and W . In principle, this would tend to lower the conditional probability of signing an informal contract, however the exact effect will depend on the properties of the distribution of shocks $G(x)$. Nevertheless, this change in the hiring policy of the firm is a candidate to explain the relative stability of the job finding rate of informal jobs. I take up this issue in the calibration of the model.

Proposition 3 *An increase in productivity, p , unambiguously decreases unemployment but it has an ambiguous effects on the relative size of formal employment.*

The unemployment rate is determined by the exit rate from unemployment $\theta q(\theta)[1 - G(W)]$, the job separation rate in each type of job and the share of formal employment. Positive productivity shocks generate lower job separation rates in all jobs and increases the rate of exit from unemployment. Therefore, in the face of a positive productivity shock unemployment can only increase if the share of formal employment falls. However, it is easy to show from equation (32) that, if unemployment increases so does the number of formal workers. Hence, it is not possible that a drop in the share of formality as that would imply an increase in the number of informal workers, incompatible with the fact that the labour force is constant. Unemployment must, therefore, fall.

The share of formal employment should follow a procyclical pattern dictated by the hiring decisions of the firm. However, the results are ambiguous, because quantitative changes in job separation rates will depend on the distribution of shocks $G(x)$.

4.2 Protecting Formal Employment: Changes in firing costs.

Next, I analyze the economy's response to changes in the firing penalties. Figures 15 and 16 show the graphical analysis. An increase in F shifts the job separation condition of formal jobs downwards. Similarly, the formal/informal hiring condition shifts upwards, capturing the fact that now formal jobs are more expensive. Now, at any given market tightness, the match has to be of higher quality for the firm to be willing to sign a formal contract. Finally, the free entry condition shifts to the left as the expected profits from vacant jobs decrease. Overall, R increases whereas

market tightness and both job separation thresholds decrease, especially for formal jobs. This implies a reduction in the meeting rate and an increase in the conditional probability of signing an informal contract (see appendix for proofs).

Proposition 4 *An increase in firing costs, F , unambiguously decreases the job separation rate in both types of jobs.*

This is analogous to the previous case since W and D decrease. The direct effect of the decrease in F lowers the job separation rate in formal jobs since now it is more expensive to fire workers. Indirectly, the decrease in market tightness decreases wage demands in all types of jobs, diminishing the job separation rate in informal jobs and further pushing downwards the job separation rate in formal jobs.

Proposition 5 *An increase in firing costs, F , unambiguously decreases the job finding rate of formal jobs, but it has ambiguous effects on the job finding rate of informal jobs.*

The decrease in the meeting rate and the higher standards for formal contracts significantly reduces the job finding rate of formal jobs. However, in this case the conditional probability of signing an informal contract increases unambiguously. This is due to the fact that the gap between R and W widens up. Note that the decrease in W comes from the lower pressure on wages generated by a depressed market tightness. The overall effect on $\theta q(\theta)[G(R) - G(W)]$ is again ambiguous, but points to the fact that there are two opposing forces determining the job finding rate of informal jobs.

Proposition 6 *An increase in firing costs, F , has ambiguous effects on the unemployment rate and the share of formal employment.*

Usually, models studying the impact of firing costs acknowledge two effects (see Bentolila and Bertola, 1990). First, a reduction of the job finding rate and second, a lower job separation rate. Hence, the overall effect on unemployment is ambiguous. In this case, there is an additional effect of substituting formal jobs, where the job separation rate is low, by informal jobs, where the job separation rate is high. This allocation effect is captured by the firm's choice of hiring standards. An increase in R implies that vacancies that previously were being transformed into formal jobs now become informal jobs when the meeting between a firm and a worker occurs. The strength of this allocation effect depends mainly on two parameters, δ and ϕ . The intuition is straight forward from equation (29). If the opportunity cost of employing

informal labour is low (big δ), or similarly, when its penalty is small (small ϕ), small changes in firing costs tend to have stronger reallocation effects. This suggests that developing countries, where the opportunity cost of informality is low, the allocation effects between the formal and informal jobs can be substantial.

5 Extensions

I consider three extension to the main framework. First, I have assumed that the initial nature of the job cannot be modified during its life. I relax this assumption here. Second, I consider the decision of becoming an informal self employed rather than an informal employee. Finally, I introduce payroll taxes in the model.

5.1 Informal upgrading and direct flows from informal to Formal jobs

I assume now that the worker-firm pair can decide the best contract at any time. It is easy to argue that only contracts initially established as informal may have the incentives to be transformed into formal contracts. The opposite is never optimal. The intuition is simple. If the worker-firm pair in an informal match receives a positive idiosyncratic shock it may decide to formalize the contract. For that to happen, the boost in idiosyncratic productivity needs to cover the fixed cost associated to the formalization of the contract (the hiring costs and the future firing costs). On the other hand, in order to downgrade a job the firm has to dissolve the previous contract, which implies paying the associated firing costs. It is easy to see that for any z , $J_f(z) + pF > J_i(z)$, and therefore, there are no profitable opportunities to take advantage of.

There are only minor modifications with respect to the basic version of the model. The flow values for the informal jobs have to take into account the potential upgrading of the job. This modifies the job separation condition for the informal workers. Now the option value of the job incorporates the possibility that the job may become formal. This implies that W is smaller, but still bigger than D . Therefore, R is greater. In other words, informal contracts are more attractive. The new equations are given by

$$\begin{aligned}
rW_i(z) &= w_i(z) + \lambda \int_R^{z^{\max}} W_f^n(z) + \lambda \int_W^R W_i(s) dG(s) \\
&\quad + \lambda G(W)U - (\lambda + \phi) W_i(z) + \phi U \\
rJ_i(z) &= \delta pz - w_i(z) + \lambda \int_R^{z^{\max}} (J_f^n(s) - pc) \\
&\quad + \lambda \int_W^R J_i(s) dG(s) - (\lambda + \phi) J_i(z) - \phi p\sigma \\
W &= \frac{b}{\delta p} + \frac{\phi\sigma}{\delta} + \frac{\eta\theta k}{(1-\eta)\delta} - \frac{\lambda}{r + \phi + \lambda} \int_W^R (s - W) dG(s) \\
&\quad - \frac{1}{\delta} \int_R^{z^{\max}} \left[\frac{(s - D)}{(r + \lambda)} - (F + c) \right].
\end{aligned} \tag{36}$$

An interesting feature of this extension is that, implicitly, the upgrading of informal jobs generates direct flows from informal to formal jobs. The probability of transition from an informal job into a formal job is given by $\lambda [1 - G(R)]$. This makes the direct transitions from informal jobs towards formal jobs procyclical, something consistent with the data. Note, however, that there is no search process involved in these transitions as workers remain in the same firm but with different status. The steady states values vary to account for these transitions from informal jobs into formal jobs and are given by

$$\begin{aligned}
u &= \frac{n_f \lambda G(D) + n_i (\lambda G(W) + \phi)}{\theta q(\theta) [1 - G(W)]} \\
n_f &= \frac{\theta q(\theta) [1 - G(R)] u + \lambda [1 - G(R)] n_i}{\lambda G(D)} \\
n_i &= 1 - n_f - u
\end{aligned} \tag{37}$$

The results on the behaviour of the job finding and job separations rates do not change.

5.2 Informal salaried vs. Informal Self-employed

Informality is an heterogeneous phenomenon. Maloney (1999 and 2004) and Yamada (1996) have strongly argued that a substantial part of the informal employment in Latin American countries is concentrated in the informal urban self-employed. Indeed, in my data between 40-50% of informal workers are informal self-employed.

This section briefly explores how to integrate the within firm margin of informality into a more general framework considering the self-employment decision. I follow Fonseca et al. (2001) to assume that agents can choose their occupation. Agents are heterogeneous in their managerial ability and they choose occupation among three different options. Agents may become formal entrepreneurs, informal entrepreneurs or workers in search for a job offered by the formal entrepreneurs. Formal entrepreneurs can manage several jobs at the same time, but informal entrepreneurs can only employ themselves.

There is a distribution of managerial ability $F(\alpha)$, which is continuous in the interval $[0, \alpha_0]$. Being α_0 the best manager in the economy. The decision between the two types of entrepreneurship and becoming a worker is taken by the individual attending to the PDV of each of the options. If the agent α opens a formal firm he would post α vacancies and has to bear some start up costs T , which account to the loss of wealth he has to undertake in order to legally set up a formal firm. If agent α chooses to be an informal self-employed he avoids the start up costs but can only employ himself enjoying a flow income of $\rho\alpha$. Finally, the individual can become a worker in search of a job and obtain U , which is independent of the managerial ability of the agent. I make two simplifying assumptions here for the shake of brevity. I assume exogenous job separations in both types of employment. As observed in the data, the job separation rate is higher in informal jobs, $\lambda_i > \lambda_f$. Stochastic job matching still applies for formal firms. However, I assume that all matches yield a positive surplus. This implies that all matches between firms and workers will result in job formation.

Figure 17 shows the PDV of the three different occupational choices according to the managerial ability, for a given θ (see the appendix for derivation of the equilibrium). First, the PDV of unemployment, U , is independent of the managerial ability of the agent. Second, as the managerial ability increases so does the number of vacancies posted by the formal entrepreneurs. This explains the upward sloping curve of $\alpha V(\theta) - T$. Finally, the better the managerial ability the higher the PDV for the informal entrepreneur. Two thresholds in the managerial ability distribution are relevant in this context. First, I define L as the managerial ability that leaves the agent indifferent between a opening a formal firm and becoming informal self-employed.

$$L = \frac{T}{V(\theta) - \rho} \tag{38}$$

Note that in order for this threshold to exist $V(\theta)$ has to be greater than ρ . Second, the managerial ability, M , that makes the agent indifferent between searching for a job and becoming an informal self-employed.

$$M = \frac{U(\theta)}{\rho} \tag{39}$$

This set up illustrates two additional margins in which informality can operate. The decision to become an informal self-employed is determined by these two margins. On the one hand, some agents may decide to be informal entrepreneurs because the returns to search are low. These are the agents just to the right of M . On the other hand, capable managers do not find it profitable to undertake the start up costs of formality, T , and therefore they become informal entrepreneurs. These are the agents just to the left of L . In summary, to the left of M , agents become workers and look for jobs. Between M and L , agents are informal self-employed. Agents with managerial ability greater than L start up a formal firm, post vacancies and exploit the within firm margin of informality. Some of those vacancies will turn into formal/informal jobs according to the match specific productivity of the match.

Consider now an increase in firing costs. In this set up changes in regulations will not only affect the within firm margin of informality, but also the other two margins of informality. Similar to the basic framework, firing costs changes the within firm margin of informality and also reduces the surpluses of formal matches reducing the value of both V and U . This modifies the other two margins of informality expanding the number of informal self-employed. The increase in the number of informal self-employment comes from two sources. First, some agents get discouraged in the face of low returns to search (decrease in M) and decide to become self-employed. Second, some formal entrepreneurs choose to go underground as the value of posting vacancies decrease (increase in L). Overall, informality expands not only because of the within firm contract decision, like in the benchmark model, but also because of the expansion of the informal self-employed.

5.3 Payroll taxes

I introduce a payroll tax t to which the employer is subject to. Thus, employers have to pay now $w_f(z)(1+t)$ in their formal labour contracts. There are two effects associated with the introduction of the payroll tax. It reduces the surplus of formal matches and it also reduces the share of the surplus allocated to the firm. This generates an asymmetry between formal and informal jobs.

All the effects on the model work through the two job separation conditions. In this case the job separation rate in formal jobs is not unambiguously higher than that of informal jobs. There is a new term in both job separation equations, Ω . This

term captures the increase of the bargaining power of the workers. Changes in the value functions are given by

$$\begin{aligned}
W &= \frac{b}{\delta p} + \frac{\phi\sigma}{\delta} + \frac{\eta}{(1-\eta)\delta} \frac{\theta k}{(1+t)} \\
&\quad - \frac{\lambda}{r+\phi+\lambda} \int_W^{z^{\max}} (s-W) dG(s) + \frac{\Omega}{(1+t)\delta} \\
D &= \frac{b(1+t)}{p} + \frac{\eta\theta k}{(1-\eta)} - \frac{\lambda}{(r+\lambda)} \int_D^{z^{\max}} (z-D) dG(s) - rF + \Omega \\
\Omega &= t\eta\theta q(\theta) \int_W^R \frac{\delta(z-W)}{(r+\lambda)} dG(z)
\end{aligned} \tag{40}$$

6 Calibration

This section explores quantitatively the predictions of the model. I take the average values of the Brazilian labour market as benchmark for the calibration.

The time span of the exercise is a month. The interest rate, r , is 0.01. I normalize the overall productivity parameter to 1. As standard in the literature, I assume a log linear matching function such as

$$\mu q(\theta)^{-\alpha}$$

where μ is a scale parameter. I set α equal to 0.5. To my knowledge, there are no estimates of an aggregate matching function for Brazil or any comparable developing country. This choice of parameter is often used in the literature and it is within the estimates reported by Petrongolo and Pissarides (2001). Similarly, the bargaining power of both types of jobs is set to 0.5. Initially, I consider the monitoring of informal jobs, ϕ , to be equal to 0. There is no evidence on what part of the job separation rate in informal jobs is due to the supervision of the government, but its size in many developing countries suggests that the effects of such punishment policies are negligible. Regarding the idiosyncratic distribution of shocks, I have assumed a uniform distribution between 0 and 1. This assumption, although arbitrary, is of little importance to the results here presented. I have also explored log normal and exponential type distributions with very similar results.

I set the arrival rate of shocks, λ , equal to 0.05. For a given λ , I set the job separation threshold of informal jobs, W , consistent with a 3.2% job separation rate in informal jobs. With knowledge of the job separation rate in informal jobs I obtain

the value of the optimal hiring decision, R , that matches the percentage of new formal to informal jobs given by $(\frac{1-G(R)}{(GR)-G(W)} = 0.42)$.

There is no evidence on the level of labour market tightness in Brazil. I initially assume that market tightness is equal to $2/3$. With this I obtain μ , matching an average job finding rate in the formal of formal jobs, $\mu q(\theta)^\alpha(1 - G(R))$, of 9.2%. The number of informal and formal workers and the job separation rate in formal jobs are chosen to match an average unemployment rate, u , of 5.5%. I also introduce payroll taxes in the calibration of the model. The World Bank estimates that the average payroll tax in Brazil is 37.3% of wages. I set $t = 0.37$.

Five parameters remain to be calibrated: F, δ, c, b and k . I use the four equilibrium equations of the model to solve for F, c, b and k . I choose δ to be the free parameter and set it to 0.625. This allows me to explore the impact of changes in δ . Table 4 shows the parameter configuration for the exercise.

Table 4: Parameter Configuration

$\lambda = 0.05$	$F = 9.30$
$r = 0.01$	$b = 0.16$
$\phi = 0$	$\mu = 1.38$
$\delta = 0.625$	$c = 0.07$
$\eta = 0.5$	$t = 0.37$
$k = 0.46$	$p = 1$

I solve for the remaining parameters of the model and check some values to assess the plausibility of the result. I obtain that k is 0.46. In the steady state, this implies that the cost of vacancies is 2.98% of output. The choice of δ implies that the average wage in informal jobs is 81% of that in the formal. Close to the 0.75 and 0.68 found for Mexico and Brazil respectively. The value for b is 0.16 which is 40% of the lowest wage in the economy. The level of firing costs, 9.3, represents 15 months of average wages of formal jobs. The World Bank estimates for firing costs in Brazil correspond to 8.5 months of average wages. Finally, the value of c , 0.07, adds up to 17% of output.

6.1 Effects of individual policies

I analyze the quantitative effects of changes in productivity and three different policies: firing costs, hiring costs and payroll taxes. This is shown in table 5.

Table 5: Calibration Results.

	<i>Baseline</i>	$\Delta p = 10\%$	$\Delta F = 0.05$	$\Delta c = 0.05$	$\Delta s = 0.05$
u	5.4	4.90	5.90	8.08	6.42
JFR_f	9.2	10.42	6.38	2.38	6.35
JFR_i	21.9	21.88	24.3	27.47	24.37
$\lambda G(D)$	0.86	0.83	0.77	0.86	0.87
$\lambda G(W) + \phi$	3.2	3.18	3.19	3.2	3.3
θ	0.66	0.70	0.64	0.61	0.64
π	60.93	64.58	52.25	24.44	47.58
R	0.89	0.88	0.92	0.97	0.92
W	0.64	0.636	0.638	0.639	0.638
D	0.172	0.166	0.153	0.171	0.183

Changes in productivity produce all the expected outcomes. A 10% increase in productivity decreases unemployment from 5.4% to 4.90%. Note that the most sensitive variable is the job finding rate of formal jobs which increases from 9.2% to 10.42%, whereas the job finding rate of informal jobs only experiences a negligible decrease of 0.02 percentage points. This is, of course, because of the change in the firms hiring standards. Market tightness, as expected, increases and so does the share of formality in the economy from 60.93% to 64.58%.

The next two columns of table 5 analyze the impact of the fixed cost of formality; firing and hiring costs. In order to allow comparable results, these two parameters are increased by the same amount, 0.05, which corresponds to 17% of the average formal wage. Both policies directly influence the hiring decision of the firm, shifting the threshold at which jobs are made formal. Moreover, they also reduce overall market tightness. As argued before, these two combined effects diminish the job finding rate of formal jobs. The job finding rate of informal jobs increases. The main difference in the effects of these two policies is related to the behaviour of the job separation rates. An increase in firing costs decrease the job separation threshold in formal jobs through two channels. A direct one, making dismissal more costly, and an indirect one, through diminished wage demands (lower market tightness). Hiring costs only activate the indirect channel. Hence, the job separation rate responds more to changes in firing costs. This implies that, reallocation from formal into informal jobs will be more intense when hiring costs change. Consequently, unemployment rate and the share of formality also vary more.

Finally, payroll taxes affect directly the incentives to hire a formal worker by reducing the net productivity of the match for the firm. Contrary to the previous two policies, it also increases wage demands affecting therefore the job separation condition of formal workers. The job finding rate of formal jobs decreases whereas

the job separation rate increases. The job finding rate of informal jobs increases accordingly responding to the new set of incentives. Overall, unemployment increases and formality decreases.

6.2 Determinants of the reallocation effect

The essence of the quantitative effects of the model lies on how much firms adjust their hiring standards when a change in policy occurs. I have argued before that this depends on the opportunity cost of using informal labour. In this model, the opportunity cost of informality is embodied in two parameters, δ and ϕ . The higher the opportunity cost of informality (lower δ and higher ϕ), the less powerful the shift in employment will be. Tables 6 and 7 present the policy experiments for a recalibrated model, where I decrease δ to 0.55 (table 6), and increase ϕ to 0.002 (table 7).

Table 6: Calibration Results ($\delta = 0.55$)

	<i>Baseline</i>	$\Delta p = 10\%$	$\Delta F = 0.05$	$\Delta c = 0.05$	$\Delta s = 0.05$
u	5.4	5.00	5.66	7.11	6.08
JFR_f	9.2	10.09	6.99	4.00	7.24
JFR_i	21.9	22.11	23.85	26.32	23.69
$\lambda G(D)$	0.86	0.84	0.76	0.83	0.87
$\lambda G(W) + \phi$	3.2	3.18	3.19	3.18	3.19
θ	0.66	0.70	0.64	0.62	0.65
π	60.93	63.47	55.15	36.72	51.87
R	0.893	0.886	0.917	0.952	0.915
W	0.64	0.636	0.637	0.636	0.637
D	0.172	0.167	0.152	0.166	0.180

Both tables report similar effects. Identical changes in productivity and policy parameters have now a lower impact in the economy. A 10% change in productivity only increases the job finding rate of formal jobs by 0.89 and 0.79 percentage points, respectively, compared to 1.22% before. Similarly, since the substitution of formal workers is less intense, the job finding rate of informal jobs presents a procyclical pattern, indicating that the meeting effect dominates the substitution effect. Moreover, both the unemployment rate and the rate of formality are less affected by changes in policy. Changes in unemployment rate are between 20% and 50% smaller and changes in the share of formality between 30% and 40% smaller.

This exercise also highlights an interesting effect. Developing countries, in which the opportunity cost of informality is likely to be small, are more likely to be subject to strong shifts of labour between informal and formal jobs in the presence of shocks and changes in regulations.

Table 7: Calibration Results ($\phi = 0.002$)

	<i>Baseline</i>	$\Delta p = 10\%$	$\Delta F = 0.05$	$\Delta c = 0.05$	$\Delta s = 0.05$
u	5.4	5.05	5.72	7.35	6.21
JFR_f	9.2	9.99	6.85	3.62	6.94
JFR_i	21.9	21.94	23.92	26.54	23.91
$\lambda G(D)$	0.86	0.84	0.76	0.84	0.91
$\lambda G(W) + \phi$	3.2	3.19	3.19	3.19	3.19
θ	0.66	0.69	0.64	0.62	0.64
π	60.93	63.35	54.48	34.09	50.4
R	0.893	0.874	0.910	0.951	0.909
W	0.60	0.597	0.598	0.599	0.598
D	0.172	0.167	0.152	0.169	0.182

In summary, all policies aimed to increase regulation in formal jobs decrease the market tightness of the economy reducing the meeting rate between vacancies and workers. Additionally, these policies change the incentives to hire and fire formal workers. Overall, the job finding rate of formal jobs is the main variable driving the reallocation between formal and informal jobs. Although the theoretical effects of policies are ambiguous, the simulations show that the net effects tend to increase unemployment and decrease the share of formality.

7 Conclusions

This paper studies the dynamics of labour markets with informal jobs. It explicitly considers the within firm margin of informality with a double purpose. First, changes in this margin can help explain the most relevant stylized facts in labour markets where the presence of informal jobs is widespread. And second, it can be used to analyze the impact of a variety of labour market regulations in a developing economy.

The essence of the model is that the divide between formal and informal jobs in developing countries does not only occur across formal and informal firms. Informal jobs can be found in, otherwise, formal firms. This suggests that firms not only choose how many vacancies to post, but also what the optimal degree of formality is. I show that positive productivity shocks produce a large impact on the job finding

rate of formal jobs, as observed in the data. This comes as a result of two separate effects. First, firms increase vacancy creation and second, firms are more willing to hire formal workers. That is, the degree of formalization in all firms of the economy increases.

This model also highlights the importance of allocation effects of policy interventions in developing countries. Stricter regulation invariably lowers the job finding rate of formal jobs. Policies have different effects on the job separation rate depending on whether they aim to protect jobs (such as firing costs) or they do not (such as payroll taxes and hiring costs). Further, the behaviour of the job finding rate in the two types of jobs transfers workers from formal jobs, where job separation is low, into informal jobs, where job separation is high. Theoretically, the overall effects on unemployment and the size of formality are ambiguous. However, this paper shows that in countries where the opportunity cost of employing informal labour is small, this reallocation effect may be very large, generating a fall in the share of formal employment and an increase in the unemployment rate.

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8 Appendix A: Proofs

8.1 Positive relationship between R and θ in the hiring decision.

Partial derivative of R , equation (29) with respect to θ is given by

$$\frac{\partial R}{\partial \theta} = \frac{1}{[(r + \lambda)(1 - \delta) + \phi]} \left[(r + \lambda + \phi) \frac{\partial D}{\partial \theta} - (r + \lambda) \frac{\partial \delta W}{\partial \theta} \right] \quad (41)$$

The derivative of job separation thresholds (27) and (25) with respect to θ is given by

$$\frac{\partial D}{\partial \theta} = \frac{(r + \lambda)\eta k}{[r + \lambda G(D)](1 - \eta)} > 0 \quad (42)$$

$$\frac{\partial \delta W}{\partial \theta} = \frac{(r + \lambda + \phi)\eta k}{[r + \phi + \lambda G(W)](1 - \eta)} > 0 \quad (43)$$

Since $W > D$ this implies that

$$(r + \lambda + \phi) \frac{\partial D}{\partial \theta} > (r + \lambda) \frac{\partial \delta W}{\partial \theta} \quad (44)$$

and therefore

$$\frac{\partial R}{\partial \theta} = \frac{1}{[(r + \lambda)(1 - \delta) + \phi]} \left[(r + \lambda + \phi) \frac{\partial D}{\partial \theta} - (r + \lambda) \frac{\partial \delta W}{\partial \theta} \right] > 0 \quad (45)$$

8.2 Effects of an increase in $\frac{b}{p}$ on θ , R , W and D

Total derivative of the free entry condition (14) with respect to $\frac{b}{p}$ is given by

$$\frac{d\theta}{d\frac{b}{p}} = \frac{-(1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \frac{b}{p}} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{d\frac{b}{p}} \right]}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \theta} \right]} < 0 \quad (46)$$

where

$$\xi(\theta) = -\frac{\partial q(\theta)}{\partial \theta} \frac{\theta}{q(\theta)} > 0 \quad (47)$$

$$\frac{\partial D}{\partial \frac{b}{p}} = \frac{(r + \lambda)}{r + \lambda G(D)} > 0 \quad (48)$$

$$\frac{\partial \delta W}{\partial \frac{b}{p}} = \frac{(r + \lambda + \phi)}{r + \phi + \lambda G(W)} > 0 \quad (49)$$

The total derivative of the job separation condition (27) of formal jobs with respect to $\frac{b}{p}$ gives

$$\begin{aligned} \frac{dD}{d\frac{b}{p}} &= \frac{(r + \lambda)}{r + \lambda G(D)} \\ &+ \frac{(r + \lambda)\eta k}{[r + \lambda G(D)](1 - \eta)} \left[\frac{-(1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \frac{b}{p}} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \frac{b}{p}} \right]}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \right] \end{aligned} \quad (50)$$

Since $(1 - \eta)\frac{\partial D}{\partial \theta} = \eta k \frac{\partial D}{\partial \frac{b}{p}}$ and $(1 - \eta)\frac{\partial W}{\partial \theta} = \eta k \frac{\partial W}{\partial \frac{b}{p}}$, equation 50 simplifies to

$$\frac{dD}{d\frac{b}{p}} = \frac{(r + \lambda)}{r + \lambda G(D)} \left[\frac{k\xi(\theta)}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{\delta[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \right] > 0 \quad (51)$$

Similarly for the job separation threshold of informal jobs,

$$\frac{d\delta W}{d\frac{b}{p}} = \frac{(r + \lambda + \phi)}{[r + \phi + \lambda G(W)]} \left[\frac{k\xi(\theta)}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{\delta[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial W}{\partial \theta} \right]} \right] > 0 \quad (52)$$

Again since $W > D$

$$(r + \lambda + \phi) \frac{dD}{d\frac{b}{p}} > (r + \lambda) \frac{d\delta W}{d\frac{b}{p}} \quad (53)$$

and therefore

$$\frac{dR}{d\frac{b}{p}} = \frac{1}{[(r + \lambda)(1 - \delta) + \phi]} \left[(r + \lambda + \phi) \frac{dD}{d\frac{b}{p}} - (r + \lambda) \frac{d\delta W}{d\frac{b}{p}} \right] > 0 \quad (54)$$

hence, a increase in $\frac{b}{p}$ decreases θ and increases all three margins R , W and D .

8.3 Effects of changes in F on θ, R, W and D

Total derivative from the free entry condition (14) with respect to F gives

$$\frac{d\theta}{dF} = -\frac{(1-\eta)\theta q(\theta) \left[\frac{[1-G(R)]}{(r+\lambda)} \frac{(r+\lambda)\lambda G(D)}{r+\lambda G(D)} \right]}{k\xi(\theta) + (1-\eta)\theta q(\theta) \left[\frac{[1-G(R)]}{(r+\lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{\partial \delta W}{\partial \theta} \right]} < 0 \quad (55)$$

In this case, the total effect of F on R depends on $\left[\frac{dD}{dF} + (r+\lambda) \right]$ and $\frac{d\delta W}{dF}$

$$\frac{dR}{dF} = \frac{[(r+\lambda+\phi) \left[\frac{dD}{dF} + (r+\lambda) \right] - (r+\lambda) \frac{d\delta W}{dF}]}{[(r+\lambda)(1-\delta) + \phi]} \quad (56)$$

The total effect of F on $\frac{dD}{dF} + (r+\lambda)$ is given by

$$\frac{dD}{dF} + (r+\lambda) = \frac{(r+\lambda)}{r+\lambda G(D)} \left(\lambda G(D) - \frac{\theta q(\theta) \frac{[1-G(R)]}{(r+\lambda)} \eta k \left[\frac{(r+\lambda)\lambda G(D)}{r+\lambda G(D)} \right]}{k\xi(\theta) + (1-\eta)\theta q(\theta) \left[\frac{[1-G(R)]}{(r+\lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \right) \quad (57)$$

Since $\lambda G(D)(1-\eta) \frac{\partial D}{\partial \theta} = \eta k \left[\frac{(r+\lambda)\lambda G(D)}{r+\lambda G(D)} \right]$, this expression simplifies to

$$\frac{dD}{dF} + (r+\lambda) = \frac{(r+\lambda)}{r+\lambda G(D)} \left[\frac{\lambda G(D) k \xi(\theta) + \theta q(\theta) \frac{[1-G(R)]}{(r+\lambda)} \left[\lambda G(D)(1-\eta) \frac{[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{\partial \delta W}{\partial \theta} \right]}{k\xi(\theta) + (1-\eta)\theta q(\theta) \left[\frac{[1-G(R)]}{(r+\lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \right] > 0 \quad (58)$$

Given that $\frac{d\delta W}{dF} = \frac{\partial \delta W}{\partial F} + \frac{\partial \delta W}{\partial \theta} \frac{d\theta}{dF} < 0$, hence

$$\frac{dR}{dF} = \frac{[(r+\lambda+\phi) \left[\frac{dD}{dF} + (r+\lambda) \right] - (r+\lambda) \frac{d\delta W}{dF}]}{[(r+\lambda)(1-\delta) + \phi]} > 0 \quad (59)$$

Therefore an increase in F decreases θ, D and W and increases R .

8.4 Effects of changes in σ on θ, R, W and D

Total derivative from the free entry condition (14) with respect to σ gives

$$\frac{d\theta}{d\sigma} = -\frac{(1-\eta)\theta q(\theta) \left[\frac{[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{(r+\lambda+\phi)\phi}{r+\phi+\lambda G(W)} \right]}{k\xi(\theta) + (1-\eta)\theta q(\theta) \left[\frac{[1-G(R)]}{(r+\lambda)} \frac{\partial D}{\partial \theta} + \frac{\delta[G(R)-G(W)]}{(r+\phi+\lambda)} \frac{\partial W}{\partial \theta} \right]} < 0 \quad (60)$$

The total effect of σ on R depends on $\frac{dD}{d\sigma}$ and $\frac{d\delta W}{d\sigma}$ given by

$$\frac{dR}{d\sigma} = \frac{[(r + \lambda + \phi)\frac{dD}{d\sigma} - (r + \lambda)\frac{d\delta W}{d\sigma}]}{[(r + \lambda)(1 - \delta) + \phi]} \quad (61)$$

The total effect of σ on W is given by

$$\begin{aligned} \frac{d\delta W}{d\sigma} &= \frac{(r + \lambda + \phi)\phi}{r + \phi + \lambda G(W)} - \quad (62) \\ &\frac{(r + \lambda + \phi)\eta k}{[r + \phi + \lambda G(W)](1 - \eta)} \frac{(1 - \eta)\theta q(\theta) \left[\frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{(r + \lambda + \phi)\phi}{r + \phi + \lambda G(W)} \right]}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \end{aligned}$$

$$\text{Since } \phi(1 - \eta)\frac{\partial \delta W}{\partial \theta} = \eta k \left[\frac{(r + \lambda + \phi)\phi}{r + \phi + \lambda G(W)} \right]$$

$$\frac{d\delta W}{d\sigma} = \frac{(r + \lambda + \phi)}{r + \phi + \lambda G(W)} \left[\frac{\phi k \xi(\theta) + \phi(1 - \eta)\theta q(\theta) \frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta}}{k\xi(\theta) + (1 - \eta)\theta q(\theta) \left[\frac{[1 - G(R)]}{(r + \lambda)} \frac{\partial D}{\partial \theta} + \frac{[G(R) - G(W)]}{(r + \phi + \lambda)} \frac{\partial \delta W}{\partial \theta} \right]} \right] > 0 \quad (63)$$

Given that $\frac{dD}{d\sigma} < 0$, hence

$$\frac{dR}{d\sigma} = \frac{[(r + \lambda + \phi)\frac{dD}{d\sigma} - (r + \lambda)\frac{d\delta W}{d\sigma}]}{[(r + \lambda)(1 - \delta) + \phi]} < 0 \quad (64)$$

Therefore an increase in σ decreases θ, D and R and increases W .

8.5 Equilibrium with Informal Self-Employed

Given a distribution $F(\alpha)$ of managerial ability L and M are defined as

$$L = \frac{T}{V(\theta) - \rho} \quad (65)$$

$$M = \frac{U(\theta)}{\rho} \quad (66)$$

The following equations represent the value functions for firms and workers under the assumption of exogenous job separations

$$\begin{aligned}
rV &= -pk + (1 - \eta)q(\theta) \left[\int_R^{z^{\max}} S_f(z)dG(z) + \int_W^R S_i(z)dG(z) \right] \quad (67) \\
rJ_f(z) &= pz - w_f(z) - \lambda_f(J_f + pF - V) \\
rJ_i(z) &= \delta pz - w_i(z) - \lambda_i(J_i - V) \\
rW_f(z) &= w_f(z) - \lambda_f(W_f(z) - U) \\
rW_i(z) &= w_i(z) - \lambda_i(W_i(z) - U) \\
rU &= b + \eta\theta q(\theta) \left[\int_R^{z^{\max}} S_f(z)dG(z) + \int_{z^{\min}}^R S_i(z)dG(z) \right]
\end{aligned}$$

The surplus for the formal and the informal workers is given by

$$\begin{aligned}
S_f(z) &= \frac{pz - rV - rU - \lambda_f pF - (r + \lambda_f) pc}{(r + \lambda_f)} \quad (68) \\
S_i(z) &= \frac{\delta pz - rV - rU}{(r + \lambda_i)}
\end{aligned}$$

Partial derivative with respect to θ gives,

$$\frac{\partial rV}{\partial \theta} < 0, \frac{\partial rU}{\partial \theta} > 0 \quad (69)$$

According to equations (66) and (65), this implies a positive relationship between θ and the two thresholds of managerial ability M and L .

The values for n_f , n_i , and u can be derived from the steady state conditions of the employment equations. The equation determining the evolution of employment in each formal (informal) firm is

$$\begin{aligned}
\dot{n}_f(\alpha) &= (\alpha - n(\alpha))q(\theta) [1 - G(R)] - \lambda_f n_f(\alpha) \quad (70) \\
\dot{n}_i(\alpha) &= (\delta\alpha - n(\alpha))q(\theta)G(R) - \lambda_i n_i(\alpha)
\end{aligned}$$

Adding up across firms

$$\begin{aligned}
\dot{n}_f &= \left(\int_L^{\alpha_0} \alpha dF(\alpha) - n \right) q(\theta) [1 - G(R)] - \lambda_f n_f \quad (71) \\
\dot{n}_i &= \left(\int_L^{\alpha_0} \alpha dF(\alpha) - n \right) q(\theta) G(R) - \lambda_i n_i
\end{aligned}$$

In the steady state $\dot{n}_f = \dot{n}_i = 0$, so the equilibrium values for n_f and n_i are

$$\begin{aligned} n_f &= \frac{(\int_L^{\alpha_0} \alpha dF(\alpha) - n)q(\theta) [1 - G(R)]}{\lambda_f} \\ n_i &= \frac{(\int_L^{\alpha_0} \alpha dF(\alpha) - n)q(\theta)G(R)}{\lambda_i} \end{aligned} \quad (72)$$

Adding up employment for the two types of jobs gives.

$$\begin{aligned} n &= n_f + n_i \\ n &= \frac{[1 - G(R)] \lambda_i + \lambda_f G(R)}{(\lambda_f \lambda_i + [1 - G(R)] \lambda_i q(\theta) + \lambda_f q(\theta) G(R))} q(\theta) \int_L^{\alpha_0} \alpha dF(\alpha) \end{aligned} \quad (73)$$

Similarly, unemployment can be defined as

$$u = \frac{\int_L^{\alpha_0} \alpha dF(\alpha) - n}{\theta} \quad (74)$$

Using (73), (74) and the fact that $G(M) = u + n$, I obtain the second equilibrium condition of the model. This gives a negative relationship between θ and L .

$$G(M) = \frac{\lambda_f \lambda_i + \theta q(\theta) [1 - G(R)] \lambda_i + \lambda_f G(R) \theta q(\theta)}{\theta (\lambda_f \lambda_i + [1 - G(R)] \lambda_i q(\theta) + \lambda_f q(\theta) G(R))} \int_L^{\alpha_0} \alpha dF(\alpha) \quad (75)$$

9 Appendix B: Data, Definitions of Informality and Computation of Transition Probabilities.

Mexico: Data on flows of workers

The data for Mexico are drawn from the National Urban Employment Survey (ENEU) that conducts quarterly household interviews in the 16 major metropolitan areas. The questionnaire is extensive in its coverage of participation in the labour market, wages, hours worked, etc. that are traditionally found in such employment surveys. The ENEU is structured so as to track a fifth of each sample across a five quarter period. I have concatenated panels from the first quarter of 1987 to the fourth quarter of 2004.

The ENEU has suffered only minor modifications during the covered period but it has substantially changed its geographical coverage. From 1988 to 1992 the survey

comprised 16 major urban areas. In 1992 18 more urban areas were introduced and throughout the following years additional cities were included in the sample to reach 44 at the beginning of 1998. The sample is constraint to the original 16 cities although all results are similar with extended the sample.

I broadly follow the ILO definition of informality by dividing employed workers into two types of employment: formal and informal workers. I classify them on the basis of lack of compliance with labour legislation- in particular lack of contributions by the employer to the social security agency, IMSS (or the equivalent for civil servants IMSTS)- as the critical distinguishing characteristic. I also consider informal workers those self-employed and owners of micro firms (less than 6 employees) with no social security contributions, excluding professionals and technicians. Owners of medium or large firms (more than 5 employees) and those professionals and technicians self-employed or with social security contributions are all considered formal.

Mexico: Data on small firms.

The second data source for Mexico is the National Survey of Microenterprises [Encuesta Nacional de Micronegocios (ENAMIN)], which reinterviews a sample of the self-employed individuals covered in previous rounds of the ENEU. These surveys ask detailed questions on the characteristics of firms with up to five employees (15 in manufacturing), including information on capital stock, time in business, and access to credit from formal and informal sources, both for starting the business and at a later time and characteristics on the employees they hire. The employer has to report on the characteristics of each one of his/her employees, as well as, weather any payment towards social security contributions is made for that worker. This way I can obtain what is the percentage of firms that are completely formal, or completely informal or the employ a mixture of both formal and informal contracts.

Brazil

The data for Brazil are draw from Monthly Employment Survey (Pesquisa Mensal de Emprego, hereafter PME) that conducts monthly household interviews in 6 of the major metropolitan regions (covering 25% of the national labour market). The questionnaire is extensive in its coverage of participation in the labour market, wages, hours worked, etc. that are traditionally found in such employment surveys. The PME is structured so as to track each household during four consecutive months and then drop them from the sample for 8 months, after which they are reintroduced for another 4 months. The rotation procedure is such that each month one fourth of the sample is substituted by households to form a new panel. Thus, after 4 months the whole initial sample has been rotated and after 8 months a third different sample is

being surveyed. After 12 months the initial sample is reencountered. Over a period of two years, three different panels of households are surveyed, and the process starts again with three new panels. I have concatenated panels from the January 1983 to December 2002. Regrettably, the PME was drastically modified in 2002 and it is not possible to reconcile the new and old definitions unemployment.

Similar to Mexico the critical factor for my definition of informality is whether the worker is in possession of a work-card or *carteira de trabalho* that entitle the worker to labour rights and benefits. In the case of Brazil the survey does not provide firm size so I consider informal only the self-employed, excluding professionals and technicians . Excluding or including owners from the data does not alter the results in any significant way.

Computation of Transition Probabilities.

I assume a homogenous Markov process $X(t)$ defined over a discrete state-space $E = \{1, \dots, K\}$ where K is the number of possible states (types of jobs) a worker could be found in. I define 4 employment statuses. Inactivity, unemployment, informal employment and formal employment. Since the data are tabulated at discrete points in time we can compute the probability $p_{ij}(t, t+n) = \Pr(X(t+n) = j, X(t) = i)$. The interpretation of p_{ij} is simply, the probability of moving from state i to state j in one step (n). Discrete time probabilities are straight forward to compute as the maximum likelihood estimator for p_{ij} is $p_{ij} = \frac{n_{ij}}{n_i}$ being n_{ij} the total number of transitions from state i to state j and n_i the total number of observations initially in state i . I then smooth the series using a moving average filter with a three quarter window.

Figures

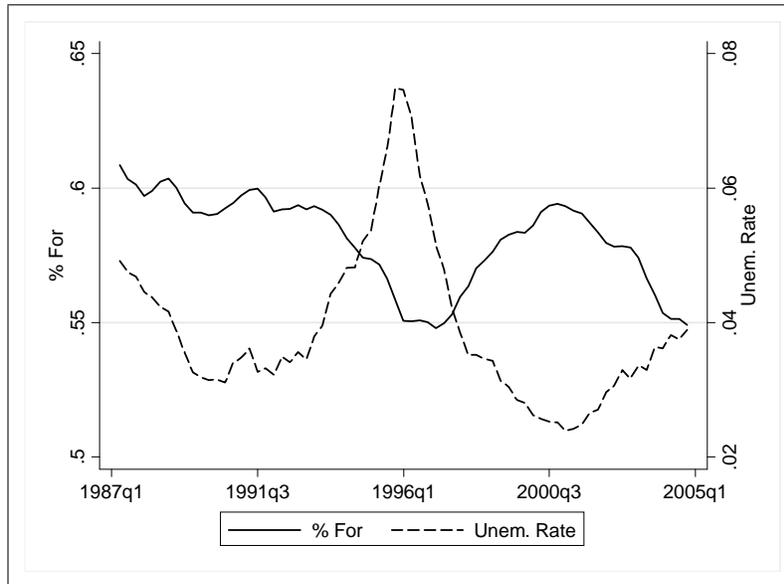


Figure 1: Unemployment Rate and Share of Formal Jobs. Mexico 1987-2004. Constructed with quarterly data from the National Urban Labor Survey (ENEU). % For is the share of formal employment constructed as number of formal workers over total employment. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.

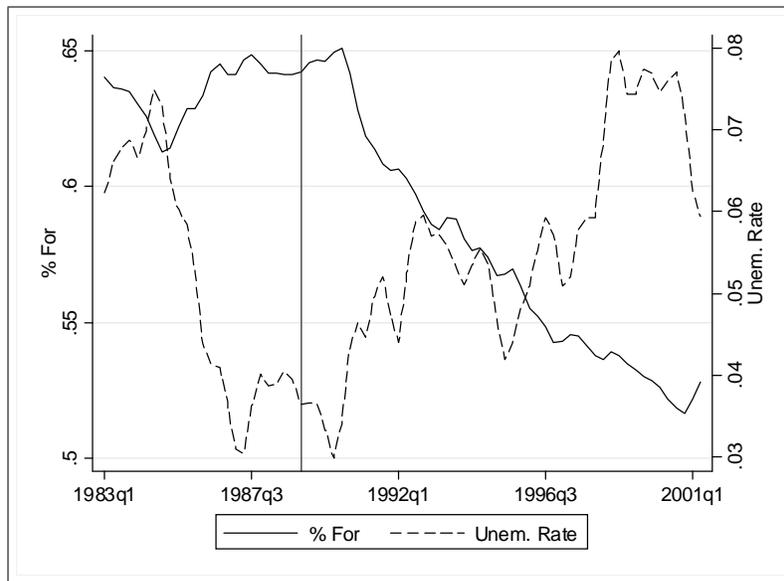


Figure 2: Unemployment Rate and Share of Formal Jobs. Brazil: 1983-2001. Constructed with monthly data, quarterly averaged, from the Monthly Employment Survey (PME). % For is the share of formal employment constructed as number of formal workers over total employment. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.

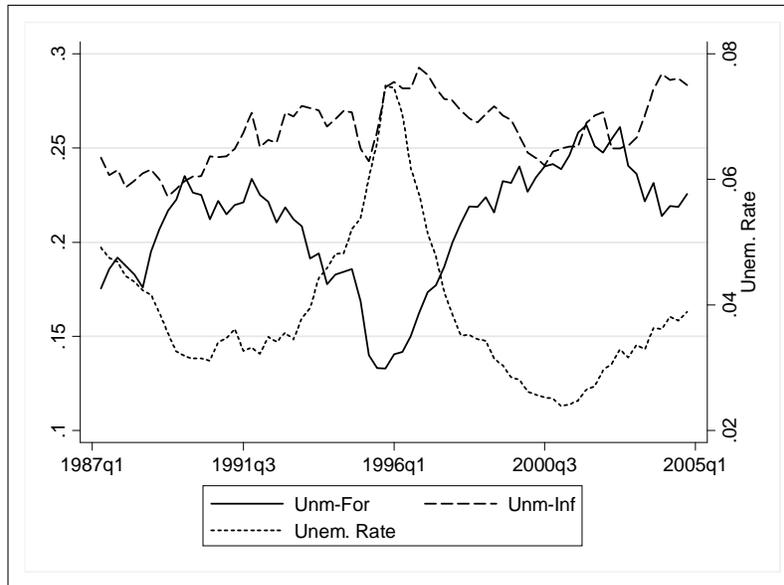


Figure 3: Job Finding Rate and Unemployment rate. Mexico: 1987-2004. Constructed with quarterly data from the National Urban Labor Survey (ENEU). Unm-For and Unm-Inf correspond to the average probability of transiting from unemployment into formal and informal employment respectively. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.



Figure 4: Job Finding Probabilities and Unemployment rate. Brazil: 1983-2001. Constructed with monthly data, quarterly averaged, from the Monthly Employment Survey (PME). Unm-For and Unm-Inf correspond to the average probability of transiting from unemployment into formal and informal employment respectively. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.

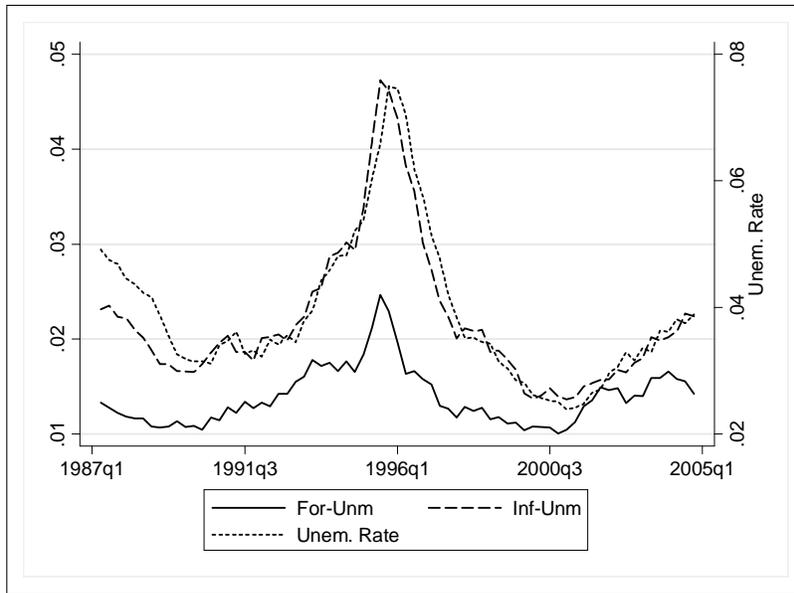


Figure 5: Job separations and Unemployment Rate.Mexico: 1987-2004. Constructed with quarterly data from the National Urban Labor Survey (ENEU). For-Unm and Inf-Unm correspond to the average probability of transiting from formal and informal employment into unemployment. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.

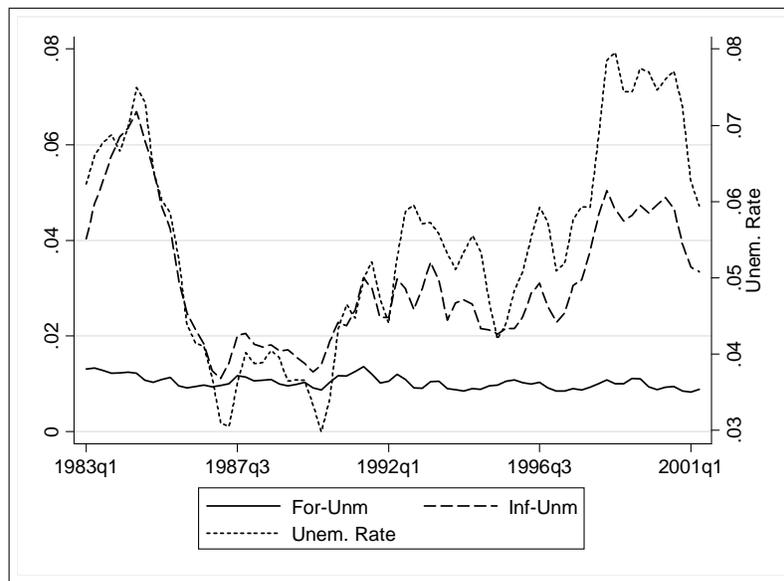


Figure 6: Job separations and Unemployment Rate. Brazil: 1983-2001. Constructed with monthly data, quarterly averaged, from the Monthly Employment Survey (PME). For-Unm and Inf-Unm correspond to the average probability of transiting from formal and informal employment into unemployment. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. The series have been smoothed using a moving average filter with a three quarter window.

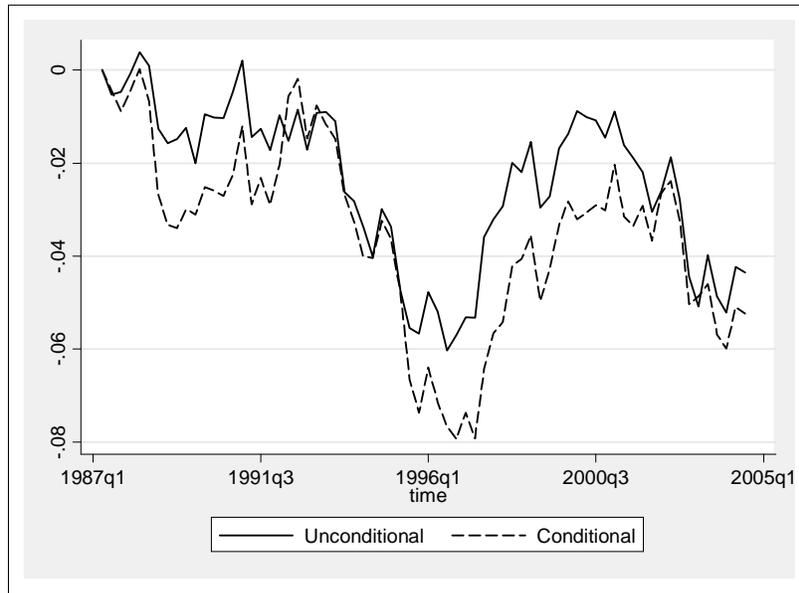


Figure 7: Compositional Contributions of Changes in the Share of Formal Employment. Mexico 1987-2004. The unconditional series corresponds to the value of the time dummies of a probit regression of the probability of being formal on time dummies. In the case of the conditional series gender, age, education, industry and occupational dummies are included in the regression.

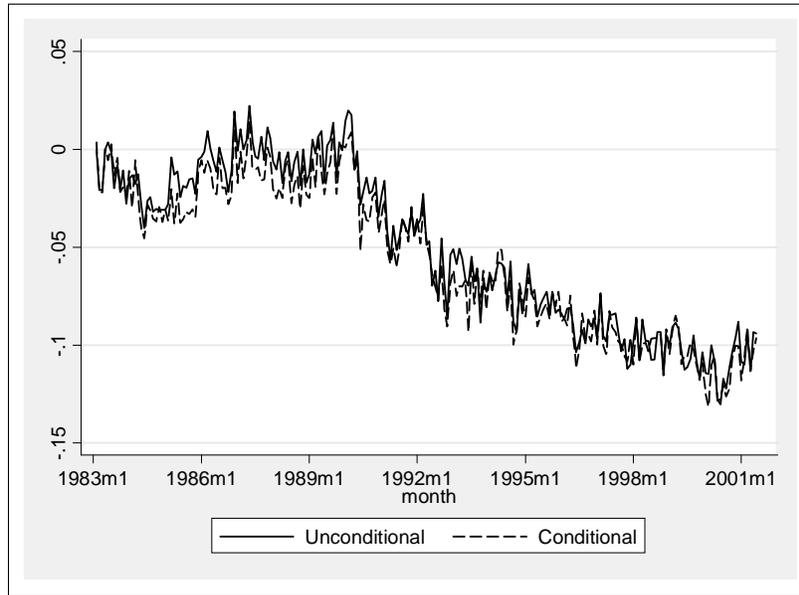


Figure 8: Compositional Contributions of Changes in the Share of Formal Employment. Brazil 1983-2001. The unconditional series corresponds to the value of the time dummies of a probit regression of the probability of being formal on time dummies. In the case of the conditional series gender, age, education, industry and occupational dummies are included in the regression.

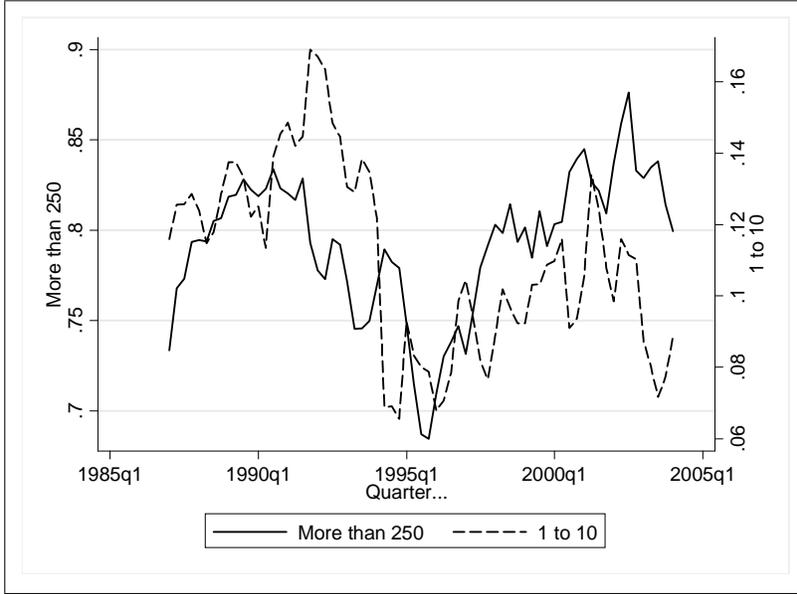


Figure 9: Relative Formal Hiring from Unemployment. Large vs Small Firms. Mexico, 1987-2004. Constructed with quarterly data from the National Urban Labor Survey (ENEU). The series show the probability of obtaining a formal contract conditional on leaving unemployment by firm size. The series have been smoothed using a moving average filter with a three quarter window.

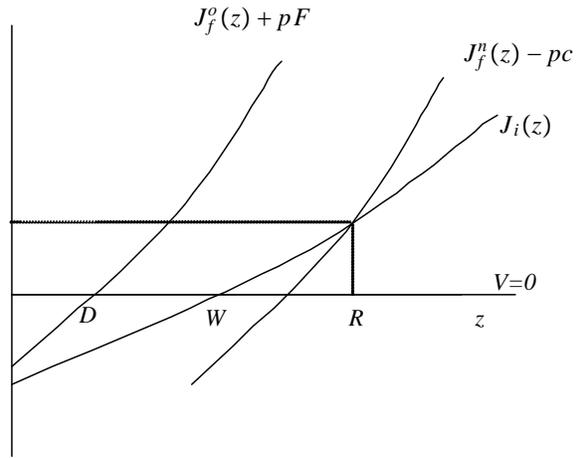


Figure 10: Equilibrium Thresholds.

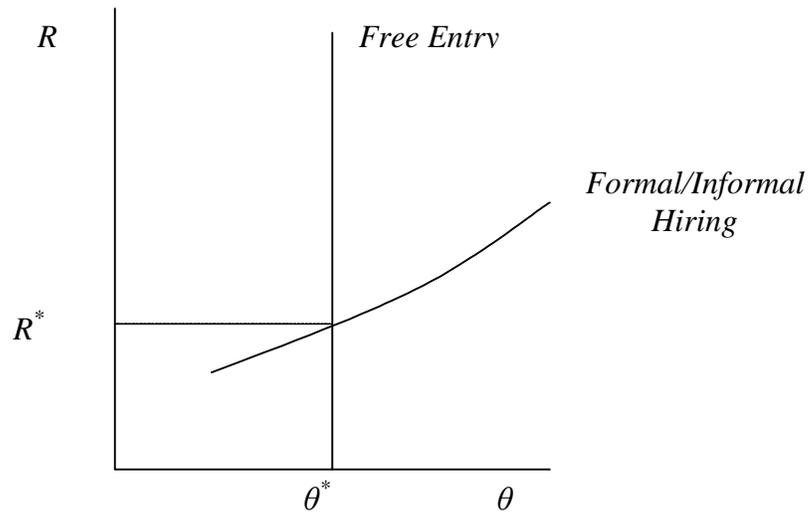


Figure 11: Equilibrium: Free Entry and Hiring

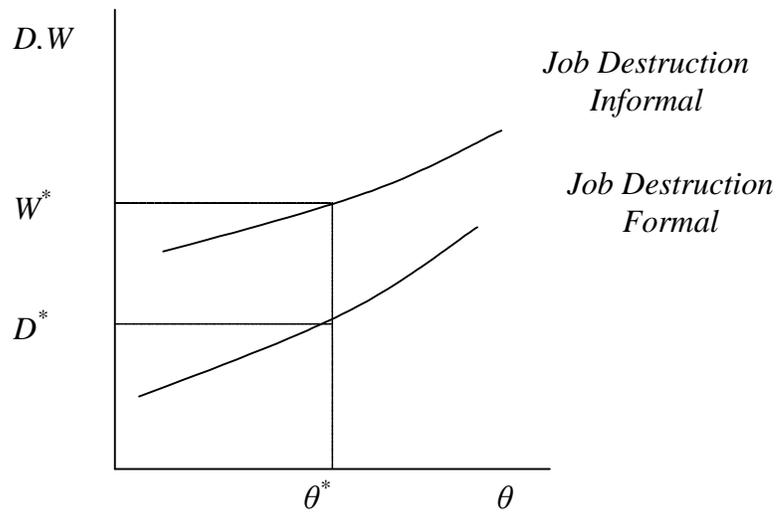


Figure 12: Equilibrium: Job Destruction

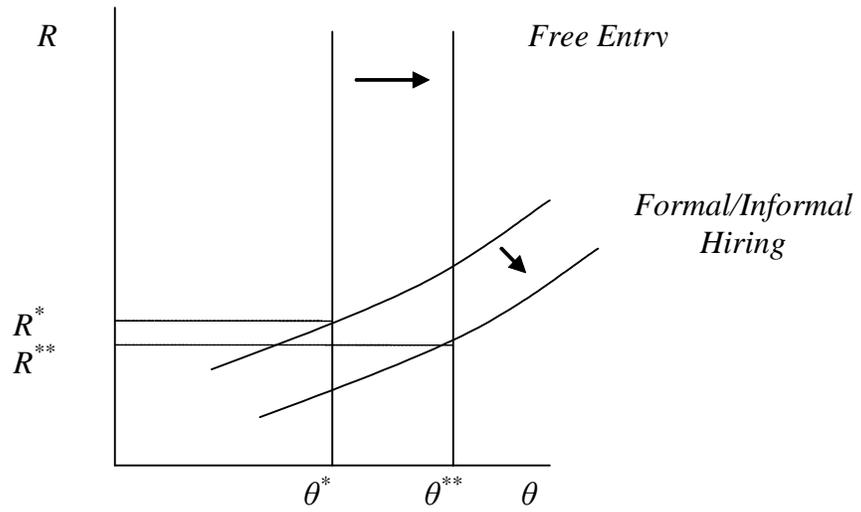


Figure 13: Increase in productivity

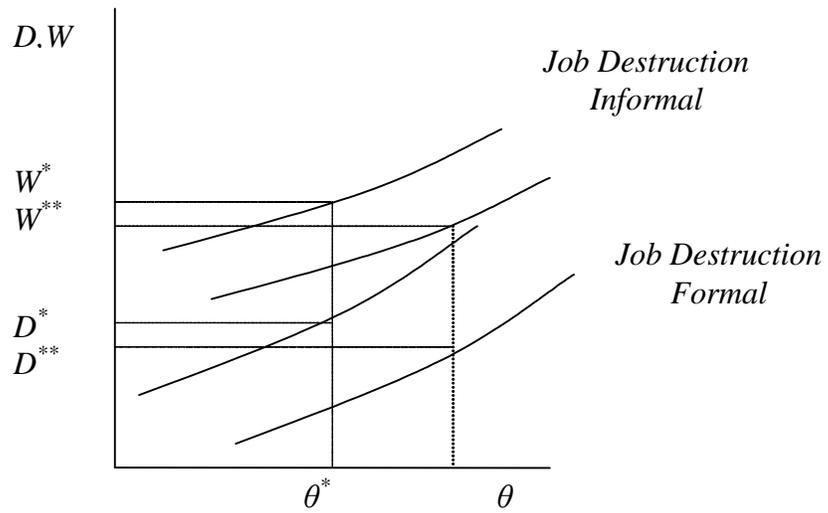


Figure 14: Increase in Productivity

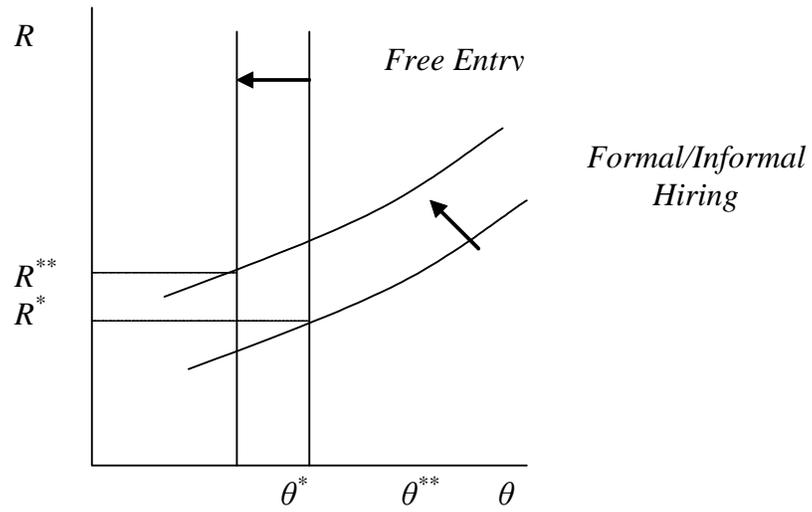


Figure 15: Increase in Firing Costs

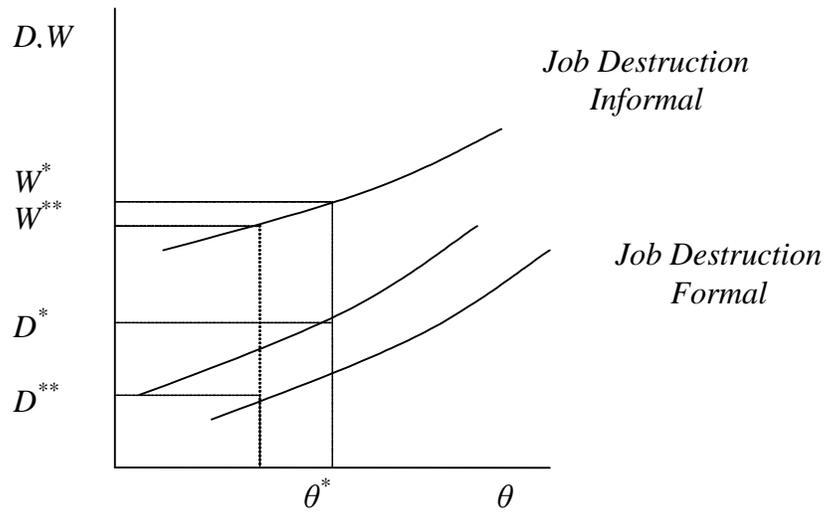


Figure 16: Increase in Firing Costs

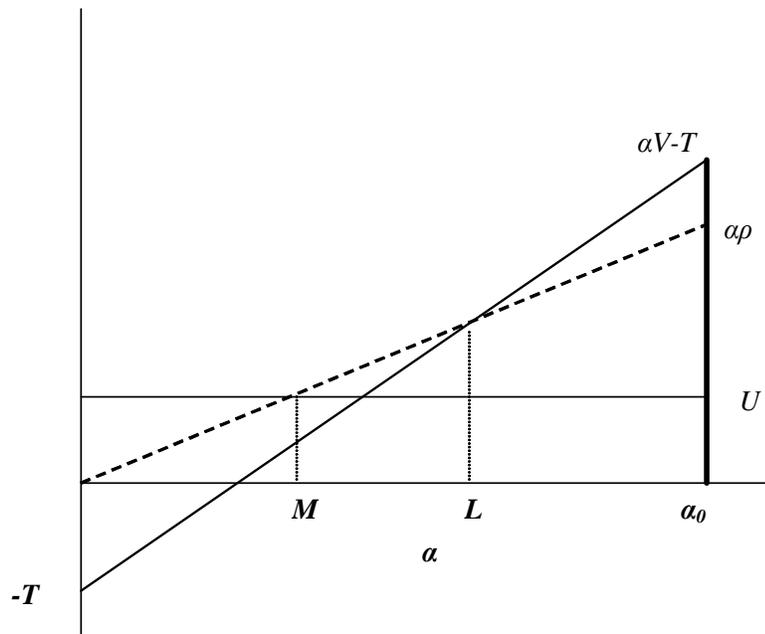


Figure 17: Equilibrium with Informal Self-Employed.

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