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Creative Destruction with On-the-Job Search

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Abstract
This paper is about the labour market consequences of creative destruction with on-the-job search. We consider a matching model in an economy with embodied technological progress and show that its dynamics are profoundly affected by allowing on-the-job search. We obtain that the elasticity of unemployment with respect to growth shrinks from 1.63 to 0.13. Moreover, the underlying transmission channels change as the flow of obsolete jobs practically disappears and is replaced by a flow of job-to-job transitions. These effects persist even if employed job seekers are significantly less efficient in the search process than the unemployed. Thus, we show that, rather than contributing to unemployment, creative destruction induces a direct reallocation of workers from low to high productivity jobs. These results could be strengthened by assuming that search efforts are unobservable by firms which induces more on-the-job search. However, the action of worker is no longer surplus maximizing and, hence, the worker's welfare is increasing in the cost of search which acts as a commitment device. Finally, we show that the model could be extended by allowing for variable search intensity.

Keywords: commitment device, creative destruction, job flows, obsolescence, on-the-job search, search equilibrium, unemployment.
JEL Classifications: E24, J41, J63, J64, O39

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Introduction

The relation between technological progress and employment has long been a popular concern. On the one hand, it is commonly believed that slow growth is one of the main causes of high unemployment, as the recent history of European economies might suggest; on the other hand, there are fears that new technologies might destroy existing jobs. These contradicting views are reflected in the economic literature which finds opposite effects of growth on unemployment depending on the nature of technological progress.

If innovations increase the productivity of existing jobs, i.e. progress is disembodied, then a faster rate of growth leads to a lower equilibrium rate of unemployment. Indeed, higher growth increases the net present value of newly created jobs without affecting job creation costs leading to more employment. This is the *capitalization effect*, Pissarides (2000, chapter 3.3). If, on the contrary, innovations only increase the productivity of newly created jobs, i.e. progress is embodied, then growth and unemployment are positively related as, after a while, workers choose to quit their obsolete job and return to unemployment in order to find a more lucrative position. This is the *creative destruction effect*, Aghion and Howitt (1994), on which we shall specifically focus in this paper.

In an economy with creative destruction, newly formed matches benefit from the best technology available and, as a consequence, the highest wages of the economy accrue to newly employed workers. As time passes, and as outside opportunities improve, the attractiveness of a job declines. We would therefore expect workers to engage into on-the-job search before their position becomes obsolete. However, to the best of my knowledge, this possibility has not seriously been considered yet. This is what I propose to do in this paper.

We extend the framework of Mortensen and Pissarides (1998) that has adapted the standard matching model of the labour market to allow for growth through creative destruction. Hence, as in their paper, the productivity of a firm is assumed to be determined by its date of creation and technological progress characterised by the ever-increasing productivity of newly established firms. Jobs eventually become obsolete when the wage that an employer needs to offer in order to retain its workers reaches its productivity. On-the-job search reduces the expected value of a match to the firm as its activity is destroyed when its employee resigns. As surplus sharing is assumed, this decreases the wage paid to the worker who therefore partially bears the cost of job destruction following a quit. Hence, workers only start looking for other jobs once outside opportunities have sufficiently improved. This reasoning implicitly relies on the assumption that a firm can observe its employee's search effort, an assumption that is subsequently relaxed. It is important to emphasize that on-the-job search is allowed rather than imposed and, as a consequence, its occurrence shows that creative destruction provide a justification for the phenomenon. It is therefore natural and legitimate to consider on-the-job search in a model of creative destruction.

As the model is not analytically solvable, we perform a calibration based on reasonable parameter values. We obtain that the positive impact of growth on unemployment is considerably reduced, although not reversed, by allowing on-the-job search. The simulated elasticity shrinks form 1.63 to 0.13 when the employed job seekers are as efficient at searching for jobs as the unemployed. What is even more surprising is that the main transmission channel at work in the traditional creative destruction model practically disappears when workers are allowed to seek jobs while employed. Indeed, the flow of obsolete jobs, which represents more than half of job destructions without on-the-job search, becomes negligible with. In fact, it is replaced by a flow of job-to-job transitions. The intuition for this result is that unemployment ceases to be a necessary step before moving to a better paid position. Moreover, on-the-job search leads to an increase in the maximum life span of a match as workers have no incentive to quit their employer to seek for a better one as long as their income, net of search costs, is above the level of unemployment benefits. These consequences of on-the-job search considerably reduce the likelihood that a match survives until obsolescence.

It is interesting to note that even if employed job seekers are significantly less efficient than the unemployed at searching for jobs, the impact of growth on unemployment is still considerably reduced by on-the-job search. Our calibration shows that, even if the employed job seekers are only half as efficient as the unemployed, the elasticity of unemployment with respect to growth is equal to 0.17, which is only a marginal increase from 0.13. This is explained by the fact that less efficient job seekers start searching earlier which helps to maintain the obsolescence flow to a very low level.

The model is slightly modified if firms cannot observe their employees' search effort. Workers cease to have an incentive to postpone their entry into the labour market in order to qualify for the high wage of non-searching employees. Instead, they treat their wage schedule as given and start prospecting for jobs earlier, as soon as the benefits from switching to a new job outweigh the search cost. In this version of the model, on-the-job search is even more widespread, which strengthens our previous results. Indeed, the resulting elasticity of unemployment with respect to growth is less than 0.1, even when employed job seekers are only half as efficient as the unemployed in the search process.

With unobservable search effort workers start searching too soon to maximize the match surplus and, thus, their welfare. The cost of search is the only thing that prevents them from beginning to apply for jobs earlier. Hence, paradoxically, the value of employment to a worker is increasing in the cost of search which acts as a commitment device.

It might seem reasonable to assume that the search effort of workers is not observed by firms. We nevertheless initially assume the opposite as this is the possibility that involves the smallest amount of on-the-job search and, therefore, the least likely to support our conclusions. Also, we might reasonably expect firms to know about the search activity of their employees with some degree of confidence. For example, an employer could find the CV of his worker in an employment agency or on the Internet. Also, a firm can guess that a worker who unexpectedly takes a day off in the middle of a week is likely to be going for an interview. The two versions of the models should therefore be seen as two benchmarks where reality is lying somewhere in between.

Creative destruction models of the labour market have often been criticized on the basis of the lack of empirical evidence of a positive impact of growth on unemployment. A first answer to those critiques was provided by Postel-Vinay (2002) who argued that the short-term dynamics of an economy with creative destruction are markedly different from those of the steady state. He showed that, following a sudden increase in the rate of growth, unemployment initially responds by a substantial decline. Thus, the positive impact of growth on unemployment is only a long-run phenomenon and it should be tested on that basis. By considering the possibility of on-the-job search, we provide another defence of the creative destruction hypothesis. Indeed, the prediction that, even in the long run, there is almost no correlation between growth and unemployment is certainly easier to reconcile with the data than the strong positive correlation that typically arises without on-the-job search.

Hence, our findings should qualify the results of Pissarides and Vallanti (2006) who estimate that nearly all technological progress is of the disembodied form. They argue that even a moderate amount of embodied progress is not compatible with the negative elasticity of unemployment with respect to growth which they find in their data. Our results might also cast doubts on the work of Hornstein, Krusell and Violante (2007) who propose an explanation for the rise in European Unemployment since the 1970's based on an acceleration of embodied technological progress. Our findings suggest that accounting for on-the-job search could significantly reduce their simulated rise in European unemployment.

This paper comprises six sections and a conclusion. The theoretical model is derived in the first section. Then, as it is not analytically solvable, a calibration is undertaken; before a sensitivity analysis is performed. The model is modified in the forth section by assuming that search efforts are not observable. In the fifth section, we derive the key characteristics of a more general model that allows for variable search intensity. Finally, in the last section, we briefly consider the consequences of having the level of unemployment benefits determined by a replacement ratio.

I/ The theoretical model

Following Mortensen and Pissarides (1998), we shall assume that, in order to produce, a firm needs to employ one worker. As time is continuous, recruiting expenses, due to search frictions, are modelled as a flow cost that has to be paid while the job is vacant. Free entry implies that, in equilibrium, the value of a newly opened vacancy is nil. When a firm fills its vacant position it adopts the most recent technology available. This choice is assumed irreversible and, hence, the same technology will have to be used throughout the existence of the match. Technological progress is therefore characterized by the ever-increasing productivity of newly established firms.

Irreversibility of investment implies that wages paid by firms erode over time in comparison with outside opportunities available on the labour market. This eventually leads to job obsolescence as the wage that an employer would need to pay in order to retain its worker exceeds its productivity. On-the-job search is allowed, but its occurrence reduces the value of a match since a firm ceases its activity when its employee quits for another job. As wages are assumed to be determined by surplus sharing, this cost of on-the-job search is partly internalized by workers who therefore only choose to start seeking for other positions once outside opportunities have sufficiently improved. This leads us to consider that, after having worked for the same firm for T' units of time, workers start searching on the job and that, after T'', they resign and return to unemployment. Thus, denoting by τ the date of job creation, there are two periods of interest:

- $t \in [\tau, \tau + T')$, when the worker is not searching;
- $t \in [\tau + T', \tau + T'')$, when the worker is searching on the job.

The unemployed are typically more efficient at searching for jobs than the employed. We therefore denote by s the efficiency of employed job seekers, relative to the unemployed, in the search process; and we could reasonably consider that $s \in [0,1]$. Hence, market tightness is given by the following ratio:

$$\theta = \frac{v}{u + s \cdot e}, \tag{1}$$

where v denotes the number of vacancies, u the number of unemployed and e the number of employed job seekers. For simplicity, the working population is normalised to one. The number of matches per unit of time is given by the following matching function:

$$m(u + s \cdot e, v), \tag{2}$$

which is increasing and concave in both of its arguments, it is equal to zero if any of its argument is nil and it satisfies the standard Inada conditions. We further assume that it is homogenous of degree one, implying that the rate at which vacant positions become filled is:

$$\frac{m(u + se, v)}{v} \begin{aligned} &= m(1/\theta, 1) \\ &= q(\theta) \end{aligned}. \tag{3}$$

Note that the function q is decreasing. The rate at which unemployed meet employers is:

$$\frac{m(u + se, v)}{u + se} \begin{aligned} &= m(1, \theta) \\ &= \theta q(\theta) \end{aligned}. \tag{4}$$

Clearly, the corresponding rate faced by employed job seekers is $s\theta q(\theta)$.

We assume that the wage rate is determined by surplus splitting at each instant[1]. Let $W(\tau,t)$ and $J(\tau,t)$ denote the asset value (i.e. the present value of expected income) at t of a job match created at τ to a worker and to a firm, respectively; similarly $U(t)$ and $V(t)$ stand for the asset value at t of unemployment and of a vacancy, respectively. As we shall soon see, $W(\tau,t)$ and $J(\tau,t)$ are both functions of the wage rate, $w(\tau,t)$, which is itself indexed by the date of job creation, τ, and current time, t. Surplus splitting implies that this wage rate is determined by:

$$W(\tau,t) - U(t) = \beta\left[W(\tau,t) + J(\tau,t) - U(t) - V(t)\right], \tag{5}$$

[1] Shimer (2006) proposes a derivation of wages from strategic bargaining in the context of on-the-job search. Although it is more satisfactory to rely on such micro-foundations, the derivation is fairly complex. It would therefore make the model much heavier, but would be unlikely to yield any additional insights to the relation between growth and unemployment.

where β is the worker's share. We shall consider the two periods of interest separately, denoting by $w^{ns}(\tau,t)$ the wage prevailing without on-the-job search at time t for a job created at τ, and by $w^{s}(\tau,t)$ the corresponding wage with on-the-job search. Thus:

$$w(\tau,t)=\begin{cases} w^{ns}(\tau,t) & if \quad t\in[\tau,\tau+T') \\ w^{s}(\tau,t) & if \quad t\in[\tau+T',\tau+T'') \end{cases}. \tag{6}$$

Productivity at time t on the technological frontier is denoted by $p(t)$. Assuming a constant rate of technological progress, g, we have:

$$\begin{aligned} p(t) &= e^{gt} \\ &= p(\tau)e^{g(t-\tau)} \end{aligned}, \tag{7}$$

where we normalize $p(0)=1$. As we shall see, the productivity of a firm created a τ is determined by the best technology available at its creation; it is therefore equal to $p(\tau)$.

A/ Wage rate before on-the-job search, $t\in[\tau,\tau+T')$

The asset value at t of a job created at τ to a worker satisfies the following Bellman equation:

$$rW(\tau,t)=w^{ns}(\tau,t)+\delta[U(t)-W(\tau,t)]+\dot{W}(\tau,t), \tag{8}$$

where r is the discount rate and δ the rate of the Poisson process that determines the occurrence of exogenous shocks that lead to job destruction. This equation states that the interest perceived from employment over a unit of time, $rW(\tau,t)$, are composed of the salary, $w^{ns}(\tau,t)$, the (negative) expected gains associated to a change of status from employed to unemployed, $\delta(U(t)-W(\tau,t))$, and the capital gains, $\dot{W}(\tau,t)$. This capital gain term is part of the Bellman equation as, even in steady state, the asset value of employment changes over time. Indeed, as a firm gets older, obsolescence gets closer and the value of employment evolves toward that of unemployment. Similarly, the asset value at t of a match made at τ to a firm satisfies:

$$rJ(\tau,t)=p(\tau)-w^{ns}(\tau,t)+\delta[V(t)-J(\tau,t)]+\dot{J}(\tau,t), \tag{9}$$

where, from the assumption of irreversible investment, the productivity of a firm, $p(\tau)$, is determined by the technology available at its creation.

The asset value of unemployment solves:

$$rU(t)=p(t)b+\theta q(\theta)[W(t,t)-U(t)]+\dot{U}(t), \tag{10}$$

where $p(t)b$ denotes the opportunity cost of employment, which could be thought of as unemployment benefits, and $W(t,t)-U(t)$ is the capital gain obtained if a job is found which occurs at the Poisson rate given by (4). The worker's opportunity cost of employment, $p(t)b$, is increasing with time as, otherwise, we would not have a steady state with a constant rate of unemployment which would be counterfactual. Also, unemployment benefits could reasonably be assumed to be equal to a fixed proportion of the average wage in the economy, justifying the indexation on the current level of productivity.

Finally, the asset value of a vacant position satisfies:

$$rV(t) = -p(t)c + q(\theta)\left[J(t,t) - V(t)\right] + \dot{V}(t), \tag{11}$$

where $p(t)c$ is the flow cost of advertising the vacancy and $J(t,t)-V(t)$ is the capital gain obtained when the vacancy is filled which occurs at the Poisson rate given by (3). Again, stationarity requires that the flow cost of advertisement is indexed on productivity which is a reasonable assumption to make. Imposing free entry, we must have $V(t)=0$ at all time; implying:

$$J(t,t) = \frac{p(t)c}{q(\theta)}. \tag{12}$$

This equation states that the value of a new match to the firm must compensate the expected cost of advertisement that needs to be incurred in order to fill the position.

The surplus sharing rule assumed for wage determination, (5), could be rewritten as:

$$W(\tau,t) - U(t) = \frac{\beta}{1-\beta} J(\tau,t). \tag{13}$$

Combining this sharing rule at time $\tau = t$ with the asset value of a newly matched firm, (12), we obtain:

$$W(t,t) - U(t) = p(t)\frac{\beta}{1-\beta}\frac{c}{q(\theta)}. \tag{14}$$

This could be substituted into the equation for the asset value of unemployment, (10), to give:

$$rU(t) = p(t)\left[b + \frac{\beta}{1-\beta}c\theta\right] + \dot{U}(t). \tag{15}$$

The first term of the right hand side of the equation corresponds to the worker's reservation wage. It is larger than the level of unemployment benefits since an unemployed worker can expect to obtain a lucrative job. Thus, the second term of the reservation wage is just the value of a new position to the worker, given by (14), multiplied by the probability of obtaining such position in any unit of time, $\theta q(\theta)$. A worker not engaged into on-the-job search refuses to work for a salary smaller than his reservation wage.

The wage rate is obtained by substituting the asset equations (8), (9) and then (15) into the sharing rule (13) and by noting that the sharing rule also applies to the capital gains. This gives:

$$w^{ns}(\tau,t) = \beta p(\tau) + (1-\beta)\, p(t)\left[b + \frac{\beta}{1-\beta} c\theta \right]. \qquad (16)$$

The wage is a weighted average of the firm's productivity and of the worker's reservation wage. It typically increases at a rate that is slower than the rate of technological progress as the employer imperfectly compensates its employee for the improvement in outside labour market opportunities.

B/ Wage rate with on-the-job search, $t \in [\tau + T', \tau + T'')$

The value at t of a job created at τ to a worker who is seeking for outside opportunities is given by:

$$rW(\tau,t) = w^{s}(\tau,t) - p(t)s\sigma + \delta[U(t) - W(\tau,t)] + s\theta q(\theta)[W(t,t) - W(\tau,t)] + \dot{W}(\tau,t), \qquad (17)$$

where $p(t)s\sigma$ denotes the opportunity cost of on-the-job search to the worker which, for stationarity, is indexed on productivity. It is also reasonable to assume that search is more costly when employed job seekers are more efficient which justifies the cost being proportional to s. In comparison with the corresponding equation without on-the-job search, (8), two new terms are added. One is the cost of on-the-job search, $p(t)s\sigma$, which could be assumed to be small relative to other variables; and the other is the capital gain obtained when moving to another job, $W(t,t) - W(\tau,t)$, multiplied by the Poisson rate at which such new jobs are found by employed job seekers, $s\theta q(\theta)$. Similarly, for a firm, the asset value at t of a match made at τ satisfies:

$$rJ(\tau,t) = p(\tau) - w^{s}(\tau,t) - \delta J(\tau,t) - s\theta q(\theta) J(\tau,t) + \dot{J}(\tau,t), \qquad (18)$$

where free entry is assumed. The value of unemployment is still given by (14) and the sharing rule, (13), still holds.

Substituting (17) and (18) and then (14) and (15) into the sharing rule (13) and noting that this rule also applies to capital gains, the wage rate that prevails with on-the-job search is:

$$w^s(\tau,t) = \beta p(\tau) + (1-\beta)p(t)\left[b + s\sigma + (1-s)\frac{\beta}{1-\beta}c\theta\right]. \quad (19)$$

If employed job seekers are as efficient as the unemployed at searching for jobs, $s=1$, then the wage is independent of outside labour market conditions, i.e. it is independent of θ. This is explained by the fact that returning to unemployment yields unemployment benefits, but, unlike in the case without on-the-job search, it does not open the possibility of finding a more lucrative job, which has a value increasing in θ, as this possibility already exists while employed. Thus, when $s=1$, the worker's reservation wage is equal to $p(t)(b+\sigma)$ where σ is the cost of searching that does not need to be paid while unemployed which explains why it is part of the gain associated to returning to unemployment. When employed job seekers are not as efficient as the unemployed at searching for jobs, $s<1$, then the worker's reservation wage is a weighted average of the two extreme cases where $s=1$, i.e. the two types of job seekers are perfect substitutes in the search process, and where $s=0$, i.e. on-the-job search is not possible. Finally, by comparing (16) to (19) it is apparent that searching while working reduces the wage paid to the employee provided that:

$$\sigma < \frac{\beta}{1-\beta}c\theta. \quad (20)$$

As we shall see in the resolution of the model this turns out to be a necessary and sufficient condition for on-the-job search to take place before obsolescence.

C/ Solving for the equilibrium

We now need to work with value functions in order to determine when the workers chooses to start searching, $\tau+T'$, and to resign, $\tau+T''$, such that the value of his job is maximised. The value of a match of vintage τ to a worker shortly after its creation, $t<\tau+T'$, is given by:

$$\begin{aligned} W(\tau,t) = & \int_t^{\tau+T'} e^{-(r+\delta)(u-t)}\left[w^{ns}(\tau,u) + \delta U(u)\right]du \\ & + \int_{\tau+T'}^{\tau+T''} e^{-(r+\delta)(u-t)-s\theta q(\theta)(u-(\tau+T'))}\left[w^s(\tau,u) - p(u)s\sigma + \delta U(u) + s\theta q(\theta)W(u,u)\right]du \\ & + e^{-(r+\delta)(\tau+T''-t)-s\theta q(\theta)(T''-T')}U(\tau+T'') \end{aligned} \quad (21)$$

By differentiating this equation with respect to T', it is straightforward to check that the first order condition for the optimal time to start searching on the job, $\tau+T'$, is given by:

$$w^{ns}(\tau,\tau+T') = w^s(\tau,\tau+T') - p(\tau+T')s\sigma + s\theta q(\theta)\left[W(\tau+T',\tau+T') - W(\tau,\tau+T')\right]. \quad (22)$$

This is rather intuitive as it indicates that search begins as soon as the instantaneous benefit from searching, the right hand side, equals the corresponding cost, the forgone wage of the left hand side. The benefit from searching are composed of the corresponding wage net of the search cost and of the expected capital gain from finding a new job.

Since the surplus is shared in fixed proportions between the employer and the worker, the problem could also be analysed from the firm's perspective. The value of a match of vintage τ to a firm shortly after creation, $t < \tau + T'$, is given by:

$$J(\tau,t) = \int_{t}^{\tau+T'} e^{-(r+\delta)(u-t)} \left[p(\tau) - w^{ns}(\tau,u) \right] du + \int_{\tau+T'}^{\tau+T''} e^{-(r+\delta)(u-t)-s\theta q(\theta)(u-(\tau+T'))} \left[p(\tau) - w^{s}(\tau,u) \right] du \,. \quad (23)$$

The value equation of the worker being much more complicated than that of the firm, it is easier, for computational purposes, to work with the latter. Now, the first order condition for the optimal time to start searching on the job, $\tau + T'$, is given by:

$$w^{ns}(\tau,\tau+T') = w^{s}(\tau,\tau+T') + s\theta q(\theta) J(\tau,\tau+T') \,. \quad (24)$$

This equation states that on-the-job search begins when, from the firm's perspective, the cost of having an employee who is not searching, i.e. the left hand side, equals the cost of employing a job seeker, i.e. the right hand side, where this latter cost comprises the instantaneous probability of loosing the positive asset value of the job.

Note that the two first order conditions, (22) and (24), are equivalent. We can therefore deduce that T' satisfies:

$$W(\tau,\tau+T') + J(\tau,\tau+T') = W(\tau+T',\tau+T') - \frac{p(\tau+T')\sigma}{\theta q(\theta)} \,. \quad (25)$$

The worker decides to start searching as soon the value of a new job net of expected search costs, the right hand side, equals the value of the match, given on the left hand side. This cannot be efficient in general as the worker fails to take into account the benefits that the newly matched firm derives from his search activity. Thus, efficiency requires:

$$W(\tau,\tau+T') + J(\tau,\tau+T') = W(\tau+T',\tau+T') + J(\tau+T',\tau+T') - \frac{p(\tau+T')\sigma}{\theta q(\theta)} \,. \quad (26)$$

In fact, it could be checked that the amount of on-the-job search is efficient if and only if $\beta = 1$. In order to see this, the previous two equations could be rearranged by subtracting $U(t)$ from both sides, by using the surplus sharing rule (13) and, finally, by multiplying both sides by $(1-\beta)$. Applying these transformations, (25) could be rewritten as:

$$J(\tau,\tau+T') = \beta J(\tau+T',\tau+T') - (1-\beta)\frac{p(\tau+T')\sigma}{\theta q(\theta)} \,. \quad (27)$$

Similarly, (26) is equivalent to:

$$J(\tau,\tau+T') = J(\tau+T',\tau+T') - (1-\beta)\frac{p(\tau+T')\sigma}{\theta q(\theta)}. \quad (28)$$

When β is equal to 1, the firm gets no surplus from the match and therefore $J(\tau+T',\tau+T') = V(\tau+T') = 0$. Hence, the failure of the worker to take into account the benefit that the newly matched firm derives from his search activity does not matter[2]. Note that this efficiency condition is not specific to the context of creative destruction and was originally derived in Pissarides (1994).

For the purpose of solving for the equilibrium of the model, the first order condition (24) could be simplified to:

$$e^{gT'}\left[\frac{\beta}{1-\beta}c\theta-\sigma\right] = \theta q(\theta)\int_{\tau+T'}^{\tau+T''} e^{-(r+\delta+s\theta q(\theta))(u-(\tau+T'))}\left[1-e^{g(u-\tau)}\left(b+s\sigma+(1-s)\frac{\beta}{1-\beta}c\theta\right)\right]du, \quad (29)$$

where the integral could be solved explicitly. It appears clearly from this equation that on-the-job search occurs if and only if condition (20), stating that the cost of on-the-job search is not too high, is satisfied[3]. As this condition is reasonable, it is natural and legitimate to allow on-the-job search in an economy with creative destruction.

Using the value function of the firm, (23), for simplicity, the first order condition for the optimal date of resignation, $\tau+T''$, is:

$$p(\tau) = w^s(\tau,\tau+T''). \quad (30)$$

Note that there is no problem of dynamic inconsistency. The firm wants to destroy the job when the wage rate reaches its productivity level, reducing the surplus to zero. Condition (30) simplifies to:

$$p(\tau) = p(\tau+T'')\left[b+s\sigma+(1-s)\frac{\beta}{1-\beta}c\theta\right]. \quad (31)$$

The condition of optimality states that the match ends when the worker's reservation wage reaches the productivity of the match. Solving explicitly for T'', using the formula for productivity growth, (7), we obtain:

$$T'' = \frac{1}{g}\ln\left(\frac{1}{b+s\sigma+(1-s)\frac{\beta}{1-\beta}c\theta}\right). \quad (32)$$

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[2] Note that there cannot be a well-behaved equilibrium when $\beta = 1$ as firms only post vacancies if their profits from forming a match cover their recruiting costs.

[3] Note that if (20) does not hold we can consider that $T' = T''$.

An interesting result is that when both types of job seekers, i.e. the unemployed and the employed, are equally efficient in the search process, i.e. $s = 1$, the maximum life span of a job is independent of market tightness, θ. Indeed, as returning to unemployment does not increase the likelihood of finding a more lucrative job, a worker remains in employment until unemployment benefits reach the productivity of the firm net of search costs. On the contrary, when on-the-job search is not allowed or when employed job seekers are not as efficient as the unemployed at finding jobs, i.e. $s < 1$, then the maximum life span of a job is decreasing in market tightness. This is explained by the fact that market tightness, which improves employment prospects, has more value to an unemployed, who is searching very efficiently, than to an employed job seeker. Finally, note that, for a given market tightness, the maximum life span of a job is increased[4] by permitting on-the-job search[5].

Finally, the equilibrium market tightness, θ, could be determined by equation (12) which could be written as:

$$\begin{aligned}\frac{1}{1-\beta}\frac{c}{q(\theta)} &= \int_0^{T'} e^{-(r+\delta)u}\left[1-e^{gu}\left(b+\frac{\beta}{1-\beta}c\theta\right)\right]du \\ &+\int_{T'}^{T''} e^{-(r+\delta)u-s\theta q(\theta)(u-T')}\left[1-e^{gu}\left(b+s\sigma+(1-s)\frac{\beta}{1-\beta}c\theta\right)\right]du\end{aligned}, \qquad (33)$$

where, again, the integrals could be solved explicitly.

The equilibrium is characterised by (T', T'', θ), which is the solution to the system composed of equations (29), (32) and (33). The wage rate is then given by equations (6), (16) and (19).

D/ Job flows and equilibrium rate of unemployment

The rate of unemployment[6], u, and the number of employed job seekers, e, could be deduced from the job flows induced by the model. Job creation could either be due to the hiring of an

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[4] The increase is strict provided (20) holds; otherwise allowing on-the-job search does not change anything at all as the worker never chooses to engage into it.

[5] In the standard creative destruction model without on-the-job search, the maximum age of a job at destruction, T'', is determined by $p(\tau) = w^{ns}(\tau, \tau + T'')$, giving:

$$T'' = \frac{1}{g}\ln\left(\frac{1}{b+\frac{\beta}{1-\beta}c\theta}\right).$$

[6] The working population being normalised to one, the rate of unemployment is also the number of unemployed.

unemployed, which occurs at rate $\theta q(\theta)u(t)$, or to the hiring of an employed job seeker, at rate $s\theta q(\theta)e(t)$. Thus, the number of new jobs created at time t, $C(t)$, is given by:

$$C(t) = \theta q(\theta)[u(t) + se(t)] . \tag{34}$$

The flow of obsolete jobs at t is equal to the number of job created at $t - T''$, $C(t - T'')$, multiplied by their survival probability from $t - T''$ to t which we shall now compute. For a job created at time τ, the probability to survive until $\tau + T'$ is equal to $e^{-\delta T'}$ since the arrival of job destruction shocks is given by an exponential distribution with parameter δ. Opportunities to move to another job are distributed according to an exponential distribution starting at time $\tau + T'$ and with parameter $s\theta q(\theta)$. Thus, in order to survive until time $\tau + T''$, the job should not be destroyed, which is satisfied with probability $e^{-\delta T''}$, and the worker should not find another job, which is satisfied with probability $e^{-s\theta q(\theta)(T''-T')}$. The two events being independent of each other, the probability to survive until $\tau + T''$ is given by $e^{-\delta T'' - s\theta q(\theta)(T''-T')}$. Thus, the obsolescence flow is equal to $C(t - T'')e^{-\delta T'' - s\theta q(\theta)(T''-T')}$.

Job destruction could either be due to an adverse exogenous shock, which occurs at rate $\delta(1 - u(t))$, to job obsolescence, at rate $C(t - T'')e^{-\delta T'' - s\theta q(\theta)(T''-T')}$, or to the resignation of an employed job seeker, at rate $s\theta q(\theta)e(t)$.

The flow into unemployment is either due to obsolescence or to the occurrence of exogenous shocks, whereas the outflow is due to the hiring of unemployed workers. Hence, the evolution of unemployment is determined by:

$$\dot{u}(t) = e^{-\delta T'' - s\theta q(\theta)(T''-T')}C(t - T'') + \delta(1 - u(t)) - \theta q(\theta)u(t) . \tag{35}$$

The flow into the set of employed job seekers is equal to the number of jobs created at $t - T'$ that survive until t, $e^{-\delta T'}C(t - T')$, whereas the corresponding outflow is either due to exogenous shocks that lead to job destruction, to the resignation of employed job seekers who receive outside offers, or to obsolescence. Thus, the evolution of the number of employed job seekers is given by:

$$\dot{e}(t) = e^{-\delta T'}C(t - T') - \delta e(t) - s\theta q(\theta)e(t) - e^{-\delta T'' - s\theta q(\theta)(T''-T')}C(t - T'') . \tag{36}$$

The rate of unemployment and number of employed job seekers in the steady state equilibrium, i.e. with $C(t) = C$, $u(t) = u$ and $e(t) = e$ for all values of t, is obtained by simultaneously solving (34), (35) and (36).

II/ Calibration

As the model is not analytically solvable, we shall now find numerical solutions under reasonable parameter values. Adopting a Cobb-Douglas matching function, q is of the form $q(\theta) = \theta^{-\alpha}$. Using the same calibration as Postel-Vinay (2002)[7], except for σ which does not appear in his model, we assume the parameter values of Table 1.

We shall now focus on the impact of growth, g, on the equilibrium of our model for different values of the search intensity of the employed job seekers, s. We report results for the cases where both types of job seekers are equally efficient in the search process, $s = 1$, where the unemployed are twice as efficient as the employed job seekers, $s = 0.5$, and, for reference, where on-the-job search is not allowed, $s = 0$. Arguably, the intermediary case, $s = 0.5$, might be the most realistic calibration. Solving for the equilibrium under the chosen parameterization, we obtain the results of Table 2. It is also interesting to compute the job flows induced by the model. These are given in Table 3.

We observe that allowing on-the-job search considerably reduces the positive impact of growth on unemployment; indeed the growth elasticity of unemployment shrinks from 1.63 to 0.13 when the employed job seekers are as efficient at searching for jobs as the unemployed. What is even more interesting is the modification of the labour market dynamics that occurs when workers are allowed to seek jobs while employed. When on-the-job search is not permitted, $s = 0$, the main explanation for the positive correlation between growth and unemployment is the flow of obsolescence, which, as can be seen from the Table 3, is

[7] Note that this parameterization is close to that chosen by Pissarides and Vallanti (2006).

responsible for over half of the job destructions. Allowing on-the-job search does not only reduce the obsolescence flow, it almost suppresses it. This is due to the combination of the large increase in the maximum life span of a match, T'', and of the possibility to move to a better paid job without intervening unemployment. Thus, in this context, a new match has a very low probability to survive until obsolescence. It is also interesting to note that the possibility of on-the-job search has important labour market consequences although only a minority of employees choose to engage into it, i.e. e remains below 10%.

We observe that the obsolescence flow is replaced by the flow of job-to-job transitions. This latter flow, contrary to the former, does not feed unemployment. In fact, when $s = 1$, the small positive impact of growth on unemployment is not due to job obsolescence. Instead, faster growth makes on-the-job search more attractive, which increases the number of employed job seekers. This decreases market tightness and, hence, the hiring of unemployed. This modification of the transmission channels at work shows that on-the-job search profoundly changes the dynamics of the matching model with creative destruction.

When employed job seekers are only half as efficient as the unemployed at searching for jobs, $s = 0.5$, we might expect the growth elasticity of unemployment to be the average of its value when $s = 0$, i.e. 1.63, and when $s = 1$, i.e. 0.13. Surprisingly, we observe that it is equal to 0.17, only marginally higher than when $s = 1$. Even when on-the-job search is not a perfect substitute for unemployment search, the obsolescence flow is considerably reduced by the presence of on-the-job search. In fact, workers compensate their lower efficiency by starting to search earlier. Figure 1 displays the relationship between the elasticity of unemployment with respect to growth and the search efficiency of employed job seekers relative to the unemployed, i.e. s.

As can be seen from the first table, allowing on-the-job search decreases the equilibrium rate of unemployment. This is essentially due to the decrease in the obsolescence flow. Indeed, on the job creation side, the rate at which unemployed are hired hardly changes as market tightness hardly changes. The evolution is therefore due to the large modifications that occur on the job destruction side.

As the outcome of a creative destruction model with on-the-job search contrast sharply with that of a model without, our results should seriously qualify some applied work realised on the topic. Pissarides and Vallanti (2006) argue that nearly all technological progress is of the disembodied form. They estimate from a panel of OECD countries that the elasticity of unemployment with respect to growth is equal to -1.49 in the United-States and to -1.31 in the European Union. Using a matching model of the labour market that allows for both embodied and disembodied technological progress, they argue that even a moderate amount of creative destruction could not be compatible with the observed negative correlation between growth and unemployment. The positive impact of growth on unemployment induced by the creative destruction effect is so strong that it could hardly be compensated by the negative impact induced by the capitalization effect. Clearly, allowing for on-the-job search should significantly alter those results in favour of the creative destruction hypothesis.

Our findings might also qualify the work of Hornstein, Krusell and Violante (2007) who argue that half the rise in European Unemployment since the 1970's could be explained by the combination of labour market rigidities and an acceleration of embodied technological progress. Although we do not consider any interaction with policies, our findings suggest that allowing for on-the-job search would reduce their simulated rise in European unemployment[8].

III/ Sensitivity analysis

In the previous section, when performing the calibration, we have implicitly assumed that allowing on-the-job search does not affect the deep parameters of the economy. While this is reasonable for most parameters, we might expect the existence of employed job seekers to affect the elasticity of the matching function, α. Also, on-the-job search might change the share of the match surplus allocated to workers, β. The elasticity of unemployment with respect to growth is given in the Table 4 for different values of α and β, where the calibration of the other parameters is left unchanged, where $s = 1$ and where the elasticity is estimated as growth increases from 3% to 4%.

Recall that in the benchmark case, i.e. $\alpha = 0.4$ and $\beta = 0.4$, without on-the-job search this elasticity was equal to 1.63. This sensitivity analysis suggests that, for a fixed share of surplus

[8] It should nevertheless be noted that their model differs from ours in that technological progress is embodied in capital rather than in labour.

allocated to the worker, a decline in the elasticity of the matching function, α, should not seriously qualify our results. It seems that our findings would only be invalidated if permitting on-the-job search has a very large positive impact on the worker's share, β.

Another parameter of interest is the opportunity cost of employment, b. Although it should not be affected by the possibility of on-the-job search, the chosen calibration for this parameter could potentially have a large impact on the resulting equilibrium. Indeed, in order to keep the obsolescence flow to a very low level, it is necessary that workers prefer to search for jobs while remaining in employment rather than choose to become unemployed. If the opportunity cost of employment is high, due to generous unemployment benefits for example, then workers on low productivity jobs will quickly choose to resign. Note that these considerations could be highly relevant for international comparisons to the extent that this parameter is likely to differ substantially across countries.

If we double the parameterized value of b and set it equal to 0.6, while leaving the rest of the calibration unchanged, then the elasticity of unemployment with respect to growth becomes equal to 3.43 without on-the-job search and 1.25 with[9]. As expected, the larger impact of growth on unemployment is essentially due to a significant decrease in the maximum life span of a job, T'', which with on-the-job search becomes inferior to 9 years for a growth rate of 4%, implying a non negligible obsolescence flow. It should nevertheless be noted that allowing on-the-job search still causes an almost threefold decrease in the impact of growth on unemployment.

IV/ Unobservable Search Effort

So far, we have implicitly assumed that the employees' search effort was observable. Indeed, if the firm cannot know about the search activity of its employee, then, given the wage schedule (6), the worker has an incentive to deviate by starting to search earlier. Note that these considerations entail that on-the-job search is even more widespread than we previously obtained and this should only strengthen our main findings from the previous section.

As could be seen from the first order conditions for T', (22) or (24), it is the decline in wage resulting from on-the-job search that prevents the worker from starting his job seeking

[9] Note that, from (16), the initial wage at time 0 is close to 0.87, which should be compared to the opportunity cost of employment of 0.6.

activity earlier. But, with unobservable search effort, he takes the wage policy (6) as given, where $\tau + T'$ corresponds to the time when the firm thinks the worker starts searching, and decides to start applying for jobs at $\tau + \tilde{T}$, as soon as the corresponding instantaneous benefit outweighs the associated cost. Thus, $\tilde{T}$ is determined by:

$$s\theta q(\theta)\left[W(\tau+\tilde{T},\tau+\tilde{T}) - W(\tau,\tau+\tilde{T})\right] = p(\tau+\tilde{T})s\sigma . \tag{37}$$

The expected capital gain on the left hand side corresponds to the benefit from search, while the search cost is on the right hand side. In a rational expectation equilibrium, the firm's belief has to be confirmed by facts and so $T' = \tilde{T}$. Hence, in equilibrium, T' is determined by:

$$\theta q(\theta)\frac{W(\tau+T',\tau+T') - W(\tau,\tau+T')}{p(\tau+T')} = \sigma . \tag{38}$$

Note that these considerations about the observability of search effort only affect the determination of T'. Thus, equation (38) replaces the first order conditions for T', (22) or (24), while the rest of the model remains unchanged.

As wages are still determined by surplus sharing, the worker would prefer search to be observable as the resulting optimal strategy leads to surplus maximization. However, given the wage schedule established by the firm, he cannot commit not to search before the time that maximizes total surplus. Hence, the search unobservability problem could also be viewed as a lack of commitment problem.

Is the amount of search effort chosen by the worker efficient in this new framework? The efficiency condition for the model is still given by equation (28). This could be compared to equation (38) which, after using the surplus sharing rule (13) and some rearrangement, gives:

$$J(\tau,\tau+T') = J(\tau+T',\tau+T') - \frac{1-\beta}{\beta}\frac{p(\tau+T')\sigma}{\theta q(\theta)} . \tag{39}$$

Comparing this equation to (28), it appears that efficiency either requires $\beta = 1$, as before, or $\sigma = 0$. If job search is costless, then the worker starts applying for new positions as soon as he is recruited, and this is efficient. However, if job search is costly, then the worker is the only one to pay for this cost which leads him to chose a level of effort that is too small to be efficient, unless $\beta = 1$.

Note that, to perform numerical simulations, it is much easier to determine T' from (39) than from (38) as the value function of the firm, (23), is much simpler than that of the worker, (21). We will nevertheless subsequently be interested in the welfare of the worker, which requires determining $W(0,0)$. The difficulty comes from the fact that the value of unemployment, $U(t)$, and of a new position, $W(t,t)$, appear in the value function of the worker, (21). The problem could be overcome by relying on the steady state properties of the model. First, it could be deduced from the value functions that:

$$U(t) = p(t)U(0), \tag{40}$$

and:

$$W(t,t) = p(t)W(0,0). \tag{41}$$

Hence, $\dot{U}(t) = gU(t)$, which, when combined with the Bellman equation for unemployment, (10), implies:

$$U(0) = \frac{b + \theta q(\theta)W(0,0)}{r - g + \theta q(\theta)}. \tag{42}$$

Finally, $W(0,0)$ could either be computed by using (40), (41) and (42) in the value equation of the worker, (21), for a new match formed at 0, or from the surplus sharing rule (13) at time 0:

$$W(0,0) = \frac{\beta}{1-\beta} J(0,0) + U(0), \tag{43}$$

where $J(0,0)$ is given by (23).

We now give the result from a calibration of the model. We keep using the same parameter values as before which are given in Table 1. Solving for the new equilibrium, were we recall that only the equation determining T' has been changed, we obtain the values of Table 5. It is also interesting to compute the job flows induced by the model. These are given in Table 6. The main difference with the results obtained under observable search effort is that on-the-job search begins much earlier. In the previous case workers were not applying for jobs before, at least, five years of tenure, whereas now they all are on the job market within a year of recruitment. Consequently, the fraction employed job seekers in the population, which was previously inferior to 20%, is now larger than 50%. Firms react by posting more vacancies.

This reaction is strong enough that market tightness does not decline by much. Finally, it should be noted that, as expected, considering unobservable search effort reinforces our main result according to which allowing on-the-job search considerably reduces the elasticity of unemployment with respect to growth, which, even when $s = 0.5$, is now inferior to 0.1.

As we previously mentioned, the search unobservability problem could also be viewed as a lack of commitment problem. An interesting, and perhaps surprising, consequence of this is that the welfare of a worker, $W(0,0)$, is increasing in the cost of search, as shown by the solid line of Figure 2 where $s = 0.5$, $g = 0.03$ and the rest of the calibration is still given by Table 1. The age of the match from which the worker starts seeking outside employment opportunities, T', is shown in Figure 3.

At the end of the interval, when $\sigma \approx 0.4435$, on-the-job search is so costly that the worker chooses not to engage into it and, hence, it could be checked that $T' = T''$. It should also be noted that, although not visible in Figure 2, the welfare of the worker is maximised slightly before on-the-job search stops occurring.

When job search is observable, the cost of search only reduces the joint surplus from the match and therefore reduces the welfare of the worker, as shown by the dashed line of Figure 2. With unobservable search, the worker would like to commit no to start searching before a distant future in order to increase the match surplus. In this case, facing a high cost of search is the only way to credibly signal to his employer that he will not start searching for a job too soon. The search cost therefore acts as a commitment device.

A practical consequence is that workers have an incentive to make job search difficult and expensive in a way that could be perceived by their employer. This could probably explain why employment contracts often specify that workers have to give their employer several months' notice of their intention to resign before they can quit. More importantly, it could lead to back loaded wage-tenure contracts. Stevens (2004) was the first to point out, in the context of the Burdett Mortensen (1998) framework, that wage profiles increasing in tenure could be used to reduce on-the-job search. With wage posting, it is indeed in the interest of firms to offer these contracts in order to increase their monopsony power. It should be emphasized that allowing these wage policies is welfare decreasing for workers. On the contrary, in the framework of this paper, with surplus splitting, back loaded wage-tenure contracts would be favoured by workers (at least up to a point). Finally, it should be emphasized that these considerations are not specific to the context of creative destruction and should also apply to simpler models of on-the-job search with surplus sharing (e.g. Pissarides 2000, Chapter 4).

V/ Variable search intensity

We have so far assumed that workers either do not search or do search with intensity s. A consequence of this characterisation is that at time $\tau + T'$, when workers re-enter the labour market, their wage suddenly slumps. This is not very realistic. The purpose of this section is therefore to generalise the model by allowing for variable search intensity. We derive the main characteristics of the equilibrium implied by this extension. Note that, although this generalisation should make the model more realistic, it should not affect our main findings.

Let $s(\tau, t) = s(t - \tau)$ denote the search intensity of a worker at t for a match created at τ. We now assume that the cost of search is given by a strictly increasing and convex function of search intensity, $\sigma(s)$, where we impose that:

$$\sigma(0) = \sigma'(0) = 0. \tag{44}$$

We therefore rule out any fixed cost of searching. We denote by $G(T) = P(t - \tau \leq T)$ the cumulative distribution of job tenure, i.e. the probability that a given job was created less than T years ago. The average search intensity across the population is therefore equal to:

$$\bar{s} = \int_0^{T''} s(\tilde{T}) dG(\tilde{T}). \tag{45}$$

Note that the parameter T' naturally disappears from this version of the model; it is replaced by the time varying level of search intensity chosen by the worker. For the model to be interesting, we also need to consider the intensity with which an unemployed searches for jobs, which we denote s_0. Market tightness is now defined a follows:

$$\theta = \frac{v}{s_0 u + \bar{s}(1-u)}. \tag{46}$$

When expressed in terms of market tightness, the matching function remains unchanged. The Bellman equation for the worker's value of employment is now given by:

$$rW(\tau,t) = \max_{s(t-\tau)\geq 0} \left\{ \begin{array}{l} w(\tau,t) - p(t)\sigma(s(t-\tau)) + \delta[U(t) - W(\tau,t)] \\ + s(t-\tau)\theta q(\theta)[W(t,t) - W(\tau,t)] + \dot{W}(\tau,t) \end{array} \right\}. \tag{47}$$

Similarly, assuming the same cost function for the unemployed as for the employed, the Bellman equation for the value of unemployment is:

$$rU(t) = \max_{s_0 \geq 0} \left\{ p(t)b - p(t)\sigma(s_0) + s_0 \theta q(\theta)[W(t,t) - U(t)] + \dot{U}(t) \right\}. \tag{48}$$

Using the same steps as in the first section, we could derive the following wage equation:

$$w(\tau,t) = \beta p(\tau) + (1-\beta)p(t)\left[b - \sigma(s_0) + \sigma(s(t-\tau)) + (s_0 - s(t-\tau))\frac{\beta}{1-\beta}c\theta \right]. \tag{49}$$

Note that, as T' was replaced by the time varying search effort, this wage equation applies throughout the duration of employment, i.e. for any $t \in [\tau, \tau + T'')$.

If search effort is observable, then it is reflected in the wage. The wage equation, (49), should therefore be substituted into the worker's value of employment, (47), before we could derive the following first order condition:

$$\sigma'(s(t-\tau)) \geq \frac{\theta q(\theta)}{\beta}\frac{W(t,t) - W(\tau,t)}{p(t)} - c\theta, \tag{50}$$

where the equality hold strictly whenever $s(t-\tau) > 0$. Although, the cost of search is initially equal to 0, as imposed by (44), the worker waits for some time after he is recruited before he starts searching. In order to understand this result, note that with surplus sharing and observable search effort, the main cost of on-the-job search is the loss in the firm's value, and hence in match surplus, caused by the worker's greater likelihood of quitting.

As the surplus is shared in fixed proportions between the two sides of the match and as there are no strategic interactions or commitment issues, the problem could also be analysed from the firm's perspective. The Bellman equation for the firm's value of employment is:

$$rJ(\tau,t) = \max_{s(t-\tau)\geq 0} \left\{ p(\tau) - w(\tau,t) - \delta J(\tau,t) - s(t-\tau)\theta q(\theta)J(\tau,t) + \dot{J}(\tau,t) \right\}. \tag{51}$$

After substituting (49) for the wage, we obtain the first order condition:

$$\sigma'(s(t-\tau)) \geq \frac{\beta}{1-\beta}c\theta - \frac{\theta q(\theta)}{1-\beta}\frac{J(\tau,t)}{p(t)}, \tag{52}$$

where again the equality holds strictly whenever $s(t-\tau)>0$. It could easily be checked using the free entry condition, (12), and surplus sharing, (13) that the two first order conditions, (50) and (52), are equivalent. Although it is simpler to solve the firm's optimality condition, (52), than the worker's, (50), it remains a difficult task. Indeed, all search efforts from now until job obsolescence, i.e. from time t to $\tau+T''$, appear in the value function of the firm, $J(\tau,t)$. Equation (52) is therefore an integral equation that cannot be solved analytically[10].

The search effort of an unemployed could be determined from the Bellman equation corresponding to the value of unemployment, (48). The first order condition gives:

$$\sigma'(s_0)=\theta q(\theta)\frac{W(t,t)-U(t)}{p(t)}. \tag{53}$$

Note that we could easily normalize the cost function such that $s_0=1$. Using the free entry condition, (12), and the sharing rule, (13), it could be verified that:

$$\sigma'(s_0)=\sigma'(s(T''))=\frac{\beta}{1-\beta}c\theta. \tag{54}$$

It follows that there is no jump in search intensity for a job that survives until obsolescence, i.e. search effort increases steadily over time until it reaches its unemployment level, s_0, just before the match is destroyed. From $s_0=s(T'')$, it could be deduce that:

$$T''=\frac{1}{g}\ln\left(\frac{1}{b}\right), \tag{55}$$

and so the maximum life span of a match is independent of market tightness. It is clear from (54) that in a model of creative destruction with variable search intensity some on-the-job search always occurs. The fact that, here, we don't even need a condition like (20) to be fulfilled further vindicates the approach defended in this paper.

Finally, we turn to variable search intensity with unobservable search effort. Now, when the worker optimises, he takes the wage rate as given. Thus, equilibrium search effort could be obtained by taking the first order condition to the worker's value of employment, (47), treating the wage rate as fixed. This yields:

$$\sigma'(s(t-\tau))=\theta q(\theta)\frac{W(t,t)-W(\tau,t)}{p(t)}. \tag{56}$$

Given the reasonable restrictions that we have imposed on the cost function, (44), the worker starts prospecting for outside opportunities as soon as he is recruited. Again, the worker

[10] A similar integral equation determines search intensity in Christensen, Lentz, Mortensen, Neumann and Warwatz (2005).

would like to credibly commit to start his job search later, but he cannot do so. Comparing (53) to (56), it is immediate that $s_0 = s(T'')$. Hence, T'' is still determined by (55) and is therefore unaffected by market tightness.

VI/ Replacement ratio

Throughout our analysis, we have assumed that the stream of income received by an unemployed is just a fraction, b, of the productivity of the economy at the technological frontier, $p(t)$. In this section, we relax this assumption by considering the possibility that the level of unemployment benefit at the time of job destruction is determined by a replacement ratio, ρ.

Solving the model in this case is not straightforward as the value of being unemployed is now a function of the last wage and, hence, of the last date of job creation, τ. Thus, we just focus on the maximum life span, T'', that a job can reach in this context. We derive the analytic result that, under plausible assumptions, a match could survive forever and, hence, there is no job obsolescence flow. This suggests that our previous numerical results are, fundamentally, very robust.

We maintain the assumption of variable search intensity. With unobservable search effort, we still have the result that search intensity at obsolescence is equal to that when unemployed[11], i.e. $s_0 = s(T'')$. Thus, at $\tau + T''$, all the terms involving search effort simplify out of the wage equation which is then given by:

$$w(\tau, \tau + T'') = \beta p(\tau) + (1-\beta) b(\tau, \tau + T''), \tag{57}$$

where $b(\tau, t)$ denotes the unemployment benefit at time t for an unemployed worker whose previous job was created at τ. We are assuming that, at destruction time, the level of benefits is determined by the replacement ratio[12], $\rho < 1$, so:

$$b(\tau, \tau + T'') = \rho w(\tau, \tau + T''). \tag{58}$$

Combining (57) and (58), we have:

[11] Note that this result no longer holds with observable search effort as, in this case, the wage rate is not treated as given and, hence, when deciding on their search intensity, workers take into account the impact that this has on their future unemployment benefits.

[12] It is also important that, after job destruction, the level of unemployment benefits does not increase too fast over time, as, otherwise, the worker would never remain in employment. A sufficient condition is that the level of unemployment benefits increases at the same rate as the wage rate, had the worker remained in employment.

$$w(\tau,\tau+T'')=\frac{\beta}{1-(1-\beta)\rho}\,p(\tau)\,. \tag{59}$$

But, this implies that, for all values of T'':

$$p(\tau)>w(\tau,\tau+T'')\,. \tag{60}$$

Thus, the first order condition for T'', (30), is never satisfied and the match could survive forever.

This result is quite intuitive. Indeed, when search is not more costly while employed than while unemployed, the opportunity cost of having a job is the forgone flow of unemployment benefits. But, if these are lower than the wage rate, then the surplus from the match is always positive and it could therefore survive forever. Note that this result could also be proved with fixed search intensity, with or without observable search effort, under the assumptions that the employed are as efficient as the unemployed at searching for jobs, i.e. $s=1$, and that searching while employed is not more costly than while unemployed, i.e. $\sigma=0$. With variable search intensity, these two conditions are naturally satisfied (at least at T'') provided that the cost of search is the same for all job seekers.

Finally, it should be emphasized that, although the obsolescence flow disappears in this version of the model, we still expect growth to have a small positive impact on unemployment. Indeed, faster growth makes on-the-job search more attractive, which decreases market tightness and, hence, the job creation flow.

Conclusion

In this paper, we have analysed the labour market consequences of allowing on-the-job search in the context of growth by creative destruction. We have shown that, in this context, workers voluntarily choose to engage into on-the-job search when they have the possibility to do so and, hence, we have argued that it is natural and legitimate to allow on-the-job search in matching models with creative destruction. Indeed, the dynamics of the model are fundamentally changed by this modification.

We have considered two different settings, one where the search efforts of workers are observable by their employer, and another one where they are not. In both cases, under a realistic calibration, the elasticity of unemployment with respect to growth is considerably reduced by allowing on-the-job search as it shrinks from more than 1.6 to around 0.1. What is even more striking is that the underlying transmission channels change as the flow of obsolescence, which is the main cause of job destruction without on-the-job search, practically vanishes. It is replaced by a flow of job-to-job transitions. In fact, the positive impact of growth on unemployment that remains is due to the increase in the number of employed job seekers induced by a rise in growth which decreases market tightness and, hence, the hiring of unemployed. Interestingly, these findings on the consequences of allowing on-the-job search in an economy with creative destruction continue to hold true even if employed job seekers are significantly less efficient at searching for jobs than the unemployed. Our main conclusion is that, rather than contributing to unemployment, creative destruction induces a direct reallocation of workers from low to high productivity jobs.

Another contribution of this paper was to show that models of on-the-job search with surplus spitting could be extended to account for non-observable search efforts. In this context, workers treat their wage schedule as fixed and start prospecting for jobs earlier. This behaviour is no longer surplus maximizing and, hence, the value of employment increases with the cost of on-the-job search which prevents workers from starting their search activity too early. The search cost therefore acts as a commitment device.

We have then considered variable search intensity which allowed us to generalise and strengthen some of our previous findings. When job search is unobservable, it begins immediately after recruitment, which is not the case when it is observable. The search intensity of employees then rises steadily until obsolescence. If prospecting for jobs is as costly for employed job seekers as it is for the unemployed, then search intensity at obsolescence is equal to that chosen by the unemployed. Perhaps surprisingly, this implies that the maximum life span of a job is independent of outside labour market conditions.

Finally, we have shown that, when the level of unemployment benefits is determined by a replacement ratio, then, under plausible assumptions, a match never becomes obsolete. This analytical result suggests that our numerical findings are, fundamentally, very robust.

A number of issues are left for further research. Following Hornstein, Krusell and Violante, (2007), it would be interesting to consider the effects of labour market policies within the framework presented in this paper. Also, there could be some further theoretical and empirical work on the result, not specific to creative destruction, according to which, under unobservable search efforts, workers prefer to face high search costs. More specifically, it would be interesting to investigate the consequences for the wage-tenure relation.

Table 1: Exogenous parameter values

r	δ	b	c	σ	β	α
0.05	0.065	0.3	0.5	0.1	0.4	0.4

Table 2: Endogenous parameter values

	u	e	v	θ	T'	T''
$s=1$						
$g=0.03$	0.0507	0.075	0.174	1.39	7.2	30.5
$g=0.04$	0.0520	0.094	0.193	1.33	6.0	22.9
$s=0.5$						
$g=0.03$	0.0508	0.145	0.171	1.38	6.6	18.1
$g=0.04$	0.0525	0.181	0.189	1.32	5.4	14.1
$s=0$						
$g=0.03$	0.1036	0	0.139	1.33	-	9.9
$g=0.04$	0.1199	0	0.150	1.25	-	8.3

Table 3: Job flows

	Total job creation (destruction) flow	Job creation		Job destruction		
		Hiring of unemployed	Hiring of employed job seekers	Exogenous shocks	Obsolescence	Resignation of employed job seekers
	C	$\theta q(\theta)u$	$s\theta q(\theta)e$	$\delta(1-u)$	$e^{-\delta T''-s\theta q(\theta)(T''-T')}C$	$s\theta q(\theta)e$
$s=1$						
$g=0.03$	0.1525	0.0617	0.0908	0.0617	$9.3\cdot10^{-15}$	0.0908
$g=0.04$	0.1728	0.0616	0.1112	0.0616	$7.5\cdot10^{-11}$	0.1112
$s=0.5$						
$g=0.03$	0.1500	0.0617	0.0882	0.0617	0.00004	0.0882
$g=0.04$	0.1689	0.0620	0.1069	0.0616	0.00040	0.1069
$s=0$						
$g=0.03$	0.1230	0.1230	0	0.0583	0.0647	0
$g=0.04$	0.1374	0.1374	0	0.0572	0.0802	0

Table 4: Elasticity of unemployment with respect to growth

β \ α	0	0.2	0.4	0.6	0.8	1
0.01	$3.0 \cdot 10^{-5}$	0.01	0.04	0.08	.	.
0.2	0.07	0.09	0.10	0.09	0.06	$6.7 \cdot 10^{-8}$
0.4	0.20	0.17	0.13	0.09	0.05	$2.3 \cdot 10^{-9}$
0.6	0.51	0.31	0.18	0.10	0.04	$4.0 \cdot 10^{-10}$
0.8	3.74	0.87	0.30	0.12	0.04	$1.2 \cdot 10^{-10}$
0.99	.	.	.	1.27	0.07	$5.0 \cdot 10^{-11}$

Table 5: Endogenous parameter values (with unobservable search effort)

	u	e	v	θ	T'	T''
$s = 1$						
$g = 0.03$	0.0646	0.511	0.520	0.90	0.80	30.5
$g = 0.04$	0.0651	0.516	0.517	0.89	0.79	22.9
$s = 0.5$						
$g = 0.03$	0.0575	0.599	0.397	1.11	0.93	20.8
$g = 0.04$	0.0583	0.609	0.394	1.08	0.90	15.8
$s = 0$						
$g = 0.03$	0.1036	0	0.139	1.33	-	9.9
$g = 0.04$	0.1199	0	0.150	1.25	-	8.3

Table 6: Job flows (with unobservable search effort)

	Total job creation (destruction) flow	Job creation		Job destruction		
		Hiring of unemployed	Hiring of employed job seekers	Exogenous shocks	Obsolescence	Resignation of employed job seekers
	C	$\theta q(\theta)u$	$s\theta q(\theta)e$	$\delta(1-u)$	$e^{-\delta T''-s\theta q(\theta)(T''-T')}C$	$s\theta q(\theta)e$
$s=1$						
$g=0.03$	0.5415	0.0608	0.4807	0.0608	$5.2\cdot 10^{-14}$	0.4807
$g=0.04$	0.5420	0.0608	0.4812	0.0608	$1.3\cdot 10^{-10}$	0.4812
$s=0.5$						
$g=0.03$	0.3802	0.0613	0.3189	0.0613	$2.4\cdot 10^{-6}$	0.3189
$g=0.04$	0.3811	0.0613	0.3198	0.0612	$5.3\cdot 10^{-5}$	0.3198
$s=0$						
$g=0.03$	0.1230	0.1230	0	0.0583	0.0647	0
$g=0.04$	0.1374	0.1374	0	0.0572	0.0802	0

Figure 1: Elasticity of unemployment with respect to growth as a function of relative search efficiency of employed job seekers

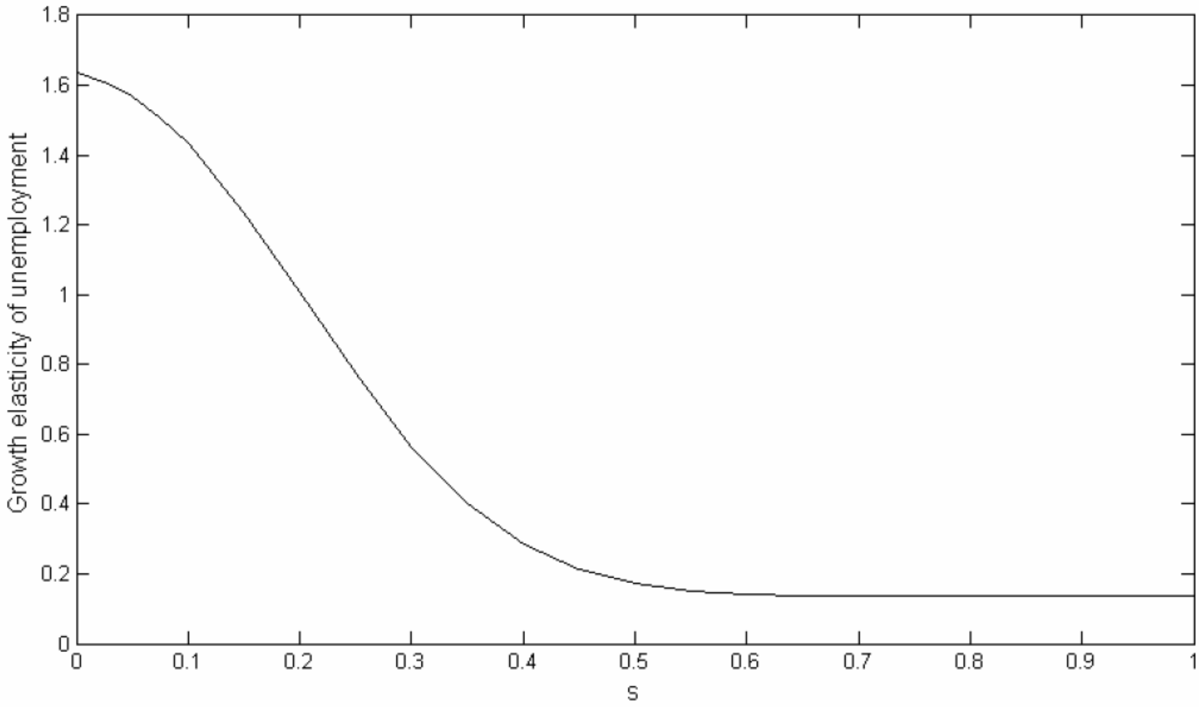


Figure 2: Welfare of worker as a function of search cost

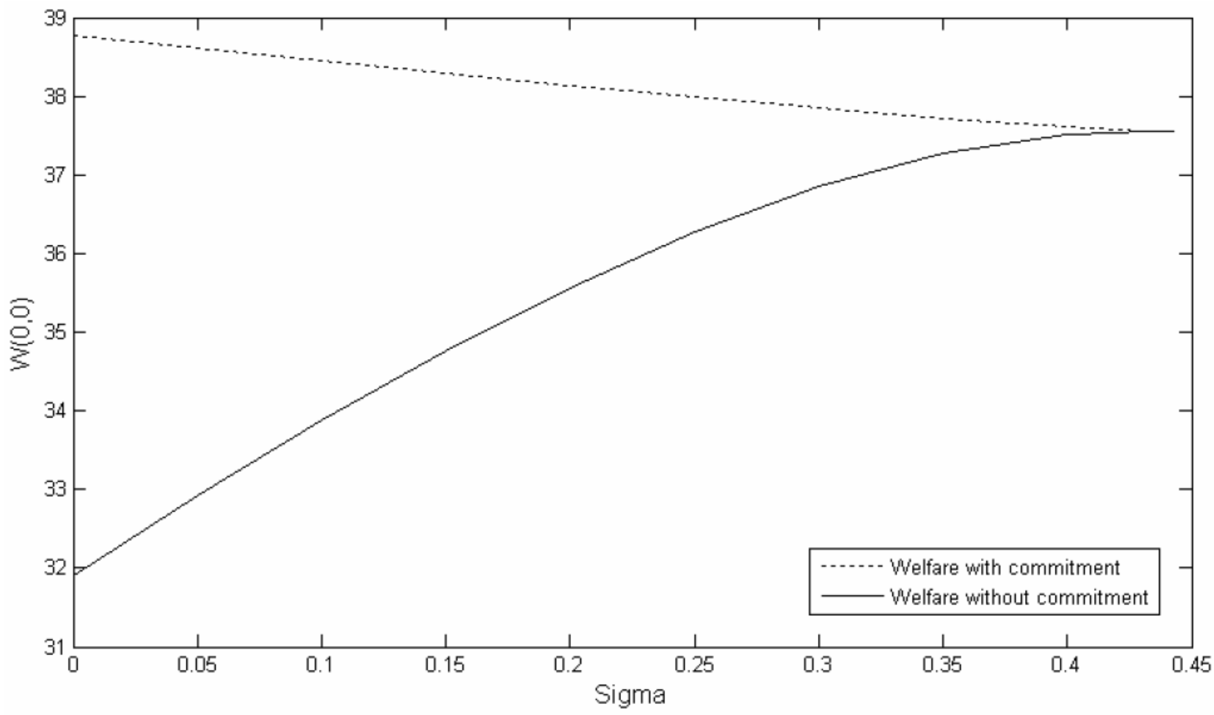
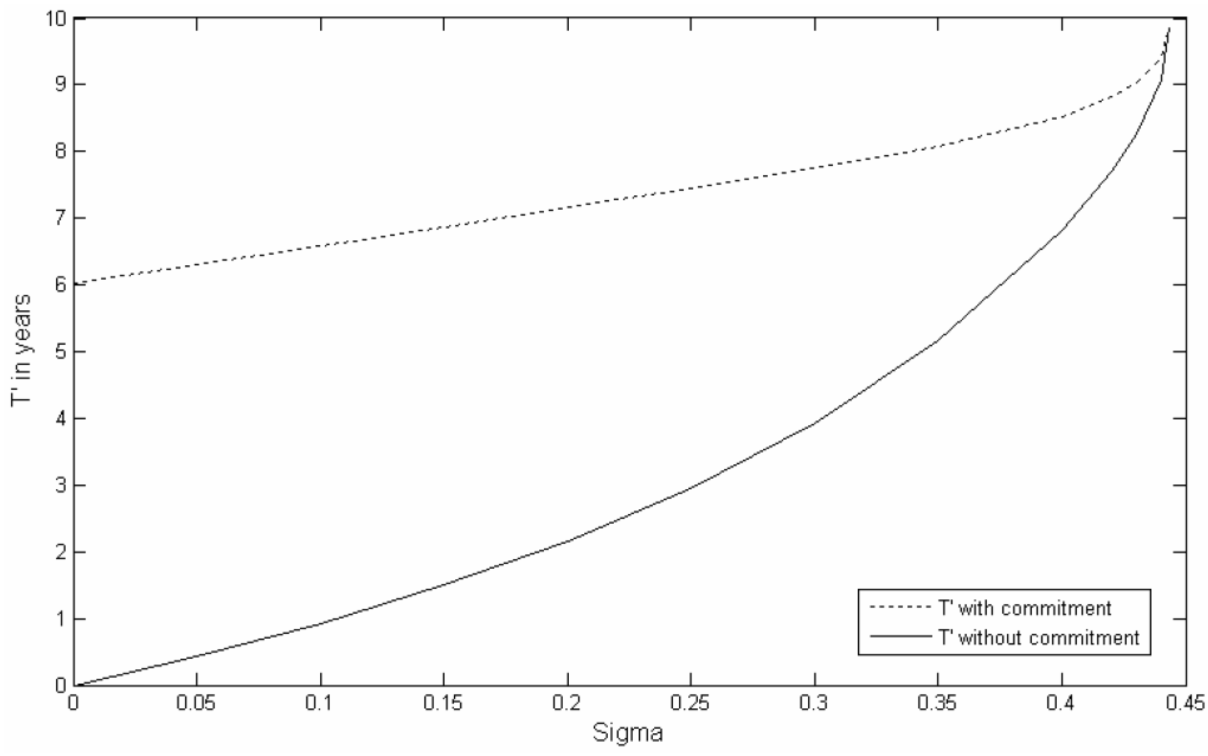


Figure 3: Age at which on-the-job search begins as a function of search cost



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