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Michael Laver

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Michael Laver[a,b]

[a]Department of Politics, New York University, New York, NY, USA; [b]London School of Economics, London, UK

ABSTRACT
I define the *substantive* 'representativeness' of elections in terms of voters' electoral welfare, the 'congruence' between voters' ideal policy positions and the stated position of their closest party. I specify and analyze a computational agent-based model of the effect of district magnitudes on voter welfare, in settings with multidimensional voter preferences. This model is dynamic and evolutionary, with endogenous entry and exit of candidates from local races, conditional on 'survival thresholds' which are a function of district magnitude. I find that increasing district magnitudes tends to increase voter welfare in two distinct ways. First, and as commonly expected, it increases the number of competing candidates – thereby offering voters more options. Second, and equally important, increasing district magnitude *combined with endogenous candidate entry and exit* affects the typical configuration of candidate positions in ways that increase voter welfare. Candidates tend to exit from over-served regions of the issue space and enter into under-served regions. Not only are there more competing candidates in larger districts, therefore, but their issue positions are distributed in ways that better reflect the preferences of voters.

KEYWORDS Agent based models; elections; district magnitude; party competition; representation

Introduction

A long-established literature, deriving in large part from the influential work of Michael Gallagher, analyzes effects of district magnitude on party-votes-to-party-seats proportionality (Benoit, 2001; Farrell, 2011; Gallagher, 1991). Districts in which larger numbers of seats are on offer are systematically associated with higher votes-seats proportionality. We can think of this as the *mathematical* representativeness of elections – an interpretation of representativeness that is blind to the policy positions offered to voters by candidates contesting the election.

CONTACT Michael Laver ml127@nyu.edu New York University, New York, NY 10022, USA; London School of Economics, London WC2A 2AE, UK

An electoral contest between a smaller number of candidates proposing a constellation of policy positions which reflects the preferred positions of most voters may better 'represent' voter preferences than a contest with more candidates proposing a constellation of policy positions that are less aligned with voter preferences. This possibility has motivated a growing literature analyzing the 'congruence' between voters' preferred policy positions and the set of party policy positions on offer. Party-voter congruence measures how close, on average, is the configuration of policy positions proposed by the electoral parties to the configuration of preferred policy positions in the electorate? (Bingham Powell, 2009; Blais & Bodet, 2006; Ferland, 2021; Golder & Ferland, 2017; Golder & Lloyd, 2014; Golder & Stramski, 2010). Congruence depends not only on the *number* of parties or candidates on offer but also on their *policy positions* relative to the distribution of voter ideal points. We can think of this as the *substantive* representativeness of elections, as the implication of the electoral offer for *voter welfare.* There is theoretical argument (e.g. Cox, 1990) that district magnitudes affect strategic party positioning on one dimension. According to this, smaller district magnitudes should be associated with more centripetal party strategies. For these reasons, *district magnitude is likely to affect electoral congruence/voter welfare, and the substantive representativeness of elections*.

Finally, a small but influential theoretical literature, (e.g. Osborne, 1993, 2000; Osborne & Slivinski, 1996; Osborne & Tourky, 2004) deals with the *endogeneity of party systems*, in particular the strategic entry of new parties at policy positions where existing parties' policy positions leave the preferences of a substantial number of voters unrepresented. Endogenous party entry at under-represented policy positions should also, therefore, have an impact on the substantive representatives of elections.

Building on these literatures, I develop a theoretical model of the *welfare (congruence) effects of district magnitudes in settings with multidimensional voter preferences* and analyze this model computationally to map these effects. I conclude by looking at the model's implications for Irish politics, narrowing down its predictions to the range of district magnitudes found in Irish elections.

Modeling effects of district magnitude on electoral congruence

Overview

The interactions I model here are complex and dynamic. I simplify by assuming that district magnitude in any particular setting is a 'given'. (If district magnitude is strategically manipulated by someone with an interest in the election outcome, then the problem becomes even more complex.) Given a district magnitude, m, candidates will enter the race. The possibility that larger district magnitudes will attract the entry of more candidates is not assumed here, but is instead an output of the model. Any given set of candidates compete with each other for votes, using decision rules I discuss below. As a result of this competition, some candidates' support over a period of time will fall below a 'survival threshold' discussed below, and they will exit the contest. Also as a result of this competition, some voters may over a period of time become 'under-represented' with no candidate offering a policy close to their preferred policy positions. In this event, new candidates may enter the race at positions close to those of under-represented voters. This dynamic process iterates continuously. The purpose of the model is to investigate, for any given district magnitude, the evolved convergence between the constellation of candidate polices and voters' ideal policy positions.

This dynamic process is too complex to be investigated using the formal logic of traditional theoretical models. It is however possible to specify a computational model using code which rigorously specifies, for each autonomous agent in the system (in this case an actual or putative candidate), the precise course of action they will take in any given situation. The result is a bottom-up 'agent based' model (ABM) which can be systematically interrogated using suites of carefully designed computer simulations. This approach is now used across a very wide range of disciplines to investigate evolving complex interactions which defeat traditional formal logic.

The agent-based model I develop here is adapted from a suite of models described and analyzed in considerable detail by Laver and Sergenti (2012) (LS).[1] I reinterpret the LS model in terms of *competition between candidates in a single multi-seat electoral district*, with the survival thresholds set by the number of seats at stake. The model's key features are:

- *Endogenous candidate exit*. Exit is determined by 'survival thresholds' set by district magnitudes. The more representatives per district, the lower the vote share needed to stay relevant in the competition, and hence the lower the survival threshold.
- *Endogenous candidate entry*. The entry of new candidates is most likely to occur at less well-represented policy positions – those which maximize the aggregate distance between the preferred policy positions of voters and the positions of their closet party.
- *Different parties can use different decision rules* to continuously adapt their policy positions, given the evolution of other parties' policy positions and the expected electoral effects of these.

The modeled environment

I assume a two-dimensional real policy space, populated by two types of agent – voters and candidates. *Voters* have preferred policy positions, uniformly distributed over the policy space, which do not change throughout the electoral process.[2] *Candidates* compete with each other for votes by proposing policy positions. These proposed policy positions continuously evolve. Candidates state their policy positions at t_1. Voters support the candidate whose proposed policy position is closest to their ideal point. The candidates' resulting vote shares are revealed, for example in an opinion poll. Given the configuration of candidate positions and vote shares revealed at t_1, candidates then use their decision rule to adapt their positions at t_2. Voters adapt to support the candidate whose proposed policy position is *now* closest to their ideal point. Given the configuration of candidate positions and vote shares revealed at t_2, candidates then use their decision rule to adapt their positions at t_3. Voters adapt to support the candidate whose proposed policy position is *now* closest to their ideal point. And so on. The result is a continuously iterating dynamic model of competitive spatial location. This iteration continues for a finite number of cycles, comprising what we can think of as an election campaign.[3] The revealed set of vote shares arising after the final cycle of the campaign is the result of what politicians like to call 'the real election' – candidates receive the vote shares indicated by that poll. Then on to the next election campaign.

Candidates' adaptive decision rules

Just as in the real world, where different people use different rules of thumb to determine their reaction to a given set of circumstances, different types of politician may use different decision rules to set their policy positions, given the policy positions of all other candidates. In this simplified version of the LS model, candidates adapt their policy positions using one of three decision rules.[4]

Sticker:	never change position.
Aggregator:	move to the centroid of your *current supporters*' positions.
Hunter:	make a random move; if this increased net support, move again in the same direction; else make a random move. (Win-stay, lose-shift).

Endogenous candidate exit

The model assumes that candidates exit the competition if their support falls below some 'survival threshold' for a period of time. Candidates do not exit the instant their support falls below the survival threshold, but rather when their support is consistently below this. A candidate's consistent support is modeled in terms of their 'evolutionary fitness'. Specifically, candidate *i*'s evolutionary fitness, f_{it}, at time t is defined in terms of their recursively updated vote share. If *i*'s vote share at time t is v_{it}, then:

$$f_{it} = \alpha_f \cdot f_{i(t-1)} + (1 - \alpha_f) \cdot v_{it}$$

In words, evolved fitness at time *t* is evolved fitness at time *t*-1, updated by the candidate's vote share at time *t*. *How much* vote share at time *t* updates evolved fitness determined α_f, the 'fitness memory' parameter. A higher alpha means evolved fitness is less affected by current vote share. (If alpha = 1, then current vote share has no effect whatsoever on evolved fitness.) A lower alpha means that evolved fitness is more sensitive to current vote share. (If alpha = 0, we have a 'goldfish memory' regime in which evolved fitness is simply current vote share, with the entire history of support during earlier phases of the campaign being completely forgotten and candidates exiting the instant their vote share falls below the survival threshold). In the computer simulations reported below, I interrogate the model for values of α_f in the range 0.2 to 0.8.

The evolutionary system for political candidates has a 'survival threshold', τ. Candidates cannot survive in the electoral competition, and exit, if their evolved 'fitness' falls below this threshold. For this interpretation of the model, I assume the survival threshold is determined by district magnitude. The more seats in the district, the lower the survival threshold. Given a district magnitude of *m* seats, one possible assumption is that $\tau = 1 / (m + 1)$, which is of course analogous to the STV quota in Ireland. Assuming this implies that a candidate in a five-seat distinct, for example, will exit if consistently supported by less than one-sixth of the voters.[5]

Endogenous candidate entry

While less successful candidates may exit the contest, new candidates may also enter. The model assumes that an incentive for a new candidate to enter arises when there is a pool of voters who are dissatisfied because, for a period of time, no current candidate offers a policy position close to their preferred positions. As with candidate fitness, voter dissatisfaction is not instantaneous, but evolves. Define voter *v*'s evolved dissatisfaction, d^*_{vt}, with the candidate configuration at time *t* as *v*'s recursively updated distance, d_{vtmin}, from their closest candidate:

$$d^*_{vt} = \alpha_d \cdot d^*_{v(t-1)} + (1-\alpha_d) \cdot d_{vtmin}$$

The updating of voter dissatisfaction defined in this way is directly analogous to the updating of candidate fitness, and α_d is the memory parameter for voter dissatisfaction. A higher value of α_d means that voters tend to have longer memories of their past (dis)satisfaction. If their closest candidate moves away from them, for example, it takes longer for them to become dissatisfied. Conversely, a lower α_d means that voters become dissatisfied much more quickly when this happens. In the computer simulations reported below, I interrogate the model for values of α_d in the range 0.2 to 0.8.

For any given location in the policy space, new candidates enter the fray at with a (tiny, scaled by a birth parameter, β) probability directly proportional to the updated dissatisfaction of voters at that location. The greater voter dissatisfaction, the greater the probability of candidate entry at this location.

Measuring congruence/voter welfare

The measure of evolved voter welfare in the population as a whole is quite simply the inverse of the mean evolved dissatisfaction of all voters, measuring dissatisfaction as d^*_{vt} above. This is, precisely, a recursively updated measure of congruence/voter welfare during the election campaign. In other words, it is a measure of aggregate voter welfare assuming that voters take some (exponentially discounted) account of the various policy positions offered by candidates during the course of the campaign. An alternative static approach, and the standard view of congruence in the literature, would be to assume that voters have no memory of the campaign, in effect assuming that α_d is zero, measuring congruence in terms of candidate positions at the instant the election actually takes place.

Model dynamics

The modeled campaign process, an evolving complex system, is described in Figure 1. Surviving candidates at each iteration use their decision rule to adapt their policy positions to the most recently revealed voter preferences and positions of other candidates. Voters then adapt their support for candidates given the new configuration of candidate positions. The campaign tick counter is advanced. Candidate and voter adaptation continues until the tick counter indicates the end of the campaign, and an election is held. Following the election result, there is a reckoning. Update voter dissatisfaction and candidate fitnesses. In light of updated voter dissatisfaction, check for candidate entry. Any new candidate who enters uses a randomly chosen decision rule.[6] In light of any entry, voters update their candidate support. In light of updated candidate fitnesses, check for candidate exit.[7] In light of any exit voters update their candidate support. The tick counter is reset and a new campaign begins. Continue *ad infinitum*; the system continuously evolves until terminated by the analyst. Burn-in diagnostics reported in the Appendix imply the system has reached a stochastic steady state after about 50 election cycles. In other words, any 'untypical' effects arising from an arbitrary random start of the model have washed out of the system after 50 election cycles, and model outputs can be considered typical for a given set of model parameters.

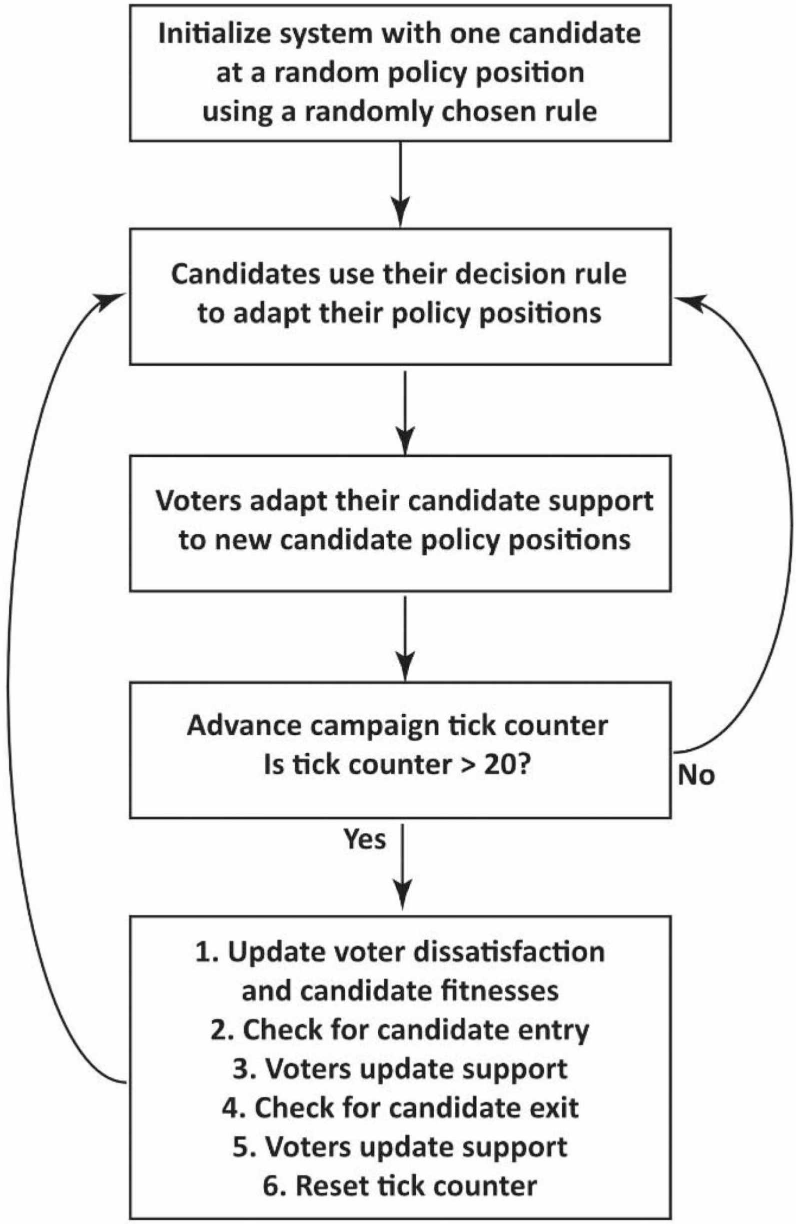

Figure 1. Recursive model dynamics.

Survival thresholds and district magnitude

The core aim of this paper is to map the relationship between *district magnitude* and the *representativeness* of elections, interpreting representativeness in terms of congruence/voter welfare. The model maps the relationship between *survival thresholds* and representativeness. Although this is ultimately an empirical matter there is a strong theoretical argument implying an inverse relationship between survival thresholds and district magnitudes. Larger districts allow more candidates to survive with smaller long-term shares of the vote. I noted above that one possible assumption is that $\tau = 1 / (m + 1)$. This assumes the survival of one perpetual runner-up candidate consistently winning fewer votes than needed to be elected. A five-seat district on this account, implies a long-run survival threshold of one-sixth of the vote. Results which follow are generated for survival thresholds, not district magnitudes, so these can be reinterpreted in terms of more 'permissive' assumptions about this relationship.

For example, the interpretation that $\tau = 1 / (M + 3)$ implies that five-seat district has a survival threshold of one-eighth of the vote. This allows the long-run survival of up to three perpetual runner-up candidates consistently winning fewer votes than needed to be elected. Figure 2 shows the (reciprocal) relationship between survival thresholds (T) and implied district

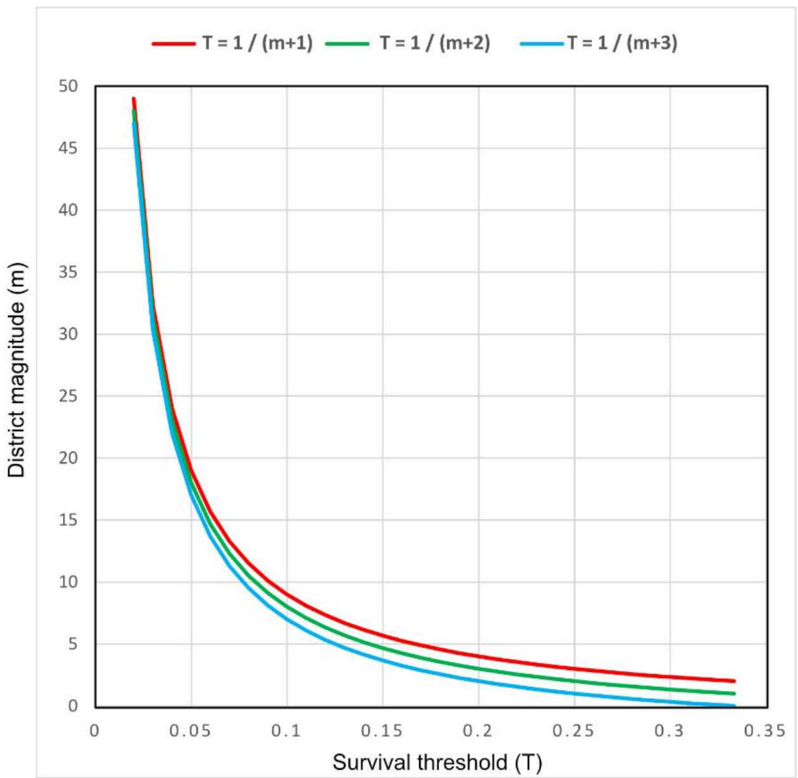

Figure 2. Relationship between survival thresholds and district magnitudes under alternative assumptions.

magnitude (m), for three different assumptions about this. The different assumptions imply somewhat different survival thresholds for very small district sizes, but they have very little effect on the big picture.

Run design

The general results reported below are generated by exercising the computational model in a suite of 1,000 model runs, each from a different random start and lasting for 50 electoral cycles, after which model output was diagnosed to have 'burnt in' to a stochastic steady state.[8] Model output is recorded at this point and a new run commenced. Each run had a Monte Carlo model parameterization, with key input parameters randomly chosen from uniform distributions within the following intervals:

- α_f, α_d, β: [0.20, 0.80]
- τ: [0.02, 0.33]

This generates a 1,000-observation artificial 'dataset' mapping model inputs into outputs. This sample of model-generated 'alternative realities' can be characterized using conventional techniques of data analysis.[9]

Results

Number of surviving candidates by survival threshold

Figure 3 plots simulation output, for the 1,000 Monte Carlo model parameterizations, mapping survival thresholds (top panel) and district magnitudes (bottom panel) onto the evolved number of surviving candidates. These results confirm conventional expectations, now extended to dynamic party systems with endogenous entry and exit, about the 'mathematical' representativeness of elections and largely speak for themselves. Lower district magnitudes/higher survival thresholds are strongly associated with fewer surviving candidates. The resulting reduction of the number of different policy positions in the electoral offer in itself reduces the potential for electoral congruence/voter welfare.

Note that results reported in Figure 3 do not, as is conventional, concern one-shot election results. They concern expectations about candidates' endogenous survival in long-run electoral dynamics. Both expectations and results, however, are strikingly similar to what is expected in one-shot elections.

Table 1 reports an OLS regression summarizing effects of key model parameters on the evolved number of surviving candidates. Over and above the strong expected effect of district magnitude, we learn two things. First,

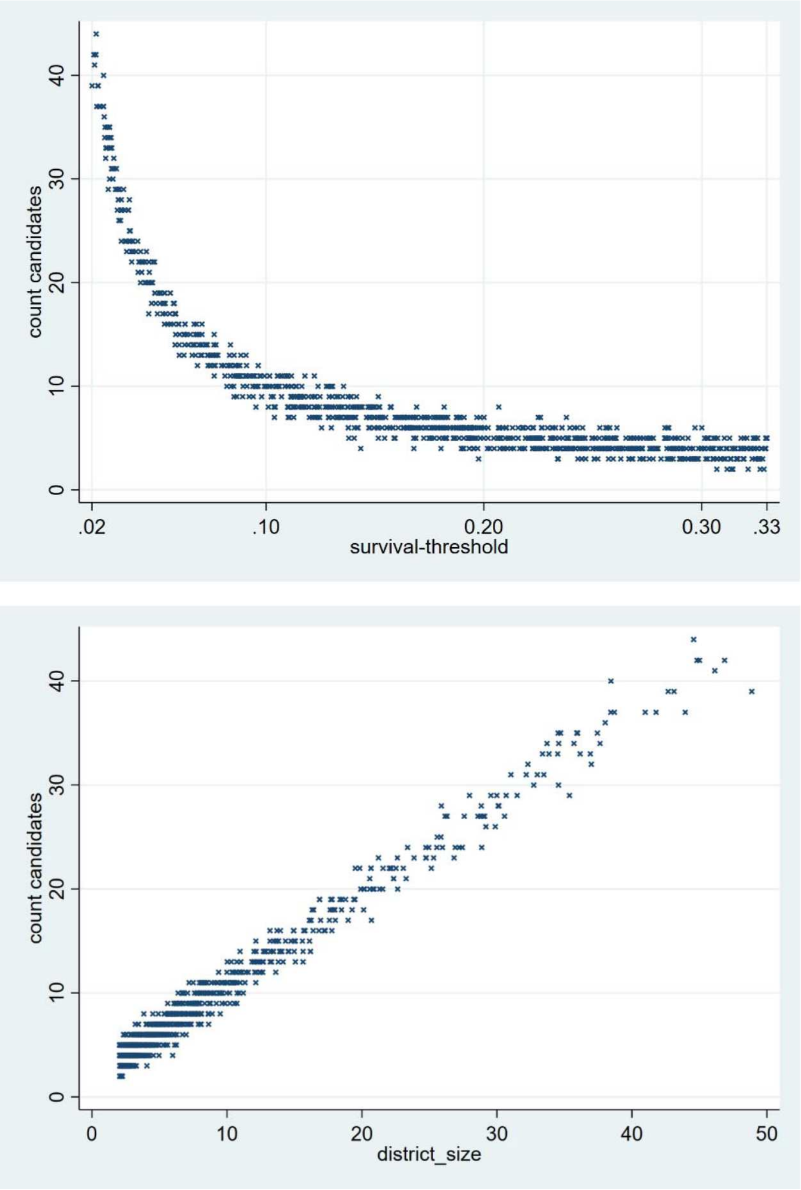

Figure 3. Modeled relationship between survival thresholds (top panel) and district magnitudes (bottom panel) on the number of surviving candidates.

as long as there is at least a small probability of endogenous candidate entry, the candidate entry parameters, α_d, β, make little difference. This is comforting, in the sense that results do not depend upon some arbitrary parameterization of the candidate entry regime. Second, fitness memory, α_f, does have a substantial effect.[10] The direction of this is intuitively plausible. Higher values

Table 1. Effects of model parameters on evolved number of surviving candidates.

	Coefficient	S.E.	p
District size (m)	0.878	0.004	0.000
Fitness alpha (α_f)	1.004	0.189	0.000
Dissatisfaction alpha (α_d)	−0.105	0.191	0.582
Birth parameter (β)	−0.114	0.190	0.548
Constant	1.552	0.171	0.000
R^2	0.980		

of α_f mean that past successes are remembered for longer, and this keeps relatively unsuccessful candidates in the competition for longer.

Candidate positioning and voter welfare

While we expect voter welfare to be a function of the number of surviving candidates, for any given number of surviving candidates, we expect voter welfare to be affected by the policy positions those candidates promote. Exploring this is the main purpose of the computer simulations. Specifically, voter welfare is maximized when candidate positions are distributed very evenly across the policy space, as opposed to clustering together close to the center of the space. Given each of the modeled decision rules (Sticker, Aggregator, Hunter) used by candidates for adapting their policy positions, these candidates will tend to distribute themselves somewhat across the policy space, rather than all clumping together near the center. Aggregators, who set their party policy positions at the centroid of the positions of their current supporters (we can think of these as internally democratic parties), are the most likely to distribute their policy positions uniformly over the policy space.

Generalizing the argument in Cox (1990) on the effect of district size on centripetal party strategies, I summarize the mean 'eccentricity' of multidimensional candidate positions as the mean absolute distance of candidate positions from the mean voter position.[11] Figure 4 plots the relationship between district magnitude and mean candidate policy eccentricity.

The results show strong heteroscedasticity – there is much more variation in policy eccentricity for low than for high district magnitudes. Nonetheless, the trend is clear. As predicted by Cox for the unidimensional case, mean policy eccentricity raises sharply with district magnitude – until district magnitude reaches about 10 or 12, after which it levels off. With larger district magnitudes, the candidate field becomes quite crowded and the scatter of proposed policy positions is as great as it is likely to get.

Voter welfare and district magnitude

Congruence/voter welfare depends not only on the number of surviving candidates but on where these candidates locate. We have just seen that

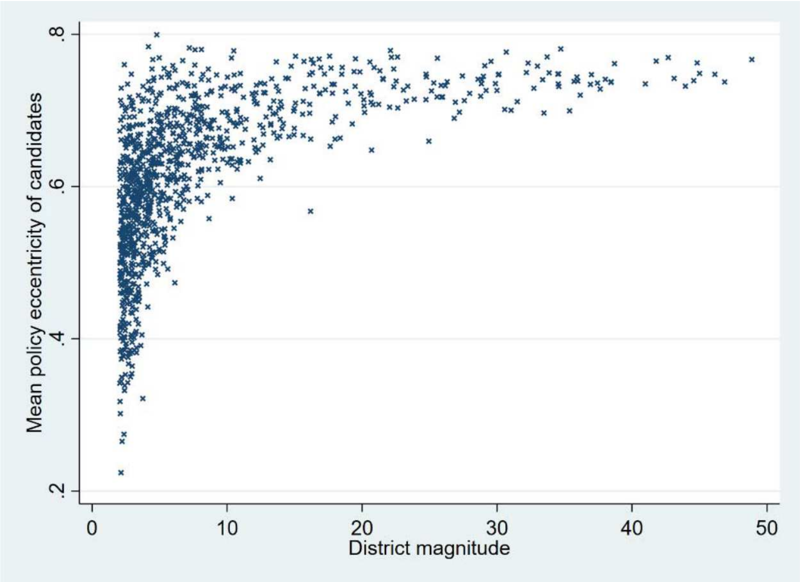

Figure 4. Modeled relationship between district magnitude and candidates' policy eccentricity.

increasing district magnitude increases not only the number of surviving candidates but the dispersion of their proposed ideal points. Figure 5 plots the net effect of district magnitude on voter welfare, resulting from both the implied number of surviving candidates and the dispersion of the policy positions they promote.

We see a sharp increase in voter welfare as district magnitude increases. This effect is particularly strong for district sizes up to about 10 or 12. *Increasing district magnitude has substantial effects on the substantive representativeness of elections,* measuring this in terms of the congruence between voters' ideal policy positions and the policy positions offered by surviving candidates.

Notwithstanding this headline result, Figure 5 also shows some of the heteroscedasticity we saw in Figure 4. Much more so for smaller districts than larger ones, *the same district magnitudes are associated with wide variations in the level of congruence/voter welfare.* District magnitude is not the full story for voter welfare. As I now show, the decision rules used by candidates for adapting their policy positions also have an important substantive effect on voter welfare.

Candidates' decision rules and social welfare

The decision rules used by candidates for adapting their policy proposals, in light of other candidates' adaptations of their own policy proposals, have a

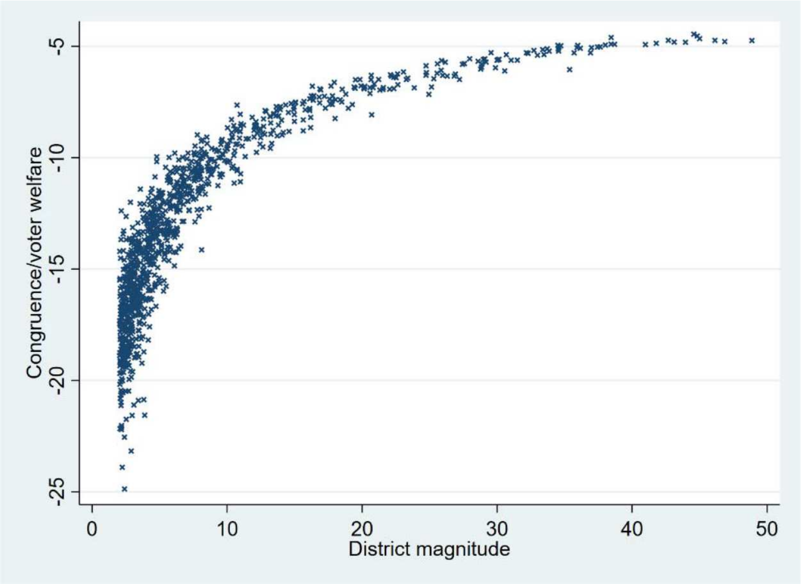

Figure 5. Modeled relationship between district magnitude and congruence/voter welfare.

big effect on where they locate in the policy space. They therefore affect congruence/voter welfare.

To take a striking example, systems in which all candidates use the Aggregator rule, each continuously (re)adapting their proposed policy position to the policy preferences of their *current supporters*, will evolve into a configuration of candidate positions which offers, mathematically, an 'optimal representation' of voter preferences for a given number of candidates. No configuration of the same number of candidate positions can improve voter welfare, seen as congruence between voter preferences and candidate positions. This is because, mathematically, candidate positions in all-Aggregator systems evolve to 'centroidal Voronoi tessellations' (CVTs) of the voter preference space, and CVTs have been shown to be optimal representations of the space. (See LS Chapter 5 for a full discussion of this).

The set of surviving candidates generated by the current model will, at any given time point, use some mix of Sticker, Hunter and Aggregator Strategies. Since Aggregator parties best represent the policy preference of their current supporters we expect that, the higher the proportion of Aggregators in the system, the higher the level of congruence/voter welfare. Table 2 tests this conjecture with an OLS regression of the effect of the proportion of Aggregators on voter welfare, holding constant the survival threshold, $\boldsymbol{\tau}$**,** the fitness memory parameter, α_f, the evolved number of surviving candidates and their mean policy eccentricity.[12]

Table 2. Effects of proportion of aggregators on voter welfare.

	Coefficient	S.E.	*p*
Survival threshold (τ)	−27.49	0.848	0.000
Fitness alpha (α_f)	0.530	0.226	0.019
N of candidates	0.121	0.009	0.000
Mean eccentricity	7.840	0.631	0.000
Proportion aggregators	1.549	0.267	0.000
Constant	−14.81	0.491	0.000
R^2	0.908		

The results reported in Table 2 support the conjecture that holding all else constant, voter welfare is higher in districts with more candidates who use the Aggregator rule, conditioning policy on the preferences of their *current* supporters, rather than candidates who endlessly change policy in search of *new* supporters. Table 2 also shows three additional independent effects on the substantive representativeness of elections. The largest effect comes from the survival threshold, and the district magnitude this implies. Lower survival thresholds, implying larger districts, are strongly associated with enhanced representativeness. While we saw above that lower survival thresholds enhance representativeness by increasing the number of surviving candidates and the dispersion of their proposed policy positions, we also saw considerable stochastic variation in this. We now see that, for a given survival threshold, having more surviving candidates and increasing the dispersion of their stated positions both independently increase substantive representation.

And so to Ireland

So far we've been looking at the big picture, where district magnitude does have a systematic large effect on the substantive representativeness of elections. Irish constituencies have only 3, 4 or 5 seats, however, comprising only a small part of the range of district sizes analyzed above. Table 3, therefore, repeats the OLS regression reported in Table 2, confining the case universe

Table 3. Effects of model parameters and proportion of aggregators on voter welfare (only for district sizes in the range 3–5).

	Coefficient	S.E.	*p*
Survival threshold (τ)	−25.47	4.233	0.000
Fitness alpha (α_f)	0.269	0.509	0.597
N of candidates	0.306	0.099	0.002
Mean eccentricity	7.105	1.348	0.000
Proportion aggregators	1.593	0.510	0.002
Constant	−15.93	0.460	0.000
R^2	0.466		

to settings in which the survival threshold implies district sizes between three and five.

All key coefficients are essentially the same in the restricted set of cases which reflects Irish constituency sizes, although modeled relationships are noticeably less sharp. If some hidden hand forced Ireland to choose between only three, four, and five seat districts, then district magnitude within this narrow range would still make a difference to the substantive representativeness of elections. Irish elections would be more representative of all districts were five seaters. There would be more surviving candidates, and their proposed policy positions would be more dispersed.

The far left of the plot in Figure 5 above refers to constituency sizes similar to those found in Ireland. Note that, while there is considerable variation in voter representativeness for any given district size, the trend in this region of the plot is for representativeness to rise steeply with increasing district size. This implies that, not only would Irish elections be more representative if all constituencies were five seaters, but that moving to six- or seven-seat districts would substantially enhance representativeness. The constituency sizes used in Ireland, in short, are of an order such that small increases in constituency size have potentially big effects on the representativeness of elections.

Conclusions

I model the dynamics of endogenously evolving competition between candidates in multi-seat districts, assuming district size sets a candidate survival threshold. On this assumption, district size affects not only the evolved number of candidates, but also electoral congruence, interpreted as voter welfare and the substantive representativeness of elections. This is a function both of the evolved number of surviving candidates and the dispersion of their evolved policy positions. Particularly for smaller distinct sizes, considerable variations in the number of candidates and voter welfare, arising from considerable variations in possible evolved candidates' positions, are consistent with the same district size. So district size, while important, is not the be all and end all of voter welfare. In particular, candidates who set policy positions which respond to their *current* supporters, rather than endlessly trying to adapt policy to attract *new* supporters, are more likely to enhance evolved voter welfare and the substantive representativeness of elections. While larger effects on substantive representation require larger district sizes than can be found in Ireland, it is nonetheless the case that five-seat districts are likely to deliver higher substantive representation than three-seaters, and that small increases to six or seven-seat districts could have potentially big effects on the substantive representativeness of Irish elections.

Notes

1. The model is implemented in the NetLogo modelling environment for ABM.
2. LS investigate a model which allows for a wide variety of distributions of voter ideal points, unimodal and multimodal. This is a natural complication of the simplified model I analyze here, which will refine but not substantially change headline results. The model can easily be extended to higher-dimensional settings.
3. The number of cycles in an election campaign is a model parameter that can be varied by the analyst. The election campaigns reported here continued for 20 cycles. In substantive terms this means an election campaign comprises a sequence of 20 periods, during each of which candidates adapt their policy positions in response to the most recent opinion poll, and voters reallocate their support on response to these adapted candidate positions. It seems not unreasonable to imagine two opinion poll cycles such as this in a typical campaign week, implying an election campaign of about 10 weeks. LS found that varying the number of cycles in an election campaign had almost no effect on results, once this number was greater than about 12. The assumption of 20 electoral cycles is, therefore, not critical.
4. These are the decision rules in the LS baseline model. LS went on to investigate two other types of decision rule. 'Explorer' is a simple hill-climbing rule. 'Predator' moves policy directly towards the other candidate with the highest current vote share. LS also investigated 'satisficing' versions of each rule, which make no policy move when a candidate's vote share is above some 'comfort threshold'. To keep things simple here, I confine the analysis to the decision rules in the LS baseline model.
5. Note that all results reported here were computed in terms of the survival threshold, τ, not in terms of the $\tau = 1 / (m + 1)$ interpretation of district magnitude. More forgiving interpretations, for example $\tau = 1 / (m + 2)$ or $\tau = 1 / (m + 3)$ would have no effect on the direction or strength of key effects, and only a small effect on their interpretation in terms of district magnitudes. See Figure 2.
6. LS developed their model to include replicator-mutator dynamics, whereby new entrants were more likely, with some error, to use historically more successful decision rules.
7. Note that the process of candidate exit means that candidates using less effective decision rules will be less likely to survive. The decision rule mix will evolve, but will be continually tested by the random rules chosen by new entrants.
8. Burn in diagnostics are reported in the Appendix.
9. It is trivial (but uninformative) to expand this dataset – simply run the computer longer. Core intuitions will not change as a result.
10. I refer here to a *substantial* rather than a *significant* effect. For computer simulations such as this, I can generate associations between variables with trivially small and substantively uninteresting, but nonetheless 'statistically' significant effects, simply by running huge numbers of trials.
11. For a uniform distribution of voter preferences, this is the origin of the space.
12. I use the survival threshold rather than the implied district magnitude, because the effect of the former on voter welfare is linear and of the latter distinctly nonlinear.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributor

Michael Laver is Emeritus Professor of Politics, New York University. Visiting Professor, Data Science Institute, London School of Economics.

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Appendix. Model burn-in

Burn-in, convergence to a (non-trending) stochastic steady state, is established using a suite of very long model runs with different values of the survival threshold. We can estimate this statistically, but here the plots speak for themselves. The first 100 cycles of 1,000-cycle diagnostic runs, below, show the dynamic model is very conservatively burnt in at 50 cycles. All reported results, therefore, are output quantities of interest for model runs stopped at 50 cycles.

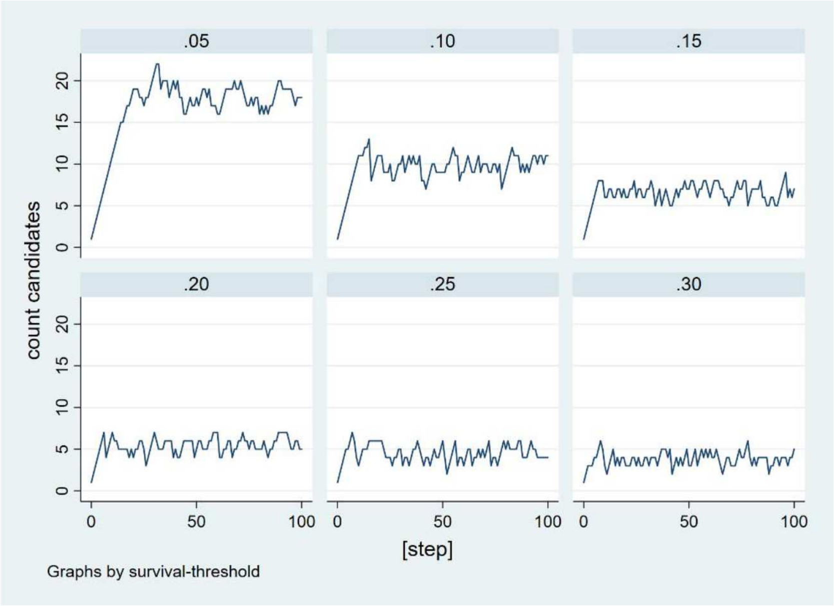

Figure A1. Number of surviving candidates, by first 100 model ticks of 1,000-tick diagnostic runs, for six values of τ (.05, .10, .15, .20, .25, .30).