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Sequential credit markets☆

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ABSTRACT

Entrepreneurs typically seek financing in decentralized markets, where they approach investors sequentially. We develop a model of sequential capital markets with privately informed investors. The sequential market creates a dynamic adverse selection externality that leads to overinvestment and excessive rents to intermediaries, even as the number of competing investors becomes arbitrary large. The resulting rents lead to excessive entry of investors and insufficient entry of entrepreneurs. Moving to a centralized market structure or reducing transparency restores competitiveness but may harm efficiency. The model also explains how even a small skill advantage for an investor can lead to preferential deal flow and outsized returns.

1. Introduction

Most primary capital markets, especially those catering to small- and medium-sized entities, function as decentralized search markets. In these markets, firms in need of funds typically sequentially approach financial intermediaries such as venture capitalists and banks. Professional investors in these markets rely heavily on their expertise in screening investment opportunities and play a crucial role in channeling resources to their most productive use.

Yet information, when privately held across competing intermediaries, can be a double-edged sword. Adverse selection and extraction of informational rents may interfere with the efficient aggregation of market information and distort investment decisions, and the extent to which this happens depends on the organizational structure of capital markets.

In this paper, we examine the impact of the sequential nature of primary capital markets on market outcomes in the presence of privately informed investors. We show that sequential markets give rise to a dynamic adverse selection externality: Entrepreneurs and investors do not fully internalize the effect of their decisions on subsequent financing rounds. As a consequence, they are often too quick to agree on marginal projects, diminishing the opportunity to gather additional market information and increasing the adverse selection faced by projects that are rejected.

We show that the dynamic adverse selection externality typically induces inefficient overinvestment, marking a departure from the typical underinvestment outcomes highlighted in the existing corporate finance literature on asymmetric information.[1] Additionally, this externality weakens competition among investors. As a result, even when the number of competing investors grows without bound so that all bargaining power sits with the entrepreneur, investors collectively earn substantial rents. On the extensive margin, the rents enjoyed by investors attract excessive entry relative to the social benefits of the information they produce, while the high cost of capital deters potential entrepreneurs from entering the market.

Our results speak to several salient facts in capital markets. As documented in Philippon (2015), the rent to financial intermediaries per dollar of intermediated capital has remained high over time, despite significant improvements in information technology which should have reduced costs, and despite deregulation, increased size, and increased competition in financial markets which should have reduced the market power of investors. This is consistent with our model, in which increased competition and reduced entry cost for intermediaries do not necessarily lead to lower rents per intermediated dollar.

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[1] The seminal paper by Myers and Majluf (1984) demonstrates how underpricing for firms possessing positive private information results in underinvestment at equilibrium. Within the mechanism design literature, Myerson and Satterthwaite (1983) illustrate how information asymmetry between buyers and sellers inevitably leads to inefficiently low levels of trade. In the auction and monopoly pricing literature, the outcome of undersupply is a standard result. We present a version of this outcome in the static version of our model and for the case of a centralized market.

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Our model also provides a rational explanation for the "success begets success" effect that has been documented in the venture capital market. For instance, Nanda et al. (2020) demonstrate that the notable persistence in outperformance by top venture capital firms is difficult to attribute solely to these firms' marginal advantages in adding value or selecting investments. Instead, it is primarily driven by preferential deal flow. This aligns with our model, where investor rents are disproportionately concentrated in the early rounds. Therefore, investors who are approached first gain a significant advantage. Consequently, any slight edge that positions an investor as a more appealing option for entrepreneurs can result in substantial rents for this investor.

Our model also provides a conceptual framework for evaluating the benefits and costs of recent financial innovation affecting market structure and transparency of primary capital markets. We show that increased transparency on the search history of firms — such as via information accessible in credit registries — typically benefit investors, but can increase firm cost of capital and have ambiguous effects on market efficiency. A move towards centralized markets tends to increase competition and lower investor rents, and increases social efficiency under some but not all circumstances.

We now describe our model and results in more detail. We consider a setting where an entrepreneur with a project idea seeks financing from investors. There is uncertainty about the viability of the project. Investors compete through directed search by posting their interest rate offers and entrepreneurs visit investors based on these offers. Each investor, upon being approached, conducts due diligence at no cost, yielding a private signal about the project's quality. Based on the signal, the investor either agrees to finance the project at the posted interest rate or reject it. The search continues until the entrepreneur either finds an investor who is willing to finance the project or runs out of options and abandons the project. As our focus is the friction introduced by dispersed information and sequential interactions, we abstract from physical costs of finding a counterparty by assuming that the entrepreneur is infinitely patient and has no search cost.

The efficiency of capital allocation depends on how well market outcomes reflect the private information generated by investors. If the individual information of investors were publicly observable, and the number of investors grew large, capital allocation would become perfect as it would be possible to perfectly separate good projects from bad. However, two problems impede information aggregation. First, whenever the entrepreneur comes to an agreement with an investor, information aggregation stops, extinguishing the option to postpone investment and seek out more information. Second, if the entrepreneur fails to secure financing with an investor and proceeds with her search, not all information in the investor's signal is passed on to the next round—only the fact that the project was rejected. As a result, each new investor faces an adverse selection problem created by the fact that the entrepreneur failed to receive financing in previous interactions with informed investors.

We assume that the length of time an entrepreneur has been on the market seeking financing is observable to investors. This informational regime may be supported either because of formal market structures, such as a credit registry recording past credit checks on the entrepreneur, or informal mechanisms, where investors obtain information through word of mouth.

While investors can observe the entrepreneur's time on the market, or equivalently, the number of times the entrepreneur has been rejected, they cannot directly observe which particular investor the entrepreneur has visited previously.[2] As a result, investors approached by the entrepreneur do not observe the financing terms at which she was previously rejected. Therefore, the entrepreneur's choice in the current round does not impact subsequent investors' beliefs. As beliefs in equilibrium should be consistent with actual financing offers, this imposes constraints on the offers that can be sustained in equilibrium.

We show that the entrepreneur chooses to bypass low-interest rate offers in early rounds, resulting in substantial rents being left to investors. If investors in subsequent rounds could observe what offers the entrepreneur chose in early rounds, the entrepreneur would internalize the effect of his choice on future adverse selection and would indeed have an incentive to choose low interest rates. However, subsequent investors do not directly observe previous entrepreneur's choices—their beliefs are based on equilibrium expectations. Therefore, the entrepreneur will favor higher interest rates with correspondingly higher probability of acceptance than if offers were directly observable.

Of course, investors in subsequent rounds anticipate this behavior of the entrepreneur, and adjust their beliefs downwards for previously rejected projects. As a result, rejections become costlier for the entrepreneur which further biases her decision towards choosing financing terms that are beneficial to investors, creating a self-reinforcing cycle leading to higher rents to investors.

We show that rents are disproportionately captured by investors who are visited early. Therefore, investors have large incentives to ensure that they meet entrepreneurs early, which is consistent with the significant effort venture capitalists invest in scouting the market and making sure they encounter prospective entrepreneurs before their competitors. We show that any advantage, however small, an investor might have in creating value for entrepreneurs will translate into significant rents, simply by making them the entrepreneurs' preferred initial contact. Whether this advantage is real or perceived is inconsequential. For example, if an investor achieves a successful exit with a portfolio company, entrepreneurs may favor visiting that investor first if they attribute the success to the skills of the investor. This, in turn, creates a positive feedback loop where the preferential deal flow secures high returns for the investor in the future—embodying the principle that "success begets success".

To evaluate the impact of the dynamic adverse selection externality on welfare, we compare market outcomes with those that a social planner would select in an otherwise identical sequential market. There are two forces in our model that distort the entrepreneur's preferred thresholds relative to that of a social planner. First, the entrepreneur cares only about the part of surplus she captures net of investor rents. This effect in isolation makes the entrepreneur prefer lower interest rates (higher thresholds) than a social planner, pushing towards lower probability of financing and hence underinvestment. This standard intuition is present in much of the literature on financing under asymmetric information, whether in a signaling or an informed investor context (see for example, Myers and Majluf (1984)).

The second effect, which is novel to our setting and acts in the opposite direction, arises from the dynamic adverse selection externality. Because the entrepreneur does not internalize the negative impact of raising thresholds on her continuation value, she tends to favor inefficiently high thresholds. This, in turn, leads to overinvestment.

We show that under a large set of circumstances, the social planner prefers as large a market as possible, where investor rents are driven to zero by setting a low interest rate that is slowly increased with each rejection, giving all surplus to the entrepreneur. For this case, there is no conflict between competitiveness and social surplus, so the underinvestment effect becomes weak. The overinvestment effect from the dynamic adverse selection externality remains strong, however, and we show that the market solution always features overinvestment.

We also show conditions on the signal distribution under which a social planner would prefer to only use a limited set of select investors and give them substantial rents. We show that in a sequential market the social planner is restricted to using the information contained in the first order statistic (highest signal) of the investors the planner includes in the market. A small market is preferred when the first order statistic from a small sample of investor signals is more informative than from a large sample. For this case, the standard underinvestment effect does

[2] Alternatively, they cannot observe whether a visited investor engaged in renegotiation over the contract terms.

not disappear, and the market solution could feature either over- or underinvestment.

Finally, we compare our market equilibrium to two other market structures: An opaque setting in which the entrepreneur's time on the market is not observed, and a centralized market in which investor compete to finance the project in an auction. We show that both these market structures become competitive, driving investor rents to zero as the number of investors grow large.

In an opaque market investors cannot directly observe the number of rejections a prospective borrower has faced. We show that even though in this case investors are faced with additional uncertainty about how many time the entrepreneur has been in the market, they are able to deduce it in equilibrium.

This follows from the fact that when investors compete by posting interest rate offers in an opaque market, only fully separating equilibria exist. Since interest rate offers are fixed, the entrepreneur is better off if she visits investors in ascending order of the interest rates posted. Consequently, if two investors quote the same interest rate, then either of these investors would be better off by quoting a slightly lower interest rate, ensuring that he is visited first by the entrepreneur.

As the number of investors increases, the desire to separate and front-run other investors forces investors to offer increasingly more favorable terms to the entrepreneur. In the limit, the market becomes competitive and investors collect no rent.

Under the circumstances when the social planner prefers a large market, both the opaque sequential market and a centralized market are more efficient than the transparent financial market as the number of investors grows large. However, when a small market is socially optimal, we show that opaque markets and the centralized market setting can no longer achieve the constrained efficient outcome, and may be less efficient than sequential markets where time on the market is observable. This may happen because in the transparent sequential market, adverse selection after rejections can be so strong that even an investor with the highest possible signal may not be able to break even. As a consequence, the entrepreneur could be endogenously excluded from the market after a few financing rounds. Since market break down is socially efficient in this case, transparent sequential markets may generate strictly more surplus than both the competitive opaque market and a centralized auction market.

Our results show that investor rents in sequential transparent markets are higher than those in either sequential opaque markets or centralized markets. Hence, investors typically benefit from any measures that increase transparency about time spent on the market, such as a credit registry. However, they do not gain from a shift towards more centralized market structures.

Our paper is related to several bodies of work. The efficiency of investment decisions in our model depends on the extent to which information is aggregated. Starting with Hayek (1945) and Grossman (1976) there is a large literature that studies information aggregation in financial markets. The closest papers in this literature are those that study herding and informational cascades in sequential decision making, see e.g., Bikhchandani et al. (1992), Welch (1992), and Avery and Zemsky (1998). Bikhchandani et al. (1992) and Welch (1992) consider nontradable assets; Avery and Zemsky (1998) focus on tradable assets. Our setup is closer to that in Bikhchandani et al. (1992) and Welch (1992) but unlike Bikhchandani et al. (1992) and Welch (1992) who assume the same exogenous offers in all rounds, we allow the entrepreneur to adjust her offers in different rounds. Therefore, in our setup, herding does not always occur in equilibrium as in Bikhchandani et al. (1992) and Welch (1992), and whether it exists or not depends on the signal distribution.

Similar to us, Bulow and Klemperer (2009), Roberts and Sweeting (2013), and Glode and Opp (2017) study relative efficiency of sequential and centralized markets. However, the economic mechanism in our paper is very different from those in the above papers. First, all three papers study selling mechanisms of an existing asset. Because information generated in a selling mechanism has no value for production, information aggregation plays no role in their models. In their settings, having as many potential buyers as possible is always good for a seller, which is not necessarily the case in our setup. Second, Bulow and Klemperer (2009) and Roberts and Sweeting (2013) assume nonzero participation costs, and Glode and Opp (2017) assume nonzero costs of information acquisition. These cost are the main sources of inefficiency in their models. In contrast, these costs are not present in our model, in which the main cause of inefficiency is imperfect information aggregation.

Another related work is Lauermann and Wolinsky (2016), who study a decentralized search setup with a seller searching for buyers. Lauermann and Wolinsky (2016) consider the case of an infinite number of buyers and assume that search history is not observable. In their analysis, Lauermann and Wolinsky (2016) focus on pooling equilibria and conclude that search markets are worse at aggregating information than the centralized markets. In contrast, we show that with finite but arbitrarily large number of buyers there is a separating equilibrium, which can be as efficient at aggregating information as centralized markets. In addition, we consider the case in which time on the market is observable and show that in this case, search markets can be more efficient than centralized markets. Our work is also related to Zhu (2012) who considers a model of opaque over-the-counter markets. Similar to Lauermann and Wolinsky (2016), Zhu (2012) considers a sale of an existing asset, assumes that search history is not observable, and studies only pooling equilibria. As a result, both the focus and analysis of Zhu (2012) are significantly different from those in our paper.

Our paper also contributes to the literature on directed search (see, Wright et al. (2021), for a recent survey of this literature). To the best of our knowledge, our paper is the first to provide a systematic analysis of the effect of competition in the presence of asymmetric information and dynamic adverse selection. The paper is also related to the literature on dynamic markets with adverse selection that studies the effects of public information on trading (see, for example, Kremer and Skrzypacz (2007) and Daley and Green (2012).) More broadly, we also relate to the large literature on search markets. Many papers in this literature focus on the friction introduced by the cost of finding a counter-party in private value environments (see, e.g., Duffie et al. (2005), Lagos and Rocheteau (2009), Vayanos and Weill (2008), Weill (2008)). We differ from this literature by focusing on the consequences of sequential interactions in a common-value environment, where the entrepreneur is infinitely patient and has no search cost.

Finally, our paper is also related to the literature on relationship lending started with the seminal paper by Rajan (1992). In common with papers in this literature, when an informed lender refuses credit in our model, he creates adverse selection for other borrowers, but in the context of a first-time borrower rather than an existing borrower.

2. Setup

The model has two sets of risk-neutral agents: a firm that seeks financing to start a new project, and a set $\{1, \ldots, N\}$ of potential investors. In our main analysis, we refer to the firm as the entrepreneur and assume that the firm has no other assets or financial resources. The project requires one unit of investment, and can be of two types: good ($\theta = G$) or bad ($\theta = B$). The good project pays $1 + X$, while the bad projects returns 0. The type of a particular project is initially unknown with investors sharing the same unconditional prior $P(G)$ that the project is good.

2.1. Signals

Investors have free access to a screening technology. When an investor makes an investigation, he gets a private signal that is informative about the project type. Conditional on the project type θ, signals are drawn identically and independently on $[0,1]$ with differentiable conditional densities $f_G(s)$ and $f_B(s)$ satisfying the strict maximum likelihood ratio property (MLRP):

Assumption 2.1.

$$\forall s > s', \quad \frac{f_G(s)}{f_B(s)} > \frac{f_G(s')}{f_B(s')}.$$

Assumption 2.1 ensures that higher signals are better news than lower signals.[3] We also assume that $f_B(1) > 0$, and that the likelihood ratio $f_G(1)/f_B(1)$ at the most optimistic signal realization $s = 1$ is bounded and equal to $\lambda < \infty$. These assumptions ensure that the observation of a single signal can never rule out the possibility of the project being bad, while an observer of all signals will be able to learn the project type perfectly as the number of investors goes to infinity.

We assume that any private information the entrepreneur herself may have is independent of investor signals conditional on the true type of the project. As we explain below, under this assumption private information held by the entrepreneur does not affect our analysis—the entrepreneur will always act *as if* the project is good.

To streamline the exposition, we also introduce the following two assumptions. First, to exclude the trivial scenario when the project is never financed, we assume that the project is positive NPV conditional on the top signal of a single investor:

Assumption 2.2. $P(G|S=1)X > P(B|S=1)$.

Additionally, we assume that the project is negative NPV conditional on the lowest signal of a single investor:

Assumption 2.3. $P(G|S=0)X < P(B|S=0)$.

Assumption 2.3 helps exclude corner solutions. It is not essential for our results, what matters is that the investment decision is non-trivial conditional on observing a sufficient number of signals, which is already guaranteed by Assumption 2.1. Assumptions 2.2 and 2.3 together imply the existence of $\underline{s} \in (0,1)$ such that the project breaks even at $S = \underline{s}$.

Finally, we make two relatively innocuous technical regularity assumptions on the signal distribution. Define function $\psi(s)$ as:

$$\psi(s) = \frac{f_B(s)}{f_G(s)} - \frac{1 - F_G(s)}{f_G(s)} \left(\frac{f_B(s)}{f_G(s)} \right)'. \tag{1}$$

We make the following assumptions:

Assumption 2.4. For $s \in [\underline{s}, 1]$, the function $\psi(s)$ is strictly decreasing.

Assumption 2.5. For $s \in [\underline{s}, 1]$, the function $\frac{F_G(s)}{F_B(s)}\psi(s)$ is either strictly decreasing or first strictly decreasing and then strictly increasing.

Assumption 2.4 is a mild regularity condition fulfilled by most standard signal distributions satisfying MLRP, and guarantees existence of equilibria. Assumption 2.4, together with Assumption 2.5, is sufficient for uniqueness of equilibria.

[3] The assumption of strict MLRP is for simplicity. It allows us to focus on pure strategy equilibria. All results go through under the weaker assumptions that signals satisfy weak MLRP: $\forall s \geq s',\ f_G(s)/f_B(s) \geq f_G(s')/f_B(s')$..

2.2. Fundraising market

The entrepreneur contacts investors sequentially and seeks to finance the project using a financial contract wherein the entrepreneur borrows one unit of capital and commits to repay $1+r$ in case of success, where $r \in [0, X]$. We refer to this as a debt contract with interest rate r, although given that there is only one positive cash flow realization, there is no distinction between equity and debt contracts in our setting.

Investors compete via directed search. At the beginning of each calendar period, before seeing their signals, investors simultaneously post interest rate menus $\{r_{i,t}\}_{t=1}^N$, where $r_{i,t}$ is the interest rate at which investor i commits to lend to a previously unencountered entrepreneur with time on the market t if the entrepreneur passes the investor's screening test.

The entrepreneur then chooses which investor to approach, whereafter the visited investor observes his private signal and either agrees to finance the project at the previously agreed interest rate or rejects it. We assume that the entrepreneur cannot revisit a previously visited investor.[4]

Investors can observe the entrepreneur's time on the market t, or equivalently, the number of times the entrepreneur has been rejected. However, they cannot directly observe which particular investor the entrepreneur has visited previously.[5]

3. Equilibrium

We look for a symmetric equilibrium in which at the beginning of each round t, remaining investors have common ex ante beliefs about project quality and post the same interest rate menu $\boldsymbol{r} \equiv \{r_t\}_{t=1}^N$ in every calendar period.

We capture investor's common beliefs at the beginning of a round t, before their private signals have been observed, by the likelihood ratio $z_{t-1} = \frac{P(G|\text{reached round } t)}{P(B|\text{reached round } t)}$ of project type for an entrepreneur that has failed to get financing in the previous $t-1$ rounds. The conditional probabilities of the project being of type G or B given z_{t-1} are:

$$P(G|z_{t-1}) = \frac{z_{t-1}}{1+z_{t-1}}, \qquad P(B|z_{t-1}) = \frac{1}{1+z_{t-1}}. \tag{2}$$

Given z_{t-1}, investor i's strategy in round t is a choice of whether to participate or wait until the next round, and if he participates, a posted interest rate $r_{i,t} \in [1, X]$ and a signal threshold $\hat{s}_{i,t} \in [0,1]$ such that if visited, the investor will extend financing when $S_i \geq \hat{s}_{i,t}$.[6]

The entrepreneur's strategy is a choice of which of the remaining investors to visit based on their postings, and if the visit results in a financing offer, whether to accept this offer or not. Note that the entrepreneur's action is not directly observable to unvisited investors.

We define a symmetric equilibrium as a triplet $\{r_t, \hat{s}_t, z_{t-1},\}_{t=1}^N$ such that all investors follow the same strategies and have the same beliefs, and such that the following holds:

[4] In particular, we do not allow the entrepreneur to "shop around" by presenting an accepted agreement to other investors in the hope of obtaining better financing terms. This assumption of exclusivity is important and represents one of the defining properties of sequential markets. Since we abstract from any search costs and costs associated with generating information, allowing the entrepreneur to take accepted offers to other investors without forfeiting them would make the resulting mechanism resemble a competitive centralized market place, which we study in detail in Axelson and Makarov (2023).

[5] One can alternatively assume that the identity of visited investors is observable, but that investors can renegotiate the posted interest rate $r_{i,t}$ once an entrepreneur arrives, where the renegotiated interest rate is not observable to other investors. Under this alternative assumption, our results are equivalent.

[6] We show in the next subsection that it is indeed optimal for investors to use cut-off strategies when choosing whether to finance or not.

1. Individual rationality at visits: Financing thresholds $\hat{s}_t$ maximize a visited investor's profits given r_t and z_{t-1}
2. Belief consistency: Beliefs z_{t-1} about credit quality are consistently updated using Bayes' rule on the equilibrium path.
3. Individual rationality at the posting stage:
   (a) Investors prefer posting r_t over waiting for the next round, and the entrepreneur prefers to accept offer r_t over waiting for the next round.
   (b) There is no alternative posting $r'_t \neq r_t$ such that the entrepreneur prefers to visit the deviating investor and such that the investor is strictly better off, under the off-equilibrium beliefs of other investors that the entrepreneur does not visit a deviating investor.

We proceed by first showing how financing thresholds are set conditional on a visit, and how beliefs are updated in equilibrium given the financing thresholds. In Section 3.4, we then show how the equilibrium at the posting stage is determined.

3.1. Threshold strategies

We first show that any visited investor will indeed use a threshold strategy under which he finances the project if his private signal is high enough. Consider an investor who is visited in round t at a posted interest rate r_t, set before the realization of his signal. After observing his signal $S = s$, the investor will agree to finance the project if

$$P(G|z_{t-1}, S = s)r_t - P(B|z_{t-1}, S = s) \geq 0, \tag{3}$$

that is, if

$$r_t \geq \frac{P(B|z_{t-1}, S = s)}{P(G|z_{t-1}, S = s)} = \frac{1}{z_{t-1}}\frac{f_B(s)}{f_G(s)}. \tag{4}$$

The last equality follows from Bayes' law and the fact that, conditional on the project type, signals are independent so that z_{t-1} and S are conditionally independent.

Since the likelihood ratio $\frac{f_G(s)}{f_B(s)}$ is strictly increasing by MLRP, the investor will finance the project for all signals above the threshold $\hat{s}_t$ where the investor just breaks even:

$$\frac{f_G(\hat{s}_t)}{f_B(\hat{s}_t)} = \min\left[\frac{1}{z_{t-1}r_t}, \lambda\right]. \tag{5}$$

The case $\frac{f_G(\hat{s}_t)}{f_B(\hat{s}_t)} = \lambda$ is where the threshold is set at the highest signal $S = 1$ and no financing takes place. As we show later, this can happen if a sufficient number of rejections reduces credit quality z_{t-1} enough such that the investor cannot break even at the highest interest rate $r_t = X$.

Using the correspondence between the optimal financing threshold and the interest rate that induces it, we can write investor strategies and value functions directly in terms of financing thresholds. Let $r(\hat{s}_t|z_{t-1})$ denote the interest rate required to implement the threshold $\hat{s}_t$ given the prior z_{t-1}. From Eq. (5), we obtain[7]

$$r(\hat{s}_t|z_{t-1}) = \frac{1}{z_{t-1}}\frac{f_B(\hat{s}_t)}{f_G(\hat{s}_t)}. \tag{6}$$

3.2. Entrepreneur and investor utility functions

Let $v^I(\hat{s}|z_{t-1})$ denote the expected profits to a visited investor who has posted an interest rate $r(\hat{s}|z_{t-1})$. Using Eqs. (2) and (6), we can write $v^I(\hat{s}|z_{t-1})$ as

$$v^I(\hat{s}|z_{t-1}) = P(G|z_{t-1})(1 - F_G(\hat{s}))r(\hat{s}|z_{t-1}) - P(B|z_{t-1})(1 - F_B(\hat{s}))$$

$$= P(B|z_{t-1})\phi(\hat{s}) = \frac{\phi(\hat{s})}{1 + z_{t-1}}, \tag{7}$$

where the function $\phi(\hat{s})$ is defined as

$$\phi(\hat{s}) = (1 - F_G(\hat{s}))\frac{f_B(\hat{s})}{f_G(\hat{s})} - (1 - F_B(\hat{s})). \tag{8}$$

The function $\phi(\hat{s})$ measures the infra-marginal informational rent captured by the investor when his signal is strictly above the threshold. This rent is strictly decreasing in the threshold $\hat{s}$ and goes to zero as $\hat{s}$ goes to one. Hence, for aggregate investor rents to go to zero as the market becomes large, all thresholds have to go to one with N—interest rates must be such that investors barely break even at the highest possible signal.

Turning to the entrepreneur, note that she never gets a payoff if her project turns out to be bad. Conditional on the project being good, let $v^E(\hat{s}|V^E_{t+1}, z_{t-1})$ denote her expected utility from a meeting with an investor posting $r(\hat{s}|z_{t-1})$, where V^E_{t+1} is her expected continuation utility if the meeting does not result in financing. We have that v^E is given by

$$v^E(\hat{s}|V^E_{t+1}, z_{t-1}) = (1 - F_G(\hat{s}))(X - r(\hat{s}|z_{t-1})) + F_G(\hat{s})V^E_{t+1}. \tag{9}$$

The first term is the probability of getting financing for a good project when the threshold is $\hat{s}$ times the residual profits $X - r(\hat{s}|z_{t-1})$ to the entrepreneur, whereas the last term is the probability of no financing times the continuation value V^E_{t+1}.

The entrepreneur's actual expected utility is given by $P(G|z^E_{t-1})v^E$, where z^E_{t-1} represents any prior the entrepreneur might have about the project. Note that v^E depends only on investor beliefs, so that the entrepreneur's own prior z^E_{t-1} only enters utility via the multiplier $P(G|z^E_{t-1})$. Hence, the entrepreneur's choices will only depend on her pay-off conditional on the project being good. She will always act "as if" the project is good, which eliminates any possibilities for signaling or screening. We can therefore use the value function v^E as the "utility" of the entrepreneur for purposes of deriving the equilibrium, while keeping in mind that we need to multiply by her prior when doing welfare analysis.

As we will show below, competition at the posting stage will lead investors to offer the rate that maximizes the entrepreneur's utility, as long as their participation constraint is satisfied. We now describe how the entrepreneur's preferred rate is determined.

The entrepreneur's continuation value V^E_{t+1} depends on the interest rates and thresholds set by the investors in the subsequent rounds, which in turn depend on their beliefs about project quality. Because unvisited investors cannot observe which investor the entrepreneur visits at round t, these beliefs are not directly affected by the entrepreneur's choice in that period. As a result, given V^E_{t+1}, the entrepreneur's preferred threshold s^*_t solves

$$s^*_t = \arg\max_{\hat{s}}\ (1 - F_G(\hat{s}))(X - r(\hat{s}|z_{t-1})) + F_G(\hat{s})V^E_{t+1}. \tag{10}$$

Taking the first order condition, we obtain

$$f_G(s^*_t)\left(X - r(s^*_t|z_{t-1}) - V^E_{t+1}\right) = -r'(s^*_t|z_{t-1})\left(1 - F_G(s^*_t)\right). \tag{11}$$

The left-hand side of Eq. (11) represents the utility loss from increasing the threshold and not being financed by the marginal investor with threshold signal s^*_t, while the right-hand side captures the interest rate savings $-r'(s^*_t|z_{t-1}) > 0$ on remaining investor types with signals above the threshold.

The entrepreneur trades off the cost of financing (which includes the opportunity cost V^E_{t+1} of stopping her search) against the probability of financing. Importantly, she does not internalize the negative effect a higher probability of financing this period has on perceived future credit quality and hence continuation utility if she gets rejected. This will drive her preferred interest rate higher than it would otherwise have been.

In general, Eq. (11) may have no solution or multiple solutions. Assumption 2.4 ensures that a unique solution exists. We formalize this in the following result.

[7] Note that $r(\hat{s}_t|z_{t-1})$ may exceed the highest possible interest rate X for sufficiently low values of z_{t-1}. However, as we will see below, this does not pose any issue since such interest rates are never chosen in equilibrium.

Lemma 3.1. *The entrepreneur's utility function $v_t^E(s|V_{t+1}^E, z_{t-1})$ is strictly quasi-concave in s for $s \in [\underline{s}, 1]$. If $z_{t-1} > \frac{1}{\lambda X}$ so that financing is possible, the entrepreneur's preferred threshold has a unique interior solution $s^* < 1$, determined by the solution to*

$$\psi\left(s_t^*\right) = z_{t-1}\left(X - V_{t+1}^E\right). \tag{12}$$

Proof. See the Appendix.

The function ψ, introduced in Assumption 2.4, is given by

$$\psi(s) = z_{t-1}\left(r(s|z_{t-1}) - r'(s|z_{t-1})\frac{(1-F_G(s))}{f_G(s)}\right). \tag{13}$$

Therefore, the condition $\psi'(s) < 0$ implies that the entrepreneur utility function $v_t^E(s|V_{t+1}^E, z_{t-1})$ is either decreasing everywhere, or initially increasing and then decreasing, which is equivalent to quasi-concavity.

3.3. Belief consistency

We next show how investor beliefs about credit quality are updated across rounds. From Bayes' law, consistency of beliefs requires that if $\hat{s}_t$ is an equilibrium financing threshold, the common equilibrium prior z_t of unvisited investors at the start of period $t+1$ evolves recursively as

$$z_t = z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}. \tag{14}$$

Eq. (14) captures how unvisited investors update their beliefs in equilibrium if the project remains unfinanced after period t, under the equilibrium belief that the investor approached in period t offers financing if and only if $S \geq \hat{s}_t$, and the entrepreneur accepts financing if it is offered. Eq. (14) shows that the perceived credit quality z_t declines as the entrepreneur remains in the market longer, since $F_G(s)/F_B(s) < 1$ by MLRP.

3.4. Equilibrium in the posting game

We next characterize equilibrium interest rates satisfying the individual rationality constraints at the posting stage. We assume and verify that we can write the equilibrium value functions V_{t+1}^E and V_t^I of the entrepreneur and investors that enter round t using investor beliefs z_{t-1} as a state variable. The value functions evolve recursively as

$$V_t^E(z_{t-1}) = (1-F_G(\hat{s}_t))(X - r(\hat{s}_t|z_{t-1})) + F_G(\hat{s}_t)V_{t+1}^E\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right), \tag{15}$$

$$V_t^I(z_{t-1}) = \frac{1}{N-t+1}\frac{\phi(\hat{s}_t)}{1+z_{t-1}} + \frac{N-t}{N-t+1}P(S_t < \hat{s}_t|z_{t-1})V_{t+1}^I\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right), \tag{16}$$

with $V_{N+1}^E = V_{N+1}^I = 0$.

Above, we have used Equation (14) to show how the continuation utilities depend on equilibrium belief formation. The first term in the investor's value function is the equilibrium probability of being picked by the entrepreneur among the remaining $N-t+1$ investors in round t times the expected profits if picked, while the second term is the probability that the investor remains unpicked and the entrepreneur remains unfinanced at the start of round $t+1$ times the equilibrium continuation utility.

The participation constraints for the entrepreneur and investors in any round t such that financing is possible can then be written as

$$r(\hat{s}_t|z_{t-1}) \leq X - V_{t+1}^E\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right), \tag{17}$$

$$\frac{\phi(\hat{s}_t)}{1+z_{t-1}} \geq P(S_t < \hat{s}_t|z_{t-1})V_{t+1}^I\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right). \tag{18}$$

Individual rationality also requires that for $\hat{s}_t$ to be an equilibrium threshold, there must be no deviating offer $r(s'|z_{t-1}) \neq r(\hat{s}_t|z_{t-1})$ such that the entrepreneur and the deviating investor are strictly better off. Using this condition, we solve for the equilibrium using backward induction, starting with the last period $t = N$.

In the last period, there is only one investor remaining and the entrepreneur has no outside option, so the last investor acts as a monopolist and sets interest rate $r_N = X$. The equilibrium threshold $\hat{s}_N(z_{N-1})$ is then given by Eq. (5):

$$\frac{f_G(\hat{s}_N(z_{N-1}))}{f_B(\hat{s}_N(z_{N-1}))} = \min\left[\frac{1}{z_{N-1}X}, \lambda\right]. \tag{19}$$

The value functions V_N^E and V_N^I of the entrepreneur and investor are given by

$$V_N^E(z_{N-1}) = 0, \tag{20}$$

$$V_N^I(z_{N-1}) = \frac{\phi(\hat{s}_N(z_{N-1}))}{1+z_{N-1}}. \tag{21}$$

For $t < N$, competition among investors shifts the bargaining power in favor of the entrepreneur. We show that the equilibrium interest rate must maximize the entrepreneur's round t utility, subject to the investors' participation constraint.

Observe that the entrepreneur's utility in Eq. (9) from visiting a deviating investor posting $r(s|z_{t-1}) \neq r(\hat{s}_t|z_{t-1})$ can now be written as

$$v_t^E(s|\hat{s}_t, z_{t-1}) = (1-F_G(s))(X - r(s|z_{t-1})) + F_G(s)V_{t+1}^E\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right). \tag{22}$$

Let $s_t^*(\hat{s}_t)$ denote the "best response" deviation for the entrepreneur to a postulated equilibrium threshold $\hat{s}_t$, that is,

$$s_t^*(\hat{s}_t) = \arg\max\, v_t^E(s|\hat{s}_t, z_{t-1}). \tag{23}$$

Let $\bar{s}_t(\hat{s}_t)$ denote the highest threshold that satisfies the investor's individual rationality constraint if other investors use $\hat{s}_t$, given by

$$\frac{\phi\left(\bar{s}_t(\hat{s}_t)\right)}{1+z_{t-1}} = P(S < \hat{s}_t|z_{t-1})V_{t+1}^I\left(z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\right). \tag{24}$$

We then have the following result:

Lemma 3.2. *For $t < N$, given z_{t-1} and continuation value functions $V_{t+1}^E(\cdot)$ and $V_{t+1}^I(\cdot)$, a symmetric equilibrium threshold $\hat{s}_t$ solves*

$$\hat{s}_t = \min\{s_t^*(\hat{s}_t), \bar{s}_t(\hat{s}_t)\}, \tag{25}$$

with the value functions $V_t^E(z_{t-1})$ and $V_t^I(z_{t-1})$ given by Eqs. (15) and (16).

Proof. See the Appendix.

Since the interest rate is strictly decreasing in the threshold, the equilibrium condition (25) is equivalent to

$$r(\hat{s}_t|z_{t-1}) = \max\{r(s_t^*(\hat{s}_t)|z_{t-1}), r(\bar{s}_t(\hat{s}_t)|z_{t-1})\}. \tag{26}$$

In other words, Lemma 3.2 says that investors will compete down the equilibrium interest to the entrepreneur's preferred rate, provided that it is above the participation interest rate of investors. Any interest rate above the entrepreneur's preferred rate and above the investor's participation constraint can be undercut by a deviating investor, creating increased expected profits to both the entrepreneur and the deviating investor.

Lemma 3.2 also shows how the equilibrium, which in general is a multi-dimensional fixed point problem, can be solved by backward induction.

We can now state the main result of this section, which is to show that even though all bargaining power will shift to the entrepreneur as the number of competing investors grows, the market never becomes competitive and investors end up earning non-vanishing rents:

Proposition 1. *There exists an equilibrium, and for sufficiently large N, the equilibrium is unique, with the investors' participation constraint binding at most in the final few periods. Investor rents decrease in t, with rents for* *early investors bounded away from zero even as $N \to \infty$. The market breaks down after a finite number of periods if and only if*

$$2\left(\frac{f_G(1)}{f_B(1)}\right)' < \lambda\left[f_G(1) - f_B(1)\right]. \tag{27}$$

Proof. See the Appendix.

As N increases, the participation thresholds of all investors go to the zero-profit level. This happens because the total surplus is bounded, so the expected profit per investor must converge to zero as N grows. Eqs. (7) and (8) for investors' surplus show that this can only happen if all participation thresholds of investors $\bar{s}_t$ converge to one.

In contrast, the entrepreneur's preferred threshold stays bounded away from one in any round where $z_{t-1} - \frac{1}{\lambda X}$ stays bounded away from zero. As explained in Lemma 3.1, this is a consequence of the fact that the entrepreneur treats the continuation value V_{t+1} as given and independent of her choice of which investor to approach.

If investors in subsequent rounds could observe the interest rate postings selected by the entrepreneur in earlier rounds, she would internalize the impact of her choices on future adverse selection. In that case, she would have an incentive to approach investors offering lower interest rates. However, since investors do not directly observe the entrepreneur's actions, their beliefs are formed based on equilibrium expectations. As a result, the entrepreneur prefers higher interest rates, which are associated with a higher probability of acceptance.

Of course, investors in subsequent rounds anticipate this behavior of the entrepreneur, and adjust their beliefs downwards for previously rejected projects. As a result, rejections become costlier for the entrepreneur which further biases her decision towards picking financing terms that are beneficial to investors, creating a self-reinforcing cycle leading to excessive rents to investors.

Because the participation thresholds of investors converge to one with N and the entrepreneur's preferred thresholds stay bounded away from one as long as credit quality z_{t-1} is bounded away from the no-financing threshold, the participation constraint of investors will be binding at most in the final few rounds—but those rounds become inconsequential as N grows large.

Proposition 1 also specifies the condition under which the entrepreneur can participate in all available rounds. Since the perceived project quality declines following each rejection, it may eventually reach a level where further financing is no longer feasible. Eq. (27) shows that market breakdown occurs when the likelihood ratio $f_G(s)/f_B(s)$ is relatively flat at the top of the signal distribution. We provide more intuition for this condition and the effects of market breakdown on social surplus in Section 5, where we discuss efficiency of the equilibrium.

4. Favored investors: Success begets success

Because rents are disproportionately captured by investors who are visited early, investors have large incentives to ensure that they are first in line to meet entrepreneurs. This is consistent with the significant effort venture capitalists exert in scouting the market to make sure they encounter prospective entrepreneurs before their competitors. We show that any advantage, however small, an investor might have in creating value for entrepreneurs will translate into significant rents, simply by making them the entrepreneurs' preferred initial contact. Whether this advantage is real or perceived is inconsequential. For example, if an investor achieves a successful exit with a portfolio company, entrepreneurs may favor visiting that investor first if they attribute the success to the skills of the investor. This, in turn, creates a positive feedback loop where the preferential deal flow secures high returns for the investor in the future—embodying the principle that "success begets success".

Consider a scenario in which one investor has a slight advantage, resulting in an excess return of $X + \delta$ upon success instead of X, where δ represents a small value-added. Suppose in the absence of such an investor, the equilibrium interest rate in the first round was r_1 and the corresponding screening threshold was $\hat{s}_1$. By posting an offer r_1' with corresponding threshold $\hat{s}_1'$ in the first round such that

$$(1 - F_G(\hat{s}_1'))\left(X + \delta - r(\hat{s}_1'|z_0) - V_2^E\right) > (1 - F_G(\hat{s}_1))\left(X - r(\hat{s}_1|z_0) - V_2^E\right),$$

the investor with an advantage can guarantee that he will be chosen by the entrepreneur over other investors. As δ approaches zero, this will lead to expected rents $\frac{\phi(\hat{s}_1)}{1+z_0}$ for the investor. From the results in Proposition 1, these rents are bounded away from zero even as N goes to infinity, while rents to the average investor converge to zero.

This result is consistent with the findings in Nanda et al. (2020), who demonstrate that the notable persistence in outperformance by top venture capital firms is difficult to attribute solely to these firms' marginal advantages in adding value or selecting investments. Instead, it is primarily driven by preferential deal flow—i.e., being the first investor entrepreneurs approach.

5. Efficiency

We now study the social efficiency of the market equilibrium. The fact that the market remains uncompetitive even as the number of investors grows without bound does not by itself imply that the equilibrium is inefficient. We show in this section that under a large set of circumstances the equilibrium is indeed inefficient—a social planner who could impose interest rates in each round would indeed drive investor profits to zero in order to maximize social surplus. We also show the conditions under which the social planner would prefer to limit market size and give a select number of investors substantial rents, which turns out to encompass the situations outlined in Proposition 1 in which the market endogenously breaks down.

Consider a social planner who maximizes welfare by setting the thresholds $\hat{s}$ (or equivalently, by imposing the interest rates $\hat{\boldsymbol{r}}$ on the market). The social planner solves:

$$\max_{\hat{s}\in[0,1]^N} P(G|z_0)\left(1 - \prod_{t=1}^{N} F_G(\hat{s}_t)\right)X - (1 - P(G|z_0))\left(1 - \prod_{t=1}^{N} F_B(\hat{s}_t)\right). \tag{28}$$

The first term in the social welfare function is the probability that a good projects gets financed given the threshold strategy (which happens when at least one of the signals is above the threshold), times the net present value X for good projects. The second term is the probability of financing bad projects, where the loss on a financed bad project is normalized to one.

Proposition 2 below shows that the behavior of the maximal social surplus depends on the properties of the function H defined as:

$$H(s) = \frac{f_G(s)}{f_B(s)}\frac{F_B(s)}{F_G(s)}. \tag{29}$$

When the function H strictly increases on the interval $[0,1]$, the social surplus increases with N, and a social planner will use the same threshold $\hat{s}_P$ in every round. In contrast, if H decreases in the neighborhood of one, the social planner may choose to only use a limited number of signals and set the threshold at one for the remaining rounds.

Proposition 2. *(i) Suppose the function H defined in Eq. (29) strictly increases on the interval $[0,1]$. Then the maximal social surplus strictly increases with N. The socially optimal screening policy is to use the same screening threshold in all rounds. The optimal screening threshold is where the project just breaks even conditional on the highest signal among investors being at the threshold:*

$$P(G|\max_{t\le N} S_t = \hat{s}_P)X - P(B|\max_{t\le N} S_t = \hat{s}_P) = 0. \tag{30}$$

As $N \to \infty$, the threshold $\hat{s}_P$ goes to one and aggregate investor rents go to zero.

(ii) If H is a decreasing function on $[\hat{s}_n, 1]$, where $\hat{s}_n$ is the break-even point conditional on the highest signal among n investors:

$$P(G|\max_{t\leq n} S_t = \hat{s}_n)X - P(B|\max_{t\leq n} S_t = \hat{s}_n) = 0 \tag{31}$$

then the maximal social surplus is achieved with no more than n screenings, and investor rents do not go to zero with N.

Proof. See the Appendix.

With a sequential screening technology, the information a social planner can use for making the investment decision is restricted to what he can learn from the first-order statistic (the highest signal) of the set of signals he chooses to pay attention to. Case (i) in the proposition is when the first-order statistic from the full set of signals is more informative than from a subset of the signals, while in case (ii) the opposite holds.

To understand the role of the function H in determining the solution, consider two screening thresholds $\hat{s}_1$ and $\hat{s}_2$ and assume that $\hat{s}_1 > \hat{s}_2$. The marginal effect of decreasing $\hat{s}_1$ on social welfare is to finance an extra project when $S_1 = \hat{s}_1$ and all other signals are below their respective thresholds. If any other signal exceeds its threshold, the project would have been financed anyway.

The attractiveness of this marginal project is driven by the likelihood ratio of the project being good, given by

$$\frac{P\left(G|S_1 = \hat{s}_1, S_2 \leq \hat{s}_2, \ldots, S_N \leq \hat{s}_N\right)}{P\left(B|S_1 = \hat{s}_1, S_2 \leq \hat{s}_2, \ldots, S_N \leq \hat{s}_N\right)} = z_0 \frac{f_G(\hat{s}_1)}{f_B(\hat{s}_1)} \frac{F_G(\hat{s}_2)}{F_B(\hat{s}_2)} \frac{\Pi_{t=3}^N F_G(\hat{s}_t)}{\Pi_{t=3}^N F_B(\hat{s}_t)}.$$

Compare this to the likelihood ratio of the marginal project financed by investor 2 at the lower threshold $\hat{s}_2$. It would improve welfare to increase $\hat{s}_2$ and decrease $\hat{s}_1$ if

$$\frac{f_G(\hat{s}_1)}{f_B(\hat{s}_1)} \frac{F_G(\hat{s}_2)}{F_B(\hat{s}_2)} > \frac{f_G(\hat{s}_2)}{f_B(\hat{s}_2)} \frac{F_G(\hat{s}_1)}{F_B(\hat{s}_1)},$$

that is, if $H(\hat{s}_1) > H(\hat{s}_2)$. In other words, the condition $H(\hat{s}_1) > H(\hat{s}_2)$ means that if observing a threshold signal from investor 1 and a rejection from investor 2 who uses a lower threshold is better information than the opposite scenario, it is better to set their thresholds closer together. Therefore, all interior thresholds should be solutions of $H(\hat{s}) = c$, where c is some constant.

For a signal not to be used, so that $\hat{s} = 1$, it must be that $H(1)$ is lower than at any interior screening thresholds. When H is a strictly increasing function, this cannot happen. Therefore, all screening thresholds must be interior and the same.

When H is decreasing at the top of the range, it is optimal to use only a finite number of internal screening thresholds and ignore the rest of the signals (setting their screening thresholds to 1). This happens when the likelihood ratio $f_G(s)/f_B(s)$ stays relatively flat at the top end of the range, and most of the increase in the likelihood ratio occurs at an intermediate range.

The two cases of large and small markets also realize in the directed search equilibrium. Comparing condition (27) for the market breakdown in the directed search model and the social planner solution, we can see that when the social planer prefers large markets, the entrepreneur will also visit all investors in the directed search setting. This follows from the fact that

$$H'(1) = \left(\frac{f_G(1)}{f_B(1)}\right)' - \lambda\left[f_G(1) - f_B(1)\right]. \tag{32}$$

Therefore, $H'(1) > 0$ implies the condition in Proposition 1 for no market break down.

It follows that when a market breakdown occurs in the directed search equilibrium, the social planner always prefers a smaller market. Both outcomes arise when the likelihood ratio is relatively flat at the top of the signal distribution. In such cases, the social planner favors fewer investors with lower thresholds, as information discovery is better further down the signal distribution. Meanwhile, the entrepreneur also prefers lower thresholds, since inframarginal rents are small when the likelihood ratio is flat.

Our results show that although market breakdown by necessity prevents information aggregation, it is not necessarily socially inefficient—in fact, it can be a feature of the constrained planner's second-best solution.

Note that the condition for market breakdown is stricter in the directed search equilibrium than in the social planner's solution. To understand why, observe that the social planner, in the final rounds of financing, always sets the interest rate at the maximum level X, thereby allocating all surplus to investors. This strategy would never be optimal for the entrepreneur. If given the choice, the entrepreneur would always prefer a lower interest rate and correspondingly higher screening threshold. As a result, being rejected at this higher threshold conveys a more favorable signal than being rejected at the maximal interest rate X, thereby preserving the possibility of obtaining financing in future rounds.

5.1. Overinvestment

As Proposition 1 shows, the equilibrium interest rate in the sequential market will be the entrepreneur's preferred rate for all but a vanishingly small set of periods at the end when N grows large. In fact, one can show that in the limit, the equilibrium converges to the equilibrium of a random search setting in which the entrepreneur sequentially approaches investors and makes take-it-or-leave-it offers.

There are two forces in our model that distort the entrepreneur's preferred thresholds relative to that of a social planner. First, the entrepreneur cares only about the part of surplus she captures net of investor rents. This effect in isolation makes the entrepreneur prefer lower interest rates (higher thresholds) than a social planner, pushing towards lower probability of financing and hence underinvestment. This standard intuition is present in much of the literature on financing under asymmetric information, whether in a signaling or an informed investor context (see for example, Myers and Majluf (1984)).

The second effect, which is novel to our setting and acts in the opposite direction, arises from the dynamic adverse selection externality. Because the entrepreneur does not internalize the negative impact of raising thresholds on her continuation value, she tends to favor inefficiently low thresholds. This, in turn, leads to overinvestment.

Our main result in this section is to show that when the social planner prefers large markets — corresponding to case (i) in Proposition 2 — the second effect dominates as N becomes large. As a result, the market equilibrium features overinvestment:

Proposition 3. *When the function H, defined in Eq. (29), is strictly increasing, then the market equilibrium in the direct search model features overinvestment in the limit as $N \to \infty$.*

Proof. See the Appendix.

As we showed in Proposition 2, when the function H is strictly increasing, there is no inherent conflict between efficiency and competitiveness. The social planner prefers as large and competitive a market as possible, extracting all rents from investors in the limit. Hence, at the social planner's solution, the first effect pushing the entrepreneur towards underinvestment disappears. Without the first effect, the dynamic adverse selection externality effect dominates, leading to overinvestment.

When the social planner prefers small markets, corresponding to case (ii) in Proposition 2, she allows investors to retain some rents. For this case, the standard underinvestment effect becomes more important in our setting, and the net effect can go either way—the entrepreneur might finance either too few or too many projects in equilibrium relative to the social optimum. The higher the rents left to investors by the social planner, the more likely it is that the entrepreneur will finance too few projects in equilibrium.

5.2. Excessive entry

In this section, we endogenize entry and examine how the equilibrium size of the market compares to the socially optimal size. We highlight two features of our model that can lead to inefficiently large financial markets.

First, as shown in Proposition 1, investors capture a non-negligible share of the total surplus. Second, the marginal social surplus — measured before accounting for entry costs — generated by an additional investor tends to decline rapidly towards zero, and may even become negative. As a result, the average profits per investor can exceed the marginal social surplus, creating incentives for excessive entry into the market.

To formalize these arguments, suppose that prospective investors face market entry costs c_i that increase with i, so that $c_1 < c_2 < \cdots < c_\infty$, and that each investor decides whether to enter before the game begins. For simplicity, assume that the signal distributions are such that social surplus is a quasi-concave function of the number of investors and is concave wherever it is increasing.

Then, holding the set of entrepreneurs fixed, the equilibrium market size N is determined by

$$\Pi_N / N \geq c_N, \tag{33}$$

$$\Pi_{N+1}/(N+1) \leq c_{N+1}, \tag{34}$$

where Π_N is aggregate expected investor profits in a market of size N.

Denoting the social surplus gross of entry costs as W_N, a social planner would choose the optimal market size, N_P, such that

$$W_{N_P} - W_{N_P-1} \geq c_{N_P} \tag{35}$$

$$W_{N_P+1} - W_{N_P} \leq c_{N_P+1}. \tag{36}$$

Comparing (33) and (35), the condition for excessive entry is

$$\Pi_N / N \geq c_N > W_N - W_{N-1}. \tag{37}$$

When competition occurs through directed search, Proposition 1 show that aggregate investor rents are non-vanishing as N grows large. Since social surplus is quasi-concave and bounded by X, it must be that

$$\lim_{N\to\infty} N(W_N - W_{N-1}) = 0. \tag{38}$$

Therefore, there must exist some N large enough such that $\Pi_N/N > W_N - W_{N-1}$. As a consequence, one can choose a sequence of entry costs such that $\Pi_N/N \geq c_N > W_N - W_{N-1}$, which will lead to an excessive entry of investors.

In contrast to investors, adding an entrepreneur to the market increases social surplus linearly. However, the entrepreneur does not fully capture this surplus when investor rents are non-vanishing. As a result, entrepreneurs facing fixed entry costs will enter at a rate below the socially optimal level, leading to a suboptimally low number of entrepreneurs in equilibrium.

The analysis presented above does not consider any positive spillovers to the broader economy that are not captured by investors and entrepreneurs. For instance, research-intensive innovative growth firms generate positive externalities by enhancing sector productivity, increasing employment, and boosting consumer surplus. These considerations have motivated industrial policies that subsidize venture capital. The tendency towards excessive entry that we highlight in this paper would need to be balanced against such positive spillovers when formulating a thoughtful industrial policy.

6. Alternative market structure: Opaque markets

In this section, we consider an alternative market structure where time on the market is not directly observable. We show that in this case, the dynamic adverse selection is alleviated because investors competing through directed search have an incentive to post low interest rates to attract fresh entrepreneurs on the market, which increases screening thresholds. We show that the market becomes competitive, eliminating investor rents as N grows large. This is also socially efficient when the social planner prefers large markets, but can be less efficient than the transparent market when the social planner prefers small markets.

As in the case where time on the market is observable, investors compete by posting interest rate offers, committing to lend if the entrepreneur passes their credit test. However, since the timing of rounds is now unobservable, investors cannot condition their offers on the number of past rejections. Each investor i therefore posts a single interest rate r_i.

We assume that investors post their offers sequentially, starting with investor $i = 1$.[8] After observing the posted offers, the entrepreneur chooses the order in which to visit the investors. Investors, however, do not observe their position in this order. We first show that the entrepreneur will prefer to choose lower interest rate offers first:

Lemma 6.1. *Entrepreneurs will visit investors in ascending order of the interest rates.*

Proof. See the Appendix.

In an opaque market, the overall probability of obtaining financing for an entrepreneur who sequentially approaches two investors posting different interest rates is unaffected by the actual order in which she visits them. But visiting the investor posting a lower interest rate first lowers the expected financing costs.

We next note that only fully separating equilibria — in which no two investors post the same interest rate — exist. If two investors quote the same interest rate r, then either of these investors would be better off by quoting an interest rate slightly lower than r and ensuring that he is visited first by the entrepreneur. Hence, even though time on the market is unobservable, investors will be able to deduce it in equilibrium.

An investor whose interest rate is higher than $t-1$ other interest rates knows he will be visited in round t. As we show in Proposition 4 below, investors will in equilibrium position themselves so as to split the market and earn equal profits.

Proposition 4. *There is a unique equilibrium. In this equilibrium, each investor $t \leq N$ selects any offer from a set of interest rates $\{r_t\}_{t=1}^{N-1}$ not chosen by previous investors and finances the project if and only if their signal exceeds the threshold $\hat{s}_t$, where the screening thresholds $\{\hat{s}_t\}_{t=1}^{N}$ are given by the unique solution to system of equations*

$$\frac{f_G(\hat{s}_N)}{f_B(\hat{s}_N)} = \frac{1}{z_{N-1}X}, \tag{39}$$

$$\frac{\phi(\hat{s}_t)}{F_B(\hat{s}_t)} = \phi(\hat{s}_{t+1}), \quad t = 1, \ldots, N-1, \tag{40}$$

where function ϕ is defined by Eq. (8), and interest rates and investors' beliefs are given by Eqs. (6) and (14). The entrepreneur visits investors in ascending order of the interest rates and all investors earn the same expected profits. As the number of investors goes to infinity, all thresholds converge to one and the entrepreneur captures all surplus.

[8] Assuming that investors post offers sequentially guarantees a pure-strategy equilibrium. The qualitative nature of our results are unchanged if investors post their offers simultaneously, but the equilibrium then involve mixed strategies and is more cumbersome to characterize.

Proof. See the Appendix.

Similar to the case when time on the market is observable, the last investor quotes the maximum possible interest rate X. Therefore, a screening threshold in the last round solves equation (39). Eq. (40) define screening thresholds to guarantee that all investors earn the same expected surplus.

The reason why all investors achieve the same rent in equilibrium is that if some investor gets a larger profits than other investors, other investors who are visited before him have incentives to charge higher interest rates. Rejections at these higher rates reduce the perceived quality of the project, thus reducing the expected profits for this investor. If the first investor obtains a larger profit, then other investors have incentives to undercut his interest rate offer to be visited before him.

Since all investors achieve the same rent in equilibrium, as the number of investors increases, the surplus that each investor earns goes to zero. We show that when the last investor is visited with a non-vanishing probability, his rent is proportional to the distance of his screening threshold to one. Therefore, a screening threshold in the last period goes to one with N. Since function ϕ is decreasing, Eqs. (40) imply that screening thresholds decrease with the number of rounds. Thus, it follows that all screening thresholds go to one with N. Therefore, investors collect no rent and the entrepreneur extracts all surplus in the limit.

It is instructive to compare the outcomes in the directed search model under observable and unobservable time on the market. In the transparent market, if in any given round the entrepreneur prefers interest rates lower than those required by the investor participation constraint (18), the equilibrium coincides with that of the opaque market. In both cases, competition drives interest rates down to the point where the participation constraint binds, and all investors earn equal expected profits regardless of the round in which they are approached.

What prevents this outcome in the transparent market is the presence of dynamic adverse selection. The entrepreneur trades off the probability of obtaining financing against the level of interest rates, while failing to internalize the negative effect of a higher interest on her continuation utility. As a result, the entrepreneur favors high interest rates, and investors have no incentives to offer her low interests in early rounds.

In contrast, when rounds are not observable, the entrepreneur is better off if she visits investors in ascending order of the interest rates. Offering low interest rates then impacts the order in which the entrepreneur visits investors. This creates powerful incentives for the investors to offer low interest rates in equilibrium.

7. Comparison to centralized markets

As we have shown, information aggregation in sequential markets is inherently restricted by the use of threshold strategies, so investment cannot be conditioned on individual signal realizations. In this section we compare the efficiency of sequential markets to centralized markets, in which no such restriction exists.

We model a centralized market as a second-price auction for simplicity, but the results are the same in a first-price or ascending-price auction. After privately observing their signals, investors simultaneously either submit interest rate bids or choose not to participate. The lowest interest rate bid wins. The winning investor finances the project at the second lowest bid, or offers the investor a cash payment for the right to abandon the project if the information in the bids indicate that the project has negative net present value.

Despite the fact that the centralized market can make the investment decision contingent on the information in all submitted bids, which reveal the signals of participating bidders, the following proposition shows that efficiency is at most the same as in the restricted social planner solution, and is strictly worse if signal are distributed as in case (ii) in Proposition 2.

Proposition 5. *The centralized market has a symmetric equilibrium in which investors participate and the project is financed if and only if their signal is above the social planner solution $\hat{s}_P$ defined in Eq. (31). Aggregate investor profits go to zero with N.*

Key to the result in the proposition is determining the participation threshold. Note that the threshold $\hat{s}_P$ in Eq. (31) is set such that the project just breaks even when the one signal is at the threshold and all others are below. But this is exactly the participation threshold an investor in the auction would choose. Bids $r(s)$ are decreasing in the signal s, so an investor that is at the threshold knows that if he participates, he will win only if no one else participates. The threshold $\hat{s}_P$ is exactly the break-even point for such an investor conditional on winning.

When signal are distributed as in case (i) in Proposition 2, the centralized market equilibrium coincides with the social planner solution, and is hence more efficient than the transparent directed search equilibrium. However, when signal are distributed as in case (ii) in Proposition 2, the centralized market equilibrium is less efficient than the social planner solution for large N. Hence, the transparent directed search market can be more efficient than centralized markets in this case, because market break down endogenously reduces the market size in the transparent market.

8. Conclusion

In this paper, we highlight the role expert financial intermediaries play in ensuring that capital flows to the most productive firms. Market architecture is an important determinant of how efficiently market information is used, and of how rents are split between intermediaries and firms seeking capital. The majority of primary capital markets still function as decentralized search markets, and the contribution of our paper is to set up a tractable model that shows how information aggregation and rents are determined in sequential markets with informed intermediaries.

We show that a main impediment to efficiency in these markets is that investors and entrepreneurs who meet do not fully internalize the effect of their decisions on adverse selection in the pool of rejected projects. The dynamic adverse selection externality leads to inefficient overinvestment and excessive rents to investors. The resulting high cost of capital can lead to overentry of intermediaries into financial markets, and can persist even when technology improvements drive down the cost of operating an intermediary.

Our model allows us to compare centralized to decentralized market architectures, as well as the effect of increased transparency. Intermediary rents are typically highest in decentralized markets where the search history of firms is observable, so one would expect market participants on the supply side of capital markets to push for such a market structure and resist attempts to move to a more centralized market.

CRediT authorship contribution statement

Ulf Axelson: Writing – original draft, Formal analysis. **Igor Makarov:** Writing – original draft, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Proof of Lemma 3.1.
Using Eq. (9), we can write $v_t^E(s|V_{t+1}^E, z_{t-1})$ as

$$v_t^E(s|V_{t+1}^E, z_{t-1}) = V_{t+1}^E + (1 - F_G(s))(X - V_{t+1}^E - r(s|z_{t-1})). \tag{A.1}$$

Direct computations show that

$$\frac{dv_t^E(s|V_{t+1}^E, z_{t-1})}{ds} = f_G(s)\left(\frac{\psi(s)}{z_{t-1}} - (X - V_{t+1}^E)\right), \tag{A.2}$$

where function ψ is defined by Eq. (1). By Assumption 2.4, the function ψ is strictly decreasing. Therefore, v_t^E is either strictly increasing or initially increasing and then decreasing on $s \in [\underline{s}, 1]$, which implies that it is quasi-concave and has a unique maximum on $s \in [\underline{s}, 1]$.

Next, suppose that $z_{t-1} > \frac{1}{\lambda X}$ so that financing is possible. We show in Lemma A.6 that the maximal expected social surplus in a sequential market with a screening technology that satisfies $f_G(1)/f_B(1) = \lambda$ is no larger than $P(G)\max(X - 1/(\lambda z_0), 0)$ and for any likelihood ratio satisfying strict MLRP is strictly lower than $P(G)\max(X - 1/(\lambda z_0), 0)$. Therefore, the continuation value conditional on the project being good must be strictly less than $\max(X - 1/(\lambda z_0), 0)$. In other words, it must be that

$$X - r(1|z_{t-1}) - V_{t+1}^E > 0. \tag{A.3}$$

Consider the F.O.C. (11). Eq. (A.3) implies that the right-hand side is strictly greater than zero at $s = 1$, while the left-hand side is equal to zero. Therefore, it must be that the entrepreneur's preferred threshold is strictly less than one. From the above, the necessary and sufficient condition for the optimum is

$$\psi(s_t^*) = z_{t-1}(X - V_{t+1}^E). \quad \square$$

Proof of Lemma 3.2.
We have shown in the text that we can use z_{N-1} as a state variable for V_N^E and V_N^I. We next show that if we can use z_t as a state variable for V_{t+1}^E and V_{t+1}^I, then the result in the lemma follows and we can use z_{t-1} as a state variable for $\hat{s}_t$, V_t^E, and V_t^I. The result in the lemma then follows by induction.

Assume the entrepreneur's participation constraint is not binding. We show at the end of the proof that this is indeed the case.

Let $\underline{r}_t \equiv r(\bar{s}_t(\hat{s}_t)|z_{t-1})$, $r_t^* \equiv r(s_t^*(\hat{s}_t)|z_{t-1})$, and $\hat{r}_t \equiv r(\hat{s}_t|z_{t-1})$. We have to have $\hat{r}_t \geq \underline{r}_t$ to satisfy the investor's participation constraint. Suppose first that $\underline{r}_t \leq r_t^*$ but, contrary to the claim in the lemma, $\hat{r}_t \neq r_t^*$. Then, there exists a deviating offer that makes both the entrepreneur and the investor offering the deviation strictly better off. Since $v_t^E(s|\hat{s}_t, z_{t-1})$ is strictly quasi concave, there exists some offer $r(s'|z_{t-1})$ arbitrarily close to $\hat{r}_t$ but closer to r_t^*such that $v_t^E(s'|\hat{s}_t, z_{t-1}) > v_t^E(\hat{s}_t|\hat{s}_t, z_{t-1})$. This implies that the entrepreneur would strictly prefer to visit an investor posting $r(s'|z_{t-1})$. Moreover, since $\hat{r}_t \geq \underline{r}$ and $r(s'|z_{t-1})$ is arbitrarily close to $\hat{r}_t$ but closer to r_t^*, it follows that $r(s'|z_{t-1}) > \underline{r}_t$ and the deviating investor is strictly better off by "undercutting" $\hat{r}_t$, thereby ensuring that the entrepreneur chooses to visit him.

Next, suppose that $r_t^* \leq \underline{r}_t$ and $\hat{r}_t > \underline{r}_t$. In this case, an offer $r(s'|z_{t-1}) < \hat{r}_t$ arbitrarily close to $\hat{r}_t$ makes both the entrepreneur and the deviating investor strictly better off.

Hence, we must have

$$r(\hat{s}_t|z_{t-1}) = \max\{r(s_t^*(\hat{s}_t)|z_{t-1}), r(\bar{s}_t(\hat{s}_t)|z_{t-1})\}. \tag{A.4}$$

Note that given that V_{t+1}^E and V_{t+1}^I by the induction hypothesis can be written as functions of z_t, and z_t in turn is a function of $\hat{s}_t$ and z_{t-1}, we have that $\hat{s}_t$ can be written as a function of z_{t-1} as in the Lemma. The same follows for V_t^I and V_t^E.

We next show by induction that the entrepreneur's participation constraint is never violated at the solution above. This is true for $t = N$. Suppose it is true for $t + 1$. We then show that it is also true for t.

Denote the participation interest rate in period t for the entrepreneur given the equilibrium threshold $\hat{s}_t$ by $\bar{r}_t$. We need to show $\bar{r}_t \geq \max\{r_t^*, \underline{r}_t\}$. Clearly $r_t^* \leq \bar{r}_t$. It remains to show that $\bar{r}_t \geq \underline{r}_t$.

Note that we must have $\underline{r}_t \leq \underline{r}_{t+1}$, since a visited investor in period t is no worse off than a visited investor in period $t + 1$ if their posted interest rate is the same. By the same logic, we must have $\bar{r}_t \geq \bar{r}_{t+1}$. But since by the induction hypothesis we have $\bar{r}_{t+1} \geq \underline{r}_{t+1}$, it follows that $\bar{r}_t \geq \underline{r}_t$. $\square$

Proof of Proposition 1.
Define function ψ_1 as follows:

$$\psi_1(s) = \frac{F_G(s)}{F_B(s)}\psi(s), \tag{A.5}$$

where the function $\psi(s)$ is defined by Eq. (1). By Assumption 2.5, ψ_1 is either strictly decreasing or first strictly decreasing and then strictly increasing. We show in Lemma A.5 that in the first case the market never breaks down, and in the second case, it breaks down for N large enough.

Define $\bar{\bar{s}}$ as the smallest solution to

$$\psi_1(\bar{\bar{s}}) = \frac{1}{\lambda}. \tag{A.6}$$

If ψ_1 is a strictly decreasing function then $\bar{\bar{s}} = 1$. This follows from the fact that $\psi(1) = \frac{1}{\lambda}$. If ψ_1 is first strictly decreasing and then strictly increasing then $\bar{\bar{s}} < 1$. $\square$

Lemma A.1. *If round t is not the last period in which financing is possible (that is, $z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)} > \frac{1}{\lambda X}$), then $\hat{s}_t < \bar{\bar{s}}$, where $\bar{\bar{s}}$ is defined by Eq. (A.6).*

Proof. The F.O.C. for problem (12) shows that s_t^*, defined by (23), must solve

$$\psi(s_t^*(\hat{s}_t)) = z_{t-1}(X - V_{t+1}^E(z_t)).$$

We showed in the proof of Lemma 3.1 that $z_t > \frac{1}{\lambda X}$ implies that $X - V_{t+1}^E(z_t) > \frac{1}{\lambda z_t}$. Therefore,

$$\psi(s_t^*(\hat{s}_t)) > \frac{1}{\lambda}\frac{z_{t-1}}{z_t} = \frac{1}{\lambda}\frac{F_B(\hat{s}_t)}{F_G(\hat{s}_t)},$$

or

$$\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\psi(s_t^*(\hat{s}_t)) > \frac{1}{\lambda}. \tag{A.7}$$

Since $s_t^*(\hat{s}_t) \geq \hat{s}_t$ and $\psi(s)$ is decreasing, we have

$$\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\psi(\hat{s}_t) > \frac{1}{\lambda}. \tag{A.8}$$

Therefore, it follows that $\frac{F_G(s)}{F_B(s)}\psi(s) < \frac{1}{\lambda}$ for $s \in (\bar{\bar{s}}, 1)$. Hence, $\hat{s}_t < \bar{\bar{s}}$. $\square$

Lemma A.2. *There exists a unique path from z_{T-1} to z_0, where T is the last period in which financing is possible and z_{T-1} is taken as given.*

Proof. In each financing round t, we have that either $\hat{s}_t = s^*(\hat{s}_t)$ or $\hat{s}_t = \bar{s}(\hat{s}_t)$. We show that there is only one solution, which is the lowest of these.

Given $\hat{s}_{t+1}, \ldots, \hat{s}_{T-1}$, define $\hat{s}_t^I$ as the threshold where the investor PC constraint (18) would hold:

$$\frac{\phi(\hat{s}_t^I)}{F_B(\hat{s}_t^I)} = \frac{1}{N-t}\sum_{n=t+1}^{N}\left(\prod_{i=t+1}^{n-1}F_B(\hat{s}_i)\right)\phi(\hat{s}_n). \tag{A.9}$$

Note that Eq. (A.9) always has a unique solution. Similarly, define $\hat{s}_t^E$ as the entrepreneur's preferred threshold compatible with z_t:

$$\frac{F_G(\hat{s}_t^E)}{F_B(\hat{s}_t^E)}\psi(\hat{s}_t^E) = z_t(X - V_{t+1}). \tag{A.10}$$

We show that $\hat{s}_t = \min\{\hat{s}_t^I, \hat{s}_t^E\}$ is the only viable solution.

Note that $z^I_{t-1} \equiv z_t \frac{F_B(\hat{s}^I_t)}{F_G(\hat{s}^I_t)} > z^E_{t-1} \equiv z_t \frac{F_B(\hat{s}^E_t)}{F_G(\hat{s}^E_t)}$ if and only if $\hat{s}^E_t > \hat{s}^I_t$. Suppose that $\hat{s}^E_t > \hat{s}^I_t$. The investor PC constraint will then be violated at t with z^E_{t-1} as prior if the entrepreneur's preferred interest rate is chosen. Hence, the only possible path is $z_{t-1} = z^I_{t-1}$ and $\hat{s}_t = \hat{s}^I_t$. We need to show that it is indeed the case at this z_{t-1} that the entrepreneur's preferred interest rate is not higher than what the participation constraint demands. The preferred threshold by the entrepreneur, taking V_{t+1} as given and with $z_{t-1} = z^I_{t-1}$ solves

$$\frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}'_t) = z_t(X - V_{t+1}). \tag{A.11}$$

Note that

$$\frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}'_t) = \frac{F_G(\hat{s}^E_t)}{F_B(\hat{s}^E_t)}\psi(\hat{s}^E_t) < \frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}^I_t), \tag{A.12}$$

where the first equality follows from Eqs. (A.10) and (A.11) and the last inequality follows from the fact that $\hat{s}^I_t < \hat{s}^E_t$ and $\frac{F_G(s)}{F_B(s)}\psi(s)$ is decreasing for $s < \hat{s}^E_t < \bar{\bar{s}}$.

Hence, we have $\psi(\hat{s}'_t) < \psi(\hat{s}^I_t)$, which implies that $\hat{s}'_t > \hat{s}^I_t$ since $\psi(s)$ is decreasing. This shows that the entrepreneur's preferred interest rate is indeed still below the interest rate required for the PC to bind, so we have $\hat{s}_t = \hat{s}^I_t$.

Next, suppose that $\hat{s}^E_t < \hat{s}^I_t$. If $\hat{s}_t = \hat{s}^E_t$ and $z_{t-1} = z^E_{t-1}$, the investor PC constraint is slack, so this is a viable solution. We need to show that it is not viable to have $\hat{s}_t = \hat{s}^I_t$ and $z_{t-1} = z^I_{t-1}$. Suppose to the contrary that $\hat{s}_t = \hat{s}^I_t$ and $z_{t-1} = z^I_{t-1}$. The entrepreneur's preferred threshold then solves

$$\frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}'_t) = z_t(X - V_{t+1}), \tag{A.13}$$

and we have

$$\frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}'_t) = \frac{F_G(\hat{s}^E_t)}{F_B(\hat{s}^E_t)}\psi(\hat{s}^E_t) > \frac{F_G(\hat{s}^I_t)}{F_B(\hat{s}^I_t)}\psi(\hat{s}^I_t). \tag{A.14}$$

The last inequality follows from the fact that $\hat{s}^E_t < \bar{\bar{s}}$ proved in Lemma A.1, so that

$$\frac{F_G(s)}{F_B(s)}\psi(s) < \frac{F_G(\hat{s}^E_t)}{F_B(\hat{s}^E_t)}\psi(\hat{s}^E_t)$$

for all $s < \hat{s}^E_t$, and from the fact that we have now assumed that $\hat{s}^E_t < \hat{s}^I_t$. But this implies that $\hat{s}'_t < \hat{s}^I_t$, so that the entrepreneur's preferred interest rate is higher than the interest rate that satisfies the participation constraint. So this is not a viable solution.

Hence, given $\hat{s}_{t+1}, \dots, \hat{s}_N$, there is a unique solution $\hat{s}_t = \min[\hat{s}^I_t, \hat{s}^E_t]$, and $z_{t-1} = \max[z^I_{t-1}, z^E_{t-1}]$. □

Lemma A.3. *Define function ψ_2 as follows:*

$$\psi_2(s) = F_G(s)\psi(s) + (1 - F_G(s))\frac{f_B(s)}{f_G(s)}, \tag{A.15}$$

where the function $\psi(s)$ is defined by Eq. (1).

Let $\tilde{s}$ be the largest $\tilde{s} \in [0, 1)$ such that

$$\psi_1(\tilde{s}) = \psi_2(\tilde{s}) \quad \Leftrightarrow \quad \psi(\tilde{s}) = \frac{F_B(\tilde{s})}{F_G(\tilde{s})}\frac{1 - F_G(\tilde{s})}{1 - F_B(\tilde{s})}\frac{f_B(\tilde{s})}{f_G(\tilde{s})}, \tag{A.16}$$

if such an $\tilde{s}$ exists, and 0, otherwise. Suppose the investor PC constraint is slack in financing rounds t and $t+1$ and $\hat{s}_{t+1} > \tilde{s}$, then

$$\tilde{s} < \hat{s}_t = \psi_1^{-1}(\psi_2(\hat{s}_{t+1})) < \hat{s}_{t+1}. \tag{A.17}$$

Proof. Since

$$z_t = z_{t-1}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)},$$

$$X - V_{t+1} = F_G(\hat{s}_{t+1})(X - V_{t+2}) + \frac{1 - F_G(\hat{s}_{t+1})}{z_t}\frac{f_B(\hat{s}_t)}{f_G(\hat{s}_t)},$$

we can write the entrepreneur's first order condition as

$$\begin{aligned}\frac{F_G(\hat{s}_t)}{F_B(\hat{s}_t)}\psi(\hat{s}_t) &= z_t\left(F_G(\hat{s}_{t+1})(X - V_{t+2}) + \frac{1 - F_G(\hat{s}_{t+1})}{z_t}\frac{f_B(\hat{s}_t)}{f_G(\hat{s}_t)}\right), \\ &= F_G(\hat{s}_{t+1})\psi(\hat{s}_{t+1}) + (1 - F_G(\hat{s}_{t+1}))\frac{f_B(\hat{s}_{t+1})}{f_G(\hat{s}_{t+1})},\end{aligned} \tag{A.18}$$

or

$$\psi_1(\hat{s}_t) = \psi_2(\hat{s}_{t+1}), \tag{A.19}$$

where the functions ψ_1 and ψ_2 are defined in (A.5) and (A.15).

Direct computations show that

$$\psi_2'(s) = F_G(s)\psi'(s) < 0. \tag{A.20}$$

Therefore, by Assumption 2.4, $\psi_2(s)$ is a strictly decreasing function. Direct computations also show that

$$\lambda^{-1} = \psi_1(\bar{\bar{s}}) = \psi_1(1) = \psi_2(1), \tag{A.21}$$

$$0 > \psi_1'(s)|_{s=\bar{\bar{s}}} \tag{A.22}$$

$$0 > \psi_2'(s)|_{s=1}, \tag{A.23}$$

where $\bar{\bar{s}}$ is defined in Lemma A.1. If the function ψ_1 is strictly decreasing, then $\bar{\bar{s}} = 1$, and

$$0 > \psi_1'(s)|_{s=1} > \psi_2'(s)|_{s=1}. \tag{A.24}$$

Therefore, the function $\eta(s) = \psi_1^{-1}(\psi_2(s))$ defines a map from a threshold in round $t+1$ to a threshold in round t. It is strictly increasing on $[\tilde{s}, 1]$, $\eta(1) = \bar{\bar{s}}$, and $\eta(s) < s$ for $s \in [\tilde{s}, 1)$. □

Lemma A.4. *Suppose the investor participation constraint binds for all rounds from T to $N-1$. Then, for all $t = T..N-1$,*

$$\frac{\phi(\hat{s}_t)}{F_B(\hat{s}_t)} = \phi(\hat{s}_{t+1}), \tag{A.25}$$

and therefore, $\hat{s}_T > \hat{s}_{t+1} > \cdots > \hat{s}_N$.

Proof. We showed in Section 3 that expected profits for an investor visited in round t who uses screening threshold $\hat{s}_t$ is given by Eq. (7). Note that the probability of reaching round $t+1$ conditional on reaching round t is equal to

$$\pi_{t-1}F_G(\hat{s}_t) + (1 - \pi_{t-1})F_B(\hat{s}_t) = F_B(\hat{s}_t)\frac{1 + z_t}{1 + z_{t-1}}. \tag{A.26}$$

Therefore, setting the expected profits equal across rounds yields Eqs. (A.25). Since the function ϕ is decreasing and $F_B \le 1$, Eqs. (A.26) define a map $\hat{\eta}$ from a threshold in round $t+1$ to a threshold in round t. It is strictly increasing and $\hat{\eta}(s) > s$. Therefore, $\hat{s}_T > \hat{s}_{t+1} > \cdots > \hat{s}_N$. □

Next, we prove the conditions for the market breakdown.

Lemma A.5. *If $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly decreasing, the market never breaks down, z_{N-1} goes to $\frac{1}{\lambda X}$, and $\hat{s}_N$ goes to one. If $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly increasing at 1, then the market breaks down for N large enough.*

Proof. Suppose first that $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly decreasing. Suppose contrary to the claim in the lemma that there is an $n < N$ such that financing is not possible after round n. Because $N > n$, there are more than one investors present in round n, and their reservation value is zero. Hence, they will offer the entrepreneur her preferred interest rate, so that

$$\frac{F_G(\hat{s}_n)}{F_B(\hat{s}_n)}\psi(\hat{s}_n) = z_n X, \tag{A.27}$$

where we used the fact that the entrepreneur's continuation utility in the final round is zero.

Since $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly decreasing, it reaches its minimum at $s = 1$, where it is equal to $\frac{1}{\lambda}$. Because we have assumed financing is possible at n, we have $\hat{s}_n < 1$. Hence,

$$z_n X > \frac{1}{\lambda}. \tag{A.28}$$

But this implies that financing is possible in round $n+1$, so we arrived at a contradiction.

To see that z_{N-1} goes to $\frac{1}{\lambda X}$, suppose to the contrary that z_{N-1} is bounded away from $\frac{1}{\lambda X}$, that is, there exists δ such that for any N,

$$z_{N-1} > \delta + \frac{1}{\lambda X}. \tag{A.29}$$

But in this case, as shown in Lemma 3.1, the entrepreneur's preferred threshold is bounded away from zero in all financing rounds. As a result, z_{N-1} must go to zero with N. Thus, we arrived at a contradiction.

Suppose now that $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly increasing at 1, and suppose contrary to the claim in the lemma that the market never breaks down for any N. We showed above that in this case, $z_{N-1} \to \frac{1}{\lambda X}$ and for any $m > 0$, $\hat{s}_{N-m} \to 1$.

We will show that in this case, the investor PC constraint in round $N-1$ is slack. Therefore, the threshold $\hat{s}_{N-1}$ in round $N-1$ must solve

$$\psi_1(\hat{s}_{N-1}) = \psi_2(\hat{s}_N),$$

where the functions ψ_1 and ψ_2 are defined in (A.5) and (A.15). However, if $\frac{F_G(s)}{F_B(s)}\psi(s)$ is strictly increasing at 1, the above equation has no solution in the neighborhood of 1, which contradicts the fact that the market never breaks down.

Consider the problem of the entrepreneur in round $N-1$. Without loss of generality, to simplify on notation, we assume that $f_B \equiv 1$ and $F_B(s) = s$, and $f_G \equiv f$, $F_G \equiv F$. Because the entrepreneur earns no rent in round N, her preferred threshold s^*_{N-1} solves

$$s_{N-1} = \arg\max_s \,(1 - F(s)) \left(1 - \frac{1}{zXf(s)}\right), \tag{A.30}$$

where we denote z_{N-2} by z. Taking the first-order conditions in Eq. (A.30) and using the Taylor series at $zX = 1/\lambda$ and $s = 1$, we obtain

$$s^*_{N-1} = 1 - \frac{\lambda}{2\gamma}(\lambda zX - 1) + O(\lambda zX - 1)^2 + O(s-1)^2, \tag{A.31}$$

where $\gamma = f'(1)$.

Suppose that $\hat{s}_{N-1} = s^*_{N-1}$. In the last round, the threshold $\hat{s}_N$ solves

$$f(\hat{s}_N) = \frac{s^*_{N-1}}{F(s^*_{N-1})}\frac{1}{zX}. \tag{A.32}$$

Using the Taylor series at $zX = 1/\lambda$ and $s = 1$, we obtain

$$\hat{s}_N = 1 - \frac{\lambda(2\gamma + \lambda - \lambda^2)}{2\gamma^2}(\lambda zX - 1) + O(\lambda zX - 1)^2 + O(s-1)^2. \tag{A.33}$$

Direct computations show that if $F_G(s)\psi(s)/F_B(s)$ increases at $s = 1$, then

$$2\gamma + \lambda - \lambda^2 < 0.$$

Therefore, the market breaks down in round N and the last investor obtains zero rent. Taking the Taylor series of $\phi(s)$ at $s = 1$, where ϕ is defined by Eq. (8), we obtain

$$\phi(s) = \frac{\gamma}{2\lambda}(s-1)^2 + O(s-1)^3. \tag{A.34}$$

Using the expression for the investor's surplus (7) and the above expressions for the thresholds, we can verify that the expected investor surplus in round $N-1$, R_{N-1}, is indeed higher than his expected surplus in round N, R_N, if

$$2\gamma + \lambda - \lambda^2 < 0.$$

We are now ready to prove the uniqueness of equilibrium. First, note that the expression for the expected profits to a visited investor (7) and Lemma A.3 imply that if the participation constraint is not binding in period T, it also does not bind for any $t \le T$. Let T be the last period when the participation constraint is not binding. Fig. 1 shows the behavior of the functions ψ_1 and ψ_2 for the two cases: when the market never breaks down and when it breaks down for sufficiently large N.

In the first case, Lemmas A.4 and A.5 guarantee that, for large enough N, $\hat{s}_T \in (\tilde{s}, 1)$. Therefore, by Lemmas A.2, A.3, A.4, and 3.2, there exists a unique sequence $z_0, \ldots, z_{N-1}$ such that z_0 is a continuous and monotone function of z_{N-1}. This implies that an equilibrium exists and that it is unique.

In the second case, for large enough N, the market breaks down. Therefore, the investor participation constraint in the last period when financing is possible is not binding. As shown in the proof of Lemma A.3, in this case the dynamics of screening thresholds are given by $\psi_1(\hat{s}_t) = \psi_2(\hat{s}_{t+1})$. Hence, the first screening threshold $\hat{s}_1$ must be large than $\tilde{s}$. Otherwise, the screening thresholds in all rounds would be below $\tilde{s}$, and the market would never breakdown. Thus, by Lemmas A.2, A.3, A.4, and 3.2, there again exists a unique sequence $z_0, \ldots, z_{N-1}$ such that z_0 is a continuous and monotone function of z_{N-1}. This implies that an equilibrium exists and that it is unique.

We now show that investors earn nonvanishing rents in equilibrium. Consider the maximization problem of the entrepreneur in the first round. Since the investor PC constraint is not binding, the equilibrium threshold coincides with the entrepreneur's preferred threshold:

$$\hat{s}_1 = \arg\max_s (1 - F_G(s))\left(X - r(s|z_0)\right) + F_G(s)V_2^E. \tag{A.35}$$

Lemma A.6 below provides an upper bound on the maximal expected surplus that can be achieved with a screening technology with finite λ. Therefore, V_2^E is less than $X - 1/(\lambda z_0)$. In the proof of Lemma A.6, we actually show that if MLRP holds strictly then the bound is strict, that is, there exists $\delta > 0$ such that $V_2 = X - 1/(\lambda z_0) - \delta$. Using Eq. (6) for the interest rate, we can rewrite the maximization problem (A.35) as

$$\max_s (1 - F_G(s))\left(\delta - \frac{1}{z_0}\left(\frac{f_B(s)}{f_G(s)} - \frac{1}{\lambda}\right)\right) + V_2.$$

It can be verified that the solution to the above problem is strictly less than one. Notice that similar arguments apply to any fixed round t, provided that z_t stays bounded away from $1/(\lambda X)$. Since we showed that the thresholds increase until the PC constraints bind and the investor rents are decreasing in the threshold, investor rents decrease with t in equilibrium. Once the investor PC constraint binds, investors earn the same rent. □

Proof of Proposition 2.

Consider maximization problem (28). Let $n \le N$ be the largest n such that the expected surplus generated with n screenings is strictly higher than that generated with $n-1$ screenings. Then for all $i > n$, $s_i = 1$. Suppose that the first n screening thresholds are not the same. Without loss of generality, assume that $s_1 = s_*$ and $s_2 = s^*$, $s_* \ne s^*$. Let

$$\Lambda = \frac{1}{z_0 X}\prod_{i>2}\frac{F_B(s_i)}{F_G(s_i)}.$$

Screening thresholds $s_1 = s_*$ and $s_2 = s^*$ maximize social surplus if and only if they solve

$$\max_{s_1,s_2} \Lambda F_B(s_1)F_B(s_2) - F_G(s_1)F_G(s_2). \tag{A.36}$$

Consider x and $y(x)$ such that $F_B(x)F_B(y(x)) = F_B(s_*)F_B(s^*)$. We have

$$y'(x) = -\frac{f_B(x)}{f_B(y)}\frac{F_B(y)}{F_B(x)}. \tag{A.37}$$

Since $s_1 = s_*$ and $s_2 = s^*$ solve (A.36), $F_G(x)F_G(y(x))$ should be minimized at $x = s_*$. Hence, it must be that

$$\left(F_G(x)F_G(y(x))\right)' = \frac{f_B(x)F_G(x)F_G(y)}{F_B(x)}\left(\frac{f_G(x)}{f_B(x)}\frac{F_B(x)}{F_G(x)} - \frac{f_G(y)}{f_B(y)}\frac{F_B(y)}{F_G(y)}\right) = 0,$$

and therefore,

$$\frac{f_G(s_*)}{f_B(s_*)}\frac{F_B(s_*)}{F_G(s_*)} = \frac{f_G(s^*)}{f_B(s^*)}\frac{F_B(s^*)}{F_G(s^*)}.$$

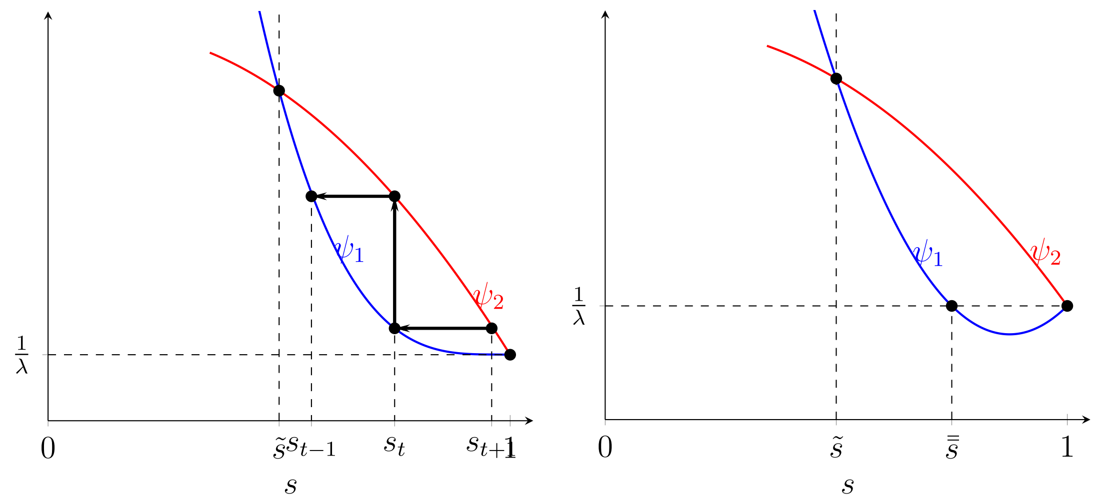

Fig. 1. The left panel shows the behavior of the functions ψ_1 and ψ_2 when the market never breaks down; the right panel shows their behavior when the market breaks down for sufficiently large N.

Thus, we showed that all interior screening thresholds are solutions of the following equation

$$H(s) \equiv \frac{f_G(s)}{f_B(s)}\frac{F_B(s)}{F_G(s)} = c,$$

where c is some constant. In particular, if H is a strictly increasing function then all screening thresholds are the same.

To prove that the expected surplus strictly increases with the number of screenings if $H(s)$ is a strictly increasing function of s we need to show that for any N the solution to the maximization problem (28) is interior. Suppose on the contrary that for some N it is optimal to set s_N to one. Let N be the lowest number of screenings when this happens. The optimal screening threshold level is the same in all $N-1$ screenings and solves the F.O.C.

$$z_0 X f_G(s)F_G(s)^{N-2} = f_B(s)F_B(s)^{N-2}.$$

Taking the derivative of the surplus with respect to s_N at $s_N = 1$ we have

$$f_B(1)F_B(s)^{N-q} - z_0 X f_G(1)F_G(s)^{N-1} = f_B(1)F_B(s)^{N-1}\left(1 - \lambda\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}\right) < 0,$$

where we have used the F.O.C and where the last inequality follows from the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s and therefore takes the lowest value λ^{-1} at $s = 1$. As a result, it is suboptimal to set s_N to 1 and the solution must indeed be interior.

Finally, we will prove that the maximal expected surplus can be achieved with no more than n screenings if $H(s)$ is a strictly decreasing function on $s \in [s_n^*, 1]$. Notice that MLRP implies that Eq. (30) has a unique solution, s_n^*, which is strictly increasing in n. Suppose on the contrary that social surplus is strictly higher with $n+1$ screenings. Each screening threshold s_j has to satisfy the F.O.C.:

$$\frac{f_G(s_j)\prod_{i\le n, i\ne j} F_G(s_i)}{f_B(s_j)\prod_{i\le n, i\ne j} F_B(s_i)} = \frac{1}{z_0 X}, \quad j = 1, \ldots, n+1. \tag{A.38}$$

By MLRP and from the definition of s_n^* there should be at least two screening thresholds above s_n^*. Since $H(s)$ is a strictly decreasing function on $s \in [s_n^*, 1]$ these two thresholds must be the same, and must maximize (A.36). Notice, however, that (A.36) can be increased if one increases one of the thresholds and decreases the other so as to keep the product $F_B(x)F_B(y(x))$ constant. Therefore, (A.36) is maximized by setting one of the thresholds to 1. Hence, we arrived at a contradiction. □

Lemma A.6. *The maximal expected social surplus in a sequential market with a screening technology that satisfies $f_G(1)/f_B(1) = \lambda$ is no larger than $P(G)\max(X - 1/(\lambda z_0), 0)$.*

Proof. We first observe that the maximal expected surplus respects the order induced by MLPR on the space of signal distributions. Consider two cases of informative signals. Suppose that in both cases if the project is bad the signal is drawn from the same distribution $F_B(s)$. In the first case, the signal is drawn from a distribution F_{G_1} with density f_{G_1}, and in the second case, from a distribution F_{G_2} with density f_{G_2}. Suppose that for all $s > s'$

$$\frac{f_{G_1}(s)}{f_{G_2}(s)} \ge \frac{f_{G_1}(s')}{f_{G_2}(s')},$$

then the maximal surplus in the first case is no less than that in the second case. This follows from the fact that MLRP implies the monotone probability ratio (Milgrom (1981)).

Suppose for now that $f_B(s) \equiv 1$. Then given λ, the maximal expected surplus is achieved with $f_G(s) = 0$ for $s \in [0, 1-\lambda^{-1})$ and $f_G(s) = \lambda$ for $s \in [1-\lambda^{-1}, 1]$. Setting a screening threshold level to $1-\lambda^{-1}$ ensures that good projects are always financed and bad projects are financed with probability λ^{-1}. Thus, with a single screening the expected surplus is $X - 1/(\lambda z_0)$. Direct computations show that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is an increasing function for $s \in [1-\lambda^{-1}, 1]$. Thus, by Proposition 2, $P(G)\left(X - 1/(\lambda z_0)\right)$ is in fact the maximal expected surplus. Finally, notice that the assumption that $f_B(s) \equiv 1$ is innocuous. For an arbitrary $f_B(s)$ the maximal surplus is achieved with $f_G(s) = 0$ for $s \in [0, \bar{s})$ and $f_G(s) = \lambda f_B(s)$ for $s \in [\bar{s}, 1]$, where $\bar{s}$ is determined by the condition that $\int_{\bar{s}}^1 \lambda f_B(s)ds = 1$. Hence, $\int_0^{\bar{s}} f_B(s)ds = 1 - \lambda^{-1}$. □

Proof of Proposition 3.

Let α^G and α^B denote the fractions of good and bad projects financed in N rounds, respectively. Therefore, $(1-\alpha^G)$ good projects and $\frac{1}{z_0}(1-\alpha^B)$ bad projects remain unfunded. We showed in the text that when the function H is strictly increasing, the entrepreneur visits all available investors in the directed search model. Therefore, as N goes to infinity, the last investor breaks even at the maximum interest rate X, and the screening threshold $\hat{s}_N$ approaches one:

$$\lim_{N\to\infty} \frac{\prod_{t=1}^{N-1} F_G(\hat{s}_t)}{\prod_{t=1}^{N-1} F_B(\hat{s}_t)}\frac{f_G(\hat{s}_N)}{f_B(\hat{s}_N)} = \frac{1}{\lambda z_0 X}. \tag{A.39}$$

Note that $(1-\alpha^G) = \prod_{t=1}^{N-1} F_G(\hat{s}_t)$ and $(1-\alpha^B) = \prod_{t=1}^{N-1} F_B(\hat{s}_t)$. Therefore, we write Equation (A.39) as

$$\lim_{N\to\infty} \frac{1-\alpha^G}{1-\alpha^B} = \lim_{\hat{s}_N\to 1} \frac{f_B(\hat{s}_N)}{f_G(\hat{s}_N)} \frac{1}{z_0 X} = \frac{1}{\lambda z_0 X}. \tag{A.40}$$

Eq. (A.40) implies that

$$\alpha^G = 1-(1-\alpha^B)\frac{1}{\lambda z_0 X}. \tag{A.41}$$

The fraction of financed good projects α^G is increasing in the fraction of financed bad projects α^B. Using (A.41) and plugging it in the expression for social surplus, we obtain

$$\alpha^G X - \frac{1}{z_0}\alpha^B = X - \frac{1}{\lambda z_0} - \alpha^B \frac{\lambda - 1}{\lambda z_0}. \tag{A.42}$$

Since $\lambda > 1$, the limiting social surplus decreases in the fraction of bad projects financed.

Also, note that Eq. (A.42) applies to the social planner solution. Proposition 2 shows that the social planner minimizes α_B using the available signal technology by using the same screening threshold in each round and letting it go to one with N. Thus, α^G and α^B are higher in the directed search equilibrium than in the social planner solution. □

Proof of Lemma 6.1.

Suppose contrary to the claim in the lemma that the entrepreneur in equilibrium does not visit investors in increasing order of posted interest rates. There must then exist rounds n and $n+1$ such that the entrepreneur chooses r_i in round n and $r_{i'} < r_i$ in round $n+1$, giving expected profits for an entrepreneur entering round n as

$$V_n = (1-F_G(\hat{s}_i)(X-r_i) + F_G(\hat{s}_i)(1-F_G(\hat{s}_{i'})))(X-r_{i'}) + F_G(\hat{s}_i)F_G(\hat{s}_{i'})V_{n+2},$$

where $\hat{s}_i$ and $\hat{s}_{i'}$ are the thresholds used by investors i and i'.

Now imagine a deviation in which she switches the order of visits of the two rounds while holding the order of other visits fixed. As investors do not observe the order, acceptance thresholds will not change. The change in profits from the switch are given by

$$(r_i - r_{i'})(1-F_G(\hat{s}_i))(1-F_G(\hat{s}_{i'})) > 0. \quad \square$$

Proof of Proposition 4.

We start by showing that when $r_N = X$, the interest rate offers $\{r_t\}_{t=1}^{N-1}$ and associated thresholds are the unique offers such that all investors earn equal rents, and that as $N\to\infty$, the surplus created is the same as in a first-price auction and accrues entirely to entrepreneurs. We then show that this choice of interest rates is the unique equilibrium in the posting game between investors.

We showed in Section 3 that expected profits for an investor visited in round n who uses screening threshold $\hat{s}_t$ is given by Eq. (7). Note that the probability of reaching round $n+1$ conditional on reaching round n is equal to

$$P(G|z_{n-1})F_G(\hat{s}_n) + P(B|z_{n-1})F_B(\hat{s}_n) = F_B(\hat{s}_n)\frac{1+z_n}{1+z_{n-1}}.$$

Therefore, setting the expected profits equal across rounds yields Eqs. (40). Since the function ϕ is decreasing and $F_B \le 1$, Eqs. (40) define s_t recursively as an increasing function of the last screening threshold $\hat{s}_N$, with $s_t > s_{t+1}$ for $t = 1,\ldots,N-1$. The equilibrium is determined uniquely by the boundary condition

$$\frac{1}{z_0}\frac{f_B(\hat{s}_N)}{f_G(\hat{s}_N)}\prod_{t=1}^{N-1}\frac{F_B(\hat{s}_t)}{F_G(\hat{s}_t)} = X. \tag{A.43}$$

Since all investors earn the same rent in equilibrium, as the number of investors increases, the surplus that each investor earns goes to zero. Therefore, a screening threshold in the last period approaches one.

We next show that this is indeed the equilibrium. Denote each investors expected profits in the proposed equilibrium by Π^*. We show that there is no other set of offers that can deliver weakly higher profits to all investors, so that the investor with the lowest expected profits cannot earn more than Π^*. We then show that each investor can guarantee himself at least Π^*. It follows that all investors must earn Π^*, which can only be done via de proposed equilibrium offers in the proposition. □

Lemma A.7. *There is no equilibrium in which each investor earns at least Π^* and some investor earns strictly more than Π^*.*

Proof. Suppose contrary to the claim in the Lemma that there exists such an equilibrium. Interest rates $\tilde{r}_t$ in such an equilibrium must be at least as high as the interest rates r_t in the proposed equilibrium, in order for each investor to earn at least Π^*. There must be one lowest t such that $\tilde{r}_t > r_t$. Since profits for investors in later rounds are strictly decreasing in r_t, all interest rates $\tilde{r}_u$ for $u > t$ must be strictly larger than r_u for these investors to earn at least Π^*. But since $r_N = X$, this is a contradiction. □

Lemma A.8. *Each investor earns at least Π^*.*

Proof. We prove the lemma via induction. Suppose that each investor after round t can guarantee himself at least Π^*, regardless of the previous offers. We show that this implies that an investor in round t can also guarantee himself at least Π^*. We then show that investor N can always guarantee himself Π^*, completing the proof.

Suppose the investor moving n^{th} with $n < N$ is faced with offers $\{r_u\}_{u=1}^{n-1}$ by the investors moving before him, with $r_t \le r_{t+1}$ for all $t \le n-2$. Let $\hat{s}_t$ be the corresponding screening thresholds. Denote the expected profits from one of these offers if no other lower offers come in as $R_t(r_t|\hat{s^{t-1}})$, given by

$$R_t(r_t|\hat{s^{t-1}}) = \Pi_{u=1}^{t-1}F_G(\hat{s}_u)(1-F_G(\hat{s}_t))r_t - \frac{1}{z_0}\Pi_{u=1}^{t-1}F_B(\hat{s}_u)(1-F_B(\hat{s}_t)), \quad t \le n-1.$$

Also, denote the profits of the n^{th} investor if investor n offers r_t by $R_n(r_t)$.

Suppose there exists at least one round before round n in which an investor expects to receive profits higher than Π^*, that is, there exists $t < n$ such that $R_t(r_t) > \Pi^*$. Define $\hat{r}$ as the lowest r_t for $t < n$ such that $R_t(r_t) > \Pi^*$, and let t be the round that corresponds to such an $\hat{r}$. Note that by the definition of $\hat{r}$, by offering an interest rate just below $\hat{r}$, the n^{th} investor can take the place of the investor offering interest rate $\hat{r}$, and therefore, achieve profits greater than Π^* (provided that later offers are higher than $\hat{r}$.) Let r', $r' < \hat{r}$ be the interest rate such that $R_t(r') = \Pi^*$. If investor n makes this offer, he will get expected profits Π^* as long as none of the later investors offer $r'' \le r'$. But such an offer would give lower profits than Π^* from the fact that r' is the lowest offer that gives Π^*. (Assuming that if two offers are tied, the entrepreneur flips a coin in the decision on who to visit first.) From the induction hypothesis, later investors can guarantee themselves Π^*, and hence would never make such an offer.

Suppose $R_t(r_t) \le \Pi^*$ for all $t < n$. Define $r' \ge r_{n-1}$ as the lowest interest rate such that $R_n(r') = \Pi^*$. Note that none of the later investors will offer $r'' \le r'$ since such an offer would give lower profits than Π^* from the fact that r' is the lowest offer that gives Π^* and $R_t(r_t) \le \Pi^*$ for all $t < n$.

It follows that if the induction hypothesis holds for $n+1$, investor n can guarantee himself at least Π^*, so that the induction hypothesis also holds for investor n.

It remains to show that investor N can guarantee himself Π^*. The proof follows the same steps as above. If $R_t(r_t) \le \Pi^*$ for all $t < N$, the last investor can offer X and can get at least Π^*, which proves the lemma. □

Hence, the equilibrium set of offers outlined in the proposition is the only feasible equilibrium candidate. The order in which investors post the offers is inconsequential, except for the fact that no investor

$t < N$ will post the highest offer $r_N = X$ in equilibrium. (Doing so would mean that there is some unposted offer $r_n < X$ with $n < N$ when the turn comes for investor N to post. But since no one moves after N, investor N then has an incentive to post an offer just below r_{n+1}, which will give him higher profits than Π^* and all investors with higher offers get lower profits than Π^*.) □

Proof of Proposition 5. We verify that it is a symmetric equilibrium for investors to participate if and only if $S \geq \hat{s}_P$, with bids for participating bidders given by

$$r^*(s) = \frac{P(B|S_i = s, \max_{j\neq i} S_j = s)}{P(G|S_i = s, \max_{j\neq i} S_j = s)} = \frac{1}{z_0}\left(\frac{f_B(s)}{f_G(s)}\right)^2 \left(\frac{F_B(s)}{F_G(s)}\right)^{N-2}.$$

When investors follows this strategy, $r^*(s)$ is the interest rate at which a bidder with signal $S = s$ who just marginally wins just breaks even (in which case the second-highest bid, which sets the interest rate, is also $r^*(s)$). Using similar steps as in, e.g., Milgrom and Weber (1982), it is straightforward to verify that for participating bidders, playing $r^*(s)$ is a best response when others follow the same strategy: A lower interest rate bid increases the probability of winning, but on that extra set the interest rate set by the second highest bidder is too low for the investor to break even, while a higher interest rate bid reduces the probability of winning on a set where expected profits are positive.

We next show that it is optimal for an investor with a signal $S < \hat{s}_P$ not to participate given the postulated equilibrium strategies by other investors. First, using the same steps as above, it is straight-forward to show that participating and winning the auction when any other bidder participates leads to negative profits for such a bidder. When no other investor participates, participating with any bid $r \leq X$ assures that the investor finances the project at the maximal interest rate X. But given the participation threshold $\hat{s}_P$, this also leads to negative profits, from the definition of $\hat{s}_P$.

Finally, we show that expected profits conditional on winning the auction and receiving an interest rate r goes to zero. Denote the first-order statistic of the signals by Y_1 and the second-order statistic by Y_2. Conditional on an outcome $Y_1 = y_1$ and $Y_2 = y_2$ with $y_1 \geq y_2 \geq \hat{s}_P$, so that at least to investors participate and the interest rate is set at $r(y_2)$, the expected profits from a project to the winner is given by

$$P(G|Y_1 = y_1, Y_2 = y_2)r(y_2) - P(B|Y_1 = y_1, Y_2 = y_2) = P(G|Y_1 = y_1, Y_2 = y_2)\left(\frac{P(B|Y_1 = Y_2 = y_2)}{P(G|Y_1 = Y_2 = y_2)} - \frac{P(B|Y_1 = y_1, Y_2 = y_2)}{P(G|Y_1 = y_1, Y_2 = y_2)}\right).$$

Since $\hat{s}_P$ goes to one with N, for any $\{y_1, y_2\}$ such that $1 \geq y_1 \geq y_2 \geq \hat{s}_P$ we must have that y_2 gets arbitrarily close to y_1 as $N \to \infty$ so that the expression in parenthesis goes to zero.

Next, consider the event $\{Y_1 = y_1, Y_2 \leq \hat{s}_P\}$ in which only one investor participates, so that the interest rate is set at X. The expected profits to the winner conditional on this event is given by

$$P(G|Y_1 = y_1, Y_2 \leq \hat{s}_P)X - P(B|Y_1 = y_1, Y_2 \leq \hat{s}_P) = P(G|Y_1 = y_1, Y_2 \leq \hat{s}_P)\left(\frac{P(B|Y_1 = \hat{s}_P, Y_2 \leq \hat{s}_P)}{P(G|Y_1 = \hat{s}_P, Y_2 \leq \hat{s}_P)} - \frac{P(B|Y_1 = y_1, Y_2 \leq \hat{s}_P)}{P(G|Y_1 = y_1, Y_2 \leq \hat{s}_P)}\right),$$

where we have used the definition of $\hat{s}_P$ to set $X = \frac{P(B|Y_1=\hat{s}_P, Y_2\leq\hat{s}_P)}{P(G|Y_1=\hat{s}_P, Y_2\leq\hat{s}_P)}$. Again, because $\hat{s}_P$ goes to one with N, $\hat{s}_P$ gets arbitrarily close to any $y_1 \geq \hat{s}_P$ as $N \to \infty$, so that the expression in parenthesis goes to zero. Hence, total expected aggregate profits to investors also goes to zero with N. □

References

Avery, C., Zemsky, P., 1998. Multidimensional uncertainty and herd behavior in financial markets. Am. Econ. Rev. 88 (4), 724–748.

Axelson, U., Makarov, I., 2023. Informational black holes in financial markets. J. Financ. 78, 3099–3140.

Bikhchandani, S., Hirshleifer, D., Welch, I., 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. J. Political Econ. 100, 992–1026.

Bulow, J., Klemperer, P., 2009. Why do sellers (usually) prefer auctions? Am. Econ. Rev. 99, 1544–1575.

Daley, B., Green, B., 2012. Waiting for news in the market for lemons. Econometrica 80, 1433–1504.

Duffie, D., Garleanu, N., Pedersen, L.H., 2005. Over-the-counter markets. Econometrica 73, 1815–1847.

Glode, V., Opp, C., 2017. Over-the-counter vs. Limit order markets: The role of expertise acquisition and market power. Working paper.

Grossman, S.J., 1976. On the efficiency of competitive stock markets where trades have diverse information. J. Financ. 31, 573–585.

Hayek, F., 1945. The use of knowledge in society. Am. Econ. Rev. 35, 519–530.

Kremer, I., Skrzypacz, A., 2007. Dynamic signaling and market breakdown. J. Econom. Theory 133, 58–82.

Lagos, R., Rocheteau, G., 2009. Liquidity in asset markets with search frictions. Econometrica 77, 403–426.

Lauermann, S., Wolinsky, A., 2016. Search with adverse selection. Econometrica 84, 243–315.

Milgrom, P., 1981. Rational expectations, information acquisition, and competitive bidding. Econometrica 49, 921–943.

Milgrom, P., Weber, R., 1982. A theory of auctions and competitive bidding. Econometrica 50, 1089–1122.

Myers, S.C., Majluf, N.S., 1984. Corporate financing and investment decisions when firms have information that investors do not have. J. Financ. Econ. 13 (2), 187–221.

Myerson, R.B., Satterthwaite, M.A., 1983. Efficient mechanisms for bilateral trading. J. Econom. Theory 29 (2), 265–281.

Nanda, R., Samila, S., Sorenson, O., 2020. The persistent effect of initial success: Evidence from venture capital. J. Financ. Econ. 137, 231–248.

Philippon, T., 2015. Has the U.S. finance industry become less efficient? On the theory and measurement of financial intermediation. Am. Econ. Rev. 105 (4), 1408–1438.

Rajan, R., 1992. Insiders and outsiders: The choice between informed and arm's-length debt. J. Financ. 47, 1367–1400.

Roberts, J.W., Sweeting, A., 2013. When should sellers use auctions? Am. Econ. Rev. 103, 1830–1861.

Vayanos, D., Weill, P.-O., 2008. A search-based theory of the on-the-run phenomenon. J. Financ. 63, 1361–1398.

Weill, P.-O., 2008. Liquidity premia in dynamic bargaining markets. J. Econom. Theory 140, 66–96.

Welch, I., 1992. Sequential sales, learning and cascades. J. Financ. 47 (2), 695–732.

Wright, R., Kircher, P., Julien, B., Guerrieri, V., 2021. Directed search and competitive search equilibrium: A guided tour. J. Econ. Lit. 59 (1), 90–148. http://dx.doi.org/10.1257/jel.20191505, URL https://www.aeaweb.org/articles?id=10.1257/jel.20191505.

Zhu, H., 2012. Finding a good price in opaque over-the-counter markets. Rev. Financ. Stud. 1255–1285.