[51]

BROWNIAN MOTION AS A MATHEMATICAL SUPERSTRUCTURE TO ORGANIZE THE SCIENCE OF CLIMATE AND WEATHER

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K. Hasselmann, "Stochastic Climate Models: Part I. Theory," *Tellus A: Dynamic Meteorology and Oceanography* 28 (6), 473–485 (1976).

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Scientific Setting

In 2021 the Nobel Prize in Physics was given, for the first time, to complexity science. Klaus Hasselmann, Syukuro Manabe, and Giorgio Parisi were cited "for groundbreaking contributions to our understanding of complex physical systems." Nobel's will stipulated that the physics prize should be given to a recipient "who shall have made the most important discovery or invention within the field," and the Nobel Committee further requires that the discovery or invention should either have had an impact on the evolution of physics as a science, or shown the usefulness of physics for society and thus have "conferred the greatest benefit to humankind." The awardees in 2021 fulfilled these requirements, with their diverse achievements being linked through the mathematics and physics of disorder, fluctuations, and variability. These have also been perennial themes at SFI, as the *Foundational Papers* volumes show.

Parisi's citation went on to recognize his "discovery of the **interplay of disorder and fluctuations** in physical systems from atomic to planetary scales."[1] In contrast Hasselmann and Manabe were recognized for foundational work on the physics of climate change, at the interface of thermodynamics, fluid mechanics, atmospheric science and geophysics. They were specifically cited "for the **physical modelling of Earth's climate, quantifying variability** and reliably predicting global warming." The importance of Hasselmann's work has long been clear to climate scientists, but it does not yet seem as well known to

[1] See chapter 54 in this volume for a key paper by Parisi.

a more general complexity audience. This is a pity for many reasons, and so for this volume I have adopted Hasselmann's paper, the first of the series with Claude Frankignoul and Peter Lemke (Frankignoul and Hasselmann 1977; Lemke 1977) that established stochastic energy-balance models as a key tool in climate science. Like many key papers in complexity science, it can be understood on several different levels. It can simply be seen as a way to add variability into the simple, and thus tractable and insightful, pioneering energy-balance models (EBM) of Mikhail Budyko (1969, see *Foundational Papers* vol. 2, ch. 32) and William Sellers (1969). On a more abstract level, it used Einstein's and Langevin's mathematical models of a physical phenomenon first observed by Robert Brown to provide a conceptual superstructure to link slow climate variability to fast weather fluctuations, and hence to posit the need for negative feedback in climate models. It thus exemplifies the different levels of abstraction highlighted for models in science by Arturo Rosenblueth and Norbert Wiener (1945, see *Foundational Papers* vol. 1, ch. 8). Since the 1970s it has, furthermore, played a key role in developing data assimilation methods to measure the fingerprint of global warming, and to predict its likely future severity, and it is still being exploited in new ways, for example by linking it to models of economic impact (Calel *et al.* 2020).

A Foundational Paper

In the mid-1970s, theoretical problems in climate science were already being attacked simultaneously from a wide variety of directions, using models at many different levels of detail. The early ambitions of John von Neumann and colleagues in the 1950s had by then been tempered after the realization, by Edward Lorenz (see *Foundational Papers* vol. 2, ch. 23) and others, of the implications for forecasting of low-dimensional chaos. Nonetheless, the insights from dynamical systems theory were developing alongside, and informing, numerical geophysical fluid dynamics, and as early as 1975 Manabe performed a pioneering computer simulation (Manabe and Wetherald 1975) of the response of the Earth's atmosphere to a doubling of CO_2 using a forerunner of today's general circulation models of weather and climate. At the other end of the scale, Budyko and Sellers had already proposed their simplified toy EBMs, which prioritized insight over detail. The all-important task of testing developing theories against nature, meanwhile, was facilitated by data sources as diverse as weather station records, ice cores, annual layers of sedimentary rocks (varves), and global ice volumes. As the data was analyzed, however, it became clear to Hasselmann that "a characteristic feature of climate records is their pronounced variability. The spectral analysis of continuous climatic time series normally reveals a continuous variance distribution, with higher variance levels at low frequencies."

Theme of Hasselmann's Paper

The problem that puzzled Hasselmann can be seen in the power spectrum of many climate variables, such as that from the Ocean Weather Ship India (fig. 5 of Frankignoul and Hasselmann 1977). Rather than the uniform "flat white" form of idealized uncorrelated random noise, they have a "red" excess of power at low frequencies and hence show autocorrelation. In this sense each measured value of a time series shows a kind of memory of at least its immediate predecessor. At least since the late 1950s it had been realized that climate and weather variables were indeed typically correlated, and showed such a low-frequency excess in their power spectra. Intriguingly, though, it seems that memory and persistence were seen as something to be modelled using the tools of statistics—such as the famous discrete first-order autoregressive model AR(1)—as opposed to those of statistical physics, despite the fact that the continuous version of the AR(1) process is mathematically equivalent to Langevin's equation of 1908. J. M. Mitchell, Jr., of the Environmental Science Services Agency and UC Berkeley was a striking exception to this, and as early as 1966 proposed two stochastic models where the autoregressive structure sought to capture the physics of the air–sea interaction (Mitchell 1966).

In the mid '70s the dominant candidates to physically explain the observed red spectra in many climatic variables were based either on external driving or internal positive feedbacks. Inconveniently, however, as Hasselmann noted, a clear signal of response to the former was not obvious in data, while models of the latter type tended to produce abrupt "flip-flop" transitions that also did not resemble what was seen, especially in the Holocene era inhabited by humans.

Hasselmann approached the problem from a different point of view, informed by his own experience in diverse areas of physics from turbulence to quantum field theory. He made the imaginative leap of mapping the slow fluctuations of climate and the fast ones of weather into those of Langevin–Brownian motion. Decades later he replied to an oral history interviewer (2006), saying:

> *I think it depends on your background training. If you are used to working with a high resolution general circulation model, looking at all the dynamics and interactions and so forth, you probably never think about Brownian motion or may not even have heard of the Langevin equation. These are simply not part of your basic research experience. If you are accustomed to only one way of thinking, you simply cannot see problems in another way. People are too specialized in the particular techniques they have learned. They are not able to cross their narrow borders and see things from a different—often simpler and more elegant—perspective.*

This cross-disciplinary synthesis, which will resonate with many scientists at SFI and elsewhere, enabled him to envision a model where the timescale separation so essential to Langevin's original formulation was embodied in that between weather and climate variables.

Hasselmann's Development of His Idea and Results

Hasselmann's paper first sets up the problem to be solved, and then in section 2 compares two paradigms—global circulation models (GCMs) and statistical dynamical models (SDMs). After explaining that the computers of that time were not adequate to model climate variability by GCM, he shows that averaging over rapid fluctuations characteristic of then current SDMs rendered them deterministic and so "statistical" was in some sense a misnomer. After a nod to Lorenz's pioneering low-dimensional models of chaos he goes on to set up a truly stochastic paradigm, in which, as he later put it (Frankignoul and Hasselmann 1977): "slow changes of climate were interpreted as the integral response to continuous random excitation by short time scale 'weather' disturbances." This is the Wiener random process used since Bachelier and Einstein in diverse ways in finance and physics, chemistry and elsewhere; in section 3 he explores how one of its hallmarks, the growth of the variance of fluctuations linearly with time, would give rise to a $1/f^2$ power spectrum in climate variables, and other detectable signatures. In section 4 he describes another corollary of the Brownian motion model, the existence of a Fokker–Planck differential equation for the time evolution of probability densities of temperature or any other variables to which his climate model could apply. Section 5 then stresses the role of negative feedbacks in changing a system from a nonstationary Wiener process into a stationary process modelled by a Langevin-type equation, turning the low-frequency power spectrum from red to white noise. Section 6 then concludes the paper with a first exploration of the importance of these new models for forecasting.

Foreshadowings and Implications

The assumption of a strong separation between weather and climate time scales initially prompted progress, as it gave the ability to map from the physics of Brownian motion, or that of climate, to the idealized random Wiener process. This allowed the importing of a results and expertise from the discipline of stochastic calculus, allowing a similarly productive interaction to that which has been seen in the development of the Kalman filter (see *Foundational Papers* vol. 1, ch. 15) in control engineering. In a real sense Hasselmann's model was itself a kind of "planetary control theory," as it has long been used in a driven version to examine the effect of different possible future CO_2 emission pathways, and coupled to economic models (e.g., Calel *et al.* 2020).

More recently though, as elsewhere in physics, the limits of the assumption have become clearer, and in particular complexity scientists (see Moon, Agarwal, and Wettlaufer 2018 and references therein, and Lovejoy 2022) have made links to Benoit Mandelbrot's pioneering work on fractals as systems (see *Foundational Papers* vol. 2, ch. 29) on which no scale could be seen as typical or dominant, and to even longer-ranged memory than the Markovian type seen in Hasselmann's original models. My own current work (Watkins *et al.* 2024) is motivated by this, exploiting the generalized Langevin equations long known in condensed matter with the aim of giving a model where one need not either assume Hasselmann's Markovian case, or a completely fractal long-ranged one, but can explore the ground in between.

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