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Competing for Contacts: Network Competition, Trade Intermediation and Fragmented Duopoly

Dimitra Petropoulou

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

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Abstract
A two-sided, pair-wise matching model is developed to analyse the strategic interaction between two information intermediaries who compete in commission rates and network size, giving rise to a fragmented duopoly market structure. The model suggests that network competition between information intermediaries has a distinctive market structure, where intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. The intermediaries' inability to price discriminate between the competitive and non-competitive market segments, gives rise to an undercutting game, which has no pure strategy Nash equilibrium. The incentive to randomise commission rates yields a mixed strategy Nash equilibrium. Finally, competition is affected by the technology of network development. The analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopolistic outcome.

Keywords: International Trade, Pairwise Matching, Information Cost, Intermediation, Networks.
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Dimitra Petropoulou is an Associate of the Globalisation Programme at the Centre for Economic Performance, London School of Economics. She is currently a Lecturer in Economics at Hertford College, University of Oxford.

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1 Introduction

This paper develops a two-sided, pairwise matching model to analyse the effects of competition between intermediaries on endogenous network-building. The simplest framework within which to undertake such an analysis is the case of two trade intermediaries competing in network size and commission rates. The model analyses the strategic interaction between two information intermediaries with symmetric access to an information technology that allows them to develop contacts with importers and exporters seeking to form unique trade matches. The intermediation analysed takes the form of information intermediation, where the role of intermediaries is to facilitate matching in a market with information frictions. Intermediaries thus seek to match members of their network of contacts for a success fee.

The related literature on competition between information intermediaries is limited to relatively few contributions, where these focus on competing 'cybermediaries' who seek to match two sides of a market on the Internet (Caillaud and Jullien, 2001, 2003). The main role of intermediaries in this literature is to gather and process information on users that visit their website so as to assist buyers and sellers in matching through their web service. This literature focuses on the effects of competition between online intermediaries in the presence of asymmetric network externalities where the value of an intermediary to a buyer depends on the number of sellers or goods that can be accessed through the intermediary (e.g. access to books through Amazon versus a smaller online seller). The literature discusses different pricing rules and contractual arrangements between users and intermediaries and contrasts the effects with the findings of the traditional literature on network competition (for example, Katz and Shapiro, 1985).

This paper examines a framework of competition in information intermediation that differs from this literature. The focus of the model is the endogenous network investment decision of competing intermediaries, which in turn affects the nature of competition between them. While the importance of network size for competition is addressed in the literature, this is explored in the context of network externalities, whereby an intermediary that offers wider access to trading partners is considered more valuable to traders. The analysis in this paper does not consider network externalities of this kind. Moreover, there are no asymmetries built into the model (although asymmetries between intermediaries may arise in equilibrium). In fact, the model assumes that traders receive intermediation offers by intermediaries *after* uncertainty about matching possibilities is resolved. That is, traders who find themselves in a position to choose between the two competing intermediaries, do so in the knowledge that a match with their unique trading partner is possible.

The model focuses on the competition between intermediaries in commission rates and the coordination game played by trading partners who must select between intermediaries. The competition between intermediaries gives rise to a distinctive market structure as a result of network overlap and the inability to price discriminate between groups of network members. In particular, intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. The paper thus models competition between information intermediaries as a fragmented duopoly with a competitive and a non-competitive segment, which gives rise to an undercutting game in commission rates with no pure strategy Nash equilibrium.

To the best of my knowledge there is no literature that examines competition in endogenous network formation in this way. A few references in the Industrial Organisation literature consider markets with similar characteristics. Baye and De Vries (1992) develop a model with brand loyal consumers and price-sensitive consumers, in a market where price discrimination is not possible. They too find no pure strategy Nash equilibrium in prices.

Beard, Ford, Hill and Saba (2005) build a model directly applicable to cable television service competition, in which cable networks overlap, but price discrimination across users is not possible. They do find a pure strategy Nash equilibrium in prices, despite the fragmented nature of the market, as a result of smoothness conditions that ensure demand is decreasing in price in both market segments. The network overlap itself is exogenous in Beard, Ford, Hill and Saba (2005), while the distribution of brand loyal versus price sensitive consumers is also arbitrary in Baye and De Vries (1992). In contrast, intermediaries' network sizes are endogenous in the model developed in this paper.

The rest of the paper proceeds as follows. Section 2 describes the economic environment and describes the timing of the game between traders and intermediaries. The subgame perfect equilibrium is characterised in Section 3. Section 4 provides two illustrative examples. Section 5 concludes.

2 Economic Environment

Consider a two-sided market where a continuum of risk-neutral importers (M) and a continuum of exporters (X), each distributed uniformly and with unit density over $[0,1]$, match uniquely to exchange a single unit of output generating joint surplus $S > 0$. There is two-sided information asymmetry as traders regarding the location of trading partner on the continuum. Due to the infinite number of importers (and exporters) along the continuum, the probability of any trader j locating her partner through random selection is 0.

Each pair (X_j, M_j) may match through a direct matching technology, which achieves successful matching with probability $q(i)$, where i reflects the level of information costs or barriers to information flow between the two sides of the market and $i \in [0,1]$. Let $q'(i) < 0$, $q(1) = 0$ and $q(0) = 1$. This direct matching technology could reflect a search process whose success hinges on the state of information technology.

Alternatively, traders may match through a trade intermediary. Suppose there are two intermediaries, A and B, with access to the same technology for developing a network of contacts. The network of intermediary I is denoted by a measure of importer contacts, P_{MI}, and exporter contacts, P_{XI}, where $P_{MI} \in [0,1]$ and $P_{XI} \in [0,1]$, respectively, for $I = \{A,B\}$. Given network size, the measure of feasible trade matches depends on the degree of overlap between importer and exporter contacts and is a random variable. For any given network investment, expected matches are maximised through symmetric contact-building in the two sides of the market. Hence intermediaries ensure $P_{MI} = P_{XI} \equiv P_I \; \forall I$, where $P_I \in [0,1]$.

Network set-up costs are assumed to be zero, for simplicity. Let $C(P_I)$ denote the total investment cost for a symmetric network of size P_I on either side of the market. The network investment decisions of intermediaries are analysed under two cost specifications:

(a) Linear costs: $C(P_I) = 2P_I c$, where $c > 0$.

(b) Convex costs: $C(P_I) = 2P_I c(i, P_I)$, where $c(i, P_I) = \gamma i^{\alpha} P_I^2$ and $\alpha \geq 1$, $\gamma > 0$.

For simplicity, it is assumed costless to match trade pairs from within the network of contacts. Hence, each intermediary has a marginal cost of intermediation equal to zero. Intermediaries receive a success fee or commission for each intermediated trade match. Let α_A and α_B denote the commission rates of intermediaries A and B, respectively, where $\alpha_A \in [0,1]$ and $\alpha_B \in [0,1]$. The marginal revenue from intermediation is thus $\alpha_A S$ and $\alpha_B S$ for A and B, respectively. Residual trade surplus is assumed to be shared equally between the importer and exporter.

The demand for each intermediary's services depends on two factors. First, the network size decisions of the two intermediaries. A larger network size gives rise to a larger measure of expected matches through the network, but also increases the expected overlap between networks. Expected overlap gives rise to an expected measure of common matches that can be intermediated through either network and for which intermediaries compete. Second, traders with access to both trade networks must choose between the intermediaries *ex post* and play a coordination game.

2.1 Timing of the Game

Intermediaries and traders interact strategically in a multi-stage game. The timing of the game is as follows.

Stage 1 - Network investment: Intermediaries A and B simultaneously and non-cooperatively choose network sizes P_A and P_B. Network investment costs are sunk.

Stage 2 - Commission setting: Intermediaries simultaneously and non-cooperatively commit to commission rates α_A and α_B, respectively.

Stage 3 - Intermediation offers: Uncertainty over which trade matches are feasible through each network is resolved. Each intermediary makes a take-it-or-leave-it intermediation offer to traders that can be matched, specifying his commission rate. Successful matching is conditional on both trade partners accepting an offer by the same intermediary. Traders accept at most one offer.

Stage 4 - Indirect trade: Indirect trade takes place through A and/or B.

Stage 5 - Direct trade: Any unmatched traders trade directly with probability $q(i)$.

2.2 Equilibrium Concept

The solution concept used is subgame perfect equilibrium (SPE) and the method used is backward induction. A strategy for intermediary I is described by a pair $\{P_I, \alpha_I\}$. An offer acceptance strategy for trader j is described by a pair $\{R_a, R_s\}$, where R_a is a rule for determining whether an intermediation offer is acceptable and R_s is a rule for selecting between acceptable offers. A set of strategy pairs, for intermediaries and traders, respectively, can be said to form an equilibrium of the game if these maximise the expected profit of each intermediary and the expected surplus from trade of each trader, given the strategies of all other players.

The subgame perfect equilibria of the game are characterised over the next sections.

3 Traders' Incentives

Traders select their offer acceptance strategy to maximise their expected payoff taking intermediaries strategies as given. Each trader in receipt of one or more offers must decide whether to accept one (or none) of the offers of intermediation. If all offers are rejected in stage 3 then trade can only take place directly in stage 5 of the game with probability $q(i)$. The expected payoff from the direct trade route represents the outside option available to all traders and forms the benchmark against which all intermediation offers are assessed. Equilibrium rule R_a summarises this assessment through a participation constraint.

Although uncertainty about available matching opportunities is resolved at the time of traders' decision-making, indirect trade between matching trade partners is not guaranteed. The uncertainty in the outcome of the model arises, in part, from the coordination game played by traders in receipt of two offers of intermediation. Equilibrium rule R_s summarises the incentives for selecting between available offers of intermediation, when both of these are acceptable. Since traders cannot communicate their intentions, there is a non-zero probability of coordination failure as a result of mismatch in coordination decisions.

The incentives of traders at each decision node of the game are examined in turn.

3.1 Stage 5 - Direct Trade

The pool of traders who attempt to match directly in stage 5 are those who either (a) receive no offers of intermediation , (b) accept no offers of intermediation, and (c) accept one offer but fail to match as a result of coordination failure. Since importers and exporters are assumed to match uniquely, any unmatched trader j can be assured that her trading partner is also a member of the pool of unmatched traders. The probability of a direct match, $q(i)$, depends on the prevailing, level of information costs, reflected in parameter i. Assuming trade surplus is shared equally between trading partners, the expected payoff from direct trade of any trader j is given by:

$$E^{DT}(\Pi_j) = \frac{1}{2}q(i)S \tag{1}$$

A monopolist intermediary sets his commission rate at $1 - q(i)$, thereby leaving traders indifferent between direct and indirect trade. Let α^M denote the monopoly commission rate, where $\alpha^M = 1 - q(i)$. Expressing (1) in terms of α^M gives:

$$E^{DT}(\Pi_j) = \frac{1}{2}\left(1 - \alpha^M\right)S \tag{2}$$

The expected payoff from direct trade reflects traders' outside option. All offers of intermediation must generate an expected payoff at least as good as $E^{DT}(\Pi_j)$ in order to be acceptable. Interpreting equation (2), duopolist intermediaries must offer traders an expected payoff from indirect trade at least as good as that which would have been received under a monopolistic market structure.

3.2 Stage 3 - Intermediation Offers

In stage 3, traders X_j and M_j find themselves in one of four positions:

(1) Pair (X_j, M_j) cannot match through either intermediary.

(2) Pair (X_j, M_j) can match through A, but not B; traders receive one offer from A.

(3) Pair (X_j, M_j) can match through B, but not A; traders receive one offer from B.

(4) Pair (X_j, M_j) can match through either A or B; traders receive two intermediation offers.

If in (1), then X_j and M_j have no option but direct trade in stage 5. If in position (2)-(4), then X_j and M_j contrast the expected payoff from each offer received against the expected payoff from direct trade.

Let Π_j^A denote the payoff of trader j from indirect matching through A and Π_j^B the payoff through B, where these are given by (3) and (4), respectively:

$$\Pi_j^A = \frac{1}{2}(1 - \alpha_A) S \tag{3}$$

$$\Pi_j^B = \frac{1}{2}(1 - \alpha_B) S \tag{4}$$

It follows directly from (2), (3) and (4), that if $\alpha_A \leq \alpha^M$, then intermediation through A is acceptable and if $\alpha_B \leq \alpha^M$, then intermediation through B is acceptable. In general, all offers must satisfy the participation constraint $\alpha_I \leq \alpha^M$ in order to be acceptable.

In the case where X_j and M_j receive only one offer, the optimal selection rule is thus to accept the unique acceptable offer. The optimal acceptance strategy conditional on one offer being received is thus 'accept offer I if $\alpha_I \leq \alpha^M$; reject otherwise'. The next section examines the selection decision of traders in receipt of two offers.

3.2.1 The Trader Coordination Subgame

Consider a trade pair (X_j, M_j) that can match through either A or B. In stage 3, X_j and M_j each receive two offers of intermediation. They apply optimal rule R_a to assess the acceptability of each offer. Since expected payoffs are symmetric for both traders, their assessment of offers is identical.

If both offers are deemed unacceptable, X_j and M_j reject both offers and can expect to receive $E^{DT}(\Pi_j)$ in stage 5. If one offer is acceptable and the other unacceptable, then the optimal decision is for X_j and M_j to reject the unacceptable offer and trade indirectly in stage 4. Hence, in the case of one acceptable offer, there is no possibility of coordination failure.

If both received offers prove to be acceptable for X_j and M_j, then each trader j must choose between them. This gives rise to a coordination game between X_j and M_j. As with all games of this class, there are three equilibria, two symmetric pure strategy Nash equilibria (both choose A or both choose B) and one symmetric mixed strategy Nash equilibrium, where both traders choose A (or B) with the same probability.

If both X_j and M_j accept A in stage 3, then each receives Π_j^A. If they both accept B, then each receives Π_j^B. If one accepts B and the other A, then indirect trade cannot take place due to coordination failure. Thus mismatch can arise in the model even though both traders are members of both networks and both offers are acceptable. If coordination failure takes place, traders can expect to receive expected payoff $E^{DT}(\Pi_j)$. Table (1) describes the payoff structure of the coordination game:

For both offers to be acceptable, it must be the case that $\alpha_A \leq \alpha^M$ and $\alpha_B \leq \alpha^M$ are satisfied. This allows the payoffs in table (1) to be ranked,

		$\mathbf{M}_j$	
		A	**B**
$\mathbf{X}_j$	**A**	$\frac{S}{2}(1-\alpha_A)$, $\frac{S}{2}(1-\alpha_A)$	$\frac{S}{2}\left(1-\alpha^M\right)$, $\frac{S}{2}\left(1-\alpha^M\right)$
	B	$\frac{S}{2}\left(1-\alpha^M\right)$, $\frac{S}{2}\left(1-\alpha^M\right)$	$\frac{S}{2}(1-\alpha_B)$, $\frac{S}{2}(1-\alpha_B)$

Table 1: Traders' Coordination Game

confirming (A, A) and (B, B) as the two pure strategy Nash equilibria of the coordination game. Either both traders accept A, or both accept B.

Allowing traders to randomise over A and B, such that X_j accepts A with probability λ (and B with $1-\lambda$) and M_j accepts A with probability μ (and B with $1-\mu$), it is straightforward to show[1] the coordination game has one symmetric mixed strategy Nash equilibrium (λ^*, μ^*) where:

$$\lambda^* = \mu^* = \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} \tag{5}$$

Combining (5) and the pure strategy payoffs allows the expected payoff for each trader j in the the mixed strategy Nash equilibrium to be computed:

$$E(\Pi_j)_{|\lambda^*,\mu^*} = \frac{S}{2}\left[\frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A}(1-\alpha_A) + \frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A}\left(1-\alpha^M\right)\right] \tag{6}$$

We can now distinguish between three cases: (a) $\alpha_A < \alpha_B$, (b) $\alpha_B < \alpha_A$ or (c) $\alpha_A = \alpha_B$. If either (a) or (b) holds, then the game between X_j and M_j becomes a Ranked Coordination game, which has the additional feature that the equilibria can be Pareto ranked.

Consider case (a) where $\alpha_A < \alpha_B$. If intermediary A offers to match pair j for a lower commission than B, then the payoffs received from pure strategy Nash equilibrium (A, A) dominate those from (B, B). By inspection of (6) it can also be observed that $E(\Pi_j)_{|\lambda^*,\mu^*} < \Pi_j^A$. Hence pure strategy Nash equilibrium (A, A) is Pareto superior to the other Nash equilibria of the coordination game. It can thus be said that although there are three equilibria, the pure strategy Nash equilibrium (A, A) offers a compelling focal point[2] of the Ranked Coordination game when $\alpha_A < \alpha_B$.

[1] A derivation of the symmetric mixed strategy Nash equilibrium, and associated expected payoff, of the coordination game is included in Appendix A.

[2] The game theory literature points to a number of mechanisms for resolving the multiplicity of equilibria in the Ranked Coordination game so that focal pure Nash equilibria emerge as the unique solution. These include communication or signalling between coordinating parties to indicate the action to be taken. In light of the information barriers that underpin the model, such communication is prohibited by assumption. If unique pairs of traders could communicate their actions to each other then there would be no need for an intermediary. Other mechanisms include mediation where an outside party imposes a solution.

Similar arguments apply in case (b), which lead to the result that (B, B) is Pareto superior to the other two Nash equilibria providing a focal point of the coordination game when $\alpha_B < \alpha_A$.

In case (c), where $\alpha_A = \alpha_B$, both pure strategy Nash equilibria yield symmetric payoffs and $\lambda^* = \mu^* = \frac{1}{2}$. Since the two intermediaries are indistinguishable and there is no way for trade partners to indicate their action to each other, the mixed strategy Nash equilibrium provides a compelling focal point of the game when $\alpha_A = \alpha_B$.

The multiplicity of equilibria implies there is a multiplicity of selection rules R_s that are optimal for each trader in pair j, given the strategies of the other players. For example, if all traders follow the selection rule 'always accept A when two acceptable offers are received', then the outcome of their actions is (A, A) and no trader j ever finds it optimal to deviate from the rule.

Let the 'focal strategy' refer to the selection rule R_s, which specifies an action for each trader j to follow in each of the three cases (a) to (c), that leads to the focal point in each case. This rule gives rise to the most intuitive and likely outcome of the stage 3 coordination game. Specifically, that each trader in pair j simultaneously and non-cooperatively accepts the most inexpensive of two acceptable indirect trade routes when commissions differ across intermediaries, and flips a coin when commission rates are the same across intermediaries. Its conceptual appeal aside, the focal rule gives rise to strategic interactions between intermediaries that do not arise if one of the two intermediaries is always selected in stage 3, irrespective of commission rates. To explore the effects of competition in overlapping matches, we examine the subgame perfect equilibrium of the game that includes the focal strategy R_s as part of the equilibrium strategy of traders.

The focal selection strategy R_s can thus be summarised as follows for trader j: If two acceptable offers are received and $\alpha_A \neq \alpha_B$, then accept the offer with the lower commission rate with probability 1; if $\alpha_A = \alpha_B$, then accept offer A with probability $\frac{1}{2}$.

To confirm that randomisation when $\alpha_A = \alpha_B$ does not give rise to an unacceptable expected payoff, consider that coordination is at A with probability $\frac{1}{4}$, and at B with probability $\frac{1}{4}$. Mismatch occurs with probability $\frac{1}{2}$. Moreover, $\alpha_A = \alpha_B$ implies that $\Pi_j^A = \Pi_j^B \equiv \Pi_j$. Hence the expected payoff when commissions are equal, which follows directly from (6), is given by (7):

$$E(\Pi_j)_{|\alpha_A = \alpha_B \leq \alpha^M} = E(\Pi_j)_{|\lambda^* = \mu^* = \frac{1}{2}} = \frac{1}{2}\Pi_j + \frac{1}{2}E^{DT}(\Pi_j) \geq E^{DT}(\Pi_j) \tag{7}$$

Hence the selection rule R_s in the case of two acceptable offers is consistent with the participation constraints of both traders.

3.3 Traders' Offer Acceptance Strategy

The analysis of traders' optimal incentives is summarised by proposition (1).

Proposition 1 *The following pair* $\{R_a, R_s\}$ *forms an optimal acceptance strategy for each trader* j*:*
R_a : *Any offer* k *is acceptable if* $\alpha_I \leq \alpha^M$ *and unacceptable otherwise.*
R_s : *If one acceptable offer is received, accept it; if two acceptable offers are received and* $\alpha_A \neq \alpha_B$*, then accept the offer with the lower commission rate with probability* 1*; if two acceptable offers are received and* $\alpha_A = \alpha_B$*, then accept offer* A *with probability* $\frac{1}{2}$*.*

Proof. The optimality of R_a follows directly from equations (2), (3) and (4). The optimality of R_s in the case of one acceptable offer also follows directly from these. The optimality of R_s given two acceptable offers and $\alpha_A \neq \alpha_B$ follows from the payoffs in table (1). The optimality of R_s when $\alpha_A = \alpha_B$ follows from the mixed strategy Nash equilibrium of the coordination game, described by (5) and from the expected payoff under randomisation given by (7). ■

4 Stage 2 - Nash Equilibrium in Commission Rates

In stage 2, intermediaries simultaneously and non-cooperatively select commission rates, α_A and α_B, respectively, to maximise their expected profit, taking each others' commission rate, network sizes $P_A \in [0,1]$ and $P_B \in [0,1]$ and $\{R_a, R_s\}$ as given.

The strategic interaction between intermediaries in the commission-setting game hinges on two conflicting incentives. On the one hand, a lower commission rate makes it more likely that the intermediary's offer is selected by traders in receipt of two acceptable offers. At the same time, a lower commission implies lower profit per successful match.

The measure of trade matches possible through a network of a given size is a random variable that depends on the degree of overlap between importer and exporter contacts. A crucial feature of the game is that intermediaries set commission rates prior to the realisation of this random variable and thus without knowing the identity of their future customers. This prevents intermediaries from price discriminating between trade pairs[3] that are exclusive to their own network and those common to both networks.

To assess intermediaries' incentives in the commission-setting game the structure of traders' demand for intermediation services needs to be characterised.

[3] All decisions of intermediaries are thus made on the basis of expectations. The *ex post* realisation of trade matches can differ markedly from the *ex ante* expectation. For example, if $P_A = P_B = \frac{1}{2}$, the measure of expected common matches is $P_A^2 P_B^2 = \frac{1}{16}$. The realised overlap between the two networks can range from 0 to $\frac{1}{2}$, however, depending on which specific importers and exporters are contacted in stage 1. The obvious exception is where $P_A = P_B = 1$ for which expected and realised trade matches coincide.

4.1 Demand for Intermediation Services

For an intermediary to be able to match pair (X_j, M_j), both partners must be members of the intermediary's network. The matching probabilities for any pair (X_j, M_j) are therefore given by (8) to (11):

$$\Pr\left[\text{Pair } j \text{ can match via } A \text{ and } B\right] = P_A^2 P_B^2 \tag{8}$$

$$\Pr\left[\text{Pair } j \text{ can match via } A\text{, but not } B\right] = P_A^2 \left(1 - P_B^2\right) \tag{9}$$

$$\Pr\left[\text{Pair } j \text{ can match via } B\text{, but not } A\right] = P_B^2 \left(1 - P_A^2\right) \tag{10}$$

$$\Pr\left[\text{Pair } j \text{ cannot match via } A \text{ or } B\right] = \left(1 - P_A^2\right)\left(1 - P_B^2\right) \tag{11}$$

Suppose both intermediaries choose network sizes between 0 and 1. The market structure that results is one of fragmented duopoly. Each intermediary has a set of exclusive matches, over which there is monopoly power. At the same time, the non-zero probability of network overlap gives rise to a set of expected common matches, over which intermediaries compete.

The success of each intermediary in gaining trade matches from the competitive network overlap depends on relative commission rates. From R_s intermediaries anticipate that all common matches are won by the intermediary with the lower of the two commission rates when $\alpha_A \neq \alpha_B$, while each expects to win $\frac{1}{4}$ of common matches when $\alpha_A = \alpha_B$.

Let $E(T_A)$ denote expected indirect trade through A or, equivalently, expected demand for A's intermediation services. Similarly, $E(T_B)$ denotes expected indirect trade through B or expected demand for B's intermediation services. Combining the matching probabilities in (8) to (11) with $\{R_a, R_s\}$ yields $E(T_A)$ and $E(T_B)$, conditional on α_A and α_B.

Consider the expected demand for A's services. Intermediary A expects a measure $P_A^2\left(1 - P_B^2\right)$ of exclusive matches while common matches between A and B are given by measure $P_A^2 P_B^2$. If $\alpha_A < \alpha_B \leq \alpha^M$, then A provides the most inexpensive trade route, so all matching traders in the competitive segment coordinate at A. This yields a total expected demand for A of P_A^2. Conversely, if $\alpha_B < \alpha_A \leq \alpha^M$ all common matching traders coordinate at B giving intermediary A an expected demand of $P_A^2\left(1 - P_B^2\right)$ only. If $\alpha_B = \alpha_A \leq \alpha^M$, then intermediaries' offers are acceptable but indistinguishable, so traders randomise over A and B in the coordination stage. $\frac{1}{4}P_A^2 P_B^2$ are expected to trade through A, another $\frac{1}{4}P_A^2 P_B^2$ through B, while the remaining $\frac{1}{2}P_A^2 P_B^2$ fail to coordinate.

Similar arguments can be applied to B. The structure of $E(T_A)$ and $E(T_B)$ summarised below gives rise to the strategic incentives discussed in the rest of the section.

$$E(T_A) = \begin{cases} P_A^2 & if \quad \alpha_A < \alpha_B \leq \alpha^M \\ P_A^2\left(1 - P_B^2\right) & if \quad \alpha_B < \alpha_A \leq \alpha^M \\ P_A^2\left(1 - \frac{3}{4}P_B^2\right) & if \quad \alpha_B = \alpha_A \leq \alpha^M \\ 0 & if \quad \alpha_A > \alpha^M \end{cases}$$

$$E(T_B) = \begin{cases} P_B^2 & if \quad \alpha_B < \alpha_A \leq \alpha^M \\ P_B^2\left(1 - P_A^2\right) & if \quad \alpha_A < \alpha_B \leq \alpha^M \\ P_B^2\left(1 - \frac{3}{4}P_A^2\right) & if \quad \alpha_B = \alpha_A \leq \alpha^M \\ 0 & if \quad \alpha_B > \alpha^M \end{cases}$$

Recall that network investment costs are sunk in stage 1. Moreover, the marginal cost of matching is assumed to be zero for simplicity. If follows that the expected operating profit of A and B, respectively, are given by (12) and (13):

$$E(\Pi_A) = \alpha_A S E(T_A) \tag{12}$$

$$E(\Pi_B) = \alpha_B S E(T_B) \tag{13}$$

Intermediaries thus choose α_A and α_B to maximise $E(\Pi_A)$ and $E(\Pi_B)$, respectively, given $E(T_A)$ and $E(T_B)$.

4.2 Polar Cases of Market Structure

The discussion above assumes network sizes between 0 and 1. In general, the commission-setting game can be analysed for three polar cases:

1. Monopoly in intermediation services, where $\{P_B = 0; P_A \in [0,1]\}$ or $\{P_A = 0; P_B \in [0,1]\}$. Inspection of $E(T_A)$ confirms that expected indirect trade collapses to P_A^2 when $\alpha_A \leq \alpha^M$ and 0 otherwise when $P_B = 0$, and *vice versa* if B is a monopolist.

2. Bertrand duopoly in intermediation services, where $\{P_A = 1; P_B = 1\}$. If both intermediaries' networks span the entire market, then networks overlap entirely giving rise to the most competitive market outcome. Intermediaries are in direct competition for all trade pairs.

3. Fragmented duopoly in intermediation services, where $\{P_A \in (0,1); P_B \in (0,1)\}$. Each intermediary's demand is partitioned between a competitive and noncompetitive segment.

The two competitive cases are analysed in turn.

4.3 Competing in Commission Rates: Bertrand Duopoly

Let $P_A = P_B = 1$. All trade pairs are common to A and B giving rise to the following demand structure:

$$E(T_A) = \begin{cases} 1 & if \quad \alpha_A < \alpha_B \leq \alpha^M \\ 0 & if \quad \alpha_B < \alpha_A \leq \alpha^M \\ \frac{1}{4} & if \quad \alpha_B = \alpha_A \leq \alpha^M \\ 0 & if \quad \alpha_A > \alpha^M \end{cases}$$

$$E(T_B) = \begin{cases} 1 & if \quad \alpha_B < \alpha_A \leq \alpha^M \\ 0 & if \quad \alpha_A < \alpha_B \leq \alpha^M \\ \frac{1}{4} & if \quad \alpha_B = \alpha_A \leq \alpha^M \\ 0 & if \quad \alpha_B > \alpha^M \end{cases}$$

Expected operating profits for A and B when $\alpha_B = \alpha_A$ are thus:

$$E(\Pi_A)_{|\alpha_B=\alpha_A} = \alpha_A \frac{S}{4} \tag{14}$$

$$E(\Pi_B)_{|\alpha_B=\alpha_A} = \alpha_B \frac{S}{4} \tag{15}$$

Expected operating profits for A and B when $\alpha_B < \alpha_A$ are thus:

$$E(\Pi_A)_{|\alpha_B<\alpha_A} = 0 \tag{16}$$

$$E(\Pi_B)_{|\alpha_B<\alpha_A} = \alpha_B S \tag{17}$$

Conversely, when $\alpha_A < \alpha_B$:

$$E(\Pi_A)_{|\alpha_A<\alpha_B} = \alpha_A S \tag{18}$$

$$E(\Pi_B)_{|\alpha_A<\alpha_B} = 0 \tag{19}$$

Intermediaries provide a homogeneous service in a price-setting Bertrand duopoly. The pattern of $E(\Pi_A)$ and $E(\Pi_B)$ provides an incentive for intermediaries to undercut each other in order to win the entire market. In contrast to the classical Bertrand duopoly, coordination failure in traders' decisions implies that intermediaries share *half* the market when $\alpha_B = \alpha_A$, instead of sharing the entire market.

Let α^C denote the 'competitive' commission rate where $E(\Pi_A) = E(\Pi_B) = 0$. In the absence of a marginal cost of matching, $\alpha^C = 0$, giving rise to a unique Nash equilibrium in commission rates at $\alpha_B = \alpha_A = \alpha^C = 0$. The Bertrand duopoly outcome is summarised by proposition (2).

Proposition 2 *If $P_A = P_B = 1$, then the commission-setting subgame has a unique, pure strategy Nash equilibrium where $\alpha_B = \alpha_A = \alpha^C = 0$.*

Proof. To prove that $\alpha_B = \alpha_A = \alpha^C = 0$ is the unique, pure strategy Nash equilibrium of the game we show that $\alpha_B = \alpha_A = \alpha_0 > \alpha^C$ can never be an equilibrium. The proof is by contradiction. Let $(\alpha_A, \alpha_B) = (\alpha_0, \alpha_0)$, where $\alpha_0 > \alpha^C$. If A deviates from (α_0, α_0) by undercutting B, then $E(\Pi_A)_{|\alpha_A=\alpha_0-\varepsilon} = S(\alpha_0 - \varepsilon)$, where $E(\Pi_A)_{|\alpha_A=\alpha_0-\varepsilon} \to S\alpha_0$, as $\varepsilon \to 0$. Since $E(\Pi_A)_{|\alpha_A=\alpha_0} = \frac{1}{4}S(\alpha_0)$, it follows that $E(\Pi_A)_{|\alpha_A=\alpha_0-\varepsilon} > E(\Pi_A)_{|\alpha_A=\alpha_0}$. It is thus profitable for intermediary A to undercut from $\alpha_0 > \alpha^C$. Likewise, $E(\Pi_B)_{|\alpha_B=\alpha_0-\varepsilon} < E(\Pi_B)_{|\alpha_A=\alpha_A-\varepsilon}$. Hence, $\alpha_B = \alpha_A = \alpha_0 > \alpha^C$ cannot be an equilibrium. ■

4.4 Competing in Commission Rates: Fragmented Duopoly

Let $P_A \in (0, 1)$ and $P_B \in (0, 1)$. This gives rise to a distinctive market structure comprised by a competitive and a non-competitive market segment between which price discrimination is not possible. Hence intermediaries are neither pure monopolists, nor pure duopolists. Consider the incentives of intermediary

A when setting α_A. In the fragmented duopoly, in contrast to the Bertrand duopoly case, A never finds it optimal to set $\alpha_A = \alpha^C = 0$, given $\alpha_B < \alpha^M$. This is due to the fact that A can always relinquish the common trade matches to B and set the monopoly commission rate. The only traders that accept A's offers at $\alpha_A = \alpha^M$, given $\alpha_B < \alpha^M$, are traders in receipt of an A offer only. Let $E^M(\Pi_A)$ denote the expected profit from A's monopolistic market segment corresponding to this strategy. It follows directly from $E(T_A)$ that:

$$E^M(\Pi_A) = \alpha^M S P_A^2 \left(1 - P_B^2\right) \tag{20}$$

Profit level $E^M(\Pi_A)$ is always an option for A, introducing a positive lower bound to the profits A receives in the commission-setting game.

If A sets $\alpha_A = \alpha_0$ then the most profit A can ever expect to earn is $E(\Pi_A)_{|\alpha_0<\alpha_B} = \alpha_0 S P_A^2$, in the event that $\alpha_0 < \alpha_B$. Contrasting $E(\Pi_A)_{|\alpha_0<\alpha_B}$ with $E^M(\Pi_A)$ reveals that $\alpha_0 \geq \left(1 - P_B^2\right)\alpha^M$ must be satisfied in order for A to find it optimal to charge α_0, given α_B. If conversely, $\alpha_0 < \left(1 - P_B^2\right)\alpha^M$, then the maximum expected profit that A can receive by setting α_0 is lower than the profit from A's outside option, and hence α_0 is never optimal.

Let $\widehat{\alpha}_A$ denote the threshold level of α_A at which $\max E(\Pi_A)_{|\alpha_A} = E^M(\Pi_A)$. Hence:

$$\widehat{\alpha}_A = \left(1 - P_B^2\right)\alpha^M \tag{21}$$

It follows that if $\alpha_A < \widehat{\alpha}_A$, then the maximum profit that A can ever expect to generate from intermediation service provision overall is less than that under monopolistic commission setting that yields $E^M(\Pi_A)$. Hence, A never charges a commission below $\widehat{\alpha}_A$.

Furthermore, given R_a, A never finds it optimal to set $\alpha_A > \alpha^M$ since all its offers are subsequently rejected. The elimination of dominated strategies of A, conditional on P_A, yields $\alpha_A \in \left[\widehat{\alpha}_A, \alpha^M\right]$.

Similar arguments for B yield the following outside option for B:

$$E^M(\Pi_B) = \alpha^M S P_B^2 \left(1 - P_A^2\right) \tag{22}$$

Let $\widehat{\alpha}_B$ denote the threshold level of α_B at which $\max E(\Pi_B)_{|\alpha_B} = E^M(\Pi_B)$. Hence:

$$\widehat{\alpha}_B = \left(1 - P_A^2\right)\alpha^M \tag{23}$$

Symmetric arguments for B allow the elimination of dominated strategies, thereby yielding $\alpha_B \in \left[\widehat{\alpha}_B, \alpha^M\right]$. Note that, in general, the threshold levels are not symmetric since network sizes can be asymmetric in stage 1.

Consider how the threshold level of B is affected by the network size of A. The lower is P_A then the smaller the measure of common matches between A and B and thus the smaller the loss of trade matches from a deviation to the monopolistic strategy. Thus the attractiveness of B's outside option is increasing as $P_A \to 0$ making B less inclined to undercut A, thereby raising the deviation threshold level $\widehat{\alpha}_B$. At the limit, when $P_A = 0$, it follows directly from equation (23) that $\widehat{\alpha}_B = \alpha^M$. In other words, the smaller is P_A, *ceteris paribus*, then the weaker are competitive forces between A and B, inducing B to set α_B more monopolistically. At the one extreme, where $P_A = 0$, there is no competition, so B sets the monopoly commission. At the other extreme, where $P_A = 1$, there is no monopolistic segment, yielding the Bertrand pure strategy Nash equilibrium in which B sets the competitive commission rate.

The partitioned market structure is thus a hybrid of the two extremes of monopoly and Bertrand duopoly that gives rise to conflicting monopolistic and competitive forces. The combined effect of these forces is to make any fixed pair of commission rates (α_A, α_B) unstable. There is thus no pure strategy Nash equilibrium in commission rates when $P_A \in (0,1)$ and $P_B \in (0,1)$.

The conflicting incentives are illustrated in figure (1). The figure illustrates the case where $P_A > P_B$ and thus $\widehat{\alpha}_A > \widehat{\alpha}_B$. Consider the incentives of A. The optimal response to $\alpha_B > \alpha^M$ is to set $\alpha_A = \alpha^M$; but if $\alpha_A = \alpha^M$, then B has an incentive to undercut A for $\alpha_A \in (\widehat{\alpha}_B, \alpha^M]$. A also has an incentive to undercut B for $\alpha_B \in (\widehat{\alpha}_A, \alpha^M]$, driving down commission rates. A's incentive to undercut is restrained, however, by the existence of A's outside option. Hence, once α_B reaches $\widehat{\alpha}_A > \widehat{\alpha}_B$, A finds it optimal to deviate to $\alpha_A = \alpha^M$; but if $\alpha_A = \alpha^M$, then B has an incentive to undercut A...and so on.

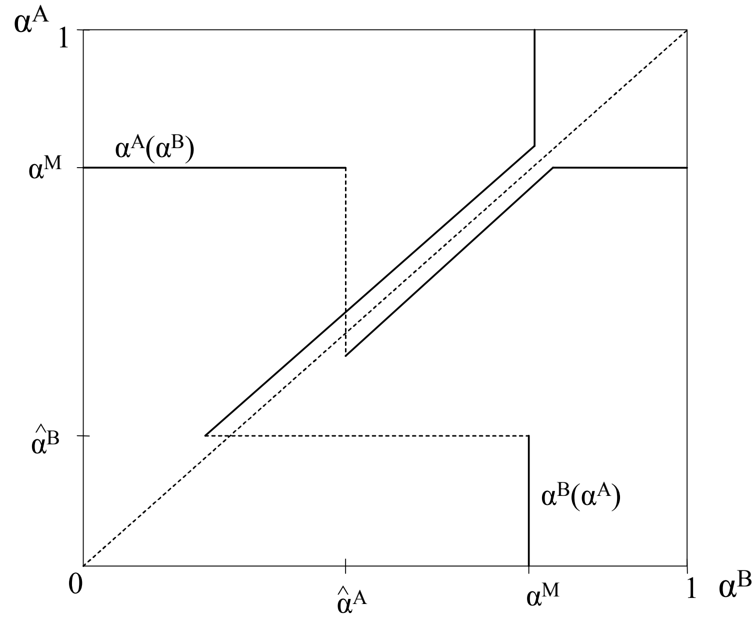

Figure 1: Strategic commission-setting in the fragmented duopoly.

The analysis is summarised by proposition (3).

Proposition 3 *If $P_A \in (0,1)$ and $P_B \in (0,1)$, then the commission-setting subgame has no pure strategy Nash equilibrium in commission rates.*

Proof. Proof is by contradiction. Let (α_A^*, α_B^*) reflect a pair of commission rates that constitute a pure strategy Nash equilibrium, where $\alpha_A^* \in \left[\widehat{\alpha}_A, \alpha^M\right]$ and $\alpha_B^* \in \left[\widehat{\alpha}_B, \alpha^M\right]$. Consider the optimal response of A to α_B^*, where α_B^* takes the following values: (a) $\widehat{\alpha}_A < \alpha_B^* \leq \alpha^M$ and (b) $\alpha_B^* = \widehat{\alpha}_A < \alpha^M$.
(a) If $\widehat{\alpha}_A < \alpha_B^* \leq \alpha^M$, then the optimal response of A is to undercut B by ε, where $\varepsilon \to 0$. The expected profit from A's undercutting strategy is $E\left(\Pi_A\right)_{|\alpha_A=\alpha_B^*-\varepsilon} = \left(\alpha_B^* - \varepsilon\right) SP_A^2 \to \alpha_B^* SP_A^2$, where $\alpha_B^* SP_A^2 > \alpha_B^* SP_A^2 \left(1 - \frac{3}{4}P_B^2\right) = \alpha_B^* \; E\left(\Pi_A\right)_{|\alpha_A=\alpha_B^*}$. Hence, A receives a higher expected profit from undercutting B by ε, than from matching α_B^*. It follows that A's optimal response to α_B^* is always $\alpha_A^* \equiv \alpha_B^* - \varepsilon$ if $\widehat{\alpha}_A < \alpha_B^* \leq \alpha^M$.
Given α_A^*, consider the incentives of B. Using an identical argument, it is optimal for B to deviate from α_B^* by undercutting α_A^* by ε, if $\widehat{\alpha}_B < \alpha_A^* \leq \alpha^M$. Hence, B's optimal response to α_A^* is $\alpha_B = \alpha_A^* - \varepsilon < \alpha_B^*$ and not α_B^*. Thus α_B^* is not an optimal response to α_A^*, where α_A^* is an optimal response to α_B^* .
(b) If $\alpha_B^* = \widehat{\alpha}_A < \alpha^M$, then the optimal response of A is to deviate to α^M, since $E\left(\Pi_A\right)_{|\alpha_A<\widehat{\alpha}_A} < E^M\left(\Pi_A\right)$; but then it follows that intermediary B finds it optimal to deviate to $\alpha_B = \alpha^M - \varepsilon > \alpha_B^*$. Hence, the optimal reply to α_B^* is $\alpha_A^* \equiv \alpha^M$, but the optimal reply of B to α_A^* not α_B^*.
Similar arguments apply for B's optimal response to α_A^*. It follows from the above that there is no pure strategy Nash equilibrium in commission rates. ■

4.5 Randomising Commission Rates

This section characterises the unique, mixed strategy Nash equilibrium in intermediaries' commission rates. Intuitively, randomisation of commission rates prevents rival intermediaries from systematically undercutting each other.

Let $H(\alpha_A)$ and $F\left(\alpha_B\right)$ denote the cumulative distribution functions used to randomise commission rates α_A and α_B, respectively, where $F(\cdot)$ and $H(\cdot)$ are continuous and have the following features:

$$\begin{aligned} H(\widehat{\alpha}_A) &= F\left(\widehat{\alpha}_B\right) = 0 \\ H(\alpha^M) &= F\left(\alpha^M\right) = 1 \\ dH/d\alpha_A &> 0;\ dF/d\alpha_B > 0 \end{aligned} \tag{24}$$

Since the distributions are continuous, the probability of A and B setting identical commission[4] rates is 0. Let α_A be a random draw from $H(\cdot)$. It follows from $F(\cdot)$ that:

$$\begin{aligned} &\Pr(\alpha_B < \alpha_A) = F(\alpha_A) \\ &\Pr(\alpha_B > \alpha_A) = 1 - F(\alpha_A) \\ &\Pr(\alpha_A = \alpha_B) = 0 \end{aligned} \tag{25}$$

[4]This implies that while coordination failure with probability $\frac{1}{4}$ is expected in stage 3 in the event where traders receive two offers and commission rates are equal, the randomisation of commission rates in stage 2 ensures that this event occurs with zero probability. Thus coordination failure does not arise in equilibrium.

Given the probabilities described in (25), we seek to find the optimal distribution $H(\cdot)$ for intermediary A, that keeps expected profits constant over the distribution, and similarly, the optimal $F(\cdot)$ that keeps B's expected profits constant.

The mixed strategy Nash equilibrium depends on the relative values of $\widehat{\alpha}_A$ and $\widehat{\alpha}_B$, which reflect the relative values of P_A and P_B. There are three cases: (i) $\widehat{\alpha}_A > \widehat{\alpha}_B$, where $P_A > P_B$ (ii) $\widehat{\alpha}_A < \widehat{\alpha}_B$, where $P_A < P_B$ and (iii) $\widehat{\alpha}_A = \widehat{\alpha}_B$, where $P_A = P_B$.

In (i) and (ii) intermediaries A and B are shown to randomise[5] over different distributions, while in case (iii) the symmetry in network sizes implies $H(\cdot) = F(\cdot) \equiv G(\cdot)$. Following the methodology used in Baye and De Vries (1992), but allowing for asymmetric network sizes[6], yields the unique, mixed strategy Nash equilibrium summarised in proposition (4).

Proposition 4 *(a) If $P_A = P_B = P \in (0,1)$, then there exists a mixed strategy Nash equilibrium, in which intermediaries choose their commission rate randomly from the same distribution:*

$$G(\alpha) = \frac{\alpha - \left(1 - P^2\right)\alpha^M}{\alpha P^2}, \text{ where } \alpha \in \left[\widehat{\alpha}, \alpha^M\right] \tag{26}$$

where $\widehat{\alpha} = \left(1 - P^2\right)\alpha^M$.
(b) If $P_A \in (0,1)$, $P_B \in (0,1)$ and $P_A \neq P_B$, then there is a unique, mixed strategy Nash equilibrium, in which intermediaries A and B choose their commission rate randomly from distributions $H(\alpha_A)$ and $F(\alpha_B)$, respectively, where:

$$H(\alpha_A) = \frac{\alpha_A - \left(1 - P_B^2\right)\alpha^M}{\alpha_A P_B^2}, \text{ where } \alpha_A \in \left[\widehat{\alpha}_A, \alpha^M\right] \tag{27}$$

$$F(\alpha_B) = \frac{\alpha_B - \left(1 - P_A^2\right)\alpha^M}{\alpha_B P_A^2}, \text{ where } \alpha_B \in \left[\widehat{\alpha}_B, \alpha^M\right] \tag{28}$$

where $\widehat{\alpha}_A = \left(1 - P_B^2\right)\alpha^M$ and $\widehat{\alpha}_B = \left(1 - P_A^2\right)\alpha^M$.

Proof. The cases of symmetric and asymmetric network sizes are examined in turn:
(a) The optimality of $G(\cdot)$ for both intermediaries when $P_A = P_B = P$, such that $H(\cdot) = G(\cdot)$ and $F(\cdot) = G(\cdot)$ in equilibrium, can be shown by examining expected operating profit of intermediaries in the ranges $[0, \widehat{\alpha}]$, $\left[\widehat{\alpha}, \alpha^M\right]$, and greater than α^M. From demand $E(T_A)$ and probabilities (25), it follows that:

[5] Note that the mixed strategy Nash equilibrium does not imply randomisation of commission rates across offers made to traders. Each intermediary sets a unique commission rate that is common to all offers made, where this unique commission rate is a random draw from the relevant distribution in proposition (4) in equilibrium.

[6] Since intermediaries' network investment decisions in stage 1 are not necessarily symmetric, we solve for the unique, mixed strategy Nash equilibrium for general P_A and P_B without imposing a restriction of symmetry.

$$E(\Pi_A) = \begin{cases} \alpha_A SP^2 & if \quad \alpha_A < \widehat{\alpha} \leq \alpha^M \\ F(\alpha_A)\left(\alpha_A SP^2(1-P^2)\right) + (1-F(\alpha_A))\left(\alpha_A SP^2\right) & if \quad \alpha_A \in \left[\widehat{\alpha}, \alpha^M\right] \\ 0 & if \quad \alpha_A > \alpha^M \end{cases}$$

Since it is never optimal for A to set α_A below $\widehat{\alpha}$, the probability that $\alpha_A < \widehat{\alpha}$ is zero. Hence, randomisation over range $\alpha_A \in \left[\widehat{\alpha}, \alpha^M\right]$ need only be considered. $E(\Pi_A)_{|\alpha_A \in [\widehat{\alpha}, \alpha^M]}$ can be simplified to $\alpha_A SP^2\left[1 - P^2 F(\alpha_A)\right]$. Hence, A chooses $H(\cdot)$ to maximise $E(\Pi_A)$ over $\left[\widehat{\alpha}, \alpha^M\right]$:

$$\max_{dH} E(\Pi_A) = \int_{\widehat{\alpha}}^{\alpha^M} \left[\alpha_A SP^2\left(1 - P^2 F(\alpha_A)\right)\right] dH \tag{29}$$

Recalling that $H(\widehat{\alpha}) = F(\widehat{\alpha}) = 0$ and $H(\alpha^M) = F\left(\alpha^M\right) = 1$, the solution to (29) yields constant expected profit for A over $\left[\widehat{\alpha}, \alpha^M\right]$ equal to $E^M(\Pi_A)$. The analysis is symmetric for B so:

$$\max_{dH} E(\Pi_A) = \max_{dF} E(\Pi_B) = \alpha^M SP^2\left(1 - P^2\right) \tag{30}$$

A's expected payoff under any random draw $\alpha_A \in \left[\widehat{\alpha}, \alpha^M\right]$ must be equal to (30). Similarly for random draw $\alpha_B \in \left[\widehat{\alpha}, \alpha^M\right]$ by B. Solving from (29) and (30) yields optimal distributions $H(\cdot) = G(\cdot)$ and $F(\cdot) = G(\cdot)$, where distribution $G(\cdot)$ is described by equation (26).
(b) The optimality of $H(\cdot)$ and $F(\cdot)$ in (27) and (28), respectively, follows similarly for the case where $P_A \neq P_B$. Let $P_A < P_B$ and hence $\widehat{\alpha}_A < \widehat{\alpha}_B$. From demand $E(T_A)$ and probabilities (25), it follows that:

$$E(\Pi_A) = \begin{cases} \alpha_A SP_A^2 & if \quad \alpha_A < \widehat{\alpha}_A \\ \alpha_A SP_A^2 & if \quad \alpha_A \in [\widehat{\alpha}_A, \widehat{\alpha}_B] \\ F(\alpha_A)\left(\alpha_A SP_A^2(1-P_B^2)\right) + (1-F(\alpha_A))\left(\alpha_A SP_A^2\right) & if \quad \alpha_A \in \left[\widehat{\alpha}_B, \alpha^M\right] \\ 0 & if \quad \alpha_A > \alpha^M \end{cases}$$

The probability that $\alpha_A < \widehat{\alpha}_A$ is zero, but there now exists a range of commission rates $[\widehat{\alpha}_A, \widehat{\alpha}_B]$, where A finds it optimal to follow an undercutting strategy but B finds the monopolistic strategy optimal. This gives rise to a positive probability that $\alpha_A < \widehat{\alpha}_B \leq \alpha^M$. Hence A chooses $H(\cdot)$ to maximise the following:

$$\max_{dH} E(\Pi_A) = \int_{\widehat{\alpha}_B}^{\alpha^M} \left[\alpha_A SP_A^2\left(1 - P_B^2 F(\alpha_A)\right)\right] dH + \int_{\widehat{\alpha}_A}^{\widehat{\alpha}_B} \left(\alpha_A SP_A^2\right) dH \tag{31}$$

Solving (31) yields constant expected profit for A:

$$\max_{dH} E(\Pi_A) = E^M(\Pi_A) = \alpha^M SP_A^2\left(1 - P_B^2\right) \tag{32}$$

B chooses $F(\cdot)$ to maximise:

$$\max_{dF} E(\Pi_B) = \int_{\widehat{\alpha}_B}^{\alpha^M} \left[\alpha_B SP_B^2\left(1 - P_A^2 H(\alpha_B)\right)\right] dF \tag{33}$$

Solving (33) yields constant expected profit for B:

$$\max_{dF} E(\Pi_B) = E^M(\Pi_B) = \alpha^M S P_B^2 \left(1 - P_A^2\right) \quad (34)$$

From (31), (32), (33) and (34) it follows that the optimal strategy of A and B is to randomise commission rates according to distributions (27) and (28), respectively. ■

Intuitively, randomisation of commission rates prevents intermediaries from systematically undercutting each other, thereby allowing the expected operating profits $E^M(\Pi_A)$ and $E^M(\Pi_B)$ to be attained in equilibrium. That is, randomisation allows each intermediary to attain expected operating profit equal to that which would arise from the monopolistic segment of their network under the monopoly commission rate.

This is summarised by Corollary (5).

Corollary 5 *In the unique, mixed strategy Nash equilibrium in commission rates:*
(a) If $P_A = P_B = P$, then each intermediary expects to receive constant operating profit equal to $\alpha^M S P^2 \left(1 - P^2\right)$.
(b) If $P_A \neq P_B$, then intermediaries A and B expect to receive constant operating profit equal to $E^M(\Pi_A) = \alpha^M S P_A^2 \left(1 - P_B^2\right)$ and $E^M(\Pi_B) = \alpha^M S P_B^2 \left(1 - P_A^2\right)$, respectively.

Proof. This follows directly from the proof of proposition (4). ■

An implication of randomised commission rates is that traders' payoffs from indirect trade through A and B, respectively, are random variables that mirror $H(\alpha_A)$ and $F(\alpha_B)$ when $P_A \neq P_B$ and $G(\alpha)$, when $P_A = P_B$. The conflicting monopolistic and competitive forces for commission-setting in the fragmented duopoly give rise to a unique, non-cooperative mixed strategy Nash equilibrium where intermediaries randomise their commission rates. The range of commission rates over which randomisation takes place has an upper bound of α^M, imposed by R_a, and a positive lower bound due to intermediaries' monopolistic outside option. While the upper bound for both commission rates is exogenously determined by traders' direct trade option, that in turn hinges on the level of information costs in the market, the lower bounds hinge on network sizes. In particular, an intermediary with a larger network, enjoys a relatively larger set of exclusive trade matches, and thus behaves more monopolistically when randomising commission rates than does an intermediary with a smaller network. Each intermediary's expected operating profit is constant and corresponds to expected monopoly profit from the monopolistic market segment. Intermediaries invest in network development in stage 1, anticipating the implications of their decisions. It is the Nash equilibrium in network sizes to which we now turn.

5 Stage 1 - Nash Equilibrium in Network Sizes

In this section we seek a Nash equilibrium, or Nash equilibria, in network sizes, where these are set simultaneously and non-cooperatively by competing intermediaries, each taking the network size of his rival, and the offer acceptance strategy of traders, as given.

5.1 Network Investment Cost

Intermediaries are assumed to have access to the same technology for developing a network of contacts, where the total investment cost for a network of size P_I is denoted by $C(P_I)$. The network investment decisions of intermediaries are analysed under two cost specifications:

(a) Linear cost: $C(P_I) = 2P_I c$, where $c > 0$. Hence:

$$\frac{\partial C_I}{\partial P_I} = 2c > 0 \tag{35}$$

The marginal cost of network expansion is constant.

(b) Convex cost: $C(P_I) = 2P_I c(i, P_I)$, where $c(i, P_I) = \gamma i^\alpha P_I^2$ and $\alpha \geq 1$, $\gamma > 0$. Hence:

$$C(P_I) = 2\gamma i^\alpha P_I^3 \tag{36}$$

$$\frac{\partial C_I}{\partial P_I} = 6\gamma i^\alpha P_I^2 > 0, \quad \frac{\partial^2 C_I}{\partial P_I^2} = 12\gamma i^\alpha P_I > 0 \tag{37}$$

$$\frac{\partial C_I}{\partial i} = 2\alpha\gamma i^{\alpha-1} P_I^3 > 0 \tag{38}$$

The marginal cost of network expansion is thus increasing monotonically in the level of information costs and network size, while convexity in network size is assumed. Cost parameter γ is a scale factor. In addition, let the probability of direct matching $q(i)$ take the functional form $q(i) = 1 - i^\delta$, where $\delta > \alpha \geq 1$.

5.2 Stage 1 Expected Profit

In stage 1, intermediary A chooses P_A to maximise stage 1 expected profit, denoted by $E^1(\Pi_A)$, taking P_B and $\{R_a, R_s\}$ as given. Similarly, intermediary B chooses P_B to maximise $E^1(\Pi_B)$, taking P_A and $\{R_a, R_s\}$ as given. Intermediaries anticipate expected operating profit levels $E^M(\Pi_A)$ and $E^M(\Pi_B)$, respectively, to arise from the stage 2 commission-setting subgame. Hence, $E^1(\Pi_A)$ and $E^1(\Pi_B)$ are given by:

$$\begin{aligned} E^1(\Pi_A) &= E^M(\Pi_A) - C(P_A) \\ &= \alpha^M S P_A^2 \left(1 - P_B^2\right) - C(P_A) \end{aligned} \tag{39}$$

and:

$$E^1(\Pi_B) = E^M(\Pi_B) - C(P_B) \quad (40)$$
$$= \alpha^M S P_B^2 \left(1 - P_A^2\right) - C(P_B)$$

where $\alpha^M = 1 - q(i)$.

Equations (39) and (40) offer a general description of expected stage 1 profit, where the three polar market structures correspond to different configurations of P_A and P_B:

1. Monopoly: if $P_B = 0$ and $P_A \in [0,1]$ then (39) and (40) yield $E^1(\Pi_A) = \alpha^M S P_A^2 - C(P_A)$ and $E^1(\Pi_B) = 0$. Thus B is inactive and A is a monopolist and *vice versa* if $P_A = 0$ and $P_B \in [0,1]$.

2. Bertrand duopoly: if $P_A = P_B = 1$ then (39) and (40) yield $E^1(\Pi_A) = E^1(\Pi_B) = -C(1) < 0$. Hence, $P_A = P_B = 1$ can never constitute a Nash equilibrium in network sizes.

3. Fragmented duopoly in intermediation services, where $P_A \in (0,1)$ and $P_B \in (0,1)$. Equations (39) and (40) allow for both symmetric and asymmetric network size selection.

It follows that Bertrand duopoly can never arise in a subgame perfect equilibrium (SPE) of the game, since it does not constitute a Nash equilibrium in stage 1. Only monopoly and fragmented duopoly are thus consistent with SPE.

5.3 Linear Network-Building Costs

Substituting $C(P_I) = 2P_I c$ into (39) and (40) yields:

$$E^1(\Pi_A) = \alpha^M S P_A^2 \left(1 - P_B^2\right) - 2cP_A \quad (41)$$
$$E^1(\Pi_B) = \alpha^M S P_B^2 \left(1 - P_A^2\right) - 2cP_B \quad (42)$$

Equations (41) and (42) show that the expected profit of an intermediary is decreasing in the network size of the rival intermediary and increasing in his own network size. Intuitively, the larger the network size of the rival, the greater the measure of common matches as a result of network overlap; and hence the lower the expected operating profit arising from the mixed strategy Nash equilibrium in commission rates.

Examination of (41) and (42) shows there is no pair of network sizes (P_A^*, P_B^*) that are best responses to each other and simultaneously satisfy $P_A^* \in (0,1]$ and $P_B^* \in (0,1]$ when network-building costs are linear.

The results are summarised by propositions (6) and (7).

Proposition 6 *If network-building cost is linear, then there is no pure strategy Nash equilibrium in which both intermediaries are active.*

Proof. Let $\overline{P}_B(P_A)$ and $\overline{P}_A(P_B)$ describe the locus of network size pairs along which $E^1(\Pi_A) = 0$ and $E^1(\Pi_B) = 0$, respectively. Hence, for a given network size P_A, $\overline{P}_B(P_A)$ gives the threshold level of P_B above which $E^1(\Pi_A)_{|P_A} < 0$. Similarly, for given network size P_B, $\overline{P}_A(P_B)$ gives the threshold level of P_A above which $E^1(\Pi_B)_{|P_B} < 0$. Rearranging $E^1(\Pi_A) = 0$ and $E^1(\Pi_B) = 0$ from equations (41) and (42) yields:

$$\overline{P}_B(P_A) = \left(1 - \frac{2c}{SP_A\alpha^M}\right)^{\frac{1}{2}} \tag{43}$$

$$\overline{P}_A(P_B) = \left(1 - \frac{2c}{SP_B\alpha^M}\right)^{\frac{1}{2}} \tag{44}$$

Proof by contradiction. Let (P_A^*, P_B^*) reflect a pure strategy Nash equilibrium in network sizes where $P_A^* \in (0,1)$ and $P_B^* \in (0,1)$.
Consider the incentives of intermediary A. If $\overline{P}_B(P_A^*) < P_B^*$, then it follows that $E^1(\Pi_A)_{|P_A^*} < 0$; but if A is making losses, P_A^* cannot be an optimal reply to P_B^*. Recall that $E^1(\Pi_A)$ is increasing in P_A. It follows that the maximum expected profit that A can attain, given P_B^*, is that which corresponds to $P_A = 1$. If $E^1(\Pi_A)_{|P_B^*, P_A=1} > 0$, then A's optimal reply to P_B^* is $P_A(P_B^*) = 1$. If $E^1(\Pi_A)_{|P_B^*, P_A=1} < 0$, then A's optimal reply is $P_A(P_B^*) = 0$.

Suppose $P_A(P_B^*) = 1$. Then B is making losses under $P_B^* > 0$. Thus B's optimal reply to $P_A(P_B^*) = 1$ is $P_B^*(1) = 0$.
Suppose instead that $P_A(P_B^*) = 0$. Then $P_B^* \in (0,1)$ is not an optimal reply to $P_A(P_B^*) = 0$ since B can raise $E^1(\Pi_B)$ by increasing P_B^* to 1. Thus B's optimal reply to $P_A(P_B^*) = 0$ is $P_B^*(0) = 1$.

If instead $P_B^* < \overline{P}_B(P_A^*)$, then it follows that $E^1(\Pi_A)_{|P_A^*} > 0$; but then interior network size $P_A^* \in (0,1)$ is not an optimal reply to P_B^*. A's optimal reply is thus $P_A(P_B^*) = 1$ and arguments apply as above.

We can thus conclude that neither is $P_A^* \in (0,1)$ an optimal reply to $P_B^* \in (0,1)$, nor is $P_B^* \in (0,1)$ an optimal reply to $P_A^* \in (0,1)$. Moreover, since $P_B^*(1) = P_A^*(1) = 0$, $(P_A^*, P_B^*) = (1,1)$ cannot be a Nash equilibrium either. Thus (P_A^*, P_B^*) cannot constitute a Nash equilibrium where both network sizes are non-zero. It follows that there is no Nash equilibrium in which both intermediaries are active. ■

Proposition 7 *If network-building cost is linear and provided $c < \frac{1}{2}S\alpha^M$, then there are two pure strategy Nash equilibria where $(P_A^*, P_B^*) = (1,0)$ and $(P_A^*, P_B^*) = (0,1)$.*

Proof. It follows from proposition (6) that (P_A^*, P_B^*) cannot constitute a Nash equilibrium where both network sizes are non-zero. The only remaining candidate Nash equilibria are $(P_A^*, P_B^*) = (1,0)$ and $(P_A^*, P_B^*) = (0,1)$. If $P_A^* = 1$, then there is no scope for B to gain exclusive trade matches from investment in P_B. All resulting trade matches are common and thus no profit can be attained. The optimal reply of B is thus $P_B^*(1) = 0$. A symmetric argument applies where

$P_B^* = 1$. Hence, $(P_A^*, P_B^*) = (1, 0)$ and $(P_A^*, P_B^*) = (0, 1)$ constitute the two pure strategy Nash equilibria in network sizes. Moreover, it follows directly from equations (41) and (42) that $c < \frac{1}{2} S\alpha^M$ must be satisfied for expected profit to be positive in equilibrium for the monopolist intermediary. ■

The analysis has shown that under linear costs of network expansion the equilibrium outcome of the game is monopolisation of the market by either A or B. The non-convexity in network-building costs provides incentives for intermediaries to increase network size without bound, other than the constraint imposed by market size. Each intermediary provides complete coverage of the market, when active, thereby preventing the rival intermediary from gaining any exclusive trade matches.

Substituting $P_A = 1$ and $P_B = 1$ into (43) and (44), respectively, yields:

$$\overline{P} = \overline{P}_B(1) = \overline{P}_A(1) = \left(1 - \frac{2c}{S\alpha^M}\right)^{\frac{1}{2}} \tag{45}$$

$\overline{P}$ denotes the threshold level of P_A below which it is optimal for B to invest in a network size that covers the whole market and above which B is inactive. By symmetry, $\overline{P}$ is also the threshold for P_B. Figure (2) illustrates the reaction functions[7] of A and B. The $E^1(\Pi_A) = 0$ and $E^1(\Pi_B) = 0$ loci pin down threshold level $\overline{P}$ for the two intermediaries and confirm the monopolisation of the market by either A or B (at NE_2 and NE_1, respectively) is the only market outcome consistent with profit maximisation under linear costs of network expansion.

5.4 Convex Network-Building Costs

This section shows how convexity in the costs of developing a network provides sufficient incentives for 'restraint' in network investment, so as to allow both intermediaries to survive in a fragmented duopoly with incomplete network overlap.

Note that in the analysis that follows we assume γ is sufficiently low relative to S so that $E^1(\Pi_A)$ and $E^1(\Pi_B)$ are positive in equilibrium. The results are summarised by proposition (8).

Proposition 8 *If network-building cost is convex, then there exists a unique, pure strategy Nash equilibrium in network sizes where:*

$$P_A^* = P_B^* = \frac{3\gamma}{2Si^{\delta-\alpha}} \left[\left(4\left(\frac{Si^{\delta-\alpha}}{3\gamma}\right)^2 + 1 \right)^{\frac{1}{2}} - 1 \right] \in (0, 1)$$

Proof. Substituting $C(P_I) = 2P_I c(i, P_I) = 2\gamma i^\alpha P_I^3$ and $q(i) = 1 - i^\delta$ into (39) and (40), where $\delta > \alpha \geq 1$ and $\gamma > 0$, yields:

$$E^1(\Pi_A) = i^\delta S P_A^2 \left(1 - P_B^2\right) - 2\gamma i^\alpha P_A^3 \tag{46}$$

$$E^1(\Pi_B) = i^\delta S P_B^2 \left(1 - P_A^2\right) - 2\gamma i^\alpha P_B^3 \tag{47}$$

[7] The figure is drawn for parameter values $S = 10$, $c = 1$ and $\alpha^M = 0.7$.

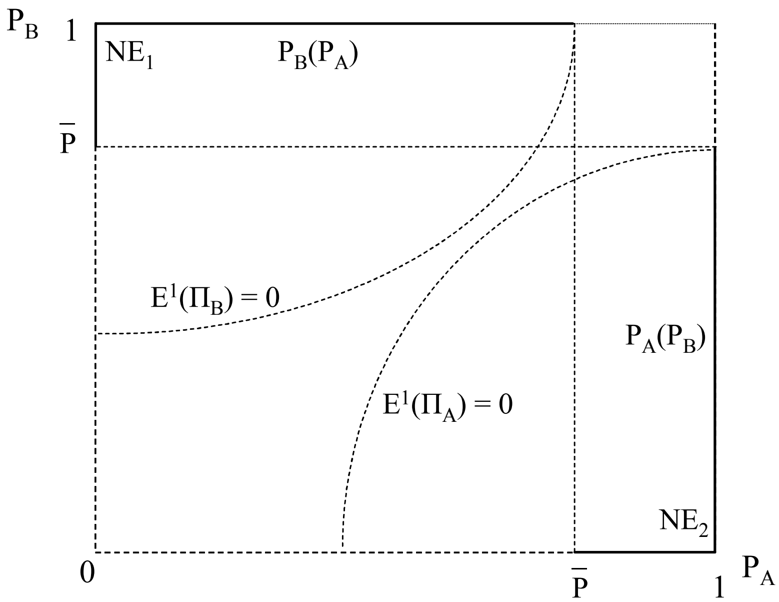

Figure 2: Monopoly Nash equilibria in network sizes.

The network size reaction functions of A and B, denoted by $P_A(P_B)$ and $P_B(P_A)$, respectively, are derived from the first order conditions:

$$\frac{\partial E^1(\Pi_A)}{\partial P_A}_{|P_B} = 2i^\delta S P_A\left(1 - P_B^2\right) - 6\gamma i^\alpha P_A^2 = 0 \tag{48}$$

$$\frac{\partial E^1(\Pi_B)}{\partial P_B}_{|P_A} = 2i^\delta S P_B\left(1 - P_A^2\right) - 6\gamma i^\alpha P_B^2 = 0 \tag{49}$$

The first order conditions (48) and (49) simplify to give:

$$P_A(P_B) = \frac{Si^{\delta-\alpha}}{3\gamma}\left(1 - P_B^2\right) \tag{50}$$

$$P_B(P_A) = \frac{Si^{\delta-\alpha}}{3\gamma}\left(1 - P_A^2\right) \tag{51}$$

Solving the reaction functions simultaneously, and confirming that we have a maximum[8], yields Nash equilibrium network sizes (P_A^*, P_B^*) in terms of information costs i, and parameters γ, S, δ and α:

$$P^* = P_A^* = P_B^* = \frac{3\gamma}{2Si^{\delta-\alpha}}\left[\left(4\left(\frac{Si^{\delta-\alpha}}{3\gamma}\right)^2 + 1\right)^{\frac{1}{2}} - 1\right] \tag{52}$$

Discarding complex and negative solutions to (48) and (49) confirms that (52) describes the unique Nash equilibrium in network sizes. ■

It follows directly from the reaction functions, (50) and (51), that network sizes are strategic substitutes. An increase in the network size of A gives rise to a strategic contraction in the network investment of B, and *vice versa.* Intuitively, when intermediary A invests in a larger network, the expected overlap between the two networks is larger, thereby lowering $E^M(\Pi_B)$. Hence, for given investment cost $C(P_B)$, B can expect a lower revenue than before, thereby inducing a network contraction.

Moreover, network sizes are increasing in trade surplus S, declining in cost parameter γ, and increasing in information cost i (since $\delta > \alpha \geq 1$).

Figure (3) illustrates[9] $P_A(P_B)$ and $P_B(P_A)$ and depicts a unique, pure strategy Nash equilibrium in network sizes, in which both intermediaries invest symmetrically in network development. This arises from the symmetry in the costs incurred by A and B. It is straightforward to show that when cost parameter γ varies across intermediaries, the intermediary with the lower cost has a larger network size in equilibrium.

Nash equilibrium network sizes (P_A^*, P_B^*) in terms of information costs i, and parameters γ, S, δ and α are:

$$P^* = P_A^* = P_B^* = \frac{3\gamma}{2Si^{\delta-\alpha}}\left[\left(4\left(\frac{Si^{\delta-\alpha}}{3\gamma}\right)^2 + 1\right)^{\frac{1}{2}} - 1\right] \tag{53}$$

[8] The second derivatives and confirmation that P_A^* and P_B^* correspond to a maximum can be found in Appendix B.

[9] The figure is drawn for parameter values $\gamma = 1$, $\delta = 4$, $\alpha = 2$, $S = 4$ and $i = 0.8$.

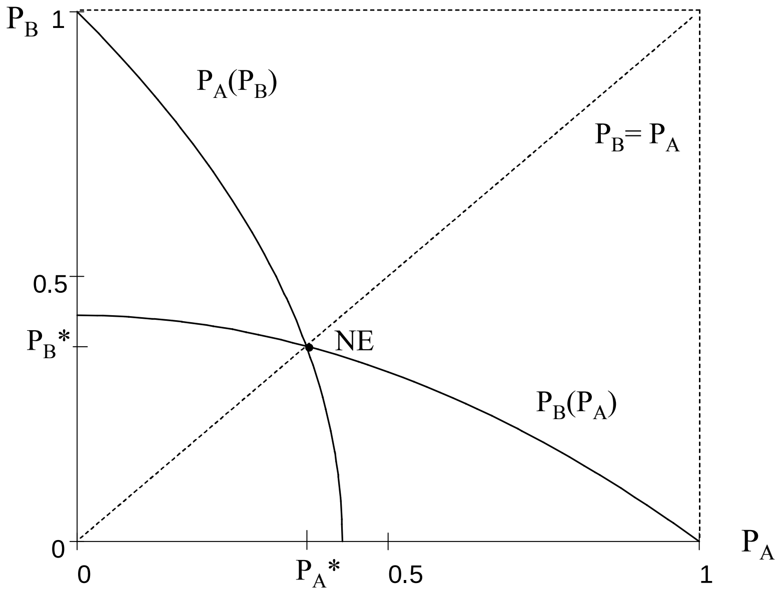

Figure 3: Fragmented duopoly Nash equilibrium in network sizes.

Let P^M denote the monopoly network size that prevails under the same cost specification, where:

$$P^M = \frac{Si^{\delta-\alpha}}{3\gamma} > P^* \tag{54}$$

Equations (53) and (54) describe the equilibrium monopoly and duopoly network sizes for network cost specification (36). Figure (4) illustrates the path of network size with information costs in the two cases for parameter values $\gamma = 1$, $\delta = 4$, $\alpha = 2$, $S = 4$.

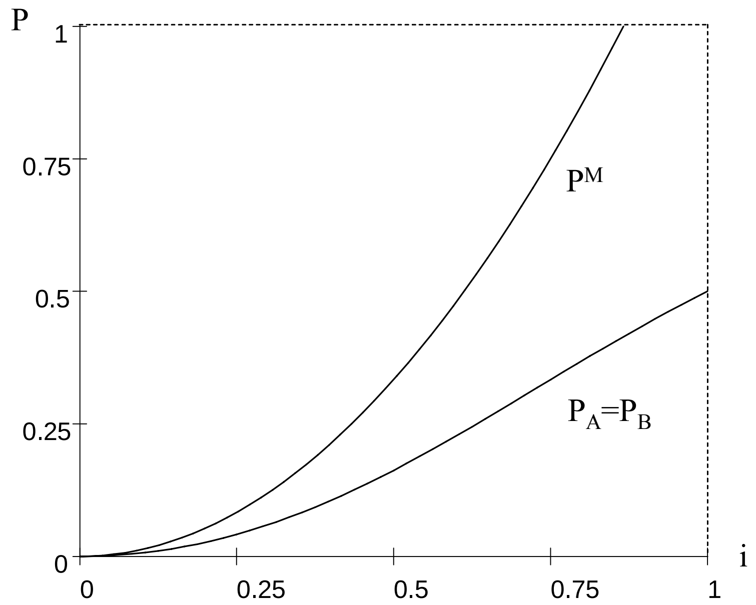

Figure 4: Network size and information cost.

To summarise, the analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.

6 Conclusion

This paper presents a new theoretical framework to analyse the strategic interaction between two information intermediaries who compete in commission rates and network size. The intermediaries are assumed to have symmetric access to an information technology that allows them to develop contacts with importers and exporters who match uniquely in pairs. Intermediaries have the opportunity to invest in a network of contacts and subsequently compete in commission rates before making offers of intermediation to members in their network. Traders select between intermediaries *ex post*, when uncertainty in the realisation of a match is resolved.

The analysis delivers the following results. First, the model suggests that network competition between information intermediaries has a distinctive market structure, where intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. Traders in the network overlap receive two intermediation offers, while other members are exclusive to one intermediary and thus receive only one offer of intermediation. Traders in receipt of two intermediation offers play a coordination game when deciding which offer to select. The information frictions in the model make it impossible for traders to signal their decisions to each other, so there are multiple equilibria to the game. The model thus emphasises the role of 'beliefs' in determining market outcomes when there are information frictions and traders gain from making coordinated decisions.

Second, the coordination game of traders presents the possibility of coordination failure between trade pairs, even though both traders are members of both networks and this is known to both.

Third, we show that if traders choose to accept the offer from the intermediary with the lower commission and randomise when commissions are the same, then intermediaries have an incentive to undercut each other. Moreover, intermediaries' inability to price discriminate between the competitive and non-competitive market segments, gives rise to an undercutting game, which has no pure strategy Nash equilibrium due to the option to charge the monopoly commission to exclusive contacts, and relinquish the overlap to the rival. Randomising over the strategy space of commission rates results in a mixed strategy Nash equilibrium yielding expected profit equal to that which would have been earned in the monopolistic outside option. In this mixed strategy Nash equilibrium, an intermediary with a larger network sets a higher commission rate, on average, than an intermediary with a smaller network. Moreover, average commission rates lie below the monopoly commission rate. Hence traders who match indirectly enjoy a trade surplus over and above their outside option.

The multiplicity of equilibria of the coordination game and the randomisation of commission rates that results shows how information problems can give rise to endogenous uncertainty in market outcomes.

Finally, competition is affected by the technology of network development. The analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.

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Appendix A. The Coordination Game

Recall the payoff matrix summarising the payoff structure of traders' simultaneous and non-cooperative coordination game in stage 3:

		$\mathbf{M}_j$: **A**	$\mathbf{M}_j$: **B**
$\mathbf{X}_j$	**A**	$\frac{S}{2}(1-\alpha_A), \frac{S}{2}(1-\alpha_A)$	$\frac{S}{2}\left(1-\alpha^M\right), \frac{S}{2}\left(1-\alpha^M\right)$
	B	$\frac{S}{2}\left(1-\alpha^M\right), \frac{S}{2}\left(1-\alpha^M\right)$	$\frac{S}{2}(1-\alpha_B), \frac{S}{2}(1-\alpha_B)$

There are two pure strategy Nash Equilibria, (A, A) and (B, B), and one symmetric, mixed strategy Nash equilibrium. Suppose X_j selects A with probability λ (and B with $1-\lambda$) and M_j selects A with probability μ (and B with $1-\mu$). For probabilities (λ^*, μ^*) to form a mixed strategy Nash equilibrium the expected payoffs from mixing between A and B must be equalised for each trader.

Equalising the expected payoff from the mixed strategy of X_j yields:

$$\lambda(1-\alpha_A)+(1-\lambda)\left(1-\alpha^M\right)=(1-\lambda)(1-\alpha_B)+\lambda\left(1-\alpha^M\right) \tag{55}$$

Rearranging (55) yields:

$$\lambda^* = \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} \tag{56}$$

Equalising the expected payoff from the mixed strategy of M_j yields:

$$\mu(1-\alpha_A) + (1-\mu)\left(1-\alpha^M\right) = (1-\mu)(1-\alpha_B) + \mu\left(1-\alpha^M\right) \tag{57}$$

Rearranging (57) yields:

$$\mu^* = \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} \tag{58}$$

Since $\lambda^* = \mu^*$, the unique mixed strategy Nash equilibrium is symmetric.

The expected payoffs of X_j and M_j in the mixed strategy Nash equilibrium are found by substituting λ^* and μ^* into each side of (55) and (57) yields:

$$\begin{aligned} E(\Pi_j)_{|\lambda^*,\mu^*} &= \frac{S}{2}\left[\frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A}(1-\alpha_A) + \frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A}\left(1-\alpha^M\right)\right] \\ &= \frac{S}{2}\left[\frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A}(1-\alpha_B) + \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A}\left(1-\alpha^M\right)\right] \end{aligned}$$

Appendix B. Second Derivatives

The second derivatives that follow from (48) and (49) are:

$$\frac{\partial^2 E^1(\Pi_A)}{\partial P_A^2}_{|P_B} = 2i^\delta S\left(1-P_B^2\right) - 12\gamma i^\alpha P_A \tag{59}$$

$$\frac{\partial^2 E^1(\Pi_B)}{\partial P_B^2}_{|P_A} = 2i^\delta S\left(1-P_A^2\right) - 12\gamma i^\alpha P_B \tag{60}$$

To have a maximum, (59) and (60) must be negative. Hence, the following must hold in equilibrium:

$$P_A > \frac{S i^{\delta-\alpha}}{6\gamma}\left(1-P_B^2\right) \tag{61}$$

$$P_B > \frac{S i^{\delta-\alpha}}{6\gamma}\left(1-P_A^2\right) \tag{62}$$

Comparing (61) and (62) with (50) and (51) confirms the constraints are satisfied in equilibrium and thus that P_A^* and P_B^* correspond to a maximum.

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