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To chain or not to chain? measuring real GDP in the US and the choice of index number

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Abstract

National Statistical Institutes (NSIs) in advanced countries have generally adopted chain-linking in their national accounts. The United States uses a chained Fisher, an example of a superlative index number, in its national accounts. However the Fisher is only one of an infinite number of superlative index numbers. So an important issue is how sensitive are the estimates of output growth to the choice of index number. This issue is analysed by examining data from the BEA/BLS industry-level integrated production account, 1987–2020. Estimates of superlative and other index numbers are presented for this dataset. The sensitivity of real GDP growth to the value of the crucial parameter in a superlative index number is tested. The extent to which the desirable characteristics of value consistency and aggregation consistency are satisfied for different superlative index numbers is also analysed. The desirability of chain-linking does not follow automatically just from the use of superlative indices. So I also compare chained and unchained versions of these same index numbers. Finally, Europe uses a different approach to output measurement to the US, chained Laspeyres versus chained Fisher. I look at how different US estimates would be if they employed European methodology.

JEL codes E01 · C43 · O47 · O51

Keywords Chain-linking · GDP · Index numbers

1 Introduction[1]

In 1996 the United States introduced annual chain-linking in its National Income and Product Accounts (NIPA), in place of a (periodically shifted) fixed base approach. At the same time it adopted the Fisher index for its main national income aggregates. Chain-linking has now become widely accepted. The 2008 System of National Accounts, the latest to be internationally approved, now recommends chain-linking (European Commission et al. 2009, chapter 15).[2]

Though chain-linking is now widely employed in advanced countries, there is no general consensus on the best index number to use. National Statistical Institutes generally use either the chained Laspeyres (mandated by Eurostat (2013) for EU countries and also still used by the UK), or the chained Fisher (as in Canada and the US). Economic modellers and productivity analysts (following Griliches and Jorgenson 1967 and Jorgenson et al. 1987) often use the Törnqvist.

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[2] In Europe (including the UK) prior to chain-linking it had been usual to update the weights every five years or so. In other words, most of Europe has always used a form of chain-linking (France was an exception in using a fixed base). In the UK case the change from what might be called quinquennial chain-linking to annual chain-linking took place in 2003.

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The case for the Fisher index was greatly advanced by the development of superlative index numbers, of which the Fisher is an example (Diewert 1976). Under this approach, it is assumed that economic behaviour can be explained exactly by utility or production functions which take the form of a "quadratic mean of order *r*". These functional forms are second order approximations to any functions acceptable to economic theory. Then there exists a superlative index number (dependent on the parameter *r*) which is exact for this particular functional form (Diewert 1976, 1978, 1980 and 1992; Caves et al. 1982; Mizobuchi and Zelenyuk 2021).

The alternative to an economic-theory-based approach to index numbers is the test approach. Superlative index numbers pass many of the tests commonly thought reasonable. But two tests are more problematic. The first test is what I call here value consistency: the product of the price index and the quantity index should equal the value index (see below, "Theory"). The other test is aggregation consistency. Aggregation consistency means that we can calculate an index of, e.g., real GDP in two ways. The first way is to calculate it directly from the basic elements at the lowest available level (e.g. value added in each industry), the one-step method. The second way is to calculate it in two (or more) steps: first calculate an index for each sub-aggregate of interest (e.g. manufacturing) and then calculate the overall index of these sub-aggregate indices, the multi-step method. Aggregation consistency means that the results of the two methods will be the same. The Fisher index passes the value consistency test (as well as other tests satisfied also by other superlative index numbers; see again Diewert (1976)) but fails the aggregation consistency test. All other superlative indices fail both these tests.

Many economists, noting the results of Diewert (1976) and (1978) that all superlative index numbers are second order approximations to each other, have concluded that the "index number problem" has been solved. However, Hill (2006) has used US data to show that the estimated growth rates of GDP are in practice quite sensitive to the value chosen for the parameter *r*. In the absence of econometric evidence there seems no empirical reason for preferring one value of *r* over another, so the issue still seems to be up in the air. Hill (2006) used what I call here two-year indices, also known as direct indices (see "Theory" below), not chained indices, so an important issue still to be resolved is whether chaining affects his conclusions.

The purpose of this paper is to study these issues empirically, using the BEA/BLS industry-level production account as a testbed. I test how sensitive chained indices of real GDP are to a range of values of the parameter defining alternative superlative index numbers. I compare chained and unchained versions of these index numbers. And I test the extent to which chained indices satisfy value and aggregation consistency approximately.

1.1 Plan of the paper

"Theory" reviews the theory of superlative index numbers and of chain-linking. Divisia index numbers are briefly discussed as an alternative justification for the use of superlative indices. "Data: the BEA-BLS industry-level production account" describes the dataset to be used for the empirical testing, the BEA/BLS integrated industry-level production account. "Results" presents the results. First the sensitivity of real GDP growth to the value of the crucial parameter *r* in a superlative index number is tested. Then the extent to which value consistency and aggregation consistency are satisfied for different superlative index numbers is analysed. I also compare chained and unchained versions of these same index numbers.[3] Finally, Europe employs a different approach to output measurement, using chained Laspeyres while the US uses chained Fisher. I look at how different US estimates would be if they employed European methodology. "Conclusions" summarises and also points to other issues in need of exploration.

2 Theory

2.1 Superlative index numbers

Diewert (1976) defined a superlative index number as one which is exact for a flexible functional form. In turn a flexible functional form is one which approximates to second order any linear homogeneous function acceptable to economic theory. A second order approximation is one where the approximating function and the other function are equal as are their first and second derivatives, *at some common point*. He developed a family of flexible functional forms called quadratic means of order *r*. In the quantity case this takes the form:

$$f_r(q) = \left[\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} q_i^{r/2} q_j^{r/2}\right]^{1/r}, \; a_{ij} = a_{ji}, \; \forall i,j, \; r \neq 0 \qquad (1)$$

where q_i is the quantity of the ith good. (The case $r = 0$ is handled by taking the limit as *r* goes to zero which yields the translog aggregator function).

Equation (1) can be interpreted as a production possibly frontier with primary input supplies and technology held constant. It arises out of an economy in which producers act as if they were maximising profits under constant returns to scale with output and input prices taken as given. (There is an analogous form for the corresponding price or unit cost possibility frontier). The corresponding (superlative)

[3] It would be desirable to extend the empirical work to include the measurement of real input and so of productivity. At the moment this cannot be done since the BEA and BLS have not published their input data at the same industry level as the output data.

quantity index for period t relative to period s is

$$Q_r^{s,t}(p^s,p^t;q^s,q^t) = \left[\frac{\sum_{i=1}^N (q_i^t/q_i^s)^{r/2}(p_i^s q_i^s/p^s\cdot q^s)}{\sum_{k=1}^N (q_k^s/q_k^t)^{r/2}(p_k^t q_k^t/p^t\cdot q^t)}\right]^{1/r}, \ r\neq 0$$
$$= \prod_{i=1}^N \left(\frac{q_i^t}{q_i^s}\right)^{\frac{1}{2}[(p_i^s q_i^s/p^s\cdot q^s)+(p_k^t q_k^t/p^t\cdot q^t)]}, \ r=0 \quad (2)$$

(The case $r=0$ can also be thought of as the limit of the first line on the right hand side as r approaches 0.) Here superscripts denote discrete time periods and $p^t(q^t)$ is the price (quantity) vector in period t; the numerator contains the expenditure shares in the earlier period s, and the denominator contains the shares in the later period t. This index is exact for the flexible functional form (1).[4] An analogous expression, with price relatives replacing quantity relatives, holds for the superlative price index.

A number of special cases of (1) and (2) are of interest. First, when $r=0$ the function takes the translog form and the corresponding index number takes the Törnqvist form. Second, when $r=2$ the index takes the Fisher "ideal" form (the geometric mean of the Paasche and Laspeyres indices). Third, when $r=1$, the result is known as the Walsh index which is used by Sweden. Fourth, when the off-diagonal coefficients in (1) are all zero, the function takes the CES form but this is now not a flexible functional form.

Hill (2006) proved an important property of superlative index numbers: The limit of the quantity (price) index number as $|r|\to\infty$ is the geometric mean of the largest and smallest quantity (price) relatives:

$$\lim_{r\to+\infty} Q_r^{s,t} = \lim_{r\to-\infty} Q_r^{s,t} = \left[\left(\min_{i=1,\ldots,N}\left(\frac{q_i^t}{q_i^s}\right)\right)\left(\max_{i=1,\ldots,N}\left(\frac{q_i^t}{q_i^s}\right)\right)\right]^{\frac{1}{2}} \quad (3)$$

An analogous expression holds for price indices. In other words as r increases in absolute value, the index becomes increasingly dominated by the extreme values.

2.2 Value and aggregation consistency

Superlative index numbers which have the quadratic mean of order r form of (2) fail the value consistency test, except in the Fisher case when $r=2$: the product of a Fisher quantity index and a Fisher price index is the value index. Superlative index numbers also fail the aggregation consistency test.[5] However Diewert (1978) showed that the class of superlative index number formulae has an approximate consistency-in-aggregation property.[6]

It is also worth noting that the Laspeyres-Paasche pair (either a Laspeyres quantity with a Paasche price index or a Laspeyres price with a Paasche quantity index) possess value consistency. Value consistency means that

$$P_{Paas}^{s,t}Q_{Lasp}^{s,t} = P_{Lasp}^{s,t}Q_{Paas}^{s,t} = \frac{\sum_{i=1}^N p_i^t q_i^t}{\sum_{i=1}^N p_i^s q_i^s} = \frac{V_t}{V_s} \quad (4)$$

which follows from the definitions of the Laspeyres and Paasche indices. Here $V_t = \sum_{i=1}^N p_i^t q_i^t$ is the aggregate nominal value in period t.

2.3 Chained versus non-chained index numbers

Developed countries have gradually shifted towards calculating their own price and quantity index numbers in chained form, e.g. EU countries and the UK use chained Laspeyres while the US and Canada use chained Fisher indices. But economists who prefer an axiomatic to an economic approach to index numbers are agnostic about the virtues of chain-linking (e.g. Balk 2008 and 2010). So what is the justification for chain-linking, beyond a rather vague desire "not to let the weights get too out of date"? Certainly, concern about substitution bias in fixed base indices has been longstanding, e.g. Jorgenson and Griliches (1971). But this concern could have been met by adopting two-period (or direct) symmetric indices like the Fisher which give equal importance to the first and last period weights. So concern about substitution bias by itself gives no support to chain-linking.

Practical considerations may have played a role. It seems likely that the US adopted annual chain-linking at least in part because of the disruptive effects of the newly introduced and rapidly falling price of computers on the national accounts. This meant that moving from an earlier fixed base to a later one caused all previously published GDP growth rates to be revised downwards (in the absence of any off-setting data

[4] Diewert (1976) showed that a quadratic mean of order r is a superlative index for measuring consumer prices. The output index is more complicated since production technology is likely shifting over time. But Caves et al. (1982) showed that a Törnqvist index covers this case and Diewert (1992) showed that a Fisher index also covers this case. More generally, Mizobuchi and Zelenyuk (2021, 2023) showed that a quadratic mean of order r can serve as a superlative index for the output index (as well as for the input and productivity indices).

[5] Montgomery-Vartia indices (Vartia 1976) do satisfy these two tests. However these indices are exact only for Cobb-Douglas aggregator functions and fail other desirable tests; e.g. rescaling the prices in a Montgomery-Vartia quantity index in either period will generally change the index (Diewert 1978).

[6] Superlative indices have found wide but not universal acceptance. O'Donnell (2016) recently advocated the so-called proper index, which is an index number with constant weights such as the Dutot, Jevons or Cobb-Douglas indexes, because these satisfy the circularity test. They have good axiomatic properties, but since they do not incorporate the economic importance of each good, they are very problematic and significantly differ from the usual Fisher, Törnqvist, Laspeyres and Paasche indexes.

revisions). This was embarrassing to the Clinton administration.[7] Chain-linking removed this embarrassment.

2.4 Chained versus non-chained superlative indices

Diewert (1976) proved an important result about chained and non-chained superlative indices when these take the form of Eq. (2). Consider the growth of quantities over the period (0, 2). From (1) we have

$$\frac{f_r(q^2)}{f_r(q^0)} = \frac{f_r(q^1)f_r(q^2)}{f_r(q^0)f_r(q^1)} \tag{5}$$

Because the index numbers Q_r are exact for the functional form (1), it follows from (5) that

$$Q_r^{0,2} = Q_r^{0,1}Q_r^{1,2} \tag{6}$$

In other words the two-period index number and the chained one produce the same result numerically, provided of course that economic agents are indeed maximizing profit or revenue using the technology described in Eq. (1).[8] In other words, superlative index numbers satisfy the circularity test. Put another way, chaining should make no difference. This result obviously generalizes to many time periods:

$$Q_r^{0,T} = Q_r^{0,1}Q_r^{1,2}\ldots Q_r^{T-2,T-1}Q_r^{T-1,T} \tag{7}$$

Diewert (1976) argued that this fact allowed an empirical test of the theory: If in practice the circularity test fails then either producers are not maximizing or they are maximizing something other than $f_r(q)$. Hence he recommended chain-linking since the aggregator function is unlikely to remain constant over long periods of time; see also Diewert (1980). This is because the slope of the production possibility frontier, at points where the ratios of quantities produced are equal (along a ray from the origin), will change over time if technical progress occurs at different rates in different industries or if supplies of primary inputs are not all growing at the same rate. In other words the a_{ij} parameters in (1) will in general be changing over time. Only if the production possibility frontier at time t is just a radial blow up of its position at any previous point will its slope at a given ratio of outputs be unchanged. That is, if the aggregator function now depends on time, now written $f_r(q,t)$, we require that

$$f_r(q,t) = g(t)f_r(q,0) \tag{8}$$

This is obviously a highly restrictive assumption since it implies that the supplies of each primary input are rising at the same rate and that TFP is growing at the same rate in every industry.

The non-chained quantity index between any two time periods s and t for a quadratic mean of order r is defined by Eq. (2) but we now write the left hand side more compactly as $Q_r(s,t)$. Under the assumption that the economy is exactly represented by Eq. (1) and that Eq. (8) holds, the chained index over the same time period is now written as

$$Q_r^{Ch}(s,t) = Q_r(s,s+1)Q_r(s+1,s+2)\ldots Q_r(t-2,t-1)Q_r(t-1,t) \tag{9}$$

It is then an empirical issue as to how similar the chained ($Q_r^{Ch}(s,t)$) and the non-chained two-period quantity indices ($Q_r(s,t)$) actually are over any given time period (s, t). Below we assess this for a range of values of r.

One advantage of chain-linking is that adding an additional time period does not require us to change the past (in the absence of data revisions). Under chain-linking the growth rate over the interval (s, t) is unchanged when we extend the overall period to $t+1$; we just add another link in the chain for the last period. But without chain-linking the growth rate over (s, t) becomes problematic. Should we continue to measure it using the weights of just s and t, while using those of s and $t+1$ to measure growth over the whole interval (s, $t+1$)? If so, then how should we measure growth from t to $t+1$? We have two choices. Either we can use the weights of t and $t+1$, i.e. $Q_r(t,t+1)$, or we can use the growth rate implied by growth over the two long intervals (s, t) and (s, $t+1$), i.e. $Q_r(s,t+1)/Q_r(s,t)$. The answers will not be the same unless (8) is satisfied.[9]

In summary, chain-linking of a superlative index can be justified if the parameters of the underlying aggregator function are shifting over time.[10]

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[7] See *The Teaching Economist*, Issue 11, Spring 1996 (apparently no longer available online).

[8] Note that just substituting into (6) from the index number formula (2) does not prove the result. One requires also maximizing behaviour on the part of producers.

[9] Another possibility is to use the GEKS index commonly applied to cross-country or cross-regional data. This approach takes a geometric mean of indices over all possible paths between year s and year t, including the direct one¸ which ensures transitivity. However, this suffers from the drawback that all the original growth rates change when an additional year is added, just as in a cross-section context the relative levels change when an additional country or region is added.

[10] Chain-linking may not be the best solution in all circumstances. The CPI is usually computed on a monthly basis and is often subject to "price bounce": the tendency of prices to first fall then revert to their previous level, usually because of sales. It is often found however that quantities do not immediately revert to their original level even though prices do. This causes monthly chained indices to be subject to "chain drift": they do not revert to their previous level even though prices and (eventually) quantities are the same as in the pre-sale period. The explanation may be a breakdown in the assumptions underlying the economic approach to index numbers. Under the latter it is assumed that households consume everything they purchase immediately. In reality they may use the opportunity of a sale to stock up on the product, running down their stock gradually before purchasing again. Similar problems arise with seasonal, including fashion, goods. See Diewert (2022), chapters 7 and 9, for discussion and suggested remedies. But for quarterly and annual data at the industry level, as used here, these problems seem less acute.

2.5 US versus European methodology

As stated above, the US NIPA use a chained Fisher for measuring real GDP growth while EU countries and the UK use a chained Laspeyres. The chained Laspeyres has some advantages over the chained Fisher since it is both value consistent (when paired with a chained Paasche index) and aggregation consistent. However in Europe price indices such as the CPI are usually chained Laspeyres not chained Paasche. A more serious drawback, one known since at least Bruno and Sachs (1985), is that a Laspeyres index predicts (wrongly) that an exogenous worsening in the terms of trade *reduces* GDP, even when all primary inputs remain fully employed; for that matter, an *improvement* in the terms of trade also reduces GDP.[11] Nor does chain-linking help since the error remains and will impact the average growth rate over any interval which includes the period when the terms of trade changed. So even changes in the terms of trade which are reversed over time will lead to a systematic underprediction of GDP growth. Nonetheless it is still of interest to see how much difference it would make had the US adopted European methodology.

2.6 Divisia index numbers

An alternative way to justify chain-linking is by invoking Divisia index numbers which are defined in continuous time.[12] The growth of the Divisia quantity index Q^D at time t with reference period 0 is defined by:

$$\widehat{Q}^D(0,t) := \sum_{i=1}^{N} w_i(t)\hat{q}_i(t) \tag{10}$$

where a hat (^) denotes a logarithmic growth rate and

$$w_i(t) := \frac{p_i(t)q_i(t)}{V(t)},\ i = 1, \ldots, N \tag{11}$$

are the point-in-time expenditure shares of the N quantities in the aggregate nominal value V; the q_i could be real value added in the ith industry and the shares w_i are then the shares of nominal value added in each industry in nominal GDP. (The corresponding Divisia price index replaces the growth rates of the quantities by the growth rates of the prices). In a Divisia index the weights shift continuously so the index exhibits a continuous form of chain-linking. It is straightforward to show from their definitions that Divisia indices satisfy both value consistency and aggregation consistency.[13] Large changes can be handled as well as small ones. Furthermore, Divisia indices have been used to prove the important result known to macroeconomists as Hulten's Theorem: the relationship between industry-level TFP growth rates and the aggregate TFP growth rate (otherwise known as Domar aggregation (Domar 1961)). See Hulten (1978), Gabaix (2011), and Baqaee and Farhi (2019).

Of course, since they are continuous, Divisia indices cannot be calculated in practice. But one could argue that Divisia indices represent the ideal to which real world, discrete indices are an approximation. On this view, the task of national income accountants is to find the best discrete approximation to the ideal, for which chained superlative indices seem the best available candidates. However (unfortunately in my view), national income accountants show no interest in Divisia index numbers. The 2008 System of National Accounts has a whole chapter devoted to price and volume measures (European Commission et al. 2009, Chapter 15) but nowhere does it mention Divisia index numbers. Despite this it can be argued that real world price and volume indices are best thought of as (more or less good) approximations to the ideal, the Divisia index (as argued by Jorgenson and Griliches (1971)). And the Divisia approach does provide a justification for chain-linking. Nonetheless, whether or not the Divisia approach is accepted does not affect the empirical results to be presented below.

3 Data: the BEA-BLS industry-level production account

The idea now is to use actual data to test the sensitivity of estimates of real GDP to variations in the parameter r in a superlative index, for both chained and unchained indices. For this purpose I employ data from the BEA-BLS industry-level production account. The advantage of the BEA/BLS dataset for index number and productivity research is that it is highly consistent with production theory and based on a massive and detailed data-gathering exercise extending over many years.

The data are constructed in accordance with the KLEMS methodology pioneered by Jorgenson and his various collaborators: Jorgenson et al. (1987), (2005), (2016) and (2018). They give annual gross output, value added, intermediate input, capital input, labour input (all in both

[11] The correct answer is that if the import whose price has risen is an intermediate input like energy, then in an efficient economy (price = marginal cost) there is no effect on GDP (abstracting from any effects on aggregate demand). If there is a positive margin of price over marginal cost, then GDP *falls* in response to a rise in the imported input's price (Oulton 2023).

[12] Divisia indices were devised by Divisia (1925–1926). They were introduced explicitly into productivity analysis by Griliches and Jorgenson (1967). They have been analysed by Frisch (1936), Richter (1966), Hulten (1973) and Balk (2005). They have been criticised for being liable to path-dependence. But Oulton (2008) and (2012) argues that path-dependence is a feature not a bug.

[13] See the earlier, discussion paper version of this paper for proof of these assertions (Oulton 2022).

nominal and real terms), and TFP for 63 industries, classified by NAICS, covering the whole economy (including federal, state, and local government). The period covered is currently 1987–2020.[14] Nominal value added in these 63 industries adds up to nominal GDP.

Real value added is double deflated. The growth of labour input is the share-weighted growth of hours worked for approximately 170 different groups of workers cross-classified by sex, eight age groups, six education groups, and employment class (payrolled vs. self-employed). The growth of capital input is the share-weighted growth rate of capital services based on about 100 types of capital including inventories and land. A full description of the BEA-BLS-industry-level production account is in Garner et al. (2020) and (2021). Further detail on methodology is available from Garner et al. (2018).

The data for 1987–2020 was downloaded from the BEA website (www.bea.gov) in the form of a spreadsheet named "BEA-BLS-industry-level-production-account-1987-2020.xlsx", available at https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems. This spreadsheet was released on May 11 2022 and comprises the latest data available at the time this research was begun. It states: "This file contains the data underlying the BEA/BLS Integrated Industry-level Production Account for the United States. The data covers the 1987–2020 period and is updated to reflect the annual update to the input output accounts released on September 30, 2021 available here: https://apps.bea.gov/scb/2021/10-october/1021-industry-annual-update.htm".

4 Results

Before turning to the results, I start with a brief descriptive analysis of the dataset, focusing in particular on the extent of structural change between 1987 and 2019. In what follows I ignore 2020 as being too distorted by the pandemic to add any light. To give an idea of the importance of each industry and of structural change, Table 1(a) in the Appendix lists the industries, together with the share of each industry's value added in total value added (nominal GDP) in three years: 1987, 2000, and 2019. Industries vary widely in importance. In 2019 the smallest shares were for industries 21 (Apparel and leather and allied products) (0.04%), 31 (Water transportation) (0.06%) and 20 (Textile mills and textile product mills) (0.07%), all in steep decline since 1987. The largest industries were 45 (Real estate) (11.13%) and 63 (State and local government) (11.74%), both rising since 1987.

[14] Extending the data back to 1947 would be highly desirable. At the moment however that cannot be done on a fully consistent basis. And the quality of the estimates for years prior to 1987 is lower (Eldridge et al. 2020).

4.1 Structural change, 1987–2019

Figure 1 shows the growth of prices in the 63 industries (measured as 100 x the log change in price) over 1987–2019. Much the largest fall in price (almost 300%) occurred in a single industry: industry 13 (Computer and electronic products). The largest rise (181%) was in industry 44 (Funds, trusts, and other financial vehicles). Figure 2 shows the growth in quantities in the same 63 industries over 1987–2019. Quantity growth is more dispersed than price growth. The standard deviation of price growth was 62.8% while that of quantity growth was 81.3%.

The outcome of price and quantity growth is changes in shares. Figure 3 shows the changes in each industry's value added share between 1987 and 2019. 28 industries experienced positive growth in share and 35 negative growth over this period. The maximum change in share was +2.1 percentage points while the minimum was −2.1 percentage points. The correlation coefficient between the shares in

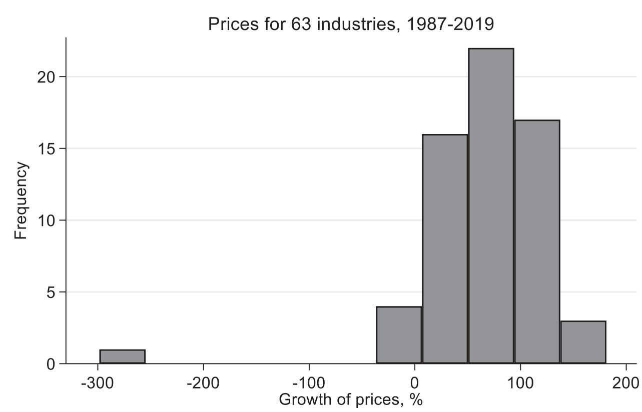

Fig. 1 Structural change in the US economy. Source: BEA-BLS Industry-level Production Account 1987–2020

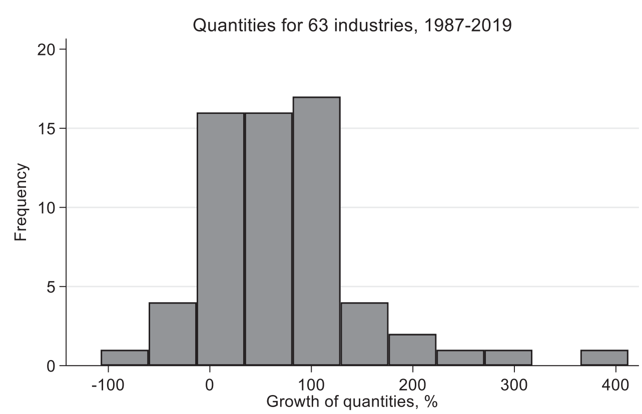

Fig. 2 Structural change in the US economy. Source: BEA-BLS Industry-level Production Account 1987–2020

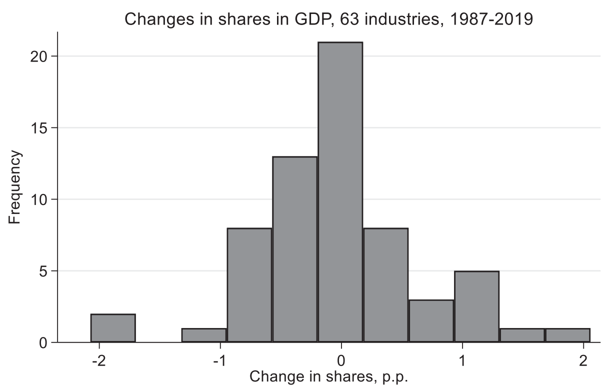

Fig. 3 Structural change in the US economy. Source: BEA-BLS Industry-level Production Account 1987–2020

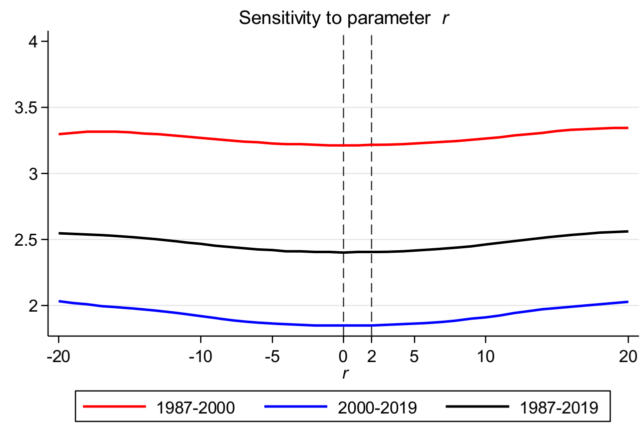

Fig. 4 Chained superlative indices: growth of real GDP, % p.a. Source Appendix Table 2

1987 and in 2000 was 0.98 while that for the shares between 1987 and 2019 was 0.95. So on this measure, changes in shares, structural change was quite limited. The proximate reason for this modest change in shares is that price growth and quantity growth are negatively correlated: the correlation coefficient is minus 0.61 over 1987–2019. A negative correlation between price and quantity growth is often found empirically. It is relevant to a study of index numbers since it makes it likely that a Laspeyres (base-weighted) quantity index will grow more rapidly than a Paasche (current-weighted) index.

Though structural change appears quite modest at the industry level, a somewhat different picture emerges if the 63 industries are aggregated into the 9 official industry groups: see Table 1(b). The major changes apparent now over 1987–2019 are a fall of 7 percentage points in the Manufacturing share and a corresponding rise in the share of Other Services.

4.2 How sensitive is the chained index of real GDP growth to the choice of *r*?

We first test how sensitive the estimated growth rate of real GDP is to the choice of the parameter r (recall that the official estimates in the US NIPAs assume in effect that $r = 2$, the Fisher case). Estimates of the average annual growth rate of real GDP over the period 1987–2019 according to the chained superlative index of Eqs. (2) and (9) appear in Table 2 in the Appendix and in Fig. 4, for a range of values of r. Here, following Hill (2006), the parameter r is allowed to vary from -20 to $+20$. This may seem an implausibly wide range, given that in practice a value of r of either 0 or 2 is usually employed. But as Hill (2006) points out, the size of r is an empirical matter and no one has in fact estimated r empirically.[15]

It turns out (Table 2) that the estimated growth rates are symmetrical around a value of r at or close to zero. The estimated growth rates are not very sensitive to the value of r: taking r to be $+20$ yields a growth rate of 2.562% p.a. while the minimum is 2.402% p.a. when $r = 0$: see Fig. 4 which also shows a similar picture for the sub-periods 1987–2000 and 2000–2019. On the other hands the volatility of the annual growth rate rises markedly as $|r|$ rises above about 10. From a practical point of view the estimated growth rate in the Törnqvist case ($r = 0$) is almost identical to the Fisher case ($r = 2$): 2.402 versus 2.404% p.a. If instead of chained Fisher the BEA employed the European method of chained Laspeyres, then growth over 1987–2019 would have been estimated as 2.481% p.a. rather than 2.404% p.a., an economically significant but still relatively minor difference.

Assuming constant returns to scale suggests that the true estimate of the annual growth rate lies between the chained Laspeyres and the chained Paasche, i.e. in the range 2.327 to 2.481% per annum. Both the Fisher and the Törnqvist satisfy this criterion. On an annual basis the chained Fisher lies within the Laspeyres-Paasche spread in every year; the chained Törnqvist lies within the spread in all but two years, 2008 and 2020, both years of severe recession.

4.3 How closely is value consistency achieved when $r \neq 2$?

We know that value consistency is achieved when $r = 2$, as in the Fisher index used officially in the US NIPAs. Here we test the extent of deviations from value consistency for a range of values of r. Specifically, we calculate an index of real GDP and an index of the price of GDP from the price and quantity data for the 63 industries, using a range of values of r. The value consistency index (VC) is then defined as the ratio of the value index to the product of the

[15] Hill (2006) also shows that as $|r| \rightarrow \infty$ the quadratic mean ceases to be flexible.

price index and the quantity index (for a given value of r):

$$VC_r(0,t) := \frac{\sum_{i=1}^{N} p_i^t q_i^t / \sum_{i=1}^{N} p_i^0 q_i^0}{P_r^{Ch}(0,t) Q_r^{Ch}(0,t)} \quad (12)$$

Note that the value consistency index is 1 in the reference year 0. Results for value consistency using chained indices appear in Appendix Table 3. For r lying between -5 and $+5$ consistency is high: the deviation from a value of 1 is less than about $\pm 1\%$. By contrast the chained Laspeyres shows a steadily increasing divergence from 1 over 1987–2019. By 2019 this index is only 0.9520.

4.4 Aggregation consistency

We now define the aggregation consistency index as the ratio of the level of the 2-step chained quantity index to the level of the 1-step chained quantity index. The first step of the 2-step index is chained quantity indices for each of 9 industry groups. The second step aggregates these to the GDP level. See Table 1(b) for the definitions of the 9 industry groups in terms of the underlying 63 industries. In symbols the aggregation consistency index (AC) is defined as

$$AC_r(0,t) := \frac{Q_r^{2,Ch}(0,t)}{Q_r^{Ch}(0,t)} \quad (13)$$

where $Q_r^{2,Ch}(0,t)$ is the 2-step index with parameter r in year t with reference year 0. Note that the index is 1 in the reference year: $AC_r(0,0) = 1$.

Appendix Table 4 shows the aggregation consistency index for selected values of r. For values of r in the interval $(-5, +5)$ deviations from aggregation consistency are very small. Only outside that range do they become significant. For example, if $r = 20$ then the minimum value of the index is 0.9717 and the maximum is 1.0370.

4.5 How much difference does chain-linking make?

The impression gained from Hill (2006) is that GDP growth is much more sensitive to the value of r than the results in Table 2 or Fig. 4 would suggest. But Hill's results (though using different data) are all based on 2-year (or direct), not chained, indices. By a 2-year or direct index is meant a superlative index which uses only the weights from the first and last years of the period, and not any of the weights from the intervening years. Thus an index for the period 1987–2000 would use the weights only of 1987 and 2000, not the weights also of 1988, 1989, …, or 1999. Figure 5 shows 2-year indices for 1987–2000, 2000–2019 and for the whole period 1987–2019. Qualitatively the picture seems very similar to Hill's.

Appendix Table 5 gives a direct comparison between chained and non-chained (2-year) superlative indices of real GDP for values of r ranging from -20 to $+20$. Three time periods are considered: 1987–2000, 2000–2019, and the whole period 1987–2019. Figure 6 compares 2-year and chained superlative indices directly for the whole period.

The first thing to note is that if r lies between about -1 and $+1$, then the 2-year and the chained indices are very similar, e.g. in the Törnqvist case the estimated growth rates over 1987–2019 are 2.350% p.a. versus 2.402% p.a. But outside that range, i.e. $r < -1$ or $r > 1$, the two types of index start to diverge; e.g. using the 2-year Fisher suggests growth was 2.893% p.a. compared to 2.404% p.a. for the chained Fisher. For $r = 5$ the 2-year index gives a growth rate of 4.788% p.a. compared to 2.415% p.a. on the chained measure. And for $r = 20$, the growth rate over 1987–2019 is 5.361% p.a. compared to 2.562% p.a. on the chained measure.

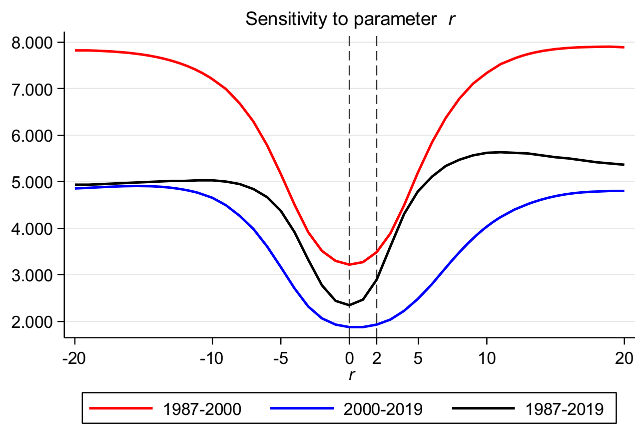

Fig. 5 Two-year superlative indices: growth of real GDP, % p.a. Source Appendix Table 5. 2-year superlative indices use only the weights from the first and last years of the period in question, not those of any intervening years

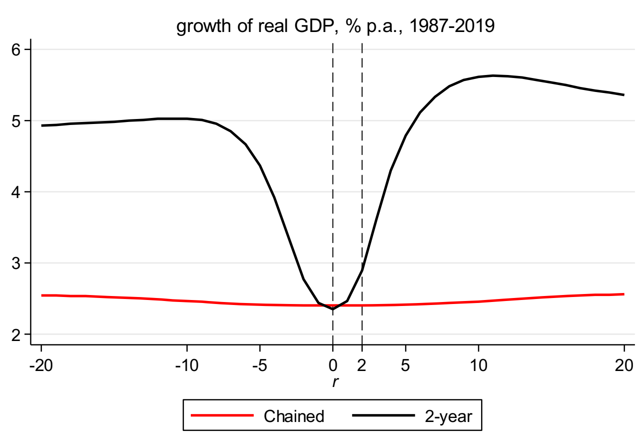

Fig. 6 Chained vs 2-year superlative indices. Source Appendix Tables 2 and 5. 2-year superlative indices use only the weights from the first and last years of the period in question (in this case 1987 and 2019), not those of any intervening years

Appendix Table 5 also compares a non-chained Laspeyres with a chained Laspeyres and a non-chained Paasche with a chained Paasche. The non-chained Laspeyres, also known as a Lowe index, uses the weights only of the first year of a given period; the non-chained Paasche uses the weights only of the last year of a given period. Without chain-linking, the Laspeyres shows growth at 3.757% p.a. over the whole period, an enormous difference from the chained measure, 2.481% p.a.

The conclusion is that chain-linking vastly reduces the sensitivity of the estimated growth rates to the choice of r. This is certainly the case for superlative indices: with chain-linking these are fairly insensitive to the value of r. With chain-linking, even the (non-superlative) Laspeyres index is not very different from the Fisher: 2.481 versus 2.404% p.a. over 1987–2019. So if the US had adopted the European method it would have made only a modest difference to the estimated growth rate. But use of an unchained Laspeyres (or Lowe) index would give us a fundamentally different view of US growth: 3.757% p.a. over 1987–2019 versus 2.404% p.a. according to the chained Fisher. This might be thought sufficient justification in itself for shifting from a fixed base (or Lowe) index to a chained one. Without chain-linking, the choice between Fisher and Törnqvist also becomes quite consequential: 2.893 versus 2.350% p.a. over 1987–2019.

In summary, the conclusions of Hill (2006) are largely replicated and confirmed for 2-year indices. But equally we see that the opposite conclusion applies to chained indices: they are fairly insensitive to the value of r.

As we have seen (Theory), there is certainly a case for chain-linking if the aggregator function is not constant over time (Diewert 1976); for example the a_{ij} coefficients in the quadratic mean of Eq. (1) may be changing over time. But it is not quite so clear why chain-linking reduces the *sensitivity* of the growth rate to r. One possibility is that in a chained index all the yearly changes are comparatively small. So we can invoke the result that all superlative indices of the form of (2) approximate each other closely for small changes around a given point (Diewert 1978).

5 Conclusions

To calculate a superlative index number we have to assume a value for the unknown parameter r in the superlative index number formula. However, using data on real value added from the BEA/BLS industry-level production account, we have found that estimates of real GDP growth are not very sensitive to the value chosen for r, provided that the estimates are chained; if the estimates are not chained, then on the contrary the results can be quite sensitive to the value of r. This is encouraging if we accept chain-linking since it reduces uncertainty about the true growth rate. We also found that with chain-linking superlative indices are very close to both value consistency and aggregation consistency (for $-5 \le r \le 5$).

In future work it would be desirable to extend the analysis to the other main aggregates, namely capital, labour and intermediate input. It would also be desirable to extend the time period back before 1987, as far as 1947, if this were to prove possible.

Finally, the estimates presented here rest on the assumption of perfect competition: prices equal marginal costs, though some distortions are still encompassed within the framework, e.g. the price for the same capital or labour input can differ across industries as in Jorgenson et al. (1987). But much of modern macroeconomics is built on the contrary assumption, imperfect competition, at least for short run analysis.[16] Recently there has been much discussion of whether margins are rising; see Basu (2019) for a survey of margin estimates in the United States which vary widely though are generally positive. On the other hand, macroeconomists of the real business cycle school still hold to the perfect competition assumption (price equals marginal cost) but they seem to be in the minority. Extracting estimates of output and productivity from the national accounts is a much more challenging task under imperfect competition since it requires the estimation of margins which are not directly observed (Basu and Fernald 2002). It also raises the possibility that aggregate TFP is affected by movements of resources towards or away from firms with high margins (Baqaee and Farhi 2020).

[16] The literature on imperfect competition and productivity goes back to Hall (1988).

6 Appendix

Tables 1–5

Table 1 (a) Shares of 63 industries in U.S. nominal GDP, percent. (b) Value added shares of 9 industry groups in GDP, percent

(a)

Number	Industry	1987	2000	2019
1	Farms	1.24	0.73	0.55
2	Forestry, fishing, and related activities	0.35	0.21	0.18
3	Oil and gas extraction	0.95	0.65	0.81
4	Mining, except oil and gas	0.38	0.27	0.26
5	Support activities for mining	0.14	0.13	0.24
6	Utilities	2.52	1.72	1.48
7	Construction	4.23	4.40	4.02
8	Wood products	0.40	0.27	0.18
9	Nonmetallic mineral products	0.51	0.41	0.29
10	Primary metals	0.66	0.45	0.29
11	Fabricated metal products	1.39	1.16	0.73
12	Machinery	1.40	1.08	0.75
13	Computer and electronic products	1.98	2.15	1.37
14	Electrical equipment, appliances, and components	0.72	0.44	0.29
15	Motor vehicles, bodies and trailers, and parts	1.43	1.31	0.71
16	Other transportation equipment	1.35	0.68	0.74
17	Furniture and related products	0.36	0.32	0.14
18	Miscellaneous manufacturing	0.49	0.56	0.43
19	Food and beverage and tobacco products	1.82	1.56	1.24
20	Textile mills and textile product mills	0.42	0.27	0.07
21	Apparel and leather and allied products	0.46	0.21	0.04
22	Paper products	0.79	0.59	0.27
23	Printing and related support activities	0.53	0.42	0.18
24	Petroleum and coal products	0.43	0.50	0.73
25	Chemical products	1.84	1.79	1.74
26	Plastics and rubber products	0.66	0.63	0.37
27	Wholesale trade	5.74	5.94	5.68
28	Retail trade	6.94	6.54	5.19
29	Air transportation	0.49	0.55	0.65
30	Rail transportation	0.42	0.22	0.19
31	Water transportation	0.08	0.08	0.06
32	Truck transportation	0.90	0.94	0.77
33	Transit and ground passenger transportation	0.15	0.18	0.24
34	Pipeline transportation	0.13	0.09	0.19
35	Other transportation and support activities	0.68	0.63	0.61
36	Warehousing and storage	0.22	0.25	0.34
37	Publishing industries, except internet (includes software)	0.91	1.11	1.30
38	Motion picture and sound recording industries	0.55	0.52	0.39
39	Broadcasting and telecommunications	2.72	2.64	2.09
40	Data processing, internet publishing, and other information services	0.28	0.23	1.27
41	Federal Reserve banks, credit intermediation, and related activities	2.93	3.07	3.22
42	Securities, commodity contracts, and investments	0.83	1.27	1.47

Table 1 (continued)

(a)

Number	Industry	1987	2000	2019
43	Insurance carriers and related activities	1.64	2.62	2.69
44	Funds, trusts, and other financial vehicles	0.15	0.15	0.11
45	Real estate	10.30	10.44	11.13
46	Rental and leasing services and lessors of intangible assets	1.04	1.30	1.19
47	Legal services	1.20	1.23	1.26
48	Computer systems design and related services	0.43	1.09	1.65
49	Miscellaneous professional, scientific, and technical services	2.86	3.90	4.41
50	Management of companies and enterprises	1.55	1.63	1.83
51	Administrative and support services	1.53	2.44	2.74
52	Waste management and remediation services	0.22	0.25	0.26
53	Educational services	0.67	0.91	1.22
54	Ambulatory health care services	2.42	2.75	3.48
55	Hospitals and nursing and residential care	2.18	2.47	2.99
56	Social assistance	0.32	0.50	0.64
57	Performing arts, spectator sports, museums, and related activities	0.32	0.47	0.65
58	Amusements, gambling, and recreation industries	0.35	0.47	0.43
59	Accommodation	0.73	0.89	0.82
60	Food services and drinking places	1.68	1.85	2.17
61	Other services, except government	2.43	2.67	2.02
62	Federal	6.86	4.74	4.78
63	State and local	9.69	10.08	11.74
	TOTAL (GDP)	100.0	100.0	100.0
	Min	0.08	0.08	0.04
	Max	10.30	10.44	11.74
	S.D.	2.11	2.11	2.25

(b)

Industry group no	Industry group code	Industries	Industry group name	1987	2000	2019
1	AFFHM	1–5	Agriculture, forestry, fishing, hunting, and mining	3.06	1.99	2.04
2	TWU	6, 29–36	Transportation, warehousing, utilities	5.59	4.65	4.54
3	CONST	7	Construction	4.23	4.40	4.02
4	MANUF	8–26	Manufacturing	17.63	14.79	10.55
5	TRADE	27,28	Trade	12.68	12.48	10.87
6	INFO	37–40	Information	4.46	4.50	5.05
7	FIRE	41–46	Finance, insurance, real estate, rental and leasing	16.89	18.84	19.82
8	OSERV	47–61	Other services	18.90	23.53	26.58
9	GOV	62,63	Government	16.55	14.82	16.53
			TOTAL (GDP)	100.00	100.00	100.00

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account (BEA-BLS-industry-level-production-account-1987–2020.xlsx, released May 11 2022)

Note Industry-level shares are industry value added as % of nominal GDP

Table 2 Chained superlative indices: annual growth rates of real GDP in the U.S., 1987–2019, % p.a

Value of r	Mean	Std. Dev.	Min	Max
−20	2.547	3.193	−12.009	9.782
−19	2.542	2.981	−11.111	8.947
−18	2.537	2.771	−10.165	8.120
−17	2.531	2.564	−9.180	7.317
−16	2.525	2.365	−8.169	6.559
−15	2.518	2.178	−7.151	5.863
−14	2.510	2.008	−6.149	5.241
−13	2.500	1.861	−5.187	4.699
−12	2.489	1.742	−4.290	4.472
−11	2.477	1.654	−3.476	4.473
−10	2.465	1.594	−2.759	4.513
−9	2.453	1.559	−2.142	4.538
−8	2.442	1.543	−1.695	4.552
−7	2.432	1.539	−2.062	4.560
−6	2.423	1.540	−2.362	4.562
−5	2.416	1.545	−2.599	4.562
−4	2.411	1.550	−2.780	4.559
−3	2.407	1.554	−2.914	4.556
−2	2.404	1.558	−3.008	4.554
−1	2.402	1.560	−3.068	4.552
0	2.402	1.560	−3.099	4.551
1	2.402	1.559	−3.104	4.552
2	2.404	1.557	−3.083	4.554
3	2.406	1.554	−3.036	4.557
4	2.410	1.549	−2.961	4.562
5	2.415	1.542	−2.854	4.567
6	2.421	1.534	−2.710	4.573
7	2.429	1.526	−2.525	4.577
8	2.438	1.517	−2.290	4.580
9	2.448	1.510	−2.002	4.580
10	2.460	1.508	−1.661	4.574
11	2.473	1.515	−1.613	4.560
12	2.486	1.538	−2.138	4.534
13	2.500	1.582	−2.758	4.493
14	2.512	1.651	−3.472	4.502
15	2.524	1.748	−4.270	4.538
16	2.534	1.872	−5.138	4.935
17	2.542	2.019	−6.054	5.515
18	2.549	2.185	−6.996	6.165
19	2.556	2.366	−7.943	6.872
20	2.562	2.556	−8.876	7.621
Memo items				
Limit as $|r| \to \infty$	2.994	10.102	−22.421	22.829
Chained Laspeyres	2.481	1.535	−2.692	4.655
Chained Paasche	2.327	1.583	−3.474	4.453

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987–2020.xlsx, released May 11 2022)

Note Quantities are real value added for 63 industries; weights are shares in aggregate nominal value added (nominal GDP). Growth rates calculated as 100 x mean annual log difference over the period. Superlative indices calculated from Eqs. (2) and (9). Limit as $|r| \to \infty$ calculated from Eq. (3)

Table 3 Value consistency index (1987 = 1): Ratio of value index to product of chained price index and chained quantity index

year	Value of r									Lasp-
	−20	−5	−2	−1	0	1	2	5	20	-eyres
1987	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1988	1.0006	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0007	0.9990
1989	1.0025	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0022	0.9982
1990	1.0024	1.0002	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0018	0.9974
1991	1.0064	1.0005	1.0001	1.0001	1.0000	1.0000	1.0000	1.0002	1.0048	0.9967
1992	1.0088	1.0007	1.0002	1.0001	1.0000	1.0000	1.0000	1.0002	1.0064	0.9959
1993	1.0052	1.0005	1.0001	1.0001	1.0000	1.0000	1.0000	1.0001	1.0035	0.9951
1994	1.0095	1.0007	1.0002	1.0001	1.0000	1.0000	1.0000	1.0002	1.0066	0.9939
1995	0.9980	1.0001	1.0001	1.0001	1.0000	1.0000	1.0000	0.9999	0.9971	0.9917
1996	0.9865	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000	0.9997	0.9883	0.9890
1997	1.0109	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	0.9997	1.0045	0.9870
1998	1.0197	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	1.0111	0.9849
1999	1.0236	1.0003	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	1.0143	0.9833
2000	0.9274	0.9969	0.9994	0.9998	1.0000	1.0001	1.0000	0.9986	0.9359	0.9794
2001	0.9078	0.9958	0.9991	0.9997	1.0000	1.0001	1.0000	0.9982	0.9211	0.9780
2002	0.9094	0.9960	0.9992	0.9997	1.0000	1.0001	1.0000	0.9982	0.9220	0.9774
2003	0.8825	0.9946	0.9989	0.9996	1.0000	1.0001	1.0000	0.9977	0.9008	0.9760
2004	0.8806	0.9944	0.9988	0.9995	1.0000	1.0001	1.0000	0.9977	0.8994	0.9751
2005	0.8738	0.9939	0.9987	0.9995	1.0000	1.0001	1.0000	0.9975	0.8941	0.9740
2006	0.8687	0.9936	0.9986	0.9995	1.0000	1.0001	1.0000	0.9974	0.8906	0.9727
2007	0.8678	0.9936	0.9986	0.9995	1.0000	1.0002	1.0000	0.9974	0.8897	0.9715
2008	0.9686	0.9997	1.0002	1.0003	1.0003	1.0002	1.0000	0.9987	0.9652	0.9709
2009	1.0854	1.0056	1.0015	1.0008	1.0003	1.0000	1.0000	1.0013	1.0651	0.9634
2010	0.9972	1.0027	1.0008	1.0005	1.0002	1.0001	1.0000	1.0004	1.0008	0.9611
2011	0.9770	1.0016	1.0005	1.0004	1.0002	1.0001	1.0000	1.0000	0.9848	0.9596
2012	0.9761	1.0015	1.0005	1.0003	1.0002	1.0001	1.0000	1.0000	0.9844	0.9588
2013	0.9751	1.0014	1.0005	1.0003	1.0002	1.0001	1.0000	1.0000	0.9838	0.9584
2014	0.9623	1.0010	1.0004	1.0003	1.0002	1.0001	1.0000	0.9999	0.9752	0.9577
2015	1.2004	1.0154	1.0034	1.0015	1.0004	0.9999	1.0000	1.0047	1.1735	0.9545
2016	1.2319	1.0169	1.0039	1.0018	1.0005	0.9999	1.0000	1.0050	1.1911	0.9542
2017	1.2196	1.0162	1.0037	1.0017	1.0005	0.9999	1.0000	1.0048	1.1826	0.9536
2018	1.2128	1.0158	1.0036	1.0017	1.0005	0.9999	1.0000	1.0047	1.1777	0.9528
2019	1.2238	1.0163	1.0037	1.0017	1.0005	0.9999	1.0000	1.0049	1.1861	0.9520

Source U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987–2020.xlsx, released May 11 2022)

Table 4 Aggregation consistency index (1987 = 1)

year	Value of *r* −20	−5	−2	−1	0	1	2	5	20
1987	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1988	1.0003	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0002
1989	1.0005	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0004
1990	1.0018	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0014
1991	1.0017	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0013
1992	1.0020	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0013
1993	1.0019	1.0002	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0012
1994	1.0010	1.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0002
1995	0.9982	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	0.9973
1996	0.9973	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9999	0.9965
1997	1.0087	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	1.0025
1998	1.0080	1.0001	1.0001	1.0000	1.0000	1.0000	1.0000	0.9998	1.0018
1999	1.0050	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992
2000	0.9977	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9936
2001	0.9892	0.9998	1.0000	1.0001	1.0000	1.0000	1.0000	0.9997	0.9890
2002	0.9866	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9868
2003	0.9843	0.9995	1.0000	1.0000	1.0001	1.0000	1.0000	0.9997	0.9858
2004	0.9840	0.9995	1.0000	1.0000	1.0001	1.0001	1.0000	0.9997	0.9856
2005	0.9839	0.9995	1.0000	1.0000	1.0001	1.0001	1.0000	0.9997	0.9853
2006	0.9789	0.9993	0.9999	1.0000	1.0001	1.0001	1.0000	0.9995	0.9816
2007	0.9773	0.9990	0.9999	1.0000	1.0001	1.0001	1.0000	0.9995	0.9802
2008	1.0558	1.0005	1.0001	1.0001	1.0001	1.0001	1.0000	0.9997	1.0370
2009	1.0303	0.9981	0.9997	1.0000	1.0000	0.9999	0.9997	0.9981	1.0127
2010	0.9775	0.9977	0.9996	0.9999	1.0000	1.0000	0.9997	0.9981	0.9764
2011	0.9784	0.9977	0.9996	0.9999	1.0000	1.0000	0.9998	0.9981	0.9774
2012	0.9767	0.9976	0.9996	0.9999	1.0000	1.0000	0.9997	0.9981	0.9759
2013	0.9755	0.9975	0.9996	0.9999	1.0000	1.0000	0.9997	0.9980	0.9750
2014	0.9695	0.9975	0.9996	0.9999	1.0000	1.0000	0.9998	0.9981	0.9718
2015	0.9756	0.9976	0.9996	0.9999	0.9999	0.9998	0.9995	0.9975	0.9717
2016	0.9855	0.9978	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9773
2017	0.9853	0.9978	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9774
2018	0.9839	0.9977	0.9997	0.9999	1.0000	0.9998	0.9995	0.9975	0.9764
2019	0.9832	0.9977	0.9997	0.9999	1.0000	0.9998	0.9995	0.9974	0.9756
Std. Dev.	0.0195	0.0011	0.0002	0.0001	0.0000	0.0001	0.0002	0.0010	0.0148
Min	0.9695	0.9975	0.9996	0.9999	0.9999	0.9998	0.9995	0.9974	0.9717
Max	1.0558	1.0005	1.0003	1.0002	1.0001	1.0001	1.0000	1.0000	1.0370

Source: U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987–2020.xlsx, released May 11 2022)

Note The aggregation consistency index is the ratio of the 2-step chained quantity index to the 1-step chained quantity index. The first step of the 2-step index is chained quantity indices for each of 9 industry groups. The second step aggregates these to the GDP level. See Table A1(b) for the definition of the 9 industry groups

Table 5 Superlative indices: average annual growth rates of real GDP, % p.a. 2-year indices compared to chained indices

	1987–2000		2000–2019		1987–2019	
Value of r	2-year	Chained	2-year	Chained	2-year	Chained
−20	7.822	3.298	4.855	2.033	4.932	2.547
−19	7.819	3.307	4.870	2.018	4.941	2.542
−18	7.810	3.312	4.883	2.006	4.950	2.537
−17	7.793	3.314	4.894	1.996	4.961	2.531
−16	7.768	3.313	4.902	1.986	4.972	2.525
−15	7.732	3.308	4.904	1.978	4.984	2.518
−14	7.680	3.302	4.896	1.968	4.997	2.510
−13	7.609	3.294	4.873	1.958	5.009	2.500
−12	7.512	3.285	4.829	1.945	5.020	2.489
−11	7.383	3.275	4.757	1.931	5.027	2.477
−10	7.211	3.266	4.647	1.917	5.025	2.465
−9	6.983	3.257	4.489	1.902	5.005	2.453
−8	6.682	3.248	4.271	1.889	4.952	2.442
−7	6.287	3.241	3.979	1.878	4.847	2.432
−6	5.780	3.234	3.605	1.869	4.663	2.423
−5	5.165	3.227	3.162	1.861	4.365	2.416
−4	4.501	3.222	2.705	1.856	3.917	2.411
−3	3.915	3.218	2.322	1.852	3.329	2.407
−2	3.509	3.215	2.068	1.849	2.769	2.404
−1	3.291	3.213	1.929	1.848	2.443	2.402
0	3.220	3.212	1.873	1.847	2.350	2.402
1	3.277	3.213	1.876	1.848	2.467	2.402
2	3.484	3.214	1.929	1.849	2.893	2.404
3	3.889	3.216	2.041	1.852	3.612	2.406
4	4.494	3.220	2.223	1.856	4.298	2.410
5	5.192	3.225	2.482	1.861	4.788	2.415
6	5.843	3.230	2.805	1.868	5.115	2.421
7	6.378	3.237	3.152	1.876	5.333	2.429
8	6.793	3.245	3.485	1.885	5.477	2.438
9	7.108	3.254	3.780	1.897	5.567	2.448
10	7.344	3.263	4.029	1.910	5.616	2.460
11	7.520	3.274	4.233	1.925	5.633	2.473
12	7.650	3.285	4.396	1.940	5.625	2.486
13	7.743	3.296	4.524	1.955	5.601	2.500
14	7.808	3.307	4.620	1.969	5.568	2.512
15	7.852	3.317	4.689	1.981	5.532	2.524
16	7.880	3.327	4.738	1.991	5.494	2.534
17	7.895	3.334	4.769	2.000	5.457	2.542
18	7.900	3.340	4.786	2.008	5.423	2.549
19	7.899	3.342	4.794	2.018	5.390	2.556
20	7.892	3.342	4.796	2.028	5.361	2.562
Memo items						
Laspeyres	4.086	3.294	2.217	1.924	3.757	2.481
Paasche	2.881	3.134	1.641	1.775	2.029	2.327

Source: U.S. Bureau of Economic Analysis, BEA/BLS Integrated Industry-level Production Account. (BEA-BLS-industry-level-production-account-1987-2020.xlsx, released May 11 2022)

Note Quantities are real value added for 63 industries; weights are shares in aggregate nominal value added (nominal GDP). Growth rates calculated as 100 x mean annual log difference over the stated period. Superlative indices calculated from Eqs. (2) and (9). 2-year superlative indices use weights of just the first and last years of the period; chained superlative indices use weights of all years of the period. 2-year Laspeyres (Paasche) uses only weights of first (last) year of period

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