Algorists, algorithms, and complexity: an exploration of the Shavian critque of discrete state computation

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Abstract

The status of computational theory in ecological psychology has been, and continues to be, a source of controversy. Over a period of more than thirty years, Robert Shaw and his colleagues have developed a powerful, negative critique of computation, based in part on the idea that computational theory cannot capture central aspects of the co-implicative structure of the relationships between animals and their environments. Two aspects of the Shavian critique are considered in this paper: the characterisation of the algorist and the problem of complexity. It is argued, contrary to the critique, that computational theory offers a properly constrained formal view of the algorist and is not defeated by complexity. Computational ideas can therefore, have a fundamental role to play in the further development of ecological psychology.

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Introduction.

In his final book, *The Ecological Approach to Visual Perception*, J.J. Gibson introduced the term "mutuality" to describe the linkages between animals and their environments. The concept of mutuality expresses the idea that the linkages are, in some ways, necessary rather than contingent. Gibson suggested, in fact, that "animal" and "environment" are inseparable terms and that each implies the other. One of the central tasks for the ecological approach is, therefore, "to explain how agents are situated, that is, functionally coupled to their environments so as to facilitate adaptive actions" Shaw (2003, p.37). This paper is about the kind of formal theory that might be developed to explain the kinds of adaptive, functional, interaction between animals and their environments that are implied by the concept of mutuality.

Mature sciences tend towards the presentation of theories in mathematical form. The many advantages of mathematical presentation include rigour, precision and clarity. If, as some have said, mathematics is the language of nature its indispensable role in scientific theories is unsurprising. It is, however, sometimes claimed that certain characteristics of sciences such as psychology and biology make them inappropriate for particular types of mathematical treatment. Penrose (1989, 1994), for example, has argued that aspects of human consciousness are provably uncomputable, and Rosen (1991, 1999) has argued that living systems in general display complexity of a kind that cannot be captured by discrete, algorithmic methods. Such arguments point to the need for careful thought about the kind of mathematics

that might best be used for the development of psychological theory in general and ecological psychology in particular.

An important strand of theorising within ecological psychology, to which Robert Shaw has made a major contribution, has examined discrete computational methods as a possible mathematical foundation for psychology. Gibson was opposed to computational accounts of information processing and Shaw and various colleagues, in papers published over a period of more than thirty years, have given reasons for thinking that Gibson's opposition was well founded. The more recent work of theorists such as Penrose and Rosen is used to support the arguments Shaw and his colleagues have developed. Shaw has been the principal architect of an account of the functional coupling of agents and their environments in terms of duality and coalitions which, he argues, provides the mathematical foundation on which ecological psychology should be built. His approach, as he has said recently, (Shaw 2003, p.102) does not entirely rule out a role for a computational form of ecological psychology but suggests that it will be peripheral. Those, like the present author, who believe that computational mathematics can have a central part to play in the development of ecological psychology need to examine the anti-computational tradition and to tackle the problems it raises.

I am going to call the body of work by Shaw and his colleagues which has examined computational methods "The Shavian critique of computation". In doing this, I am in no sense wishing to minimize the important contributions of Shaw's co-authors. Some of the papers in which the critique has been centre stage include, as co-authors, Claudia Carello, Peter Kugler, Michael McIntyre, Michael Turvey and James Todd. However, the major themes with which the present paper is concerned are recurrent aspects of Shaw's oeuvre from 1969 onwards and his contributions to the

critique of computation and to the alternative approach via coalitions have been pivotal.

The major foci of this paper are two aspects of the Shavian critique: the characterisation of the algorist or knower and the problem of complexity. In outline the Shavian critique presents the following argument. The formal theory of computation developed by Turing and others in the 1930s fails to specify the role of the agent or knower because abstract automata do not satisfy the natural constraints that must be satisfied by real agents. At the very least, therefore, computational methods must be supplemented by an account of these constraints. However, even if such an account can be given, the standard computational approach to psychological functioning remains vulnerable because it assumes that simple and complex systems are equally amenable to explanation by computational modelling. Von Neumann's conjecture strongly suggests that computational methods are viable only for systems of low complexity. Since the situated human agent is a highly complex system it is suggested that computational methods cannot provide the right type of explanation. Instead, it is argued, the appropriate theoretical foundation for ecological psychology is a formalized account of coalitions.

The response made to the Shavian critique, again in outline, is the following. The importance of the algorist and the challenge of von Neumann's conjecture are accepted. However, it is argued contrary to the critique, that Turing's analysis of computation in his famous paper of 1937 does in fact provide a rigorous account of the algorist. Once this is understood, the ground is cleared for a computational account of ecological psychology which is properly constrained. It is also argued that von Neumann's conjecture can be understood in a way that is consistent with the use of computational methods. Such an interpretation of the conjecture is more consistent

with other aspects of von Neumann's thinking than the interpretations that Shaw discusses. The new approach to the conjecture rests on a distinction between structural and behavioural descriptions of complex systems. Structural descriptions are finite but behavioural descriptions may be infinite. The existence of undecidable predicates shows that explanations of the behaviour of complex systems cannot be given in full a priori, but this does not imply that complex systems must exhibit uncomputable behaviour. Seen from this fresh perspective the limits of formal enquiry provide the basis for an integration of the 'what', 'how', and 'who' questions that Shaw & McIntyre (1974) posed for psychologists. The agent or "epistemic who" is modelled by the finite control of the Turing machine. What is known, and how it is known are facets of the situated agent and the relation of mutuality between the agent and the environment. Von Neumann's insights also suggest that the study of the functional architecture of the brain should be part of the research agenda of ecological psychology.

Characterising the Algorist.

"*What* is known and *how* it is known are relative questions that make no sense independent of the question of *who* knows. Indeed, our opinion is that the central question of cognitive psychology concerns the essential nature of a knowing-agent, rather than just what is known or even how what is known *is* known." (Shaw & McIntyre, 1974, p.305). Quite so! Thirty years on we should applaud the clarity of vision which this opening sentence of *Algoristic Foundations to Cognitive Psychology* demonstrates. Psychological research methods, the demands of career development and the narrow targeting of research funds all tend to lead to a focus on details, a

focus which can be to the detriment of theoretical concern for the whole person, the ultimate subject of psychological theory. It was a characteristically bold move for Shaw and McIntyre to locate the whole person at the centre of their paper. They suggested that there was no consensus about the inter-relationship of the 'what', 'how', and 'who' questions or about how to tackle them and thus no clear picture of the whole person. They described how existing approaches to the 'what' question were couched broadly in terms of information and those to the 'how' question broadly in terms of algorithms but suggested that the neglected 'who' question could serve as a theoretical fulcrum to provide an integrated attack on all three. "If we can even roughly decide on the nature of the epistemic-who we will, at the same time, *have* to take a stand on the nature of the information processed from and about the environment, as well as on the nature of the psychological processes required to do so." Shaw & McIntyre (1974, pp.307-8). Shaw and McIntyre characterized Gibson's ecological optics as an attempt to study the 'what' of perception and contrasted his approach with those of a number of constructivist theorists, including Sperling, Neisser and Broadbent, which they characterized as concerned with the 'how' question. They argued that the union of these two types of approaches left the 'who' question untouched. To tackle the 'who' question they proposed to relate the class of knowing systems to various classes of automata, making en route a crucial distinction between what they called 'algorithmic' and 'algoristic' approaches to psychological processes. The two terms distinguished a large set of processes (the algorithms) which is characterised solely by the satisfaction of mechanistic relations between input and output, from a subset (the algorisms) whose members are those algorithms which also satisfy the constraints under which human agents function, for example, the constraints of evolution and natural law. The class of algorithms, according to

Shaw and McIntyre, is "obviously much larger than the class of machines that may be physically realized by construction as artefacts or by natural evolution." (Shaw & McIntyre, 1974, p.310) and, in consequence, "the algorithmic bases of cognitive processes can only be defined relative to what we wish to call their *algoristic* bases." (Shaw & McIntyre, 1974, p.315). Another way to make this point is to say that the algorithmic level of analysis cannot be fundamental for psychology because it presupposes the algoristic level. Shaw and McIntyre confirmed this interpretation as their intended meaning a little further on in the paper where they said that "the intuitive notion of algorithm rests ultimately on what is meant by an epistemic agent, or algorist." (Shaw & McIntyre, 1974, p.316). It follows, if this line of argument is correct, that it is essential to characterise the algoristic bases of cognitive processes, that is the set of constraints that picks out the algorisms from the larger set of algorithms. Moreover, as Shaw and McIntyre also noted, there are constraints arising from the properties of energy distributions in the environment which further limit the set of humanly realisable algorithms. The ultimate target sought is the specification of a system that computes "natural" algorithms, that is those "functions satisfying both the natural cost parameters and intentionality of physically, biologically, and psychologically realizable systems." (Shaw & McIntyre, 1974, p.320). Shaw and McIntyre argued that the development of an adequate theory of the algorist had to go beyond the theory of computation and might transcend the then current understanding of physical and biological laws.

An immediate consequence of this argument is that algorithmic, i.e. computational, specifications of processes do not satisfy the constraints under which natural agents function. Computational processes are therefore, at best, incomplete as a foundation for psychology. Shaw and McIntyre were quite explicit about this with

respect to Turing machines. They discussed the Church-Turing thesis and acknowledged the fundamental logical result that every scheme proposed as a formal model of the intuitive notion of an effective procedure turns out to compute exactly those functions computable by a universal Turing machine. They then posed the question, "Does this mean then that the universal Turing machine and the class of abstract automata equivalent to it provide a rigorous instantiation of what we intuitively mean by an algorist?" The rhetorical tone of the question suggests the negative answer that swiftly followed. Turing machines, they said, do not capture the properties of algorists because "such abstract automata do not satisfy the natural constraints that must be satisfied by any real agent. For instance, Turing machines are assumed to possess infinite memory capacity, to be perfectly reliable, and to compute as fast as you please – all ideal properties not representative of any organism or actual machine." (Shaw & McIntyre, 1974, p.317).

The algorist paper has been influential in ecological psychology and its arguments have contributed to the rejection of computational theory as a suitable mathematical basis for the ecological approach. I agree completely with Shaw and McIntyre about the need for a formal theory which respects the constraints under which agents operate and their paper is an early and important statement of the naturalistic constraints that any psychological theory needs to respect. I part company with them, however, with respect to the status of the Turing machine. Our difference in this respect is fundamental. I shall demonstrate that Turing's theory does in fact contain a model of the algorist which satisfies natural constraints. The algorisms are co-extensive with the algorithms, as these were defined by Turing, not a proper subset of them as Shaw and McIntyre suggest. Turing's theory thus provides, in principle at least, a suitable mathematical foundation for ecological psychology.

The difference of approach stems largely, I believe, from the fact that contemporary scholars have greater access than did our colleagues in the 1970s to Turing's own paper. It may also be relevant that Shaw and McIntyre are members of the American rather than the British academic community. It is noteworthy, for example, that they cite directly the papers of Church (1936) and Kleene (1936) which were published in American journals, but do not cite Turing's equally fundamental paper (Turing, 1937) which was published in the Proceedings of the London Mathematical Society, a journal which was, presumably, much less accessible to American scholars. My supposition is that when Shaw and McIntyre wrote their paper they knew of Turing's work from sources other than his own writings. Had they had the chance to study and think about his paper directly they might, I surmise, have come to different views about the relation between algorithms and algorists and ecological psychology might have taken a different, more computational, turn. Turing's work is unique among the founding documents of computer science for the way in which it relates the function computed to the agent doing the computing. For this reason, the distinction between algorithm and algorist does not apply to Turing in the way that it appears to apply to the systems of Church and Kleene despite the formal equivalences between their approaches and that of Turing.

Turing's analysis of computation and the modelling of the algorist.

Shaw and McIntyre (1974) were critical of the Turing machine as a basis for the formal specification of the algorist because the machine model was said not to satisfy the natural constraints that must be satisfied by any natural agent. The Turing machine's infinite memory and perfect reliability were cited as instances of this

failure. A different understanding can be derived from Turing's fundamental paper *On Computable Numbers with an application to the Entscheidungsproblem*, Turing (1937). The paper is now available in the volume of Turing's collected works containing his papers in mathematical logic, (Gandy &Yates, 2001). Turing's analysis of the process of routine computation was an ecological analysis which gave equal weight to the environment and to the algorist. Moreover, the design of the Turing machine specifically reflects natural constraints arising from both the agent and the environment. It will become apparent, strange as it may sound, that the infinite memory and the perfect reliability of the machine are quite consistent with the satisfaction of natural constraints. A short account of Turing's analysis is given here. A more detailed exposition can be found in Wells (2004). The best starting point is Turing's own description of a machine whose purpose was to compute exactly those real numbers which could be computed "effectively" by a human working with paper and pencil. He said,

> We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions q_1, q_2, …, q_R which will be called "*m*-configurations". The machine is supplied with a "tape" (the analogue of paper) running through it, and divided into sections (called "squares") each capable of bearing a "symbol". At any moment there is just one square, say the *r*-th, bearing the symbol S(*r*) which is "in the machine". We may call this square the "scanned" square. The symbol on the scanned square may be called the "scanned symbol". The "scanned symbol" is the only one of which the machine is, so to speak, "directly aware". However, by altering its *m*-configuration the machine can effectively remember some of

> the symbols which it has "seen" (scanned) previously. The possible behaviour of the machine at any moment is determined by the *m*-configuration q_n and the scanned symbol S(*r*). This pair q_n, S(*r*) will be called the "configuration": thus the configuration determines the possible behaviour of the machine. In some of the configurations in which the scanned square is blank (*i.e.* bears no symbol) the machine writes down a new symbol on the scanned square: in other configurations it erases the scanned symbol. The machine may also change the square which is being scanned, but only by shifting it one place to right or left. In addition to any of these operations the *m*-configuration may be changed.
>
> Turing (1937, p.231).

This excerpt provides the key to understanding the ecological basis of Turing's analysis of paper and pencil calculations. A person carrying out a calculation is compared to a machine with a finite number of "conditions" as Turing calls them. In modern terminology they are called functional states. Each is given a unique name. The finite state machine is Turing's model of the mind of the algorist. The paper on which the workings and results of the calculation are written down is replaced by a one-dimensional tape divided into squares. The tape is Turing's model of the environment. Each square of the tape is either blank or contains a symbol from a finite alphabet which the machine is able to recognise. It cannot be emphasized too strongly that the tape is distinct from the finite machine although the two are connected. A person doing a calculation is distinct from the paper on which the calculation is worked although they are connected via the processes of reading and writing. So it is with the finite machine and its tape. The machine is "directly aware" of one square of

the tape at a time and can read what is on that square and write something on it. It can move the tape, one square at a time, so as to change the square of which it is directly aware. The machine can also change its functional state. The behaviour of the machine is determined by its current functional state and the contents of the square of the tape of which it is currently aware. This pair is called a "configuration". It relates both to the algorist and to the environment and provides a striking formal model of a Gibsonian affordance. See (Wells, 2002) for a detailed exposition of this idea.

Turing machines have a single perceptual capacity: the capacity to read or recognise symbols from a fixed, finite alphabet. This capacity is a model of the perceptual processes of a human algorist carrying out a paper and pencil calculation. The capacity is defined in a purely functional fashion. Nothing is said about how symbol recognition might be instantiated in a real physical machine. Turing machines have two action capacities: the capacity to write or print symbols from the alphabet and the capacity to move the machine relative to the tape. These capacities are models of the activity of the algorist. Nothing is said about how these capacities might be physically instantiated. Reading, writing and moving are capacities that connect the machine to the tape. Turing machines are also able to change from one functional state to another, modelling changes of state of mind of the algorist, in a way which is described by rules. Turing says nothing about the physical realization of the functional states or of the means by which changes of state are achieved.

The differences between physical and functional descriptions of a system and the nature of the abstraction involved in a functional description are clearly illustrated by the everyday light switch. A light switch is a physical system with many parts. Many physically different kinds of switch can be made which are designed to have two distinct functional states, ON and OFF. Turing described the nature of the

abstraction involved in moving from physical to functional description. “Everything really moves continuously. But there are many kinds of machine which can profitably be *thought of* as being discrete state machines. For instance in considering the switches for a lighting system it is a convenient fiction that each switch must be definitely on or definitely off. There must be intermediate positions, but for most purposes we can forget about them.” (Turing, 1950, p.439).

A functional description can be used as the basis for a physical realisation, or a functional description can be abstracted from an existing physical system. The latter was the route that Turing took in the analysis of computation that he carried out for the 1937 paper. The physical system was the human algorist performing a calculation and the functional description that Turing abstracted was a machine with a finite number of functional states. The output of a Turing machine is of two kinds. There is the sequence of zeroes and ones which constitutes the binary representation of the real number computed by the machine. Once a zero or a one has been printed it is not erased and forms a permanent part of the sequence. Turing machines also print a variety of other symbols for various purposes much as one might write down a carry digit when doing a large addition. These temporary symbols are eventually erased. Turing made an important distinction between machines which keep printing zeroes and ones ad infinitum, which he called “circle-free” machines, and those which print only a finite number of zeroes and ones which he called “circular” machines. The potentially infinite sequence of digits output by a circle-free machine is produced by the interaction between the finite state control and the tape. Since the finite state control has a fixed number of functional states the machine has to cycle through its functional states many times in order to produce its potentially infinite output.

The determination of behaviour by configurations (ordered pairs of functional states and tape symbols) is straightforward to state but easy to misinterpret. Each possible configuration of a Turing machine has a set of actions associated with it. The combination of a configuration and its associated actions is called an "instruction" and the complete set of instructions defining the behaviour of a machine is called a "machine table". A starting configuration is specified for each Turing machine and the actions associated with it lead to the next configuration with its associated actions and so on. It is customary to describe Turing machines as "rule governed" because each instruction can be called a rule of behaviour. However, it is misleading to think of a Turing machine's behaviour as being governed by explicit rules, except in the special case of universal machines. The concept of a rule of behaviour should be understood in the sense that Gibson intended when he talked about rules for the control of behaviour. In *The Ecological Approach to Visual Perception*, he described the problem of expressing the appropriate sense of the term "rule" with respect to the visual control of locomotion;

> I asserted that behavior was controlled by *rules*. Surely, however, they are not rules enforced by an authority. The rules are not commands from a brain; they emerge from the animal-environment system. But the only way to describe rules is in words, and a rule expressed in words is a command. I am faced with a paradox. The rules for the control of locomotion will sound like commands, although they are not intended to. I can only suggest that the reader should interpret them as rules *not formulated in words*.
>
> Gibson (1979/1986, pp.232-3)

The rules for control of the behaviour of a Turing machine express the actions needed to produce a specific outcome. They are not rules formulated in symbols, to adapt Gibson's phrase, but are emergent properties of the combined system of the machine and its tape. This characteristic makes the Turing machine a suitable mathematical foundation for ecological psychology. Looking out of my window, I can see a Swift, *Apus apus*, hawking insects in the air above the building on the other side of the road. Its behaviour is controlled by an interaction between the states of its brain and the states of its environment. One fundamental contention of this paper is that Turing machines provide a suitable foundation for the study of such interactions. The brain of the swift can be described in terms of a set of functional states, the states of its environment can be described in terms of locations and their contents, and the behaviour of the swift can be described as a sequence of configurations and associated actions. There is much to be added to this basic account, including the important fact that the swift's behaviour is non-deterministic, but that does not undermine the fundamental appropriateness of computational description. Please notice also that the characterisation of the neural states of the swift as the functional states of a biological machine does not imply that those states consist of internal symbolic rules or that the swift has an internal symbolic representation of its environment.

The characterisation of functional states and the distinction between universal Turing machines and others are topics that require discussion. The two are related as will become clear. Cooking provides familiar situations that can be used as examples. Consider, first, the striking fact that I can carry out a sequence of operations that results in a boiled egg without having recourse to explicit instructions. I do this by putting the egg in a pan, adding water, putting the pan on the hob, turning on the gas,

and so on. These observable operations are controlled by activity in my central nervous system. The nature of that internal activity is a topic of fundamental debate.

From a standard computational perspective my brain might be characterised as instantiating an egg boiling production system whose productions break down the task into a sequence of elementary actions such as (PUT (EGG, PAN)) which have explicit symbolic representations somewhere in my memory and are evoked by the state of the environment and my current goal structure. I am not, I repeat NOT, advocating a computational approach of this kind.

From the Gibsonian perspective, my brain might be characterised as resonating to the learned affordances of the egg boiling situation. The resonance concept is much closer to the kind of computational approach advocated in this paper than is the production system model. Resonance, however, implies a rather passive role for the agent and it is hard to see how to use it to model the exploratory, orienting activities that are characteristic of the behaviour of organisms. A fundamental suggestion of the computational approach described here is that the concept of resonance can be explored and elaborated within the discrete state framework of the Turing machine formalism. Behaviour in the Turing machine model is controlled jointly by the functional states of the finite machine and by the currently perceived part of the environment. Such joint determination seems ideally suited to the task of modelling the situated control of behaviour which is characteristic of ecological psychology.

Functional states are discrete because the egg-boiling task consists of a sequence of discrete elementary operations and the sequence has at least a partial ordering. Putting an egg in the pan is one discrete operation, putting water in the pan is another. It doesn't much matter whether the egg goes in the pan before the water or

vice versa, but the water must be in the pan before the heat source is applied, hence the existence of a partial ordering of the operations for a successful outcome. The characterisation of functional states as discrete is compatible with a wide range of modes of operation of the underlying physical system in which the abstractly described system might be instantiated. It is, for example, perfectly possible for discrete functional states to be described in systems whose underlying control dynamics are continuous. The different macro-states of water resulting from continuous temperature changes are a simple example and the phenomena of categorical perception (Harnad, 1987) provide numerous human examples.

Turing's account of discrete functional states abstracts from the details of physical instantiation but is compatible with any physical realization that preserves the functionality described in a given model. Thus the description of the states of my egg-boiling brain as discrete functional states is compatible with a continuous dynamical description of the underlying neural hardware. It is also compatible with a discrete, internal, symbolic representation of the task but does not entail one. The activities that I undertake when boiling an egg could be modelled by a non-universal Turing machine.

In addition to boiling an egg, I can carry out a sequence of operations that results in a dish of green beans in a spicy tomato sauce called *Masaledar sem* (Jaffrey, 1995). *Masaledar sem* requires a much longer sequence of operations than that needed to boil an egg and I need to follow instructions to achieve the desired result. In this case, there is an explicit symbolic representation involved in my performance of the task. It is called a recipe. It is located in the environment and my actions result from reading and carrying out the instructions that constitute the recipe. The capacity to read and execute recipes enormously increases the scope of my cooking without

requiring permanent modifications to my mental contents. It is not necessary for me to memorise recipes or to remember them from one occasion of use to another. Provided I have the book available, I have, as it were, a source for many sauces. A universal machine is one whose functional states are organised in such a way that it is able to do something very similar. A universal machine interprets the description of another Turing machine which is written on its tape, and behaves as though it were that Turing machine. By changing the description its behaviour is changed in essentially the way that changing the recipe with which I work, changes what I cook. A universal Turing machine is, nevertheless, just like any other Turing machine with regard to the character of its own functional states. The difference comes in how they are organized and the way that the machine uses its tape.

Turing's analysis is striking in its simplicity. It asserts that all that is needed for a mathematical model of a person doing a paper and pencil calculation is a set of functional states modelling the algorist, a tape divided into squares modelling the environment, an alphabet of symbols and a set of actions determined by the machine's configurations. In what sense can this model be said to satisfy the natural constraints that impinge on a situated agent? Four crucial points of Turing's analysis are relevant here.

First, the set of functional states modelling the mind of the algorist is finite. This, as Turing said, reflects the fact that "the human memory is necessarily limited." (Turing 1937, p.231). Turing made little in the way of a defence of this contention but it may be interpreted as a commitment to a physical basis for mental states. Since the brain is a finite organ, if mental states are physical states it follows that there can only be finitely many of them. It is important, in this respect, to note that the requirement for an infinite tape does not contradict this suggestion. The tape is a model of the

environment, not of the algorist. Moreover, the tape is not required to be infinite at the start of a computation. It is simply assumed that more squares can be added if required. The specification of an unbounded tape reflects the fact that availability of paper is not part of the definition of an effective calculation. If, for example, we want to write out the decimal expansion of π to an arbitrary degree of precision we will need an infinite supply of paper because π has infinitely many digits.

Second, the number of symbols in the alphabet of a Turing machine has to be finite. This constraint reflects the fact that symbols are to be written on tape squares and squares are of a fixed size. As Turing said, "If we were to allow an infinity of symbols, then there would be symbols differing to an arbitrarily small extent". (Turing 1937, p.249). This is a straightforward perceptual constraint. The human visual system has a finite resolving capacity. If we had to deal with an infinity of symbols confined to squares of fixed size, the symbols would differ to an arbitrarily small extent. In consequence, effective calculation would cease to be possible because errors of symbol discrimination would occur. It is important to notice the human character of this constraint. It is easy to describe machines which are not subject to human perceptual limitations and it is possible to imagine machines with infinite resolving capacity, for example machines with registers that can store numbers with arbitrary precision. Some theorists have explored the possibility that Turing computability does not exhaust the possibilities of natural computation (cf. Siegelmann, 1999; Stannett, 2004; Eberbach, Goldin, & Wegner, 2004). Super-Turing or hyper-Turing models as they are sometimes called result from relaxing one or more of the constraints that Turing identified as necessary for a formal model which could compute all and only those numbers that could be computed by a human algorist

working with paper and pencil. It was of the essence of the Turing machine that it respected the constraints on human algorists.

Third, Turing specified a bound on the number of symbols or squares of which a machine could be "directly aware". This reflects the fact that humans can observe only a certain number of symbols at one moment. If we want to observe more we have to use successive observations. In the simplest case, we observe one symbol at a time and it is for this reason that Turing constrained his machines to be "directly aware" of only a single square of the tape at a time. Nothing fundamental is lost by doing this, it simplifies the analysis, and it further reflects the fact that humans working with paper and pencil almost invariably write only one symbol at a time.

Fourth, Turing specified a bound on what he called "changes of distribution of observed squares". The point of this constraint is to reflect the fact that there are physical limits on human shifts of attention. If I am attending to the computer screen on my desk at home, I cannot immediately shift my attention to the computer screen on my desk at work which is four miles away. My perceptions and actions are bounded by constraints of place and it is this that Turing's fourth constraint reflects. He implemented it by specifying that his machines could move only one square at a time. Again, nothing fundamental is lost by doing this.

It should be clear from the above that, far from being an arbitrary construction, the Turing machine was carefully designed to reflect the constraints on human algorists and their interactions with their environments. The Turing machine is, it is true, a model of a very narrow slice of general human competence but that is not the issue of immediate concern. The immediate issue is its plausibility as a formal model of a human algorist. Let me finally briefly address the points about reliability and speed made by Shaw and McIntyre. It is true that Turing machines are assumed to be

perfectly reliable and to work as fast as one wishes, but neither point invalidates the Turing machine as a satisfactory model of the algorist. Turing machines incorporate the fundamental principles of effective calculation whereas reliability and speed are adventitious, practical considerations. They are important but separable from the theoretical requirements. Turing's analysis addresses the question of the conditions which enable a human to carry out a calculation effectively. One of those conditions is the ability to read and write symbols reliably. The analysis does not rule out an account of the conditions which might interfere with effective calculation but that was not part of Turing's purpose. Reliability and speed are performance constraints rather than competence constraints.

Von Neumann's Conjecture.

Turing's analysis provides an account of the process of computation which includes elements modelling the algorist and elements modelling the environment. It is generally accepted as the most natural formalization of the intuitive notion of an effective procedure for calculation and, as the previous section shows, it respects the constraints on human algorists. The question remains whether Turing's analysis can form the foundation for a more general theory of automata or whether that path is blocked by von Neumann's conjecture as Shaw suggests. This aspect of the Shavian critique has had its most recent statement in Shaw (2003). In that paper Shaw listed a number of theses or claims which he attributed to Herbert Simon's work on the study of artificial systems. One of these is the claim that complex systems can be reduced to simpler form by proper description. Shaw argues that some ideas of the mathematician John von Neumann can be used to make an alternative case which

asserts that “for systems beyond a certain level of finite complexity, no verbal description or mathematical formulation can reduce its [sic] complexity.” (Shaw 2003, p.61) Shaw goes on to say that von Neumann’s work may justify “an even more pessimistic conjecture” (Shaw, 2003, p.61). It is not entirely straightforward to state this more pessimistic conjecture precisely but, roughly speaking, it amounts to the assertion that the question “What can this object do?” is of a higher logical type than the question “What is the structure of this object?”. The difference this makes is that it can take infinitely longer to answer higher type questions about behaviour.

Shaw made a more detailed analysis of the conjecture many years ago in a paper which was originally published in 1971 before the algorist paper was written, and later republished as Shaw (1976). In that paper, he distinguished strong and weak versions of the conjecture which he described as “fundamentally different”. The strong interpretation maintains that

> a formal understanding of truly complex natural phenomena is inaccessible to us due to the logical impossibility of codifying significant properties of complex systems in terms of simpler systems of abstract principles (e.g., a model).
>
> (Shaw, 1976, p.162)

The weak interpretation, which Shaw suggested was “more in keeping with von Neumann’s intent” asserts that

> some (but not necessarily all) approaches to modelling are inherently inadequate for providing explanations of complex phenomena which pass

> muster under the criterion of conceptual economy, although the same approaches may be quite adequate when directed toward explaining simple to moderately complex phenomena.
>
> (Shaw, 1976, p.162)

Shaw's view in 1976 was that the strong interpretation of the conjecture might foster a cynical attitude towards psychology, was unwise on pragmatic grounds and was not forced on us on logical grounds. He appears to have hardened his view somewhat since then and now leans more towards the strong version of the conjecture because in Shaw (2003) he cites the work of Penrose (1989, 1994) and that of Rosen (1991, 1999) both of whom, in somewhat different ways, argue that "living systems are complex exactly because they exhibit behaviors not algorithmically computable" (Shaw, 2003, p. 63).

Several of von Neumann's works are relevant to questions about the study of complex systems and the development of a theory of complex automata. It is quite clear that von Neumann thought a properly founded theory would be applicable to both natural and artificial systems. He did not make the distinction between theories of living systems and theories of artefacts that Rosen (1991, 1999) treated as fundamental. The discussion here draws on three of von Neumann's works: the paper he read at the Hixon symposium in Pasadena, California in September 1948 which is reprinted in Aspray & Burks (1987); a series of five lectures given at the University of Illinois in December 1949 which were published posthumously as von Neumann (1966); this is the main source used by Shaw in his discussions of von Neumann; finally, a short book making comparisons between computers and brains, also

published posthumously, von Neumann (1958). The book was started in 1955 and was unfinished when von Neumann died in 1957. The chronology is of importance in reaching conclusions about how best to interpret some difficult aspects of von Neumann's thought.

In the Hixon symposium paper, as in the other works discussed here, von Neumann's primary concern was to sketch an outline for a theory of complex automata. He said that "a detailed, highly mathematical, and more specifically analytical, theory of automata and of information is needed. We possess only the first indications of such a theory at present. In assessing artificial automata…of only moderate size, it has been possible to get along in a rough, empirical manner without such a theory. There is every reason to believe that this will not be possible with more elaborate automata." (Aspray & Burks, 1987, pp.407-8). The problem, he suggested, was that existing logical methods would lead to theoretical statements which were simpler than their objects, as theoretical statements must be if they are to be of any use, only when those objects were of modest complexity. With a highly complex object like a human brain or a very large computer, he thought that "any attempt to describe it by the usual literary or formal-logical method may lead to something less manageable and more involved." (Aspray & Burks, 1987, p.414). He went on to suggest that this view was buttressed by some results in "modern logic". These results are discussed in due course.

The fundamental reason why a complex automaton might prove resistant to simplification via existing logical methods lay, von Neumann thought, in the relationship between structure and function which was different for simple and complex systems. It was, he said, the mark of a simple automaton that its behaviour was of "a lower degree of complication than the automaton itself." (Aspray & Burks,

1987, p.415) By contrast, it was possible for a complex automaton to exhibit functioning of a higher degree of complexity than its own structure. Von Neumann said that organisms are clear examples of automata which are complex in this way because their capacity for reproduction is a form of behaviour which produces systems of equal or greater complexity.

To illuminate the discussion, von Neumann drew attention to the work of McCulloch and Pitts (1943) who had proved the equivalence of logical formulae and finite networks of idealized neurons. The equivalence proof shows that every description of a system in terms of logical formulae can, in principle, be translated into a neural network description and vice versa. However, the equivalence proof did not demonstrate that logical and network descriptions would be equally perspicuous. Von Neumann suggested that in simple cases, the logical description of a system would simplify the network description but that in complex cases the reverse might be true. Taking as an example the visual brain, whose fundamental activity he thought to be the making of analogies, he argued that "it is perfectly possible that the simplest and only practical way to say what constitutes a visual analogy consists in giving a description of the connections of the visual brain." (Aspray & Burks, 1987, p.414).

The conclusion of the Hixon symposium paper was that logic might have to change to rise to the challenge of developing a theory of complex systems. Von Neumann did not suggest that logical methods were intrinsically unsuitable for the study of complex systems. He said "[A] new, essentially logical, theory is called for in order to understand high-complication automata and, in particular, the central nervous system. It may be, however, that in this process logic will have to undergo a pseudomorphosis to neurology to a much greater extent than the reverse." (Aspray & Burks, 1987, p. 414).

The discussion of the Hixon symposium paper was sharpened and clarified in the Illinois lectures. The assertion that a description of the brain might be the simplest way to approach the complex concept of visual analogy was stated with striking force: "It is absolutely not clear a priori that there is any simpler description of what constitutes a visual analogy than a description of the visual brain." (von Neumann, 1966, p.47) The connection of this point with the results in logic mentioned above was made more explicit. Von Neumann introduced the connection by reiterating the concept of an inverse relationship between structural and functional descriptions for simple and complex automata. "There is a good deal in formal logics to indicate that the description of the functions of an automaton is simpler than the automaton itself, as long as the automaton is not very complicated, but that when you get to high complications, the actual object is simpler than the literary description." (von Neumann, 1966, p.47). He went on to say that the logical work he had in mind was a theorem of Gödel's which showed, as he put it, that "the description of an object, is one class type higher than the object and is therefore asymptotically [?][1] infinitely longer to describe." (von Neumann, 1966, p.47). A few pages later he made some further remarks which bear on the issue but which make a slightly different point. The further remarks begin with a brief discussion of the significance of Turing's proof of the unsolvability of the halting problem for Turing machines. Turing had shown that although there was a single machine, the universal machine, which could simulate the processing of any other Turing machine by interpreting a symbolic description of it, there was no single machine which could tell from the description of a given machine whether it would halt or would continue its processing indefinitely. Using Turing's

[1] The brackets and question mark indicate that the transcript of the lectures from which von Neumann (1966) was put together was incomplete or incomprehensible at this point. The conclusions one can safely draw about von Neumann's intended meaning must, therefore, be considered tentative and it is important to try to find an interpretation which is consistent with other points he made.

own terminology, his proof showed that there was no single, general method to distinguish circular from circle-free machines. Von Neumann said Turing's proof showed that "you can build an organ which can do anything that can be done, but you cannot build an organ which tells you whether it can be done." (Von Neumann, 1966, p.51). He then related Turing's proof to the theory of types, to Gödel's work, and to his earlier remarks. "It is connected with the theory of types and with the results of Gödel. The feature is just this, that you can perform within the logical type that's involved everything that's feasible, but the question of whether something is feasible in a type belongs to a higher logical type…in the complicated parts of formal logic it is always one order of magnitude harder to tell what an object can do than to produce the object. The domain of the validity of the question is of a higher type than the question itself." (Von Neumann, 1966, p. 51).

Shaw (1976, p.156; 2003, p.61) has taken this passage to be the key to von Neumann's conjecture and I think he is right to do so. The difficulty, as Shaw himself has said, is to come to a clear understanding of exactly what von Neumann meant and what the logical results imply for psychology. In the earlier of the two papers, in which he discussed the topic in detail, Shaw suggests that von Neumann's intended meaning is clarified by a passage in one of the later Illinois lectures in which he discussed a specific kind of complexity. "It is effectivity in complication, or the potentiality to do things. I am not thinking about how involved the object is, but how involved its purposive operations are. In this sense, an object is of the highest degree of complexity if it can do very difficult and involved things." (von Neumann, 1966, p.78). Notice again here the distinction between structural issues (how involved the object is) and functional issues (how involved its purposive operations are).

In his later paper, Shaw takes a somewhat different line and relates the conjecture directly to the theorem of Gödel, also proved by Tarski, that von Neumann mentioned. But Shaw goes further than this and aligns von Neumann's thinking with the arguments of Penrose and Rosen, to the effect that "living systems are complex exactly because they exhibit behaviors not algorithmically computable" (Shaw, 2003, p.63). Later in the paper, he asks the question "What if complexity is by nature rather than artifice (formal description) a limitless source of generatively specified impredicativities, that is, undecidable predicates, as von Neumann, Penrose, and Rosen all suspected? What then?" (Shaw, 2003, p.98). This pregnant question is left unanswered, but Shaw's later conclusion is that "some radical, ecological version of science must replace the mechanistic science most psychologists adopt uncritically." (Shaw, 2003, p.101).

Let us recall at this point von Neumann's conclusion in the Hixon symposium paper, which was not that logical methods were unsatisfactory for the study of complex automata, as Shaw's analysis suggests, but that logic would need to become more like neurology. It is, of course, possible that von Neumann had changed his mind by the time the Illinois lectures were prepared but his short book on computers and brains, which was written later still, suggests otherwise. At the end of that book, von Neumann made some remarks which support the idea that his vision of automata theory involved a transformation of logic rather than its abandonment.

> Just as languages like Greek or Sanskrit are historical facts and not absolute logical necessities, it is only reasonable to assume that logics and mathematics are similarly historical, accidental forms of expression. They may have essential variants, i.e. they may exist in other forms than the ones to which we

> are accustomed. Indeed, the nature of the central nervous system and of the message systems that it transmits indicate positively that this is so…the outward forms of *our* mathematics are not absolutely relevant from the point of view of evaluating what the mathematical or logical language *truly* used by the central nervous system is."
>
> (Von Neumann, 1958, pp.81-2)

Given von Neumann's continued emphasis on the logical language used by the central nervous system, it seems that Shaw's view of the conjecture in 1976 was probably closer to von Neumann's intended meaning than the somewhat stronger view of Shaw (2003). I shall argue that von Neumann's work does not indicate the need for a non-computational theory. Moreover, although I shall not develop the arguments here, I think it can be shown that neither Penrose nor Rosen has made a conclusive case against the use of computational methods in ecological psychology. Once again, though, while I come to different conclusions about the implications of von Neumann's work, I am indebted to Shaw's instinct for the fundamental questions. I am not aware of any other psychological theorist who saw so early the significance of the issues concerning complexity that von Neumann's work invites us to consider.

An alternative interpretation of von Neumann's conjecture.

I shall offer an interpretation of von Neumann's writings which takes a much more optimistic view than Shaw (2003) of the possibilities for computational methods in psychology. Computing has illuminated many areas of psychology and biology as

well as mathematics. It is noteworthy, for example, that computational methods have been of great importance in facilitating the study of non-linear dynamical systems which authors such as Port & van Gelder (1995) have proposed as a replacement for computational thinking in psychology. Thompson and Stewart (1986), for example, in their textbook on non-linear dynamics and chaos remarked on "a spectacular blossoming of nonlinear dynamics, made possible…by the wide availability of powerful digital and analogue computers." (Thompson & Stewart, 1986, p. ix). This casts doubt on the idea that non-linear dynamical systems thinking will replace computational methods.

The starting point for the alternative understanding of von Neumann's conjecture is, once again, the theorem of Gödel that von Neumann mentioned in the Illinois lectures. Arthur Burks, the editor of von Neumann's manuscript, was puzzled by the reference because he was not aware of a theorem that had the characteristics described by von Neumann, i.e. a proof that the functional description of an automaton might be infinitely longer than its structural description and require a higher type of logical construct. Burks wrote to Gödel to ask if he could clarify the matter. His letter and Gödel's reply are both reported in von Neumann (1966). Gödel's reply was characteristically careful and suggested that the reference might have been to his proof that a complete description of a formal language *A* cannot be given in *A* because the concept of truth of sentences of *A* cannot be defined in *A*. However, he warned that this might not have been the answer because higher logical types do not necessarily involve longer symbolic descriptions. He proposed that "what von Neumann perhaps had in mind appears more clearly from the universal Turing machine." (Von Neumann, 1966, p.56).

To understand the significance of Gödel's remarks it is essential to have in mind the fact that the behaviour of a universal machine is determined by the description of the machine on its tape. If the description of the machine is changed the behaviour of the universal machine also changes. The point Gödel made about the universal machine was that because there was no decision procedure to predict its behaviour a complete description could only be given by enumerating all its instances. That would be an infinitely long task because a universal machine can simulate the processing of a countable infinity of Turing machines. The structural description of a universal machine is finite, however, because, like every other Turing machine, it has a finite machine table. For this reason Gödel said "The universal Turing machine, where the ratio of the two complexities is infinity, might then be considered to be a limiting case of other finite mechanisms. This immediately leads to von Neumann's conjecture." (Von Neumann, 1966, p.56).

The statement that "the ratio of the two complexities is infinity" links structural and behavioural issues. The two complexities are the structural and behavioural descriptions of a universal machine. The ratio is infinity because a complete description of the behaviour of a universal machine is infinite whereas a complete description of the structure of its control automaton is finite. This leads to von Neumann's conjecture as Gödel said, but does so in a way that is consistent with the possibility that every complex system is a mechanism. It does not show that complex systems exhibit uncomputable behaviours. A universal machine is complex by von Neumann's definition but does not, of course, do anything uncomputable.[2]

Gödel's point about the ratio of infinities stems from the fact that the behaviour of a universal machine has to be studied by enumerating its instances

[2] Rosen (1991, 1999) treats complexity in a way that makes a universal machine simple by definition. Complex systems, in his terminology, are defined to be those that exhibit uncomputable behaviour.

because there is no decision procedure to predict its behaviour. Shaw takes the existence of undecidable predicates to support the case for a non- mechanistic psychology. However, the existence of undecidable predicates does not undermine the claim that mechanistic explanation is an appropriate goal for psychology. The starting point for an explanation of this somewhat counter intuitive fact is the simple but important distinction between an individual Turing machine and the countably infinite set of all the Turing machines. Proofs such as the unsolvability of the halting problem, which is a famous instance of undecidability, concern what finite methods can demonstrate about the set as a whole. This is quite distinct from what can be said about individual members of the set. The halting problem arises with respect to the distinction (mentioned earlier) that Turing made between machines which keep printing zeroes and ones ad infinitum, which he called "circle-free" machines, and those which print only a finite number of zeroes and ones which he called "circular" machines. The problem can be posed as a question: "Is there a general method which can be used to determine, in a finite number of steps, whether an arbitrary Turing machine is circular or circle-free"? The proof that the problem is unsolvable shows that the answer to the question is "No". This is what it means to say that the predicates "circular" and "circle-free" are undecidable. The crucial instance on which the proof is based involves self-reference and this feature is characteristic of other limitative results in logic such as Gödel's famous incompleteness theorem.

The general fact of undecidability does not show that the predicate in question is undecidable for every Turing machine nor does it show that there must be machines for which it is undecidable. What it shows is just that there is no single method which can be applied to decide the question in every possible instance. Some instances, for example self-referential ones, will require different methods and it is this that leads to

the requirement for the enumeration of instances. An example of an everyday predicate which is undecidable in general but has clearly decidable instances is the predicate "prints zero". Turing was able to prove that this predicate is undecidable because its decidability would imply that the halting problem could be solved. However, there are many machines for which the predicate "prints zero" is decidable. Any machine which does not have an instruction to print zero in its description is a decidable instance as is any machine which can be shown to print zero after a finite number of steps.

How then, should we understand von Neumann's conjecture in relation to psychology? If humans are complex for the same sorts of reasons as universal machines, their behaviour will give rise to undecidable predicates which show that no single, finite method of psychological enquiry can be specified in advance to give correct answers to all psychological questions. Psychology cannot, therefore, be a purely theoretical science. It has inescapable empirical or observational content. There are behaviours which are not entirely predictable in advance and can only be understood by observation or experiment. Few psychologists will be surprised by this result. Perhaps more interestingly for ecological psychology, von Neumann's conjecture also suggests that the structural descriptions of complex systems, although they do not fully explain their behaviour, are a useful and important source of information about them. In the case of humans this can be interpreted to mean that the study of the structure of the nervous system is an important source of psychological information.

Back to the 'what', the 'how' and the 'who'.

The alternative interpretation of von Neumann's conjecture, taken in conjunction with the understanding that Turing's analysis of computation does give an appropriate account of the algorist, has some natural and pleasing consequences for ecological psychology. These can be set in the context of Shaw and McIntyre's 'what', 'how', and 'who' questions.

Starting with the 'who' question, Turing's analysis describes the algorist abstractly, in terms of a finite set of functional states which are related to each other and to the states of the environment. The structure of this system of states and the types of behaviour in which it can engage are finitely specified but its actual behaviour, understood as a sequence of interactions with the environment, may be indefinitely long. Turing's analysis suggests, and later theorists proved, that the isolated algorist, i.e. a finite automaton, is computationally less powerful than a Turing machine which has access to an unbounded environment. It is, therefore, a key postulate of Turing's theory that the behavioural complexity of the algorist is a function, not just of its internal states, but also of the use it makes of its environment. Turing's algorist, in other words, is essentially situated.

The answer to the question, 'What does the algorist do?' is that the algorist perceives a constrained portion of the environment and acts on it. Each perception-action cycle changes the algorist, the environment, or both and leads without a break to the next cycle. Although the algorist and the environment are distinct entities they are not separable as far as the analysis of behaviour is concerned. Von Neumann's conjecture suggests that behaviour may be indefinitely extended and thus, that it cannot be predicted fully in advance. That is the conclusion to draw from Turing's proof of the unsolvability of the halting problem. It does not imply that any specific, situated instance of behaviour is uncomputable.

The potentially infinite extent of the 'what' of behaviour fits well with observations that Gibson made repeatedly about what the environment affords the perceiver. In his statement of the theory of information pickup in *The Senses Considered As Perceptual Systems*, Gibson said "The environment provides an inexhaustible reservoir of information. Some men spend most of their lives looking, others listening, and a few connoisseurs spend their time in smelling, tasting, or touching. They never come to an end." Gibson (1966, p.269). In his later statement of the theory in *The Ecological Approach to Visual Perception* he said "The information in ambient light, along with sound, odor, touches, and natural chemicals, is inexhaustible. A perceiver can keep on noticing facts about the world she lives in to the end of her life without ever reaching a limit." Gibson (1979/1986, p.243).

The general answer to the 'how' question is that knowledge is obtained from ongoing cycles of interaction between the algorist and the environment. Von Neumann's conjecture suggests that an important part of the answer to the 'how' question is to be found in investigations of the logical functioning of the nervous system. This aspect of ecological psychology is least well developed at present. Turing's work provides only the barest outline of how to proceed. There are two reasons for this: the first is that Turing's immediate concern was with only a small part of the behavioural capability of the situated agent, namely the capacity to calculate. The second is that Turing's analysis of this limited capacity was purely abstract and functional.

Von Neumann discussed the issues involved in studying the nervous system in his Hixon symposium paper. The problem, he thought, was to account for what he called "general syndromes" of behaviour as opposed to "special phases". A special phase of behaviour is, for example, something like the capacity to treat two objects as

instances of the class “triangle”. Von Neumann thought that any special phase could be described exhaustively and that it would be a form of “logical mysticism” to deny it, but he qualified this assertion in the following important way. “It is, however, an important limitation, that this applies only to every element separately, and it is far from clear how it will apply to the entire syndrome of behavior.” (Aspray & Burks, 1987, p.413). The point he was making was further elucidated in the discussion following the paper. In response to a question he said, “The problem, then is not this: How does the central nervous system effect any one, particular thing? It is rather: How does it do all the things that it can do, in their full complexity? What are the principles of its organization? How does it avoid really serious, that is, lethal, malfunctions over periods that seem to average many decades?” (Aspray & Burks, 1987, p.424).

Von Neumann seems to be making two slightly different points. One is that it may be a very complicated matter to describe everything the nervous system can do because it has such a wide range of capabilities. Thus its behavioural description may be indefinitely long. This is the case for a universal machine as Gödel indicated. However, there is a second point which is that it is a complex problem to understand how the nervous system is organised so as to support the error free functioning of so many different capabilities. Von Neumann did not specifically mention the fact that the human agent is a complex system in which multiple activities proceed simultaneously, but the obvious challenge which this behavioural virtuosity presents to the psychological theorist is implicit in what he says.

Gibson’s concept of a perceptual system is relevant to questions about the multiple activities that human agents engage in simultaneously and may be interpreted as the foundation for a theory of the relations between “special phases” of behaviour

and “general syndromes”. The senses may be thought of as instances of special phases of behaviour whereas perceptual systems are instances of general syndromes which co-ordinate the activities of the senses. Support for this view can be found in Gibson (1966, p.49).

> When the “senses” are considered as active systems they are classified by modes of activity not by modes of conscious quality…Some of the systems, moreover, will pick up the same information as others, redundant information, while some will not, and they will cooperate in varying combinations.

The study of specific affordances elucidates the individual things that agents can do but leaves untouched the general question of how it is that agents can reliably perceive, select among, and act on the countless affordances that human environments offer. This question can also be tackled within a theory of perceptual systems or general syndromes of behaviour.

The fact that agents sometimes misperceive the affordances of the environment or fall prey to illusions is another aspect of complexity. Gibson was, of course, aware of the significance of errors in the study of perception. He was clear that “a concept of information is required that admits of the possibility of illusion. Illusions are a theoretical perplexity in any approach to the study of perception.” (Gibson, 1979/1986, p.243).

Von Neumann’s suggestion that the structure of the patterns of connectivity in the visual brain might be the best way to understand the complexities of visual processing may be slow to achieve acclaim among ecological psychologists but does, nevertheless, point in a fruitful direction. When seen in the light of Turing’s

ecological analysis of the relation between the algorist and the environment the functional study of the nervous system supports rather than threatens ecological analysis. The converse is also true and may help to explain why some neuroscientists, for example Nakayama (1994), have found Gibson's approach important for their work.

Turing's analysis leaves the question about general syndromes of behaviour unanswered because the computation of numbers is clearly a "special phase". Traditional computational theory of mind uses the universal machine concept to answer the general question via the notions of simulation and internal representation, but that is not a plausible solution for a range of reasons which have been discussed in the ecological literature. What we need to understand is how the functional states that model the algorist in the performance of one particular task are related to those that model the performance of another task. It was to answer this question that von Neumann thought logic would need to undergo a change and become more probabilistic and neurological. The statistical character of contemporary neural network theory suggests that logic has indeed progressed in that direction. However, there are also developments in the area of concurrency theory that suggest the continued value of discrete state methods. In particular, process algebras such as the π-calculus, (Milner, 1999, Sangiorgi & Walker, 2001) that have been developed to study systems of interacting, non-deterministic processes, may prove suitable for the study of interacting functions generally. This kind of study may make a significant contribution to the kind of theory that von Neumann thought was needed for an understanding of general syndromes of behaviour and that Gibson was arguing for with the concept of a perceptual system.

Conclusions.

Turing's analysis of computation provides a rigorous model of the situated algorist which specifically takes account of the constraints under which real agents function. It provides a formal answer to the 'who' question that Shaw and McIntyre (1974) placed at the heart of their vision of psychology. Turing's proof of the unsolvability of the halting problem demonstrates that the 'what' question cannot be answered a priori with full generality. There is no single process that can tell, from the structural description of a system alone, exactly how that system will behave. To understand a system fully one must study it as its behaviour unfolds. This form of knowledge is different from the knowledge that can be acquired by studying the structure of the system, but both forms are needed. Turing's analysis does not provide a full answer to the 'how' question because it abstracts from the details of systems under study and characterises them in terms of functional states. Von Neumann indicated that an adequate theory of complex automata would still be logical in character but that our understanding of logic would most likely be transformed into something more probabilistic and neurological.

The theoretical significance of the algorist has been clear to ecological psychologists since Shaw and McIntyre discussed the issue in 1974. The theoretical significance of von Neumann's conjecture is more difficult to gauge immediately but may prove equally important in the long term. Shaw has done a service to ecological psychology and to the broader psychological community by being the first to explore in detail some of the ramifications of von Neumann's complex thinking about complexity. It is appropriate that the last words, with whose sentiments I thoroughly agree, should come from Shaw. "The positive import derived from a serious

consideration of von Neumann's conjecture leads us then to reopen some old doors and admonishes us to peer more deeply into the nature of complexity." Shaw (1976, p.167).

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