



Measuring office productivity: A model accounting for the dependence of outflows on inflows[☆]

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ABSTRACT

The measurement of productivity in the public sector is challenging, in part because of the difficulties associated with defining and quantifying outputs. Even when outputs are observable, their proper evaluation remains complex. This paper proposes a parsimonious yet generalizable model, using judicial courts as a case study, that assumes a linear production function in which each case has the same weight. The model shows that the number of resolved cases is systematically shaped by both the volume and the composition of newly filed cases. Consequently, standard productivity indicators that fail to account for the characteristics of incoming workloads may be severely biased.

1. Introduction

A central challenge in empirical economics is the accurate measurement of output and productivity. A substantial body of literature has focused on distinguishing between Revenue TFP (TFPR) and Physical TFP (TFPQ) to analyse cross-country differences, within-country reallocation, and firm turnover (Foster et al., 2008; Hsieh and Klenow, 2009). As Haltiwanger (2016) notes, “distinguishing between TFPR... and TFPQ is not a bug but a feature of the literature”, highlighting the need to account for demand-side factors when studying firm dynamics. However, a potential gap remains in understanding output and productivity in contexts where prices do not reflect demand, such as in the public sector.

A growing strand of research examines the productivity of the public sector in various domains, including schools (Bloom et al., 2015), hospitals (Chandra et al., 2013), administrative agencies (Fenizia, 2022), and the public service (Rasul and Rogger, 2018). In these settings, productivity is typically modelled as the efficiency with which bureaucratic labour and capital are converted into public services, often measured as output per unit of input. However, measuring productivity in the public sector presents unique challenges: inputs tend to be rigid and slow to adjust to fluctuations in demand; moreover, output is not priced, making performance comparison difficult. For example, a judge resolving 10 cases may appear more productive than one resolving 5 cases, but if the latter handles more complex legal disputes, the

comparison becomes misleading. Finally, and relatedly, the volume and composition of incoming cases may influence the volume and composition of resolved cases.

A simple example illustrates these points. Suppose a judge works 200 days per year and receives 20 new cases annually, half of which are “easy” (requiring 20 days each) and half are “difficult” (requiring 100 days each). If the judge aims to maximize the number of resolved cases, she will prioritize simple cases, resolving a total of 10 cases per year. If the case inflow is halved the following year while maintaining the same ratio of easy to difficult cases, the judge, working at the same pace, will resolve 6 cases, spending 100 days on 5 simple cases and another 100 days on one difficult case.

As a result, standard productivity measures can offer an incomplete and potentially biased perspective.

To address these issues, we develop a simple model based on courts, in which the judge aims to maximize the number of resolved cases, which enter linearly into her production function and carry identical weights (i.e., “prices”). Although the model focuses on judicial activity, the results are easily generalizable to other administrative offices and also the private sector (e.g., warehouse management) if similar assumptions hold. The model shows that the number of resolved cases depends on the volume and composition of newly filed cases. Specifically, a reduction (increase) in the number of new cases corresponds to a reduction (increase) in the number of resolved cases, with an elasticity

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between 0 and 1, and the elasticity increasing in the complexity gap between cases. Using annual data on case inflows and outflows in Italian courts, we provide empirical evidence supporting our theoretical results. The paper also includes an extension that discusses the generalizability of the results when relaxing some of the assumptions on the production function.

Our model highlights the importance of distinguishing between shifts in demand and real productivity changes when measuring public sector output and efficiency. Accounting for these factors enables a more accurate assessment of supply and performance, particularly in environments where inputs remain fixed, and traditional market mechanisms are absent.

2. Model

Consider the maximization problem of the production function of a judge who is faced with the choice of allocating her working hours over new cases of different types. Specifically, there are a total of $D > 0$ new cases, each belonging to one type $i \in \{1, \dots, N\}$ with probability $p_i \in (0, 1)$. Without loss of generality, we assume there are two types of proceedings, i.e., $N = 2$. Given E total working hours and knowing the required hours per case type θ_i , the judge must decide how many cases x_i to handle for each type. The required hours per case type θ_i determines the case resolution complexity, and we assume, without loss of generality, that it is growing in i , i.e., $\theta_1 < \theta_2$, and $\theta_i \neq 0$. The number of cases processed for each type i , x_i , cannot exceed the number of new cases of that type, $p_i D$. The judge's production function is linear in the number of proceedings, assigning equal weight to each case, irrespective of its complexity. Then, the judge's maximization problem is as follows:

$$\begin{aligned} \max_{\{x_1, x_2\}} \quad & x_1 + x_2 \\ \text{s.t.} \quad & x_i \leq p_i D \quad \forall i \in \{1, 2\} \\ & p_1 + p_2 = 1 \\ & \theta_1 x_1 + \theta_2 x_2 \leq E \end{aligned} \quad (1)$$

Given the strictly monotone production function, at least one between the budget constraint and the demand constraint of the proceedings holds with equality:

$$\begin{aligned} \forall i \quad & x_i = \frac{E}{\theta_i} - \frac{1}{\theta_i} \theta_{j \neq i} x_{j \neq i} \quad \text{and} \quad x_i \leq p_i D \\ \text{or} \quad & \\ & x_i \leq \frac{E}{\theta_i} - \frac{1}{\theta_i} \theta_{j \neq i} x_{j \neq i} \quad \text{and} \quad x_i = p_i D \end{aligned} \quad (2)$$

Since the judge assigns equal weight to each case in the maximization function, but cases differ in processing time, cases are resolved in order of complexity. With $p_1 = (1 - p)$, the optimal number of resolved cases is:

$$\begin{aligned} x_1^* &= \min\{(1 - p)D, \frac{E}{\theta_1}\} \\ x_2^* &= \min\{pD, \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 x_1^*\} \\ &= \min\{pD, \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 \min\{(1 - p)D, \frac{E}{\theta_1}\}\} \end{aligned} \quad (3)$$

Initially, the judge chooses to resolve x_1^* cases from the total $(1 - p)D$, whose difficulty is θ_1 , (1) closing all the new cases, i.e., $(1 - p)D$, if $(1 - p)D \leq \frac{E}{\theta_1}$, or (2) exhausting the available hours of work, i.e. $\frac{E}{\theta_1}$, if $(1 - p)D \geq \frac{E}{\theta_1}$. After fixing x_1^* , x_2^* is determined accordingly and similarly, as pD , if $pD \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 x_1^*$, or $\frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 x_1^*$ if $pD \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 x_1^*$.

For a comparative statics analysis, we assume the judge's total working hours are not exhausted by type 1 proceedings, i.e., $\theta_1(1 - p)D < E$. This simplifies the optimal case resolution to:

$$\begin{aligned} x_1^* &= (1 - p)D \\ x_2^* &= \min\{pD, \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p)D\} \end{aligned} \quad (4)$$

2.1. Demand level

We analyse how total resolved cases $F(x_1, x_2) = x_1^* + x_2^*$, vary with demand D :

$$\frac{\partial F(x_1, x_2)}{\partial D} = \begin{cases} (1 - p) + p = 1 & \text{if } pD' \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p)D' \\ (1 - p)(1 - \frac{\theta_1}{\theta_2}) < 1 & \text{if } pD' \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p)D' \end{cases} \quad (5)$$

When the optimal number of resolved cases does not exhaust the judge's available working hours, a decrease in demand leads to an equal decrease in supply. However, when the judge operates at full capacity, a reduction in demand results in a smaller reduction in resolved cases. Under no circumstances does a decrease in demand correspond to an increase in supply ($-1 \leq \frac{\partial F(x_1, x_2)}{\partial D} < 0$).

The left panel of Fig. 1, using data from the empirical analysis, shows, at the court-year level, the variation in resolved cases as a result of changes in new cases for courts, which, in line with the prediction of the model, is $0 < \frac{\partial F(x_1, x_2)}{\partial D} \leq 1$. Note also that when the judge is at full capacity, the greater the complexity gap between cases $\frac{\theta_1}{\theta_2}$, the greater the elasticity of resolved-to-filed cases. This result is intuitive: if the judge is working at full capacity and the number of cases submitted to the court increases, a higher percentage of 'easy' cases will be resolved, especially when 'easy' cases are very simple compared to others.

Examining court productivity, measured as the output per unit of input, $\frac{x_1^* + x_2^*}{E}$, it is straightforward to realize that a reduction in demand leads to a smaller reduction in productivity:

$$\begin{aligned} \frac{\partial}{\partial D} \left[\frac{x_1^* + x_2^*}{E} \right] &= \begin{cases} \frac{1}{E} & \text{if } p'D' \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p')D' \\ \frac{(1 - p)}{E} (1 - \frac{\theta_1}{\theta_2}) & \text{if } p'D' \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p')D' \end{cases} \\ \Rightarrow 0 &< \frac{\partial}{\partial D} \left[\frac{x_1^* + x_2^*}{E} \right] < 1 \end{aligned} \quad (6)$$

If demand decreases while holding all else constant, the per-unit-of-input output also decreases. However, total factor productivity (TFP), denoted as A in the production function $f(A, E)$, remains unaffected by changes in demand. A reduction of x newly filed cases in the court would leave productivity unchanged only if the judge's hours worked decreased by $x \frac{E}{D}$. However, since E is fixed — reflecting the rigidity of labour in the public sector — measuring productivity as output per unit of input would lead to a biased assessment of actual productivity when demand fluctuates.

2.2. Demand composition

We also examine how the elasticity of resolved-to-filed cases varies with the distribution of cases.

$$\frac{\partial^2 F(x_1, x_2)}{\partial D \partial p} = \begin{cases} 0 & \text{if } p'D' \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p')D' \\ -(1 - \frac{\theta_1}{\theta_2}) < 0 & \text{if } p'D' \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1(1 - p')D' \end{cases} \quad (7)$$

Partial derivatives in (7) suggest that when new cases do not exhaust the available workforce, the way supply changes in response to demand does not depend on the distribution of cases. By contrast, when the judge exhausts the available workforce before resolving all new cases, an increase in demand is followed by a decrease in supply if the probability of 'difficult' cases increases (but always with a $|x| < 1$ multiplier).

Finally, we consider how the resolved-to-filed cases elasticity at the subject-matter level (define $pD = D_2$ and $(1 - p)D = D_1$) varies with case complexity:

$$\frac{\partial^2 F(x_2)}{\partial D_2 \partial \theta_2} = \begin{cases} 0 & \text{if } D_2' \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 D_1' \\ -\frac{\theta_1}{\theta_2^2} < 0 & \text{if } D_2' \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 D_1' \end{cases} \quad (8)$$

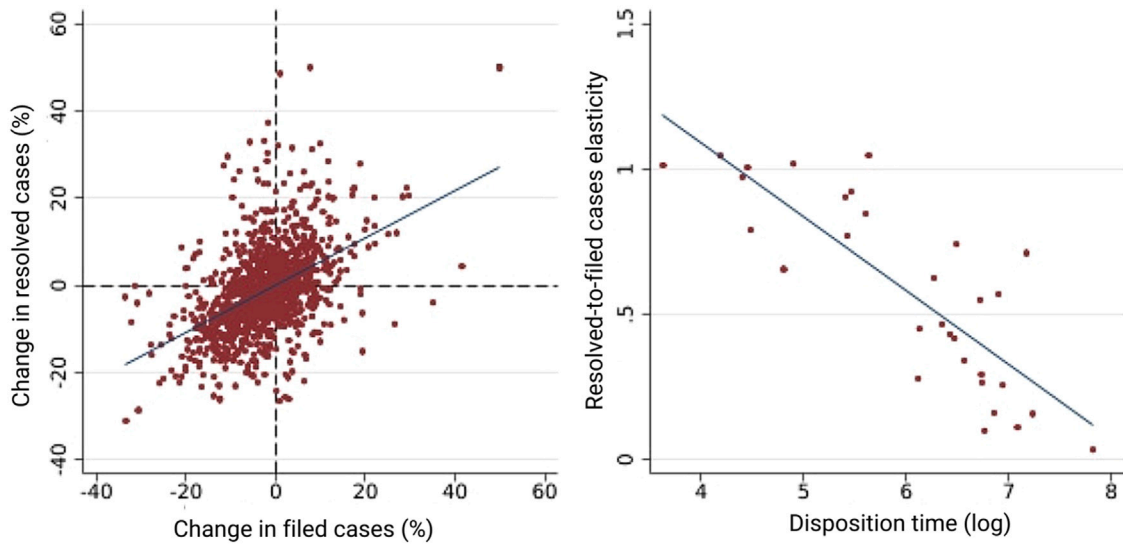


Fig. 1. Elasticity of resolved-to-filed cases.

Notes: The figure on the left shows the relationship between the variation in resolved and new cases at the court and year level. The figure on the right shows the estimated elasticity for each subject matter, which is linked to the disposition time.

Empirical data in Fig. 1 (right panel) illustrates the resolved-to-filed cases elasticity in response to changes in disposition time at the subject-matter level. Disposition time, defined as the average time required to settle a case, serves as a proxy for case complexity, θ_i . As expected, elasticity decreases with increasing disposition time: as the newly filed cases become more difficult, the resolution of new cases becomes slower and less proportional.

Examining the role of demand composition in productivity, we observe that as the fraction of ‘difficult’ cases increases, the per-unit-of-input output decreases:

$$\frac{\partial}{\partial p} \left[\frac{x_1^* + x_2^*}{E} \right] = \begin{cases} 0 & \text{if } p'D' \leq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 (1-p')D' \\ -\frac{D}{E} (1 - \frac{\theta_1}{\theta_2}) < 0 & \text{if } p'D' \geq \frac{E}{\theta_2} - \frac{1}{\theta_2} \theta_1 (1-p')D' \end{cases} \quad (9)$$

Output per unit of input would remain unchanged if working hours were adjusted accordingly, given the new fraction of difficult cases. However, under labour stickiness, measuring productivity as output per unit of input results in a biased assessment, as changes in demand composition are misinterpreted as reductions in productivity. Furthermore, since productivity is assessed at the provider level, this approach obscures variations in individual case resolution efficiency:

$$\begin{aligned} \frac{\partial}{\partial p} \left[\frac{x_1^*}{E} \right] &= -\frac{D}{E} < 0 \\ \frac{\partial}{\partial p} \left[\frac{x_2^*}{E} \right] &= \frac{\theta_1}{\theta_2} \frac{D}{E} > 0 \end{aligned} \quad (10)$$

2.3. Extensions

The court production model above assumed a linear production function with equal weights across cases. The assumptions are consistent with the metrics commonly used to evaluate the efficiency of the justice system, especially in cross-country comparisons, where the number of overall resolved cases, independent of the complexity or type, is a key indicator. The targets assigned to courts in terms of reducing trial duration — measured by disposition time, i.e., the ratio of pending to resolved cases — also implicitly incentivize maximizing the number of resolved cases.

To account for judges prioritizing certain types of cases, diminishing marginal returns to effort, and potential complementarities between different case types, we can relax our assumptions by introducing weights in the production function, such as $F(x_1, x_2) = \eta x_1 + (1 - \eta)x_2$

with $\eta \in [0, 1]$,¹ or considering concave functions, such as Cobb–Douglas, $F(x_1, x_2) = x_1^\alpha * x_2^\beta$ with $\alpha, \beta \in (0, 1)$. Under these alternative specifications, our main result still holds: when demand decreases, holding all else constant, both total output and output per unit of input decrease. Furthermore, if type 1 cases carry more weight than type 2 cases in the weighted additive function (i.e., $\eta > \frac{1}{2}$), or if type 1 cases contribute relatively more to the production relative to their frequency in demand in the Cobb–Douglas function (i.e., $\frac{\alpha}{\beta} > \frac{1-p}{p}$), then total output and output per unit of input decline as demand for the more difficult cases increases.

3. Conclusion

This work highlights the importance of distinguishing between shifts in demand and actual productivity when evaluating public sector output. Proposing a simple model of courts, we show that the number of resolved cases (and, therefore, office productivity) is determined by the volume and composition of the newly filed cases. Our empirical findings, based on Italian court data, confirm the theoretical predictions: when demand fluctuates, measured output adjusts accordingly, but traditional productivity metrics can misrepresent real efficiency.

A key implication of our model is that standard productivity measures in the public sector, which typically rely on output per unit of input, may be biased in settings where labour is rigid, and outputs are not priced. Specifically, declines in demand mechanically lead to lower measured productivity, while increases in complex cases can reduce measured productivity even if efficiency remains unchanged. These findings call for the need to account for demand when measuring public sector performance, especially in domains where market-based pricing mechanisms are absent, and labour adjusts slowly to demand fluctuations.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

¹ In the absence of a dynamic structure, this would be equivalent to first-in-first-out rule under the assumption that case types and weights map to arrival order.

Data availability

The authors do not have permission to share data.

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