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Correlation concern

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ABSTRACT

In many choice problems, the interaction between several distinct variables determines the payoff of each alternative. I propose and axiomatize a model of a decision maker who recognizes that she may not accurately perceive the correlation between these variables, and who takes this into account when making her decision. She chooses as if she calculates each alternative's expected outcome under multiple possible correlation structures, and then evaluates it according to the worst expected outcome.

1. Introduction

In many decision problems, the overall payoff of each alternative depends on multiple, distinct variables; for instance, the return of a stock portfolio depends on the return of the underlying stocks. Understanding the correlations between these underlying objects is difficult both conceptually and econometrically.[2] Concern about her own (or another agent's) lack of a good understanding of these interdependencies can materially change the behavior of a decision maker (DM).

An individual may choose an index fund over the corresponding stocks because she does not recognize their connection and is uncertain about the correlation between the stocks. A financial institution may choose a suboptimal loan portfolio in order to pass a stress test that ensures it is not subject to too much systematic risk. A principal may offer a simple contract to ensure that it is robust to the agent's perception of the correlations between the payoffs it offers, the agent's own information, and the private information and actions of other agents.

I propose and axiomatize a model of a DM who recognizes that she may not accurately perceive the correlation between the underlying sources of uncertainty, and who takes this into account when making her decision. The DM expresses preferences over (lotteries over) portfolios of assets whose payoffs all depend on a common state space, known to the modeler but not necessarily the DM. I consider a DM who may misperceive the correlation between assets, and propose axioms that characterize a preference for alternatives with payoffs that do not depend on the correlations. As in Ellis and Piccione (2017) (henceforth, EP), misperception of correlation is identified by a strict preference over two portfolios that always yield the same consequence, but, unlike EP, the DM is averse to her uncertainty about these correlations. The main result shows that a DM's behavior satisfies the axioms if and only if she can be represented as if she considers a set of possible correlation structures between the actions and evaluates each alternative by the worst expected utility in that set.

To illustrate the setting and approach, consider a DM choosing between collections of assets whose returns depend on tomorrow's high temperature. This common source of uncertainty defines an objective relationship between them, and if they are all expressed in Celsius, then understanding their correlations is easy. However, if some are expressed in Celsius, others in Fahrenheit, and still more in Kelvin, then a DM who does not remember how to convert between units may be uncertain about how strongly correlated some are with others. I identify whether the DM understands these connections from her choice between two portfolios that lead to the exact

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[2] Throughout, I use the term "correlation" interchangeably with the more accurate "joint distribution".

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same payoff for every high temperature using the correct conversions, but where the first contains a single bet and the second several bets expressed in both Celsius and Fahrenheit. For example, she may choose between a bet that pays $100 for sure and a pair of bets, one paying $100 if and only if the temperature is above 30° Celsius the other paying $100 if and only if it is below 86° Fahrenheit. If the DM expresses a strict preference for one or the other, then she reveals that she is uncertain (or has incorrect beliefs) about the conversion between units and so the correlation between the bets. For a more realistic choice, an agent may strictly prefer an S&P 500 index-tracking fund to a portfolio of the 500 underlying stocks of the S&P 500 (in the right proportions and without transaction costs) because she does not recognize their connection and is uncertain about the underlying correlations between the stocks.

The novel *Negative Uncorrelated Independence* axiom captures concern about correlation in the alternatives. Intuitively, it says that if she prefers an alternative where correlation matters over one where it does not, then introducing potential correlation to both without making either better or worse does not lead to a preference reversal. Formally, I capture this by introducing lotteries and weakening the independence axiom. If the DM prefers a portfolio P (such as the pair of bets above) to an individual asset a (such as a sure amount of money), then she also prefers a lottery between P and a second portfolio P' to a lottery between a and P'. I also impose EP's *Weak Monotonicity* axiom that requires that whenever one portfolio is better than another for every possible joint distribution over outcomes, that portfolio is preferred.

These two axioms, jointly with other standard axioms, are necessary and sufficient for the DM's preference to have a *Correlation Concern Representation (CCR)*. The representation consists of a set of joint distributions over the payoffs of assets, each consistent with the same underlying distribution when restricted to a given asset. She evaluates each portfolio by its worst expected utility according to one of these distributions. When the DM understands sufficiently rich subsets of assets, then her perception of possible correlation structures can be uniquely recovered from her choices.

More formally, I take as given an objective state space Ω that pins down both the possible returns of each asset and how they are correlated with each other (in the above example, Ω is the temperature). A DM with a CCR acts as if she considers a larger, subjective state space rich enough to express correlations between assets not captured by Ω. Following EP, I use a state space that has many copies of Ω where the return of each asset is determined by one copy (the temperature in Celsius, in Fahrenheit and in Kelvin). Any belief about correlations between assets can be represented as a probability measure on such a state space. A set of these probability measures, each with the same marginal distribution over every copy of Ω, captures the DM's uncertainty about correlations. Her choice of profile maximizes the minimum expected utility across the set, as in the literature on ambiguity aversion. When enough assets are mapped to the same copy, the set can be uniquely identified from these choices.

A DM with a CCR is not probabilistically sophisticated (Machina and Schmeidler, 1992), even on the larger state space. This is important because, as noted by Epstein and Halevy (2019), probabilistic models of correlation misperception (including Eyster and Weizsäcker (2010); Levy and Razin (2015); Enke and Zimmerman (2018); EP) do not allow "awareness of the complexity of her environment and self-awareness of her cognitive limitations [to] lead the decision-maker to doubt that her wrong beliefs are correct." In contrast, a DM with a CCR may recognize that she does not understand certain correlations and choose accordingly; for instance, she may refuse to take either side of a trade that she does not understand, or she may pay a premium to avoid more complex prospects that require her to evaluate more correlations.

The paper concludes by studying how the CCR captures some behavior of particular interest. Consider a DM who always prefers an individual action to a profile that yields the same outcome in every state. This DM prefers "simpler" alternatives that do not require her to think about correlations. A CCR captures this behavior when the set of priors includes the correctly specified one. The components of the CCR reflect patterns of behavior in the natural way. For example, one DM is more concerned about correlation than another if whenever the first prefers a profile to an action, then so does the second. This is equivalent to the first considering a larger set of probability measures than the second, once the two are projected onto a state space in which they are comparable.

1.1. Related literature

This paper extends and generalizes EP. The setting, detailed in Section 2, is essentially identical to theirs. EP allow the DM to express uncertainty about the objective but unknown relationship between actions, but they require her to do so in a manner consistent with expected utility. My model allows the DM to be averse to this source of uncertainty to capture concern about potentially misunderstanding correlations. To do so, I replace the standard independence axiom with the novel Negative Uncorrelated Independence axiom discussed above. The other axioms and definitions in Section 3 have counterparts in EP. In particular, Weak Monotonicity, the key new axiom of EP, is exactly the same, and I strengthen non-singularity slightly relative to EP. Theorems 1 and 2 generalize their counterparts in EP (Theorems 1 and 2) in the same way that Gilboa and Schmeidler (1989) generalizes Anscombe and Aumann (1963): they deliver representations on a subjective state space capturing all possible correlations where the DM maximizes her minimum expected utility across a set of priors, as opposed to maximizing her expected utility for a given prior. More explicitly, EP is the special case of CCR where the set is a singleton and is distinguished by satisfying the independence axiom (Corollary 1).

There is ample experimental evidence for correlation misperception, including Eyster and Weizsäcker (2010), Rubinstein and Salant (2015), Enke and Zimmerman (2018), and Hossain and Okui (2024). Experimental evidence that agents are aware of and concerned about correlation can be found in Epstein and Halevy (2019). They study an environment with explicit uncertainty about the correlation between two events whose joint realization determines the payoff of a bet. A majority of subjects are inconsistent with a probabilistic model of correlation, and a majority of that majority at least weakly prefer bets that do not depend on correlation. Indirect evidence comes from theoretical study of asset markets. In this context, Jiang and Tian (2016), Liu and Zeng (2017), and Huang et al. (2017) consider such a model and show it explains some stylized facts including under-diversification and limited participation in the market.

Epstein and Seo (2010, 2015) explore axiomatically the consequences of introducing ambiguity in the classic exchangeable model of de Finetti. An exogenous product state space describes the outcome of a sequence of experiments that are indistinguishable and possibly related but not identical. They provide a model where the DM perceives ambiguity about the relationship between experiments. The functional form, especially in the 2010 paper, is similar to the one I consider, but acts depend exogenously on a collection of experiments.

Epstein and Halevy (2019) consider a related model. They argue that, in the setting above, an ambiguity averse agent may prefer bets on only a single experiment to bets that depend on multiple experiments. As noted above, they conduct an experiment that confirms that a number of subjects have this preference.

Heo (2020) provides a different perspective on DM averse to acts that depend on different components of an (objectively given) product state space, or issues. He argues that such a DM may strictly prefer an act that depends on a single issue to a mixture of that act with an equally good act that depends on a different issue. As a consequence, she may violate the classic uncertainty aversion axiom. Adapted to this setting, NUI requires that a DM who prefers a multi-issue act to a single issue act does not reverse that preference when both are mixed with a common third act. Moreover, a DM with a correlation concern representation satisfies the uncertainty aversion axiom, though I do not explicitly impose it. These approaches provide complementary perspectives on the issue.

To apply the above papers that take a product state as given, the modeler must know what the DM perceives the state space to be and observe her ranking of acts on this state space. In settings where she misperceives her options, this may be more difficult. For instance in the thought experiment in Section 3.1, this modeling requires that the DM can be observed to rank bets on impossible events like "temperature is greater than $0\,^{\circ}C$ but lower than $32\,^{\circ}F$."

Concern for robustness has other applications in mechanism design. For instance, Carroll (2017) considers a seller uncertain about the correlation between the buyer's values of different goods that can be bundled. The CCR captures the behavior of such a principal nicely. Another strand of the behavioral mechanism design literature focuses instead on a principal concerned that the agent does not correctly understand the game that she is playing. Most notably, Li (2017) considers obviously strategy proof mechanisms: mechanisms played correctly even agents who do not understand the relationship between the other agents' actions and information with her own payoff. This is conceptually connected to the results herein, but in the CCR an agent evaluates each mechanism with the "worst" beliefs about the relationship to others. One can model this by considering incomplete preferences and maintaining independence instead of maintaining completeness and NUI.

Levy et al. (2022) and Laohakunakorn et al. (2019) consider an agent who is exposed to information from multiple sources. She considers all priors "close enough" to a benchmark when making her decision. As in this paper, she considers the worst of these priors when evaluating acts. The benchmark is full independence.

2. Primitives

There is a set $\mathcal{A}$ of *actions*, with typical elements a, a_i, b, b_i. Each action results in an *outcome* or consequence in $X = \mathbb{R}$, with typical elements x, y, z. This outcome is determined by a *state of the world* drawn from the finite set Ω, with typical elements ω, ω'. I interpret the state space Ω as a description of the "objectively possible" joint realizations of the outcomes of any set of actions, against which the DM's subjective perceptions of joint realizations are evaluated.

A map $\rho : \mathcal{A} \times \Omega \to X$ describes the relationship between actions, states, and outcomes, with the action a yielding the outcome $\rho(a, \omega)$ in state ω. The set $\mathcal{A}$ includes every Savage act, i.e. for any $f : \Omega \to X$ there is an action yielding the outcome $f(\omega)$ in state ω for every $\omega \in \Omega$. Slightly abusing notation, I write $x \in X$ for an action that yields x in every state.

An *action profile* (or profile) is a finite vector of actions for which the order does not matter – i.e., a multiset of actions. A profile that consists of taking the n actions $a_1, \ldots, a_n$ is denoted $\langle a_1, \ldots, a_n \rangle$ or $\langle a_i \rangle_{i=1}^n$. To save notation, the range of indices is omitted when the number of actions is unimportant, i.e. $\langle a_i \rangle$ instead of $\langle a_i \rangle_{i=1}^n$. An agent who chooses the profile $\langle a_i \rangle_{i=1}^n$ receives the outcomes of all n actions $a_1, \ldots, a_n$, that is, she receives $\sum_{i=1}^n \rho\left(a_i, \omega\right)$ in state ω. Let $\mathcal{F}$ be the set of all action profiles.

The DM chooses by maximizing a preference relation $\succsim$ over probability distributions on $\mathcal{F}$ having finite support, the set of which is denoted by $\Delta\mathcal{F}$. A typical element of $\Delta\mathcal{F}$ is $p = \left(p_1, \langle a_i^1 \rangle; \ldots; p_m, \langle a_i^m \rangle\right)$, interpreted as the lottery where profile $\langle a_i^j \rangle$ occurs with probability p_j. As usual, the symbol $\sim$ denotes indifference and $\succ$ strict preference. The set of lotteries over actions, $\Delta\mathcal{A} = \{p \in \Delta\mathcal{F} : p(\langle a_i \rangle_{i=1}^n) > 0 \text{ only if } n = 1\}$, plays an important role in the axioms, and includes as a subset the lotteries over outcomes, $\Delta X = \{p \in \Delta\mathcal{A} : p(\langle a \rangle) > 0 \implies a \in X\}$. It will be convenient to write $p(\langle a_i \rangle) > 0$ for the set of profiles $\langle a_i \rangle$ in the support of p. Given $p, q \in \Delta\mathcal{F}$, a mixture $\alpha p + (1-\alpha) q$, $\alpha \in [0,1]$, is the lottery in $\Delta\mathcal{F}$ in which the probability of each profile in the support of p and q is determined by compounding the probabilities in the obvious way.

Endow $\Delta\mathcal{F}$ with the weak* topology for the space $\mathcal{F}$ endowed with metric d defined as follows. Let $X^* = \{\langle x \rangle : x \in X\}$. The metric d is discrete on $\mathcal{F} \setminus X^*$ and agrees with the usual metric on X on X^*. That is, for any $\langle a_i \rangle_{i=1}^n, \langle b_i \rangle_{i=1}^{n'} \in \mathcal{F}$, $d(\langle a_i \rangle_{i=1}^n, \langle b_i \rangle_{i=1}^{n'}) = 1$ when $\left\{ \langle a_i \rangle_{i=1}^n, \langle b_i \rangle_{i=1}^{n'} \right\} \not\subset X^*$ and $\langle a_i \rangle_{i=1}^n \neq \langle b_i \rangle_{i=1}^{n'}$, $d(\langle x \rangle, \langle y \rangle) = |x - y|$, and $d(\langle a_i \rangle_{i=1}^n, \langle b_i \rangle_{i=1}^{n'}) = 0$ only if $\langle a_i \rangle_{i=1}^n = \langle b_i \rangle_{i=1}^{n'}$. According to d, a sequence of profiles converges only if it is eventually constant, or every profile therein is a single, constant outcome, and the sequence of outcomes converges.[3]

[3] A sequence of profiles (F_n) d-converges to the profile G only if there exists N so that either $F_n = G$ for all $n > N$ or $F_n = \langle x_n \rangle$ for all $n > N$, $G = \langle y \rangle$, and the sequence $(x_n)_{n>N}$ approaches y in the usual sense.

3. Foundations

This section begins by presenting a thought experiment illustrating the novel behavior the model captures. It then introduces and discusses the axioms on the DM's preference relation invoked by the main result. Finally, the assumptions necessary for the identification result are introduced.

3.1. Illustration of behavior

Consider a DM choosing between bets that depend on τ, tomorrow's high temperature. The DM can have either \$100 or the sum of the outcomes of bets b_C and b_F, where b_C pays \$100 if τ is less than 30° Celsius (\$0 otherwise) and b_F pays \$100 if τ is at least 86° Fahrenheit (\$0 otherwise). On another occasion with the same weather forecast, the DM must choose between b, which pays \$100 if τ is less than 30° Celsius and −\$100 otherwise, and the combination of b_C and $-b_F$, which is a "short" position on b_F that pays −\$100 if τ is at least 86° Fahrenheit (\$0 otherwise). Formally, the DM makes a choice from each of the sets $\{\langle 100\rangle, \langle b_C, b_F\rangle\}$ and $\{\langle b\rangle, \langle b_C, -b_F\rangle\}$. As 30° Celsius equals 86° Fahrenheit, a DM who knows this and easily converts Fahrenheit to Celsius expresses indifference in both choices. However, a DM who does not know exactly how to convert from one unit to the other may not exhibit such indifference and reasonably express $\langle 100\rangle \succ \langle b_C, b_F\rangle$ and $\langle b\rangle \succ \langle b_C, -b_F\rangle$.

A probabilistic approach to correlation can capture only one of the two preferences. For instance, a risk-averse DM with consistent probabilistic beliefs may express $\langle 100\rangle \succ \langle b_C, b_F\rangle$ because she believes that b_C and b_F may not be perfectly hedged and thus riskier than \$100 for sure. However, that same DM believes that b_C and $-b_F$ are negatively correlated and thus less risky than b. Hence under expected utility, $\langle 100\rangle \succ \langle b_C, b_F\rangle$ implies $\langle b_C, -b_F\rangle \succ \langle b\rangle$. This section outlines restrictions on behavior consistent with the thought experiment and equivalent to a representation of a DM concerned about correlation.

3.2. Preference for avoiding correlation

The key axiom reflects a DM who dislikes exposure to correlation. It weakens the Independence Axiom, which holds that for any $p, q, r \in \Delta\mathcal{F}$ and $\alpha \in (0, 1]$, $p \succsim q$ implies that $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.

Axiom 1 *(Negative Uncorrelated Independence, NUI). For any $p, q, r \in \Delta\mathcal{F}$ and $\alpha \in (0, 1]$, if $p \succsim q$ and $q \in \Delta\mathcal{A}$, then $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.*

NUI requires that preference between two lotteries remains the same when both are mixed with a third lottery only when the less preferred lottery attaches probability exclusively to single action profiles. To illustrate, suppose that the DM is indifferent between $\langle a, b\rangle$ and $\langle c\rangle$. While her evaluation of $\langle a, b\rangle$ depends on her perception of the correlation between the two stocks, $\langle c\rangle$ does not involve any correlation at all. Hence, the absence of exposure to correlation by $\langle c\rangle$ exactly offsets a better expected outcome from $\langle a, b\rangle$. After mixing both with an arbitrary lottery r over profiles, evaluating either lottery requires computing correlations. Moreover, neither alternative becomes objectively better: the mixture changes the expected utility of both profiles in the same way for any given correlation between actions by standard usual independence axiom arguments. Nonetheless, the "simplicity" advantage that $\langle c\rangle$ had is lost. A DM who dislikes thinking about correlations should (weakly) prefer the mixture of $\langle a, b\rangle$ and r to the mixture of $\langle c\rangle$ and r.

NUI implies that violations of independence do not occur when comparing lotteries over actions. Moreover, any violation of independence favors a lottery over profiles. Fixing a correlation structure, there is no expected utility based reason to reverse preference after mixing, as the mixture should not change the relative desirability of the two alternatives. However, mixing with a lottery over profiles may hedge against the possibility of a different correlation structure being realized. This hedging is potentially valuable in a lottery over profiles but not in one over actions, since the outcome of the former does not depend on the correlation.

A similar logic to NUI underlies axioms that appear in Gilboa et al. (2010), Dillenberger (2010), and Cerreia-Vioglio et al. (2015), with certain or unambiguous alternatives playing the role of uncorrelated profiles. The first assumes that the DM defaults to certainty: if an act is not objectively better than a lottery, then subjectively it should not be better. The other two assume that the DM prefers sure outcomes: if a lottery is not objectively better than a sure outcome, then a mixture of the lottery is better than a mixture of the sure outcome.

3.3. Basic axioms

The first two remaining axioms are standard.

Axiom 2 *(Weak order, WO). The preference relation $\succsim$ is complete and transitive.*

Axiom 3 *(Continuity, C). For any sequences $p_n, q_n \in \Delta\mathcal{F}$, if $p_n \to p$, $q_n \to q$, and $p_n \succsim q_n$ for all n, then $p \succsim q$.*

The DM can compare any pair of lotteries over action profiles, and her pairwise comparisons are sufficiently consistent with each other to form an ordering. Moreover, the ranking is sufficiently continuous. According to the topology introduced in Section 2, it is continuous in probability (and only probability) when the lotteries involve profiles that have non-constant payoffs, but it is continuous in the usual sense when restricting to lotteries over constant payoffs. In particular $(1, \langle x_n\rangle) \to (1, \langle x\rangle)$ whenever $x_n \to x$.

Misperception of correlation occurs whenever the DM violates Monotonicity. In this context, Monotonicity holds if

$$\left(p\big(\langle a_i\rangle_{i=1}^n\big), \langle \sum_{i=1}^n \rho(a_i,\omega)\rangle \right)_{p(\langle a_i\rangle)>0} \succsim \left(q\big(\langle a_i\rangle_{i=1}^n\big), \langle \sum_{i=1}^n \rho(a_i,\omega)\rangle \right)_{q(\langle a_i\rangle)>0}$$

for every $\omega \in \Omega$ implies that $p \succsim q$. EP suggests a weakening that allows the behavior in the thought experiment, yet rules out other violations that cannot be attributed to misperception of correlation, such as expressing $\langle 49, 50\rangle \succ \langle 100\rangle$.

Stating the axiom formally requires some notation. For any finite subset of actions $\{c_1, \ldots, c_n\} = C \subset \mathcal{A}$, the set of all *plausible realizations* of C equals

$$range(c_1) \times range(c_2) \times \ldots \times range(c_n).$$

I denote by x^{c_i} the value that $\vec{x}$ takes in the coordinate corresponding to c_i. A vector of outcomes $\vec{x}$ is a *plausible realization of the lotteries p and q* if it is a plausible realization of the set of all the actions included in profiles that are assigned positive probability by either p or q. For a plausible realization $\vec{x}$ of p and q, p *induces the lottery*

$$p_{\vec{x}} = \left(p\big(\langle a_i\rangle_{i=1}^n\big), \langle \sum_{i=1}^n x^{a_i}\rangle \right)_{p(\langle a_i\rangle)>0}$$

in which the constant action yielding the outcome $\sum_{i=1}^n x^{a_i}$ has probability $p\left(\langle a_i\rangle_{i=1}^n\right)$.

Axiom 4 *(Weak Monotonicity, WM). For any $p, q \in \Delta\mathcal{F}$, if $p_{\vec{x}} \succsim q_{\vec{x}}$ (respectively, $p_{\vec{x}} \succ q_{\vec{x}}$) for every plausible realization $\vec{x}$ of p and q, then $p \succsim q$ (respectively, $p \succ q$).*

In words, if the DM prefers the lottery induced by p to that induced by q for all of their plausible realizations, then she prefers p to q. To illustrate, consider three actions, a, b, c, and consider a DM who must choose between the profile containing a and b, denoted $\langle a, b\rangle$, and the one containing only c, denoted $\langle c\rangle$. In this case, if the minimum payoff of action a added to the minimum payoff of b exceeds the maximum payoff of c, then $\langle a, b\rangle$ is preferred to $\langle c\rangle$. The axiom thus implies that $\langle 100\rangle \succ \langle 49, 50\rangle \sim \langle 99\rangle$. However, it *does not* require that \$100 for sure is preferred to the combination of b_C and b_F in the thought experiment, since the latter could also yield \$200 (or \$0).

Finally, the DM makes comparisons between (lotteries over) actions without difficulty.

Axiom 5 *(Simple Monotonicity, SM). For any $p, q \in \Delta\mathcal{A}$, if $(p(\langle a\rangle), \rho(a,\omega))_{p(\langle a\rangle)>0} \succsim (q(\langle b\rangle), \rho(b,\omega))_{q(\langle b\rangle)>0}$ for all $\omega \in \Omega$, then $p \succsim q$, strictly whenever the preference is strict for each state.*

This is a standard Anscombe-Aumann monotonicity condition. It only applies to lotteries over profiles consisting of a single action. These profiles correspond to Savage acts, and lotteries over them to Anscombe-Aumann acts. Restricted to these objects the DM behaves as a standard expected utility maximizer.

3.4. Understanding and richness

The previous axioms are necessary and sufficient for the representation theorem. With the goal of identifying a "coarsest" state space and a unique set of beliefs, I introduce an assumption about what the DM understands. When a DM accurately perceives the correlations within a subset of actions, she rules out implausible realizations. Specifically, if a DM understands the connections between actions in the set C, she should disregard any plausible realization of p and q that fails to align the outcomes for actions in C as dictated by ρ and Ω. Formally, say that a plausible realization $\vec{x}$ is C*-synchronous* if there exists $\omega \in \Omega$ such that $x^a = \rho(a, \omega)$ for all $a \in C$.

Definition 1. *The preference $\succsim$* understands *$C \subseteq \mathcal{A}$ if for any $p, q \in \Delta\mathcal{F}$, $p \succsim q$ whenever $p_{\vec{x}} \succsim q_{\vec{x}}$ for all C-synchronous plausible realizations $\vec{x}$ of p and q.*

The definition builds on Weak Monotonicity. A DM who understands C only needs to consider C-synchronous plausible realizations, not all plausible realizations. That is, the logic behind the usual Monotonicity Axiom applies for actions in that subset. For the identification result, I require that every action belongs to a rich, understood set of actions. Richness is defined as follows.

Definition 2. *A set $B \subset \mathcal{A}$ is* rich *if for any function $f : \Omega \to X$, there exists $a \in B$ with $\rho(a, \omega) = f(\omega)$ for all $\omega \in \Omega$.*

A rich set contains an action that has an outcome agreeing with any given function from states to outcomes. That is, ρ maps a rich set of actions onto the set of all acts. This is a slightly stronger definition than that used by EP, which held that a set is rich if it contains every function measurable with respect to a given algebra. I can now state the assumption.

Assumption 1 *(Non-Singularity). Each $a \in \mathcal{A}$ belongs to a rich, $\succsim$-understood subset of actions.*

Non-Singularity is in the vein of the Savage (1954) assumption that the domain of preference contains all possible acts. It ensures that the choice domain is for any two DMs with different representations but the same understanding, there exists some pair of lotteries that the two rank differently. It is a joint assumption on both the preference $\succsim$ and the set $\mathcal{A}$.

4. Representation

4.1. Correlation concern representation

A DM who misperceives correlation perceives additional uncertainty beyond that captured by Ω. The DM is represented using a subjective state space rich enough to capture her perception of uncertainty. Formally, let $\Omega^{\mathcal{A}} = \prod_{a\in\mathcal{A}} \Omega$ be the Cartesian product where one copy of Ω is assigned to each action in $\mathcal{A}$, $\Sigma^{\mathcal{A}} = \otimes_{a\in\mathcal{A}} \sigma(a)$ be the product σ-algebra on $\Omega^{\mathcal{A}}$ where $\sigma(a)$ is the smallest algebra by which a is measurable, Ω^a be the copy of Ω assigned to $a \in \mathcal{A}$, and for any $\vec{\omega} \in \Omega^{\mathcal{A}}$, ω^a be the component of $\vec{\omega}$ in Ω^a. The DM is represented as if she considers events in the larger state space $\Omega^{\mathcal{A}}$. Every $\vec{\omega} \in \Omega^{\mathcal{A}}$ determines a joint realization of the outcomes of the corresponding actions, with a yield $\rho(a, \omega^a)$, b yielding $\rho(b, \omega^b)$, and so on. Hence, all additional uncertainty corresponds to the perception of correlations.

Definition 3. *A preference $\succsim$ has a* Correlation Concern Representation (CCR) *if there exists*

- *a continuous utility index $u : X \to \mathbb{R}$,*
- *a probability measure μ on Ω,*
- *and a closed, convex set of probability measures Π on $\left(\Omega^{\mathcal{A}}, \Sigma^{\mathcal{A}}\right)$ whose marginals agree with μ: for any $a \in \mathcal{A}$ and all $\pi \in \Pi$,*

$$\int_{\Omega^{\mathcal{A}}} u(\rho(a, \omega^a))d\pi = \int_{\Omega} u(\rho(a, \omega))d\mu$$

such that for any $p, q \in \Delta\mathcal{F}$, $p \succsim q \iff V(p) \geq V(q)$ where

$$V(p) = \min_{\pi\in\Pi} \int_{\Omega^{\mathcal{A}}} \left[\sum_{p(\langle a_i \rangle)>0} p(\langle a_i \rangle_{i=1}^n) u\left(\sum_{i=1}^n \rho\left(a_i, \omega^{a_i}\right) \right) \right] d\pi. \tag{1}$$

While she acts as if she maximizes expected utility with probability measure μ when comparing individual actions, she does not when comparing profiles. When evaluating them, she considers a set of possible joint distributions possible, represented by the set Π. A lottery over profiles is evaluated by its expectation according to the measure that minimizes the resulting expected utility, as in Gilboa and Schmeidler (1989). Consequently, she acts as if she is averse to uncertainty about correlations but not about the returns of individual actions.

Because the mapping between the objects of choice and the product state space is subjective and derived from behavior, the model captures correlation misperception endogenously. With an objective rather than subjective state space, a similar model could only capture aversion to *correctly perceived* correlations. Most notably, Epstein and Seo (2010, 2015) use non-expected utility models with an objective product state space as above to capture ambiguity about the relationship between distributions of different variables. While their agent is averse to uncertainty about correlation, that correlation is determined by the objectively given mapping from components of the state-space to the act's outcome.

The axioms introduced above are necessary and sufficient for the DM's behavior to be represented by a CCR.

Theorem 1. *The preference $\succsim$ satisfies Weak Order, Continuity, Weak Monotonicity, Simple Monotonicity, and Negative Uncorrelated Independence if and only if it has a correlation concern representation.*

The properties of the representation relate naturally to the axioms imposed. Simple Monotonicity and NUI imply that the DM acts a standard subjective expected utility maximizer when dealing with single action profiles. Weak Monotonicity allows the DM to misunderstand correlation between actions, as captured when the representation has a $\pi \in \Pi$ so that $\pi(\omega^a \neq \omega^b) > 0$ for some $a, b \in \mathcal{A}$. It nonetheless implies that the DM ignores "implausible" outcomes, so each probability measure in the set Π attaches zero probability to such outcomes, e.g. $\langle b_C, b_F \rangle$ yielding \$300 or −\$400, as captured by the subjective state space $\Omega^{\mathcal{A}}$. The representation captures recognition and dislike that the DM does not understand correlation by allowing Π to be a non-singleton set. As shown by Corollary 1, this follows from NUI.

Corollary 1 *(Ellis and Piccione (2017)). The preference $\succsim$ satisfies Weak Order, Continuity, Weak Monotonicity, Simple Monotonicity, and Independence if and only if it has a CCR where Π is a singleton.*

The result highlights that NUI implies a concern for correlation not captured by the independence axiom. A consequence of the violation of independence is a strict preference for randomization. Formally, there may exist lotteries so that $p \sim q$ but $\frac{1}{2}p + \frac{1}{2}q \succ q$. This occurs in much of the ambiguity aversion literature and underlies the logic of the uncertainty aversion axiom. In the CCR model, preference for randomization follows from NUI. Here, the randomization is explicit since the ordering of the horse-race and roulette wheel is reversed relative to the usual Fishburn (1970) formulation of the Anscombe-Aumann model.

While CCR provides a simple model equivalent to the axioms, the set of priors is not unique, nor is the state space. The next subsection develops a representation on a subjective state space with fewer dimensions for which the set of priors is unique. The CCR state space is canonical, in the sense that any joint distribution over the returns of actions can be expressed by a probability measure on it. The representation is thus an "as if" one, as opposed to a literal description of how the DM thinks about uncertainty. Other state spaces may make more sense in a given context, and these can be embedded in the CCR state space. For example, the thought experiment has one-dimensional uncertainty, namely the temperature, so the DM should realize that higher temperatures in one unit correspond to higher temperatures in another. An alternative representation could have one component representing the "true" temperature and other components representing the possible conversions. For example, the subjective state space $\Omega^* = \mathbb{R} \times \mathbb{F}$, where $\mathbb{F}$ is a set of increasing functions such as $\{x \mapsto 1.8x + 32, x \mapsto 2x + 30\}$, directly captures uncertainty about both the temperature and the conversion between units: the first component is the temperature in Celsius and the second is the how it converts to Fahrenheit. That is, the return of b_C equals $\rho(b_C, x)$ and the return of b_F equals $\rho(b_F, f(x))$ in state (x, f). This state space can be embedded into the appropriate coordinates of the CCR state space via the mapping $(x, f) \mapsto (x, f(x))$.

The formal proof of Theorem 1 can be found in the appendix. The following outlines the main arguments showing sufficiency of the axioms for the representation. NUI implies independence for lotteries over individual actions, which allows identification of a utility index and the marginal probability measure μ by following Anscombe and Aumann (1963). Each lottery over profiles is mapped to a real valued act on the product state space $\Omega^{\mathcal{A}}$. A utility value is assigned to each by setting the utility equal to that of an action equivalent, with Weak Monotonicity and Continuity insuring that one exists. By NUI, this utility function is homogeneous of degree one and superlinear, but is defined only on a convex subset of acts on $\Omega^{\mathcal{A}}$. The key step extends it to all bounded, measurable functions while maintaining the above properties. Arguments following Gilboa and Schmeidler (1989) then establish the result.

4.2. Identification and rich representation

This section proposes a more tractable special case of CCR with a more parsimonious state space that has a unique set of priors. The DM acts as if she undergoes the following procedure. First, she groups together certain actions that she understands as per Definition 1 into an *understanding class*. The classes are revealed from her choices. Then, she forms beliefs about the return within each class. Finally, she constructs a set of beliefs about the potential relationships across classes. When comparing any two profiles, she evaluates each according to the worst of its possible expected utilities from these beliefs. The main result of this section shows that this representation exists whenever the DM exhibits the behavior implied by the correlation concern representation as long as there exist sufficiently rich subsets of actions that the DM understands. Moreover, the components of the representation are unique for most utility indexes, unlike the CCR.

The understanding classes are contained in a *correlation cover* $\mathcal{U}$ *for* $\succsim$; formally, $\mathcal{U}$ is a collection of subsets of $\mathcal{A}$ so that $\mathcal{U}$ covers $\mathcal{A}$, each $C \in \mathcal{U}$ is understood by $\succsim$, and no $C \in \mathcal{U}$ contains a distinct $C' \in \mathcal{U}$. The correlation cover $\mathcal{U}$ is *rich* if each $C \in \mathcal{U}$ is rich. If for any $C' \in \mathcal{U}'$, there exists $C \in \mathcal{U}$ so that $C' \subset C$, then $\mathcal{U}$ is *coarser* than $\mathcal{U}'$. This mirrors the definition of when one partition is coarser than another (though the elements of $\mathcal{U}$ may intersect one another).

In principle, multiple correlation covers could represent the same preference. I focus on the *coarsest* rich correlation cover $\mathcal{U}$: a rich correlation cover for which no other is coarser than it. The coarsest rich correlation cover has the largest (in the sense of set inclusion) understanding classes, and so captures a maximal view of the DM's understanding of connections. Examples of correlation covers include $\{\{a\} : a \in \mathcal{A}\}$ and $\{\mathcal{A}\}$. The former is always a correlation cover but never rich, while the latter is the coarsest rich correlation cover provided that $\mathcal{A}$ is understood. EP show that there exists a unique coarsest rich correlation cover under non-singularity.

Beliefs are defined on the subjective state space $\Omega^{\mathcal{U}} = \prod_{C \in \mathcal{U}} \Omega$, with the C-coordinate denoted by Ω^C, endowed with the product σ-algebra $\Sigma^{\mathcal{U}} = \otimes_{C \in \mathcal{U}} \Sigma$. All actions in the same class are assigned to the same copy of Ω, so the possible joint realizations within a class are identical to the objective ones. Given a state $\vec{\omega} \in \Omega^{\mathcal{U}}$ and a class $C \in \mathcal{U}$, ω^C denotes the component of $\vec{\omega}$ assigned to C, and given an event $E \in \Sigma^{\mathcal{U}}$, E_C is the projection of E onto the C component.

Definition 4. *A preference* $\succsim$ *has a* Rich CCR $(u, \mu, \mathcal{U}, \Pi)$ *if*

- $u : X \to \mathbb{R}$ *is a continuous utility index,*
- μ *is a probability measure on* Ω*,*
- $\mathcal{U}$ *is the coarsest rich correlation cover for* $\succsim$*,*
- *and* Π *is a closed, convex set of probability measures on* $\left(\Omega^{\mathcal{U}}, \Sigma^{\mathcal{U}}\right)$ *where the marginal of every* $\pi \in \Pi$ *on every* $C \in \mathcal{U}$ *equals* μ

such that for any $p', q' \in \Delta\mathcal{F}$*,* $p' \succsim q' \iff V(p') \geq V(q')$ *where*

$$V(p) = \min_{\pi \in \Pi} \int_{\Omega^{\mathcal{U}}} \left[\sum_{p(\langle a_i \rangle) > 0} p(\langle a_i \rangle_{i=1}^n) u\left(\sum_{i=1}^n \rho\left(a_i, \omega^{C(a_i)}\right) \right) \right] d\pi \tag{2}$$

for any $C : \mathcal{A} \to \mathcal{U}$ with $a \in C(a)$ for all $a \in \mathcal{A}$.

If one assumes Non-Singularity, then the CCR axioms hold if and only if the DM has a Rich CCR.

Theorem 2. *Under Non-Singularity, the preference $\succsim$ satisfies Weak Order, Continuity, Weak Monotonicity, Simple Monotonicity, and Negative Uncorrelated Independence if and only if it has a Rich Correlation Concern Representation. Moreover, μ and $\mathcal{U}$ are unique, u is unique up to a positive affine transformation, and if u is not a polynomial, then Π is unique.*

A DM represented by a rich CCR behaves according to the same axioms as one who has a CCR. Non-singularity plays an analogous role to the usual assumption in decision theory that every act is conceivable and ranked by the DM. This allows construction of a richer representation in which the parameters are unique. As standard, the DM's risk preference is identified from her preference over lotteries. But here her perception of possible correlations is identified uniquely as well, unless her risk attitude is such that she does not care about it. This allows cleaner interpretation of the set of probability measures in the representation.

4.3. Other non-expected-utility models

Other non-EU models are also compatible with the rankings in Section 3.1. One could consider either a more general model, such as one derived from variational preferences (Maccheroni et al., 2006), or a non-nested model, such as one based on non-additive probabilities (Schmeidler, 1989) or smooth ambiguity (Klibanoff et al., 2005). The axioms, in particular NUI, justify the preference having a Correlation Concern Representation.

To illustrate, consider a version of the above model where, following Schmeidler (1989), a Choquet integral replaces the minimum over a set of measures, called *Correlation Concerned Choquet Expected Utility*, or *CCCEU* for short.[4] A capacity v on $\Omega^{\mathcal{U}}$ is a set function so that $v(\emptyset) = 0$, $v(\Omega^{\mathcal{U}}) = 1$, and $E \subset E'$ implies $v(E) \le v(E')$. Say that $\succsim$ is *CCCEU* if there exists a strictly increasing utility index u, a rich correlation cover $\mathcal{U}$, and a capacity v on $\Omega^{\mathcal{U}}$ so that for any $p, q \in \Delta\mathcal{F}$, $p \succsim q \iff W(p) \ge W(q)$ where

$$W(p) = \int_{\Omega^{\mathcal{U}}} \left[\sum_{p(\langle a_i \rangle)>0} p(\langle a_i \rangle_{i=1}^n) u \left(\sum_{i=1}^n \rho\left(a_i, \omega^{C(a_i)}\right) \right) \right] dv \tag{3}$$

for any C as in Definition 4 and the integration is a la Choquet.

CCCEU can accommodate the behavior in the thought experiment when there are two understanding classes, $\mathcal{U} = \{C, F\}$, the bets in Celsius belong to C, and the bets in Fahrenheit belong to F. There are four relevant events: $E_1 = \{\vec{\omega} : \omega^C < k \& \omega^F \ge k\}$, $E_2 = \{\vec{\omega} : \omega^C < k \& \omega^F < k\}$, $E_3 = \{\vec{\omega} : \omega^C \ge k \& \omega^F \ge k\}$, and $E_4 = \{\vec{\omega} : \omega^C \ge k \& \omega^F < k\}$. One can find a capacity v and a utility function u that explain the choices; for instance, $u(x) = x$ for all outcomes and $v(E_i) = \frac{1}{4}$, $v(E_i \cup E_j) = \frac{1}{3}$ and $v(E_i \cup E_j \cup E_k) = \frac{2}{3}$ for any distinct $i, j, k \in \{1, 2, 3, 4\}$.

However, a CCCEU $\succsim$ that satisfies NUI cannot explain these choices (provided that $\succsim$ understands all actions expressed in the same temperature scale).

Proposition 1. *Under non-singularity, a CCCEU preference relation $\succsim$ that satisfies NUI cannot exhibit the choices in the thought experiment. Moreover, if u is not a polynomial, then v is additive.*

The result, along with the example above, establishes that CCR and CCCEU are non-nested models. While the above example shows that CCCEU can explain the thought experiment when u is linear, CCR cannot: risk neutral agents do not care about correlation, so being uncertain about correlation does not affect behavior. At the same time, CCR satisfies NUI whiles CEU satisfies it only if it also satisfies independence and thus exhibits no aversion to uncertainty about correlation. Intuitively, NUI implies that the DM acts as a standard subjective expected utility maximizer when prospects do not involve correlation, and that mixing with uncorrelated prospects preserves preference. These put enough additivity requirements on the capacity to reduce it to a probability measure. I leave a full behavioral characterization of CCCEU, as well as other models, to future work.

5. Special case and comparatives

This section begins by exploring a special case of the model where an agent considers the true correlation structure but is nonetheless concerned that there may be a different one. Then, a notion of more concerned about correlation is introduced. One agent is more concerned about correlation than another if she understands fewer connections and whenever she prefers an action profile to an individual action, the other also does. The former is represented by a larger set of priors, representing more possible ways in which actions are related to one another.

Throughout this section, I assume without explicitly mentioning that every utility index is not a polynomial and that all correlation covers are countable or finite.

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[4] Thanks to a referee for suggesting this.

5.1. Concern about correlation

Consider a DM who prefers to avoid "complex" profiles whose evaluation requires her to compute correlations in favor "simple" profiles that would yield the same outcome if all correlations are understood. In its simplest manifestation, this DM expresses

$$\langle a\rangle \succsim \langle b,c\rangle$$

whenever $\rho(a,\omega)=\rho(b,\omega)+\rho(c,\omega)$ for every state ω. If the DM understands the correlation between b and c, then the two alternatives always yield the same outcome and she would be indifferent. However, the single-action profile has no exposure to correlation whereas the multi-action profile might. A DM who does not understand the correlation between b and c may strictly prefer to choose a and thereby avoid this exposure.

I capture this in general with the following axiom.

Axiom *(No False Hedging). For any $p\in\Delta\mathcal{A}$ and $q\in\Delta\mathcal{F}$, if*

$$(p(\langle a\rangle),\langle\rho(a,\omega)\rangle)_{p(\langle a\rangle)>0}\succsim\left(q(\langle a_i\rangle),\sum_i\rho(a_i,\omega)\right)_{q(\langle a_i\rangle)>0}$$

for all $\omega\in\Omega$, then $p\succsim q$.

No False Hedging requires that the DM opts for a lottery over actions over any lottery over action profiles whenever the lottery over actions yields at least as good an outcome in every state of the world according to the true structure. To interpret the axiom, note that the lottery p involves no proper profiles, only individual actions. In this sense, p is less complex than q. Moreover, p yields at least as good an outcome as q according to the true model. The axiom says that these two advantages are sufficient for the DM to prefer p to q. Note that Monotonicity implies No False Hedging which in turn implies Simple Monotonicity.

A DM with a rich CCR satisfies No False Hedging if and only if the true correlation structure belongs to her set of possible priors.

Proposition 2. *Let $\succsim$ have a rich CCR $(u,\mu,\mathcal{U},\Pi)$. Then, $\succsim$ satisfies No False Hedging if and only if there exists $\pi'\in\Pi$ so that $\pi'(\{\vec{\omega}:\omega_C=\omega^*\text{ for all }C\in\mathcal{U}\})=\mu(\{\omega^*\})$ for every $\omega^*\in\Omega$.*

The claimed measure π' can be thought of as the correctly specified probability distribution. It attaches zero probability to realizations of actions not possible according to the objective state space. Therefore, the DM takes into account the true correlation structure. No False Hedging cannot be exhibited by a DM who takes a probabilistic approach to correlation unless she correctly perceives every lottery. Such a DM necessarily overvalues some misunderstood profiles while overvaluing others. For instance, consider a risk averse DM who misunderstands the connection between two stocks a and b. She overvalues the profile $\langle a,b\rangle$ whenever she underestimates their correlation, and undervalues it when she over estimates the correlation. But if she overestimates the correlation between a and a long position on stock b, she underestimates the correlation between a and a short position on b. Consequently, she overvalues one if and only if she undervalues the other.

5.2. Comparatives

The DM's perception and attitude towards the correlation between actions is captured by two objects, $\mathcal{U}$ and Π. In this section I discuss the behavior associated with changes in these parameters. In particular, I provide behavioral definitions of when DM1 understands more connections than DM2 and of when DM1 is more concerned about correlation than DM2. The first definition implies that DM2's correlation cover is coarser than that of DM1, and the second that the set of probability measures entertained is larger in the sense of set inclusion when projected onto a suitable space.

I first provide a definition of when one DM understands more connections than another.

Definition 5. *Say that $\succsim_1$* understands more connections than *$\succsim_2$ if, for any rich $B\subseteq\mathcal{A}$, $\succsim_2$ understands B implies that $\succsim_1$ understands B.*

This definition relates to the representation in the natural way.

Proposition 3. *If each $\succsim_i$ has a Rich CCR $(u_i,\mu_i,\mathcal{U}_i,\Pi_i)$, then $\succsim_1$ understands more connections than $\succsim_2$ if and only if $\mathcal{U}_1$ is coarser than $\mathcal{U}_2$.*

When DM1 understands more connections than DM2, her correlation cover is coarser (as defined in Section 4.2). A DM who satisfies monotonicity and so is represented with $\mathcal{U}=\{\mathcal{A}\}$ thus understands more connections than any other DM. Note that the comparison does not imply anything about the relationship between the two DMs' tastes or beliefs.

When the definition holds, $\succsim_1$ can be represented with the same correlation cover as $\succsim_2$, namely $\mathcal{U}_2$. More formally, if a rich correlation cover $\mathcal{U}$ is coarser than $\mathcal{U}'$ and $\succsim$ has a Rich CCR $(u,\mu,\mathcal{U},\Pi)$, then there exists a unique set of probability measures Π'

on $\left(\Omega^{\mathcal{U}'}, \Sigma^{\mathcal{U}'}\right)$ so that $\succsim$ also a representation analogous to Definition 4 with $\mathcal{U}'$ replacing $\mathcal{U}$ and Π' replacing Π. Call such a Π' the projection of Π onto $\Omega^{\mathcal{U}'}$. This follows from the arguments in Theorem 2, which apply to any rich correlation cover for $\succsim$, not just the coarsest one.

I now provide my definition of more concerned about correlation.

Definition 6. *Say that $\succsim_2$ is* more concerned about correlation than *$\succsim_1$ if for any $p \in \Delta\mathcal{F}, q \in \Delta\mathcal{A}$, $p \succsim_2 q$ ($p \succ_2 q$) implies that $p \succsim_1 q$ ($p \succ_1 q$) and $\succsim_1$ understands more connections than $\succsim_2$.*

Consider two DMs, DM1 and DM2, and profiles $\langle a, b\rangle, \langle c\rangle$. As above, evaluating $\langle a, b\rangle$ requires taking a position on how a and b are related to each other, while evaluating $\langle c\rangle$ does not. If DM2 is more concerned about correlation than DM1 is, then whenever she prefers $\langle a, b\rangle$ to $\langle c\rangle$, so should DM1. The above extends this logic to lotteries and formalizes it. The definition adds one condition to the above, namely that DM1 must understand more connections than DM2. That is, whenever DM1 views a profile as involving unknown correlations, so does DM2. Adding it to the definition allows us to establish the following characterization.

Proposition 4. *If each $\succsim_i$ has a Rich CCR $(u_i, \mu_i, \mathcal{U}_i, \Pi_i)$, then $\succsim_2$ is more concerned about correlation than $\succsim_1$ if and only if $\mu_1 = \mu_2$, $u_1 = \alpha u_2 + \beta$ for some $\alpha > 0$ and $\beta \in \mathbb{R}$, $\mathcal{U}_1$ is coarser than $\mathcal{U}_2$, and $\Pi_1' \subset \Pi_2$ where Π_1' is the projection of Π_1 onto $\Omega^{\mathcal{U}_2}$.*

When DM1 is more concerned about correlation than DM2, the former perceives less uncertainty about correlation. Her set of probability measures is smaller in the sense of set-inclusion after they are projected onto the same state space. This means that every probabilistic relationship between actions considered by DM1 is also taken into account by DM2. Nevertheless, the two rank individual actions in the same way, so they have the same tastes and the same priors about actions.

Notice that an increase in concern for correlation leads to both a finer correlation cover and a larger set of priors. The former is necessary for the latter. Otherwise, the support of the larger set of priors would not include the support of the smaller set. One could strengthen the comparative static to require that both preferences understand the same sets of actions. Then, they would have the same correlation cover and one would have a larger set of priors.

A preference with a Rich CCR satisfies No False Hedging if and only if it is more concerned about correlation than some preference with a Rich CCR that satisfies Monotonicity. Intuitively, a DM satisfies Monotonicity only if she understands all connections or is completely unconcerned with correlation, e.g., because she is risk neutral. Such a DM trivially has the coarsest possible correlation cover. By Proposition 4, the projection of the measure μ belongs to the set representing the more concerned preference.

Declaration of competing interest

Andrew Ellis declares that he does not have any relevant interests or relationships that could inappropriately influence his work.

Appendix A. Proofs

Throughout the appendix, I sometimes write $p\alpha q$ instead of $\alpha p + (1-\alpha)q$ for two lotteries p, q and $a(\omega)$ instead of $\rho(a, \omega)$.

Lemma 1. *Under Axioms 2 and 3, the set $\{\alpha \in [0,1] : \alpha p + (1-\alpha)q \succsim \alpha p' + (1-\alpha)q'\}$ is closed for all $p, q, p', q' \in \Delta\mathcal{F}$.*

Proof. Follows from observing that $\alpha_n p + (1-\alpha_n)q \to \alpha p + (1-\alpha)q$ according to the weak* topology whenever $\alpha_n \to \alpha$. See e.g. Theorem 15.3 of Aliprantis and Border (2006). □

Lemma 2. *Under Axioms 1-4, for any $p, q, r \in \Delta\mathcal{F}$ and $\alpha \in (0,1]$, if $r \in \Delta\mathcal{A}$, then $p \succsim q \iff \alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.*

Proof. Fix any $r \in \Delta\mathcal{A}$ and $\alpha \in (0,1]$.

First, $p \succ q \implies \alpha p + (1-\alpha)r \succ \alpha q + (1-\alpha)r$. If not, then $p \succ q$ and $\alpha q + (1-\alpha)r \succsim \alpha p + (1-\alpha)r$ for some p, q. Axioms WO, C, and WM imply there exists $p' \in \Delta\mathcal{A}$ with $p' \sim p$. By NUI and WO, $q\alpha r \succsim p\alpha r \succsim p'\alpha r$. Let

$$\tau = \sup\{\beta \in [0,1] : q\beta r \succsim p'\beta r\}.$$

By Lemma 1, $q\tau r \succsim p'\tau r$. Since $p'\tau r \in \Delta\mathcal{A}$, $(q\tau r)\beta q \succsim (p'\tau r)\beta q$ and $(q\tau r)\beta p' \succsim (p'\tau r)\beta p'$ by NUI for any $\beta \in (0,1)$.[5]

Observe $(p'\tau r)\frac{1}{1+\tau}q = (q\tau r)\frac{1}{1+\tau}p'$. By WO,

$$q\frac{2\tau}{1+\tau}r = (q\tau r)\frac{1}{1+\tau}q \succsim (p'\tau r)\frac{1}{1+\tau}q = (q\tau r)\frac{1}{1+\tau}p' \succsim (p'\tau r)\frac{1}{1+\tau}p' = p'\frac{2\tau}{1+\tau}r.$$

Thus $\tau \geq 2\tau/(1+\tau)$, which can hold only if $\tau = 1$ or $\tau = 0$. Since $\tau \geq \alpha > 0$, $\tau = 1$. But then $q = q\tau r \succsim p'\tau r = p'$ by Lemma 1, implying that $q \succsim p$ since $p \sim p'$.

[5] The remainder of the argument is based on one that appears in Shapley and Baucells (1998).

Now, $p \sim q \implies \alpha p + (1-\alpha)r \gtrsim \alpha q + (1-\alpha)r$.[6] Fix any $p \sim q$, and pick a lottery $x \in \Delta X$ s.t. $x \succ p_{\vec{y}}$ for all plausible realizations $\vec{y}$ of p. For all $\epsilon \in (0,1]$, $p\epsilon x \succ q$ by WM and WO, and so by the above, $(p\epsilon x)\alpha r \succ q\alpha r$. By C and sending ϵ to 0, $p\alpha r \gtrsim q\alpha r$ as well.

Now, one has that $p \succ q \implies p\alpha r \succ q\alpha r$ and $p \gtrsim q \implies p\alpha r \gtrsim q\alpha r$. The second, combined with the contrapositive of the first and completeness, is that $p \gtrsim q \iff p\alpha r \gtrsim q\alpha r$. This completes the proof. □

Lemma 3. *Under Axioms 1-4, if $p, q \in \Delta A$ and $r \in \Delta F$, then*

$$p \gtrsim q \iff \alpha p + (1-\alpha)r \gtrsim \alpha q + (1-\alpha)r.$$

Proof. Pick $r' \in \Delta A$ with $r' \sim r$; this exists by WO, C, and WM. Observe $p\alpha r \sim p\alpha r'$ and $q\alpha r' \sim q\alpha r$ by Lemma 2. Then, $p \gtrsim q \iff p\alpha r' \gtrsim q\alpha r'$ also by Lemma 2. By WO, $p\alpha r' \gtrsim q\alpha r' \iff p\alpha r \gtrsim q\alpha r$. □

Lemma 4. *Under Axioms 1-4, there exists a continuous, convex ranged $u : X \to \mathbb{R}$ so that for any $p, q \in \Delta X$, $p \gtrsim q$ if and only if $\sum_{p(\langle x \rangle)>0} p(x)u(x) \geq \sum_{q(\langle x \rangle)>0} q(x)u(x)$.*

This follows from the above lemmas and Grandmont (1972).

Proof of Theorem 1. Necessity is trivial. For sufficiency, assume that $\gtrsim$ satisfies the axioms. Let u be utility index shown to exist by Lemma 4. Normalize so that $range(u) \supset [-1,1]$, and that $u(0) = 0$. Recall that $B_0(Y, \Sigma)$ is the simple, real-valued, Σ-measurable functions on the set Y. For any $p \in \Delta\mathcal{F}$ define $f_p \in B_0(\Omega^{\mathcal{A}}, \otimes_{a\in\mathcal{A}}\sigma(a))$ so that

$$f_p = \vec{\omega} \mapsto \sum_{p(\langle a_i \rangle)>0} p\big(\langle a_i \rangle_{i=1}^n\big) u\left(\sum_{i=1}^n a_i(\omega^{a_i})\right).$$

By WM, $f_p \geq f_q$ (resp., $f_p \gg f_q$) implies that $p \gtrsim q$ (resp., $p \succ q$).

Define $W = \{f_p : p \in \Delta\mathcal{F}\}$, noting that W is convex. For ϕ in W, define $\tilde{I}(\phi) = \int u(x)dr$ for some $p \in \Delta\mathcal{F}$ s.t. $f_p = \phi$ and a lottery r over X satisfying $r \sim p$. Such an r exists for every p by Weak Monotonicity, Completeness, and Continuity, so $\tilde{I}$ is well-defined. Denote $f_{(1,\langle 0 \rangle)} = 0$.

The function $\tilde{I}$ has the following properties for any $\alpha \in (0,1]$, $\phi = f_q$, $\psi = f_r$ and $g = f_p$ with $p \in \Delta\mathcal{A}$, $q, r \in \Delta\mathcal{F}$, $x \in \Delta X$ with $u(x) = x$ and $f_x = x$.

- $\tilde{I}(\cdot)$ is normalized: $\tilde{I}(x) = x$ by construction. Based on this, I abuse notation slightly by also identifying $\tilde{I}(\theta)$ with a lottery over X that yields utility $\tilde{I}(\theta)$ for any $\theta \in W$.
- $\tilde{I}(\cdot)$ is monotone: $\phi \geq \psi$ implies $\tilde{I}(\phi) \geq \tilde{I}(\psi)$. Follows from WM.
- $\tilde{I}(\cdot)$ is action invariant: $\tilde{I}(\alpha\psi + (1-\alpha)g) = \alpha\tilde{I}(\psi) + (1-\alpha)\tilde{I}(g)$. To see this, note that by Weak Monotonicity and Continuity, there exists $z, z' \in \Delta X$ such that $r \sim z$ and $p \sim z'$; for this z, $\int u(x)dz = \tilde{I}(\psi)$. By Lemma 3, $r\alpha p \sim z\alpha p \sim z\alpha z'$. By transitivity, $r\alpha p \sim z\alpha z'$. Since $z\alpha z' \in \Delta\mathcal{A}$ and $f_{r\alpha p} = \alpha\psi + (1-\alpha)g$, $\tilde{I}(\alpha\psi + (1-\alpha)g) = \tilde{I}(f_{z\alpha z'}) = \alpha\tilde{I}(f_z) + (1-\alpha)\tilde{I}(f_{z'}) = \alpha\tilde{I}(\phi) + (1-\alpha)\tilde{I}(g)$.
- $\tilde{I}(\cdot)$ is concave: $\tilde{I}(\alpha\phi + (1-\alpha)\psi) \geq \alpha\tilde{I}(\phi) + (1-\alpha)\tilde{I}(\psi)$. To see this, note $q \sim \tilde{I}(\psi)$. By NUI, $r\alpha q \gtrsim r\alpha\tilde{I}(\psi)$. Since $\tilde{I}$ is action invariant and normalized,

$$\tilde{I}(\alpha\phi + (1-\alpha)\tilde{I}(\psi)) = \alpha\tilde{I}(\phi) + (1-\alpha)\tilde{I}(\psi).$$

- $\tilde{I}(\cdot)$ is Homogeneous of Degree 1: $\tilde{I}(\alpha\psi) = \alpha\tilde{I}(\psi)$. This follows from action invariant and normalized.
- $\tilde{I}(\cdot)$ is supnorm continuous. Suppose $\phi^n \to \phi$ for some sequence ϕ^n and ϕ that belong to W. Let $x^n = \max_{\vec{\omega}}[\phi^n(\vec{\omega}) - \phi(\vec{\omega})]$ and $y^n = \max_{\vec{\omega}}[\phi^n(\vec{\omega}) - \phi(\vec{\omega})]$. Pick $\kappa, \kappa' \in \Delta X$ with $u(\kappa) = 1$ and $u(\kappa') = -1$. For n large enough that $|x_n|, |y_n| < 1$,

$$\tilde{I}\left(y_n[\frac{1}{2}\phi + \frac{1}{2}f_{\kappa'}] + (1-y_n)[\frac{1}{2}\phi + \frac{1}{2}0]\right) \leq \tilde{I}\left(\frac{1}{2}\phi^n + \frac{1}{2}0\right)$$

and

$$\tilde{I}\left(\frac{1}{2}\phi^n + \frac{1}{2}0\right) \leq \tilde{I}\left(x_n[\frac{1}{2}\phi + \frac{1}{2}f_{\kappa'}] + (1-x_n)[\frac{1}{2}\phi + \frac{1}{2}0]\right)$$

since $\tilde{I}$ is monotone. By continuity and that

$$z_n\left[\frac{1}{2}q + \frac{1}{2}\kappa''\right] + (1-z_n)\left[\frac{1}{2}q + \frac{1}{2}0\right] \to \frac{1}{2}q + \frac{1}{2}0$$

for any $\kappa'' \in \Delta X$ in the weak* topology whenever $z_n \to 0$, $\tilde{I}(\phi\frac{1}{2}0) = \lim \tilde{I}(\phi^n\frac{1}{2}0)$. Action invariance of $\tilde{I}$ establishes the result.

[6] A symmetric argument obtains indifference.

Given the above and that $0 \in W$, extend $\tilde{I}$ to the cone generated by W (which is simply denoted by $\tilde{I}$ and W for convenience) using the identity that $\tilde{I}(\alpha\phi) = \alpha\tilde{I}(\phi)$. Clearly, all the above properties are maintained. The set W is a convex cone contained in the vector space $B(\Omega^{\mathcal{A}}, \otimes_{a\in\mathcal{A}}\sigma(a)) = W^*$, the bounded, $\otimes_{a\in\mathcal{A}}\sigma(a)$-measurable functions. Extend $\tilde{I}$ to W^* as follows.

For any $x \in W^*$, define

$$I(x) = \sup\left\{\tilde{I}(w) : x \geq w,\ w \in W\right\}.$$

The function I inherits the following properties from $\tilde{I}$:

- $\phi \in W$ implies $I(\phi) = \tilde{I}(\phi)$: First, $\phi \in W$ and $\phi \leq \phi$ immediately imply that $I(\phi) \geq \tilde{I}(\phi)$. Second, $w \leq \phi$ immediately yields $\tilde{I}(w) \leq \tilde{I}(\phi)$ by monotonicity of $\tilde{I}$, so $\tilde{I}(\phi) \geq I(\phi)$ also.
- I is concave: fix $\phi, \psi \in W^*$ and $\lambda \in (0,1)$. For any $\epsilon > 0$, there exist $w_1, w_2 \in W$ with $w_1 \leq \phi$ and $w_2 \leq \psi$ such that $\tilde{I}(w_1) > I(\phi) - \epsilon/2$ and $\tilde{I}(w_2) > I(\psi) - \epsilon/2$. Now, $\lambda w_1 + (1-\lambda)w_2 \leq \lambda\phi + (1-\lambda)\psi$. Then, $I(\lambda\phi + (1-\lambda)\psi) \geq \tilde{I}(\lambda w_1 + (1-\lambda)w_2) \geq \lambda\tilde{I}(w_1) + (1-\lambda)\tilde{I}(w_2) > \lambda I(\phi) + (1-\lambda)I(\psi) - \epsilon$. Letting ϵ go to zero establishes the result.
- I is Monotone and $I(x) < \infty$ for all x: Monotone is trivial. Since $y = \min_\omega x(\omega) \leq x$ belongs to W, $I(x) > \tilde{I}(y) = y$. Letting $z = \max_\omega x(\omega)$, for any $w \in W$ with $x \geq w$, $z \geq w$. Thus $z = \tilde{I}(z) \geq \tilde{I}(w)$ by monotonicity; hence $I(x) \leq z$.
- I is Homogeneous of Degree 1: fix $x \in W^*$ and $\alpha > 0$. If $\alpha I(x) > I(\alpha x)$, then there is $w \in W$ such that $x \geq w$ and $\alpha\tilde{I}(w) > I(\alpha x)$. Observe that $\alpha w \leq \alpha x$, $\alpha w \in W$ and so $\tilde{I}(\alpha w) = \alpha\tilde{I}(w)$, immediately leading to a contradiction; reversing the argument leads to a contradiction if $\alpha I(x) < I(\alpha x)$.
- I is action invariant: $I(\alpha\phi + (1-\alpha)g) = \alpha I(\phi) + (1-\alpha)I(g)$ when $g = f_p$ for $p \in \Delta\mathcal{A}$. Notice that $w \in W \iff \alpha w + (1-\alpha)g \in W$, and that if $\phi \geq w$, then $\alpha\phi + (1-\alpha)g \geq \alpha w + (1-\alpha)g$. The rest follows from $\tilde{I}$ being action invariant.
- I is supnorm continuous: Suppose not, so $x_n \to x$ in supnorm and, first, $\liminf I(x_n) < I(x)$. There is $\epsilon > 0$ and a sub-sequence, WLOG the whole sequence, such that $I(x_n) + \epsilon < I(x)$ for all n. By definition, there exists $x \geq w \in W$ such that $\tilde{I}(w) \geq I(x) - \epsilon/3$. Also, for n large enough, $x_n \geq x - \epsilon/3$ in every state. Thus $x_n \geq w - \epsilon/3$, but then $I(x_n) \geq \tilde{I}(w - \epsilon/3) = \tilde{I}(w) - \epsilon/3 \geq I(x) - 2\epsilon/3$, a contradiction. Second, if $\limsup I(x_n) > I(x)$, then there exists $\epsilon > 0$ a sub-sequence, WLOG the whole sequence, such that $I(x_n) > I(x) + \epsilon$ for all x_n. Pick n such that $x \geq x_n - \epsilon/3$. There exist $x_n \geq w \in W$ such that $\tilde{I}(w) \geq I(x_n) - \epsilon/3$. Then $x \geq w - \epsilon/3$ and $I(x) \geq \tilde{I}(w - \epsilon/3) \geq I(x_n) - 2\epsilon/3 > I(x)$, a contradiction.

To finalize the proof, I adapt the Gilboa and Schmeidler (1989) (GS) arguments to construct a set of priors representing the preference as in GS but with the additional property that $\int f_{(1,\langle a\rangle)} d\pi = \int f_{(1,\langle a\rangle)} d\pi'$ for all $\pi, \pi' \in \Pi$ for any $a \in \mathcal{A}$. Let W_A be the cone generated by $\{f_p : p \in \Delta\mathcal{A}\}$.

For any $\phi \in W^*$ with $I(\phi) > 0$, define $D_1 = \{\psi \in W^* : I(\psi) > 1\}$ and

$$D_2 = co(\{\psi \in W^* : \psi \leq a,\ I(a) = 1, \text{ and } a \in W_A\} \bigcup \{\psi \in W^* : \psi \leq \phi/I(\phi)\}).$$

To apply the GS arguments, I show that $D_1 \bigcap D_2 = \emptyset$. By I action invariant and convexity of the constituent sets, any $d_2 \in D_2$ equals $\alpha a_1 + (1-\alpha)a_2$ where $a_1 \leq a$ for $a \in W_A$ and $I(a) = 1$, $a_2 \leq \phi/I(\phi)$ and $\alpha \in [0,1]$. Then, $I(d_2) \leq I(\alpha a + (1-\alpha)a_2)$ by WM, which equals $\alpha I(a) + (1-\alpha)I(a_2)$ by action invariant, which is less than

$$\alpha I(a) + (1-\alpha)I(\phi/I(\phi)) = 1$$

by WM. Conclude $I(d_2) \leq 1$ for any $d_2 \in D_2$ and hence $D_1 \bigcap D_2 = \emptyset$. Moreover, note $1 \in D_2$ and 1 in $cl(D_1)$. A separating hyperplanes argument gives a finitely additive measure π_ϕ such that $\int d_1 d\pi_\phi \geq 1 \geq \int d_2 d\pi_\phi$ for all $d_1 \in D_1$ and $d_2 \in D_2$.

Applying the GS arguments shows that π_ϕ is a finitely additive probability measure, $I(\phi) = \int \phi d\pi_\phi$, and $\int \psi d\pi_\phi \geq I(\psi)$ for all $\psi \in W^*$. This π_ϕ must have $\int a d\pi_\phi = I(a)$ for all $a \in W_A$, since $I(a/I(a)) = 1$ implies that $a/I(a) \in D_2$ and $1 \geq \int a/I(a) d\pi_\phi$. Since Ω is finite and each $\phi \in W^*$ is measurable with respect to finite cylinder events, I can take each π_ϕ to be countably additive by the Kolmogorov Extension Theorem (Aliprantis and Border, 2006, Theorem 15.26; see the arguments following Lemma 3 of EP). As in GS, for $\Pi = \bar{co}\{\pi_\phi : I(\phi) > 0\}$, $p \succsim q$ if and only if

$$\min_{\pi\in\Pi} \int f_p d\pi \geq \min_{\pi\in\Pi} \int f_q d\pi.$$

Since

$$\min_{\pi\in\Pi} \int f_p d\pi = \min_{\pi\in\Pi} \int_{\Omega^{\mathcal{A}}} \mathbb{E}_{p(\langle a_i\rangle)} \left[u\left(\sum_{i=1}^{n} a_i(\omega^{a_i}) \right) \right] d\pi = V(p),$$

the function V represents $\succsim$.

To complete the proof, I show existence of a $\mu \in \Delta\Omega$ so that

$$V((1,\langle a\rangle)) = \int a d\mu.$$

For any $p \in \Delta\mathcal{A}$, let $h_p \in \mathbb{R}^\Omega$ satisfy $h_p(\omega) = \sum_{p(\langle a\rangle)>0} u(a(\omega))p(\langle a\rangle)$. By SM, $h_p \geq h_q \implies p \succsim q$. By Lemma 3, $\succsim$ satisfies independence when restricted to $\Delta\mathcal{A}$. By the usual arguments, for any $p, q \in \Delta\mathcal{A}$, there is a μ so that $p \succsim q$ if and only if $\int h_p d\mu \geq \int h_q d\mu$. Since

V is an affine function on $\Delta\mathcal{A}$ that also represents $\succsim$, there exists $\alpha > 0$ and β so that $V(p) = \alpha \int h_p d\mu + \beta$ for all $p \in \Delta\mathcal{A}$. Moreover, $\alpha = 1$ and $\beta = 0$ since V and $\int h_p d\mu$ agree on lotteries over constants, completing the proof. □

Proof of Theorem 2. By Proposition 3 of EP, a unique coarsest correlation cover $\mathcal{U}$ exists. For any $C : \mathcal{A} \to \mathcal{U}$, let $f_p^C \in B_0(\Omega^{\mathcal{U}}, \Sigma^{\mathcal{U}})$ be defined by

$$f_p^C = \sum_{p(\langle a_i \rangle)>0} p\big(\langle a_i \rangle_{i=1}^n\big) u\left(\sum_{i=1}^n a_i \left(\omega^{C(a_i)}\right)\right).$$

The result follows from the same arguments as in Theorem 1 if $f_p^{C_1} \geq f_q^{C_2}$ implies that $p \succsim q$ for any $C_1, C_2 : \mathcal{A} \to \mathcal{U}$ with $a \in C_i(a)$ for $i = 1, 2$.

By Theorem 1, $\succsim$ has a CCR (u, μ, Π) where $u(0) = 0$. Let $V(p)$ be the utility of $p \in \Delta\mathcal{F}$ according to this representation, and let f_p be as in Theorem 1. If u is a linear function, then for any $p \in \Delta\mathcal{F}$

$$V(p) = \min_{\pi} \int \sum p(\langle a_i \rangle) \sum_i a_i(\omega^{a_i}) d\pi = \sum p(\langle a_i \rangle) \sum_i \int a_i(\omega) d\mu,$$

because $\int a(\omega^a) d\pi = \int a(\omega) d\mu$ for all $\pi \in \Pi$ and $a \in \mathcal{A}$. Therefore, Monotonicity holds, $\mathcal{A}$ is understood, and $\mathcal{U} = \{\mathcal{A}\}$. The result follows immediately from the usual Anscombe-Aumann Theorem. Otherwise, there exist $x, y \in X$ such that $u(x + y) \neq u(x) + u(y)$.

Write $\Omega = \{1, \dots, K\}$ and Na for N copies of the action a, where N is a positive integer. For $x \in X$ and $B \in \mathcal{U}$ choose an action $\beta_x^{B,k} \in B$ so that $\beta_x^{B,k}(\omega)$ equals x if $\omega = k$ and 0 otherwise and define the corresponding event

$$\mathcal{E}^{B,k,x} = \{\vec{\omega} \in \Omega^{\mathcal{A}} : \omega^{\beta_x^{B,k}} \in E_B^k\}.$$

Note such actions exist because B is rich. Let $\Theta_\epsilon = (-\epsilon, 0) \bigcup (0, \epsilon)$, i.e. an open interval of size ϵ around 0 that excludes 0.

Lemma 5. *Suppose that there exist $x, y \in X$ such that $u(x + y) \neq u(x) + u(y)$. There exists $\epsilon > 0$ such that for every collection $\{\beta_{x_1}^{B_1,k_1}, \dots, \beta_{x_n}^{B_n,k_n}\}$ with $x_i \in \Theta_\epsilon$, $B_i \in \mathcal{U}$, and $k_i \in \Omega$ for each i, and any $p \in \Delta\mathcal{F}$, there exists*

$$\pi_0 \in \arg\min_{\pi\in\Pi} \int f_p d\pi$$

such that

$$\pi_0(\mathcal{E}^{B_i,k_i,x_i}) = \mu(k_i) \tag{4}$$

$$k_i \neq k_j,\ B_i = B_j \implies \pi_0(\mathcal{E}^{B_i,k_i,x_i} \bigcap \mathcal{E}^{B_j,k_j,x_j}) = 0 \tag{5}$$

$$\text{and } k_i = k_j,\ B_i = B_j \implies \pi_0\left(\mathcal{E}^{B_i,k_i,x_i} \bigcap \mathcal{E}^{B_j,k_j,x_j}\right) = \mu(k_i) \tag{6}$$

for all distinct $i, j \in \{1, \dots, K\}$ and every $B \in \mathcal{U}$.

In words, for any p, there is minimizing probability measure π_0 with the following properties. Eq (4) requires that the marginals of π_0 agree with μ. Eq (5) implies that the DM believes it impossible that bets on distinct states in the same class pay off jointly. Eq (6) implies that if one bet on state i pays off, then all bets on state i in the same class pay off. In sum, within the same understanding class, all the bets on one and only one of the elements of its finest partition pay off jointly.

Proof of Lemma 5. Following the proof of Lemma 4 from EP, for any non-zero $x', y' \in (-\epsilon, \epsilon)$ for $\epsilon > 0$ small enough, the absolute value of

$$u(Nx' + My' + z_0) + u(z_0) - u(Nx' + z_0) - u(My' + z_0) \tag{7}$$

is sufficiently close to $u(x + y) + u(0) - u(x) - u(y)$ for some positive integers N and M and an appropriately chosen z_0. In particular, one can find $\epsilon > 0$ so that (7) does not equal zero for every non-zero $x', y' \in (-\epsilon, \epsilon)$.

To ease notation, set $\beta_i = \beta_{x_i}^{B_i,k_i}$ and $\mathcal{E}^i = \mathcal{E}^{B_i,k_i,x_i}$.

First, observe that for $\pi \in \Pi$, $\pi(\mathcal{E}^i) \geq \mu(k_i)$, since

$$\mu(k_i)u(x_i) = V((1, \beta_i)) = \min_{\pi\in\Pi} \pi(\mathcal{E}^i)u(x_i).$$

Second, for any $p \in \Delta\mathcal{F}$, there exists $\pi \in \arg\min_{\pi\in\Pi} \int f_p d\pi$ with $\pi(\mathcal{E}^i) = \mu(k_i)$ for all i. Fix any $p \in \Delta\mathcal{F}$, and note $\alpha V(p) + (1 - \alpha)V(1, \beta_i) = V(\alpha p + (1 - \alpha)(1, \beta_i))$. The former equals $\alpha V(p) + (1 - \alpha)\mu(k_i)u(x_i)$. The latter equals $\int [\alpha f_p] d\pi + \pi(\mathcal{E}^i)u(x_i)$ for some $\pi \in \Pi$. If $\pi(\mathcal{E}^i) > \mu(k_i)$, then $\int f_p d\pi < V(p) = \min_{\pi'\in\Pi} \int f_p d\pi'$, contradicting the definition of V. Conclude there is a minimizer with $\pi(\mathcal{E}^1) = \mu(k_1)$. Now, suppose for n there is $\pi \in \arg\min_{\pi\in\Pi} \int f_p d\pi$ with $\pi(\mathcal{E}^i) = \mu(k_i)$ for $i < n$. Repeat the above arguments with $i = n$, but choose π to be the minimizer claimed by the IH. Conclude that this minimizer must also have $\pi(\mathcal{E}^n) = \mu(k_n)$. Induction implies this must be the case for all $\pi(\mathcal{E}^i)$. Hence, for any p, there is a minimizer satisfying Equation (4) for all i.

Third, claim that this minimizer can also be taken to have $\pi_0(\mathcal{E}^i \bigcap \mathcal{E}^j) = 0$ when $B_i = B_j$ and $k_i \neq k_j$. There are a finite number of these pairs of events; order them arbitrarily. Assume (IH) that there is a minimizer π_0 for any $p \in \Delta\mathcal{F}$ satisfying Eq (4) and for which Eq (5) also holds for the first $n-1$ pairs. The base case holds by the above.

Let (i,j) be pair n. Since $x_i, x_j \in \Theta_\varepsilon$, by the above, there exists N, M, z_0 such that

$$u(Nx_i + Mx_j + z_0) + u(z_0) - u(Nx_i + z_0) - u(Mx_j + z_0) = D \neq 0.$$

Define lotteries

$$p_1 \equiv \left(\frac{1}{2}, \langle N\beta^i, z_0\rangle; \frac{1}{2}, \langle M\beta^j, z_0\rangle\right)$$

and

$$p_2 \equiv \left(\frac{1}{2}, \langle N\beta^i, M\beta^j, z_0\rangle; \frac{1}{2} z_0\right).$$

Since B understood,

$$q_1 = \frac{1}{2}p + \frac{1}{2}p_1 \sim \frac{1}{2}p + \frac{1}{2}p_2 = q_2$$

Since V is action independent,

$$V(q_1) = \frac{1}{2}I(p) + \frac{1}{2}V(p_1)$$

and

$$V(p_1) = \mu(k_i)[u(Nx_i + z_0) - u(z_0)] + \mu(k_j)[u(Mx_j + z_0) - u(z_0)] + u(z_0).$$

By IH, there exists $\pi_0 \in \Pi$ satisfying (4) so that

$$\begin{aligned}
V(q_2) = & \int f_{q_2} d\pi_0 \\
= & \frac{1}{2}\int f_p d\pi_0 + \frac{1}{2}\pi_0(\mathcal{E}^j \bigcap \mathcal{E}^i)[u(Nx_i + Mx_j + z_0) - u(z_0)] + \frac{1}{2}u(z_0) \\
& + \frac{1}{2}[\pi_0(\mathcal{E}^i) - \pi_0(\mathcal{E}^j \bigcap \mathcal{E}^i)][u(Nx_i + z_0) - u(z_0)] \\
& + \frac{1}{2}[\pi_0(\mathcal{E}^j) - \pi_0(\mathcal{E}^j \bigcap \mathcal{E}^i)][u(Mx_j + z_0) - u(z_0)] \\
= & \frac{1}{2}\int f_p d\pi_0 + \frac{1}{2}V(p_1) + \frac{1}{2}\pi_0(\mathcal{E}^j \bigcap \mathcal{E}^i)D
\end{aligned}$$

If $\pi_0(\mathcal{E}^j \bigcap \mathcal{E}^i) > 0$, then $V(q_1) \neq V(q_2)$, contradicting the claimed indifference. Moreover, $V(q_2) = V(q_1) = \frac{1}{2}V(p) + \frac{1}{2}V(p_1)$ by action independence, so $\pi_0 \in \arg\min_\Pi \int f_p d\pi$. Conclude the IH holds for the first n pairs as well. Conclude by induction that there is a minimizer satisfying Eq (5) for any $p \in \Delta\mathcal{F}$.

Fourth, suppose $\beta_i, \beta_j \in B \in \mathcal{U}$. Let $b \in B$ be a bet yielding x_i on $\Omega \setminus \{k_i\}$ and 0 otherwise.

$$\mathcal{E}^b = \{\vec{\omega} \in \Omega^{\mathcal{A}} : \omega^b \neq k_i\}.$$

Because B is understood, one has, for any $N \in \mathbb{N}$ and $z \in X$, that

$$\begin{aligned}
\frac{1}{2}p + \frac{1}{2}\left(\frac{1}{2}, \langle N\beta_i, z\rangle; \frac{1}{2}, \langle Nb, z\rangle\right) & \sim \frac{1}{2}p + \frac{1}{2}\left(\frac{1}{2}, \langle N\beta_i, Nb, z\rangle; \frac{1}{2}, \langle z\rangle\right) \\
& \sim \frac{1}{2}p + \frac{1}{2}\left(\frac{1}{2}, \langle Nx_i, z\rangle; \frac{1}{2}, \langle z\rangle\right).
\end{aligned}$$

By above, there is a minimizer satisfying Eqs. (4) and (5), and similar arguments to those establishing Eq. (5) show the minimizer π_0 for f_p satisfying Eqs. (4) and (5) can be taken to also satisfy

$$\pi_0\left(\mathcal{E}^i \bigcap \mathcal{E}^b\right) = \pi_0\left(\mathcal{E}^j \bigcap \mathcal{E}^b\right) = 0.$$

Picking $N \in \mathbb{N}$ and $z \in X$ such that $u(z + Nx') \neq u(z)$, one also has that

$$\left[\pi_0(\mathcal{E}^i) + \pi_0(\mathcal{E}^b)\right]\left(u\left(Nx_i + z\right) - u(z)\right) = u\left(Nx_i + z\right) - u(z)$$

and so

$$\pi_0(\mathcal{E}^i) + \pi_0(\mathcal{E}^b) = 1.$$

The inclusion-exclusion formula gives that

$$1 \geq \pi_0\left(\mathcal{E}^i \bigcup \mathcal{E}^j \bigcup \mathcal{E}^b\right) = 1 + \pi_0(\mathcal{E}^j) - \pi_0\left(\mathcal{E}^i \bigcap \mathcal{E}^j\right)$$

and thus $\pi_0(\mathcal{E}^j) = \pi_0\left(\mathcal{E}^i \bigcap \mathcal{E}^j\right)$. A symmetric argument with b' defined using x_j instead of x_i shows $\pi_0(\mathcal{E}^i) = \pi_0\left(\mathcal{E}^i \bigcap \mathcal{E}^j\right)$. Inductively extending as above yields a minimizing π_0 satisfying Eq. (6). □

Lemma 6. *Given any $\varepsilon > 0$ and profile $F = \langle a_i \rangle_{i=1}^n$ and allocation $C : \mathcal{A} \to \mathcal{U}$, there exist $\beta_1, ..., \beta_T \in \mathcal{A}$, $B_1, ..., B_T \in \mathcal{U}$ and $N_1, ..., N_T \in \mathbb{N}_+$ such that:*
(i) for any B_j, $j = 1, ..., T$, there exists a_i such that $C(a_i) = B_j$;
(ii) for any $j = 1, ..., T$, $\beta_j = \beta_x^{B_j,k}$ for some $k \in \{1, ..., K\}$ and $x \in \Theta_\varepsilon$;
(iii) For any $C \in \mathcal{U}$ and all $\omega \in \Omega$,

$$\sum_{\{j : B_j = C\}} N_j \beta_j(\omega) = \sum_{\{i : C(a_i) = C\}} a_i(\omega).$$

The proof of Lemma 6 follows the same arguments as Lemma 5 of EP.

Pick any $p, q \in \Delta\mathcal{F}$ and any allocations $C_1, C_2 : \mathcal{A} \to \mathcal{U}$ satisfying $f_p^{C_1} \geq f_q^{C_2}$. Fix ε as per Lemma 5. Write $p = (p_i, F_i)_{i=1}^n$ and $q = (q_i, F_i)_{i=n+1}^N$. For $i = 1, \ldots, N$ there are bets $\beta_1^i, \ldots, \beta_{T_i}^i$, understanding classes $B_1^i, \ldots, B_{T_i}^i$, and positive integers $N_1^i, \ldots, N_{T^i}^i$ as in Lemma 6 so that

$$p \sim \left(p_i, \langle N_t^i \beta_t^i \rangle_{t=1}^{T_i}\right)_{i=1}^n \equiv p^\beta$$

and

$$q \sim \left(q_i, \langle N_t^i \beta_t^i \rangle_{t=1}^{T_i}\right)_{i=n+1}^N \equiv q^\beta$$

by construction of understanding classes and the conclusion of Lemma 6. By Lemma 5,

$$V(p) = V(p^{\beta,C}) = \int f_{p^\beta,C}\, d\pi_p$$

for some $\pi_p \in \Pi$ satisfying Eqs. (4), (5) and (6) for $\left\{\beta_1^1, \beta_2^1, \ldots, \beta_{T_N}^N\right\}$. By construction, for any measurable event $E \subset \Omega^{\left\{\beta_1^1, \beta_2^1, \ldots, \beta_{T_N}^N\right\}}$ with $\pi_p(E) > 0$, there exists a measurable event $E' \subset \Omega^{\mathcal{U}}$ so that $f_{p^\beta}(\omega) = f_p^{C_1}(\omega')$ and $f_{q^\beta}(\omega) = f_q^{C_2}(\omega')$ for $\omega' \in E'$ and $\omega \in E$. Therefore,

$$V(p^\beta) = \int f_{p^\beta}\, d\pi_p \geq \int f_{q^\beta}\, d\pi_p \geq V(q^\beta),$$

and by transitivity, $p \succsim q$. Then, I can repeat the arguments of Theorem 1 to find a rich CCR of $\succsim$.

If $\left(u, \mu, \mathcal{U}, \Pi_1\right)$ and $\left(u, \mu, \mathcal{U}, \Pi_2\right)$ are both rich CCR's of $\succsim$, and $\Pi_2 \not\subseteq \Pi_1$, I can find $\pi_2 \in \Pi_2 \setminus \Pi_1$. Since all these measures are countably additive, there is some finite $E \subset \mathcal{U}$ so that the marginal on Ω^E of π_2 does not equal the marginal on Ω^E of any $\pi_1 \in \Pi_1$ by Kolmogorov's Extension Theorem (Theorem 15.26 of Aliprantis and Border (2006)). By the Separating Hyperplane Theorem, one can find a $\phi \in \mathbb{R}^{\Omega^E}$ so that $\int \phi d\pi_2 < \int \phi d\pi_1$ for all $\pi_1 \in \Pi_1$. As shown in Theorem 2 of EP, the collection $\left\{f_p^C : p \in \Delta\mathcal{F} \text{ \& } C \text{ an allocation}\right\}$ has dimensionality equal to $\min\{|\mathcal{U}|, |\mathbb{N}|\}$ when u is not a polynomial, so one can find a p and a C so that $f_p^C = \phi$. Then, for some $q \in \Delta X$ so that $\sum_{q(\langle x \rangle) > 0} q(\langle x \rangle) u(x) = \int \phi d\pi_2$, the utility of q according to $\left(u, \mu, \mathcal{U}, \Pi_2\right)$ exceeds that of p while the utility of p according to $\left(u, \mu, \mathcal{U}, \Pi_2\right)$ strictly exceeds that of q, contradicting that both represent the same $\succsim$. Similar arguments give that $\Pi_1 \subseteq \Pi_2$, so the two are equal. □

For any a finite $F \subset \mathcal{U}$ and probability distribution π over $\left(\Omega^{\mathcal{U}}, \Sigma^{\mathcal{U}}\right)$, let π_F be the marginal distribution over the coordinates indexed by F.

Lemma 7. *For any Rich CCR $(u, \mu, \mathcal{U}, \Pi)$ and probability distribution π^* over $\left(\Omega^{\mathcal{U}}, \Sigma^{\mathcal{U}}\right)$, if for every finite $F \subset \mathcal{U}$, there exists $\pi(F) \in \Pi$ so that $\pi_F^* = \pi(F)_F$, then $\pi^* \in \Pi$.*

Proof of Lemma 7. Suppose that $\pi_F^* = \pi(F)_F$ for some $\pi(F) \in \Pi$ for all finite F. Then, $(\pi(F))_F$, indexed by $F \subset \mathcal{U}$ and $|F| < \infty$, is a net directed by $\subseteq$. The set $\Omega^{\mathcal{U}}$ is compact by the Tychonoff Theorem and metrizable in the product topology because $\mathcal{U}$ is at most countable. By Theorem 15.11 of Aliprantis and Border (2006), the set of probability measures on it is also compact and metrizable with the weak*-topology. As a closed subset thereof, Π is also compact. Therefore, $(\pi(F))_F$ has a subnet that converges to some $\pi^\dagger \in \Pi$. But $\pi_F^\dagger = \pi_F^*$ for any finite F, so $\pi^* = \pi^\dagger$ by the Kolmogorov Extension Theorem (Theorem 15.26 of Aliprantis and Border (2006)). □

Proof of Proposition 1. Observe that a CCCEU preference satisfies Weak Monotonicity and is complete, transitive, and continuous. If it also satisfies NUI, then one can apply Lemmas 2 and 3. NUI and the Lemmas give that W is concave: if $p \succsim q$, pick $p', q' \in \Delta X$ with $p' \sim p$ and $q' \sim q$. Then $\alpha p + (1-\alpha)q \succsim \alpha p' + (1-\alpha)q$ by NUI, and $\alpha p' + (1-\alpha)q \sim \alpha p' + (1-\alpha)q'$ by Lemma 2. Moreover, W is linear on ΔX, so $W(\alpha p + (1-\alpha)q) \geq W(\alpha p' + (1-\alpha)q') = \alpha W(p) + (1-\alpha)W(q)$. This implies that v is convex: for any $A, B \in \Sigma^{\mathcal{U}}$, $v(A \cup B) + v(A \cap B) \geq v(A) + v(B)$.

Consider first a linear u. Then, for any $a, b \in A$, $(\frac{1}{2}, \langle a, b\rangle; \frac{1}{2}, \langle 0\rangle) \sim (\frac{1}{2}, \langle a\rangle; \frac{1}{2}, \langle b\rangle)$ by Weak Monotonicity. For the $C_1 \in \mathcal{U}$ with $b_C, b \in C_1$, there exists $b'_C, -b'_C \in C_1$ so that $-\rho(-b'_C, \omega) = \rho(b'_C, \omega) = \rho(b_F, \omega)$ for all ω by non-singularity. By Lemma 2, $\succsim$ satisfies the independence axiom when restricted to lotteries over action. Because u is linear, $W(-b'_C) = -W(b'_C)$ and $W(b_F) = -W(-b_F)$. Now, $b \sim \langle -b'_C, b_C\rangle$ and $100 \sim \langle b'_C, b_C\rangle$. Therefore, $\frac{1}{2}\langle b\rangle + \frac{1}{2}\langle 0\rangle \sim \frac{1}{2}\langle -b'_C\rangle + \frac{1}{2}\langle b_C\rangle \succsim \frac{1}{2}\langle b_C\rangle + \frac{1}{2}\langle -b_F\rangle$. Lemma 2 gives that $-b'_C \succsim -b_F$. Also, $\frac{1}{2}\langle 100\rangle + \frac{1}{2}\langle 0\rangle \sim \frac{1}{2}\langle b'_C\rangle + \frac{1}{2}\langle b_C\rangle \succsim \frac{1}{2}\langle b_C\rangle + \frac{1}{2}\langle b_F\rangle$. Lemma 2 gives $b'_C \succsim b_F$. Therefore, $b'_C \sim b_F$ and $-b'_C \sim -b_F$, which implies $\langle 100\rangle \sim \langle b_C, b_F\rangle$ and $\langle b\rangle \sim \langle b_C, -b_F\rangle$.

Consider now a non-linear u, so there are $x, y \in X$ and $\alpha \in (0,1)$ so that $u(\alpha x + (1-\alpha)y) \neq \alpha u(x) + (1-\alpha)u(y)$. By continuity, there is no loss in setting $\alpha = \frac{1}{2}$ and normalizing so that $u(x) = u^* > u(z^*) = 1 > u(y) = 0$ where $z^* = \frac{1}{2}x + \frac{1}{2}y$ and where $u^* \neq 2$.

Denote $\bar{E} = \Omega \setminus E$ for any $E \in \Sigma$. Pick any $A, B \in \Sigma$ and $C_1, C_2 \in \mathcal{U}$. Denote $A^1 = \{\vec{\omega} : \omega^{C_1} \in A\}$ and $B^2 = \{\vec{\omega} : \omega^{C_2} \in B\}$. I show that $v(A^1 \cup B^2) = v(A^1) + v(B^2) - v(A^1 \cap B^2)$.

By richness, there are bets C_1 and C_2 on $A, B, \bar{A}, \bar{B}$ at stakes $z = z^* - y > 0$ and 0. Let E_i be the bet in C_i on E for $E \in \{A, B, \bar{A}, \bar{B}\}$ for $i = 1, 2$. Then for $E, F \in \{A, B, \bar{A}, \bar{B}\}$ the utility of $\langle E_1, F_2, y\rangle$ in $\vec{\omega}$ is u^* if $\vec{\omega} \in E \times F$, 1 if $\vec{\omega} \in \bar{E} \times F$ or $\vec{\omega} \in E \times \bar{F}$, and 0 if $\vec{\omega} \in \bar{E} \times \bar{F}$. Then viewing the utilities of

$$\langle A_1, B_2, y\rangle, \langle \bar{A}_1, B_2, y\rangle, \langle A_1, \bar{B}_2, y\rangle, \langle \bar{A}_1, \bar{B}_2, y\rangle$$

as vectors in $\mathbb{R}^4$, the vectors are linearly independent (in matrix form their determinant is $u^{*4} - 4u^{*2}$). Then for any $x_1, \ldots, x_4 \in \mathbb{R}$, there exist a $p \in \mathcal{F}$, $\alpha > 0$ and $\beta \in \mathbb{R}$ so that

$$f_p^C = \alpha \sum_{i=1}^{4} x_i \mathbb{1}_{E_i}(\cdot) + \beta$$

where $(E_i)_{i=1}^4 = (A^1 \cap B^2, A^1 \cap [B^2]^c, [A^1]^c \cap B^2, [A^1]^c \cap [B^2]^c)$. In what follows all vectors are $\sigma(\{E_i\}_i)$ measurable, so identify them with vectors in $\mathbb{R}^4$ where the i the component indicates the value in E_i.

Consider a bet r on A^c r' on A, both in C_1: $r = (1, g)$ with $g \in C_1$ s.t. $g(\omega)$ equals z on $\bar{A}$ and y otherwise and $r' = (1, g')$ with $g' \in C_1$ s.t. $g'(\omega)$ equals z on A and y otherwise. I combine these with more complicated profiles to show that v is additive.

First, pick any $p, q \in \Delta(F)$ and C so that $f_p^C = v$ with $v_1 > v_2 = v_3 > v_4$ and $f_q^C = v'$ with $v'_1 < v'_2 = v'_3 < v'_4$ and find $p', q' \in \Delta\mathcal{X}$ so that $p \sim p'$ and $q \sim q'$. Consider $p(\alpha) = \alpha p + (1-\alpha)r$ and $p'(\alpha) = \alpha p' + (1-\alpha)r$. Lemma 2 gives that $p(\alpha) \sim p'(\alpha)$ for every α. In particular, for all sufficiently large $\alpha < 1$,

$$\begin{aligned}\alpha^{-1}W(p(\alpha)) =& W(p) + (1 - v(E_1 \cup E_2 \cup E_3) + v(E_1 \cup E_3) - v(E_1))\frac{1-\alpha}{\alpha} \\ =& \alpha^{-1}W(p'(\alpha)) = W(p) + v(E_3 \cup E_4)\frac{1-\alpha}{\alpha}.\end{aligned}$$

This must hold for all α in an interval, so

$$v(E_3 \cup E_4) = 1 - v(E_1 \cup E_2 \cup E_3) + v(E_1 \cup E_3) - v(E_1).$$

Repeating the above with q replacing p and r' replacing r yields $q(\alpha) = \alpha q + (1-\alpha)r' \sim q'(\alpha) = \alpha q' + (1-\alpha)r'$ for all α. This requires that

$$v(E_1 \cup E_3) = 1 - v(E_2 \cup E_3 \cup E_4) + v(E_3 \cup E_4) - v(E_4)$$

Adding the two yields

$$2 = v(E_1 \cup E_2 \cup E_3) + v(E_1) + v(E_2 \cup E_3 \cup E_4) + v(E_4)$$

which can only hold if

$$v(E_1 \cup E_2 \cup E_3) + v(E_4) = v(E_2 \cup E_3 \cup E_4) + v(E_1) = 1 \tag{8}$$

by convexity of v.

Now, pick a new $p, q \in \Delta\mathcal{F}$ and C so that $f_p^C = w'$ where $w'_3 > w'_1 > w'_2 > w'_4$ and so that $f_q^C = w$ where $w_2 > w_3 > w_4 > w_1$. Again find $p', q' \in \Delta X$ be so that $p' \sim q$ and $q' \sim q$. Define $q(\alpha) = \alpha q + (1-\alpha)r$ and $q'(\alpha) = \alpha q' + (1-\alpha)r$. Lemma 2 implies that $q(\alpha) \sim q'(\alpha)$ for every α, and so for all sufficiently large $\alpha < 1$,

$$\begin{aligned}\alpha^{-1}W(q(\alpha)) =& W(q) + (v(E_2 \cup E_3 \cup E_4) - v(E_2))\frac{1-\alpha}{\alpha} \\ =& \alpha^{-1}W(q'(\alpha)) = W(q) + v(E_3 \cup E_4)\frac{1-\alpha}{\alpha}.\end{aligned}$$

Therefore using Equation (8),

$$\begin{aligned} v(E_3 \cup E_4) =& v(E_2 \cup E_3 \cup E_4) - v(E_2) \\ =& 1 - v(E_1) - v(E_2). \end{aligned}$$

Repeating the above with p replacing q, and p' replacing q', and r' replacing r to get that $p(\alpha) = \alpha p + (1-\alpha)r' \sim \alpha p' + (1-\alpha)r' \sim p'(\alpha)$ for all α, which holds if only if

$$\begin{aligned} v(E_1 \cup E_2) =& v(E_1 \cup E_2 \cup E_3) - v(E_3) \\ =& 1 - v(E_4) - v(E_3) \end{aligned}$$

using Equation (8). Since $v(E_3 \cup E_4) = 1 - v(E_1 \cup E_2)$ to satisfy independence on actions, $v(E_3 \cup E_4) = v(E_3) + v(E_4)$ and $v(E_1 \cup E_2) = v(E_1) + v(E_2)$.

To conclude, notice

$$v(A^1 \cup B^2) = v(E_1 \cup E_2 \cup E_3) = 1 - v(E_4)$$

by Eq (8). Also, $1 = v(E_1 \cup E_2) + v(E_3 \cup E_4) = \sum_i v(E_i)$ by the above. Combining, $v(A^1 \cup B^2) = \sum_{i=1}^{3} v(E_i)$. Since $v(E_1) = v(A^1 \cap B^2)$, v is additive for A^1, B^2. But A, B and C_1, C_2 were arbitrary, so v is a probability measure when restricted to pairs of understanding classes. By the logic at the end of Section 3.1, $\succsim$ cannot exhibit the choices in that experiment.

The proof for non-linear u extends inductively to an events generated finite collection of understanding classes when u is not a polynomial. To do so, let B^2 be an event in the first n understanding classes and A^1 an event in the $n+1$st. When u is not a polynomial and non-singularity holds, the dimensionality f_p^C across lotteries and assignments is large enough so that the analogues of p and q exist. Thus, the above arguments apply, so v must be additive and thus be a probability measure. □

Proof of Proposition 2. Necessity is trivial. Suppose $\succsim$ has a rich CCR $(u, \mu, \mathcal{U}, \Pi)$ where u is not a polynomial. Since u is not a polynomial, there exists x, y, z so that $u(x+z) + u(y+z) - u(x+y+z) - u(z) \neq 0$. To save notation, set $z = 0$; adding z to each of the profiles in the lotteries compared below covers the case where $z \neq 0$. To save notation, write EF instead of $E \bigcap F$ for events $E, F \in \Sigma^{\mathcal{U}}$.

First consider $K = u(x) + u(y) - u(x+y) - u(0) < 0$. For $E \in \Sigma$, let $a_E^i = xE0 \in C_i$ and $b_E^j = 0Ey \in C_j$ and $a_E^i + b_E^j = xEy \in C_1$. Then $V(\langle a_E^i + b_E^j \rangle) = \mu(E)u(x) + \mu(E^c)u(y)$ and there is $\pi \in \Pi$ so that

$$\begin{aligned} V(\langle a_E^1, b_E^2 \rangle) =& [\pi(E_{C_1}) - \pi(E_{C_1} E_{C_2}^c)]u(x) + [\pi((E^c)_{C_2}) - \pi(E_{C_1}^c E_{C_2})]u(y) + \\ & [\pi(E_{C_1} E_{C_2}^c)]u(x+y) + [\pi(E_{C_1}^c E_{C_2})]u(0) \\ =& \mu(E)u(x) + \mu(E^c)u(y) - \pi(E_{C_1}^c E_{C_2})K \end{aligned}$$

since $\pi(E_{C_1}) = \pi(E_{C_2}) = \mu(E)$, $\pi(E_{C_1}^c) = \pi(E_{C_2}^c) = \mu(E^c)$, and

$$\pi(E_{C_1} E_{C_2}^c) = \pi(E_{C_1}^c E_{C_2}) = \pi(E_{C_i}) - \pi(E_{C_1} E_{C_2}) = \mu(E) - \pi(E_{C_1} E_{C_2}).$$

No False Hedging implies $V(\langle a_E^i + b_E^j \rangle) \geq V(\langle a_E^i, b_E^j \rangle)$, which holds only if $\pi(E_{C_1}^c E_{C_2}) = 0$ because $K < 0$. Similarly, for $E_1, \ldots, E_n \in \Sigma$ and $j_1, \ldots, j_n, k_1, \ldots, k_n \in \mathcal{U}$ so that $k_i \neq j_i$, there exists $\pi \in \Pi$ so that

$$V\left(\left(\frac{1}{n}, \langle a_{E_i}^{j_i} + b_{E_i}^{k_i} \rangle\right)_{i=1}^n\right) - V\left(\left(\frac{1}{n}, \langle a_{E_i}^{j_i}, b_{E_i}^{k_i} \rangle\right)_{i=1}^n\right) = -\frac{1}{n} \sum_{i=1}^{n} \pi\left((E_i)_{C_{j_i}} (E_i^c)_{C_{k_i}}\right) K.$$

No False Hedging holds only if $\pi\left((E_i)_{C_{j_i}} (E_i^c)_{C_{k_i}}\right) = 0$ for each i.

Pick any finite $F \subset \mathcal{U}$. Choosing events and indexes so that for every ω and every $C \in F$, $E_i = \{\omega\}$ and $C_{j_i} = C$. This implies there is $\pi \in \Pi$ with $\pi(\{\vec{\omega} : \omega^* = \omega_C \text{ for all } C \in F\}) = \mu(\omega^*)$ for all $\omega^* \in \Omega$. Lemma 7 implies that the extension to all $C \in \mathcal{U}$ lies in Π, establishing the result.

If instead $K = u(x) + u(y) - u(x+y) - u(0) > 0$ repeat instead with $c_E^i = yE0 \in C_i$ and $a_E^i + c_E^j = (x+y)E0 \in C_1$ replacing b_E^i, noting $V(a_E^i + c_E^j) = \mu(E)u(x+y) + \mu(E^c)u(0)$, and there is $\pi' \in \Pi$ so that

$$\begin{aligned} V(\langle a_E^1, c_E^2 \rangle) =& [\pi'(E_{C_1}) - \pi'(E_{C_1} E_{C_2}^c)]u(x+y) + [\pi'((E^c)_{C_2}) - \pi'(E_{C_1} E_{C_2}^c)]u(0) + \\ & [\pi'(E_{C_1} E_{C_2}^c)]u(x) + [\pi'(E_{C_1}^c E_{C_2})]u(y) \\ =& \mu(E)u(x+y) + \mu(E^c)u(0) + \pi'(E_{C_1}^c E_{C_2})K \\ =& V(\langle a_E^1 + c_E^2 \rangle) + \pi'(E_{C_1}^c E_{C_2})K \end{aligned}$$

and No False Hedging requires $\pi'(E^c_{C_1}E_{C_2})=0$. Similar arguments to the other case imply existence of the measure claimed by the result. □

Proof of Proposition 3. Suppose that $\succsim_1$ understands more connections than $\succsim_2$ and pick any $B \in \mathcal{U}_2$. Now, B is rich and $\succsim_2$-understood, so B is also $\succsim_1$-understood. Hence, it is contained in a maximal, rich, $\succsim_1$-understood subset B'. By construction of $\mathcal{U}_1$ in Theorem 2, $B' \in \mathcal{U}_1$. Conversely, suppose that for any $C \in \mathcal{U}_2$, there exists $C' \in \mathcal{U}_1$ with $C \subseteq C'$. Pick any rich $B \subset \mathcal{A}$ so that $\succsim_2$ understands B. Then B is contained in a maximal element $C \in \mathcal{U}_2$, which is in turn contained in $C' \in \mathcal{U}_1$. Since C' is $\succsim_1$-understood and $B \subseteq C'$, B is $\succsim_1$-understood, completing the proof. □

Proof of Proposition 4. Suppose that $\succsim_2$ is more concerned about correlation than $\succsim_1$. When $p,q \in \Delta\mathcal{A}$, $p \succsim_2 q \iff p \succsim_1 q$, so the usual Anscombe-Aumann uniqueness result gives that $\mu_1=\mu_2$ and that u_1 is a positive affine transformation of u_2. By Proposition 3, $\mathcal{U}_1$ is coarser than $\mathcal{U}_2$. As described in the text following that proposition, one can project Π_1 onto $\mathcal{U}_2$; for convenience, slightly abuse notation by denoting its projection by Π_1.

If $\Pi_1 \not\subseteq \Pi_2$, then there exists $\pi^* \in \Pi_1 \setminus \Pi_2$. Lemma 7 implies there exists a finite $F \subset \mathcal{U}$ so that $\pi^*_F \notin \Pi_{2,F} = \{\pi_F : \pi \in \Pi_2\}$. Now, $\Pi_{2,F}$ is compact, convex and disjoint from $\{\pi^*_F\}$. By a separating hyperplane theorem, there is a function $\phi : \Omega^F \to \mathbb{R}$ so that $\int f d\pi^* < 0 \leq \int \phi d\pi$ for all $\pi \in \Pi_{2,F}$. There is a positive affine transformation g so that $g\circ\phi(\vec{\omega}), g(0) \in range(u_1)$ for every $\vec{\omega} \in \Omega^{\mathcal{U}_2}$. Following the proof of Theorem 2 and using that u_1 is not a polynomial, there exists a $p \in \Delta\mathcal{F}$ so that $f_p = g\circ\phi$ and $q \in \Delta X$ so that $f_q = g(0)$. Then, $p \succsim_2 q$ since

$$\min_{\pi\in\Pi_2} \int f_p d\pi \geq g(0),$$

but $q \succ_1 p$ since

$$\min_{\pi\in\Pi_1} \int f_p d\pi \leq \int f_p d\pi^* < g(0).$$

This contradicts that $\succsim_2$ is more concerned about correlation then $\succsim_1$. □

Data availability

No data was used for the research described in the article.

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