Combinatorial auctions in practice

LSE Research Online URL for this paper: http://eprints.lse.ac.uk/124108/

Version: Accepted Version

Article:

Palacios-Huerta, Ignacio, Parkes, David C. and Steinberg, Richard ORCID: 0000-0001-9636-472X (2024) Combinatorial auctions in practice. Journal of Economic Literature, 62 (2). 517 - 553. ISSN 0022-0515

https://doi.org/10.1257/jel.20221679

Combinatorial Auctions in Practice

Ignacio Palacios-Huerta[1], David C. Parkes[2], and Richard Steinberg[3]

[1]Department of Management, London School of Economics and Political Science,
London, WC2A 2AE, United Kingdom
`i.palacioshuerta@gmail.com`

[2]John A. Paulson School of Engineering, Harvard University,
Cambridge, MA 02138
`parkes@eecs.harvard.edu`

[3]Department of Management, London School of Economics and Political Science,
London, WC2A 2AE, United Kingdom
`r.steinberg@lse.ac.uk`

March 19, 2024

Abstract

We survey the uses of combinatorial auction that have been deployed in practice, giving emphasis to their key representational and economic aspects. In addition, we discuss behavioral economics considerations on both the bidder and auctioneer sides of the market, and the interrelated topics of simplicity and trust, highlighting key opportunities for future work.

1 Introduction

In a *combinatorial auction* (CA) multiple items are offered either for sale or purchase and bidders can express non-additive valuations for items by placing bids on sets of specific items. Considerable advances have been made in the economic theory of CAs since they were formally introduced in 1982. Against this backdrop, we reflect upon the practical uses that have been found for this special type of auction, where we aim to capture the state of the art. An enormous number of applications have been proposed for CAs. Here, we have restricted our attention to those CAs that have not only been proposed, but have actually been put into practice.[1] Beyond the high-level design characteristics of such CAs, we also detail some of the key representational and economic aspects of deployed CAs. In addition, we discuss behavioral economics considerations on both sides of the market, and the interrelated topics of simplicity and trust, highlighting key opportunities for future work.

The need for combinatorial auctions. If multiple items are to be auctioned off, why not auction them individually one-by-one? A problem with this is that if some items are worth more

---

[1] CAs are also used for resource allocation within organizations, with bid units specified in terms of a virtual currency allocated to bidders. For example, CAs have been used to allocate cloud computing resources to product groups within Google (Stokely et al. 2009).

to a bidder as a set than the sum of their individual valuations, then it is difficult for the bidder to know how much to bid for each item. Since there is no guarantee that the bidder would win every item in his set, the bidder faces what is known as "exposure risk", i.e., the danger of ending up paying more for an incomplete set of items than they are worth to him. Why doesn't the auctioneer bundle the items into packages in advance to be auctioned sequentially? Unfortunately, the auctioneer's packages might not correspond to the packages in which the bidders are interested, and in fact the desired packages could differ from bidder to bidder.

Beyond package bidding, modern applications of CAs allow bidders to express other types of non-additive valuations on items. For example, a CA may allow a supplier to bid a volume discount for quantities above a specified threshold. Package bidding by itself may be unnatural and insufficiently succinct to use in large-scale applications such as those in the supply chain. CAs need not allow bidders to express every possible valuation function on packages, but must provide some ability to express non-additive valuations, e.g., substitutes or complements. This is the crucial requirement in meeting our definition of a CA. However, the design of a CA is not obvious, and it is delicate matter to design an auction that allows packages of interest to the bidders, without placing an undue computational burden on the auctioneer. The problem of determining winning bids in a CA is computationally intractable for some *bidding languages*, i.e., ways in which bidders can communicate their bids to the auctioneer. For example, if the bidding language allows bids on all packages of items, then the Winner Determination Problem is *NP-hard* and thus intractable in the worst case.[2]

Types of auctions, and their objectives. In a *forward auction*, the auctioneer is the seller and the bidders are buyers, and a possible design objective is to maximize revenue. When the bidders are sellers and the auctioneer is the buyer, the auction is a *reverse auction*—also called *procurement auction*—and a possible design objective is to minimize the cost of purchasing the specified goods. However, a more typical objective in auctions for the strategic sourcing of direct and indirect materials and for transportation logistics would be some measure of *total value to the buyer*, which considers not just cost but also supplier risk and business value. Another possible design objective for both forward and reverse auctions is *allocative efficiency*, which is the maximization of the total value of the allocation to all bidders and the seller.

CAs may be single-round events, in which sealed bids are placed simultaneously, or they may proceed across multiple rounds, with new bids placed or existing bids refined in response to new information available to the bidders. An example of such new information would be *ask prices* quoted by the auctioneer.

A short history of combinatorial auctions. An early reported use of a CA in practice was a real estate auction, for the Liberty Plant at Alameda, California, held on November 14, 1922 (U.S. Congress 1925). The auction was for 70 residence sites comprising the plant, and allowed for a package bid to be placed on all items—an "entirety bid"—in addition to bids on individual items. There was some confusion as to whether an entirety bid for $405,000 was placed (U.S. Congress 1925), but it seems that no deposit was recorded and no such bids were initially considered. When individual parcels were auctioned, the total amount of the winning bids was $164,243.50. Following

---

[2] NP-hardness is a concept from the field of computational complexity, which studies the practical difficulty of solving problems involving many items. A strongly conjectured implication of a problem being NP-hard is that, as the size of the problem grows (e.g., the number of bids in the auction increases), then the worst-case number of steps that would be required to solve the problem (e.g., determine the allocation of items to bidders) increases not polynomially but exponentially in the size of the problem (Rothkopf et al. 1998).

a later appeal an entirety bid of $350,000 was accepted. Thirty years later, bankruptcy auctions that allowed for entirety bids were reported in the literature (McLean 1955).

Almost another thirty years passed before CAs were formally introduced by Rassenti, Smith and Bulfin (1982). Their motivation was the allocation of airport runway slots at congested airports. When an airline requires a take-off slot for a flight, of course it requires a landing slot at the destination airport too; when there are interconnecting legs in the flight path, there will also be demands for slots at intermediate airports. Rassenti et al. proposed addressing this problem via a sealed-bid CA, where airlines submit package bids for flight-compatible combinations of individual airport runway slots. They formulated the auctioneer's problem as an integer programming problem, and provided an algorithm to find an allocation that maximizes efficiency and determines prices.

By 1992 CAs were being used for the sourcing of transportation services at Global 1000 companies, and in 1993 they were used for the sourcing of bus services in Manchester, England and domestic flight routes in Sweden. In 2000, CAs were deployed for industrial procurement at Mars, Inc. In 2001, CAs were used for the sale of spectrum, and soon after that for many other applications as well. These are discussed in Section 3.

Related work. Cramton et al. (2006) provides a thorough survey of CAs, focusing mainly on theory. However, the final four chapters discuss applications to airspace system resources (Ball, Donohue & Hoffman 2006), truckload transportation (Caplice & Sheffi 2006), bus routes (Cantillon & Pesendorfer 2006), and industrial procurement (Bichler et al. 2006). For a non-technical overview of auction theory—including CAs—see Steinberg (2012). There is very little empirical work on CAs. Two significant contributions, each of which takes a structural estimation approach, is work by Cantillon and Pesendorfer which analyzes bid data from first-price sealed-bid CAs for bus routes in London (in the book chapter cited above, (Cantillon & Pesendorfer 2006)), and Kim et al. (2014), which measures the performance of large-scale first-price sealed-bid CAs for school meals in Chile. Hortaçsu & McAdams (2018) survey developments in the empirical analysis of multiple-item auctions, both combinatorial and non-combinatorial.

In the private sector there are a significant number of proprietary deployments of CAs; however, for most of these, reliable information is not available. Thus, we have been able to include only a small sample of private sector applications here.

In addition, one topic that is not discussed in detail is that of *combinatorial exchanges*. A combinatorial exchange is a two-sided combinatorial auction with multiple buyers and multiple sellers that is used to trade multiple heterogeneous goods (Parkes et al. 2001, Lubin et al. 2008, Mittelmann et al. 2021). One successful application is to the trade of fishery access rights in New South Wales (Bichler et al. 2018). However, the major use is in wholesale electricity markets, which are spot markets that aim to maximize clearing value subject to network and resource constraints. Through the use of expressive bids, resources can bid their underlying economic drivers, which include start-up cost, minimum energy cost, and the marginal cost for electricity generation. The largest markets are the *day-ahead markets (DAM)*, which determine electricity prices for each of the 24 hours of the following day and come in two types: *pool-type* models and *exchange-type* models. Pool-type models are commonly used in U.S. markets (such as PJM, ERCOT, and MISO), in which market participants submit their bids to the market operator and self-schedule their units according to dispatched amounts determined by the operator. Exchange-type models are commonly used in European markets, typically with a uniform pricing rule.[3]

---

[3] For a detailed description of the operation of day-ahead wholesale electricity markets, see the first section of Derinkuyu et al. (2020).

Outline of the Paper In Section 2, we discuss CA designs; specifically, auction design variations, cross-cutting features, and payment rules. In Section 3 we present our findings and what we view as being the most successful, and least successful, deployments of CAs to date. We begin Section 4 by connecting design choices with theory. We then integrate a number of behavioral economics considerations into the study of CAs by reviewing insights from the economics and psychology literature that operate on the sides of regulators, auctioneers and bidders, which we view as important for future theoretical and empirical research. This includes a consideration of the interrelated topics of simplicity and trust. We conclude in Section 5 with some final thoughts.

A catalog of applications of CAs in practice can be found in Appendix A, which we separate out into Reverse CAs (A.1), and Forward CAs (A.2). Appendix B is the technical appendix, which covers Core Constraints (B.1), the Vickrey-Clarke-Groves Mechanism (B.2), and the Vickrey-nearest Core-selecting Payment Rule (B.3). Finally, we should point out that the paper includes a comprehensive list of references by academics, practitioners, and government agencies worldwide on combinatorial auctions in practice.

2 Combinatorial Auction Designs

2.1 Auction Design Variations

To date there are well over 8000 papers in the academic literature on CAs. Many if not most of these papers propose new designs for CAs; however, *in practice* there have been only three main variations of CAs actually deployed:

- *Sealed-bid combinatorial auction (SB)*: A single-round auction, with bidders submitting bids on individual items and packages without knowledge of the bids placed by others. After the auction closes, the Winner Determination Problem is solved to determine the allocation. Some criteria that are adopted for the selection of bids include: maximize total reported value or minimize total reported cost; maximize total reported value or minimize total reported cost subject to business rules and quality constraints. Various payment rules can be adopted (see §2.3), including *first-price*, *VCG*, and *Vickrey-nearest core-selecting.*

- *Multi-round combinatorial auction (MR)*: An auction where bids are submitted simultaneously on individual items and packages in each of multiple rounds. A *provisional allocation* is determined at the end of each round, and generally some feedback is available to bidders between rounds about the allocation, and including bid prices or how to improve bids. The feedback process across rounds, and determination about when to close can be *ad hoc*, especially in the private sector. A MR auction may close after a fixed number of rounds, when demand is less than or equal to supply, or at the discretion of the auctioneer. The final allocation may be determined based on the bids submitted across all rounds, or just the final bids. Various payment rules can be adopted, with first price the most common.

- *Combinatorial clock auction (CCA)*: An auction including a *clock stage*, composed of multiple rounds with prices on items set by the auctioneer. In each round of the clock stage, each bidder places a bid for a single package of items at the current prices. At the end of each round, the auctioneer increases the price on each item whose aggregate demand exceeds its supply. The clock stage ends when the aggregate demand on every item is less than or equal to its supply. The CCA was introduced by Porter et al. (2003).

The CCA is ordinarily used with a second stage, which might take the form of a *supplementary bids round*, consisting of a single round in which bidders can submit one or more package bids at prices they themselves specify, both to improve on clock stage bids and to bid for new packages. In this format it becomes the *clock-proxy auction* (Ausubel et al. 2006). The final allocation is determined by placing the bids from the clock rounds and the second stage (if it exists) into the Winner Determination Problem, with the restriction that only at most one bid from each bidder can be selected as winning; i.e., the *XOR bidding language*, discussed in Section 2.2, is used. Payments are typically determined via second-price principles, most notably the Vickrey-nearest core-selecting payment rule.

In the CCA, *activity rules* are often used to constrain the bids that can be placed in each successive round, including *revealed-preference based activity rules* and *eligibility-point based activity rules*. A revealed-preference based activity rule requires that bids placed by a bidder in each round must be consistent with their bids in all previous rounds. Bids are consistent when there exists a valuation for which bids in each round would be a best-response for a utility-maximizing, non-budget-constrained bidder.[4] An eligibility-point based activity rule prevents a bidder from holding back information and increasing demand later in the auction—sometimes referred to as the "snake in the grass" strategy. A bidder is allocated eligibility points at the start of the auction that define the maximum number of items on which the bidder is allowed to bid. If the number of items the bidder wins in a round and the new bids the bidder submits require fewer eligibility points than those in the previous round, then the bidder loses eligibility points.

The CCA was designed for use by national governments to sell spectrum licenses, the radio frequency portion of the electromagnetic spectrum that is used for wireless communication, including radio, television, cellular phones, as well as other applications. The CCA has been in constant evolution (Ausubel & Baranov 2014*a*). In particular, CCAs can include a third stage of bidding. In the three-stage CCA, items are first distinguished by category (e.g., spectrum band), with items in a category (e.g., blocks of frequencies in the spectrum band) treated as identical during the first two stages. The third stage, called the *assignment stage*, allows bidders to express values for specific items within each category (e.g., blocks of specific frequencies within the spectrum band), and results in the assignment of those specific items to bidders. The third stage is typically conducted as a sealed-bid auction. In spectrum auctions, assignments are usually constrained to require contiguous spectrum licenses within a geographic region, with bidding restricted accordingly.

In 2016, for its 1800 MHz auction, Denmark introduced the *combinatorial multi-round ascending auction (CMRA)*, which modifies the CCA to allow multiple packages to be bid during each round of the clock stage, rather than just a single package, including some at or below the clock prices, and is used with a first-price payment rule and without a proxy round (see Section 2.3). The CMRA was employed again by Denmark in 2019 and by Norway in 2020.

2.2 Cross-cutting Features

Having described the high-level design variations found in practice, we now describe various cross-cutting features that provide additional specificity to the CAs used in practice.

The first two cross-cutting features are defining features of a CA, i.e., a CA must possess one or both of them. They relate to the bidding language, which determines the bids that can be placed:

---

[4] For example, if the price in round 2 is 5 for A and 3 for B and the bidder bids on A, then that bidder cannot bid on B in round 4 when the prices are 10 for A and 10 for B. This is because the price on B has increased by more than the price on A, and B cannot be a best-response in round 4 when A was a best-response in round 2.

(F1) *Package bids*: A bid on a set of items, requiring that if any item in the set is allocated to a bidder, then every item in the set must be allocated to that same bidder. The two most prominent in practice are the *OR language*, in which any number of package bids can be selected as winning, although these can sometimes be restricted to pre-defined packages by the auctioneer, and the *XOR language* (Sandholm 2002*a*), in which only one bid from each bidder can be selected as winning. Another bidding language, although used much less frequently for CAs in practice, is the *OR-of-XOR language* (Sandholm 2002*b*), in which each bidder can submit an arbitrary number of XOR bids, where an arbitrary number of bids can be accepted from each bidder that appear in distinct XOR bids.

(F2) *Expressive bids*: A bid for which the bidder can compactly indicate a willingness to purchase or sell a possibly large number of specified sets of items at specified prices (Sandholm 2013). In a procurement auction, two possible types of expressive bids include *conditional discounts*, which trigger when quantity or volume thresholds are met over some set of goods, and *capacity constraints*, which limit the total quantity of items that can be supplied. In an auction to procure transportation logistics, an illustrative kind of expressive bid is "*I will provide up to ten vehicles on leg one and up to twenty vehicles on leg two, at a cost of $5,000 per vehicle, as long as the number of vehicles on leg one is within 50% of the number of vehicles on leg two, and the total number of vehicles is at most fifteen.*"

The third cross-cutting feature reflects whether the bid taker (the seller in a forward auction, the buyer in a reverse auction) can provide specific criteria to define the objective and constraints guiding winner determination:

(F3) *Business rules*: A business rule is a specific constraint or adjustment imposed by the auctioneer on the optimization criterion that is used to selecting winning bids. Examples include constraints to limit the number of suppliers that can win in a procurement auction, or the maximum quantity that can be allocated to any one supplier. Business rules are distinct from feasibility requirements, e.g., leaving band gaps when selling spectrum, or leaving space to accommodate utility shafts when configuring rental units in a new building. In some settings, rules are used together with scenario navigation, where the bid taker can modify such rules after bids have been submitted, re-solving the Winner Determination Problem given the modified rules (Sandholm 2006, Catalán et al. 2009).

In application instances where there were multiple auction runs, this taxonomy of cross-cutting features will be used to indicate *typical use*, not the capability of the particular system. For example, in Table 2, *Reverse CAs, Private Sector (sample)*, under Industrial Procurement and Sourcing of Direct and Indirect Materials, we include "Materials at Global 1000 companies," which made use of a technology developed by CombineNet. Although package bidding was included in the software suite, this feature was rarely used. Rather, side constraints, adjustments, discounts and other forms of expressiveness were used for most of the auctions run using this software. There were also limits set on the maximum and minimum number of winners, preferences given to existing suppliers and minority-owned business, and quality-of-service constraints. One or more of these business rules applied to essentially all auctions run, and often combined with scenario navigation by the bid taker. For this reason, we choose to adopt features F2 and F3 but not F1 for this application, as reflects typical use.

2.3 Payment Rules

CAs can be distinguished through the design of their *payment rule*, which is used to determine payments made by (or made to, in the case of procurement) bidders.

The simplest payment rule is first price:

- *First-price (FP)*: In a forward auction, the bidder with the highest bid pays his bid. In a reverse auction, the bidder with the lowest bid is paid his bid. The first-price payment rule is also known as *pay-as-bid*. This definition relates to the calculation of the payment: the payment is simply set equal to the bid amount. A first-price rule in a sealed-bid or multi-stage auction might provide quite different incentives and thus result in quite different final bids.

Another kind of payment rule that has been proposed in the theoretical literature but not, to our knowledge, used much in practice, is a *linear-pricing rule*, which may also be called *pay-as-clear* (or, in the context of multi-item auctions, *uniform pricing*). As suggested in Rassenti et al. (1982) and further developed by Kwasnica et al. (2004) for multi-round CAs and Lubin et al. (2008) for multi-round combinatorial exchanges, the idea is to find approximate clearing prices given bidder demand.[5] For CAs, a simpler form of linear prices, i.e., those used in the first stage of the CCA, are now widely adopted (as explained above, the pricing in the first stage of the CCA is pay-as-bid, with no use of optimization to find approximate clearing prices).

An alternative is to use a second-price payment rule, where the payment made by a bidder is calculated using the bids of other bidders, and is minimal in some sense (or maximal in a reverse auction). We need to introduce two definitions. The *core* is the subset of feasible allocations where there does not exist any coalition of bidders and the seller (the buyer, in a reverse auction) willing to renegotiate the outcome of the auction. Clearly, for reasons of stability, it is highly desirable that the outcome of the auction be in the core. The *core constraints* are constraints that ensure that the outcome of an auction is in the core. (For more details, see Appendix B.1.) Two variations of second-price rules are used in practice for combinatorial auctions:

- *Vickrey-Clarke-Groves (VCG)*: In a forward auction, the payment made by a bidder is the amount by which its presence in the auction reduces the total value of other bidders, as represented through their bids. In a reverse auction, the payment to a selected seller is equal to the effect that the seller's participation in the auction has in reducing the total cost to other sellers, as represented through their bids. (For more details, see Appendix B.2.)

- *Vickrey-nearest core-selecting (CORE)*: A modification of the VCG payment rule that requires bidder payments to satisfy the core constraints and minimize a distance metric to the VCG payments as represented through their bids. (These core constraints are imposed on bids, with the effect that the auction is no longer incentive aligned. There is a general impossibility result on achieving core outcomes in incentive-compatible auctions if VCG is not in the CORE (Goeree & Lien 2016).) In a forward auction, these constraints have the effect of placing lower bounds on seller revenue, considering revenue available from unallocated bids, for example. (For more details, see Appendix B.3.)

3 Survey of Applications

3.1 Summary of CAs in Practice

Reverse Combinatorial Auctions. We begin by reporting on the applications of Reverse Combinatorial Auctions, i.e., CAs for sourcing and procurement, first in the public sector (Table 1) and

---

[5] The one application of a linear-pricing, pay-as-clear rule that we are aware of is to a course registration system at Chicago University's Graduate School of Business and was used at least during the 1980s (Graves et al. 1993).

then in the private sector (Table 2). The term *sourcing* refers to the use of CAs for strategic decision making, where the output of an auction is a long-term agreement or contract, providing terms against which goods and services can be purchased going forward. A typical example is to determine which suppliers will provide which ingredient goods, and at what cost, with purchases made subsequent to contracting. The term *procurement* refers to the use of CAs for tactical decision making, where the output of an auction is the timely purchase of goods or services from suppliers, rather than a contract that specifies purchases in the future.

In the simplest cases for reverse CAs, the winners are the suppliers with bids that, taken in combination, provide goods or services at the lowest total price. However, in practice there are often important additional considerations. For example, in the sourcing of school Bus Services in the UK there was a quality-of-service constraint. Details of each reverse CA application are provided in the Electronic Companion in Section 5.

Table 1: Reverse CAs, Public Sector.†

Location	Application	Auction Design	Features				Date
			F1:pack	F2:expr	F3:rules	Payment Rule	
	Sourcing of Bus Services						
Manchester, UK	School bus services	SB	◇	◇	◇	FP	93-
London, UK	Municipal bus services	SB	◇		◇	FP	95-01
Helsinki, Finland	Municipal bus services	SB	◇		◇	FP	97-09
Sweden	Municipal and regional bus services	SB (MR once)	◇	◇	◇	FP	03-04
	Sourcing of School Meal Services						
Chile	School meals to public schools	SB	◇	◇	◇	FP	97-
	Sourcing of Miscellaneous Government Services						
Sweden	Domestic flight routes	SB	◇	◇	◇	FP	03
Ireland	Vehicle fleets	SB	◇		◇	FP	05
Sweden	Cleaning of schools and offices	SB	◇	◇	◇	FP	05-11
Sweden	Road resurfacing	SB	◇	◇	◇	FP	05-11
Sweden	Elderly care	SB	◇	◇	◇	FP	05-11
Denmark	Snow removal	SB	◇	◇	◇	FP	13
	Sourcing of Electricity Generation Capacity						
Chile	Forward capacity for distribution companies	SB	◇	◇	◇	FP	06,08,10

† F2: Bidders can adjust the bid price. Allocation also limited by financial capacity, other considerations. F3: Various bid taker constraints, e.g., quality-of-service, total number of tasks, total quantity constraints.

Reverse CAs in the public sector have been applied in four areas, all of them sourcing, specifically, bus services in the UK, Finland, and Sweden; school meal services and electricity generation capacity in Chile; and miscellaneous government services in Sweden, Denmark, and Ireland.

The School Meal Services auction involved 2.5 million children in public schools in Chile. The items for auction were three-year contracts for each of around one hundred "territorial units" (TU's) into which the country was divided. The design was that of a first-price sealed-bid CA. Bids were selected to minimize the government's cost, where constraints were imposed on the number of meals, and the number of TU's, that each bidder would be permitted to supply. Typical packages selected in the final allocation contained three TU's. The Chilean government credits the auction as saving $40 million per year on spend of around $500 million per year.

Reverse CAs in the private sector have been applied in three areas: industrial procurement and sourcing of direct and indirect materials worldwide, sourcing of transportation services worldwide, and sourcing of miscellaneous non-industrial services in the UK, viz., roadside assistance services. Specific applications here are quite diverse and include, under direct materials, food ingredients and chemicals; under indirect materials, industrial equipment; and under transportation services, truckload transport, intermodal transport, ocean transport, and air transport.

Table 2: Reverse CAs, Private Sector (sample).‡

Location	Application	Auction Design	Features				Date
			F1:pack	F2:expr	F3:rules	Payment Rule	
	Sourcing of Transportation Services						
Worldwide	Truckload, Rail, Inter-Modal & Ocean Transport at Global 1000 companies	SB	◇	◇	◇	FP	92-02
US	Truckload Transport at Sears Logistics Services	MR	◇			FP	93-02
Worldwide	Truckload Transport at Mars	MR	◇	◇	◇	FP	00-02
Worldwide	Air Transport at Motorola	MR		◇	◇	FP	02-03
	Industrial Procurement and Sourcing of Direct and Indirect Materials						
Worldwide	Materials at Mars, e.g., sugar (direct), in-store display boxes (indirect)	MR	◇	◇	◇	FP	00-02
Worldwide	Materials at Motorola, e.g., semiconductors (direct), paper (indirect)	MR		◇	◇	FP	02-03
Worldwide	Materials at Global 1000 companies	MR		◇	◇	FP	00-
	Sourcing of Miscellaneous Non-Industrial Services						
UK	Roadside Assistance Services at Direct Line Insurance	SB	◇		◇	FP	12

‡ F2: Capacity constraints. Discount schedules. F3: Preference for minority-owned business, quality-of-service adjustments. Various other constraints (e.g., min/max winners, total quantity).

For reverse CAs, bidder expressiveness (F2) allows suppliers to specify capacity limits, volume discounts, and alternate products. Firms surveyed employing reverse CAs generally place emphasis on the use of sophisticated modeling and optimization, e.g., to enable the re-optimization of an entire, multi-modal transportation network. Cross-cutting feature (F3) is especially important in the application of CAs to large-scale sourcing events. This feature allows a bid taker to specify business rules and to perform interactive scenario navigation in order to make various tradeoffs and value adjustments prior to selecting a final allocation. Details of each reverse CA application are provided in the Electronic Companion in Section 5.

Forward Combinatorial Auctions. We next survey the application of Forward Combinatorial Auctions, i.e., CAs for the sale or rental of goods or services, first in the public sector—Table 3 for non-spectrum applications and Tables 4 and 5 for the sale of wireless spectrum licenses—and then in the private sector (Table 5). For obvious reasons, sale auctions in the public sector usually have names. Also, despite the vaunted claim by governments that their objective is efficiency rather than the generation of revenue, the very significant amounts of revenue that can be raised by these auctions—especially from the sale of spectrum—cannot be ignored, and is often taken as a proxy of the auction's success by the public and government alike. Forward CAs in the public sector have included the sale of government plant in California (the first CA in practice), sales of wireless spectrum licenses worldwide (the most remunerative CA in practice), and rental of floor space in Amsterdam (arguably, the most unusual CA in practice). With forward CAs, a seller might seek to maximize revenue subject to additional considerations, such as giving preferential treatment to small businesses through *bid adjustments*, i.e., modifications made to bids by the auctioneer in order to achieve policy goals, or he might place limits on the quantity available to purchase by

Table 3: Forward CAs, Public Sector, Non-Spectrum Applications.§

Location	Auction Name	Auction Design	Features				Dates (yy-yy)
			F1:pack	F2:expr	F3:rules	Payment Rule	
	Wholesale electricity markets (sample)						
France	EDF Generation Capacity	CCA	♢			FP	01
Texas	PUC of Texas Quarterly Capacity Auctions	CCA	♢			FP	01-06
New Jersey	New Jersey BGS (Basic Generation Service)	CCA	♢			FP	02-
Belgium	Electrabel generation	CCA	♢			FP	03-
Netherlands	Nuon generation capacity	CCA	♢			FP	04
Illinois	Illinois Annual Capacity Auctions	CCA	♢			FP	04-07
Denmark	Elsam VPP (Virtual Power Plant)	CCA	♢			FP	05
Germany	RWE VPP (Virtual Power Plant)	CCA	♢			FP	06
Portugal	REN-EDP	CCA	♢			FP	07-09
Spain	Spanish Capacity Auctions (Endesa-Iberdrola)	CCA	♢			FP	07-08
Pennsylvania	Penelec Electricity Auctions	CCA	♢			FP	09-12
Germany	E.ON VPP (Virtual Power Plant)	CCA	♢			FP	10
	Sale of Government Plant						
California	Liberty Yard Auction	SB	♢			FP	11/1922
	Rental of Floor Space						
Amsterdam	Furore Solids Auction	MR	♢		♢	FP	5/11
Amsterdam	IJburg Solids Auction	MR	♢		♢	FP	6/11

§ F3: Minimum percentage of floor area required for each bidder type (for rental of floor space).

any one bidder, such as *spectrum caps* in spectrum auctions, which are limits on the quantity of spectrum that a bidder is permitted to acquire during the auction.

Wholesale electricity markets (Table 3) were discussed in Section 1. Two significant on-going wholesale electriicity markets are the New Jersey Basic Generation Service and Electrabel in Belgium. The sale of government plant and the Liberty Yard auction was discussed in Section 1.

One of the most unusual applications of combinatorial auctions was for the configuration and rental of floor space in a public housing project. Stadgenoot, a Dutch public housing corporation, devised the idea of constructing buildings in which the tenants determine the allocation of space within the building. The building under consideration is divided into lots, which tenants use as "building blocks" to specify spaces, and prices, through the use of package bidding. The first of two auctions was held in May 2011 for a single building in the west of Amsterdam having 7000 square meters of floor space partitioned into 125 lots over seven floors. The second auction was held in June 2011, for two buildings in IJburg, a new planned neighborhood in the east of Amsterdam.

For the sale of wireless spectrum licenses, we have included for each application in Tables 4 (non-CCA format) and 5 (CCA format) both the name of the auction and the revenues raised. Some spectrum auctions have the frequency band included in their names, e.g., the "3.6 GHz Band Spectrum Auction" held in Ireland in 2017, while some spectrum auctions do not, e.g., the "AWS Spectrum Auction" held in Mexico in 2016 for the 1.7 GHz and 2.1 GHz bands. In the latter case, we have included in Tables 4 and 5 the relevant spectrum bands in brackets after the auction name.

An explanation of some of the technical terms associated with spectrum auctions would be in order here. The terms *3G*, *4G*, and *5G* refer, respectively, to the third, fourth, and fifth generation of wireless communication networks, introduced about every ten years, which have brought successive improvements with regard to speed, performance, and security, as well as other technological

Table 4: Forward CAs, Public Sector, Sale of Wireless Spectrum Licenses (non-CCA format).¶

Location	Auction Name	Features				Dates	Revenue (millions)
		F1:pack	F2:expr	F3:rules	Payment Rule		
	Sealed-Bid Combinatorial Auction						
Norway	900 MHz Auction (Auction #1)	◇		◇	FP	10/01	\$1.3+\$1.5pa
Norway	1800 MHz Auction (Auction #2)	◇		◇	FP	12/01	\$0.6+\$1.4pa
Nigeria	Fixed Wireless Access Auction [3.5 GHz]	◇			FP	06/02	\$38
UK	Auction of 412-414 MHz paired with 422-424 MHz	◇		◇	FP	10/06	\$2.3
Ireland	26 GHz Auction[1]	◇		◇	CORE	10/08	\$1.7
Portugal	Auction [3.4-3.6, 3.6-3.8 GHz]	◇			CORE	04/10	\$4.6
France	2.6 GHz Spectrum Auction	◇		◇	FP	09/11	\$854
France	800 MHz Spectrum Auction	◇		◇	FP	12/11	\$3,353
Norway	800, 900 and 1800 MHz Auction (Auction #14)	◇		◇	FP	12/13	\$292
Ireland	26 GHz Auction	◇		◇	CORE	04-06/18	\$1.7+\$1.8pa
	Multi-Round Combinatorial Auction						
US	Regional Narrowband PCS Auction (Auction 51) [901, 930 MHz]	◇		◇	FP	09/03	\$0.13
US	700 MHz Auction (Auction 73)	◇		◇	FP	01-03/08	\$19,120

¶ F3: Spectrum Caps, bidder adjustments, credits for small business. Various other constraints e.g., must-allocate-item, max winners.

[1] Used a modified CORE payment rule involving adjusting all bids at once with a sum-of-squares penalty.

advances. *AWS* stands for Advanced Wireless Services, which is a specific spectrum band intended for 3G and 4G wireless phone and broadband services. *Broadband* refers to a high-capacity transmission technique using a wide range of frequencies that enables a large number of messages to be communicated simultaneously. *BWA* stands for Broadband Wireless Access, a catch-all term to describe some specific wireless broadband technologies including fixed and mobile applications. *Fixed wireless access* refers to the provision of wireless communication between two fixed locations, e.g., building to building. *Narrowband PCS (Personal Communications Service)* refers to two-way paging and other services such as the monitoring of utility meters from off-site locations. The *digital dividend* is the spectrum that becomes available when existing analogue television services are converted to digital technology; for each analog TV channel that is changed to digital, the digital video compression technology can transmit numerous digital subchannels using the same amount of spectrum.

CAs have been used to sell widely differing types of spectrum. The first two CAs for spectrum were held in 2001 in Norway, for 900 MHz and 1.8 GHz, respectively; both auctions employed a first-price sealed-bid design. Currently, the most popular CA design for spectrum is the CCA, and the second-most popular is the sealed-bid auction. The first CCA spectrum auction was held in Trinidad in 2005.

The payment rule used most often for spectrum is the Vickrey-nearest core-selecting payment rule, followed by the first-price payment rule. A modification of the Vickrey-nearest core-selecting payment rule was used for the sale of 26 GHz spectrum in Ireland in 2008. It employed a complex pricing rule involving adjusting all bids at once with a sum-of-the-squares adjustment penalty.

For its 2016 auction of 1800 MHz spectrum, Denmark used the CMRA, where a bidder submitting a *headline bid* at linear clock prices may also submit additional package bids at or below

Table 5: Forward CAs, Public Sector, Sale of Wireless Spectrum Licenses (CCA format).¶

Location	Auction Name	Features				Date (mm/yy)	Revenue (millions)
		F1:pack	F2:expr	F3:rules	Payment Rule		
	Combinatorial Clock Auction						
Trinidad	Spectrum Auction [800 MHz, 1.9 GHz]	♢		♢	FP	06/05	$25.1
Trinidad	BWA Auction [700 MHz; 2.6, 28 GHz]	♢		♢	FP	10/07	$1.3pa
UK	10-40 GHz Auction	♢			CORE	02/08	$2.8
UK	L-Band Auction [1.4 GHz]	♢			CORE	05/08	$16
Trinidad	Second BWA Auction [700 MHz; 2.3, 2.6 GHz]	♢		♢	FP	04/09	$0.74pa
Netherlands	2.6 GHz Spectrum Auction	♢		♢	CORE	04/10	$3.3
Denmark	2.5 GHz* Spectrum Auction	♢		♢	CORE	05/10	$175
Austria	2.6 GHz Spectrum Auction	♢		♢	CORE	09/10	$53
Sweden	1800 MHz Auction[1]	♢			VCG	06/11	$204.5
Switzerland	Auct. of Mobile Radio Frequencies [800, 900 MHz; 1.8, 2.1, 2.6 GHz]	♢		♢	CORE	02/12	$1,085
Denmark	800 MHz Spectrum Auction	♢		♢	CORE	06/12	$130
Romania	Spectrum Auction [800, 900 MHz; 1.8, 2.6 GHz]	♢		♢	FP	09/12	$891
Netherlands	Multi-Band Spectrum Auction [800, 900 MHz; 1.8, 1.9, 2.6 GHz]	♢		♢	CORE	10-12/12	$4,950
Ireland	Multi-Band Spectrum Auction [800, 900 MHz; 1.8 GHz]	♢		♢	CORE	11/12	$631+$16.7pa
UK	4G Spectrum Auction [800 MHz; 2.6 GHz]	♢		♢	CORE	01-02/13	$4,550
Australia	Digital Dividend Spectrum Auction [700 MHz; 2.6 GHz]	♢		♢	CORE	05/13	$2,020
Austria	Multiband Auction [800, 900 MHz; 1.8 GHz]	♢		♢	CORE	10/13	$2,756
Slovakia	800, 1800 & 2600 MHz Spectrum Auction	♢			CORE	01/14	$222
Canada	700 MHz Spectrum Auction	♢		♢	CORE	02/14	$4,770
Slovenia	Multi-Band Spectrum Auction [800, 900 MHz; 1.8, 2.1, 2.6 GHz]	♢		♢	CORE	04/14	$206
New Zealand	700 MHz Auction (Auction 12)	♢		♢	FP	10/13-06/14	$234
Canada	2500 MHz* Auction	♢		♢	CORE	04-05/15	$624
Mexico	AWS Spectrum Auction [1.7, 2.1 GHz]	♢		♢	CORE	02/16	$2,400
Montenegro	Auct. for Spectrum of Mobile Networks [800, 900 MHz; 1.8, 2, 2.6 GHz]	♢		♢	CORE	07-08/16	$5.7
Denmark	1800 MHz Spectrum Auction[2]	♢		♢	FP	09/16	$154
Ireland	3.6 GHz Band Spectrum Auction	♢		♢	CORE	05/17	$67.7+$19.8pa
Denmark	700 MHz, 900 MHz and 2300 MHz Auction[2]	♢		♢	FP	03-04/19	$385.5
Norway	Auct. of Fixed Link Microwave Bands [10, 13, 18, 23, 28, 32, 38 GHz][2]	♢		♢	FP	05/20	$4.2

¶ F3: Spectrum Caps, bidder adjustments, credits for small business. Various other constraints e.g., must-allocate-item, max winners.

*The 2.6 GHz band (2500 to 2690 MHz) is also known as the 2.5 GHz (2500 MHz) band.

[1] Used a degenerate CCA where the items on offer are identical during the clock stage.

[2] This was conducted through the CMRA format.

Table 6: Forward CAs, Private Sector (sample).††

Location	Application	Auction Design	Features				Date
			F1:pack	F2:expr	F3:rules	Payment Rule	
	Bankruptcy and Real Estate Sales						
Worldwide	Sale of general property (commonplace)	SB+Single-item	◇			FP	1922-
	Sale of Contextual Display Ads						
Worldwide	Facebook	SB	◇	◇	◇	VCG	10-
Worldwide	Google, AdSense	SB	◇	◇	◇	VCG	12-

†† F2: XOR bid on items imputed from per-click bid, using click-through rates. Budget constraints. Various constraints (e.g., quality thresholds, within-show crowding and separating constraints.)

the clock prices (and above the reserve). The CMRA has no supplementary bidding, or proxy stage, and the payment rule is first-price. Moreover, it closes whenever every active bidder can be allocated.

Spectrum licenses can have widely differing durations, but usually for a limited term, which typically range from between ten and twenty years. For the sale of spectrum in Montenegro in 2016, lots were sold in both ten- and fifteen-year terms in the same auction.

Forward CAs in the private sector (Table 6) have included bankruptcy and real estate sales worldwide, and contextual display ads on the Internet. A display ad is typically an image, sometimes including some form of animation or video, together with ad copy. Viewers can then click on the ad, and be taken to the corresponding advertiser's web page. Contextual display ads are targeted based on the text content on the web page, together with additional user context (including behavioral context). Such ads were introduced by Google in 2003 ("AdSense"), and by Facebook in 2007. The VCG mechanism was adopted by Facebook in 2010, and by Google in 2012.

It is interesting to observe that while expressive bids are not used in any of the public sector forward CAs surveyed in this paper, expressive bids are a feature of almost all applications of forward CAs in the private sector. Details of each forward CA application are provided in the Electronic Companion in Section 5.

3.2 Most and Least Successful CAs in Practice

Below, we review the CAs in practice that were the most successful and least successful, with success measured in terms of frequency of use and ongoing use, or in terms of revenue generated and costs saved.

Most Successful. In the public sector, CAs have been used in the sale of government plant, for the sourcing of bus services, and of miscellaneous government services, including domestic flight routes, meals to public schools, cleaning of schools and offices, road resurfacing, elderly care, snow removal, and vehicle fleets. However, the major success stories, in terms of frequency of use and revenue generated, have been in two areas: for the sourcing of electricity generation capacity, and especially for the sale of spectrum licenses. As of the closing of the Norwegian Fixed Link Microwave Band Auction in May 2020, the use of CAs for the sale of spectrum has raised over fifty billion dollars world-wide. Of this, forty-four billion dollars was generated in just eight auction events.

It is interesting to observe that essentially all the significant uses of CAs for sourcing in the public sector have been for services. Why the public sector has not embraced CAs more broadly for procurement and sourcing of goods is somewhat of a mystery, given that standard auctions have proved to be a highly useful tool for the selling of goods in the public sector. For example, in the U.S., the Federal and state governments use auctions for the sale of surplus government equipment, abandoned property, troubled assets (SIGTARP 2015), and even ships.

In the private sector, CAs have been used in bankruptcy and estate sales, direct and indirect industrial procurement, and for the sourcing of transportation services. In terms of frequency of use, the major success stories of CAs in the private sector have been for the sourcing of transportation and direct materials, and for contextual display ads on the Internet.

Least Successful. We have clear explanations for two applications of CAs that failed to fulfill expectations: truckload transportation and electricity generation capacity.[6]

For truckload transportation, Caplice et al. (2003) collected data to 2002 from three sources: from reviews of fourteen CAs from three software vendors; from interviews with eight truckload carriers, four software vendors, and one shipper, and from surveys conducted with 57 truckload carriers. Caplice et al. found that CAs had not been applied as extensively to transportation procurement as had been expected because shippers who ran CAs did not receive a large number of package bids, while carriers participating in CAs found that package bids were rarely awarded. Caplice et al. identified three key reasons for this: (1) a robust set of package bids is difficult and time consuming to construct; (2) package bids only rarely offer shippers the least-cost alternative to covering a given set of lanes, and (3) even when package bids are awarded, many shippers fail to actually tender those loads because they lack the implementation technology (i.e., to operationalize the synergies). As of 2018, bidding subject to minimum quantities is more common than package bidding (C. Caplice, personal communication, 2017).

For the sourcing of electricity generation capacity in Chile, between 2006 and 2010, there were high price differences among awarded contracts and distribution areas along with general city high prices and poor coverage. The consensus is that it was difficult to balance the dual objectives of minimizing cost and maximizing demand coverage (Moreno et al., 2010).

4 Discussion: Design, Theory, and Behavioral Economics

Our emphasis in this paper on practical applications of CAs suggests several directions potentially worthy of additional theoretical development and empirical research.

4.1 Connecting Design Choices with Theory

The evidence we have surveyed shows a number of distinct patterns of use of CAs in the public and private sector, which in turn suggest various avenues for research worthy of investigation. A distinction that we find salient in the practice of CAs is in regard to choices made in the use of

---

[6] Two other CAs were discontinued, but nonetheless should be viewed as successful. One was for the sourcing of bus services in Helsinki, which began in 1997 and ended in 2010. The use of CAs ended because the responsibility to run the auctions had moved from the City of Helsinki to a larger regional unit whose algorithms and software apparently could not handle CAs (J. Tukiainen, personal communication, 2017). The other was for the configuration and rental of floor space in Amsterdam, which was deployed only twice, in May and in June 2001. The explanation here is that Stadgenoot, the Dutch housing association that designed and ran the auctions, had wanted to use this auction for their new buildings in Amsterdam; however, it is not often that a housing agency builds a building of that size, and none was constructed in Amsterdam after June 2011 (F. Spiekssma, personal communication, 2017).

simple vs. complex designs. In regard to multi-round CAs, and the design choices as they affect the complexity placed on the auctioneer, one distinction is in regard to the choice between adopting simple, *ad hoc* feedback across rounds vs. the use of structured price-adjustment rules and activity rules. As it relates to design choices in regard to bidding languages, whether for sealed-bid or multi-round CAs, and their effect on the complexity facing the auctioneer, the main distinction that we have in mind is in regard to the use of the standard XOR bidding language vs. the relative complexity that comes through the use of a more succinct but less standard and more specialized language. In regard to pricing, and whether for sealed-bid or multi-round CAs, first-price rules are simpler for an auctioneer to operationalize than second-price or core-selecting designs.

For Sourcing and Procurement. Public sector CAs for procurement and sourcing tend to be simpler than their private sector counterparts. This can be explained by the legal requirements of public sector auctions, which often include mandatory public consultation, as well as concerns with fairness and perception of fairness. In particular, public sector procurement auctions tend to be sealed bid, whereas private sector auctions tend to be multiple-round events. Turning to the private sector, the significant theoretical development of price-adjustment rules does not seem to be reflected in industrial practice. Rather, it is typical to run an unstructured, multi-round auction with simple and *ad hoc* feedback between rounds. It would be worthy of further investigation to explain why auction designs in practice tend to be simpler than theory would propose in this sense, with coarse price feedback and a small number of bidding rounds.

Practical private-sector designs make use of various forms of expressiveness in addition to package bidding, such as bid curves, constraints, volume discounts, and business rules and constraints, and are in this sense relatively complex. These auctions also tend to have sophisticated winner determination procedures, for example needing to optimize the outcome of an auction given the use of complex bid taker adjustments and constraints.

The use of CAs in the private sector includes settings where the bid taker can change the winner determination criteria as bids are received. For example, the bid taker may adjust a constraint on the number of winning suppliers, a criterion that bounds the maximum quantity that can be allocated to any one supplier, or a coefficient that controls the cost to the bid taker from replacing a preferred kind of material with an alternate material. This departs from the study of auctions with endogenous quantity, in that the criteria by which these decisions are made arise as an additional input from the bid taker (coming, for example, as the result of interactive scenario navigation by the bid taker with a winner determination algorithm). This suggests as an opportunity for future work to develop a theory for this phenomenon in which the method for determining winners is not explicitly stated and may in fact vary across rounds of an auction, in response to changing bid taker considerations. A key question to address concerns the implications for the equilibrium bidding behavior of auction participants.

All surveyed applications of CAs for procurement and sourcing, in both the public and private sectors, have been first price. This is despite considerable theoretical development of second-price payment rules. One salient reason is that well-defined item prices are required for the contracting phase (while second-price rules adjust the total payment, not the per-item prices). Another possible reason relates to reasons of trust on the part of the bid taker (and the topic of *credibility*, to which we return in Section 4.3). In the case of government sourcing and procurement, it may be that it is difficult to tell the public that a bidder was paid more than what the bidder had offered to accept. This speaks directly to the preferences of the government, or “behavioral regulator,” which we discuss below.

For Sale. Public sector CAs for the sale of goods have become more sophisticated over the years and, in this sense, less simple. It is also here where theory has informed practical CA design, and especially in regard to the use of activity rules and the choice of core-selecting payment rules. For spectrum auctions, for example, the combinatorial-clock auction with a core-selecting payment rule has become popular. The use of the XOR language and package bidding in these public sector applications may be driven by the relative structure that can be provided in defining the goods space in these economic environments compared to the use of CAs for sourcing and procurement in the private sector. In the private sector, CAs for the sale of goods have been dominated by the VCG mechanism, for contextual display ads on the Internet, and by sealed-bid first price auctions, which find widespread application to bankruptcy and real estate sales.

Design, generally. A first overarching observation is that it has proved important to provide structure to the preference elicitation process of CAs, and most typically this is done through multi-round designs that enable price discovery, guiding bidders towards the parts of the allocation space where they can be especially competitive. In the particular case of spectrum auctions, this is coupled with the use of activity rules so that multi-round designs do not afford new kinds of strategic behaviors (beyond refining a bidder's revealed preferences as prices change). Another aspect of this is that many private sector sourcing and procurement auctions make use of non-package forms of bid expressiveness. Beyond the sheer scale of the goods spaces in many of these supply chain applications, which in itself necessitates the use of specialized languages, another reason is that the outcome of a sourcing event most typically needs to be operationalized through a procurement system, where these systems require item prices (i.e., with well defined prices as procurement is done against a sourcing agreement that is generated through auction). A third aspect of preference elicitation is that while the XOR language is the most typical choice in multi-round, public-sector applications such as to wireless spectrum licenses (where its disadvantages in terms of succinctness are mitigated through the use of price discovery), there is more use of the OR language and other variants in few or single-round settings.

In regard to simplicity vs. complexity, it is forward auctions and not reverse auctions that tend to be relatively complex and sophisticated in the public sector. This is flipped in the private sector, where we see relatively sophisticated reverse auctions, and in contrast, less application of forward auctions and where they are used more association with simple designs. A possible explanation is that complexity tends to increase when it is justified by the potential economic value associated with an event (i.e., going along with wireless spectrum and other high-stakes forward auctions in the public sector, and all kinds of sourcing and procurement events in the private sector).

Turning to questions of implementation, and the difficulty in operationalizing CAs, a key point that bears emphasis is that the winner determination problem is NP-hard, and thus there is complexity in deciding which bids to take as winning. Beyond the auctioneer, this complexity of winner determination also affects the bidders in regard to their ability to model the strategic setting and develop sophisticated bidding strategies, with high computational complexity for example in simulating different strategies, bidder values, and auction outcomes. Another key point relates again to the need for care around the design of bidding languages, which are important because of the exponentially-large space of possible bundles, and the inherent difficulty in covering the goods space and the associated need for languages in which a relatively small number of bids can describe bid valuations over a relatively large number of possible bundles.[7]

---

[7] These kinds of implementation and operational issues are supported through various providers of CA technology, including IBM Emptoris, Jaggaer, Keelvar, Determine, and Coupa. In addition, firms such as Market Design and Auctionomics offer design and implementation expertise for specialized applications such as wireless spectrum auctions and electricity capacity auctions.

4.2 Behavioral Economics

Which auction format is best suited to allocate multiple objects depends on two issues: First, what are the objectives of the auctioneer and how well can the objectives be accomplished?; second, how do bidders behave in a given auction design? The answers to these questions involves "behavioral economics" considerations on both sides of the CA market. This is an area that integrates insights from psychological research into economic science, especially concerning judgement and decision-making under risk and uncertainty. Not only do institutional details matter, but real-world complexities associated with rationality, motivation, cognition and adaptation also come to the foreground. Mechanisms that work well from a theoretical perspective under simplifying assumptions may fail under more realistic assumptions. We argue that this represents a likely fruitful area of future research both theoretically as well as empirically (on the sides of regulators, auctioneers and bidders).

4.2.1 Behavioral Regulators and Auctioneers

Revenue is often an important objective in the auction literature. However, regulators are typically bound by their statutory duties, which express that they should maintain a competitive and innovative market, and allocate items efficiently. Usually, an important secondary goal is that companies acquiring these items pay a fair market price. The goal of maximizing revenue is rarely mentioned. See, for example, in spectrum auctions, Loertscher et al. (2015, pp. 863) who note that "in 1993 Congress granted the FCC authority to auction licenses with multiple objectives, including "efficient and intensive use of the electromagnetic spectrum" and recovering "a portion" of the value of the licenses for the public;" similarly, the National Audit Office (Morse 2014) noticed that in the 2013 UK auction "maximizing proceeds for the taxpayer was not an objective."

Further, a regulator's behavior may deviate from optimality in various ways, and instead be better characterized as "imitative" and "satisficing." Simon (1955) argued long ago that full optimality is an implausible assumption for the "administrative man." Rather, managers (regulators, auctioneers, designers) may optimize locally and try to achieve satisficing outcomes.

This is important because even small departures from rationality on the part of regulators can be as important as much larger departures from rationality on the part of individual participants (Ellison 2006, Armstrong & Huck 2010), and matter for designs observed in practice. Also, regulators typically face no competition, and this limits the market forces that will tend to eliminate behavioral firms. In a recent study, Kittsteiner et al. (2021) compared the choice of auction format by two competing auctioneers with that of a single auctioneer, and found that, even if a single auctioneer offers a CA, competing auctioneers in a comparable setting will not.

As noted earlier, it is somewhat of a mystery why the public sector has not embraced CAs more broadly for the procurement and sourcing of goods. Departures from optimality arising from these behavioral considerations represent potentially important reasons why some CA design should be offered but are not, and how some CAs are actually implemented in practice.

Take for instance, Fox & Bajari (2013)'s study of the C block auction of the 1900 MHz spectrum band held in the U.S. in 1995-96. The C block divided the U.S. into almost 500 graphically distinct license areas. Fox and Bajari found that the licenses had strong complementarities, which in turn meant that the FCC's decision to split the country into hundreds of separate zones was very inefficient. They conclude that efficiency would have been 48% higher had the FCC simply awarded four large, regional licenses to the four bidders with the highest valuations for these licenses. The auction would probably have achieved an even higher efficiency had it allowed for combinatorial bidding. Fox and Bajari write: "Evaluating all possible packages is not a tractable estimation strategy in the C block environment." Of course, the auction took place several years before the development of the CCA.

From a practical perspective, there are a number of pressing issues regulators face (Ausubel & Baranov 2017). We briefly discuss them next and argue that we should expect to see the impact of both design and behavioral considerations in future research:

1. DESIGN COMPLEXITY

1.1 RESERVE PRICES. Regulators often set reserve prices using non-transparent *ad hoc* procedures with very little thought given to the implementation of these prices. As a result, "many CCAs have adopted implementations that unnecessarily distort bidders' incentives" (Ausubel & Baranov (2017), p. F336). Take for example the two main approaches used in CCAs, bundle reserve prices and incremental reserve prices. (Bundle reserve prices specify that the payment for any bundle must be at least the sum of the reserve prices established for the items in the bundle, while incremental reserve prices require that an increase in payment for winning extra items is always at least the reserve prices for those items.) With complex and sometimes competing considerations coming from adopting the viewpoints of optimality of revenues, number of allocated items, and incentives, it is unclear which design approach different behavioral regulators will find suitable. In fact both of these reserve pricing approaches have been observed in practice. Understanding these considerations would require a rich model of regulator's behavior.

1.2 CHOICE OF ACTIVITY RULES. They are intended to make the early rounds of dynamic auctions informative, speed up the process, generate straightforward biding, and curtail 'bid sniping' opportunities. Of course, some goals interfere with others.[8] Given the trade-offs, regulators with different preferences will choose differently.

1.3. CHOICE OF BIDDING LANGUAGE. Regulators implementing CCAs have always imposed a maximum number of bids that can be submitted in the supplementary round. However, this leads to the question of how the limitation on the number of bids will impact the activity rules and the treatment of this reduced set of bids by the winner determination problem.

2. COMPETITION POLICY. The most frequent instruments for competition policy in spectrum auctions have been the set-aside (one or more spectrum blocks are reserved for a particular class of bidders) and the spectrum cap (a limit is placed on the quantity of spectrum within a given group of bands that a bidder is permitted to acquire or hold). But which exact frequencies should be reserved for entrants? And why apply caps individually to each incumbent when this impedes competition among incumbent bidders and depresses revenues? This issue is more general as it broadly concerns the reasons why we may expect behavioral phenomena and imperfections on the supply side, especially in the behavior of institutions that are not necessarily profit-oriented. Becker (1958) already argued that these may be at least as important, and perhaps substantially more so, in the policy sector as in the market place. Recent work points in this direction. Heidhues & Kószegi (2018, section 7) study general behavioral policy implications regarding the regulation of markets. Nuñez (2017) and Walker (2017) argue for developing behavioral economics as a practical tool for market authorities and discuss the lessons that competition authorities need to learn in a behavioral world. Kasdan (2020) further explores connections between public administration and behavioral economics. When theoretical models are confronted with the behavioral complexities of real-life situations, economists, regulators and public administration may think of themselves more as engineers. Whether behavioral bias may arise or not is determined endogenously in equilibrium (see, e.g., Esponda (2008) for an example with adverse selection). Examples of "engineering practices" range from the design of entry labor market clearinghouses and undergraduate college

---

[8] See Ausubel et al. (2006), Ausubel & Baranov (2014*a*, 2016), Janssen & Karamychev (2016), and Ofcom (2012).

admissions to the FCC spectrum auctions.

3. Preferences for Fairness. It is well known that the allocation of the difference between Vickrey-Clarke-Groves and core prices affects the fairness of how large versus small bidders are treated. But what determines the preferences for fairness of the regulator? Beginning with Rabin (1993) and Fehr & Schmidt (1999), there is an extensive behavioral economics literature on fairness. Recent work by Keniston et al. (2021), for example, uses detailed data from vastly different real-world settings that document a very robust pattern: agents favor offers that "split the difference" in potential surplus. They also show how this split-the-difference rule can be viewed as fair, and how this is a strong social norm in both low-stakes and high-stakes settings. de Clippel & Rozen (2021) experimentally study fairness through the lens of cooperative game theory. They find that a simple one-parameter model (mixing equal split and Shapley value) arises as a parsimonious description of the data. As such, understanding the determinants of fairness may prove quite valuable in future empirical CA work.

4.2.2 Behavioral Bidders

Auction theory is often presented as a prime example of the success of game theory. A reason is that the rules and set of strategies of the theoretical auction game are identical to the real-world game played by firms. Nevertheless, there are still significant difficulties for game theorists to predict how bidders will behave. As increasingly detailed evidence will become available in the future, we expect that a better understanding of the nature and formation of bidders' preferences will be a key ingredient in the evaluation of the design and performance of CAs. Theoretical foundations of CAs will have to integrate insights from the behavioral economics literature concerning human judgement and decision making.

Bidders' Objectives: Exogenous and Endogenous. As noted in Janssen & Karamychev (2017), "there is at least one important element where the real-world game may depart from the formal game that is studied, namely the objectives of the bidders. One may assume a certain type of a payoff function, but the objectives of the real-world bidders may well be different." Rapid changes in the technological and regulatory environment in which bidders operate may lead to substantial uncertainty about the value of objects for sale. Bidders may also have preferences over the full auction outcome, and not only on what they acquire. The value a bidder attaches to acquiring a certain item may also depend on what other bidders acquire and on how much they pay (e.g., spiteful bidders). The answer, for example, to the question as to whether the CCA fares better in terms of bidders paying a market price than the simultaneous ascending auction depends on the objectives bidders have, and thus on the nature of their preferences and their behavior towards competitors. This links to other areas of interest. For example, conduct that unreasonably excludes competitors from the marketplace is a concern of antitrust law. This includes predatory pricing (setting low prices in the short run in order to induce a rival to exit and then recouping the lost revenues by raising prices after exit), which focuses on conduct that lowers revenues. Similarly, cost-raising strategies can be used to disadvantage rivals or drive them out of the market with or without predatory pricing (Janssen & Karamychev 2017).[9]

---

[9] The origin of the concept of "rising rivals' costs" has been attributed to Director & Levi (1956); see also Salop & Scheffman (1983, 1987). This motive is of real concern in real-world auction design consultation phases; see, e.g., Ofcom (2012, page 122, point 7.9)]. In a multi-object auction, bidders can be winners and raise rivals' cost by placing bids on packages that are not winning themselves. In a multi-unit context, Janssen & Karamychev (2013) argue that the raising rivals' cost motive may stem from principal-agent issues within a firm (bidder) or from the fact that (in spectrum auctions) bidders face weaker competitors in the market after an auction if competitors have paid more for their licenses (see also Levin & Skrzypacz (2016)).

Bidders' Cognition, Ability and Errors in Strategic Reasoning. Often participants in CAs are highly experienced, profit-oriented professionals with significant payoffs at stake (who sometimes employ auction theorists to assist with their bidding decisions). However, as the number of packages grows exponentially with the number of objects, it may be difficult for bidders to reason about all packages of interest, and even with expressive bidding languages. Bounded cognition and errors matter. Bids need not be chosen optimally to maximize bidders' expected payoff, and thus an equilibrium may not be reached.[10]

Scheffel et al. (2012) experimentally study the CCA and other formats. They find that the main source of inefficiency in all formats is the bidders' selection of a limited number of packages. They conjecture that this has to do with cognitive limits, which in turn may lead them to "satisficing behavior" (Simon 1955) as a tendency to choose only a few options that meet some criteria using simple heuristics such as the relative value of a package and the number of adjacent items in the package.

Lab experiments have also yielded low revenue and low efficiency for the CCA in a market with a larger number of licenses (Bichler et al. 2013). Again, one reason was that bidders submitted only a small subset of the thousands of packages they could bid on. Focusing on a small subset can arise as a result of strategic and also behavioral reasons, as noted above. Interestingly, the CCA conducted in the UK in 2013 achieved revenue below the expectations, leading to an investigation by the UK National Audit Office (Arthur 2013), whereas some other CCAs such as the one in Austria in 2013 achieved high revenue. As it is unclear how bidders select a small number of bundles, and thus we may expect a substantial degree of heterogeneity in this selection, we may also expect to observe CCAs in the field with varying amount of revenues relative to expectations (see Kagel et al. (2010, 2014) for examples of studies that address such questions).

The role of *cognition* has been shown to be critical in other auction formats (see Goldfard & Xiao (2011), Hortaçsu & Puller (2008)). Strategic ability affects efficiency and this is likely to be important in CAs as well.[11] For instance, Hortaçsu et al. (2019) find substantial heterogeneity in how firms deviate from Nash equilibrium bidding in the Texas electricity market. These deviations, in turn, increase the cost of production. They also find that manager education affects sophistication, and that mergers that increase sophistication can increase efficiency. As noted earlier, complexity was a reason for the limited success in some of the least successful CAs (on the demand side, sourcing of electricity and the exchanges for financial securities, and on the supply side, with truckload transportation). Enke (2020), for example, provides causal evidence showing how the computational complexity of a decision problem is related to the frequency of incorrect mental models and the mechanisms behind errors in statistical reasoning. Neglecting relevant information is context-dependent and endogenous to the computational complexity of the environment. These insights are potentially relevant for CA policy in terms of developing effective methods to correct biased beliefs in these complex environments.

Related to the role of cognitive and practical limitations, errors in strategic reasoning may also play an important role. These errors are well-documented in experiments and central in behavioral

---

[10] Non-equilibrium behavior and alternative solution concepts have been able to explain the evidence in non CA settings. See, e.g., an incomplete model of bidding (Haile & Tamer 2003), and solution concepts such as quantal response equilibrium, cognitive hierarchy models, or stability (Bajari & Hortaçsu 2005, Gillen 2015, Fox & Bajari 2013). When auction data are not generated by equilibrium play, researchers have considered two approaches. First, data can be interpreted as generated by non-equilibrium play; second, researchers can restrict bidders' strategy space and focus on equilibrium play in a restricted game. Kim et al. (2014) show that this can be a tractable alternative approach. Similarly, behavioral considerations discussed in this section can be interpreted from this perspective and thus represent a tractable way to restrict bidder's strategy spaces.

[11] More generally, an important recent literature shows that cognition can be a crucial determinant in strategic settings (see Gill & Prowse (2016), Proto et al. (2019), and other references therein).

game theory,[12] and there are many instances that document these failings both in the lab and in other auctions formats.[13]

Kroemer et al. (2016) show that, apart from missing bids in the supplementary phase, *inconsistent* bidding in the clock phase can be a major source of error. They show that bidders in the lab and in the field (Canada and UK) do not bid straightforwardly in the clock phase. They conjecture that this may be due to budget constraints or interdependent endogenous values during the auction.[14] They report lab evidence, as well as data from the British 4G 2013 auction and the 2014 Canadian 700 MHz auction, showing that bidders deviate significantly from straightforward bidding in the clock phase, which can restrict the bids they can submit in the supplementary phase. Inconsistent bidding with respect to their true valuations can have a significant negative impact on efficiency and revenue.[15] It may also reflect, at least in part, the intrinsic nature of participants. Amador et al. (2006) develop a general framework to study the tradeoffs between commitment and flexibility (which they then apply to a specific setting). Dynamically inconsistent subjects value the commitment afforded by smaller choice sets and, at the same time, value the flexibility afforded by larger choice sets since they expect to receive new information relevant to their future decisions.

Bidders' Cognitive Uncertainty. A simple way to summarize the above ideas is to note that even in many ideal settings subjects may exhibit *cognitive uncertainty*, defined as subjective uncertainty about what the optimal action is. This would intuitively seem to be an essential characteristic in CA settings. Enke & Graeber (2021*a*) show that this concept provides a unifying lens for understanding a large set of behavioral anomalies in how people choose under risk and think about probabilities. It also helps predict beliefs and actions: "The key idea is that noise and bias are linked: when people are cognitively noisy, they revert more to a cognitive default, which introduces systematic bias." Enke & Graeber (2021*b*) then extend the study of cognitive uncertainty for understanding and predicting intertemporal choice. They document how it explains differences in impatience over long and short horizons, why impatience is greater when the decision environment is more complex, and how cognitive uncertainty matters for choice architecture: greater cognitive uncertainty is associated with a greater likelihood to follow expert advice to be more patient. Cognitive uncertainty partly reflects the subjective complexity of decision problems. See also Alaoui & Penta (2022) on cost-benefit reasoning. As such, it is directly relevant for the auctions literature in general, and the CA literature in particular. In a CA environment, cognitive uncertainty could reflect uncertainty over future preferences or the cognitive difficulty of computing discounted utility because of the complexity of the setting. Importantly, this concept helps to predict various judgment and decision errors and anomalies that are traditionally viewed as distinct in the literature. This means that it holds potential for future research interested in testing the predictions of CA models, on both the supply and demand sides, that would be difficult to test otherwise.

---

[12] See Camerer (2003). Eyster (2019) describes in detail the evidence on underinference (subjects learn too little from observing and conditioning on other people's actions), misinference (subjects may infer the wrong information content from other people's actions), social learning (people may make better inferences when they observe other people's actions than when they simply condition upon those actions), and on failures to best respond to own beliefs and understand payoffs. For lab evidence on errors in reasoning about others' rationality, see Kneeland (2015) and Dal Bó et al. (2018).

[13] For an important pair of early papers in natural-resource auction settings, see Hendricks et al. (1987), Hendricks & Porter (1988). They provide strong evidence of systematic overbidding.

[14] As discussed earlier, even without these effects, there can be incentives in the clock phase to drive up payments of competitors (Bichler et al. 2013, Janssen & Karamychev 2017, Levin & Skrzypacz 2016, Harsha et al. 2010).

[15] Rubinstein (2021), however, notes that there may be scenarios in which the bounded rationality of the subjects could be advantageous for a designer: "If all candidates were fully rational, as is commonly assumed in the mechanism design literature, all of them would be able to game the system if necessary. It is the cognitive imperfection of the individuals which opens a door to obtaining a desirable outcome."

Summing up, these considerations suggest the need to simplify CA designs from bidders' perspective, at least given current capabilities. A suitable design response, however, would require a careful understanding of the benefits and costs of simplification, which are likely driven by the number of packages selected and the dynamic consistency of bidding across phases.

Rationality of Other Bidders and Common Knowledge of Rationality. The economics literature on bounded rationality has focused on models in which reasoning procedures (limited abilities, complexity of strategies, belief formation) are the cornerstones of the analysis (Rubinstein 1998). Much like this early body of work, however, the CA literature has not given sufficient attention to the role of *knowledge* of rationality. It has typically argued that in many CA where the stakes are high and a bidders' bidding teams spend many months preparing the bid strategy, one may expect strong forms of irrationality to be ruled out. But this is not sufficient, even if all forms of irrationality are ruled out. The reason is that it is not only rationality but the knowledge of rationality that is often key to expect equilibrium outcomes. In the behavioral game theory literature, the "paradoxes of backward induction" involve sequential games that imply very counterintuitive behavior. The best example is the centipede game. Aumann (1992) shows that in this game it is possible for rationality to be *mutually known* to a high degree and still for both players to violate backward induction.[16] Aumann (1995) then shows that it is not rationality, nor even mutual knowledge of rationality, but *common knowledge of rationality* that implies the backward induction outcome. This, however, is an ideal condition that is rarely met in practice.[17] As in other environments, common knowledge of rationality is unlikely to be satisfied in CA settings. This represents a potential reason why some CA designs will not transfer to practice, and opens up a potentially fruitful avenue for research. At the very least, it suggests making any information potentially associated with the rationality of bidders publicly available at no cost. For instance, the experimental literature shows that stakes, experience and payoffs correlate with equilibrium outcomes under the hypothesis of common knowledge of rationality. Thus designers might provide free information about the behavior and outcomes of every bidder in previous auctions (e.g., stakes, bids, revenues).

Multiplicity, Focal Points and Coordination. Even in an ideal world in which rationality and preferences are perfectly understood, the game that bidders play may be characterized by multiplicity of equilibria. For example, Janssen & Karamychev (2016), Levin & Skrzypacz (2016) provide a full analysis of the CCA rules, and show that there are multiple equilibria with no guarantee for efficiency.[18] The equilibria depend on assumptions about bidders' incentives to drive up prices of competitors (Bichler et al. 2013). Janssen & Kasberger (2019) consider the interaction between the clock and the supplementary phase in a setting in which bidders care about their own payoff and lexicographically about how much rivals pay. They find that there is no efficient equilibrium of the CCA that fully reveals the type of the competitor in the clock phase, and that the CCA is inefficient if the uncertainty concerning final allocations is relatively large. They further argue that qualitative evidence from the 2013 Austrian auction is consistent with their results.[19]

---

[16] See also Reny (1992) and Ben-Porath (1997).

[17] Palacios-Huerta & Volij (2009) vary the closeness to common knowledge of rationality across different experimental treatments, and find strong evidence that the predictive power of subgame-perfect equilibrium hinges mainly on the common knowledge of players' rationality. Baghestanian & Frey (2016) replicate these findings.

[18] Levin & Skrzypacz (2016) and Janssen & Karamychev (2016) both work in a model where VCG prices are in the core. Levin & Skrzypacz (2016) consider problems that arise resulting from a dynamic implementation of Vickrey pricing. Janssen & Karamychev (2016) focus on the supplementary phase of the CCA. They show that, given a particular clock phase behavior, this phase can be solved using iterative elimination of dominated strategies, resulting in bidders raising rival's costs without running the risk of winning undesired packages.

[19] The clock phase ended with excess supply in key spectrum bands, but this did not prevent the bidders from

From a behavioral perspective, it is difficult to predict which equilibrium (if any) will be selected. In some settings, focal points may operate in terms of equilibrium selection. However, there might not be a focal point that all parties can determine and agree upon. As far back as Schelling (1960), there has been an ample behavioral economics literature on focal points in game theory: "(p)eople can often concert their intentions or expectations with others if each knows that the other is trying to do the same" (Schelling 1960, p. 157). Choices may then converge to some focal points that have some kind of prominence in the environment, which may be determined here by other characteristics of a CA.[20] As in other settings, however, there is a lack of clear guidance on what the focal point may be in many CAs, and this creates a major degree of freedom in future empirical work. Relatedly, important recent theories propose that preferences are *reference dependent* in that individuals evaluate outcomes relative to a reference point such as, for example, the current state (the *status quo*), past states, expectations about future states, or social comparisons.[21] In light of their impact in the behavioral economics literature, we expect that focal points and reference dependence will play an important role in future theoretical and empirical work in the CA literature, for both bidders and regulators.

For CCAs, a special consideration is the setting where uncertainty concerning final allocations is relatively low. Would then efficiency be expected? Dal Bó & Fréchette (2011) study several different games with multiple equilibria, and show that there remains a challenge with coordination—even when efficiency can be supported in equilibrium, subjects may still fail to reach it.

4.3 Simplicity and Trust on Both Sides of the Market

We have argued that the prominence of simple CAs, e.g., clock-style, with item prices and simpler languages, as opposed to more complex designs (bundle prices, richer languages, etc) may reflect the impact of both behavioral regulators as well as regulators' recognition of the reality of behavioral bidders. This is reflected not only in practice but also in current theoretical research. Teytelboym et al. (2021) note that in contexts such as CA, difficulties in implementing market-clearing mechanisms "in practice have motivated research to seek alternatives that have less demanding computation and/or communication requirements" while still making it possible to find nearly efficient prices.

The discussion in the preceeding subsections often points to the role that trust and simplicity plays both in the demand and supply sides of CAs. For instance:

(a) One way to interpret the evolution of CA designs from *SB* to *MR* to *CCA* is not only from the perspective of efficiency, but also from the viewpoint of simplicity. For instance, relative to a *SB* design the feedback processes in *MR* help bidders in terms of common knowledge and rationality (see above discussion). And the assignment stage in CCAs is added to *simplify* bidding in the first two stages significantly (the main idea is to treat several closely related items as completely

---

bidding aggressively in the supplementary round. Bidders may have created excess supply purposefully to obfuscate their type to prevent rivals from raising their costs.

[20] For example, strategic demand reduction in multi-band auctions requires that there is a focal point for how to allocate the spectrum. In a non-CA auction, Cramton & Ockenfels (2017) find that bidding in the 2010 German 4G spectrum auction was competitive and the final assignment was efficient. Yet: "Coordination was difficult, though, because of a multiplicity of focal points ... When the auction ended, the industry as a whole was more than €4 billion poorer and all bidders worse off compared to what might have been possible early in the auction." As a counterpoint, coordination (tacit collusion) on a focal point that shared the market succeeded in the 1999 German spectrum auction (Klemperer 2002).

[21] Contributions to the literature include expectations-based reference points (Köszegi & Rabin 2006, 2007, 2009) and models of salience (Bordalo et al. 2012, 2013, 2019).

identical during the clock and supplementary rounds).[22]

(b) More broadly, we have also touched upon the role of trust and simplicity in the deployment of CAs. One reason that first-price rules are used in the private sector is because the bidders may have little reason to trust the bid-taker to respect a second-price payment rule[23] (and even if the rule could be trusted, bidders may fear that information truthfully-revealed could be used against them in the future (Rothkopf et al. 1990)).

(c) There exists an ongoing debate regarding the most appropriate format for auctioning spectrum, with concerns about the complexity of the CCA design, including its payment rule; these concerns include those of unintended consequences, such as the possibility of bidding to drive up a rival's prices so that one's own payment may decrease (Beck and Ott 2013), or for predatory reasons (Levin & Skrzypacz 2016, Janssen & Kasberger 2019, Janssen & Karamychev 2016).[24]

(d) A primary reason for the adoption of a CA auction along with the relatively complex VCG mechanism, in place of what had been a non-CA, in the context of contextual display ads, is its flexibility; this was adopted at Google, for example, to facilitate optimal tradeoffs in regard to advertisements of various sizes and shapes, and with the flexibility of VCG in regard to pricing different kinds of tradeoffs cited as a chief advantage (Varian & Harris 2014).[25] It would be helpful to develop a theory for design flexibility, to capture the "optionality" that is afforded by different kinds of designs, perhaps following the insights in Amador et al. (2006) about how to model the tradeoffs between flexibility and different reasons why smaller (and possibly simpler) choice sets may be valuable.

These examples and ongoing discussions testify to the importance of designing simple mechanisms. Dominant-strategy mechanisms, for example, are desirable. They reduce participation costs, cognitive costs, and provide protection from some strategic errors. But this may not be enough in particular in CA settings. Consider the following example. When bidders have private values, the sealed-bid second-price auction is strategically equivalent to the ascending auction. Choosing when to quit in the latter is the same as choosing a bid in the former. The two formats are strategically equivalent and have the same reduced normal form. And yet, lab subjects behave substantially more closely to equilibrium theory under a clock auction than under sealed bids (Kagel et al. 1987).

Li (2017) shows that *obviously dominant strategies* are played more often than mere dominant strategies. He models what it means for a mechanism to be *obviously strategy-proof* (OSP) and implements a laboratory experiment that compares pairs of mechanisms that implement the same choice rule (see also Breitmoser & Schweighofer-Kodritsch (2019)). One mechanism in each pair is SP, but not OSP. The other mechanism is OSP. Standard game theory predicts that both mecha-

---

[22] See Benisch et al. (2008), Milgrom (2010), Dütting et al. (2011), Bichler et al. (2014), Dütting et al. (2014) for related discussions.

[23] There is also a growing literature on the use of cryptographic methods to establish trust, by proving that the rules have been correctly followed without revealing any information about the bids beyond that implied by the outcome (Thorpe & Parkes 2009, Parkes et al. 2009, 2008). At the same time, it remains technically challenging for the bid taker to commit to the use of a second-price payment rule.

[24] These authors study predatory bidding, to drive up the price of another bidder, and in the special case that core pricing corresponds to VCG payments. Kasberger & Teytelboym (2022) also provide a theoretical analysis of the CMRA, characterizing different equilibria, and find that it can be an effective auction format for selling sufficiently complementary goods. A concern, though, is that there is the possibility of collusive demand reduction when goods are not sufficiently complementary. They find that in the Danish spectrum auctions held between 2016 and 2021, outcomes changed as bidders became more acquainted with the auction format, with behavior consistent with demand reduction and coordination.

[25] Varian & Harris (2014) state the advantage of the VCG in their application to contextual advertising in regard to its flexibility in handling different kinds of allocation problems over the *status quo*, generalized second price auction rule (see Edelman et al. (2007), Varian (2007), Aggarwal et al. (2006)).

nisms will produce the same outcome. And yet subjects play the dominant strategy at significant higher rates under OSP mechanisms. As predicted, OSP mechanisms are easier for real people to understand.

We argue that the behavioral implications for future developments moving towards OSP CA formats should not be underestimated. In a CA, as the welfare-optimal mechanism must elicit valuations on a power set of objects, it can be that no OSP mechanism yields first-best welfare.[26] As Li (2017, p. 3284)) notes, *"when designing mechanisms, the appropriate constraints depend on the intended application. ... If the agents are expert strategists, and the mechanism will be thoroughly audited by a trusted third-party, then OSP is too demanding. Strategically sophisticated agents and full commitment power are the ideal conditions for mechanism design. These conditions do not always obtain, so it is valuable to design mechanisms that are robust to weaker assumptions."* Golowich & Li (2021) present an algorithm that determines, given some choice rule, whether there exists an OSP mechanism for that choice rule.[27]

CAs represent the quintessential complex environment where the level of simplicity in practice is likely to be a major determinant of observed designs, choices on both sides of the market, and outcomes. We expect a great deal of future research from this perspective. Further, we also view the recent research on trust and credibility as directly relevant for the implementation of CA in practice. Trust plays an important role in all types of human relationships, especially in economic exchanges (see Fehr & Schmidt (1999) and Fehr (2009)). During the last two decades, there has been a surge of empirical research on trust, from measuring trust and its determinants to the impact of institutions on trust and comparisons across countries. Trust plays a causal role for important outcomes (e.g., trading volume, gains from trade) and cannot be captured by beliefs about others and risk preferences alone: social preferences also play a key role. From the perspective of CA design, trust is also important. For instance, Akbarpour & Li (2020) study the issue of credibility by analyzing mechanisms run by an auctioneer who can deviate from the rules provided that no single bidder detects the deviation. A mechanism is credible if it is incentive-compatible for the auctioneer to follow the rules. Also, trust and cooperative behavior naturally relate to a demand for reducing the complexity of strategies in the game theory literature, suggesting that the auction needs to be simple enough for cooperation (collusion) to unravel with bidders who can only use simple strategies.[28]

5 Final thoughts

On October 12th, 2020, the importance of combinatorial auctions was recognized by the Royal Swedish Academy of Sciences, who awarded the Nobel Prize in Economic Sciences to Paul Milgrom

---

[26] However, when objects are substitutes and types are independently distributed, there always exists an OSP mechanism that delivers at least half of first-best expected welfare (Feldman et al. 2014, Dütting et al. 2016). In these settings, the "cost of simplicity" is bounded by a half.

[27] Akbarpour et al. (2021) study investment incentives for participants to report their input values truthfully in strategy-proof mechanisms that use algorithms that approximate efficient allocations. In sharp contrast with the VCG mechanism, for which individual returns on investments are aligned with social welfare, they find that some such algorithms can nevertheless create misaligned investment incentives that lead to arbitrarily bad overall outcomes. However, if a near-efficient algorithm excludes "bossy negative externalities," (defined as if raising the value of a packed bidder or reducing the value of an unpacked bidder can decrease the total value of the allocation for the other bidders) then its outcomes remain near-efficient even after accounting for investments.

[28] See Rubinstein (1986) and Abreu & Rubinstein (1988). Neyman (1985) finds that cooperation in the classic finitely repeated prisoner's dilemma is justified under the assumption that there are bounds to the complexity of the strategies that the players may use (measured by the size of the minimal finite automaton (Moore machine) that can implement it). See also Neyman (1997, 1998). Megiddo & Wigderson (1986) and Chen et al. (2017) study the model of repeated games in which players use Turing machines instead of finite automata.

and Robert Wilson "for improvements to auction theory and inventions of new auction formats.... Their discoveries have benefitted sellers, buyers and taxpayers around the world." In particular, "Milgrom and Wilson invented new formats for auctioning off many interrelated objects simultaneously." In his Nobel lecture, Milgrom (2021) pointed out: "Most of my work published in academic journals is theoretical, proving theorems about the properties of abstract models, but developing and participating in real-world mechanisms requires more than that."

There is some irony that, even with the overwhelming success of CAs in practice across a wide variety of applications, the specific application that motivated the first paper on CAs, access to airport runway slots, has never been implemented. This is especially striking given that, if anything, the problem is even more severe than it was forty years ago when CAs were first proposed in Rassenti, Smith and Bulfin (1982). Ball et al. (2018) updates the issues involved, concluding that it is "more than obvious that transitioning any airport from an existing administration slot control system to a system based on slot auctions represents a major political challenge." Karla Hoffman, a long-time consultant to the Federal Aviation Administration on the use of auctions, is even more pessimistic, suggesting that runway access via CAs looks unlikely to occur anytime in the near future (K. Hoffman, personal communication, 2021). The lesson here is that, beyond a natural and intuitive specific application, there must also be a receptive institutional framework to drive adoption of combinatorial auctions in practice.

References

Abreu, D. & Rubinstein, A. (1988), 'The structure of Nash equilibrium in repeated games with finite automata', *Econometrica* **56**, 1259–1282.

Aggarwal, G., Goel, A. & Motwani, R. (2006), Truthful auctions for pricing search keywords, *in* 'Proceedings 7th ACM Conference on Electronic Commerce (EC)', pp. 1–7.

Akbarpour, M., Kominers, S. D., Li, S. & Milgrom, P. (2021), Investment incentives in near-optimal mechanisms, Technical report, Stanford University.

Akbarpour, M. & Li, S. (2020), 'Credible auctions: A trilemma', *Econometrica* **88**, 425–467.

Alaoui, L. & Penta, A. (2022), 'Cost-benefit analysis in reasoning', *Journal of Political Economy* **130**, 881–925.

Amador, M., Werning, I. & Angeletos, G.-M. (2006), 'Commitment vs flexibility', *Econometrica* **74**, 365–396.

ANACOM (2010), *Relatorio Final do Procedimento de Selaccao por Leilao de espectro para BWA*, ANACOM Autoridade Nacional de Communicacoes (Portugal). Undated document, circa 2010.

ANCOM (2012), Terms of reference for the organization of the competitive selection procedure in view of awarding the rights to use the radio frequencies in the 800 MHz, 900 MHz, 1800 MHz and 2600 MHz bands, Technical report, Nat. Auth. for Manag. and Reg. in Comm. of Romania.

Armstrong, M. & Huck, S. (2010), 'Behavioral economics as applied to firms: A primer', *Competition Policy International* **6**(1), 3–45.

Arthur, C. (2013), *4G Auction To Be Investigated by Audit Office After Poor Return*, The Guardian. April 24.

Aumann, R. J. (1992), Irrationality in game theory, *in* P. Dasgupta, D. Gale, O. Hart & E. Maskin, eds, 'Economic Analysis of Markets and Games: Essays in Honor of Frank Hahn', MIT Press, pp. 214–227.

Aumann, R. J. (1995), 'Backward induction and common knowledge of rationality', *Games and Economic Behavior* **8**(1), 6–19.

Ausubel, L., Cramton, P. & Milgrom, P. (2006), The Clock-Proxy Auction: A practical combinatorial auction design, *in* Cramton et al. (2006), chapter 5, pp. 115–138.

Ausubel, L. M. & Baranov, O. (2014*a*), 'Market design and the evolution of the combinatorial clock auction', *American Economic Review* **104**(5), 446–451.

Ausubel, L. M. & Baranov, O. (2016), Revealed preference and activity rules in auctions, Technical report, University of Maryland and University of Colorado.

Ausubel, L. M. & Baranov, O. (2017), 'A practical guide to the Combinatorial Clock Auction', *Economic Journal* **127**, F334–F350.

Ausubel, L. M. & Baranov, O. V. (2014*b*), A practical guide to the combinatorial clock auction, Technical report, University of Maryland.

Baghestanian, S. & Frey, S. (2016), 'GO Figure: Analytic and strategic skills are separable', *Journal of Behavioral and Experimental Economics* **64**, 71–80.

Bajari, P. & Hortaçsu, A. (2005), 'Are structural estimates of auction models reasonable? Evidence from experimental data', *Journal of Political Economy* **113**(4), 703–741.

Ball, M. O., Berardino, F. & Hansen, M. (2018), 'The use of auctions for allocating airport access rights', *Transportation Research Part A* **114**, 186–202.

Ball, M. O., Donohue, G. L. & Hoffman, K. (2006), Auctions for the safe, efficient, and equitable allocation of airspace system resources, *in* Cramton et al. (2006), chapter 20, pp. 507–538.

Bazelon, C. (2009), 'Too many goals: Problems with the 700 MHz Auction', *Info., Econ., and Pol.* **21**, 115–127.

Becker, G. S. (1958), 'Competition and democracy', *Journal of Law and Economics* **1**, 105–109.

Ben-Porath, E. (1997), 'Rationality, Nash equilibrium and backwards induction in perfect-information games', *Review of Economic Studies* **64**(1), 23–46.

Benisch, M., Sadeh, N. & Sandholm, T. (2008), A theory of expressiveness in mechanisms, *in* 'Proc. 23rd AAAI Conf. on Artificial Intelligence', pp. 17–23.

Bichler, M., Davenport, A., Hohner, G. & Kalagnanam, J. (2006), Industrial procurement auctions, *in* Cramton et al. (2006), chapter 23, pp. 596–612.

Bichler, M., Fux, V. & Goeree, J. K. (2018), 'A matter of equality: Linear payment rules in combinatorial exchanges', *Information Systems Research* **20**, 779–1068.

Bichler, M., Goeree, J., Mayer, S. & Shabalin, P. (2014), 'Spectrum auction design: Simple auctions for complex sales', *Telecommunications Policy* **38**, 613–622.

Bichler, M., Shabalin, P. & Wolf, J. (2013), 'Do core-selecting combinatorial clock auctions always lead to high efficiency? An experimental analysis of spectrum auction designs', *Experimental Economics* **16**, 511–545.

Bordalo, P., Gennaioli, N. & Shleifer, A. (2012), 'Salience theory of choice under risk', *Quarterly Journal of Economics* **127**(3), 1243–1285.

Bordalo, P., Gennaioli, N. & Shleifer, A. (2013), 'Salience and consumer choice', *Journal of Political Economy* **121**(5), 803–843.

Bordalo, P., Gennaioli, N. & Shleifer, A. (2019), Memory, attention and choice, Technical report, Harvard University.

Breitmoser, Y. & Schweighofer-Kodritsch, S. (2019), Obviousness around the clock, Discussion Papers, Research Unit: Market Behavior SP II 2019-203, WZB Berlin Social Science Center.

Brusco, S., Lopomo, G. & Marx, L. (2009), 'The 'Google effect' in the FCC's 700 MHz Auction', *Information, Economics, and Policy* **21**, 101–114.

Camerer, C. (2003), *Behavioral Game Theory. Experiments in Strategic Interaction.*, Princeton University Press.

Cantillon, E. & Pesendorfer, M. (2006), Auctioning bus routes: The London experience, *in* Cramton et al. (2006), chapter 22, pp. 573–591.

Caplice, C. & Sheffi, Y. (2003), 'Optimization-based procurement for transportation services', *J. of Business Logistics* **24**(2), 109–128.

Caplice, C. & Sheffi, Y. (2006), Combinatorial auctions for truckload transportation, *in* Cramton et al. (2006), chapter 21, pp. 539–571.

Caplice, C., Sheffi, Y. & Plummer, C. (2003), Applications of combinatorial auctions in truckload transportation, Technical Report 2004-01, MIT Center for Transportation and Logistics Working Paper.

Cassady, Jr., R. (1967), *Auctions and Autioneering*, University of California Press.

Catalán, J., Epstein, R., Guajardo, M., Yung, D., Martínez, C. & Weintraub, G. Y. (2009), 'Solving multiple scenarios in a combinatorial auction', *Computers & Operations Research* **36**(10), 2752–2758.

Chen, L., Tang, P. & Wang, R. (2017), Bounded rationality of restricted turing machines, *in* 'Proc. Thirty-First AAAI Conference on Artificial Intelligence', pp. 444–450.

COMREG (2008), The award of national block point to point and point to multipoint assignments in the 26 GHz band, Technical Report 07/93R, Commission for Communications Regulation (Ireland).

COMREG (2012), Results of the multi-band spectrum auction. Information notice, Technical Report 12/123, Commission for Communications Regulation (Ireland).

COMREG (2017), Results of the 3.6 GHz band spectrum award. Information notice, Technical Report 17/38, Commission for Communications Regulation (Ireland).

Cramton, P. (2013), 'Spectrum auction design', *Review of Industrial Organization* **42**, 161–190.

Cramton, P., Kwerel, E., Rosston, G. & Skrzypacz, A. (2011), 'Using spectrum auctions to enhance competition in wireless services', *J. of Law and Economics* **54**, S167–S188.

Cramton, P. & Ockenfels, A. (2017), 'The German 4G spectrum auction: Design and behaviour', *Economic Journal* **127**, F305–F324.

Cramton, P., Shoham, Y. & Steinberg, R., eds (2006), *Combinatorial Auctions*, Cambridge, MA: MIT Press.

Dal Bó, E., Dal Bó, P. & Eyster, E. (2018), 'The demand of bad policy when voters underappreciate equilibrium effects', *Review of Economic Studies* **85**, 964–998.

Dal Bó, P. & Fréchette, G. R. (2011), 'The evolution of cooperation in infinitely repeated games: Experimental evidence', *American Economic Review* **101**(1), 2205–2229.

de Clippel, G. & Rozen, K. (2021), Fairness through the lens of cooperative game theory: An Experimental Approach, Technical report, Brown University.

DEA (2016), 1800 MHz Auction. Award of frequencies in the frequency bands 1720.1-1785.0 MHz and 1815.1-1880.0 MHz, Technical report, Danish Energy Agency Information Memorandum.

Derinkuyu, K., Tanrisever, E., Kurt, N. & Ceyhan, G. (2020), 'Optimizing day-ahead electricity market prices: Increasing the total surplus for Energy Exchange Istanbul', *Manufacturing & Service Operations Management* **22**, 700–716.

Director, A. & Levi, E. H. (1956), 'Law and the future: Trade regulation', *Northwestern University Law Review* **51**, 281–296.

DotEcon (2012), *Spectrum Value of 800, 1800 MHz and 2.6 GHz. A DotEcon & Aetha Report for Ofcom.*

Dütting, P., Feldman, M., Kesselheim, T. & Lucier, B. (2016), Posted prices, smoothness, and combinatorial prophet inequalities, Technical Report 1612.03161, arXiv.

Dütting, P., Fischer, F. A. & Parkes, D. C. (2011), Simplicity-expressiveness tradeoffs in mechanism design, *in* 'Proc. 12th ACM Conf. on Electronic Commerce (EC-2011)', pp. 341–350.

Dütting, P., Fischer, F. A. & Parkes, D. C. (2014), Expressiveness and robustness of first-price position auctions, *in* 'Proc. ACM Conf. on Economics and Computation, EC '14', pp. 57–74.

Edelman, B., Ostrovsky, M. & Schwarz, M. (2007), 'Internet advertising and the Generalized Second-Price auction: Selling Billions of dollars worth of keywords', *American Economic Review* **97**, 242–259.

EKIP (2016), Information document on the procedure of awarding radio-frequencies in the 800 MHz, 900 MHz, 1800 MHz, 2 GHz and 2.6 GHz bands for the implementation of public mobile electronic communications networks, Technical report, Montenegro Agency for Electronic and Postal Services.

Ellison, G. (2006), 'Bounded rationality in industrial organization', *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress* **2**, 142–174.

Elmaghraby, W. & Keskinocak, P. (2003), Combinatorial auctions in procurement, *in* C. Billington, T. Harrison, H. Lee & J. Neale, eds, 'The Practice of Supply Chain Management', Springer-Verlag, pp. 245–258.

Enke, B. (2020), 'What you see is all there is', *Quarterly Journal of Economics* **135**, 1363–1398.

Enke, B. & Graeber, T. (2021*a*), Cognitive uncertainty, Technical report, Harvard University.

Enke, B. & Graeber, T. (2021*b*), Cognitive uncertainty in intertemporal choice, Technical report, Harvard University.

Epstein, R., Henríquez, L., Catalán, J., Weintraub, G. Y. & Martínez, C. (2002), 'A combinatorial auction improves school meals in Chile', *Interfaces* **32**(6), 1–14.

Esponda, I. (2008), 'Behavioral equilibrium in economies with adverse selection', *American Economic Review* **98**, 1269–1291.

Eyster, E. (2019), Errors in strategic reasoning, *in* 'Handbook of Behavioral Economics', Vol. 2, North Holland, pp. 187–259.

FCC (2003), Auction of Regional Narrowband PCS licenses scheduled for September 24, 2003, Technical report, US Federal Communications Commission.

FCC (2007), Auction of 700MHz Band Licenses Scheduled for January 24, 2008, Technical report, US Federal Communications Commission.

Fehr, E. (2009), 'On the economics and biology of trust', *Journal of the European Economic Association* **7**, 235–266.

Fehr, E. & Schmidt, K. M. (1999), 'A theory of fairness, competition, and cooperation', *Quarterly Journal of Economics* **114**(3), 817–868.

Feldman, M., Gravin, N. & Lucier, B. (2014), Combinatorial auctions via posted prices, Technical Report 1411.4916, arXiv.

Fox, J. T. & Bajari, P. (2013), 'Measuring the efficiency of an FCC spectrum auction', *American Economic Journal: Microeconomics* **5**(1), 100–146.

Gill, D. & Prowse, V. (2016), 'Cognitive ability, character skills and learning to play equilibrium: A level-k analysis', *Journal of Political Economy* **124**, 1619–1676.

Gillen, B. (2015), Identification and estimation of k-level auctions, Technical report, CalTech.

Goeree, J. K. & Lien, Y. (2016), 'On the impossibility of core-selecting auctions', *Theoretical Economics* **11**, 41–52.

Goldfard, A. & Xiao, M. (2011), 'Who thinks about the competition? Managerial ability and strategic entry in US local telephone markets', *American Economic Review* **101**(7), 3130–3161.

Golowich, L. & Li, S. (2021), On the computational properties of obviously strategy-proof mechanisms, Technical Report 2101.05149v2, arXiv.

Goossens, D. R., Onderstal, S., Pijnacker, J. & et al., F. C. R. S. (2014), 'Solids: A combinatorial auction design for real estate', *Interfaces* **44**(4), 351–363.

Graves, R. L., Schrage, L. & Sankaran, J. (1993), 'An auction method for course registration', *Interfaces* **23**(5), 81–92.

Haile, P. & Tamer, E. (2003), 'Inference with an incomplete model of English auctions', *Journal of Political Economy* **111**(1), 1–51.

Harsha, P., Barnhart, C. & Parkes, D. C. (2010), 'Strong activity rules for iterative combinatorial auctions', *Computers and Operations Research* **37**, 1271–1284.

Heidhues, P. & Kószegi, B. (2018), Behavioral industrial organization, *in* D. B. Bernheim, S. della Vigna & D. Laibson, eds, 'Handbook of Behavioral Economics', Vol. 1, North Holland, chapter 6, pp. 517–612.

Hendricks, K. & Porter, R. H. (1988), 'An empirical analysis of an auction with asymmetric information', *American Economic Review* **78**(5), 865–883.

Hendricks, K., Porter, R. H. & Boudreau, B. (1987), 'Information, returns and bidding behavior in OCS auctions: 1954-1969', *Journal of Industrial Economics* **35**(4), 517–542.

Hohner, G., Rich, J., Ng, E., Reid, G., Davenport, A. J., Kalagnanam, J. R., Lee, H. S. & An, C. (2003), 'Combinatorial and quantity-discount procurement auctions benefit Mars Incorporated and its suppliers', *Interfaces* **33**(1), 23–35.

Hortaçsu, A., Luco, F., Puller, S. & Zhu, D. (2019), 'Does strategic ability affect efficiency? evidence from electricity markets', *American Economic Review* **109**(12), 4302–4342.

Hortaçsu, A. & McAdams, D. (2018), 'Empirical work on auctions of multiple objects', *J. Economic Literature* **56**, 157–184.

Hortaçsu, A. & Puller, S. (2008), 'Understanding strategic bidding in multi-unit auctions: A case study of the Texas electricity spot market', *RAND Journal of Economics* **39**, 86–114.

IWS (2015), Rules proposal for the 2015-2016 AWS spectrum tender process in Mexico, Technical report, Instituto Federal de Telecommunicaciones.

Janssen, M. & Karamychev, V. A. (2013), Gaming in combinatorial clock auctions, Technical Report 13-027/VII, Tinbergen Institute.

Janssen, M. & Karamychev, V. A. (2016), 'Spiteful bidding and gaming in combinatorial clock auctions', *Games and Economic Behavior* **100**, 186–207.

Janssen, M. & Karamychev, V. A. (2017), 'Raising rivals' cost in multi-unit auctions', *International Journal of Industrial Organization* **50**, 473–490.

Janssen, M. & Kasberger, B. (2019), 'On the clock of the combinatorial clock auction', *Theoretical Economics* **14**, 1271–1307.

Kagel, J. H., Harstad, R. M. & Levin, D. (1987), 'Information impact and allocation rules in auctions with affiliated private values: A laboratory study', *Econometrica* **55**(6), 1276–1304.

Kagel, J. H., Lien, Y. & Milgrom, P. (2010), 'Ascending prices and package bidding: A theoretical and experimental analysis', *American Economic Journal: Microeconomics* **2**, 160–185.

Kagel, J. H., Lien, Y. & Milgrom, P. (2014), 'Ascending prices and package bidding: Further experimental analysis', *Games and Economic Behavior* **85**, 210–231.

Kasberger, B. & Teytelboym, A. (2022), The combinatorial multi-round ascending auction, Technical report, University of Oxford.

Kasdan, D. O. (2020), 'Toward a theory of behavioral public administration', *International Review of Administrative Sciences* **86**, 605–621.

Keniston, D., Larsen, B. J., Li, S., Prescott, J., Silveira, B. S. & Yu, C. (2021), Fairness in incomplete information bargaining: Theory and widespread evidence from the field, Technical Report 21-042, Stanford Institute for Economic Policy Research.

Kim, S. W., Olivares, M. & Weintraub, G. Y. (2014), 'Measuring the performance of large-scale combinatorial auctions: A structural estimation approach', *Management Science* **60**(5), 1180–1201.

Kittsteiner, T., Ott, M. & Steinberg, R. (2021), 'Competing combinatorial auctions', *Information Systems Research* p. to appear.

Klemperer, P. (2002), 'What really matters in auction design', *Journal of Economic Perspectives* **16**, 169–189.

Kneeland, T. (2015), 'Identifying higher-order rationality', *Econometrica* **83**(5), 265–279.

Koboldt, C., Maldoom, D. & Marsden, R. (2003), The first combinatorial spectrum auction: Lessons from the Nigerian auction of fixed wireless licences, Technical Report Issue 03/01, Dotecon, London.

Köszegi, B. & Rabin, M. (2006), 'A model of reference-dependent preferences', *Quarterly Journal of Economics* **121**(4), 1133–1166.

Köszegi, B. & Rabin, M. (2007), 'Reference-dependent risk attitudes', *American Economic Review* **97**(4), 1047–1073.

Köszegi, B. & Rabin, M. (2009), 'Reference-dependence consumption plans', *American Economic Review* **99**, 909–936.

Kroemer, C., Bichler, M. & Goetzendorff, A. (2016), '(un)expected bidder behavior in spectrum auctions: About inconsistent bidding and its impact on efficiency in the combinatorial clock auction', *Group Decision and Negotiation* **25**, 31–62.

Kwasnica, A. M., Ledyard, J. O., Porter, D. & DeMartini, C. (2004), 'A new and improved design for multiobject iterative auctions', *Management Science* **51**, 419–434.

Ledyard, J., Olson, J., Porter, D., Swanson, J. & Torma, D. (2002), 'The first use of a combined-value auction for transportation services', *Interfaces* **32**(5), 4–12.

Letchford, A. N. (1996), 'Allocation of school bus contracts by integer programming', *J. of the Operational Research Society* **47**, 369–372.

Levin, J. & Skrzypacz, A. (2016), 'Properties of the combinatorial clock auction', *AER* **106**(9), 2528–2551.

Li, S. (2017), 'Obviously strategy-proof mechanisms', *American Economic Review* **107**(11), 3257–3287.

Loertscher, S., Marx, L. & Wilkening, T. (2015), 'A long way coming: Designing centralized markets with privately informed buyers and sellers', *Journal of Economic Literature* **53**(4), 857–897.

Lubin, B., Juda, A. I., Cavallo, R., Lahaie, S., Shneidman, J. & Parkes, D. C. (2008), 'ICE: An Expressive Iterative Combinatorial Exchange', *J. of Artificial Intelligence Research* **33**, 33–77.

Lunander, A. & Lundberg, S. (2012*a*), 'Combinatorial auctions in public procurement – Experiences from Sweden', *J. of Public Procurement* **12**(1), 81–109.

Lunander, A. & Lundberg, S. (2012*b*), 'Different design – different cost: An empirical analysis of combinatorial public procurement bidding of road maintenance', *J. of Public Procurement* **12**(3), 407–422.

Lunander, A. & Lundberg, S. (2013), 'Bids and costs in combinatorial and non-combinatorial procurement auctions – Evidence from procurement of public cleaning contracts', *Contemp. Econ. Policy* **31**, 733–745.

Marinescu, C. (2012), Results of the spectrum auction for mobile electronic communications, Technical report, Nat. Auth. for Manag. and Reg. in Comm. of Romania (press conf., 24 September), Bucharest, Romania.

Marsden, R., Sexton, E. & Siong, A. (2010), Fixed or Flexible? A survey of 2.6G spectrum awards, Technical Report 10/01, DotEcon.

Mastropietro, P., Batlle, C., Barroso, L. A. & Rodilla, P. (2014), 'Electricity auctions in South America: Towards convergence of system adequacy and RES-E support', *Renew. and Sustain. Energy Rev.* **40**, 375–385.

Maurer, L. T. A. & Barroso, L. A. (2011), *Electricity Auctions: An Overview of Efficient Practices*, The World Bank.

MBIE (2013), 700 MHz Auction. Consultation on Auction Design and Implementation Requirements, and the Execution, Technical Report MBIE-MAKO-1867806, Min. of Bus. Inno. & Employ. (New Zealand).

McLean, L. (1955), 'Auction anecdotes', *The Shingle* **18**(3), 65–70.

Megiddo, N. & Wigderson, A. (1986), On play by means of computing machines, *in* 'Proceedings of the 1986 Conference on Theoretical Aspects of Reasoning About Knowledge', pp. 259–274.

Metty, T., Harlan, R., Samelson, Q., Moore, T., Morris, T., Sorensen, R., Shneur, A., Raskina, O., Schneur, R., Kanner, J., Potts, K. & Robbins, J. (2005), 'Reinventing the supplier negotiation process at Motorola', *Interfaces* **35**(1), 7–23.

Milgrom, P. (2010), 'Simplified mechanisms with an application to sponsored-search auctions', *Games and Economic Behavior* **70**, 62–79.

Milgrom, P. (2021), 'Auction research evolving: Theorems and market designs', *American Economic Review* **111**(5), 1383–1405.

Mittelmann, M., Bouveret, S. & Perrussel, L. (2021), A general framework for the logical representation of combinatorial exchange protocols, *in* 'Proc. Int. Conf. on Autonomous Agents and Multi-Agent Systems (AAMAS'21)'.

Mochon, A. & Saez, Y. (2017), 'A review of radio spectrum combinatorial clock auctions', *Telecommunications Policy* **41**, 303–324.

Moreno, R., Barroso, L. A., Rudnick, H., Mocarquer, S. & Bezerra, B. (2010), 'Auction approaches of long-term contracts to ensure generation investment in electricity markets: Lessons from the Brazilian and Chilean experiences', *Energy Policy* **38**, 5758–5769.

Morse, A. (2014), 4G radio spectrum auction: Lessons learned, Technical Report HC 968, U.K. National Audit Office, March 6. `http://www.nao.org.uk/wp--content/uploads/2015/4G-radio-spectrum-auctions-lessons-learned-summary.pdf`.

Neyman, A. (1985), 'Bounded complexity justifies cooperation in the finitely repeated prisoners' dilemma', *Economics Letters* **19**, 227–229.

Neyman, A. (1997), Cooperation, repetition, and automata, *in* S. Hart & A. Mas-Colell, eds, ‘Cooperation: Game-Theoretic Approaches’, pp. 233–255.

Neyman, A. (1998), ‘Finitely repeated games with finite automata’, *Mathematics of Operations Research* **23**, 513–552.

NKOM (2001*a*), Auction #1 (900 MHz), Technical report, Norwegian Communications Authority.

NKOM (2001*b*), Auction #2 (1800 MHz), Technical report, Norwegian Communications Authority.

NKOM (2013), 800, 900 and 1800 MHz auction. Auction Rules, Technical report, Norwegian Communications Authority.

Nuñez, B. J. R. (2017), ‘Developing behavioural economics as a practical tool for market authorities’, *Journal of Antitrust Enforcement* **5**, 375–406.

Ofcom (2012), Assessment of future mobile competition and award of 800 mhz and 2.6 ghz, Technical report, Ofcom. `https://www.ofcom.org.uk/__data/assets/pdf_file/0031/46489/statement.pdf` (Accessed May 1, 2021).

Olivares, M., Weintraub, G. Y., Epstein, R. & Yung, D. (2012), ‘Combinatorial auctions for procurement: An empirical study of the Chilean school meals auction’, *Management Science* **58**(8), 1458–1481.

Palacios-Huerta, I. & Volij, O. (2009), ‘Field centipedes’, *American Economic Review* **99**(4), 1619–1635.

Parkes, D. C., Kalagnanam, J. & Eso, M. (2001), Achieving budget-balance with Vickrey-based payment schemes in exchanges, *in* ‘Proc. 17th Int. Joint Conf. on Art. Intell., IJCAI’, pp. 1161–1168.

Parkes, D. C., Rabin, M. O., Shieber, S. M. & Thorpe, C. (2008), ‘Practical secrecy-preserving, verifiably correct and trustworthy auctions’, *Electronic Commerce Research and Applications* **7**(3), 294–312.

Parkes, D. C., Rabin, M. O. & Thorpe, C. (2009), Cryptographic combinatorial clock-proxy auctions, *in* ‘Proc. 13th Int. Conf. Financial Cryptography and Data Security’, pp. 305–324.

Porter, D., Rassenti, S., Roopnarine, A. & Smith, V. (2003), ‘Combinatorial auction design’, *Proceedings of the National Academy of Sciences* **100**(19), 11153–11157.

Proto, E., Rustichini, A. & Sofianos, A. (2019), ‘Intelligence, personality, and gains from cooperation in repeated decisions’, *Journal of Political Economy* **127**, 1351–1390.

PTS (2011), Open invitation to the 1800 MHz auction, Technical report, Swedish Post and Telecom Agency.

Rabin, M. (1993), ‘Incorporating fairness into game theory and economics’, *American Economic Review* **83**, 1281–1302.

Rassenti, S. J., Smith, V. L. & Bulfin, R. L. (1982), ‘A combinatorial auction mechanism for airport time slot allocation’, *The Bell J. of Economics* **13**(2), 402–417.

Reny, P. J. (1992), ‘Rationality in extensive-form games’, *Journal of Economic Perspectives* **6**(4), 103–18.

Rothkopf, M. H., Pekeč, A. & Harstad, R. M. (1998), ‘Computationally manageable combinational auctions’, *Management Science* **44**(8), 1131–1147.

Rothkopf, M. H., Teisberg, T. J. & Kahn, E. P. (1990), ‘Why are Vickrey auctions rare?’, *Journal of Political Economy* **98**, 94–109.

Rubinstein, A. (1986), ‘Finite automata play the repeated prisoner’s dilemma’, *Journal of Economic Theory* **39**, 83–96.

Rubinstein, A. (1998), *Modelling Bounded Rationality*, MIT Press.

Rubinstein, A. (2021), Modeling bounded rationality in economic theory: Four examples, *in* R. Viale, ed., 'Routledge Handbook of Bounded Rationality', Routledge, pp. 423–435.

Salop, S. C. & Scheffman, D. T. (1983), 'Raising rivals' costs', *American Economic Review* **73**(2), 267–271.

Salop, S. C. & Scheffman, D. T. (1987), 'Cost-raising strategies', *Journal of Industrial Economics* **36**(1), 19–34.

Sandholm, T. (2002*a*), 'Algorithm for optimal winner determination in combinatorial auctions', *Art. Intell.* **135**, 1–54.

Sandholm, T. (2002*b*), 'emediator: A next generation electronic commerce server', *Computational Intelligence* **18**, 656–676.

Sandholm, T. (2006), 'Expressive commerce and its application to sourcing: How we conducted $35 billion of generalized combinatorial auctions', *AI Magazine* **28**(3), 45–58.

Sandholm, T. (2013), Very-large-scale generalized combinatorial multi-attribute auctions: Lessons from conducting $60 billion of sourcing, *in* N. Vulkan, A. E. Roth & Z. Neeman, eds, 'The Handbook of Market Design', Oxford University Press, chapter 16, pp. 379–412.

Sandholm, T., Levine, D., Concordia, M., Martyn, P., Hughes, R., Jacobs, J. & Begg, D. (2006), 'Changing the game in strategic sourcing at Proctor & Gamble: Expressive competition enabled by optimization', *Interfaces* **36**(1), 55–68.

Scheffel, T., Ziegler, G. & Bichler, M. (2012), 'On the impact of package selection in combinatorial auctions: An experimental study in the context of spectrum auction design', *Experimental Economics* **15**(4), 667–692.

Schelling, T. C. (1960), *The Strategy of Conflict*, Harvard University Press.

Shakers (2012), 'SHAKERS[R] Vodka Brand and Equipment to be Auctioned online through June 26', *The Free Library* .

Sheffi, Y. (2004), 'Combinatorial auctions in the procurement of transportation services', *Interfaces* **34**(4), 245–252.

SIGTARP (2015), Quarterly Report to Congress, Technical Report SIG-QR-15-01, Office of the Special Inspector General for the Troubled Asset Relief Program.

Simon, H. A. (1955), 'A behavioral model of rational choice', *Quarterly Journal of Economics* **69**(1), 99–118.

Steinberg, R. (2012), Auction pricing (chapter 27), *in* 'Oxford Handbook of Pricing Management, *O. Ozer and R. Phillips, eds*', Oxford University Press, pp. 679–712.

Stokely, M., Winget, J., Keyes, E., Grimes, C. & Yolken, B. (2009), Using a market economy to provision compute resources across planet-wide clusters, *in* 'Proc. IEEE Int. Symp. on Par. and Distr. Proc.', pp. 1–8.

TATT (2005), *Public Release of Bidding Data from the Auction of 23 June 2005*, Telco. Auth. Trinidad & Tobago.

TATT (2007), *Statement by Mr Cris Seecheran, Executive Director, Telecommunications Authority of Trinidad and Tobago following the Auction for Radio Spectrum to Provide Public Broadband Wireless Access (BWA) Services*, Telco. Auth. Trinidad & Tobago. Press statement, October 5.

TATT (2009), *Auction for Radio Spectrum to Provide Public Broadband Wireless Access (BWA) Services*, Telco. Auth. Trinidad & Tobago. Press release, April 3.

Taylor, G. (2015), 'Spectrum policy in canada', *IEEE Wireless Communications* **22**(6), 8–9.

Teytelboym, A., Li, S., Kominers, S. D., Akbarpour, M. & Dworczak, P. (2021), 'Discovering auctions: Contributions of Paul Milgrom and Robert Wilson', *Scandinavian Journal of Economics*

123, 709–750.

Thorpe, C. & Parkes, D. C. (2009), Cryptographic combinatorial securities exchanges, *in* 'Proc. 13th Int. Conf. Financial Cryptography and Data Security', pp. 285–304.

Tukiainen, J. (2008), 'Testing for common costs in the city of Helsinki bus transit auctions', *Int. J. of Industrial Organization* **26**, 1308–1322.

TUSR (2013), Public consultation on the tendering procedure for spectrum licences from the 800 MHz, 1800 MHz and 2600 MHz frequency bands by electronic auction, Technical report, Telecomm. Regul. Auth. of the Slovak Republic.

U.S. Congress (1925), Exhibit B-1, Exhibits to Testimony. U.S. Shipping Board and Emergency Fleet Corp., *in* 'Select Comm. to Inquire into Oper., Policies, and Affairs of the U.S. Shipping Board and the U.S. Emergency Fleet Corp.', Government Printing Office: Washington, D.C., pp. 3440–3443.

Varian, H. & Harris, C. (2014), 'The VCG auction in theory and practice', *AER* **104**(5), 442–445.

Varian, H. R. (2007), 'Position auctions', *International Journal of Industrial Organization* **25**(6), 1163–1178.

Walker, M. (2017), 'Behavioural economics: The lessons for regulators', *European Competition Journal* **13**, 1–27.

Appendix A: Applications

In the taxonomy we use for summarizing the design of CAs across the various applications, each application is specified by *auction type*, which consists of three components: (i) *payment* rule (FP | VCG | CORE), (ii) auction *design* (SB | MR | CCS), and (iii) cross-cutting *features* {F1, F2, F3}. The notation takes the form: *payment-design*(*features*). Here are two illustrative auction types:

- FP-SB(F1,F2,F3): a first-price sealed-bid CA with package bids, non-package expressiveness, and business rules.
- CORE-CCA(F1,F3): a CCA with package bids and business rules, employing a Vickrey-nearest core-selecting payment rule.

Below, we survey applications of CAs in practice, first for reverse auctions and then for forward auctions.

A.1 Reverse Combinatorial Auctions

PUBLIC SECTOR

Sourcing of Bus Services

- **Sourcing of School Bus Services in Manchester, UK** (Letchford 1996)

  Organization: Greater Manchester Passenger Transport Executive (GMPTE)(1993-)[29]
  Auction Type: FP-SB(F1,F2,F3).
  Description: Annual tendering for school bus contracts, with one contract per vehicle, for the 38 weeks of the normal school year, from Monday to Friday. Winner determination via integer programming.

(F1) *Group bids*, where a discount is offered if certain contracts are awarded to the bidder simultaneously.

(F2) Bidders free to impose almost any restriction onto the basic bid price schema.

(F3) Quality-of-service constraints.

Outcome: In the first three years of operations, there were around 300 contracts per year. The auction was reported to be a success, but quantitative measures were unavailable (Letchford 1996). GMPTE was superseded by Transport for Greater Manchester in 2001, which continues to accept group bids.

- **Sourcing of Municipal Bus Services in London, UK** (Cantillon & Pesendorfer 2006)

  Organization: London Regional Transport (LRT) (1995-01).
  Auction Type: FP-SB(F1,F3).
  Description: Annual tendering of gross-cost contracts, with revenue from fares going to LRT. Contracts for five years, representing 15-20% of the eight hundred route, 3.5M passengers/day network. Auctions of 1 to 21 routes conducted in sequence, usually for routes in a single geographic area.

---

[29] Letchford (1996) incorrectly refers to the GMPTE as the “Greater Manchester Public [*sic*] Transport Executive”. We thank Adam Letchford for pointing out this correction to us.

(F1) Package bids, OR language.
(F3) Quality-of-service adjustments.

Outcome: The total value of auctioned routes approximately $900M/year. For larger auctions with seven or more routes, most bidders submitted at least one package bid, with discounts relative to bids on individual routes of 5-8%.

- **Sourcing of Municipal Bus Services in Helsinki, Finland** (Tukiainen 2008) (J. Tukiainen, personal communication, 2017)

  Organization: City of Helsinki Intra-city Bus Services, Finland (1997-2009).
  Auction Type: FP-SB(F1,F3).
  Description: Annual tendering of gross-cost, typically five-year contracts representing some portion of the city network, conducted through a sequence of smaller auctions. Bidders could specify a bid price for a route by specifying the unit cost of service per km, per hour, and per vehicle-day.

(F1) Package bids, OR language.
(F3) Quality-of-service adjustments.

Outcome: Sixty-four contracts, some covering multiple routes, auctioned between 1997 and 2005. Package bids in a sequence of seven auctions for the entire network in a sequence of auctions between 1997-01, with around 9% of bids placed on packages. (When the responsibility to run the auctions for bus transit moved from Helsinki to a larger regional unit in 2010, the use of CAs ceased. At the time of this writing, the use of CAs is rare in Finnish public procurement, perhaps because in many cases the government unit conducting the auctions has not adopted software that can accommodate CAs.)

- **Sourcing of Municipal and Regional Bus Services in Sweden** (Lunander & Lundberg 2012*a*)

  Organizations: Local governments in Sweden (2003-04).
  Auction Type: FP-SB(F1,F2,F3); one auction FP-MR(F1,F2,F3).
  Description: Tendering of gross-cost contracts. One auction was multi-round with the chance to adjust bids based on feedback about ranking relative to other bids. Winner determination via integer programming.

(F1) Package bids, OR language.
(F2) Maximum distance by vehicle type.
(F3) Quality-of-service adjustments.

Outcome: Between 30-85% of the contracts were allocated to package bids. The package discount relative to item bids was between 1.7% and 4.95%. Constraints on maximum distance were not used.

Sourcing of School Meal Services

- **Sourcing of School Meal Services to Public Schools in Chile** (Epstein et al. 2002, Catalán et al. 2009, Kim et al. 2014, Olivares et al. 2012)

  Organizations: National School Assistance and Scholarship Board (JUNAEB) and pre-school boards JUNJI and INTEGRA (1997-present)

Auction Type: FP-SB(F1,F3).
Description: Three-year contracts auctioned for 100 geographical areas, with tenders on around 1/3 each year. Winner determination via integer programming. Scenario navigation allows the bid taker to vary projected demand.

(F1) Package bids, XOR language.

(F2) Allocation to a bidder limited by financial capacity. Bids specified through prices on different meals at different nutritional quality levels, linked to price and food indexes, and with price adjustments based on realized demand.

(F3) Constraints on the total number of meals and geographic areas each bidder can supply. Constraints on the number of winners in each region.

Outcome: Typical winning packages on three areas. Electronic bid submission in 2004 saw number of bids increase from 43,000 to 190,000. Approx. \$500M annual spend from 1997 to 2017, with meals served to around 2.5M children each year. Credited with savings of around 20%, i.e., \$100M/year.

Sourcing of Miscellaneous Government Services

- **Sourcing of Domestic Flight Routes in Sweden** (Lunander & Lundberg 2012*a*)

Organization: Swedish National Public Transport Agency (2003).
Auction Type: FP-SB(F1,F2,F3).
Description: Tendering for flight service contracts, with regulated ticket prices and with revenue from fares collected by winning bidders. Winner determination via integer programming.

(F1) Package bids, OR language.

(F2) Maximum number of awarded contracts.

(F3) Quality-of-service constraints.

Outcome: Three of six flight contracts were allocated to a package bid, with the discount from the package bid at a cost saving of 0.3% relative to bids on individual contracts. Constraints on maximum number of contracts were not used.

- **Sourcing of Vehicle Fleets in Ireland** (A. Holland, Keelvar, personal communication, 2017).

Organization: Cork City Council, Ireland (2005), auction technology developed by University College Cork.
Auction Type: FP-SB(F1,F3).
Description: Tendering for 360 vehicles across 16 categories that included both light and heavy equipment. Winner determination via integer programming.

(F1) Package bids.

(F3) Quality-of-service adjustments.

Outcome: The auction was reported to have resulted in comparative prices for similar specification vehicles at 5-10% lower prices than Dublin City Council. Niche suppliers also received contracts for specialized vehicles, and thus public sector objectives for supplier inclusion were also reached.

- **Sourcing in Sweden of Cleaning of Schools and Offices, of Road Resurfacing, and of Elderly Care** (Lunander & Lundberg 2013, 2012*a*,*b*)

  Organizations: Local governments, the National Social Insurance Agency, and the Swedish Road Administration (2005-11).
  Auction Type: FP-SB(F1,F2,F3).
  Description: Sourcing of various services, with bids selected to maximize total value based on quality-adjusted scores for bids. Winner determination via integer programming.

(F1) Package bids, OR language. (Sometimes on restricted packages, perhaps size-restricted; must also bid on items in a package, sometimes with total price on items close to package bid.)

(F2) Capacity constraints on the total value (or total quantity) of allocated business, and the total number of winning bids.

(F3) Minimum quality requirements. Quality-of-service adjustments.

Outcome: Package bids for cleaning services at a 2-9% discount from item bids, and tended to be selected. A single package bid won in a nationwide cleaning services auction for forty-two contracts in 2007, at an estimated cost saving relative to item bids of 6%.

- **Sourcing of Snow Removal in Denmark** (A. Lunander and F. Ygge, Coupa (formerly Trade Extensions), personal communication, 2017).

  Organization: Danish National Road Administration (*Vejdirektoratet*) (2013), technology developed by Coupa.
  Auction Type: FP-SB(F1,F2,F3).
  Description: Sourcing of snow removal services for roads for the coming three years, comprising 97 separate geographic contracts, covering all governmental roads in Denmark. Winner determination via integer programming.

(F1) Package bids, specified as a discount on a set of bids on items.

(F2) Maximum number of awarded contracts. Conditional discounts that triggered based on a quantity threshold.

(F3) Miscellaneous business rules.

Outcome: A total of 103 suppliers submitted bids, 75% of whom submitted bids on three or fewer contracts, with only five using package bids. Two suppliers used a discount on packages of size two or more. Contracts awarded to 51 suppliers.

Sourcing of Electricity Generation Capacity

- **Sourcing of Forward Capacity for Electricity Distribution Companies in Chile** (Moreno et al. 2010, Maurer & Barroso 2011, Mastropietro et al. 2014)

  Organizations: Chilectra, Chilquinta, EMEL, CGE, SAESA (Chilean electricity distribution companies) (2006-10).
  Auction Type: FP-SB(F1,F2,F3).
  Description: Three auctions in total, with an average capacity auctioned of 28 KW hours/year for years 2010-2025, and multiple distributors per event. The government determines an inflation-indexed price for the amount of reserved capacity.

(F1) Bids are peak-load generation capacity to each distributor, along with a price-indexed amount per MWh.

(F2) Maximum on the allocated generation capacity.

(F3) Price caps, while also seeking to balance the allocation across distribution companies.

Outcome: There were high price differences among awarded contracts and distribution areas along with generally high prices and poor coverage. There was difficulty in defining the criteria to balance the dual objectives of cost minimization and demand coverage maximization (Moreno et al. 2010).

PRIVATE SECTOR

Sourcing of Transportation Services

- **Sourcing of Truckload, Rail, Inter-modal and Ocean Transportation Logistics at Global 1000 companies** (Sheffi 2004, Caplice & Sheffi 2006, Elmaghraby & Keskinocak 2003, Caplice & Sheffi 2003)

  Organizations: Walmart, Compaq, Staples, The Limited, K-Mart, Ford, among others (1992-02); technology developed by Logistics.com (acquired by Manhattan Associates, Inc. in 2002).
  Auction Type: FP-SB(F1,F2,F3).
  Description: Sourcing contracts for truckload, rail, inter-modal and ocean logistics. Bids selected to maximize total value subject to business rules. Winner determination via integer programming.

(F1) Package bids. (E.g., for truckload transportation, bids on a specified fraction of the total volume in each of several lanes.)

(F2) Capacity constraints. (E.g., for truckload transportation, a limit on the total number of vehicles used in a particular allocation.)

(F3) Capacity limits on winners. Preferences for minority-owned businesses. Quality-of-service based adjustments. Other business rules (e.g., for truckload transportation, maximum and minimum number of allocated bidders on sets of lanes).

Outcome: E.g., for truckload transportation, in excess of $8B from 1988-2002, with a combined savings of over $500M and typical savings reported of 3-15%. More than half of the truckload auctions allowed package bids.

Less than 10% of lanes were bid in packages, perhaps because of the difficulty to utilize back-haul synergies operationally.

- **Sourcing of Truckload Transportation at Sears Logistics Services, Inc.** (Ledyard et al. 2002)

  Organization: Sears Logistics Services, Inc. (1993-02).
  Auction Type: FP-MR(F1).
  Description: Buy 3-year contracts for truckload services on shipping lanes. An auction was held in 1993 on 800 trucking lanes, followed by five auctions over the next three years, covering an additional 500 lanes. Provisional winners reported each round. Winner determination via integer programming.

(F1) Package bids, OR language, at most 5000 bids per bidder in each round.

Outcome: Reported savings over the first six auctions was \$85M on a \$587M spend. Some rounds included more than 2000 package bids, and typically around 30% of the package bids in the final round were successful.

- **Sourcing of Truckload Transportation at Mars, Inc.** (Hohner et al. 2003, Bichler et al. 2006)

  Organization: Mars, Inc., Worldwide (2000-2002); technology developed by IBM, Inc.
  Auction Type: FP-MR(F1,F2,F3).
  Description: Sourcing and procurement of indirect materials (e.g., in-store display boxes) and direct materials (e.g., sugar). Winner determination via integer programming.

(F1) Package bids, OR language.

(F2) Discount schedules.

(F3) Maximum and minimum number of winners. Maximum and minimum on the total quantity allocated to a single bidder.

Outcome: 60 CAs were conducted by end of 2002, credited with providing return-on-investment in less than a year. When package bids were enabled, typically less than 5% of bids were on packages. When non-package expressiveness was enabled, typically this was used by more than 80% of bids.

- **Sourcing of Air Transport at Motorola, Inc.** (Metty et al. 2005)

  Organization: Motorola, Inc. (2002-2003); technology developed by Emptoris, Inc., acquired by IBM in 2012.
  Auction Type: FP-MR(F2,F3).
  Description: Winner determination via integer programming.

(F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.

(F3) Maximum and minimum number of winners. Preferred number of winners. Quality-of-service constraints.

Outcome: As of 2003, \$600M saving on volume over \$16B for Materials and air transport. Auctions credited with facilitating faster more focused negotiation, and providing more transparency for suppliers. Around 80% of spend went through CAs, compared with 20% going through simple, non-combinatorial reverse auctions.

Industrial Procurement and Sourcing of Direct and Indirect Materials

- **Sourcing and Procurement of Direct and Indirect Materials at Mars, Inc.** (Hohner et al. 2003, Bichler et al. 2006)

  Organization: Mars, Inc., Worldwide (2000-2002); technology developed by IBM, Inc.
  Auction Type: FP-MR(F1,F2,F3).
  Description: Sourcing and procurement of indirect materials (e.g., in-store display boxes) and direct materials (e.g., sugar). Winner determination via integer programming.

(F1) Package bids, OR language.

(F2) Discount schedules.

(F3) Maximum and minimum number of winners and on the total quantity allocated to a single bidder.

Outcome: 60 CAs were conducted by end of 2002, credited with providing return-on-investment in less than a year. When package bids were enabled, typically less than 5% of bids were on packages. When non-package expressiveness was enabled, typically this was used by more than 80% of bids.

- **Sourcing of Materials at Motorola, Inc.** (Metty et al. 2005)

  Organization: Motorola, Inc. (2002-2003); technology developed by Emptoris, Inc., acquired by IBM in 2012.
  Auction Type: FP-MR(F2,F3).
  Description: Sourcing events for direct materials (e.g., cables, displays, semiconductors), and indirect materials (e.g., paper, printed materials). Winner determination via integer programming. CAs were used to run the sourcing events for "over 50% of [Motorola's] annual spending."

(F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.

(F3) Maximum and minimum number of winners and of preferred winners. Quality-of-service constraints.

Outcome: As of 2003, $600M saving on volume over $16B for materials and air transport. Auctions credited with facilitating faster more focused negotiation, and providing more transparency for suppliers. Around 80% of spend went through CAs, compared with 20% going through simple, non-combinatorial reverse auctions.

- **Sourcing and Procurement of Materials at Global 1000 companies** (Sandholm et al. 2006, Sandholm 2013).

  Organizations: More than sixty companies, mostly in the Global 1000, including Proctor & Gamble, Walmart and Target (2000-present); Jaggaer (rebranded from SciQuest), technology and products developed at CombineNet.
  Auction Type: FP-MR(F2,F3).
  Description: Sourcing and procurement for transportation, direct materials, packaging, indirect materials, chemicals, technology, services, health care and telecommunications. Activity rules sometimes used. Winner determination via integer programming.

(F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.

(F3) Maximum and minimum number of winners. Preferences to existing suppliers and minority-owned businesses. Quality-of-service constraints. Scenario navigation by the bid taker.

Outcome: As of 2006, $35B spend had been sourced in events ranging in size from $2M to $1.6B, involving over 12,000 bidders, and with reported year-on-year cost savings of 11.1%. Non-package forms of expressive bidding were used more often than package bids.

Sourcing of Miscellaneous Non-Industrial Services

- **Sourcing of Roadside Assistance Services in the UK** (A. Holland, Keelvar, personal communication, 2017).

Organization: Direct Line Insurance (2012), auction technology developed by Keelvar Systems Ltd.
Auction Type: FP-SB(F1,F3).
Description: Tendering for roadside assistance services for broken down vehicles. Winner determination via integer programming.

(F1) Package bids.

(F3) A quality-of-service constraints.

Outcome: The speed of contract award was reported to have improved by a factor of four, supplier consolidation goals were achieved, and suppliers reported winning bundles of contiguous regions that permitted improved operational efficiency.

A.2 Forward Combinatorial Auctions

PUBLIC SECTOR, NON-SPECTRUM APPLICATIONS

Wholesale Electricity Markets

- **Sale of Electricity Generation Capacity in Virtual Power Plant auctions**

  Organization: EDF (Electricité) de France); Electrabel, Belgium; Nuon, Netherlands; Elsam, Denmark; Endesa and Iberdrola, Spain; REN and EDP, Portugal; Texas Capacity Auctions, Texas, U.S.; E.ON and RWE, Germany; Annual Capacity Auctions, Illinois, U.S.; Penelec, Pennsylvania, U.S.
  Auction Type: FP-CCA(F1)
  Description: Multiple products, i.e., generating capacity, are sold in a single auction, each representing base load or peak load and different contract duration, which is typically six months to two years. A typical CCA design for these auctions has the following features: feedback is provided regarding excess demand for each product across rounds, the auction closes when there is no excess demand, and the activity rules adopt to constrain bids in terms of bids placed in previous rounds, thus preventing bidders from jumping into the bidding towards the end of the auction. Intra-round bids are also adopted, to allow a bidder to express his demand at all price vectors along a price segment between the start-of-round and end-of-round prices. The idea is to allow larger price changes between rounds, and to enable fewer rounds and faster auctions.

(F1) Bidders can place a bid on a single package in each round, the bid specifying demand for a particular quantity of each product at current prices.

Outcome: The first VPP auction was used by EDF (France) in 2001 to divest 6 GW of generating capacity following an acquisition; quarterly auctions have been held by EDF since then, typically allocating capacity to more than fifteen bidders and with an average contract volume of US \$430 million per auction, and auctions completing in less than ten days and within a single day. Almost all capacity is typically sold, and any unsold capacity added to the next auction in the sequence. Electrabel (Belgium) has run quarterly VPP auctions since 2003 with a typical contract volume of \$130m. Eslam (Denmark) and E.ON and RWE (Germany) have run quarterly auctions, and Endesa and Iderdola (Spain) run semi-annual auctions. Texas Capacity Auctions has been running VPP auctions five times a year since 2001.

Sale of Government Plant

- **Liberty Yard Auction (1922)** (U.S. Congress 1925, Cassady, Jr. 1967)

  Organization: U.S. Shipping Board Emergency Fleet Corp.
  Auction Name, Date: Liberty Yard Auction
  Auction Type: FP-SB(F1).
  Description: Sale of 70 residence sites comprising the Liberty Plant, Alameda, California

(F1) Package bids; specifically, entirety bids only. (Typical auction designs for government real estate allow for a first stage of package bids on all items, and a second stage with an auction on individual items, with these bidders winning if the sum total bid exceeded the best package bid. Package bids on all items remain a feature of real estate auctions.)

Outcome: An entirety bid of $350,000 won on appeal.

Rental of Floor Space

- **Configuration and Rental of Floor Space in Amsterdam (2011)** (Goossens et al. 2014)

  Organization: Stadgenoot (2011).
  Auction Name: Furore Solids Auction; IJburg Solids Auction
  Auction Type: FP-MR(F1,F3).
  Description: To configure and rent space in "Solids," buildings in which the tenants determine the allocation of space within the building. A Solid is divided into lots, which tenants use as "building blocks" to specify spaces. Winner determination via integer programming. Two auctions were held: May 2011, for a single building in the west of Amsterdam, called Furore, with 7000 square meters of floor space partitioned into 125 lots over seven floors; various constraints reduced the number of valid packages to 1,214; and June 2011, for two buildings in IJburg, a new planned neighborhood in the east of Amsterdam. (Further information on the June 2011 auction is unavailable.)

(F1) Package bids, OR-of-XOR language. Package bids subject to restrictions, including composed of adjacent lots on a single floor, and having access to the central hallway and utility shaft.

(F3) Minimum percentage of floor area required for each of the three different bidder types (residential, commercial and social). Unsold lots must not be too fragmented.

Outcome: May 2011 auction: 100 bidders placed 700 bids, with 30 winning bidders and 95% of space allocated. Monthly rental revenue generated was between €80,000 and €120,000 ($100,000 and $160,000)(F. Spieksma, personal communication, 2014 & 2017).

PUBLIC SECTOR, SALE OF WIRELESS SPECTRUM LICENSES

Here are some technical terms used in this section that did not arise in the main text: *GSM*, Global System for Mobile Communication, a wireless technology originally developed for Europe, but now widely used in other parts of the world; *E-GSM*, Extended GSM, a spectrum band that includes an extension of an additional 10 MHz at the lower end of the GSM 900 MHz band; *paired spectrum*, two associated blocks of spectrum, one used to transmit in one direction, and the other used to simultaneously transmit in the opposite direction, via a technology called *FDD*, Frequency Division Duplexing; *TDD* Time Division Duplexing, a technology for unpaired spectrum that makes use of a single frequency band, with transmitting and receiving occurring at different times; *MVNO*, Mobile Virtual Network Operator, a company that does not own any radio spectrum but nevertheless provides mobile telephone service.

Sealed-Bid Combinatorial Auction

- **Sale of 900 MHz and Sale of 1.8 GHz Spectrum in Norway (2001)** (NKOM 2001*a*,*b*)

  Organization: Norwegian Communications Authority (Nkom)
  Auction Name, Date: 900 MHz Auction (Auction #1), October 2001.
  Auction Name, Date: 1800 MHz (Auction #2), December 2001.
  Auction Type: FP-SB(F1,F3).
  Description: 900 MHz band divided into six 2x1.15 MHz lots comprising 11 E-GSM channels. 1.8 GHz band divided into seven lots comprising 32 GSM channels.

(F1) Bidders could submit one bid on a combination of two or more lots, and any number of bids on individual lots where the price offered for each lot may vary.

(F3) Spectrum caps.

  Outcome of 900 MHz Auction: Three bidders won two adjacent lots each.
  Total of lump sum payments: NOK 11.5M ($1.3M). Total of annual fees: NOK 13.8M ($1.5M).

  Outcome of 1800 MHz: Two bidders won one license each, leaving five lots unsold.
  Total of lump sum payments: NOK 56,800 ($6,369). Total of annual fees: NOK 12.8M ($1.4M).

- **Sale of 3.5 GHz Spectrum in Nigeria (2002)** (Koboldt et al. 2003)

  Organization: Nigerian Communications Commission (NCC)
  Auction Name, Date: Fixed Wireless Access Auction, June 2002.
  Auction Type: FP-SB(F1).
  Description: 80 licenses in the 3.5 GHz band, with two or three licenses available in each of 37 regions.

(F1) Package bids, XOR language.

  Outcome: Around 40% of bids were on packages. 67 of the 80 licenses allocated. The contested licenses initially generated $3.78B Nigerian naira ($38M), but five of the twenty-five winners defaulted.

- **Sale of 400 MHz Spectrum in the UK (2006)**

  Organization: Ofcom
  Auction Name, Date: Auction of 412-414 MHz paired with 422-424 MHz, October 2006.
  Auction Type: FP-SB(F1)
  Description: 4 MHz of paired spectrum in four frequency lots, hence fifteen possible packages.

(F1) Package bids, OR language.

(F3) Spectrum caps.

  Outcome: There were five bidders. One bidder outbid all the others by a considerable margin and won all four lots for a bid of £1,500,025 ($2.3M).

- **Sale of 26 GHz Spectrum in Ireland (2008)** (COMREG 2008)

  Organization: Commission for Communications Regulation (ComReg)
  Auction Name, Date: 26 GHz Auction, June 2008.
  Auction Type: CORE-SB(F1,F3).
  Description: 18 lots of 2x28 MHz, in 27 possible packages. Auction design allowed market to determine split between point-to-point (P2P) and point-to-multipoint (PMP) applications. (This was one of the first auctions that allowed the market to determine how bands are to be divided between competing technologies.) Complex pricing rule involved adjusting all bids at once with a sum-of-squares adjustment penalty.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

  Outcome: Multiple winners in both P2P and PMP categories. Thus, a second stage, sealed-bid round was required to determine frequency assignments to bidders. Total revenue: €1,100,000 (US $1.7M).

- **Sale of 3.4-3.6 GHz AND 3.6-3.8 GHz Spectrum in Portugal (2009–2010)** (ANACOM 2010)

  Organization: ANACOM
  Auction Name, Date: Broadband Wireless Access (BWA) Auction, October 2009–April 2010.
  Auction Type: CORE-SB(F1).
  Description: 36 2x28 MHz licenses. Two-stage auction. Variation on CORE payment rule. Auction design allowed market to determine how spectrum bands are to be divided between FDD and TDD technologies.

(F1) Each bidder could submit up to 100 package bids, OR language.

  Outcome: There were three bidders; two were allocated licenses. 50% of the lots were allocated. Total revenue: €3,400,000 ($4.6M).

- **Sale of 2.6 GHz Spectrum (2011) AND 800 MHz Spectrum in France (2011)** (DotEcon 2012)

  Organization: ARCEP (French Regulatory Authority for Electronic Communications and Post)
  Auction Name, Date: 2.6 GHz Spectrum Auction, September 2011.
  Auction Name, Date: 800 MHz Spectrum Auction, December 2011.
  Auction Type: FP-SB(F1,F3).

  Description: 2.6 GHz auction: Two lots of 2x20 MHz and two blocks of 2x5 MHz. (The 2.6 GHz spectrum band, 2500 to 2690 MHz, is also known as the 2.5 GHz spectrum band (Marsden et al. 2010).) 800 MHz auction: Two lots of 2x10 MHz and two 2x5 MHz lots. Licensees to pay annual fees of 1% of revenue from allocated spectrum.

(F1) Package bids, XOR language.

(F3) 2.6 GHz auction: Bidders agreeing to provide MVNO access had a multiplier applied to their bid. 800 MHz auction: Bidders agreeing to provide MVNO access and/or commit to accelerated rollout schedule had a multiplier applied to their bid.

Outcome: 2.6 MHz auction: 70 MHz of paired spectrum awarded. All four bidders won licenses. Total revenue: €936M ($854M). 800 MHz auction: 30 MHz of paired spectrum awarded. Three of the four bidders won licenses. Total revenue: €2.649B ($3.353B).

- **Sale of 800 MHz, 900 MHz and 1.8 GHz Spectrum in Norway (2013)** (NKOM 2013)

  Organization: Norwegian Post and Telecommunications Authority
  Auction Name, Date: 800, 900 and 1800 MHz Auction (Auction #14), December 2013.
  Auction Type: FP-SB(F1,F3).
  Description: Spectrum in the 800MHz, 900MHz and 1800MHz bands. Specifically: 2x30MHz in the 800MHz band, 2x15 MHz in the 900MHz band and 2x55MHz in the 1800MHz band.

(F1) Package bids, XOR language.

(F3) One of the lots (A2) required to be in the winning allocation. Spectrum caps.

Outcome: Three of the four bidders won licenses. Two each allocated 2x10 MHz in 800MHz band, 2x5 MHz in 900MHz band, 2x10 MHz in 1800MHz band. The third was allocated 2x10 MHz in 800MHz band, 2x5 MHz in 900MHz band, 2x20 MHz in 1800MHz band. Total revenue: NOK 1.8B ($292M).

- **Sale of 26 GHz Spectrum in Ireland (2018)** (COMREG 2008)

  Organization: Commission for Communications Regulation (ComReg)
  Auction Name, Date: 26 GHz Auction, April–June 2018.
  Auction Type: FB-SB(F1,F3).
  Description: Spectrum offered for 19 2x28 MHz blocks of spectrum in the range 24.275–25.277 GHz paired with 25.753-26.285 GHz. The auction was in two-stages. A sealed-bid combinatorial auction was first run to assign the spectrum as frequency-generic lots, with prices set on the basis of opportunity cost. Bidders were restricted to winning no more than five 2x28 MHz blocks.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were three winners: Vodafone Ireland, Three Ireland, and Meteor Mobil Communications, each of which won five 2x28 MHz blocks, which was the maximum possible. The duration of each of the licenses is ten years. Lump sum payments: Three and Meteor €£350K each; Vodaphone: €550 (higher due to an additional payment for being awarded a specific, requested frequency range). Annual fees: All three winners had a requirement to pay approx. €1.25 million each over the 10-year license duration.

Multi-Round Combinatorial Auction

- **Sale of 900 MHz Spectrum in the US (2003)** (FCC 2003)
  Organization: Federal Communications Commission (FCC)
  Auction Name, Date: Regional Narrowband PCS Auction (Auction 51), September 2003.
  Auction Type: FP-MR(F1,F3).
  Description: Six regional licenses in Channels 16 and 17. Channel 16: 901.8125–901.8250 MHz & 930.65–930.70 MHz. Channel 17: 901.8250–901.8375 MHz & 930.70–930.75 MHz.

(F1) Package bids, OR language (maximum 12 packages per bidder).

(F3) Bidding credits for small and very small businesses, or consortia of either, corresponded to a discount on the winning bid. (In this paper, revenue is reported net of bidding credits, if any.)

Outcome: There were two bidders, with three rounds of bidding over two days, with only a single bid: A package bid for the five items consisting of the five regions for Channel 16. Total revenue: $134,250.

- **Sale of 700 MHz Spectrum in the US (2008)** (FCC 2007, Bazelon 2009, Brusco et al. 2009, Cramton et al. 2011)

Organization: Federal Communications Commission (FCC) (2008)
Auction Name: 700 MHz Auction (Auction 73), January–March 2008.
Auction Type: FP-MR(F1,F3)
Description: Five blocks of licenses: A, B, C, D, and E. Combinatorial bidding on the twelve licenses of 22 MHz of spectrum in block C using hierarchical package bidding involving three pre-defined packages.

(F1) Package bids, OR language.

(F3) Bidding credits for small and very small businesses, or consortia of either. This corresponded to a discount on the winning bid.

Outcome: There were 214 bidders. There were 261 rounds over 38 days, where 101 bidders won 1090 of the 1099 licenses. Only a small number of package bids were submitted, only one of which was in the final allocation. C Block raised $5B (with reserve price $4.5B). Total revenue: $19.12 billion.

<u>Combinatorial Clock Auction</u>

- **Sale of 800 MHz and 1.9 GHz Spectrum in Trinidad and Tobago (2005)** (TATT 2005)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)
Auction Name, Date: Spectrum Auction, June 2005.
Auction Type: FP-CCA(F1,F3). A first-price two-stage CCA, consisting of a clock stage and an assignment stage, with no supplementary bid stage. (In the specification of design features for spectrum auctions, F1, F2, and F3, we will ignore the assignment stage for the purpose of taxonomy and just focus on bidding languages for the main part of the auction.)
Description: 10 lots of 5 MHz spectrum. 800 MHz band: 2 lots in of 2x5 MHz. 1.9 GHz band: 8 lots of 2x5 MHz. In Stage 2, the bidders could choose to bid for 3-lot, 4-lot, or 5-lot packages.

(F1) Package bids in the second stage, XOR language.

(F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were two bidders. Both won licenses. One firm won a package of 5 lots at US $15,756,003. The other firm won a package of 3 lots at $9,300,007. Total revenue: $25.1M.

- **Sale of 12 GHz, 700 MHz, and 28 GHz Spectrum in Trinidad and Tobago (2007)** (TATT 2007)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)
Auction Name, Date: BWA Auction, October 2007.
Auction Type: FP-CCA(F1,F3). Similar structure to Trinidad and Tobago Spectrum Auction

of 2005.
Description: Spectrum in the Lower 700 MHz (698-746 MHz), 12 GHz, and 28 GHz bands.

(F1) Package bids in the second stage, XOR language.
(F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were three bidders. All won licenses. In the 12 GHz band, one bidder won all twelve blocks at TT$650,000 per block p.a. each. In the Lower 70 MHz band, one bidder won three blocks at TT$ 170,000 per block p.a. In the 28 GHz band, there was no winner, and thus no spectrum was allocated. Total revenue: TT$8,331,000 ($1.3M) p.a.

- **Sale of 10-40 GHz Spectrum (2008)** and **1.4 GHz Spectrum in the UK (2008)** (Cramton 2013, Ausubel et al. 2006)

Organization: Ofcom.
Auction Name, Date: 10-40 GHz Auction, February 2008.
Auction Name, Date: L-Band Auction, May 2008.
Auction Type: CORE-CCA(F1).
Description: The 10-40 GHz band was divided into 27 lots; the L-band (1.4 GHz) was divided into 17 lots. The L-band auction design allowed the market to determine how bands are to be divided up between high power vs. low power technologies.

(F1) Package bids, XOR language.

Outcome: 10-40 GHz auction: Each of the ten bidders were allocated spectrum. Total revenue: £1,430,000 ($2.8M). L-Band auction: One of the eight bidders won all the lots. Total revenue: £8,330,00 ($16M).

- **Sale of 2.3 GHz, 2.6, and 700 MHz Spectrum in Trinidad and Tobago (2009)** (TATT 2009)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)
Auction Name, Date: Second BWA Auction, April 2009.
Auction Type: FP-CCA(F1,F3). Similar structure to Trinidad and Tobago BWA Auction of 2007.
Description: Lots in the 2.3 GHz and 2.6 GHz bands, as well as spectrum remaining in the 700 MHz band left over from the first BWA auction.

(F1) Package bids in the second stage, XOR language.
(F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were three bidders. All won licenses. One won 1 lot in the Lower 700 MHz band, 4 lots in the 2.3 GHz band. The second won 4 lots in the Lower 700 MHz band, 10 lots in the 2.6 MHz band. The third won 8 lots in the 2.6 MHz band. Revenue: Lower 700 MHz band TT$1,000,000; 2.3 GHz band TT$80,000; 2.6 GHz band TT$3,6000,000. Total revenue: TT$4,680,000 ($0.74M) p.a.

- **Sale of 2.6 GHz Spectrum in the Netherlands (2010)** (Marsden et al. 2010)

Organization: Ministry of Economic Affairs
Auction Name, Date: 2.6 GHz Spectrum Auction, April 2010.

Auction Type: CORE-CCA(F1, F3).
Description: 37 blocks of 5 MHz to be awarded as paired or unpaired spectrum, or as unpaired guard blocks. The actual number of lots in each category to be determined by the auction. Also, one lot of unpaired spectrum at 2010-2019.7 MHz.

(F1) Package bids. Restrictions on some lots being contiguous. XOR language.

(F3) Spectrum caps.

Outcome: All five bidders who participated were awarded spectrum, with the amounts of frequency awarded per bidder the maximum permitted according to the caps imposed by the government. The total revenue from the auction was €2,627,000 ($3.3M).

- **Sale of 2.5 GHz Spectrum in Denmark (2010)** (Marsden et al. 2010)

Organization: National IT and Telecom Agency
Auction Name, Date: 2.5 GHz Auction, May 2010.
Auction Type: CORE-CCA(F1,F3).
Description: 14 lots of 2x5 MHz and 9 lots of 5 MHz.

(F1) Each bidder could submit a single package bid in each clock round, together with multiple supplementary package bids; XOR language.

(F3) Spectrum caps.

Outcome: There were four winners. One won 2 blocks of paired and 5 blocks of unpaired spectrum; a second won 4 blocks of paired and no unpaired spectrum, and the third and fourth each won 4 blocks of paired and 2 blocks of unpaired spectrum. Revenue of DKK 1.01 billion ($175M).

- **Sale of 2.6 GHz Spectrum in Austria (2010)** (Marsden et al. 2010, DotEcon 2012)

Organization: Austrian Regulatory Authority for Broadcasting and Telecommunications (RTR).
Auction Name, Date: 2.6 GHz Auction, September 2010.
Auction Type: CORE-CCA(F1, F3).
Description: 14 lots of 2x5 MHz and 10 lots of 5 MHz. Auction design was essentially the same as the Danish (2010) design.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: The auction had four winners, with revenue €39.5M ($53M).

- **Sale of 1.8 GHz Spectrum in Sweden (2011)** (PTS 2011)

Organization: PTS (National Post & Telecom Agency)
Auction Name, Date: 1800 MHz Auction, October 2011.
Auction Type: VCG-CCA(F1).
Description: 2x70 MHz of spectrum divided into 14 lots of 2x5 MHz each. Two-stage CCA consisting of: (1) Clock Stage; and (2) Assignment Stage with winner and price determination for specific lots, employing VCG pricing rule—*not* Vickrey-nearest core-selecting pricing rule.

(F1) Package bids, XOR language.

Outcome: There were three bidders, two winners, each winning 7 lots. Total revenue: SEK 1.35B ($204.5M).

- **Sale of Multi-Band Spectrum in Switzerland (2012)**

  Organizations: Swiss Federal Communications Commission (ComCom) and Swiss Federal Office of Communications (BAKOM)
  Auction Name, Date: Auction of Mobile Radio Frequencies, February 2012.
  Auction Type: CORE-CCA(F1, F3). *Description*: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz, and 2.1 GHz, and 2.6 GHz bands.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: The three eligible bidders each won licenses, generating revenues of CHF 996.3M ($1.085B).

- **Sale of 800 MHz Spectrum in Denmark (2012)** (DotEcon 2012)

  Organization: Danish Business Authority
  Auction Name, Date: 800 MHz Spectrum Auction, June 2012.
  Auction Type: CORE-CCA(F1,F3).
  Description: 2x30 MHz divided into five lots: one lot of 2x10 MHz, four lots of 2x5 MHz.

(F1) Package bids, XOR language.

(F3) Coverage obligations.

Outcome: Two of the three bidders won licenses. One won 2x10 MHz without coverage obligation; the other the remaining 2x20 MHz with the coverage obligation. Total revenue: DKK 793.2M ($130M).

- **Sale of Multi-Band Spectrum in Romania (2012)** (ANCOM 2012, Marinescu 2012)

  Organization: ANCOM
  Auction Name, Date: Spectrum Auction, September 2012.
  Auction Type: FP-CCA(F1, F3).
  Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz and 2.6 GHz bands, available in 2x5 MHz lots, other than unpaired 2.6 GHz spectrum, where there were three 15 MHz lots. No supplementary bids round for winner determination, with one further round only for any undersell.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligations.

Outcome: All five bidders were awarded spectrum. Of the 545 MHz auctioned, 485 MHz was awarded. Total revenue: €6.82M ($891M).

- **Sale of Multi-Band Spectrum in the Netherlands (2012)** (DotEcon 2012)

  Organization: Agentschap Telecom
  Auction Name, Date: Multiband Spectrum Auction, October–December 2012.

Auction Type: CORE-CCA(F1, F3).
Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz, 1.9 GHz, and 2.6 GHz bands. 41 licenses.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Four of the five bidders won licenses. All licenses were sold. Total revenue: €3.80b ($4.85B).

- **Sale of 800 MHz, 900 MHz, and 1.8 GHz Spectrum in Ireland (2012)** (COMREG 2012)

Organization: ComReg
Auction Name, Date: Multi-Band Spectrum Auction, November 2012.
Auction Type: CORE-CCA(F1, F3).
Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz bands. Existing license holders could relinquish existing GSM-only licenses and simultaneously receive a liberalized license for same spectrum if this exchange is included in their winning bid. Also, licenses could be short-term or long-term.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were four bidders, all of whom were allocated spectrum, paying €481.7m ($631m) upfront and the remaining €372.95m ($284.7m) paid in annual fees over the 17 year license period. Total revenue: $1.088B.

- **Sale of 800 MHz and 2.6 GHz Spectrum in the UK (2013)** (Morse 2014)

Organization: Ofcom

Auction Name, Date: 4G Spectrum Auction
Auction Type: CORE-CCA(F1,F3)
Description: 250 MHz of spectrum. 800 MHz band: One 2x10 MHz lot plus four 2x5 MHz lots. 2.6 GHz band: Paired spectrum in 2x5 MHz lots, unpaired spectrum in 5 MHz lots. Eligibility-point based activity rule.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligations.

Outcome: Five of the seven bidders were allocated spectrum. Total revenue: £2.4B ($4.55B).

- **Sale of 700 MHz and 2.6 GHz Spectrum in Australia (2013)** (Ausubel & Baranov 2014*b*)

Organization: ACMA (Australian Communications and Media Authority)
Auction Name, Date: Digital Dividend Spectrum Auction, May 2013.
Auction Type: CORE-CCA(F1, F3)
Description: 700 MHz band: 9 generic lots of 2x5 MHz. 2.6 GHz band: 14 generic lots of 2x5 MHz across 11 geographic regions. Variation of CORE payment rule minimized a weighted Euclidean distance to the Vickrey payments.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: All three bidders won licenses, with a total revenue of AUS\$1.965 billion (US\$2.02 billion).

- **Sale of 800 MHz, 900 MHz, and 1.8 GHz Spectrum in Austria (2013)** (Mochon & Saez 2017)

  Organization: (RTR) Austrian Regulatory Authority for Broadcasting and Telecommunications
  Auction Name, Date: Multiband Auction, October 2013.
  Auction Type: CORE-CCA(F1,F3).
  Description: Paired spectrum in the 800, 900, and 1.8 GHz bands. There were 28 blocks of spectrum for auction: 6 blocks of 800 MHz, one of which had increased coverage obligations; 7 blocks of 900 MHz, and 15 blocks of 1.8 GHz.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Three of the four bidders were allocated spectrum. However, two filed complaints in the high court against the auction results. On 4 December 2014, the Austrian Constitutional Court ruled confirming the official decisions of the multiband auction. Revenue raised €2B (\$2.8B).

- **Sale of 800 MHz, 1.8 GHz and 2.6 GHz Spectrum in Slovakia (2014)** (TUSR 2013)

  Organization: Telecommunications Regulatory Authority of the Slovak Republic
  Auction Name, Date: 800, 1800 and 2600 MHz Spectrum Auction, January 2014.
  Auction Type: CORE-CCA(F1).
  Description: 800 MHz band: 6 blocks of 2x5.0 MHz. 1.8 GHz band: 3 blocks of 2x5.0 MHz, 1 block of 2x2.2 MHz, 1 block of 2x1.2 MHz, 1 block of 2x1.0 MHz, 1 block of 2x0.6 MHz, 1 block of 2x0.4 MHz. 2.6 GHz band: 14 blocks of 2x5.0 MHz, 10 blocks of 5.0 MHz.

(F1) Package bids, OR language.

Outcome: Four companies won licenses. Total revenue: €163.9M (\$222M).

- **Sale of 700 MHz Spectrum in Canada (2014)** (Taylor 2015)

  Organization: Industry Canada
  Auction Name, Date: 700 MHz Spectrum Auction, February 2014.
  Auction Type: CORE-CCA(F1,F3).
  Description: 68 MHz of spectrum: 3 blocks of 2x6 MHz, 2 blocks of 2x5 MHz, 2 blocks of 6 MHz, covering 14 license areas, 98 licenses in total. Auction design similar to UK 4G auction (2013). However, here there were revealed-preference based activity rules, which were not in the UK design.

(F1) Package bids, OR language.

(F3) Spectrum caps.

Outcome: There were ten bidders and eight winners, with 97 of the 98 licenses awarded. Total revenue: CA\$5.27B (\$4.8B).

- **Sale of Multi-Band Spectrum in Slovenia (2014)** (Mochon & Saez 2017)

  Organization: AKOS (Agency for Communication Networks and Services of the Republic of Slovenia)
  Auction Name, Date: Multi-Band Spectrum Auction, April 2014.
  Auction Type: CORE-CCA(F1,F3).
  Description: Licenses offered in the 800 MHz, 900 MHz, 1.8 GHz, 2.1 GHz, and 2.6 GHz bands.

(F1) Package bids, XOR language.

(F3) Reserved spectrum for new entrants or existing operators with market share at most 15%. Coverage obligations.

  Outcome: All three bidders were awarded spectrum. Total revenue: €148,741,000 ($206M).

- **Sale of 700 MHz Spectrum in New Zealand (2014)** (MBIE 2013)

  Organization: Ministry of Business, Innovation & Employment
  Auction Name, Date: 700 MHz Auction (Auction 12), October 2013–June 2014.
  Auction Type: FP-CCA(F1,F3).
  Description: Nine lots of 2x5 MHz in the 700 MHz band. Supplementary bids round to be held only if aggregate number of bids at end of clock stage does not exceed the number of lots available, in which case a supplementary bids round may be used to offer any remaining lots unsold in the clock rounds.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

  Outcome: Eight lots were sold in the clock stage, with the 9th lot sold in a run-off between two bidders. No supplementary bids round was held. Three bidders were allocated spectrum. Total revenue: NZ$270,100,174 ($234M).

- **Sale of 2.6 GHz Spectrum in Canada (2015)**

  Organization: Industry Canada
  Auction Name, Date: 2500 MHz Auction, April–May 2015.
  Auction Type: CORE-CCA(F1,F3).
  Description: 318 blocks of spectrum in the 2.6 GHz band across 61 regions of Canada. This was the first CCA to allow OR bidding.

(F1) Package bids, XOR language and OR language.

(F3) Spectrum caps.

  Outcome: Total revenue: C$755M ($624M).

- **Sale of 1.7 GHz and 2.1 GHz Spectrum in Mexico (2016)** (IWS 2015)

  Organization: IFT (Ifetel, the Federal Telecommunications Institute)
  Auction Name, Date: AWS Spectrum Auction, February 2016.
  Auction Type: CORE-CA(F1, F3).
  Description: 80 MHz of 2x5 MHz spectrum across two AWS bands for the deployment of 4G networks: 30 MHz for AWS-1 (1710-1725 MHz/2110-2125 MHz); 50 MHz for AWS-3 (1755-1780 MHz/2155-2180 MHz). Allowed for replanning of entire AWS band so that bidders received contiguous spectrum.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were two winners. One paid 31.0B pesos ($1.7B); the other paid 12.7B pesos ($698M). Unassigned at conclusion: AWS-3 bands 1755-1760 MHz and 2155-2160 MHz. Total revenue: 43.7B Mexican pesos ($2.4B).

- **Sale of 800 MHz, 900 MHz, 1.8 GHz, 2 GH and 2.6 GHz Spectrum in Montenegro (2016)** (EKIP 2016)

  Organization: Agency for Electronic Communications and Postal Services
  Auction Name, Date: Auction for Spectrum of Mobile Networks, July–August 2016.
  Auction Type: CORE-CCA(F1,F3).
  Description: 625 MHz of spectrum in 5 bands. 800 MHz: 6 lots of 2x5 MHz for 15 years. 900 MHz: 5 lots of 2x5 MHz for 15 years. 1800 MHz: 11 lots of 2x5 MHz for 15 years, 4 lots for 10 years. 2 GHz: 3 lots of 2x5 MHz for 15 years, 9 lots of 2x5 MHz for 10 years, 4 lots of 5 MHz for 15 years, 3 lots of 5 MHz for 10 years. 2.6 GHz: 14 lots of 2x5 MHz for 15 years, 10 lots of 5 MHz for 15 years.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Reserved spectrum for existing operators in bands already in use and for new operators in newly available bands.

Outcome: There were three bidders, all of whom won spectrum. The clock stage ran 50 rounds. There was a supplementary bids round. The assignment stage ran 4 rounds. The auction raised €50,650,000 ($57M). However, unallocated spectrum with a total reserve price of €8,305,000 ($9M) was left unsold.

- **Sale of 1.8 GHz Spectrum in Denmark (2016)** (DEA 2016)

  Organization: Danish Energy Agency
  Auction Name, Date: 1800 MHz Auction, September 2016.
  Auction Type: FP-CCA(F1, F3).
  Description: 2x64.9 MHz paired frequencies (1720.1–1785.0 MHz paired with 1815.1–1880.0 MHz). The format is a CMRA (and thus, first price, with no proxy stage).

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligation.

Outcome: There were three winners, and raised DKK 1.025 billion ($154M).

- **Sale of 3.6 GHz Spectrum in Ireland (2017)** (COMREG 2017)

  Organization: ComReg
  Auction Name, Date: 3.6 GHz Band Spectrum Auction, May 2017.
  Auction Type: CORE-CCA(F1, F3).
  Description: 350 MHz of spectrum in the 3.6 GHz band in four rural regions (Borders Midland and West, South West, East, South East) and five cities and their suburbs (Dublin, Cork, Limerick, Galway, Waterford). 66 lots were offered: one 25 MHz frequency-specific lot and sixty-five 5 MHz generic lots.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were five winning bidders. The first won 25 MHz in each of the rural regions, and 60 MHz in each of the cities. The second won 60 MHz in the rural regions. The third won 80 MHz in the rural regions and 85 MHz in the cities. The fourth won 100 MHz nationally. The fifth won 85 MHz in the rural regions and 105 MHz in the cities. Total revenue: €60,466,312 ($67.7M) in total up front fees plus €17,707,200 ($19.8M) in total spectrum usage fees paid annually for 15 years.

- **Sale of 700 MHz, 900 MHz and 2300MHz Spectrum in Denmark (2019)**

  Organization: ComReg.
  Auction Name, Date: 700 MHz, 900 MHz and 2300 MHz, March–April 2019.
  Auction Type: FP-CCA(F1, F3).
  Description: All three Danish Mobile Network Operators participated in the auction. TDC paid DKK 1.6 billion for 2x10 MHz of paired spectrum in the 700 MHz band, 20 MHz of paired spectrum of 700 MHz supplemental downlink spectrum, 2x10 MHz in the 900 MHz band, and 60 MHz in the 2.3 GHz band. Hi3G paid DKK 485.2 million for 2x10 MHz in each of the 700 MHz and 900 MHz bands. TT-Network paid DKK 107.6 million for 2x5 MHz in the 700 MHz band and 2x10 MHz in the 900 MHz band. The remaining spectrum was assigned through a CMRA format, where an additional assignment stage allowed bidders to express preferences over different locations within each band to establish the specific frequencies assigned.

(F1) Package bids, XOR language.
(F3) Spectrum caps. Regional coverage obligation.

Outcome: All three bidders were winners. The auction raised in excess of DKK 2.2 billion ($385.5 million).

- **Sale of Microwave Bands, 10, 13, 18, 23, 28, 32, 38 GHz in Norway (2020)**

  Organization: Nkom.
  Auction Name, Date: Auction of Fixed Link Microwave Bands, May 2020.
  Auction Type: FP-CCA(F1, F3).
  Description: The CMRA was used for the assignment of frequency-generic lots grouped into 24 lot categories, with bidders able to place multiple package bids in each round, and optimization used in each round to seek an assignment that included all active bidders; otherwise, the prices were increased to resolve demand conflicts. The specific frequencies were assigned in a subseqent assignment stage and with an additional sealed-bid process, with the objective of maximizing contiguity of holdings.

(F1) Package bids, XOR language.

Outcome: The auction raised NOK 34.3 million ($4.2 million).

PRIVATE SECTOR

Bankruptcy and Real Estate Sales

- **Sale of General Property** (McLean 1955, Shakers 2012)

Organization: Various (from prior to 1955, to the present).
Auction Type: FP-SB(F1); followed by single-item auctions.

Description: A common design allows for a first stage of sealed-bid package bidding (on all items only), followed by a second stage with ascending-price auctions on individual items. A variation changes the order of the two stages. Package bids on all items remain a feature of bankruptcy auctions.

(F1) Package bids, entirety bids only.

Outcome: Early examples include the sale of a machine shop (a package bid of \$60,000 losing to a single bidder with bids on individual lots totalling \$95,150), the sale of prefab houses (a package bid of \$325,000 losing to a combination of bids on individual lots), and the sale of machinery and equipment of a manufacturing plant (a package bid of \$285,000 winning on appeal).

Contextual Display Advertising on the Internet

- **Contextual Display Ads** (Edelman et al. 2007, Varian & Harris 2014)

  Organization: Major search engines and social media platforms (2000-present).
  Auction Type: VCG-SB(F1,F2,F3)

  Description: Ads auctioned to appear next to content when viewed in web browsers and on mobile devices. A typical auction is for multiple items, each item representing a different position on the page. The VCG payment rule has been adopted by Facebook (2010) and Google (2012) for contextual display ads. Auction designs vary across platforms, but can include:

(F1) Bids for multiple positions on a page to allow a more prominent ad.

(F2) Bids state a per-click willingness-to-pay, used to impute an XOR bid on each of various positions on the page via estimated click-through rates.

(F3) Quality thresholds prevent low quality or inappropriate ads from winning the auction.

Appendix B: Technical Appendix

B.1 Core Constraints

We illustrate core constraints to the utilities of bidders and the seller in a forward CA. An allocation that is consistent with core constraints is a *core allocation*. Payments that are consistent with core constraints are *core payments*. Collectively, this allocation and payments are referred to as a *core outcome*.

Formally, let $N = \{1, \dots, n\}$ denote the bidders, with 0 to denote the seller. Let π_i denote the *utility* for bidder i. This is the value for the allocated items (if any) minus the bidder's payment. Let π_0 denote the utility to the seller, which is the sum total of the payments. Let G denote the set of indivisible items. Let value $v_i(S) \in R_+$ denote the value to bidder i for package $S \subseteq G$. The core constraints are stated on utilities, and are:

$$\sum_{i \in L} \pi_i \geq W(L), \qquad \forall L \subseteq N \cup \{0\}, \tag{1}$$

$$\sum_{i \in N \cup \{0\}} \pi_i = W(N \cup \{0\}) \tag{2}$$

$$\pi_i \geq 0, \qquad \forall i \in N \cup \{0\} \tag{3}$$

where $W(L)$ is the value of the allocation to the bidders in $L \setminus \{0\}$ that maximizes the total value when $0 \in L$, and $W(L) = 0$ otherwise. Since payments cancel, (2) implies that the core allocation must be the value-maximizing (efficient) allocation.

For any $i \in N \cup \{0\}$, the core constraint on $L = \{i\}$ implies $\pi_i \geq 0$, so that a bidder's payment is no greater than its value for a package. Because $\pi_i \geq 0$, the binding core constraints are those on *critical coalitions*, which are any $L \subseteq N \cup \{0\}$, and with $0 \in L$, for which $W(L) > W(L \setminus \{k\})$ for every $k \in L$. We have $\pi_k = 0$ for any unallocated bidder k, and zero core payment for any such bidder. If $\pi_k > 0$, then $\sum_{i \in (N \setminus \{k\}) \cup \{0\}} \pi_i(S_i) < \sum_{i \in N \cup \{0\}} \pi_i = W(N \cup \{0\}) = W(N \setminus \{k\} \cup \{0\})$, where the final equality is because k is not allocated.

In the context of defining the rules that define an auction, the core constraints are imposed with respect to the bids rather than the value functions. In particular, the *bid functions* associated with bids take the place of value functions, with the core definition otherwise unchanged.

Consider the following example. Suppose there are goods $G = \{A, B\}$, and three bidders with bids,

	A	B	$\{A, B\}$
bidder 1	0	0	10
bidder 2	8	0	8
bidder 3	0	6	6

The core allocation S^* is $(\emptyset, A, B)$, which is value-maximizing given these bids. The core payments are $p_1 = 0$, since this bidder is unallocated. To determine the constraints on the payments for bidders 2 and 3, we have critical coalitions $\{0, 1\}$, $\{0, 2\}$, $\{0, 3\}$ and $\{0, 2, 3\}$, and core constraints:

$$\pi_0 + \pi_1 \geq 10 \tag{4}$$

$$\pi_0 + \pi_2 \geq 8 \tag{5}$$

$$\pi_0 + \pi_3 \geq 6 \tag{6}$$

$$\pi_0 + \pi_2 + \pi_3 \geq 14 \tag{7}$$

Since $\pi_1 = 0$ (the bidder is unallocated), then $\pi_0 \geq 10$ and (4) implies (5) and (6). We already have $\pi_0 + \pi_2 + \pi_3 = 14$ by (2) (and allocation S^*), and thus (7). Putting this together, we need $\pi_0 \geq 10$, $\pi_0 \leq 14$ (since $\pi_0 + \pi_2 + \pi_3 = 14$, and by non-negativity), and $\pi_2, \pi_3 \geq 0$. Translating into core payments, these are any payments with $p_1 = 0$, $p_2 + p_3 \geq 10$, $p_2 + p_3 \leq 14$, $p_2 \leq 8$, and $p_3 \leq 6$. For example, $p_2 = 5, p_3 = 5$, or $p_2 = 8, p_3 = 2$, or $p_2 = 8, p_3 = 6$ are all core payments.

B.2 Vickrey-Clarke-Groves Mechanism

The Vickrey-Clarke-Groves (VCG) mechanism provides a format for auctioning multiple, non-identical, items. In a forward auction, bidders simultaneously submit sealed bids that define their willingness-to-pay for different possible allocations. The auctioneer then determines an efficient assignment based on the bids.

The payment rule specifies that the payment by a bidder is the total negative externality imposed on the other bidders by the bidder's presence in the auction. For this, let $W(K \cup \{0\})$ denote the total value (as reported in bids) for the allocation that is value maximizing (again based on bids) to some set of K bidders, with $K \subseteq N$. The VCG payment rule, for each bidder $i \in N$, is

$$p_{\mathrm{vcg},i} = W(N \cup \{0\}) - W((N \setminus \{i\}) \cup \{0\}). \tag{8}$$

In the example from Section 5, the payments would be

$$p_{\mathrm{vcg},1} = 14 - 14 = 0, \tag{9}$$

$$p_{\mathrm{vcg},2} = 10 - 6 = 4, \quad \text{and} \tag{10}$$

$$p_{\mathrm{vcg},3} = 10 - 8 = 2. \tag{11}$$

Because $p_{\mathrm{vcg},2} + p_{\mathrm{vcg},3} = 4 + 2 = 6 < 10$, this is not a core outcome. Although the VCG outcome is outside the core in this example, this need not be the case. As an illustration, if the example is modified to remove bidder 3, then the allocation is $(AB, 0)$ and the payment by bidder 1 is $p_{\mathrm{vcg},1} = 8 - 0 = 8$, and can be verified to be in the core.

B.3 Vickrey-nearest Core-selecting Payment Rule

The Vickrey-nearest core-selecting payment rule is used to define payments together with an allocation rule that allocates items, as in the VCG mechanism, to maximize the total bid value. The payment rule selects payments that are in the core, and minimize a distance metric to VCG payments (both the core and VCG payments defined with respect to bids).

To illustrate this, we assume truthful bids and consider the core payments from Section 5. There, the set of core payments are,

$$\mathit{Core} = \{(p_1, p_2, p_3) : p_1 = 0, p_2 + p_3 \in [10, 14], p_2 \in [0, 8], p_3 \in [0, 6]\}. \tag{12}$$

In this instance, the payments $\vec{p} \in \mathit{Core}$ that minimize the total squared distance to $p_{\mathrm{vcg},2} = 4, p_{\mathrm{vcg},3} = 2$ are $p_1 = 0, p_2 = 6, p_3 = 4$ and these are the Vickrey-nearest core-selecting payments. Note that the Vickrey-nearest core-selecting payments are just the VCG payments when the VCG payments are already in the core.