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The Immobile Incumbent Problem in a Model of Short-Term Wage-Posting

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Abstract: This paper takes the canonical Burdett-Mortensen model of wage-posting and relaxes the assumption that wages are set once-for-all, instead assuming they can only be committed one period at a time. It derives a closed-form solution for a steady-state Markov Rank-Preserving Equilibrium and shows how this relates to the canonical model and performs some comparative statics on it. But it is shown that a Rank-Preserving Equilibrium may fail to exist because employers have more monopsony power over existing workers than new recruits and that this non-existence can be a problem for plausible parameter values. It is shown how a Rank-Inverting Equilibrium may exist. It is argued that this problem is likely to occur in a wide range of search models.

Keywords: wage-posting; search; monopsony

1 Introduction

The model proposed by Burdett and Mortensen (1998) (henceforth BM) has rightly become regarded as a canonical way in which to analyze labour markets with frictions in which employers post wages. BM showed how in a world where all workers and employers are ex ante identical, frictions can lead to ex post differences in wages across workers and firms with larger firms paying higher wages. The economic mechanism underlying the model – that employers who pay higher

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wages find it easier to recruit and retain workers – is intuitively very appealing. Given this it is not surprising that labour economists have wanted to use the model to address a wider range of phenomena than were the subject of the original paper. But, the assumptions that wages are set once-for-all and that firms want to maximize steady-state profits are obviously problematic for using the model more widely and in empirical applications.

This paper analyzes a version of the canonical model in which firms and workers are identical, but (in contrast to the canonical model) employers can only commit to wages for the current period and forward-looking workers base their decisions not just on the current wage offered but on what they rationally expect to happen in the future. Those future expectations are tied down by the assumption that firms follow a stationary Markov strategy in which their wages depend only on the state variables (the level of employment in the last period for them and other firms) and the wages paid by other firms. In essence this is a finite-period version of the canonical model. If the economic environment is static, it is natural to seek a steady-state equilibrium in which the wages paid by individual firms are constant over time and, as a result, employment levels in every firm are as well. Such an equilibrium would be – to use the terminology of Moscarini and Postel-Vinay (2013) – a rank-preserving equilibrium (RPE) as a firm's position in the wage distribution is constant through time. Using a 'first-order' approach based on necessary conditions the paper derives a unique closed form solution for a steady-state RPE, shows, as one would expect, this corresponds to the equilibrium in the canonical Burdett-Mortensen model when employers do not discount the future, but that when there is discounting of the future, wages are lower than in the canonical model. If workers and firms have different discount factors equilibrium wages are higher the more myopic are workers and the more forward-looking are firms.

However, all of these results are derived under the assumption that an RPE exists. But the paper also shows that an RPE may not exist i.e. if we start from the proposed rank-preserving equilibrium some firms may increase profits by deviating from the proposed equilibrium strategy. The paper shows how a sufficient condition for the non-existence of a RPE is that the probability of workers losing their job per period is below some critical value. If workers and firms are myopic this critical value is 0.5. As we get arbitrarily close to the canonical model (i.e. as the discount factor of both employer and workers approaches one) the critical value becomes $1-\sqrt{\frac{1}{2}} \approx 0.293$. As actual labour market flows are lower than this for a reasonable definition of a 'period' (which is best thought of as the length of time for which employers can commit to wages), this suggests that non-existence could be more than a hypothetical concern. The origin of this potential non-existence problem is what we call the 'immobile incumbent' problem, which is that an employer may have more monopsony power over their existing workers than new recruits and that – if constrained to pay the same wage to incumbents and new recruits – this provides an incentive for large employers (who have many incumbents) to pay lower wages.

The intuition for the 'immobile incumbent' problem is most easily explained using the case in which both firms and workers are myopic so only care about current period pay-offs. If a firm pays a wage w it will have a quit rate $q(w)$ and a flow of recruits $R(w)$ (both depending only on the current wage because of the myopia assumption).[1] Consider a firm that has inherited an employment level of N_0 and wants to choose its wage to maximize current profits $[p - w]\{[1 - q(w)]N_0 + R(w)\}$. Like any monopsonist the employer will choose a higher wage the higher the elasticity of the labour supply curve facing it. That labour supply elasticity will be a weighted average of the elasticity of $[1 - q(w)]$ with respect to the wage and the elasticity of $R(w)$ with respect to the wage with the weight on the elasticity of $[1 - q(w)]$ being higher the higher the inherited level of employment. So if the elasticity of $[1 - q(w)]$ with respect to the wage is lower than the elasticity of $R(w)$ with respect to the wage (and the paper shows how that can easily be the case) then a higher inherited level of employment leads to a lower elasticity in the labour supply curve facing a firm and this induces it to choose a lower wage. This example has been for myopic firms and workers and the non-myopic case is more complicated but the basic intuition for why the existence of an RPE is problematic remains. The paper then investigates the nature of the equilibrium if an RPE does not exist. The paper does not provide a complete characterization of equilibrium but shows by construction how a Rank-Inverting equilibrium may exist in which a high-wage firm one period is a low-wage firm the next. Although wages in individual firms cycle, the aggregate wage distribution is constant through time.

The sixth section shows how heterogeneity in productivity makes it more likely that a rank-preserving equilibrium exists – essentially because high productivity firms have a permanent incentive to pay higher wages. The seventh section then discusses whether the conclusion about the possible non-existence of an RPE is specific to the model considered here or likely to be more generic. First, we consider the role played by commitment in the model – this is particularly pertinent given that an RPE has been shown to exist in outwardly similar models developed by Moscarini and Postel-Vinay (2013) who assume full commitment and Coles and Mortensen (2011) who assume less commitment in the sense that wages are set in continuous time so the length of a period can be thought of as very small. We argue that the generic existence of an RPE in both cases derives from assumptions other

1 These functions will depend on what every other firm in the market is doing but can be taken as exogenous from the perspective of an individual firm.

than the extent of commitment. Second, we show how non-existence of an RPE may arise in a model of directed search as opposed to the undirected search of the canonical model. Thirdly, the paper considers what happens if firms are allowed to offer wage-tenure contracts (as in Burdett and Coles 2003) as opposed to the single wage model of the canonical model. In this case it is shown that an RPE will exist but that a different problem arises – in the absence of commitment the ‘immobile incumbent’ problem may provide an incentive for employers to pay lower wages to incumbent workers than new recruits (because they have more monopsony power over incumbent workers) which is not in line with empirical evidence for most labour markets though Ransom (1993) argues it is relevant for the academic labour market.

The plan of the paper is as follows. The second section lays out the discrete-time version of the Burdett-Mortensen model analyzed in this paper. The third section then shows how one can derive a unique steady-state RPE under the assumption that this equilibrium exists. The fourth section then discusses the existence of the RPE, the fifth the possibility of a rank-inverting equilibrium. The sixth section discusses how heterogeneity in productivity makes an RPE more likely to exist and the seventh section whether the non-existence problem might be expected to occur in other related models. The final section of the paper presents a simple model in which an RPE exists. In this model it is assumed that employers can commit to future wages for existing workers but each period they set wages for new workers.

2 The Model

The model presented here can be thought of as a discrete-time version of the Burdett-Mortensen model in which wages are only set for a single period in a constant economic environment. The firms have constant returns to scale with marginal revenue of labour equal to p. There are L identical workers and M identical firms – we can, without loss of generality assume that $L, M = 1$ – the constant returns assumption means this has no consequence for the equilibrium wage distribution. Workers get a flow utility of b when unemployed.

The timing of each period is the following

1. at the start of each period, workers are either attached to a firm or unemployed. The state variable for each firm is the number of workers inherited from the previous period.
2. employers then set wages for that period which they must pay to all their workers.
3. employed workers lose their job with probability δ and become unemployed.

4. those workers who did not suffer job loss, whether previously employed or unemployed receive a job offer with probability λ, drawn at random from the firms. Job loss and a job offer are assumed to be mutually exclusive probabilities so we must have $(\delta + \lambda) < 1$. This assumption implies that workers who become unemployed do not have a chance to be re-employed within the period.
5. workers with job offers accept or reject job offers, there is production and wage payments.

3 Constructing a Rank-Preserving Equilibrium

In a RPE there will be a distribution of wages across firms, denoted by $w(f)$ where f takes all values in the unit interval, and firms pay the same wage every period. If the wages offered by firms are constant through time, employment levels will be as well and the higher-wage firms will have higher levels of employment. There will be a distribution of employment across firms, denoted by $N(f)$ and each firm occupies the same position in the wage and employment distributions. A proposed wage and employment distribution will be an RPE if no firm can increase the present discounted value of profits by deviating from $w(f)$. We now show how to construct an RPE if it exists. An argument familiar from the canonical Burdett and Mortensen model shows that the equilibrium wage distribution cannot contain any mass point and we will take this as a given.

3.1 The Distribution of Employment and Employment Flows in a RPE

First we will show that in any RPE the equilibrium distribution of employment across firm can be solved for independent of the wage distribution because workers will always want to move from a lower to a higher wage firm. We now derive the steady-state distribution of employment in an RPE.

In steady-state, the unemployment rate will be given by:

$$u = \frac{\delta}{\delta + \lambda} \tag{1}$$

The fraction of employed workers employed in firms at position f in the wage distribution or lower, $G(f)$ will, equating employment flows into and out of the mass of firms below f, in steady-state solve:

$$\left[\delta + \lambda\left(1 - f\right)\right] G\left(f\right)(1 - u) = \lambda f u \tag{2}$$

that can be solved to yield:

$$G(f) = \frac{\delta f}{\delta + \lambda(1-f)} \tag{3}$$

A firm at position f in the wage offer distribution will, in equilibrium, have quits equal to recruits so will have the following level of employment:

$$[\delta + \lambda(1-f)]N(f) = \lambda[u + (1-u)G(f)] \tag{4}$$

that can be solved to yield:

$$N(f) = \frac{\delta\lambda}{[\delta + \lambda(1-f)]^2} = \frac{\delta\lambda}{q(f)^2} \tag{5}$$

where the quit rate of the firm is given by:

$$q(f) = [\delta + \lambda(1-f)] \tag{6}$$

and the flow of recruits is:

$$R(f) = \frac{\delta\lambda}{[\delta + \lambda(1-f)]} = \frac{\delta\lambda}{q(f)} \tag{7}$$

Note that we have been able to solve for these distributions without reference to the equilibrium wage distribution; this is a consequence of the assumption in the Burdett-Mortensen model that workers will always move for a wage increase, however small, and that there can be no mass points in the equilibrium wage distribution. The implication is that all possible RPEs must have the same distribution of employment across firms. Additional useful results are:

$$R'(f) = \frac{\delta\lambda^2}{q(f)^2} = \lambda N(f) \tag{8}$$

and:

$$\frac{R'(f)}{R(f)} = \frac{\lambda}{q(f)} = \frac{-q'(f)}{q(f)} \tag{9}$$

which implies that the elasticity of recruits with respect to the position in the wage distribution is equal to minus the elasticity of quits with respect to the same variable.[2] What follows below will make extensive use of the equations derived in this section. These equations are identical to those obtained in the canonical Burdett-Mortensen model which is set in continuous time. This is useful as it makes comparison easy.

2 This is true in a wider class of search models and has been proposed by Manning (2003) to estimate the elasticity of recruits to the firm.

3.2 The Value Functions of Workers

Denote the value function for an unemployed worker by V^u, and for a worker employed at the start of the period in a firm at position f in the wage distribution by $V(f)$. Define the value functions to be the value of that state before any job offers or unemployment shocks arrive i.e. at the beginning of a period. Assume the discount factor for workers is β_w. Given this, we will have in a steady-state:

$$V^u = (1-\lambda)\left[b + \beta_w V^u\right] + \lambda \int \left[w(f) + \beta_w V(f)\right] \mathrm{d}f \tag{10}$$

and:

$$\begin{aligned} V(f) = {} & \delta\left[b + \beta_w V^u\right] + \left[1 - \delta - \lambda(1-f)\right]\left\{w(f) + \beta_w V(f)\right\} \\ & + \lambda \int_f \left[w(f') + \beta_w V(f')\right] \mathrm{d}f' \end{aligned} \tag{11}$$

Differentiating (11) we have that:

$$V'(f) = \left[1 - q(f)\right]\left\{w'(f) + \beta_w V'(f)\right\} \tag{12}$$

that solves to:

$$V'(f) = \frac{\left[1 - q(f)\right] w'(f)}{1 - \beta_w\left[1 - q(f)\right]} \tag{13}$$

This is useful in what follows. Note that the reservation wage of workers will be equal to b because of the assumption that job offers arrive at the same rate whether employed or unemployed.

3.3 The Employer's Decision

The state variable for an employer is its inherited level of employment[3] and the assumption that firms follow Markov strategies means that this is a sufficient statistic for the strategy followed by the firm. As we are interested in the steady-state and not transitional dynamics one can – without loss of generality – use as the state variable the position of a firm in the steady-state employment distribution, f_0 – this makes the derivation of the equilibrium a lot easier. Denote by $\Pi(f_0)$ the value function of a firm at position f_0. This value function will obviously depend on the strategies used by other firms but as these are constant in a steady-state we

3 Also the distribution of past employment levels across firms as a whole but we have already shown this cannot vary across possible RPEs so is suppressed in the interests of notational simplicity.

do not make this dependence explicit in the interests of notational simplicity. Each period the firm will choose the wage it pays, w. Workers will then quit from and be recruited to this firm according to a rule that will be derived shortly. Given these decisions, next period, the firm will end up with a level of employment that must be in the range $[N(0), N(1)]$ – so that one can think of the firm as ending up at some, possibly different point, point in the steady-state employment distribution. The reason is that $N(0)$ is the level of employment in a firm that was at that level last year, only ever recruits from unemployment and always loses workers to other firms when they have alternative job offers – as long as $w \geq b$ (and all firms will do this) one can never do worse than this. Similarly one can never do better than $N(1)$ as the highest wage firm can do no better than only having quits to unemployment (which are exogenous) and recruiting all workers, employed and unemployed, whenever they have a job offer.

We will assume that employers set wages to maximize the present discounted value of profits, treating the wages offered by all other firms as exogenous. The position in the employment distribution next period will be a function of the initial position and the wage offered this period – denote this function by $\psi\left(f_0, w\right)$ – we derive this function below. So under the assumption that firms follow a stationary Markov strategy i.e. that workers expect a firm that deviates from the equilibrium strategy this period to return next period to the equilibrium strategy (which will not be the same wage if employment has changed), the value function for the employer can be written as:

$$\Pi\left(f_0\right) = \max_w \quad [p - w]N\left(\psi\left(f_0, w\right)\right) + \beta\Pi\left(\psi\left(f_0, w\right)\right) \tag{14}$$

where β is the discount factor of the employer (we retain the possibility that this differs from the workers' discount factor). In a steady-state rank-preserving equilibrium we will have:

$$\Pi(f) = \frac{[p - w(f)]N(f)}{1 - \beta} \tag{15}$$

Substituting (15) into (14) we have, after some re-arrangement, that:

$$(1 - \beta)\Pi\left(f_0\right) = \max_w \quad \left[p - (1 - \beta)w - \beta w(\psi\left(f_0, w\right))\right]N\left(\psi\left(f_0, w\right)\right) \tag{16}$$

so one can think of the firm as maximizing a level of current profits where the employment it will have is the current employment level and the 'wage' is a weighted average of the current wage and the future wage with the weight on the future wage being the discount factor.

To make any progress we need to derive an expression for the function $\psi\left(f_0, w\right)$ – this comes from the mobility rule followed by workers.

3.4 Workers' Mobility Decision

Now let us consider how we can derive $\psi(f_0, w)$. Suppose all workers think that a firm currently at position f_0 and offering w will be at position $\psi(f_0, w)$ next period. Suppose that workers can observe the current employment level in the firm and the current wage offered – we return to this below. If workers are assumed to believe that employers are following a Markov strategy and they expect reversion to equilibrium behaviour, the state variable of the firm next period will be $\psi(f_0, w)$ and the value of a job at such a firm is $V(\psi(f_0, w))$. Now define f as the solution to the following equation:

$$w(f) + \beta_w V(f) = w + \beta_w V(\psi(f_0, w)) \tag{17}$$

The right-hand side is the value one obtains from a job in the deviating firm – the current offered wage and then the future value of a job at the position the firm ends up with. The left-hand side is the value from accepting a job at a firm that is following the equilibrium strategy and is at position f. The implication of (17) is that a worker in the deviating firm will accept a wage offer from a firm at a position above f and, symmetrically, a worker in a firm at position below f will accept a wage offer from the deviating firm. Equation (17) assumes a solution in the unit interval but there is no point in a firm making setting a wage w so high that (17) implies a solution in which even $f = 1$ has the left-hand side less than the right-hand side as the current wage can be lowered without affecting quit and recruitment rates. Similarly there is no point in a firm making setting a wage w so low that (17) implies a solution in which even $f = 0$ has the left-hand side greater than the right-hand side as the lowest equilibrium offer will be the utility from unemployment and offering lower than this will make all workers quit and an inability to recruit anyone which cannot be profit-maximizing.

Equation (17) has three notions of the position of a firm – its initial position f_0, the position it is viewed as being equivalent to by workers when making job decisions and ψ, the position that the firm will end up at the end of the period. If the firm pays a wage $w = w(f_0)$, then we will have $f = \psi = f_0$. If $w > w(f_0)$, then we will have $f > \psi > f_0$ while $w < w(f_0)$ implies $f < \psi < f_0$. Although (17) has been written to derive f given (f_0, w, ψ) it is more convenient to think of the firm having f as the choice variable and (17) then tells us the wage required to have this level of attractiveness to workers. Using this idea, change notation to let us write $\psi(f_0, f)$. From the mobility decision for workers one can derive what the relationship between f, ψ, and f_0 will be. From (17) we know that the offer from the deviating firm will be regarded as equivalent to a firm at position f so that quit and recruitment decisions will be based on this and hence employment will satisfy:

$$N(\psi(f_0, f)) = [1 - q(f)]N(f_0) + R(f) \tag{18}$$

And with the change in the decision variable to f, and using (17), (16) becomes:

$$(1-\beta)\Pi\left(f_0\right) = \max_f \pi\left(f, f_0\right) \equiv \left\{p - (1-\beta)w(f) - \beta w(\psi) - (1-\beta)\beta_w[V\left(f\right) - V(\psi)]\right\}N(\psi) \tag{19}$$

where $\psi = \psi\left(f_0, f\right)$. If employers do not discount the future so that $\beta = 1$, the maximand in (19) reduces to $\{p - w(\psi)\}\ N(\psi)$ i.e. it is only the steady-state level of profits that is relevant as one would expect. But if there is some discounting of the future this is not the case. And if workers discount the future then if a firm is currently at a position f_0 but wants to be at position ψ next period it has to make sure workers evaluate its current job offer at a position $f > \psi > f_0$ so that the term $[V(f) - V(\psi)]$ is positive and this adds to the employer costs. Intuitively, the labour supply curve facing an employer is less elastic in the wage when there is discounting and short-term wage contracts – this intuition will be important later.

Figure 1 shows how the labour supply curve facing an employer is affected by the assumption of no-commitment in wage-setting – this is helpful for understanding the intuition of the results. Figure 1 first shows the steady-state relationship between wages and employer size – this can be thought of as the long-run labour supply curve facing the firm as if it paid a wage w for ever it would end up with a level of employment on this curve. Now consider a firm starting on this long-run labour supply curve and considering paying a wage that deviates from it. There are two reasons for why the short-run labour supply curve will be less elastic than the long-run labour supply curve. First, because it takes time for employment to adjust. Suppose that workers expect the new wage to be paid for ever (or that workers are myopic) so that the quit rate and recruitment flow faced by a firm

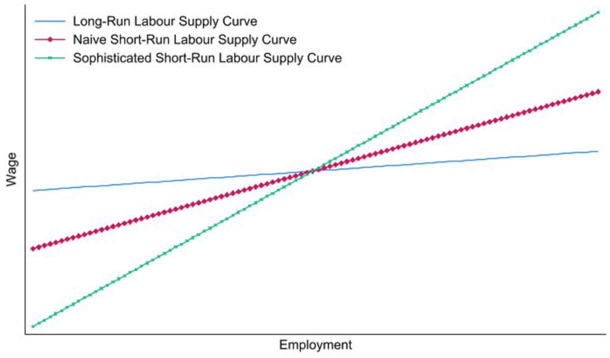

Figure 1: Long- and short-run labour supply curves.

doing this deviation is the same as that faced by a firm offering the new wage on a permanent basis. But employment in the deviating firm this period will be $[1-q]N_0+R$ and this will be lower than the employment level in a firm that has always paid the new wage because the initial level of employment is lower – call this the naive short-run labour supply curve. But there is a second effect at work if workers are forward-looking and not myopic. Because employment is lower at the end of the period in a firm that has only paid w this period compared to one that has always paid it, workers do not expect the new higher wage to be sustained – they expect the wage to fall back in the subsequent period. This means that the quit rate will be higher and flow of recruits lower giving a second reason why employment will be less sensitive to current wages than the long-run labour supply curve would suggest. The size of this second effect depends on how forward-looking are workers – if they do not care about the future at all they will only pay attention to the current wage in making mobility decisions. This is represented by the 'sophisticated' labour supply curve in Figure 1 This way of understanding the model will be of use in giving an intuition for some comparative statics results later.

In deriving the above, we assumed that workers can observe both the current level of the offered wage and the current level of employment in the firm. They can then deduce whether the firm is deviating from the steady-state and then use this information to derive what they expect future wages to be. But, suppose that workers only observe the current wage offer and not the current level of employment or the history of wages. Workers would then be unable to detect deviation from the steady-state as long as the offered wage is an equilibrium offer for some firm. As no firm will deviate from the equilibrium, workers would simply assume that the offered wage is the equilibrium offer for the firm. Decisions would then be based on the current wage alone as workers would assume this wage will be maintained in the future. From the employer perspective, this case is then isomorphic to the case $\beta_w = 0$ where workers are myopic. This makes the myopic worker case of more interest than one might have thought.

3.5 Solving for the Rank-Preserving Equilibrium

We are now in a position to derive the RPE, assuming that it exists (an issue we return to later). We are looking for an RPE equilibrium in which each firm chooses its current wage to maximize the present discounted value of profits treating the decisions of all other firms as given and these decisions have the rank-preserving property. We will use the first-order conditions that are necessary for profit maximization and show that these have a unique solution.

Proposition 1. *An RPE, if it exists must be the unique solution to the differential equation:*

$$\frac{\partial \ln\left[p - w(f)\right]}{\partial f} = -2\lambda \frac{1 - \beta_w[1 - q(f)]}{1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]} \tag{20}$$

with the initial condition $w(0) = b$

Proof. See Appendix. ■

We can use the differential equation (20) to obtain some comparative static results.

Proposition 2. *Assuming that an RPE exists, the equilibrium wage distribution is:*

(a) *increasing (in the sense of first-order stochastic dominance) in β holding β_w fixed.*

(b) *decreasing (in the sense of first-order stochastic dominance) in β_w holding β fixed.*

(c) *increasing (in the sense of first-order stochastic dominance) in β if $\beta = \beta_w$.*

Proof. See Appendix. ■

Let us provide some intuition for these results. First, consider part (a). If employers have a higher discount factor, they put greater weight on future relative to current profits. This reduces the temptation to exploit the short-run immobility of workers by cutting current wages so tends to make employers choose higher wages. The higher wages of one employer then spills over to raise the wages of other employers. There is a certain paradox here – one can interpret (somewhat loosely) a higher β as a longer period of committed wages. It would seem in the interest of an individual employer to commit to wages for a longer period but this is to the collective disadvantage of employers.

Now, consider part (b). If workers have a higher discount factor, they put greater weight on future wages relative to current wages. This makes their mobility decision less responsive to current wage offers from employers as was explained earlier. This has the effect of making the labour supply curves to employers less elastic and this leads to lower wages. Again there is a certain paradox here – workers want to make employers think they are more responsive to current wages than they really are and being myopic is one way to do this.

Part (c) shows that when one puts the two effects together it is that in part (b) that dominates. Wages are higher the lower is the discount factor.

It is perhaps useful to compare the RPE with the equilibrium in the canonical B-M model in which employers make a one-off decision about wages to maximize steady-state profits. The equilibrium in this case is well-known - all firms must make the same level of profits that is given by:

$$\left[p - w(f)\right]N(f) = \left[p - b\right]N(0) \tag{21}$$

Using (5) and (6) this can be written as:

$$\left[p - w(f)\right] = \left[p - b\right]\frac{q(0)^2}{q(f)^2} \tag{22}$$

which, differentiating, can be written as:

$$\frac{\partial \ln\left[p - w(f)\right]}{\partial f} = -\frac{2\lambda}{q(f)} \tag{23}$$

Comparison of (20) and (23) shows that the equilibria are identical when $\beta = 1$ (though note that no assumption is needed on β_w). This is what we would expect – if firms do not discount future profits they only care about steady-state profits and this is essentially the canonical model. More generally, if we compare the two equilibria we have the following result:

Proposition 3. *Wages are lower (in the sense of first-order stochastic dominance) in the RPE than in the canonical equilibrium.*

Proof. See Appendix. ■

These results have been derived assuming that job offer arrival rates are the same whether employed or unemployed – what would we find if they were different? As is well-known the lowest wage would be the reservation wage and that this is itself a function of the wage offer distribution. If (as most estimates suggest) the offer arrival rate is higher for the unemployed than the employed then a result like Proposition 2 would probably be true as the reservation wage is higher the higher are wages generally.

For what it is worth,[4] the closed-form solution for the equilibrium wage distribution is given by:

4 Probably not much except that it involves the differentiation of an inverse trigonometrical function, knowledge not used since secondary school.

$$\left[p - w(f)\right] = \left[p - b\right] \frac{1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]}{1 - \beta[1 - q(0)] - \beta_w q(0)[1 - q(0)]} . \exp[Z(f) - Z(0)] \tag{24}$$

where:

$$Z(f) \equiv \sqrt{\frac{2 - \beta - \beta_w}{(1 - \beta)\beta_w}} \arctan\left(\sqrt{\frac{\beta_w}{1 - \beta}} q(f) + \frac{\beta - \beta_w}{2\sqrt{(1 - \beta)\beta_w}} \right) \tag{25}$$

All of this has assumed that an RPE exists, and used necessary conditions to show that there is a unique RPE if one exists. But as the next section shows this is problematic.

4 The Existence of the Rank-Preserving Equilibrium

So far, we have used first-order conditions to derive necessary conditions that an RPE must satisfy. The easiest way to demonstrate a lack of existence is to show that the second-order conditions are not satisfied for some firm at the proposed equilibrium. This means that a sufficient condition for non-existence of an RPE is that at some point f_0, the profit function (19) is convex in f at $f = f_0$. The following result provides some useful information on this.

Proposition 4. *At the constructed RPE if $\beta < 1$:*

(a) *A necessary (sufficient) condition for the existence (non-existence) of an RPE is that:*

$$\delta \geq (<) q^*(\beta, \beta_w) \tag{26}$$

where $q^*(\beta, \beta_w)$ is the unique solution in the unit interval to the equation:

$$[2q - 1] - \beta[2q - 1](1 - q)^2 + 2\beta_w q(1 - q)^3 = 0 \tag{27}$$

(b) *$q^*(\beta, \beta_w)$ is decreasing in β and β_w, reaching a minimum value of* $1 - \sqrt{\frac{1}{2}} \approx 0.293$ *when $\beta = \beta_w = 1$ a maximum value of 0.5 when $\beta_w = 0$.*

Proof. See Appendix. ■

While this result shows that the existence of an RPE is not guaranteed, it is natural to ask whether this non-existence result is likely to be relevant in practice. To answer that question needs a definition of a ‘period’. The most natural interpretation of a period is that it represents the time for which employers can commit to a certain level of wages. That would suggest a period of a year at most. Even over the time horizon of a year, turnover rates are below the critical value for non-existence suggesting this may be more than a hypothetical concern. However, this definition of a period is somewhat loose as the model assumes that workers can move at most once within a period, while in reality they are moving in continuous time even if wages are set in discrete time.

The general intuition for why a RPE may fail to exist is that given in the introduction – that the elasticity of the labour supply curve facing a firm may be lower if it has more current workers inducing it to want to pay a lower wage. But it is perhaps useful to see a particular case worked out – the myopic case is easiest for which we can also provide a necessary condition for non-existence of an RPE. Considering the myopic case also helps provide some intuition for the non-existence problem.

Proposition 5. *If* $\beta = \beta_w = 0$, $\delta < 0.5$ *is a necessary and sufficient condition for non-existence of an RPE.*

Proof. See Appendix. ■

If the firm inherits employment at position f_0 and chooses f then the Appendix shows profits can be written as:

$$\pi(f, f_0) = [p - w(f)]N(f, f_0) \tag{28}$$

where:

$$N(f, f_0) = [(1 - q(f)]N(f_0) + R(f)] \tag{29}$$

One way to write the first-order condition for the maximization of the profit function in (28) is:

$$\frac{\partial \ln[p - w(f)]}{\partial \ln f} = \frac{\partial \ln N(f, f_0)}{\partial \ln f} \tag{30}$$

This is a version of the standard first-order condition for the optimal wage of a monopsonist – the right-hand side of (30) is the elasticity of the labour supply curve facing the employer. The left-hand side of (30) does not vary with the initial position, f_0, so the question of how the optimal choice of position relates to the initial position depends on how the elasticity of the labour supply curve varies with the initial level of employment. Using (29) the labour supply elasticity can be written as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = f\frac{-q'(f)N(f_0) + R'(f)}{N(f, f_0)} = \frac{-\varepsilon_{qf}q(f)N(f_0) + \varepsilon_{Rf}R(f)}{N(f, f_0)} \tag{31}$$

where ε_{qf} is the elasticity of the quit rate with respect to f, and where ε_{Rf} is the elasticity of recruitment with respect to f. Neither of these elasticities depend on f_0.

Now define $\rho(f, f_0)$ to be the share of total current employment that is from retained workers $\rho(f, f_0) = [1 - q(f)]N(f_0)/N(f, f_0)$. Then, using these bits of notation we can re-write (31) as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = f\frac{-q'(f)N(f_0) + R'(f)}{N(f, f_0)} = \left[-\varepsilon_{qf}\frac{q(f)}{1-q(f)} - \varepsilon_{Rf}\right]\rho(f, f_0) + \varepsilon_{Rf} \tag{32}$$

When one increases the initial level of employment the only part of the right-hand side of (32) that changes is that there is a rise in $\rho(f, f_0)$. Equation (32) then tells us that the effect of a rise in the initial level of employment on the elasticity of the labour supply curve facing the employer depends on the sign of $[-\varepsilon_{qf}\frac{q(f)}{1-q(f)} - \varepsilon_{Rf}]$. But from (9) we have that $-\varepsilon_{qf} = \varepsilon_{Rf}$ so that (32) can be written as:

$$\frac{\partial \ln N(f, f_0)}{\partial \ln f} = \varepsilon_{Rf}\left[\frac{q(f)}{1-q(f)} - 1\right]\rho(f, f_0) + \varepsilon_{Rf} = \varepsilon_{Rf}\frac{2q(f)-1}{1-q(f)}\rho(f, f_0) + \varepsilon_{Rf} \tag{33}$$

so that a rise in the initial level of employment raises the elasticity of the labour supply curve facing the employer if $q(f) > 0.5$, and reduces it otherwise. This is exactly the condition that came out of (27).

Given this non-existence result one might wonder whether an RPE ever exists. In the myopic case note from (33) that the sign of $\frac{\partial^2 \ln N(f, f_0)}{\partial \ln f \partial f_0}$ depends only on whether $q(f)$ is above or below half. So if this is above 0.5 for all f then it is always the case that initially large employers will prefer to choose higher wages. And the condition that $q(f) \geq 0.5$ for all f corresponds to the condition that $\delta \geq 0.5$ so this is a necessary and sufficient condition for the RPE to exist in the myopic case.

Proposition 4 has as a condition that $\beta < 1$ i.e. it applies arbitrarily close to but not at the canonical equilibrium with $\beta = 1$ – one might wonder what happens in the limit. When $\beta = 1$, the objective function of the firm does not depend on its initial condition – see (19) and in the RPE every derivative of the profit function with respect to f is equal to zero. This can be thought of as the canonical equilibrium being on a knife-edge between existence and non-existence for some values of (λ, δ). And if we are arbitrarily close to $\beta = \beta_w = 1$, then whether the RPE exists or not depends on whether $\delta > 0.293$ or not. So there is a discontinuity in the existence of a rank-preserving equilibrium at $\beta = 1$.

5 The Possibility of a Rank-Inverting Equilibrium

The discussion above has shown that a RPE may not exist. This raises the obvious question of what is the equilibrium in this case. This section does not provide a full characterization of equilibrium but shows, by means of example, that the exact opposite type of equilibrium to an RPE may exist – call this a Rank-Inverting Equilibrium (RIE). In a rank-inverting equilibrium those firms that inherit a large stock of employment from last period choose the lowest wages this period. Note that in a RIE we cannot have a steady-state in which the employment levels in a particular firm remain constant (except for the median firm). To understand why, suppose the contrary, that firms' employment levels are constant through time which also means their position in the employment distribution must also be constant through time. Consider for clarity the largest firm. This must every period be setting the lowest wage in a RIE so must have the lowest recruitment rate and highest quit rate. But this means that it cannot remain the largest firm for ever, contradicting the steady-state assumption.

The particular example constructed in this section is the myopic model introduced above because that is the simplest case. Because of the argument above, a RIE must have non-constant employment levels for firms and, for the myopic case, it must have a cycle lasting two periods. However this is consistent with the overall distribution of firm sizes and wages being constant through time as firms simply swap places from one period to the next (with the exception of the median firm). A firm that starts period 0 as the largest firm sets the lowest wage and becomes the smallest firm entering period 1. It then sets the highest wage in period 1 and becomes the largest firm entering period 2, reproducing the situation in period 0. More generally a firm starting at position f in the distribution will be at position $(1 - f)$ next period before returning once more to position f. The following result derives the distribution of workers across firms.

Proposition 6. *If (δ, λ) are small enough, a Rank-Inverting Equilibrium exists. As $\lambda \to 0$ wages collapse to the reservation level.*

Proof. See Appendix. ■

6 Heterogeneity in Productivity

An RPE is appealing not just because it simplifies the mathematics (though it does) but because it is empirically appealing – firm wages are highly correlated over time and there is a large literature documenting the existence of firm wage effects (see Kline 2025, for a review). If the model presented here does not guarantee existence for realistic parameter values then one might wonder whether the non-existence result derives from some specific features of the model or might be expected to occur in more general models. In this section we show that the non-existence problem is less severe if there is heterogeneity in firm productivity. The intuition is simple – if there is heterogeneity in productivity across firms (for which there is ample empirical evidence) an RPE is more likely to exist as high productivity firms have an ever present incentive to be the larger firms. Consider the myopic model. Denote by $p(f)$ the marginal product of labour in a firm at position f in the productivity distribution. In any RPE it must be the case that the most productive firms have more workers so that:

$$\pi(f, f_0) = [p(f_0) - w(f)][(1 - q(f))N(f_0) + R(f)] \tag{34}$$

Each employer will maximize this with respect to f – this leads to the first-order condition:

$$-w'(f)[(1 - q(f))N(f_0) + R(f)] + [p(f_0) - w(f)][R'(f) - q'(f)N(f_0)] = 0 \tag{35}$$

In an RPE this must be satisfied for $f = f_0$. Using (7), (6) and (5) this can be written as:

$$2\lambda w(f) + w'(f) = 2\lambda p(f) \tag{36}$$

a differential equation with solution:

$$e^{2\lambda f} w(f) = w(0) + \int_o^f 2\lambda e^{2\lambda g} p(g)\mathrm{d}g \tag{37}$$

Integrating by parts and using the fact that $w(0) = b$, this can be written as:

$$p(f) - w(f) = e^{-2\lambda f}[p(0) - b] + \int_o^f e^{2\lambda(g-f)} p'(g)\mathrm{d}g \tag{38}$$

For the heterogeneous productivity case $p'(f) = 0$, this reduces to the previous solution. Substituting into (34) leads to the following expression:

$$\pi(f, f_0) = \left[p(f_0) - p(f) - e^{-2\lambda f}[p(0) - b] - \int_o^f e^{2\lambda(g-f)} p'(g)dg \right][(1 - q(f))N(f_0) + R(f)] \tag{39}$$

and, for the RPE to exist we need to check that $f = f_0$ is a global maximum of this function. One can do this through simulation. In Figure 2 we compare the regions of the parameter space where an RPE exists for the case when there is no heterogeneity in productivity and the case when there is productivity heterogeneity and

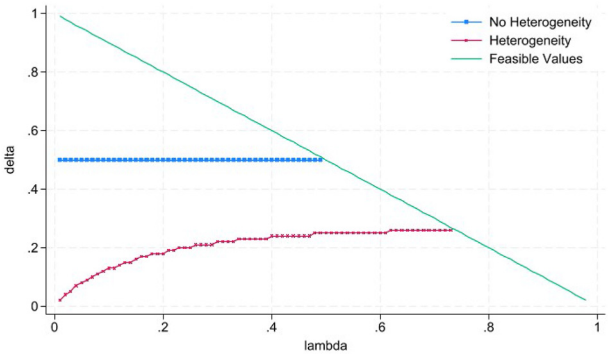

Figure 2: Existence of a rank-preserving equilibrium with and without heterogeneity in productivity. The area below the diagonal line and above the relevant line shows the parameter values for which a rank-preserving equilibrium exists in the myopic model where $p(0) - b = 1$ and $p'(f) = 1$ for the heterogeneous productivity case.

$p(0) - b = 1$ and $p'(f) = 1$. As $\delta + \lambda \leq 1$ the only possible parameter values are below the diagonal. In the case where all firms have the same productivity an RPE exists if $\delta > 0.5$ which is the area above the horizontal line. But with heterogeneity in productivity, the area where an RPE exists is larger, being above the curved line. So heterogeneity in productivity does make it more likely that a RPE exists but needs to be large enough.

7 The Existence of a Rank-Preserving Equilibrium in Other Search Models

One concern is that there is something special about the model here that leads to the immobile incumbent problem. This section shows that the non-existence problem can occur in models with different assumptions about commitment, and in models with directed as opposed to undirected search. Because the intention in this section is to show how the non-existence problem can occur in other models, we only need to consider cases in which the problem occurs and it is simplest to do that by considering the case where workers are myopic so their mobility decisions depend only on current wages. If there is a strict non-existence problem with myopic workers there will, by continuity, be a problem when workers do value the future but only to a small extent. The root cause of the non-existence – which is that the employer might be expected to have more monopsony power over existing than new workers is likely to occur in a wider range of models than the one considered in detail here.

7.1 The Commitment Assumption

It has been assumed that firms can commit to wages for a period of finite length – so the model can be thought of as one of limited commitment. Given that Moscarini and Postel-Vinay (2013) assume full commitment is possible and Coles and Mortensen (2011) assume no commitment is possible (their model being set in continuous time) and both have existence of an RPE one might conclude there is something unusual about the limited commitment case. However, in this section, we show that the guaranteed existence of an RPE in these models derives from features other than the different level of commitment that is assumed feasible for firms.

To see this, first consider the case where all other features of the model of this paper are retained but firms are assumed to be able to commit to the future path of wages but constrained to pay the same wage to all workers in each period – this is the assumption made by Moscarini and Postel-Vinay (2013). Commitment on future wages is only of value to employers if workers are not myopic. From this we can deduce that the commitment and no-commitment equilibria coincide if workers are myopic. This in turn implies that there can be non-existence in a model with full commitment of wages. If workers are not myopic the commitment and no-commitment equilibrium differ but for β_w close enough to zero there will, by continuity, be non-existence of an RPE if there is non-existence in the myopic worker case.

Given that Moscarini and Postel-Vinay (2013) – henceforth MPV – prove existence of an RPE in a much more general economic environment (e.g. they allow for aggregate productivity shocks and transitional dynamics), this raises the question of the source of the difference in conclusions. The difference stems from one seemingly innocuous difference in assumption about the timing of wage payments. In this paper it is assumed that production occurs and wage payments are made at the end of each period. In contrast, in MPV it is assumed that wages are paid at the beginning of the next period. This might be thought to be a trivial difference but it has the consequence in MPV of introducing into their model an initial wage (which they call the time 0 wage), which transfers value between workers and firms before the firm chooses an initial contract (quite what this time 0 wage corresponds to in the real world is unclear). This time 0 wage has no role in influencing mobility decisions of workers and, hence, no allocational role. MPV discuss the role played by this initial wage and emphasize correctly that the precise mechanism by which it is determined does not influence the nature of the equilibrium in later periods. The RPE result does not necessarily apply to this initial wage – MPV discuss a case where "if the firm has some employees at time 0 and all the bargaining power, it optimally pays the lowest possible wage….by making them just indifferent between staying or quitting into unemployment" (MPV 2013, p. 1557). Because larger firms will pay higher wages from time 1 onwards from the RPE results – this implies that the time 0 wage must be lower in larger firms in order to make sure the value of a job in all firms is equal to the value of unemployment.[5]

Although the mechanism by which the time 0 wage is determined is unimportant for the nature of the continuation equilibrium and so might be thought a mere technicality, the existence of the pre-contractual time 0 wage is vital for their result on the existence of an RPE – if there is no pre-contractual wage then the proof of the existence of a rank-preserving equilibrium fails. This can be seen most clearly in the two-period example considered by MPV where the removal of what they term the period 0 wage causes the main result to fail. What happens in the MPV model is that there is an immobile incumbent problem but it shows up solely in the initial wage as it is efficient to exploit incumbent workers using this initial wage as it has no allocational consequences. But if one assumes that no such initial wage exists then there is no guarantee of the existence of an RPE in their model because firms can then only use current wages to exploit the relatively immobile incumbents. To summarize, the existence of an RPE in Moscarini and Postel-Vinay (2013) rests on the availability, at the beginning of time before any contracts have been signed, of a transfer between workers and employers that has no allocational role.

Now consider Coles and Mortensen (2011) who assume that wages are set in continuous time. This can be thought of as the limit of the model of this paper as the length of the period goes to zero. If we restrict attention to Markov strategies then, as discussed at the end of the previous section, the equlibrium converges to one in which all employers pay b. The intutition is that in continuous time firms are unable to commit to paying wages higher than b for more than an instant, so workers' mobility decisions do not respond to wage changes and the labour supply to employers becomes completely inelastic inducing employers to pay the minimum possible wage. Coles and Mortensen point out that if there is Markov behaviour wage deviation will have (almost) no effect on the expected value of the worker's future earnings and thus there will be no turnover response. But if turnover does not respond to the wage deviation, then cutting wages is a profitable strategy. This can be thought of as a version of the Diamond paradox (Diamond 1971) – if workers have no threat to leave, wages get driven down to reservation level. Coles and Mortensen want to avoid this prediction but one could argue that it is sensible. Somewhat loosely (because transitions are certainly happening in continuous time so more than one transition might be made per 'period') one can interpret the length

5 This can also be seen in the two-period deterministic example considered in MPV where it is shown that the second period wage is increasing in the level of initial employment. But it is possible that the first period wage is decreasing in the initial level of employment e.g. if the value of a job promised to workers does not vary across firms.

of a period as the length of time for which employers are able to commit to paying a certain level of wages. Then the limiting result is what we might expect to happen if firms cannot commit to wages for more than an instant (a second, a millisecond?) – one goes to work in the morning and the employer changes the wage in the course of the working day. If this equilibrium does not correspond to what we observe this is because in the real world firms are able to commit to wages for a certain period of time so that the finite time period model is the correct one to use and the Diamond paradox problem does not arise in that model. The continuous time model is not simply a convenient way to represent a finite time model – there is a substantive difference in the nature of the equilibrium. We should not be surprised by this conclusion that the extent of possible commitment affects the real equilibrium.

But Coles and Mortensen (2011) do not assume that agents pursue Markov strategies and this allows them to prove the existence of an RPE. In particular they assume that an employer who deviates from the steady-state equilibrium is assumed to pay the reservation wage thereafter. Given these expectations there is no point in an employer doing anything different so this is an equilibrium.[6] However even allowing such punishment strategies does not overcome the potential non-existence problem when firms can commit to wages for a finite period. The simplest way to see this is to consider the case where workers are myopic – in this case it is the current wage alone that influences workers' decisions and beliefs about what might follow deviations from the equilibrium strategy are irrelevant. So, models with non-Markov strategies cannot be guaranteed to have an RPE if employers are able to commit to wages for any finite period.

This section has argued that the problem of potential non-existence of an RPE is likely to occur whenever current wages have some allocational role (which seems plausible) and the restriction to non-Markov strategies is not crucial.

7.2 Directed Search

The model here has assumed that search is undirected and one might wonder whether similar problems arise if search is directed (Delacroix and Shi 2006; Kircher 2009; Menzio and Shi 2010, 2011; Moen 1997) which can be thought of as the case where it is costless to sample all vacancies in the market. This section gives an example of a very stylized directed search model in which the non-existence of a RPE occurs and for similar reasons as in the undirected search model. Assume

6 They do not claim the equilibrium is unique and it probably is not though Coles (2001) does show that the proposed equilibrium tends to the canonical equilibrium as the interest rate goes to zero so one could argue that this gives the choice of their non-Markov strategy some plausibility.

the number of firms and workers to be fixed and equal. The timing of events and actions in each period is assumed to be the following:

1. Each firm has only one job which can be filled or vacant at the start of each period. The state variable for the firm is whether the job was filled the previous period.
2. Employed workers lose their job with probability δ and become unemployed.
3. Employers then set wages for that period which they must pay if their job is filled at the end of the period. Employers are assumed myopic so choose the wage to maximize expected profits in the current period.
4. All workers, both employed and unemployed can, with probability λ, see all the wages offered. Workers are assumed myopic so apply for the vacant job that offers the highest expected utility in the current period.
5. The probability of filling a vacant job, $h(a)$ depends on the number of applicants, a according to the usual formula in directed search models $h(a) = 1 - e^{-a}$. A worker who applies to a job with a applicants will get the job with probability $\mu(a) = h(a)/a$. Workers with job offers accept or reject job offers, there is production and wage payments.

In this model firms with filled jobs are trying to set wages to deter their workers from applying to other jobs and quitting while firms with vacant jobs are trying to set wages to attract recruits. Our aim is not to provide a complete characterization of equilibrium but to give an example of an equilibrium where firms with filled jobs choose to pay lower wages than firms with vacant jobs (within the set-up this is the only dimension of employer size). That this is possible is summarized in the following result.

Proposition 7. *If λ is small enough there is an equilibrium in which:*

a. *employers with filled jobs pay a wage equal to b*
b. *employers with vacant jobs pay a wage $w^* > b$*
c. *all workers, whether employed or unemployed apply to the vacant jobs*

Proof. See Appendix. ■

This Propostion shows that something akin to the lack of an RPE can occur in directed as well as undirected search models and for the same reasons – firms have more monopsony power over existing workers than over new recruits so there is a temptation to pay lower wages to existing workers.

7.3 Wage Discrimination

This section considers what happens when we relax the assumption that employers have to pay the same wage to all workers in each period and allow the employer to pay different wages to different workers although we maintain the assumption that wages must be set before any other wage offers have been received. In this situation the only wage differentiation of interest to employers will be that the employer will choose different wages for new recruits and existing workers. Allowing employers to vary wages with seniority has been considered by Burdett and Coles (2003, 2010) in the context of undirected search and Shi (2009) in the context of directed search – both papers assume that the employer can commit to wage-tenure contracts. We assume workers are myopic so recruitment and quits depends only on the current wage. Suppose that the employer offers a wage w_R to new recruits, that this leads to a flow of recruits $R(w_R)$ (which will depend on what other firms are offering), and a wage w_S to senior existing workers that leads to a quit rate $q(w_S)$. If the firm has an initial level of employment N_0 (which will be its state-variable) the value function for the firm in a steady-state will be given by:

$$\Pi(N_0) = (p - w_R)R(w_R) + (p - w_S)[1 - q(w_S)]N_0 + \beta\Pi[R(w_R) + [1 - q(w_S)]N_0] \tag{40}$$

A very useful result (proved in the Appendix) is that $\Pi(N_0)$ is linear in N_0 i.e. can be written as $\Pi(N_0) = \pi_0 + \pi_1 N_0$. Equation (40) can then be written as:

$$\Pi(N_0) = (p - w_R + \beta\pi_1)R(w_R) + (p - w_S + \beta\pi_1)[1 - q(w_S)]N_0 + \beta\pi_0 \tag{41}$$

(41) makes it clear that there is a separability in the profits to be obtained from new recruits and from existing workers. This has a number of implications. First it means that all offered wages to new recruits must yield the same equilibrium level of profits from new recruits – denote this by π_R^* i.e. we must have:

$$(p - w_R + \beta\pi_1)R(w_R) = \pi_R^* \tag{42}$$

Secondly it means that all offered wages to senior workers must yield the same equilibrium level of expected profits per worker – denote this by π_S^* i.e. we must have:

$$(p - w_S + \beta\pi_1)[1 - q(w_S)] = \pi_S^* \tag{43}$$

Thirdly the separability in the profit function between the part of profits from new recruits and the part from senior workers that can be noted in (41) means there is no reason why a firm that pays high wages to new recruits should also pay high wages to existing workers. But the smallest amount of heterogeneity in productivity across firms will resolve this indeterminacy – high productivity firms will want to pay high wages both to new recruits and to exisitng workers. We will assume that those firms which pay high wages to new recruits also pay high wages to senior workers and the equilibrium we derive can then be thought of as the limit as productivity differentials between firms go to zero. If this is the case we would then appear to have something that resembles a RPE – large firms pay higher wages than small firms to both new recruits and senior workers and these differentials will persist through time. But though the problem of the lack of an RPE has been solved by allowing contracts of this type, another potential problem arises, namely that it may be optimal for an employer to pay a lower wage to senior workers than to new recruits something that does not seem plausible given that senior workers are generally paid more than new recruits.

The following Proposition provides a sufficient condition for this to be the case.

Proposition 8. *(a) If $\delta < \frac{1}{2}$ then some employers will pay lower wages to senior workers than new recruits.*

(b) If $\left(\delta + \lambda\right) < \frac{1}{2}$ then all employers will pay lower wages to senior workers than new recruits.

Proof. See Appendix. ■

The root of this result is the same as the cause of the non-existence of an RPE in the canonical model – if turnover rates are low, employers have more monopsony power over existing workers and they will pay them lower wages. The immobile incumbent problem takes a different form here, but it remains a problem.

8 A Simple Model with a Rank-Preserving Equilibrium

This section develops a very simple model, embodying elements of commitment and wage discrimination that can deliver an RPE. Assume that an employer can commit – at the time of hiring – to a contract for workers that specifies not just the current wage but also future wages.[7] There are some reasons for this wage contract to have wages increasing with job tenure (e.g. to limit turnover – see Burdett and Coles 2003), to have stable wages over time (because of worker risk aversion and

7 This is a more plausible form of commitment than that assumed in many models where future wages can be promised to workers who are not yet hired.

imperfect capital markets) or even to have wages that decrease with job tenure (e.g. because workers are more impatient than employers). Here, we do not wish to focus on these issues and simply assume that the employer can only commit to a contract that pays the worker a constant wage during their tenure with the firm. But we also assume that new recruits can be offered a different wage from incumbent workers (though this ability will not be exercized in the steady-state equilibrium considered here).

In this set-up each period the employer only has a decision about the wage to pay the new recruits as the wage paid to senior workers has already been determined. If the offer of a wage w to new workers, leads to a flow $R(w)$ of recruits, who in later years quit at a rate $q(w)$, the present discounted flow of profits from the current recruits will be:

$$\pi = \frac{(p-w)R(w)}{1-\beta[1-q(w)]} \tag{44}$$

where β is the employer's discount factor. Note that this does not depend on the number of senior workers the firm has or the wage which they are paid so the profit function is separable. If firms all have the same level of productivity this means that there is no reason for a firm that paid its recruits a high wage in the past to pay a high wage today i.e. to have an RPE. But, as in the previous section, the smallest amount of productivity differentials will lead to a determinate outcome in which there is a RPE.

The equilibrium in this case must have all offered wages yielding the same present discounted value of profits. As is usual in this sort of model, the lowest offered wage will be b. In a steady-state, the wages offered each period will be the same so that the expressions given in (6) and (7) will be valid and one can read off the equilibrium wage distribution as being the solution to:

$$\frac{(p-w(f))R(f)}{1-\beta[1-q(f)]} = \frac{(p-b)R(0)}{1-\beta[1-q(0)]} \tag{45}$$

If $\beta = 1$ this corresponds to the canonical model but if $\beta < 1$, wages are lower as one would expect. The reason this model does not suffer from the 'immobile incumbent' problem comes from the combination of the assumed commitment and the wage discrimination. The assumed commitment means there is never a period in which the firm is choosing a wage for senior workers and the wage discrimination means that the firm does not have to choose a single wage to maximize the sum of rent extraction from incumbents and new recruits.

9 Conclusions

This paper has taken the classic canonical Burdett and Mortensen (1998) model of wage-posting and changed the assumption that firms set wages once-for-all. Instead it has been assumed that firms set wages one period at a time without commitment. Workers take account of this fact in making their mobility decisions. The focus has been on the steady-state equilibrium of this model. It was shown how one can derive the unique Rank-Preserving Equilibrium but that the RPE may fail to exist. The source of the problem is what is termed the 'immobile incumbent' problem that, for low turnover rates, employers would be expected to have more monopsony power over incumbent workers than new recruits. It has been argued that this problem may be quite common in search models, though taking different forms in different models. Those models without the problem have generally solved the problem by introducing a contractual richness (that may or may not be plausible) that allows the problem to be concentrated in one variable. The model presented here does have its limitations. Because this paper is just about the steady-state it cannot be directly applied to questions like the cyclical variability of wages and employment. Nor does it produce more realistic predictions about things like the equilibrium distribution of wages than the canonical model. But until the steady-state is properly understood, those applications should perhaps wait.

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Appendix

A Proof of Proposition 1

Taking the derivative of the log of (19) with respect to f, leads to the following first-order condition:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{N'(\psi)}{N(\psi)}\frac{\partial \psi}{\partial f} - \frac{(1-\beta)w'(f) + \beta w'(\psi)\frac{\partial \psi}{\partial f} + (1-\beta)\beta_w\left[V'(f) - V'(\psi)\frac{\partial \psi}{\partial f}\right]}{\{p - (1-\beta)w(f) - \beta w(\psi) - (1-\beta)\beta_w[V(f) - V(\psi)]\}} \tag{46}$$

Using (13) this can be simplified to:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{N'(\psi)}{N(\psi)}\frac{\partial \psi}{\partial f} - \frac{\frac{(1-\beta)w'(f)}{1-\beta_w[1-q(f)]} + \frac{[\beta-\beta_w+\beta_w q(\psi)]w'(\psi)}{1-\beta_w[1-q(\psi)]}\frac{\partial \psi}{\partial f}}{\{p - (1-\beta)w(f) - \beta w(\psi) - (1-\beta)\beta_w[V(f) - V(\psi)]\}} \tag{47}$$

By differentiating (18) we have that:

$$N'(\psi)\frac{\partial \psi}{\partial f} = -q'(f)N(f_0) + R'(f) = \lambda[N(f_0) + N(f)] \tag{48}$$

A necessary condition that an RPE must satisfy is that the first-order condition for the maximization of (19) must be satisfied at $f = \psi = f_0$. Putting this into (47) leads to the following:

$$2\lambda = \frac{(1-\beta) + q(f)[\beta - \beta_w + \beta_w q(f)]}{1 - \beta_w[1 - q(f)]}\frac{w'(f)}{[p - w(f)]} \tag{49}$$

and re-arrangement leads to (20). The lowest wage offered must be equal to the reservation wage b. Given this, the RPE must be unique. ■

B Proof of Proposition 2

The general pattern of proof is the following. Because all equilibrium wage distributions start from $w(0) = b$, one can rank them in terms of stochastic dominance if the right-hand side of (20) is higher or lower for all f. A higher value of the right-hand side is associated with profit per worker that declines faster which implies wages rise faster so are higher.

Proof. (a) By inspection one can see that the right-hand side of (20) is decreasing in β holding β_w fixed proving the result.

(b) Differentiating the right-hand side of (20) with respect to β_w we obtain:

$$\frac{\partial^2 \ln\left[p - w(f)\right]}{\partial f \partial \beta_w} = 2\lambda \frac{[1 - q(f)]^2(1 - \beta)}{[1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]]^2} > 0 \tag{50}$$

so this proves the result.

(c) If $\beta = \beta_w$ then (20) can be written as:

$$\frac{\partial \ln\left[p - w(f)\right]}{\partial f} = -2\lambda \frac{1 - \beta[1 - q(f)]}{1 - \beta[1 - q(f)^2]} \tag{51}$$

and differentiating leads to:

$$\frac{\partial^2 \ln\left[p - w(f)\right]}{\partial f \partial \beta} = -2\lambda \frac{q(f)[1 - q(f)]}{[1 - \beta[1 - q(f)^2]]^2} < 0 \tag{52}$$

■

C Proof of Proposition 3

Given that $w(0) = b$ in both cases, wages will be lower (in the sense of first-order stochastic dominance) in the RPE than in the canonical equilibrium if profit per worker declines slower in the canonical case than the short-run case. As long as $\beta < 1$ as we have:

Proof

$$-2\lambda \frac{1 - \beta_w[1 - q(f)]}{1 - \beta[1 - q(f)] - \beta_w q(f)[1 - q(f)]} > -2\lambda \frac{1}{q(f)} \tag{53}$$

Re-arranging this can be written as:

$$(1 - \beta)[1 - q(f)] > 0 \tag{54}$$

which is true if $\beta < 1$. ■

D Proof of Proposition 4

First, let us change the variable of choice for the firm from f to ψ. There will be a mapping from ψ and f_0 to f that will be given by (18) – denote this by $f(f_0, \psi)$. From (19) the log of profits can be written as:

$$\begin{aligned} &\ln \pi(f_0, \psi) \\ &= \ln N(\psi) + \ln\left[p - (1-\beta)w(f) - \beta w(\psi) - (1-\beta)\beta_w\{V(f) - V(\psi)\}\right] \\ &\equiv \ln N(\psi) + \ln Z(f_0, \psi) \end{aligned} \tag{55}$$

The first-order condition for the choice of ψ can be written as:

$$\frac{\partial \ln \pi(f_0, \psi)}{\partial \psi} = \frac{1}{Z(f_0, \psi)} \frac{\partial Z(f_0, \psi)}{\partial \psi} + \frac{N'(\psi)}{N(\psi)} \tag{56}$$

and that, in the RPE this must be zero at $\psi = f_0$ for all f_0 in the unit interval. This means that we must have:

$$\frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} + \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} = 0 \tag{57}$$

when evaluated at the RPE and $\psi = f_0$. One implication of (57) is that:

$$\operatorname{sgn} \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = -\operatorname{sgn} \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} \tag{58}$$

when evaluated at the RPE and $\psi = f_0$. One implication of this is that the log profit function will be convex in ψ if and only if the cross-partial is positive. The RPE will fail to exist if, for a firm at any position, the log profit function is convex at the proposed RPE so this result tells us to look for the sign of the cross-partial. This is Proposition 4a.

Differentiating (56) we have that:

$$\frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = -\frac{1}{Z(f_0, \psi)^2} \frac{\partial Z(f_0, \psi)}{\partial \psi} \frac{\partial Z(f_0, \psi)}{\partial f_0} + \frac{1}{Z(f_0, \psi)} \frac{\partial^2 Z(f_0, \psi)}{\partial \psi^2} \tag{59}$$

from which, after some rearrangement one can derive that:

$$\operatorname{sgn} \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi^2} = -\operatorname{sgn} \frac{\partial^2 \ln \pi(f_0, \psi)}{\partial \psi \partial f_0} = \operatorname{sgn} \left(\frac{\partial \ln\left(\frac{\partial Z(f_0, \psi)}{\partial f_0}\right)}{\partial \psi} - \frac{\partial \ln Z(f_0, \psi)}{\partial \psi} \right) \tag{60}$$

Now consider the last two terms in (60). First, $\frac{\partial \ln\left(\frac{\partial Z(f_0,\psi)}{\partial f_0}\right)}{\partial \psi}$ – from (55) we have that:

$$\frac{\partial Z(f_0,\psi)}{\partial f_0} = -(1-\beta)\left[w'(f) + \beta_w V'(f)\right]\frac{\partial f}{\partial f_0} \tag{61}$$

Using (13) this can be written as:

$$\frac{\partial Z(f_0,\psi)}{\partial f_0} = \frac{-(1-\beta)w'(f)}{1-\beta_w+\beta_w q(f)}\frac{\partial f}{\partial f_0} \tag{62}$$

and, at the proposed RPE (20) this can be written as:

$$\frac{\partial Z(f_0,\psi)}{\partial f_0} = \frac{-2\lambda(1-\beta)[p-w(f)]}{1-\beta[1-q(f)]-\beta_w q(f)[1-q(f)]}\frac{\partial f}{\partial f_0} \tag{63}$$

Taking logs we have that:

$$\begin{aligned}\ln\frac{\partial Z(f_0,\psi)}{\partial f_0} &\\ = \ln 2\,\lambda(1-\beta) + \ln[p-w(f)] - \ln\left[1-\beta[1-q(f)]-\beta_w q(f)[1-q(f)]\right] &\\ + \ln\left(-\frac{\partial f}{\partial f_0}\right)&\end{aligned} \tag{64}$$

Now, differentiating this with respect to ψ as (60) says we need to we have that:

$$\begin{aligned}&\frac{\partial \ln\frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial\psi}\\ &= \frac{\partial \ln[p-w(f)]}{\partial f}\frac{\partial f}{\partial\psi} + \frac{\lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]}{1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2}\frac{\partial f}{\partial\psi} + \frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial\psi}\end{aligned} \tag{65}$$

Using (20) this can be written as:

$$\begin{aligned}&\frac{\partial \ln\frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial\psi}\\ &= \frac{-2\lambda[1-\beta_w+\beta_w q(f)]\frac{\partial f}{\partial\psi} + \lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]\frac{\partial f}{\partial\psi}}{1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2} + \frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial\psi}\end{aligned} \tag{66}$$

Now consider the values of $\frac{\partial f}{\partial\psi}$ and $\frac{\partial \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial\psi}$ - these can be derived from (18). From (18) we have that:

$$N'(\psi) = \lambda\left[N(f_0)+N(f)\right]\frac{\partial f}{\partial\psi} \tag{67}$$

We are interested in evaluating this expression at $\psi = f = f_0$ (the steady-state) when (67) becomes:

$$\frac{\partial f}{\partial \psi} = \frac{N'(f)}{2\lambda N(f)} = \frac{1}{q(f)} \tag{68}$$

where the final equality comes from differentiation of (5).

Now differentiate (18) with respect to f_0 to give:

$$0 = \lambda\left[N(f_0) + N(f)\right]\frac{\partial f}{\partial f_0} + [1 - q(f)]N'(f_0) \tag{69}$$

from which we can derive:

$$\ln\left(-\frac{\partial f}{\partial f_0}\right) = \ln[1 - q(f)] + \ln N'(f_0) - \ln\ \lambda - \ln\left[N(f_0) + N(f)\right] \tag{70}$$

Differentiating (70) with respect to ψ (as required for (66)) leads to:

$$\frac{\partial\ \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi} = \frac{\lambda}{[1 - q(f)]}\frac{\partial f}{\partial \psi} - \frac{N'(f)}{\left[N(f_0) + N(f)\right]}\frac{\partial f}{\partial \psi} \tag{71}$$

We are interested in evaluating this expression at $\psi = f = f_0$ (the steady-state) when (71) becomes, after using (5) and (68):

$$\frac{\partial\ \ln\left(-\frac{\partial f}{\partial f_0}\right)}{\partial \psi} = \frac{\lambda\left[2q(f) - 1\right]}{q(f)^2[1 - q(f)]} \tag{72}$$

Now putting (72) into (66) we have that at $\psi = f = f_0$

$$\frac{\partial\ \ln\frac{\partial Z(f_0,\psi)}{\partial f_0}}{\partial \psi} = \frac{-2\lambda[1 - \beta_w + \beta_w q(f)] + \lambda\left[(\beta - \beta_w) + 2\beta_w q(f)\right]}{q(f)\left[1 - \beta + (\beta - \beta_w)q(f) + \beta_w q(f)^2\right]} + \frac{\lambda\left[2q(f) - 1\right]}{q(f)^2[1 - q(f)]} \tag{73}$$

Now consider the other term in (60), $\frac{\partial\ \ln\ Z(f_0,\psi)}{\partial \psi}$. This is simple to evaluate at the proposed steady-state because, from the first-order condition for profit maximization we must have:

$$\frac{N'(\psi)}{N(\psi)} + \frac{\partial\ \ln\ Z\left(f_0, \psi\right)}{\partial \psi} = 0 \tag{74}$$

so that at the steady-state:

$$\frac{\partial\ \ln\ Z\left(f_0, \psi\right)}{\partial \psi} = -\frac{N'(\psi)}{N(\psi)} = -\frac{2\lambda}{q(f)} \tag{75}$$

Combining (73) and (75) with (60) we have that the second-order conditions for a maximum at the profit functioning the proposed steady state is that:

$$\frac{-2\lambda[1-\beta_w+\beta_w q(f)]+\lambda\left[(\beta-\beta_w)+2\beta_w q(f)\right]}{q(f)\left[1-\beta+(\beta-\beta_w)q(f)+\beta_w q(f)^2\right]}+\frac{\lambda\left[2q(f)-1\right]}{q(f)^2[1-q(f)]}\geq-\frac{2\lambda}{q(f)} \tag{76}$$

A necessary condition for the existence of an RPE is that this is true for all f. Hence a sufficient condition for the non-existence of an RPE is that the inequality in (76) is violated for any f. After some rearrangement the sufficient condition for non-existence can be written as:

$$A(q,\beta,\beta_w)=[2q-1]-\beta[2q-1](1-q)^2+2\beta_w q(1-q)^3<0 \tag{77}$$

for some $q(f)$. Inspection reveals that $A(0,\beta,\beta_w)<0$ and $A(1,\beta,\beta_w)>0$. We will show that there is only one value of q at which $A(0,\beta,\beta_w)=0$. We will show that at the point where $A(0,\beta,\beta_w)=0$ we have $\frac{\partial A(q,\beta,\beta_w)}{\partial q}>0$. Note that when $A(0,\beta,\beta_w)=0$ we must have, from (77) $q\leq 0.5$. Differentiate (77) to give:

$$\frac{\partial A(q,\beta,\beta_w)}{\partial q}=2\left[1-\beta(1-q)^2\right]+\beta[2q-1](1-q)+2\beta_w(1-q)^2[1-4q] \tag{78}$$

Using (77) to eliminate the term in β_w we can write (78) as:

$$\frac{\partial A(q,\beta,\beta_w)}{\partial q}=2\left[1-\beta(1-q)^2\right]+\beta[2q-1](1-q)+\frac{A(q,\beta,\beta_w)-[2q-1]\left[1-\beta(1-q)^2\right]}{q(1-q)} \tag{79}$$

At the point where $A(q,\beta,\beta_w)=0$, we have, from (79) that, after some rearrangement:

$$q(1-q)\frac{\partial A(q,\beta,\beta_w)}{\partial q}=\left[1-\beta(1-q)^2\right](1-2q^2)+\beta[2q-1]q(1-q)^2 \tag{80}$$

(80) is decreasing in β when $q<0.5$. Hence:

$$q(1-q)\frac{\partial A(q,\beta,\beta_w)}{\partial q}\geq[2-q]q(1-2q^2)-[1-2q]q(1-q)^2>0 \tag{81}$$

where the final inequality follows from the fact that $[2-q]>(1-q)^2$ and $(1-2q^2)>[1-2q]$.

This shows that there is a unique value $q^*(\beta,\beta_w)$ such that $A(q,\beta,\beta_w)<0$ when $q<q^*(\beta,\beta_w)$ and $A(q,\beta,\beta_w)>0$ when $q>q^*(\beta,\beta_w)$. A necessary condition for existence (or sufficient condition for non-existence) of an RPE is that

$A(q(f), \beta, \beta_w) > 0$ for all f – this is hardest to satisfy when the quit rate is lowest which is in the highest wage firm when the quit rate is δ. Hence the condition in (27) and this proves Proposition 4b.

To prove the first part of Proposition 3c note that from $A(q^*(\beta, \beta_w), \beta, \beta_w) = 0$ we must have:

$$\frac{\partial A(q^*, \beta, \beta_w)}{\partial q} \frac{\partial q^*}{\partial \beta} + \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta} = 0 \tag{82}$$

As $\frac{\partial A(q^*, \beta, \beta_w)}{\partial q} > 0$, this implies that:

$$\operatorname{sgn} \frac{\partial q^*}{\partial \beta} = -\operatorname{sgn} \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta} = -\operatorname{sgn}[1 - 2q](1 - q)^2 < 0 \tag{83}$$

as, from (77) we must have $2q^*(\beta, \beta_w) - 1 < 0$.

To prove the second part of Proposition 4c a similar argument shows that:

$$\operatorname{sgn} \frac{\partial q^*}{\partial \beta_w} = -\operatorname{sgn} \frac{\partial A(q^*, \beta, \beta_w)}{\partial \beta_w} = -\operatorname{sgn} 2q(1 - q)^3 < 0 \tag{84}$$

E Proof of Proposition 5

In the myopic case, $\beta = \beta_w = 0$, (20) then becomes:

$$\frac{\partial \ln[p - w(f)]}{\partial f} = -2\lambda \tag{85}$$

with solution

$$[p - w(f)] = [p - b]e^{-2\lambda f} \tag{86}$$

This has been derived using the first-order conditions for profit maximization i.e. necessary conditions that must be satisfied in an RPE. But they are not sufficient conditions – for that we need to ensure global profit maximization. So let us now consider whether those conditions are satisfied. In the myopic case workers will base their mobility decisions only on the current wage offered so that $\psi(f_0, f) = f$. So we can think of a firm as choosing its current position in the wage offer distribution. If it inherits employment of position f_0 and chooses f then profits will be given by:

$$\begin{aligned} \pi(f, f_0) &= [p - w(f)]N(f, f_0) = [p - w(f)][(1 - q(f))N(f_0) + R(f)] \\ &= [p - b]e^{-2\lambda f}[(1 - q(f))N(f_0) + R(f)] \end{aligned} \tag{87}$$

Using (86). It is easiest to analyze this by taking logs to give us:

$$\ln \pi(f, f_0) = \ln[p - b] - 2\lambda f + \ln[(1 - q(f))N(f_0) + R(f)] \tag{88}$$

Differentiating with respect to f leads to:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = -2\lambda + \frac{-q'(f))N(f_0) + R'(f)}{(1-q(f))N(f_0) + R(f)} \tag{89}$$
$$= -2\lambda + \lambda \frac{N(f_0) + N(f)}{(1-q(f))N(f_0) + R(f)}$$

This is equal to zero when evaluated at $f = f_0$. But another necessary condition is that the second-order conditions are satisfied. Differentiating (89) again with respect to f, we have that:

$$\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} \tag{90}$$
$$= -\frac{\lambda^2[N(f_0) + N(f)]^2}{[(1-q(f))N(f_0) + R(f)]^2} + \frac{\lambda N'(f)}{[(1-q(f))N(f_0) + R(f)]}$$
$$= -\left[2\lambda + \frac{\partial \ln \pi(f, f_0)}{\partial f}\right]^2 + \frac{2\lambda^2 N(f)/q(f)}{[(1-q(f))N(f_0) + R(f)]}$$

The sign of (90) depends on the sign of the numerator. At $f = f_0$, we have that the first derivative is zero so that:

$$\operatorname{sgn} \frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} = \operatorname{sgn}\left\{-4\lambda^2 + \frac{2\lambda^2}{q(f)}\right\} = \operatorname{sgn}[1 - 2q(f)] \tag{91}$$

If $q(f) < 0.5$ for any f, which corresponds to $\delta < 0.5$ then (91) shows that the profit function is convex in f, so that, far from the proposed RPE picking out a maximum of the profit function it actually picks out a minimum. In this case the RPE fails to exist.

F Proof of Proposition 6

Before proving the main Proposition the following result about the distribution of workers across firms in a RIE is useful.

Result 6a: If $G(f)$ is the fraction of workers at position f or below in last period's wage distribution, then in a RIE $G(f)$ must satisfy:

$$G(f) = \frac{f\{[1-q(f)](\delta+\lambda) + \delta\}}{1 - [1-q(f)][1-q(1-f)]} \tag{92}$$

where $q(f)$ is given by (6).

Proof. Consider the fraction of workers who, in a rank-inverting equilibrium will end up at position f or below in this period's wage distribution. In the steady-state this must also be equal to $G(f)$. This will, as usual, be the fraction of workers who are in this group who do not get a shock that causes them to leave it plus the recruits into this group from unemployment. Because of the rank-inverting nature of the proposed equilibrium the proportion of workers who are in this group is not $G(f)$ as in a RPE but $[1 - G(1 - f)]$. Putting this together we have that, in a steady state:

$$(1-u)G(f) = [1-(\delta+\lambda(1-f))](1-u)[1-G(1-f)] + \lambda fu \tag{93}$$

Using the expression for the unemployment rate (1) this can be written as:

$$G(f) = [1-(\delta+\lambda(1-f))][1-G(1-f)] + \delta f \tag{94}$$

Now invert the roles of f and $(1 - f)$ and write this as:

$$G(1-f) = [1-(\delta+\lambda f)][1-G(f)] + \delta(1-f) \tag{95}$$

(94) and (95) can be solved to give (92). ■

Now consider the Proof of Proposition 6. From (92) one can derive the employment level in a firm at position f as $N(f) = (1 - u)G'(f)$. Now consider a firm that inherits a level of employment $N(f_0)$ and chooses a current position in the wage offer distribution of f. In the myopic model its log profits must be given by:

$$\ln \pi(f, f_0) = \ln[p - w(f)] + \ln[R(f) + (1-q(f))N(f_0)] \tag{96}$$

Taking first-order conditions leads to:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{\partial \ln[p - w(f)]}{\partial f} + \frac{R'(f) + q'(f)N(f_0)}{[R(f) + (1-q(f))N(f_0)]} \tag{97}$$

Now in a RIE we have that $R'(f) = \lambda N(1 - f)$ so (97) can be written as:

$$\frac{\partial \ln \pi(f, f_0)}{\partial f} = \frac{\partial \ln[p - w(f)]}{\partial f} + \frac{\lambda[N(1-f) + N(f_0)]}{[R(f) + (1-q(f))N(f_0)]} \tag{98}$$

In a RIE this first-order condition must be equal to zero at $f = 1 - f_0$ or, equivalently, $f_0 = 1 - f$. Putting this in (98) leads to:

$$\frac{\partial \ln[p - w(f)]}{\partial f} = -\frac{2\lambda N(1-f)}{N(f)} \tag{99}$$

that has, as a solution:

$$[p - w(f)] = [p - b]e^{-2\lambda \int_0^f \frac{N(1-x)}{N(x)}\, \mathrm{d}x} \tag{100}$$

where we have used the fact that the lowest offered wage must be equal to b.[8] This RIE has been derived using the first-order conditions. But one has to check that at this proposed solution the choice of $f = 1 - f_0$ is a global maximum. Substituting (100) into (96) leads to:

$$\ln \pi(f, f_0) = \ln[p - b] - 2\lambda \int_0^f \frac{N(1-x)}{N(x)}\, dx + \ln[R(f) + (1 - q(f))N(f_0)] \tag{101}$$

Differentiating this with respect to f we have that:

$$\begin{aligned} \frac{\partial \ln \pi(f, f_0)}{\partial f} &= -2\lambda \frac{N(1-f)}{N(f)} + \frac{[R'(f) - q'(f)N(f_0)]}{[R(f) + (1 - q(f))N(f_0)]} \\ &= -2\lambda \frac{N(1-f)}{N(f)} + \frac{\lambda[N(1-f) + N(f_0)]}{[R(f) + (1 - q(f))N(f_0)]} \end{aligned} \tag{102}$$

Differentiating this again with respect to f we have that:

$$\begin{aligned} \frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} &= 2\lambda \frac{N'(1-f)N(f) + N'(f)N(1-f)}{N(f)^2} \\ &\quad - \frac{\lambda N'(1-f)}{[R(f) + (1 - q(f))N(f_0)]} - \frac{\lambda^2[N(1-f) + N(f_0)]^2}{[R(f) + (1 - q(f))N(f_0)]^2} \end{aligned} \tag{103}$$

Now, in a RIE, we must have that:

$$N(f) = R(f) + [1 - q(f)]N(1-f) \tag{104}$$

and, differentiating this with respect to f we have that:

$$\begin{aligned} N'(f) &= R(f) - q'(f)N(1-f) - [1 - q(f)]N'(1-f) \\ &= 2\lambda N(1-f) - [1 - q(f)]N'(1-f) \end{aligned} \tag{105}$$

Using (105) to eliminate $N'(f)$ from (103) we have that:

$$\begin{aligned} &\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} \\ &= 2\lambda \frac{N'(1-f)[N(f) - (1 - q(f))N(1-f)] + 2\lambda N(1-f)^2}{N(f)^2} \\ &- \frac{\lambda N'(1-f)}{[R(f) + (1 - q(f))N(f_0)]} - \frac{\lambda^2[N(1-f) + N(f_0)]^2}{[R(f) + (1 - q(f))N(f_0)]^2} \end{aligned} \tag{106}$$

8 This is a feature of the myopic case but if workers are not myopic, the lowest wage will be lower than b because workers expect the lowest-wage firm this period to be a higher-wage firm next period.

$$= 2\lambda \frac{N'(1-f)R(f) + 2\lambda N(1-f)^2}{N(f)^2}$$
$$- \frac{\lambda N'(1-f)}{[R(f) + (1-q(f))N(f_0)]} - \frac{\lambda^2[N(1-f) + N(f_0)]^2}{[R(f) + (1-q(f))N(f_0)]^2}$$

Using (102), and after some rearrangement (106) can be written as:

$$\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} \tag{107}$$
$$= \frac{\lambda N'(1-f)\left[2R(f)[R(f) + (1-q(f))N(f_0)] - N(f)^2\right]}{N(f)^2[R(f) + (1-q(f))N(f_0)]}$$
$$+ \frac{4\lambda^2 N(1-f)^2}{N(f)^2} - \left[\frac{\partial \ln \pi(f, f_0)}{\partial f} + \frac{2\lambda N(1-f)}{N(f)}\right]^2$$

Using (104) this can be re-arranged to give:

$$\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} \tag{108}$$
$$= \frac{\lambda N'(1-f)[N(f)[2R(f) - N(f)] + 2R(f)(1-q(f))[N(f_0) - N(1-f)]]}{N(f)^2[R(f) + (1-q(f))N(f_0)]}$$
$$- \frac{\partial \ln \pi(f, f_0)}{\partial f}\left[\frac{\partial \ln \pi(f, f_0)}{\partial f} + \frac{4\lambda N(1-f)}{N(f)}\right]$$

We can use (108) to provide necessary and sufficient conditions for an RIE to exist. A necessary condition is that at $f_0 = 1 - f$, the second derivative of the profit function is negative. Substituting in $f_0 = 1 - f$ and using the fact that $\frac{\partial \ln \pi(f, f_0)}{\partial f} = 0$ at this point we have that:

$$\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} = \frac{\lambda N'(1-f)N(f)[2R(f) - N(f)]}{N(f)^2[R(f) + (1-q(f))N(f_0)]} \tag{109}$$

and this will be negative if $2R(f) < N(f)$ for all f. This is a condition that can be satisfied if λ and δ are small enough.

This is only a necessary condition for the RIE to exist – now consider how we can provide a sufficient condition. If $2R(f) < N(f)$ for all f the above argument shows that the profit function must have at least a local maximum at $f = 1 - f_0$. If this is not a global maximum then there must also be a local interior minimum of the profit function. At a local minimum we have that $\frac{\partial \ln \pi(f, f_0)}{\partial f} = 0$ and that $\frac{\partial^2 \ln \pi(f, f_0)}{\partial f^2} > 0$. But this will not be possible if the first term in (108) is everywhere negative. So the first term in (108) being everywhere negative is a sufficient condition for the existence of an RIE. This can be written as the condition:

$$N(f)[2R(f) - N(f)] + 2R(f)(1 - q(f))[N(f_0) - N(1 - f)] \leq 0 \tag{110}$$

This condition is harder to satisfy the larger is f_0 so a sufficient condition is:

$$N(f)[2R(f) - N(f)] + 2R(f)(1 - q(f))[N(1) - N(1 - f)] \leq 0 \tag{111}$$

After some re-arrangement, (111) can be written as:

$$N(f) \geq 2R(f)\left[1 + \frac{(1 - q(f))[N(1) - N(1 - f)]}{N(f)}\right] \tag{112}$$

One can choose the parameters of the model so that this is satisfied e.g. if λ and δ are small enough.

G Proof of Proposition 7

Denote the equilibrium unemployment rate at the end of each period by u^*, if everyone behaves as described in the Proposition, then each period the number of vacant jobs will be given by $\left[u^* + \delta(1 - u^*)\right]$. If all workers who see jobs apply for vacant jobs then the number of applicants per job will be given by:

$$a^* = \frac{\lambda}{\left[u^* + \delta(1 - u^*)\right]} \tag{113}$$

The equilibrium level of unemployment must be the level that equates flows into and out of unemployment i.e. satisfies:

$$\delta\left(1 - u^*\right) = \mu\left(a^*\right)\lambda\left[u^* + \delta\left(1 - u^*\right)\right] \tag{114}$$

Using (113) to eliminate u^* from (114) we have the following expression for a^*:

$$\mu\left(a^*\right) = \frac{\delta}{1 - \delta}\frac{a^* - \lambda}{\lambda^2 a^*} \tag{115}$$

The left-hand side of (115) is decreasing in a^*, the right-hand side increasing so there is a unique solution which must have $a^* > \lambda$. It can readily be checked that a^* must be decreasing in δ and increasing in λ.

Now, consider the optimal strategy for employers. If all employers with vacant jobs offer a wage w^* then the expected gain in utility of a worker who applies for one of these jobs (as compared to not applying) is, in equilibrium, given by:

$$E^* = \mu\left(a^*\right)\left[w^* - b\right] \tag{116}$$

Note that this will be true both for unemployed workers and workers in filled jobs under the assumption (to be justified later) that their employers only pay a wage b. Now consider the behavior of an employer with a vacant job who is considering paying a wage w. As is usual in directed search models such a job will attract applicants until the expected utility from applying to this job is equal to E^*. This implies that wages will be chosen to solve the following profit maximization problem:

$$\max\left(p-w\right)h(a) \quad \text{s.t.} \quad \mu(a)(w-b)=\mu\left(a^*\right)\left(w^*-b\right) \tag{117}$$

The first-order condition for this optimization problem can be written as:

$$-h(a)-h'(a)\frac{\mu(a)}{\mu'(a)}\frac{\left(p-w\right)}{(w-b)}=0 \tag{118}$$

In the hypothesized equilibrium this derivative must be zero at (a^*, w^*) which, after some re-arrangement, leads to the following expression for w^*:

$$w^*=b+\frac{a^*h'(a^*)}{h(a^*)}\left[p-b\right] \tag{119}$$

Now consider the behaviour of employers with a filled job. If they offer any wage strictly above w^* then their workers will not apply to the vacant jobs. So one option for them is to pay w^*. If they do this then their profits will be given by:

$$\pi_0=\left(p-w^*\right)=\left[1-\frac{a^*h'(a^*)}{h(a^*)}\right]\left[p-b\right] \tag{120}$$

If, on the other hand, they offer a wage strictly below w^* then their workers will apply for the vacant jobs and will quit if they get it. As the worker's decision does not depend on the level of the wage the best option for the employer in this case is to pay b. If they do this then their profits will be given by:

$$\pi_1=\left(p-b\right)\left[1-\lambda\mu\left(a^*\right)\right]=\left(p-b\right)\left[1-\lambda\frac{h(a^*)}{a^*}\right] \tag{121}$$

A comparison of the two profit levels shows that the employers will choose to offer b if:

$$\lambda<h'\left(a^*\right)\left[\frac{a^*}{h(a^*)}\right]^2 \tag{122}$$

The right-hand side of (122) can be shown to be decreasing in a^*. Because a^* is increasing in λ this means that the inequality in (122) can be satisfied for small enough λ as claimed in the Proposition.

H Proof of Proposition 8

First, consider some useful results that simplify the analysis.

Result 8a: The value function $\Pi(N_0)$ as defined in (40) is linear in N_0.

Proof. From the form of (40) one can see that for a given wage policy (both current and into the future) the present discounted value of profits is linear in N_0. This implies that the value function is also linear in N_0. ■

Result 8b: The highest wage offered to recruits must be the same as the highest wage offered to senior workers.

Proof. Suppose the highest wage offered to recruits is above that offered to senior workers. Then the highest wage offered to recruits can be reduced without lowering the flow of recruits to the firm and, as this increases profit per recruit, profits must rise. Similarly if the the highest wage offered to senior workers is above that offered to recruits, the highest wage offered to senior workers can be reduced without raising the quit rate and, as this increases profit per recruit, profits must rise. ■

Result 8c: The lowest wage offered to both recruits and senior workers must be equal to b.

Proof. Suppose the lowest wage for recruits, $w_R(0) > b$. Then all firms that pay their senior workers in the range $[b, w_R(0))$ must have the same quit rate so will make the highest profits by paying b. But if there are no firms paying their senior workers a wage in the region $[b, w_R(0))$, the firm that pays its recruits the wage $w_R(0)$ can increase profits by lowering its wage to b as the flow of recruits will not be altered by this. Similarly, suppose that the lowest wage for senior workers, $w_S(0) > b$. Then all firms that pay their recruits in the range $[b, w_S(0))$ must have the same recruitment rate so will make the highest profits by paying b. But if there are no firms paying their recruits a wage in the region $[b, w_S(0))$, the firm that pays its senior workers the wage $w_S(0)$ can increase profits by lowering its wage to b as the quit rate will not be altered by this. ■

Result 8d: There can be no mass points in the distribution of wages for either new recruits or senior workers at a wage strictly above b.

Proof. Suppose there is a mass point in the wage distribution for recruits that is at a wage strictly above b, denoted it by w. Then there must be no firms paying senior workers a wage in the gap $[w-\varepsilon, w]$ for some $\varepsilon > 0$. To see this note that a firm can discontinuously reduce its quit rate to other firms by a paying a wage to its senior workers that is epsilon above w. This will be a profitable deviation if the initial wage is sufficiently close to w. But if there is a hole in the wage distribution of senior workers below w then it will pay the firms who are paying their recruits the wage of w to reduce their wage as doing so increases profits per worker while having no impact on the flow of recruits.

The argument for why there cannot be a mass point in the distribution of wages for senior workers at a wage strictly above b is similar. Suppose there is a mass point in the wage distribution for senior workers that is at a wage strictly above b, denoted it by w. Then there must be no firms paying recruits a wage in the gap $[w-\varepsilon, w]$ for some $\varepsilon > 0$. To see this note that a firm can discontinuously raise its flow of recruits from other firms by a paying a wage to its recruits that is epsilon above w. This will be a profitable deviation if the initial wage is sufficiently close to w. But if there is a hole in the wage distribution of recruits below w then it will pay the firms who are paying their senior workers the wage of w to reduce their wage as doing so increases profits per worker while having no impact on the quit rate.

Note that this argument does not work if there is a mass point of workers being paid equal to b though there cannot be a mass point there in both the recruit and senior worker wage distributions and one has to assume that all workers at the mass point move when receiving an offer of b from the side of the market where there is no mass point.

We are now in a position to prove Proposition 8. It is convenient – as before – to work with the decision variable for a firm being the position in the wage offer distribution. Denote the wage offered to new recruits by a firm at position f in the recruits wage distribution by $w_R(f)$, and the wage paid to senior workers by $w_S(f)$. Consider the level of recruits that a firm will have if it is at position f in the wage distributions. Note that the flow of new recruits must come from unemployment and from workers who are senior workers in other firms so will not be influenced by the wages that other firms pay to their new recruits but will be influenced by the wages that other firms pay their senior workers. Suppose that a firm at position f in the wage distribution for recruits attracts senior workers from other firms that are at position $f_S(f)$ or lower. In this case the flow of recruits will be given by:

$$R(f) = \lambda\left[u + \int_0^{f_S(f)} N_S(x)\mathrm{d}x\right] \tag{123}$$

where $f_S(f)$ must satisfy

$$w_R(f) = w_S(f_S(f)) \tag{124}$$

if there is an interior solution. If there is no interior solution then $f_S(f) = 0$ if $w_R(f) < w_S(0)$ and $f_S(f) = 1$ if $w_R(f) > w_S(1)$. In the equilibrium the lowest and highest wages paid to recruits and senior workers will be equal so the interior solution is the relevant one. Now consider the number of senior workers at the start of a period in a firm that pays a wage $w_S(f)$. This will be given by the number of recruits it made in the previous period plus the number of senior workers it previously had who did not quit i.e. will satisfy:

$$N_S(f) = R(f) + [1 - q(f)]N_S(f) \tag{125}$$

The quit rate will be both to unemployment and to other firms where the worker must be a new recruit. So the quit rate will not be influenced by the wages paid by other firms to their senior workers but will be influenced by the wages they pay to their recruits. We must have:

$$q(f) = [\delta + \lambda(1 - f_R(f))] \tag{126}$$

where $f_R(f)$ must satisfy

$$w_S(f) = w_R(f_R(f)) \tag{127}$$

for all wages that are chosen in equilibrium if there is an interior solution. If there is no interior solution then $f_R(f) = 0$ if $w_S(f) < w_R(0)$ and $f_R(f) = 1$ if $w_S(f) > w_R(1)$. Comparing (124) and (127) one can see that we must have $f_R(f_S(f)) = f$. Now from (41) we must have that:

$$\pi_1 = \frac{(p - w_S(f))[1 - q(f)]}{1 - \beta[1 - q(f)]} = \pi_S^* \tag{128}$$

Let us denote the highest wage offered by w^* i.e. $w_R(1) = w_R(1) = \ w^*$. In this case (124) and (127) imply that $f_S(1) = f_R(1) = 1$. In this case we then have from (126) that $q(1) = \delta$ and from (123) that $R(1) = \lambda$. Equation (128) then implies that:

$$\pi_S^* = \pi_1 = \frac{(p - w^*)[1 - \delta]}{1 - \beta[1 - \delta]} \tag{129}$$

From (42) we then have that:

$$\pi_R^* = \frac{(p - w^*)\lambda}{1 - \beta[1 - \delta]} = \frac{\lambda}{1 - \delta}\pi_S^* \tag{130}$$

Now consider the equilibrium wage distribution for senior workers. From (43) we must have that:

$$-q'(f)(p - w_S(f) + \beta\pi_1) - w_S'(f)[1 - q(f)] = 0 \tag{131}$$

which can be written as:

$$w_S'(f) = -\frac{q'(f)\pi_S^*}{[1-q(f)]^2} \tag{132}$$

Now, from (126) and (124) we have that:

$$q'(f) = -\lambda f_R'(f) = -\lambda \frac{w_S'(f)}{w_R'(f_R(f))} \tag{133}$$

Substituting (133) into (132) and re-arranging leads to:

$$w_R'(f_R(f)) = \frac{\lambda \pi_S^*}{[1-\delta-\lambda(1-f_R(f))]^2} \tag{134}$$

and we can change the variable $f_R(f)$ to f to yield:

$$w_R'(f) = \frac{\lambda \pi_S^*}{[1-\delta-\lambda(1-f)]^2} \tag{135}$$

Now consider the optimal choice of the wage for recruits. From (42) we must have that:

$$w_R'(f)R(f) = (p - w_R(f) + \beta\pi_S^*)R'(f) = \pi_R^* \frac{R'(f)}{R(f)} = \frac{\lambda\pi_S^* R'(f)}{(1-\delta)R(f)} \tag{136}$$

where the final equality follows from (130). From (123) we have that:

$$R'(f) = \lambda N_S(f_S(f))f_S'(f) = \lambda N_S(f_S(f))\frac{w_R'(f)}{w_S'(f_S(f))} \tag{137}$$

Substituting (137) into (136) and re-arranging leads to:

$$w_S'(f_S(f)) = \frac{\lambda^2\pi_S^* N_S(f_S(f))}{(1-\delta)R(f)^2} \tag{138}$$

Using (123) we can change the variable $f_S(f)$ to f to yield:

$$w_S'(f) = \frac{\pi_S^* N_S(f)}{(1-\delta)\left[u + \int_0^f N_S(x)\mathrm{d}x\right]^2} \tag{139}$$

Using (135) and (139) and evaluating at $f = 1$, we have that:

$$w_R'(1) = \frac{\lambda\pi_S^*}{[1-\delta]^2} \tag{140}$$

and that:

$$w_S'(1) = \frac{\pi_S^* N_S(1)}{(1-\delta)} = \frac{\lambda \pi_S^*}{\delta(1-\delta)} \tag{141}$$

Comparing (140) and (141) we have that $w_R'(1) < w_S'(1)$ as $\delta < \frac{1}{2}$. As we know from Result 8b that $w_R(1) = w_S(1)$, this implies that $w_R(f) > w_S(f)$ for f sufficiently close to 1.

Now consider how we can provide a sufficient condition for never having $w_R(f) \leq w_S(f)$ for any $f < 1$. Suppose, to the contrary this condition is satisfied and denote by $\widetilde{f}$ the highest value of $f < 1$ at which $w_R\left(\widetilde{f}\right) = w_S\left(\widetilde{f}\right)$. By definition we must have $w_R(f) > w_S(f)$ for $f \in (\widetilde{f}, 1)$ which implies that $w_R(f)$ must cut $w_S(f)$ from below so that $w_R'\left(\widetilde{f}\right) > w_S'\left(\widetilde{f}\right)$. We will provide a sufficient condition for this not being possible. From (124) and (127) we must have that:

$$f_R\left(\widetilde{f}\right) = f_S\left(\widetilde{f}\right) = \widetilde{f} \tag{142}$$

In this case (135) can be written as:

$$w_R'\left(\widetilde{f}\right) = \frac{\lambda \pi_S^*}{\left[1-\delta-\lambda\left(1-\widetilde{f}\right)\right]^2} = \frac{\lambda \pi_S^*}{\left[1-q\left(\widetilde{f}\right)\right]^2} \tag{143}$$

and (138) can be written as:

$$w_S'\left(\widetilde{f}\right) = \frac{\lambda^2 \pi_S^* N_S\left(\widetilde{f}\right)}{(1-\delta) R\left(\widetilde{f}\right)^2} = \frac{\lambda^2 \pi_S^*}{(1-\delta) R\left(\widetilde{f}\right) q\left(\widetilde{f}\right)} \tag{144}$$

Comparing (143) and (144) we have that:

$$\frac{w_R'\left(\widetilde{f}\right)}{w_S'\left(\widetilde{f}\right)} = \frac{(1-\delta) R\left(\widetilde{f}\right) q\left(\widetilde{f}\right)}{\lambda\left[1-q\left(\widetilde{f}\right)\right]^2} \tag{145}$$

as $R\left(\widetilde{f}\right) < \lambda$, and $(1-\delta) > 1 - q\left(\widetilde{f}\right)$, $q\left(\widetilde{f}\right) < \frac{1}{2}$ is a sufficient condition for $w_R'\left(\widetilde{f}\right) < w_S'\left(\widetilde{f}\right)$ But if this is a case $w_R(f)$ cannot cut $w_S(f)$ from below so there can be no point $\widetilde{f} < 1$ at which $w_R\left(\widetilde{f}\right) = w_S\left(\widetilde{f}\right)$. ■

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