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Implications of Brazilian Institutional Guidelines on Educational Efficiency

ABSTRACT This paper investigates the relation between inefficiency in the Brazilian education system and municipal wealth, discussing how the current legislation possibly influences it. To that end, we apply a stochastic frontier model that accommodates covariates in the asymmetric error component to analyze the impact of per capita GDP on inefficiency. This methodology is applied to a data set on the Rio Grande do Sul municipalities for the years 2007 and 2017. The results indicate a positive effect, suggesting that richer municipalities are less efficient in allocating their resources.

JEL Codes: I20, H75, D61, C11

Keywords: Inefficiency; FUNDEB; stochastic frontier analysis; Bayesian inference

Basic education is provided, albeit partially, by governments around the world. This investment is funded by taxpayers and is associated with the economy's productive capacity to generate wealth, that is, per capita gross domestic product (GDP). Because of its relevance and the limited resources available for providing public services—including not only education but also health care, law enforcement, social security, and so forth—it is important to ensure an efficient allocation of this capital.

The education economics literature provides empirical evidence that an increase in financial funding for education does not necessarily imply a better performance on standardized assessments of educational attainment (Glewwe and others, 2011; Hanushek and Luque, 2003; Monteiro, 2015). However, *Education at a Glance 2017* (OECD, 2017) presents an association between developed countries in the Organization for Economic Cooperation and Development (OECD) and better results on the Program for International Student Assessment (PISA) test. This suggests that school achievement is related not only to the total amount available but also to an efficient allocation of this resource. Afonso and St. Aubyn (2006) explore these ideas using a two-stage approach in which they first obtain the educational efficiency of a sample of countries and then estimate the relationship between efficiency and per capita GDP. Based on PISA data for twenty-five OECD countries, the authors conclude that efficiency is strongly correlated with GDP; in other words, the richer the country, the more efficient it is in providing a better education.

Does this relationship hold for municipalities within a country? Oliveira and Santos (2005) evaluated Portuguese school efficiency and analyzed the influence of municipal GDP per capita, but the relation was not significant. In the Brazilian case, this question is particularly interesting because municipalities face severe fiscal restrictions and major challenges in this area. In addition, Article 212 of the Constitution of the Federative Republic of Brazil establishes that municipalities must allocate at least 25 percent of their budget revenue to public education.

The main hypothesis of this paper is that wealthy municipalities are less efficient in allocating their resources because of a legal obligation to invest an amount that is possibly higher than necessary. To test our hypothesis, we investigate the relation between municipal GDP per capita and inefficiency. We propose an extension of the spatial stochastic frontier model introduced by Schmidt and others (2009) to accommodate covariates in the asymmetric error component, and then apply the methodology to a data set on municipalities in the state of Rio Grande do Sul for the years 2007 and 2017.

The state is located in the southern region of Brazil and has a historically high education level, relative to other regions of the country, as a consequence of its European heritage. Based on its economic capacity, geographic size, and population, it is one of the most representative Brazilian states. Over the last ten to fifteen years, it has recorded a decline in educational achievement: in 2007, the state's public education system ranked fifth in the nation, with a score of 3.7 on the Basic Education Development Index (IDEB); by 2017, it had fallen to eleventh place, with an IDEB score of 4.4. Although the state's score had improved 19 percent in the period, the national average rose 26 percent, resulting in the drop in ranking. Despite having the fifth-highest state-level GDP per capita in 2017, Rio Grande do Sul has been facing a fiscal crisis in recent years caused by its debt with the federal government and repeated deficits.

The remainder of the paper is organized as follows. The next section provides a brief literature review of the main methods to measure efficiency in education economics, highlighting some interesting outcomes. It also outlines the Brazilian legislation and its connection with the concept of adequacy in school finance. We then introduce the methodology and describe the inference process, followed by a presentation of the data and results. The final section lays out our main conclusions.

Background

"What matters more are the choices countries make in how to allocate that spending and the policies they design to improve the efficiency and relevance of the education they provide" (OECD, 2013). With this statement, Ángel Gurría, OECD secretary-general, underlines the importance of efficient public spending and the rational allocation of these resources. Since education is relevant for promoting a number of outcomes, such as cognitive and non-cognitive skill development and economic growth (Cunha, Heckman, and Schennach, 2010; Hanushek and Kimko, 2000), this is a topic of intense debate among policymakers, teachers, and other stakeholders.

Analyzing the efficiency of educational provision involves defining a technology function for knowledge production, which represents the maximum output that can be achieved with a given provision. A system is considered efficient if its producers make an effective use of the available inputs. In an inefficient system, it is possible to increase attainment for a given spending level or decrease expenditure for a given attainment level (de Witte and López-Torres, 2017).

However, defining and estimating a production function is not a trivial task, since it is necessary to specify the relevant inputs. Glewwe and others (2011) review the literature on school resources and educational outcomes in developing countries and conclude that most school and teacher characteristics are not statistically significant for explaining the learning process. In addition, the results are influenced by several factors that are beyond the control of the evaluated observation. Coleman and others (1966) observe that investment explains only 10 percent of academic achievement, while the remainder depends on other economic variables and students' family environment, which are known as nondiscretionary variables.

Thus, various specifications and methods have been applied to study the importance of structural, institutional, and socioeconomic variables for educational achievement and efficiency scores. Nonetheless, it is possible to identify two main modeling techniques that are implemented in the literature: data envelopment analysis (DEA) (Charnes, Cooper, and Rhodes, 1978) and stochastic frontier analysis (SFA) (Aigner, Lovell, and Schmidt, 1977; Meeusen and van den Broeck, 1977). Both techniques are commonly employed as a first step in a two-stage empirical strategy in which the second stage is based on a regression-type model of the efficiency scores and explanatory variables.

Bradley, Johnes, and Millington (2001) and Worthington (2001) provide a list of studies conducted in several countries, illustrating different applications of the DEA methodology. Agasisti (2013), for example, measures the performance of Italian secondary schools, investigates which factors affect efficiency through a Tobit regression, and concludes that there is a potential role for better results by increasing competition. There is also a broad literature on the SFA methodology (Izadi and others, 2002; Kuo and Ho, 2008; Lenton, 2008). For instance, Lewis, Pattinasarany, and Sahn (2011) analyze the public elementary schools in Indonesia; their results suggest that outcomes could be enhanced even with a reduction in total spending.

There are also some alternative methods. Deutsch, Dumas, and Silber (2013) apply the corrected least squares method (Richmond, 1974) to estimate the educational efficiency of five Latin American countries; they report that individual efficiency is likely to be influenced by increases in public debt caused by the expansion of educational access. Thieme, Giménez, and Prior (2012), in turn, use directional distance functions (DDF) to evaluate Chilean urban schools and find that the most important source of inefficiency is the resource endowment effect. The authors also argue that when specific variables concerning the amount allocated are disregarded, the performance is undervalued.

Regarding the Brazilian case, Carvalho and Sousa (2014) and Gonçalves and França (2013) apply the DEA methodology to a data set of Brazilian municipalities and northeastern and southeastern public schools, respectively. The first paper indicates that, even when environmental factors are discounted, improvements can be made. The second establishes a positive relation between efficiency gains and decentralized management. Adopting an approach based on quantile estimators, Oliveira, Souza, and Annegues (2018) suggest that management autonomy is not a determining factor for efficiency in Brazilian public schools. Ferraz, Finan, and Moreira (2012), on the other hand, look at the resource allocation problem and student outcomes from the perspective of corruption, using variation in corruption across municipalities to explore whether missing resources due to corruption affect performance. They find a significant negative impact on primary school achievement.

Nevertheless, there is no consensus in the literature, and conclusions vary according to the method, period, and country analyzed. Kirjavainen (2012) fits different stochastic frontier models for panel data to estimate a production function and the efficiency of Finnish general upper secondary schools. The estimates indicate that inefficiency and score-based rankings diverge considerably depending on the type of model applied.

Adequacy in School Finance and Brazilian Legislation

Adequacy in school finance is a term used in education economics to define the amount of funding required to produce a desired level of student performance. According to Odden (2003), determining sufficient revenue levels involves the following steps: identifying the costs of effective programs and strategies; converting these investments into appropriate school finance structures; and certifying that the resources are used in schools to produce the desired results. These levels vary in accordance with the socioeconomic characteristics of municipalities (Ruggiero, 2007). For example, locations in which pupils face precarious conditions should invest more.

This concept is applied to the design of public policies in order to guarantee a minimum expenditure on education (Hanushek, 1994). The post-1990 school finance reforms in the United States, for example, were strongly grounded in adequacy concepts. In this regard, a topic of concern is the impact on absolute and relative spending and achievement in low-income school districts. Lafortune, Rothstein, and Schanzenbach (2018) demonstrate that the U.S. reforms led to immediate and sustained increases in spending in these districts and that they had a large positive impact on student achievement. Lee (2012) assesses the achievement gap in mathematics proficiency standards from the perspective of adequacy and equity and finds that the required school funding varies by poverty status.

In Brazil, the re-democratization process of the 1980s promoted several reforms in the legislation and financing of public education. Article 212 of the 1988 Constitution of the Federative Republic of Brazil establishes that states and municipalities must allocate at least 25 percent of their budget revenue to the maintenance and development of the basic public education system. The education system is regulated under Law 9,394 of 1996, on the Guidelines and Bases of National Education (LDB). For example, Articles 70 and 71 detail how municipalities must and must not invest their resources. Law 9,424 of 1996 created an education fund that aimed to guarantee a minimum investment per pupil and promote the distribution of resources across municipalities within the same state. This fund was initially focused on elementary and middle schools (FUNDEF—Fund for Elementary and Middle School Education and for Enhancing the Value of the Teaching Profession). However, in 2007, FUNDEF was replaced by FUNDEB (Fund for Basic Education and for Enhancing the Value of the Teaching Profession) with the promulgation of Law 11,494, which extended the fund's coverage to include kindergarten, high school, and basic education for adults who did not complete their schooling at the usual age. FUNDEB consists in a state account in which the municipalities deposit 20 percent of the revenue collected from eight specific taxes. To comply with the 25 percent allocation mandated by the federal constitution, an additional 5 percent of the amount collected from the same eight taxes must be earmarked for an account dedicated to education. Moreover, at least 25 percent of the revenue collected from the remaining taxes must be set aside in the same account. Finally, Articles 70 and 71 of the LDB limit how resources from this account are spent, and Articles 21, 22, and 23 of Law 11,494 define how FUNDEB resources are invested.

The state's FUNDEB account is redistributed across municipalities according to the number of students enrolled in public schools, without taking into account other municipal sources of school funding. Consequently, both wealthy and poor municipalities receive a similar amount per pupil. Based on a socioeconomic indicator ranking, Bertoni and others (2018) show that FUNDEB represents almost the same relative share of per student spending in all municipalities. According to the authors, this implies a neutral transfer rule that does not equalize municipal resources, given that the more developed municipalities raise more own funds than the lower-income ones.

Finally, Monteiro (2015) evaluates the enhancement of education spending in oil-producing municipalities that benefited from higher royalty revenues in 2000–10. The author concludes that a 15 percent increase in revenues and therefore in education funding did not translate into better results relative to results from other municipalities on the Brazilian coast. This suports the argument that municipalities with a higher per capita GDP have less incentive for efficient management under the current legislation.

Methodology

Suppose that observations are available in the form of balanced panel data for N municipalities across T periods. Let y_{it} be the logarithm of the output of municipality i in period t. The stochastic frontier model is defined by the following equation:

$$y_{it} = g\left(\mathbf{r}_{it}, \boldsymbol{\theta}\right) - u_{it} + \epsilon_{it}, \tag{1}$$

where $g(\mathbf{r}_{it}, \boldsymbol{\theta})$ is the production function, $\mathbf{r}_{it}$ is a vector of inputs, and $\boldsymbol{\theta}$ is a vector of parameters that describe the effect of each input on the output, y_{it}. The component u_{it} follows an asymmetric positive distribution and models the inefficiency of unit i in period t. The random error, ϵ_{it}, is assumed independent of u_{it} and follows a Gaussian distribution centered at zero with variance σ^2, that is, $\epsilon_{it} \sim N(0, \sigma^2)$.

With regard to the distribution of the inefficiency component, the literature offers a number of proposals: the exponential (Meeusen and van den Broeck, 1977), the half-normal (Aigner, Lovell, and Schmidt, 1977), the truncated normal (Stevenson, 1980), and the gamma (Greene, 1990). Here we adopt the truncated normal distribution, and its mean is a function of municipality effects and covariates. More specifically, we have

(2) $$u_{it} \big| \alpha_i, \mathbf{z}_{it}, \boldsymbol{\eta}, \tau^2 \sim N^+\left(\mu_{it}, \tau^2\right) \text{ and}$$

(3) $$\mu_{it} = \alpha_i + \mathbf{z}_{it}\boldsymbol{\eta},$$

where $N^+(a, b)$ denotes the normal distribution truncated at zero, whose associated normal has mean a and variance b.

The above specification is similar to the one introduced by Schmidt and others (2009). The difference consists in the possibility of modeling the inefficiency not only as a function of $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$, but also as a function of covariates. In accordance with Schmidt and others (2009), α_i is allowed to represent a process that spreads through spatial contagion, such as social and economic conditions. This process is frequently represented by priors that vary smoothly across space, and, in several applications, it is assumed that α follows a conditional autoregressive distribution that depends on its neighbors. Therefore, this specification enables the spatial structure to be naturally imposed in the model (Besag, York, and Mollié, 1991).

Our prior belief about this structure is motivated by empirical evidence presented in Power and Rodrigues-Silveira (2019). The authors calculate a measure of vote-revealed ideology called the municipal ideological score (MIS) over the course of thirteen electoral cycles between 1994 and 2018. The results suggest that nearby municipalities share similar ideologies. Therefore, we are making a prior assumption that these similarities have an impact on the education policy and governance mechanisms adopted by the elected politicians. Moreover, we can alternatively interpret the latent effects, α_i, as an attempt to capture the unobservable particularities of each municipality.

Inference Procedure

Let $\mathbf{y} = (y_{11}, \ldots, y_{1T}, \ldots, y_{N1}, \ldots, y_{NT})'$ be a random sample of the logarithm of the outputs and $\mathbf{u} = (u_{11}, \ldots, u_{1T}, \ldots, u_{N1}, \ldots, u_{NT})'$ be the vector of unobserved inefficiencies. Assuming the model presented in equations 1–3, the likelihood function is given by

$$f\left(\mathbf{y}, \mathbf{u} \middle| \mathbf{r}, \sigma^2, \boldsymbol{\alpha}, \mathbf{z}, \boldsymbol{\eta}, \tau^2\right) \propto \left(\sigma^2\right)^{-\frac{NT}{2}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{it} - \mathbf{r}_{it}\boldsymbol{\theta} + u_{it}\right)^2\right\}$$
$$\times \left(\tau^2\right)^{-\frac{NT}{2}} \exp\left\{-\frac{1}{2\tau^2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(u_{it} - \alpha_i - \mathbf{z}_{it}\boldsymbol{\eta}\right)^2\right\} 1_{\{u_{it}>0\}}.$$

When performing a Bayesian analysis, an important step is the prior distribution selection, in this case the prior distribution of the latent effects, $\boldsymbol{\alpha}$. Since the data consist of observations made across municipalities, a certain spatial correlation is expected from these effects. Given this geographic component, it is intuitive to think that inefficiencies in neighboring municipalities share some common characteristics. For these reasons, as in Schmidt and others (2009) and following Besag, York, and Mollié (1991), we assume a conditional autoregressive (CAR) prior for $\boldsymbol{\alpha}$.

The CAR prior distribution is described as

$$p\left(\boldsymbol{\alpha} \middle| \psi^2\right) \propto \exp\left\{-\frac{1}{2\psi^2}\sum_{i=1}^{N}\sum_{j<i} W_{ij}\left(\alpha_i - \alpha_j\right)^2\right\}, \tag{4}$$

and is denoted by $\boldsymbol{\alpha} \sim \text{CAR}(\psi^2)$. The matrix $\mathbf{W}$ is an adjacency matrix, and since the spatial phenomenon observed in Power and Rodrigues-Silveira (2019) is not coincident with any regional division for the state of Rio Grande do Sul, we assume a standard specification in which $W_{ij} = 1$ if municipality i shares a border with municipality j and $W_{ij} = 0$ otherwise. Additionally, we also assume two other specifications in which $W_{ij} = 1$ if municipality i belongs to the same immediate/intermediate region as municipality j and $W_{ij} = 0$ otherwise. Both immediate and intermediate regional divisions are defined by the Brazilian Geographical and Statistical Institute (IBGE) following a criterion based on urban networks. The distribution in equation 4 is an improper joint distribution for α in the sense that it is possible to add a constant to all α_i without affecting it (Banerjee, Carlin, and Gelfand, 2004). To guarantee that the posterior is proper, each sample from α obtained through Markov chain

Monte Carlo (MCMC) methods (Gamerman and Lopes, 2006) is centered, following Besag and Kooperberg (1995) and Gelfand and Sahu (1999).

Thus far, we have discussed the prior distribution of the latent effects, α. However, from a Bayesian perspective, the model specification is complete only after a prior distribution is assigned to all unknowns in the model. Thus we now turn to the prior distribution of the other parameters. Let $\boldsymbol{\vartheta}$ be the parametric vector $\boldsymbol{\vartheta} = (\boldsymbol{\theta}, \sigma^2, \boldsymbol{\alpha}, \boldsymbol{\eta}, \tau^2, \psi^2)$, and assume that all of its components are independent a priori. Hence the joint prior distribution for $\boldsymbol{\vartheta}$ is given by

$$(5) \qquad p(\boldsymbol{\vartheta}) = \prod_{l=1}^{p}\left[p(\theta_l)\right]p(\sigma^2)p\left(\boldsymbol{\alpha}\middle|\psi^2\right)p(\psi^2)\prod_{k=1}^{q}\left[p(\eta_k)\right]p(\tau^2).$$

In this paper, we follow a conjugate prior analysis. Therefore, considering the coefficients θ_l, $l = 1, \ldots, p$, and η_k, $k = 1, \ldots, q$, we specified a normal prior distribution, $N(\mu_0, \sigma_0^2)$, in which the hyperparameters $\mu_0 = 0$ and $\sigma_0^2 = 100$. For the scale parameters σ^2 and τ^2, we chose an inverse gamma prior distribution, $\mathrm{IG}(\phi, \phi)$, with $\phi = 0.01$. Special care must be taken when assigning the prior distribution for ψ^2, as it is an unidentifiable parameter in the sense of Dawid (1979), so it is not recommended to be too uninformative (Besag and Kooperberg, 1995). We therefore adopt the same strategy as Schmidt and others (2009) and use an inverse gamma prior distribution, $\mathrm{IG}(\phi_0, \phi_0)$, in which the mean is equal to the ordinary least squares (OLS) variance estimate based on an independent stochastic frontier model and the variance is fixed. From the conjugate prior analysis, we obtain full conditional posterior distributions in closed form. Therefore, we use an MCMC algorithm based on the Gibbs sampler (Gelfand and Smith, 1990).[1]

Empirical Analysis

We used three different databases: the school census from the Anísio Teixeira National Institute for Educational Studies and Research (INEP), the SIDRA database from the IBGE, and the National Treasury Secretariat database. From the first, we obtained the municipalities' Basic Education Development Index (IDEB) for students in the ninth grade (the last year of lower secondary

1. See appendix A for a step-by-step description.

TABLE 1. Summary Statistics

Variable	*No. observations*	*Mean*	*Std. deviation*	*Minimum*	*Maximum*
IDEB	935	4.35	0.70	2.50	6.70
Teacher-student ratio	992	0.29	0.07	0.13	0.68
FUNDEB	991	1,819.13	911.01	71.82	4,700.70
School infrastructure index	992	6.88	0.96	3.34	10.00
Municipal GDP per capita	993	29,083.06	21,398.81	7,711.18	393,569.40

school), the infrastructure available in the schools, and the student-teacher ratio. From the second, we collected the municipalities' GDP. From the last, we accessed the amount of FUNDEB resources allocated to each municipality.

As mentioned, we focus on Rio Grande do Sul, so our data encompass the municipalities in this state. The analysis covers the years 2007 and 2017. We chose these dates because 2007 is the year FUNDEB was established, and 2017 represents a decade of program implementation. Furthermore, there are relatively few missing observations for these two years, yielding a final data set comprising 445 municipalities out of a total of 497. We considered an alternative analysis with a more extended panel (for example, a biannual panel from 2007 to 2017), but the sample was reduced to a small number of municipalities because of the frequency of missing observations.

In our analysis, the IDEB is specified as output. This index is a product of two variables that evaluate education quality: proficiency in mathematics and Portuguese, and the passing rate. Thus a municipality is considered efficient based not only on its proficiency score but also on its ability to graduate students from lower secondary school. This choice reflects the fact that an educational system in which students systematically fail is not desirable, yet high passing rates could be correlated with insufficient learning among certain students.

As model inputs, we have the following variables: the teacher-student ratio, the school infrastructure index, and FUNDEB resources. The teacher-student ratio represents the labor input in our production function. The infrastructure index is an indicator of the total resources available at the schools, including sports facilities, science and computer laboratories, libraries, internet access, projectors, and so forth. This variable, in turn, represents physical capital. The FUNDEB resources allocated to each municipality were normalized by the total number of students registered according to the school census, to represent public education spending. Table 1 summarizes the descriptive statistics.

Afonso and St. Aubyn (2006) state: "We have considered the option of using education spending per student as an input. However, results would be hardly interpretable, as they would reflect both inefficiency and cost provision differences. For example, countries where teachers are better paid would tend to show up as inefficient, irrespective of the intrinsic performance of the education system." Thus the choice of FUNDEB as an input rather than total education spending seems a good option, since the two variables are highly correlated and the former is not affected, for example, by differences in teacher remuneration.

Results

We consider three different specifications of the adjacency matrix, **W**: the first is based on municipalities that share a border (M1); the second, on municipalities that belong to the same immediate region (M2); and the third, on municipalities that belong to the same intermediate region (M3). We adopt the Watanabe-Akaike information criterion (WAIC) (Watanabe, 2010) for model selection, which is defined as a function of the posterior predictive density and a correction for the effective number of parameters to adjust for overfitting. Smaller WAIC values indicate better fit. Details are provided in appendix B.

Table 2 summarizes the results obtained from the inference process for M1, M2, and M3, presenting the posterior mean and the 95 percent highest posterior density credible interval for the parameters. These results were obtained after running 100,000 iterations of the MCMC scheme in appendix A, discarding the first 20,000 as a burn-in period, and storing only every eightieth value in order to reduce the autocorrelation between successive values of the simulated chain. Consequently, we have a final sample of 1,000 draws. The WAIC suggests that M1 has the best fit, so we focus our discussion and analysis of the results on this specification.

Figure 1 presents a map with the latent spatial effects, α_i. The figure shows that most of the latent effects are significant, giving support to their presence in the model. Figure 2 illustrates the inefficiency (u_{it}) of municipalities in the state of Rio Grande do Sul in 2007 and 2017. As the figure shows, there was almost no variation in efficiency between the two years. In other words, municipalities with higher levels of inefficiency have not significantly improved their investment policy over the years, and a feasible explanation for that is the lack of incentives for better practices under the current legislation.

TABLE 2. Mean and Highest Posterior Density (HPD) Interval

Parameter	*M1*	*M2*	*M3*
Intercept (θ_0)	10.0000	1.5903	1.7791
	(1.4940, 1.9091)	(1.3896, 1.7719)	(1.5897, 1.9870)
Dummy variable for 2017 (θ_1)	0.1455	0.1416	0.1544
	(0.1214, 0.1695)	(0.0755, 0.1579)	(0.1297, 0.1796)
Teacher-student ratio (θ_2)	0.1087	0.1160	0.1448
	(0.0613, 0.1429)	(0.0755, 0.1579)	(0.1003, 0.1852)
FUNDEB (θ_3)	−0.0043	−0.0041	−0.0253
	(−0.0272, 0.0196)	(−0.0262, 0.0182)	(−0.0482, −0.0011)
School infrastructure index (θ_4)	0.0790	0.1320	0.1229
	(0.0229, 0.1371)	(0.0682, 0.1813)	(0.0723, 0.1925)
σ_2	0.0061	0.0087	0.0067
	(0.0034, 0.0079)	(0.0058, 0.0111)	(0.0039, 0.0095)
Municipal GDP per capita (η)	0.0298	0.0247	0.0292
	(0.0253, 0.0388)	(0.0200, 0.0315)	(0.0218, 0.0360)
τ_2	0.0029	0.0026	0.0029
	(0.0014, 0.0055)	(0.0011, 0.0053)	(0.0013, 0.0058)
WAIC	**−1,837.1550**	−1,513.6240	−1,692.243

Notes: Based on a sample of 1,000 draws from the posterior distribution for all three models and their respective WAIC. M1, municipalities that share a border; M2, municipalities that belong to the same immediate region; M3, municipalities that belong to the same intermediate region.

Of the three inputs, only FUNDEB is not significant, since its credible interval contains zero (table 2). There is empirical evidence in the literature supporting the hypothesis that an increase in financial funding for education does not necessarily imply a better performance in standardized assessments of educational attainment (Glewwe and others, 2011; Hanushek and Luque, 2003). With regard to the Brazilian case, Monteiro (2015) evaluates the impact of higher spending in the oil-producing municipalities from 2000 to 2010, concluding that an increase in education funding was not converted into better results in comparison with other municipalities on the Brazilian coast.

The school infrastructure index has a positive and significant coefficient. In the literature, the effects of school resources on student performance are not well understood (Glewwe and others, 2011). Card and Krueger (1996) observe that while most of the literature on test scores points to little, if any, effect of school resources, some observational studies and experiments do find a connection. Figlio (1999) argues that these differences may be attributable, in part, to the functional form assumptions of the school production function used in the existing literature.

Our third input is the teacher-student ratio, which is broadly applied in similar contexts to ours (Afonso and St. Aubyn, 2006; Agasisti, 2013; Kirjavainen,

FIGURE 1. Significance of Spatial Effects: M1 Specification

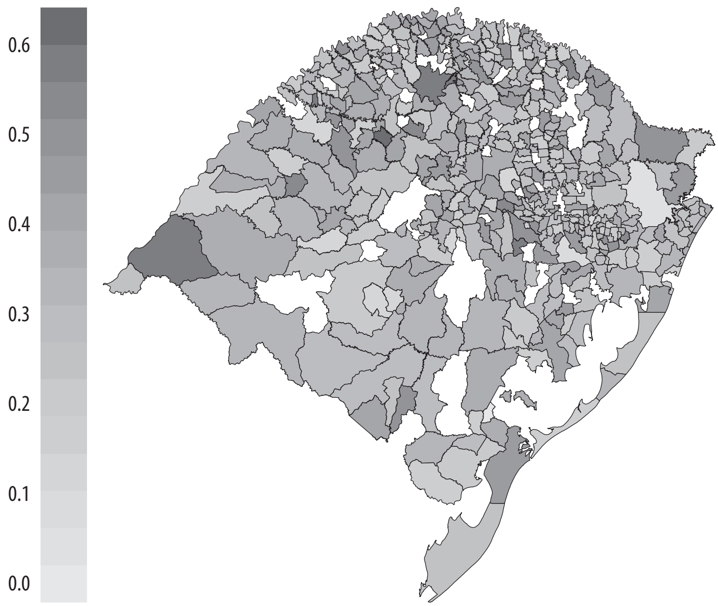
Notes: Black indicates positive significance (lower credible limit of 2 SD above zero); white, negative significance (upper credible limit of 2 SD below zero). The intermediate categories are shaded in gray, from darker to lighter: lower credible limit of 1 SD above zero; credible interval, including zero; and upper credible limit of 1 SD below zero.

2012). Our estimates suggest that this ratio has a positive and significant effect, meaning that the supply of teachers contributes to a better educational system. De Witte and López-Torres (2017) interpret similar results as evidence that a larger supply of teachers enables more individualized work with students.

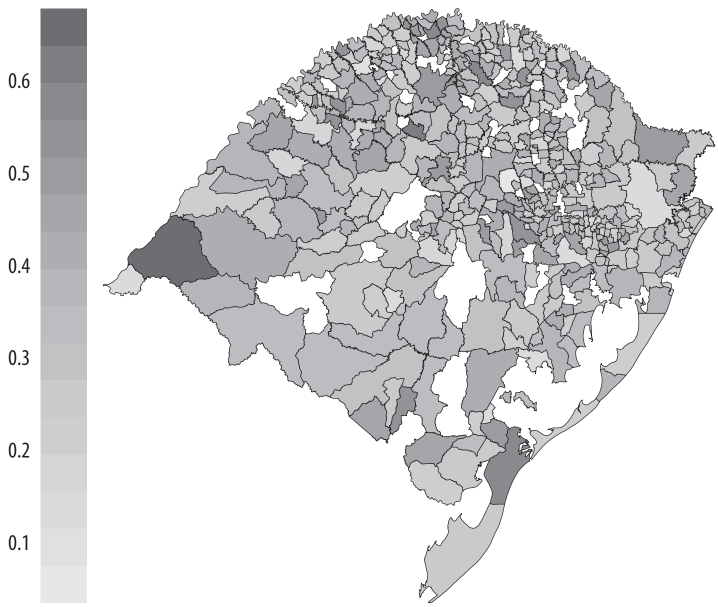
The main hypothesis of this paper is that wealthy municipalities are less efficient in allocating their resources because of the legislation and mechanisms described earlier. To test this hypothesis, we extended the model proposed by Schmidt and others (2009) to accommodate municipal GDP per

FIGURE 2. Inefficiency Point Estimates: M1 Specification

A. 2007



B. 2017



capita as a covariate that explains inefficiency. As shown in table 2, this variable has a positive relation with inefficiency and is statistically significant, thus supporting our hypothesis. It is also in line with Monteiro (2015), who concludes that Brazilian oil-producing municipalities that benefited from royalty revenues are less efficient than others with similar characteristics. While this result must be interpreted with care since it derives from states other than Rio Grande do Sul, it is an indication of a wider phenomenon. These points also demonstrate that the outcomes obtained by Afonso and St. Aubyn (2006) and Fonchamnyo and Sama (2016), who report that higher GDP per capita results in more efficiency at the country level, might not hold when we focus on municipalities, in particular in the presence of rigid legislation dictating the amount that must be invested in education.

Although caution should be taken in extrapolating our findings to the national level, the joint analysis of our results can contribute to the debate about the current design and effectiveness of FUNDEB. Since FUNDEB legislation expires in December 2020, policymakers and civil organizations are discussing the possibility of making it permanent, with modifications to the resource distribution strategy. As discussed earlier, the current distribution policy is mainly centered on the number of students enrolled in the public basic education system, without taking into account, for example, the special needs of poor municipalities. We also discussed results that point to the importance of allocating more resources to low-income districts and its positive impact on educational achievements and completed years of education (Jackson, Johnson, and Persico, 2016; Lafortune, Rothstein, and Schanzenbach, 2018). In that sense, our results support the design of a FUNDEB distribution mechanism based on proportional allocation, in which more resources should be directed to low-income municipalities. This alternative would contribute to improving school results and efficiency in resource allocation.

Conclusion

A common idea in the Brazilian public debate is that advances in educational quality are directly proportional to the amount of investment in the area. Although this argument might be appealing, the education economics literature presents some evidence in a different direction (Glewwe and others, 2011; Hanushek and Luque, 2003; Monteiro, 2015), exposing the need for well-designed public policies and rigorous evaluations of their effectiveness. Under the current economic scenario and the serious fiscal crisis in Brazil, a particular topic of interest rises from the discussions: namely, efficiency in education management and, especially, in education spending.

This paper contributes to the literature on the relation between inefficiency in the Brazilian education system and municipal wealth, discussing how the current legislation possibly influences it. We underscore the current legislation because it imposes rigid regulations that disregard the economic capacity of each municipality and does not introduce incentives for efficient policies, which is critical when local governments have limited budgets. To explore this issue, we applied a stochastic frontier model to a panel data set on municipalities in the state of Rio Grande do Sul in 2007 and 2017. The results indicate that municipal GDP per capita has a positive effect on inefficiency, suggesting that richer municipalities are less efficient in allocating their resources, which corroborates our main hypothesis. In addition, there were no significant improvements in efficiency over the period under analysis, indicating a lack of incentives. For future research, this model might be applied to a larger number of municipalities in other regions of the country, in order to facilitate generalizations of our results.

Appendix A. MCMC Algorithm

The MCMC algorithm is based on the following full conditional distributions:

- Sample from the conditional distribution $\boldsymbol{\theta}|\mathbf{y}, \mathbf{u}, \mathbf{r}, \sigma^2 \sim N_p(\mu_1, \Sigma_1)$, where

$$\Sigma_1 = \left[\Sigma_0^{-1} + \frac{1}{\sigma^2}(\mathbf{r}^T\mathbf{r})\right]^{-1} \text{ and}$$

$$\mu_1 = \Sigma_1\left\{\Sigma_0^{-1}\boldsymbol{\mu}_0 + \frac{1}{\sigma^2}\left[\mathbf{r}^T(\mathbf{y}+\mathbf{u})\right]\right\},$$

in which $\Sigma_0 = \sigma_0^2\mathbf{I}_p$ and $\boldsymbol{\mu}_0$ is a p-dimensional vector of μ_0.

- Sample from the conditional distribution $\sigma^2|\mathbf{y}, \mathbf{u}, \mathbf{r}, \sim \text{IG}(\phi_1, \phi_2)$, where

$$\phi_1 = \phi + \frac{NT}{2} \text{ and}$$

$$\phi_2 = \phi + \frac{1}{2}(\mathbf{y} - \mathbf{r}\boldsymbol{\theta} + \mathbf{u})^T(\mathbf{y} - \mathbf{r}\boldsymbol{\theta} + \mathbf{u}).$$

• For $i = 1, \ldots, N$ and $t = 1, \ldots, T$, sample from $U_{it}|\mathbf{y}, \mathbf{u}, \mathbf{r}, \boldsymbol{\theta}, \sigma^2, \boldsymbol{\alpha}, \mathbf{z}, \tau^2 \sim TN_{[0,+\infty)}(a_1, a_2)$, where

$$a_1 = \frac{\sigma^2(\alpha_i + \mathbf{z}_{it}\boldsymbol{\eta}) + \tau^2(y_{it} - \mathbf{r}_{it}\boldsymbol{\theta})}{\sigma^2 + \tau^2}, \text{ and}$$

$$a_2 = \frac{\sigma^2\tau^2}{\sigma^2 + \tau^2},$$

in which $TN_{[0,+\infty)}(\cdot,\cdot)$ is the truncated normal distribution over the interval $[0, +\infty)$.

• Sample from the conditional distribution $\boldsymbol{\eta}|\mathbf{u}, \boldsymbol{\alpha}, \mathbf{z}, \tau^2 \sim N_q(\mu^*, \Sigma^*)$, where

$$\sum{}^* = \left[\sum{}_0^{-1} + \frac{1}{\tau^2}(\mathbf{z}^T\mathbf{z})\right]^{-1} \text{ and}$$

$$\mu^* = \sum{}^*\left\{\sum{}_0^{-1}\boldsymbol{\mu}_0 + \frac{1}{\tau^2}\left[\mathbf{z}^T(\mathbf{u} - \boldsymbol{\alpha})\right]\right\}.$$

in which $\Sigma_0 = \sigma_0^2\mathbf{I}_q$ and $\boldsymbol{\mu}_0$ is a q-dimensional vector of μ_0.

• Sample from the conditional distribution $\tau^2|\mathbf{u}, \mathbf{z}, \boldsymbol{\eta} \sim \mathrm{IG}(\phi_1^*, \phi_2^*)$, where

$$\phi_1^* = \phi + \frac{NT}{2} \text{ and}$$

$$\phi_2^* = \phi + \frac{1}{2}(\mathbf{u} - \boldsymbol{\alpha} - \mathbf{z}\boldsymbol{\eta})^T(\mathbf{u} - \boldsymbol{\alpha} - \mathbf{z}\boldsymbol{\eta}).$$

• For $i = 1, \ldots, N$, sample from $\alpha_i|\mathbf{u}, \mathbf{z}, \boldsymbol{\eta}, \tau^2, \psi^2 \sim N(b_1, b_2)$, where

$$b_1 = \frac{\tau^2\sum_{j\in\mathcal{J}}\alpha_j + \psi^2\sum_{t=1}^{T}(u_{it} - \mathbf{z}_{it}\boldsymbol{\eta})}{\tau^2 + \psi^2} \text{ and}$$

$$b_2 = \frac{\tau^2\psi^2}{\tau^2 + \psi^2}$$

in which $\mathcal{J}$ is a set of indexes with the neighbors of i.

- Sample from the conditional distribution $\psi^2|\boldsymbol{\alpha} \sim \mathrm{IG}(\hat{\phi}, \underline{\phi})$, where

$$\hat{\phi} = \phi_0 + \frac{NT - c}{2} \text{ and}$$

$$\underline{\phi} = \phi_0 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j<i} W_{ij}\left(\alpha_i - \alpha_j\right)^2,$$

in which c is the number of blocks.

Appendix B. Watanabe-Akaike Information Criterion

Following Gelman, Hwang, and Vehtario (2014), the Watanabe-Akaike information criterion (WAIC) has an alternative adjustment, as follows:

$$\mathrm{WAIC}^* = 2\sum_{i=1}^{n}\left\langle \log\left[E_{(\boldsymbol{\theta}|\mathbf{y})} p\left(y_i|\boldsymbol{\theta}\right)\right] - E_{(\boldsymbol{\theta}|\mathbf{y})}\left\{\log\left[p\left(y_i|\boldsymbol{\theta}\right)\right]\right\}\right\rangle.$$

Therefore, we have

$$\mathrm{WAIC} = 2\left[p\left(\mathbf{y}\right) - \mathrm{WAIC}^*\right],$$

in which $p(\mathbf{y}) = \sum_{i=1}^{n} \log \int p(y_i|\boldsymbol{\theta})p(\boldsymbol{\theta}|\mathbf{y})d\boldsymbol{\theta}$.

In practice, p($\mathbf{y}$) and WAIC* are calculated using the draws obtained from the posterior simulations, which means

$$\overline{p\left(\mathbf{y}\right)} = \sum_{i=1}^{n} \log\left[\frac{1}{T}\sum_{t=1}^{T} p\left(y_i|\boldsymbol{\theta}^{(t)}\right)\right];$$

$$\overline{\mathrm{WAIC}^*} = 2\sum_{i=1}^{N}\left\{\log\left[\frac{1}{T}\sum_{t=1}^{T} p\left(y_i|\boldsymbol{\theta}^{(t)}\right)\right] - \frac{1}{T}\sum_{t=1}^{T}\log\left[p\left(y_i|\boldsymbol{\theta}^{(t)}\right)\right]\right\}.$$

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