Granular DeGroot Dynamics – a Model for Robust Naive Learning in Social Networks

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We study a model of opinion exchange in social networks where a state of the world is realized and every agent receives a zero-mean noisy signal of the realized state. It is known from Golub and Jackson [6] that under DeGroot [3] dynamics agents reach a consensus that is close to the state of the world when the network is large. The DeGroot dynamics, however, is highly non-robust and the presence of a single "stubborn agent" that does not adhere to the updating rule can sway the public consensus to any other value. We introduce a variant of DeGroot dynamics that we call $\frac{1}{m}$*-DeGroot*. $\frac{1}{m}$-DeGroot dynamics approximates standard DeGroot dynamics to the nearest rational number with m as its denominator and like the DeGroot dynamics it is Markovian and stationary. We show that in contrast to standard DeGroot dynamics, $\frac{1}{m}$-DeGroot dynamics is highly robust both to the presence of stubborn agents and to certain types of misspecifications.

CCS Concepts: • **Theory of computation** → **Algorithmic game theory and mechanism design**.

Additional Key Words and Phrases: DeGroot dynamics, robust naive learning , social networks

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Social networks play a very important role as a conduit of information. Individuals repeatedly interact with their peers and adjust the choices they make by responding to their observed behavior. One explanation for such adaptive behavior is informational effects; i.e., individuals infer the underlying information that gives rise to the observed behavior and based on this *social learning* change their own behavior (e.g., [2], [5], [1]). Making fully rational inferences about the private information of *all* agents given the observed behavior, however, is known to be conceptually and computationally complex.[1] In practice, agents are often unaware of the structure of the network. This unawareness prevents the agents from tracing the information path in an environment of repeated interaction.

In order to overcome the complexity of rational decision making, an alternative approach has been developed. The most prominent such approach is the one proposed by DeGroot [3] and brought into the economics literature by DeMarzo et al. [4]. The DeGroot framework comprises a social network that is represented by a (finite) graph. The vertices of the graph represent agents who are in

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[1]See, e.g., [7].

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direct relations with their neighbors in the graph. The true state of the world is represented by a real number μ. At each time period $t \geq 0$ each agent i obtains a subjective opinion $A_{i,t} \in \mathbb{R}$ regarding the state of the world. A common assumption is that the initial opinions satisfy $A_{i,0} = \mu + \varepsilon_i$, where $\{\varepsilon_i\}_{i \in V}$ are i.i.d. with expectation zero. As time progresses, the agents simultaneously revise their opinions while relying only on the opinions of their neighbors. An important property of the DeGroot updating procedure, as established by DeGroot [3] is that in the long run agents reach consensus.

When the graph is large, one can infer μ from the initial opinions $\{A_{i,0}\}_{i \in V}$, but no single agent can make such an inference by herself, since she observes her own opinion and the opinions of her neighbors only. However, Golub and Jackson [6] demonstrated the following striking property of the DeGroot dynamics, which they termed *naive learning*. Despite the simplicity of the DeGroot rule, when the graph is large and no agent is too central, the agents' opinions converge to a consensual opinion that is close to the true state of the world with high probability.

A weakness of the standard DeGroot dynamics as a tool to aggregate private information is its sensitivity to minor deviations. The source of such deviations may be either malicious agents, who have an incentive to change public opinion, or agents who suffer from behavioral biases. For example, suppose one of the agents does not follow the updating rule, while all the others do. Specifically, say that one of the agents is "stubborn"; that is, she starts with a certain opinion and never changes it. Applying DeGroot dynamics in the presence of such a stubborn agent results in everyone adopting the opinion of the stubborn agent in any finite connected graph. Another possible modification of the model is distorted monitoring: the agents do not observe the exact opinion of each of their neighbors, but rather a slight modification of it. Say, for example, that everyone sees $x + \beta$ when the true opinion is x. Then, even if β is very small (yet positive), the resulting limiting opinions of all of the agents will be ∞ regardless of the state of the world.

The core research question we address in this paper is the following: does there exist a simple boundedly rational heuristics that enable naive learning that is immune to the above-mentioned deviations? We provide a positive answer to this question in a setting with either a large or a countably infinite graph of bounded degree.

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