Chapter 1
Introduction

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Abstract This chapter provides an introduction to the book. The twelve essays in the book fall into three groups. Essays in the first group address problems in the philosophy of mathematics; essays in the second group investigate foundational questions concerning Lakatos's philosophy of science; and essays in the third group apply Lakatos's concept of Methodology of Scientific Research Programmes (MSRP) to medicine. The book ends with an epilogue.

Although Lakatos is nowadays primarily known for his work in philosophy of natural science, and in particular for his Methodology of Scientific Research Programmes (MSRP), his first major contribution was his *Proofs and Refutations*—a groundbreaking work in the philosophy of mathematics. The central thesis of *Proofs and Refutations* is that the development of mathematics does not consist in the steady accumulation of eternal truths, as conventional philosophy of mathematics suggests. Mathematics develops, according to Lakatos, in a much more dramatic and exciting way, via a process of conjecture, followed by attempts to "prove" the conjecture (in his view, to reduce it to other conjectures) followed by criticism via attempts to produce counterexamples both to the conjectured theorem and to the various steps in the proof, resulting in the proof of a much modified version of the original conjecture.

Among the still open questions about Lakatos's views are: Does Lakatos's account really amount to a fully "quasi-empirical" view of the epistemology of mathematics to rival the traditional philosophies of logicism, formalism and intuitionism? Or is it instead "merely" an—albeit fascinating—account of how mathematical theorems are arrived at, an account which has no consequences for

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the epistemological status of those eventually arrived-at theorems? Is Lakatos's central example—the Descartes-Euler conjecture about polyhedra—itself too "quasi-empirical" to be representative of mathematics in general? Finally, did Lakatos outgrow his Hegelian roots? Or is *Proofs and Refutations* best, or perhaps even *only*, understandable as a thoroughly Hegelian work? Some of these issues are touched on in the contributions to the philosophy of mathematics section of this volume.

Turning, then, to his philosophy of science, Lakatos famously presented MSRP as a synthesis of the views of Karl Popper and Thomas Kuhn—preserving from the former the claim that theory change in science is a rational process, while allowing that the latter's account of how scientists regard and deal with experimental difficulties is altogether more true-to-scientific-life than Popper's. There is no consensus as to whether or not this "synthesis" succeeds. Nor is there any consensus about how to interpret Lakatos's central notion of progress and the associated concept of "novel fact". Another open issue is whether the insights underlying Lakatos's MSRP can be captured and thereby given a more solid foundation by the Bayesian approach to scientific reasoning. The view that those insights can be given a Bayesian justification was argued by Howson and Urbach in their *Scientific Reasoning: The Bayesian Approach*.

A large part of the continuing influence of Lakatos's ideas consists in attempts to apply his MSRP to identify and evaluate research programmes in special sciences such as medicine, psychology, economics, and sociology, as well as in disciplines like educational theory, informatics, and international relations, which otherwise receive scant attention in philosophy of science. These attempts often originate in the sciences themselves and are driven by practitioners' desire to understand developments in their fields, rather than by traditional philosophical concerns. Being relevant beyond the confines of professional philosophy is probably the best marker of a lasting influence.

The book consists of 12 essays, which fall into three groups. Essays in the first group address problems in the philosophy of mathematics; essays in the second group investigate foundational questions concerning Lakatos's philosophy of science; and essays in the third group apply his MSRP to medicine. The book ends with an epilogue.

The first group of essays begins with Philip Kitcher's "Mathematical Methodology". Lakatos regarded his *Proofs and Refutations* as a study in the "methodology of mathematics" or the logic of mathematical discovery. Philip Kitcher agrees that philosophy of mathematics has—both before and after Lakatos—concentrated on issues about the status of mathematical results and ignored issues about how those results emerged in the first place; and it has done so to its cost. Accordingly, Kitcher's contribution develops a mathematical methodology. He outlines the major changes that resulted in the mathematics of the late nineteenth century, indicates how those results emerged, and appraises them in terms of a notion of pragmatic progress (progress *from*) as opposed to any notion of teleological progress (progress *to*). Kitcher's methodology transcends Lakatos in many ways but is recognisably Lakatosian in spirit.

In her "Proofs as Dialogues: The Enduring Significance of Lakatos for the Philosophy of Mathematical Practice", Catarina Dutilh Novaes focuses on what Lakatos's ideas have to offer for contemporary philosophical work on mathematical practice. In particular, she highlights the influence of Lakatos's *Proofs and Refutations* on the development of her dialogical account of deduction and mathematical proof, which relies on so-called Prover-Skeptic dialogues. Similarities and differences between Prover-Skeptic dialogues and Lakatosian Prover-Refuter dialogues are discussed with special attention to the roles of cooperation and adversariality. The article closes with a reflection on the broader philosophical differences between Lakatos's "Hegelian" approach and Dutilh Novaes's dialogical pragmatism: Lakatos aims at the dialectic development of mathematical *concepts,* disregarding individual human activities, while Dutilh Novaes's dialogical account of mathematical proof is primarily about *human agents* and their interactions.

In their "Lakatos and the Euclidean Programme", Alexander Paseau and Wesley Wrigley critically examine and revise Lakatos's account of the Euclidean Programme (EP), which is a foundationalist account of mathematical knowledge inspired by Euclid's Elements. In Lakatos's view, a system of mathematical knowledge that is organised according to the EP starts from a finite set of trivially true axioms with perfectly well-understood primitive terms, and truth then "flows" from axioms to theorems via deductive channels. The authors critically examine various aspects of Lakatos's account and suggest modifications that lead to an improved characterisation of the EP, consisting of seven principles. The proposed characterisation inherits some core ideas from Lakatos's account (e.g. the idea of flow) but differs in various other respects. The outcome is an updated reconstruction of the EP in the spirit of Lakatos.

In "Proofs and Refutations, Non-Classically and Game Theoretically", Can Başkent argues that the reasoning in Lakatos's *Proofs and Refutations* is not governed by the rules of classical logic but instead exemplifies paraconsistent logic—a type of logic where it is not the case that everything follows from a contradiction. Başkent points out that inconsistencies play a fundamental role in the Lakatosian method of proofs and refutation. Crucially, when contradictions arise (e.g. due to counterexamples to a conjecture), one is not permitted to draw arbitrary conclusions. How one can move forward in the face of a contradiction is precisely what defines the method of proofs and refutations. Furthermore, Başkent argues that the strategic way in which inconsistencies should be handled according to Lakatos can be fruitfully analysed through the lens of game theory. This is illustrated using concrete examples from *Proofs and Refutations*.

Vincenzo Crupi's "The Case of Early Copernicanism: Epistemic Luck versus Predictivist Vindication" is the first contribution of the second group, which concerns foundational questions about Lakatos's philosophy of science. In his paper, Crupi investigates the issue of whether the adoption of the Copernican theory by Kepler and Galileo (as well as by Copernicus himself) was, as many have claimed, a matter of "epistemic luck": these luminaries happened to make what was by later lights the correct choice but had no empirical justification for that choice at the time when they initially made it. The idea that Kepler and Galileo were 'lucky' has

generally been based on the claim that—allegedly—any empirical phenomenon that might be taken to support Copernican theory could in fact equally well be accounted for on the rival Ptolemaic theory. In a widely read paper, Lakatos and Zahar argued that, to the contrary, once the notion of prediction is properly understood, the initial Copernican theory is seen to have enjoyed predictive successes not shared by its Ptolemaic rival and hence Kepler's and Galileo's theory-choices are vindicated. Crupi investigates whether Lakatos and Zahar's view stands up to historical and philosophical analysis.

The paper "The Bayesian Research Programme in the Methodology of Science, or Lakatos Meets Bayes" by Stephan Hartmann argues that, when understood correctly, Bayesianism is an instance of a progressive Lakatosian research programme in the methodology of science. This stands in stark contrast to Lakatos's own rather sceptical view about Bayesianism. To support its claim, the paper considers and then dismisses three challenges to Bayesianism. These arise in connection with indirect evidence, new types of evidence, and genuinely new evidence. Hartmann shows how these challenges can be met within the Bayesian Research Programme. He also shows that in order to be able to handle these challenges, one has to abandon a core tenet of traditional Bayesianism: that belief change has to be made via standard conditionalization. Instead of relying on standard conditionalization, belief change should be based on the "Principle of Conservativity": the requirement that belief change should minimize a certain distance between the probability measures representing beliefs.

Thodoris Dimitrakos' "Lakatos's Naturalism(s): Distinguishing between Rational Reconstructions and Normative Explanations" examines Lakatos's concept of "rational reconstruction" in the philosophy of science, defending its use against critics like Kuhn who claim it distorts historical records. After briefly discussing, and setting aside, some uncharitable criticisms of Lakatos's account, Dimitrakos identifies the real problem it faces: that Lakatos's attempt to provide both a historically informed philosophy of science and an account of scientific rationality led to problems of circularity. Dimitrakos argues that these problems can be resolved in three steps. First, one needs to distinguish between rational reconstruction, a philosophical tool for evaluating different theories of scientific rationality, and normative explanation, a historiographical category. Second, one has to reject Popper's "three worlds" conception, situating Lakatos's approach within a liberal naturalism. And, finally, one must replace Lakatos's inter-methodology evaluation process with a suitable intra-methodology process. In doing so, the chapter aims to show how Lakatos's work remains relevant to contemporary debates about the relationship between history and philosophy of science.

In his "Heuristic, Physics Avoidance, and the Growth of Knowledge", Jack Ritchie examines the notion of positive heuristic in Lakatosian philosophy of science, particularly in *Methodology of Scientific Research Programmes*. He begins by setting aside an alternative view of heuristic due to John Worrall (claiming that it departs too far from the source text), and then offers a different interpretation inspired by the work of Mark Wilson. On Ritchie's account, the positive heuristic fosters the growth of knowledge through a process often best understood as "model-

making and improving". On this view, a central driver of progress is the construction and refinement of scientific models. The aim of these models is to convert empirical difficulties into mathematical difficulties. These difficulties include the construction of mathematically tractable models and providing plausible bridges between higher and lower-level models of the same phenomena. On the view that Ritchie provides, refutation is less essential to Lakatosian progress than sometimes supposed, with the incremental improvement of models playing a more central role.

Samuel Schindler's "Beyond Footnotes: Lakatos's Meta-Philosophy and the History of Science" revisits Lakatos's approach to historical facts. Lakatos infamously claimed that the actual history of science could be recorded in the footnotes of rational reconstructions of science. Schindler points out that Lakatos's approach to actual history was more reasonable than that, not least because he argued that a philosophical methodology of science should aim to maximise rationally explainable facts, even though there should be no expectation that all historical facts will turn out to be rational. Schindler examines this idea in the context of the contemporary discussion about meta-philosophy. The paper then compares Kuhn's and Lakatos's approaches to science and argues that Lakatos's account, contrary to what he himself thought, doesn't have a more legitimate claim to rationality than Kuhn's.

The next two contributions form the third group of papers, which are dedicated to the philosophy of medicine. In his "Cholesterol and Cardio-Vascular Disease: Degenerating Research Programmes in Current Medical Science", John Worrall argues that the mini research programmes built to defend two extremely influential claims in current medicine have both consistently degenerated. If so, as he remarks, one would have expected those two claims to have been rejected as not evidence-based. But in fact, although the consensus on the first claim now shows some signs of breaking up, it remained in place for many years after degeneration set in; while the second remains almost universally accepted in medicine and remains the basis for accepted medical advice and treatment. The second part of his paper analyses this clash between expectation and reality, leading to a re-examination of Lakatos's distinction between internal and external history.

Anya Plutynski's "Trade-offs and Progress in Cancer Science" begins with the observation that almost all examples of research programmes analysed in terms of progress and degeneration by Lakatos and those influenced by him were from physics (or occasionally chemistry). One might therefore be tempted to object that MSRP, while a useful tool for analysing developments in basic sciences like physics and chemistry, is not usefully employed in other, more "special" or applied sciences. Plutynski raises this question and concludes that appropriately analysing developments in Cancer Science may require replacing Lakatos's notion of progress in science with one that recognizes the prevalence of trade-offs intrinsic to the culture of science.

The book ends with an epilogue, John Worrall's Scientific Theory-Change and Rationality: Lakatos and the "Popper- Kuhn Debate" in which he takes a look back at the "Popper-Kuhn" debate and Lakatos's attempt to resolve it. The Popper-Kuhn debate was one of the foci of attention at the famous Bedford College Colloquium

held in the summer of 1965. What exactly was at issue in this debate? Was Lakatos right that Kuhn's account of theory-change in science denies that change is a rational affair by reducing change to "a matter of mob psychology"? Was Lakatos right that his MSRP provides a satisfactory "synthesis" of the views of Popper and Kuhn—one that preserves the rationality of theory-change? How has the debate progressed since 1965 and where does it currently stand? The chapter is the written version of a lecture Worrall gave at the conference *Centenário Imre Lakatos: matemática e ciência* in Sao Paolo in November 2022. We have kept the lecture in its original form. The points come across most vividly in this talk by a PhD student supervised by Lakatos himself.