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Bayesian Markov switching model for BRICS currencies' exchange rates

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Abstract
Exchange rate modeling has always fascinated researchers because of its complex macroeconomic dynamics. This study documents the exchange rate dynamics of major emerging economies after accounting for their macroeconomic cycles and explores the Bayesian Vector Error Correction Model (VECM) Markov Regime switching model, which uses time-varying transition probabilities. The main objective is to study the exchange rate dynamics of Brazil, Russia, India, China, and South Africa (BRICS) vis-à-vis the US dollar. The Bayesian setup uses two hierarchal shrinkage priors, the normal-gamma (NG) prior and the Litterman prior, for parameters' estimation. These shrinkage priors allow for a more comprehensive assessment of the regime-specific coefficients. The model performed well in differentiating between the two regimes for all currencies. The Russian ruble was identified to be the most depreciated currency, whereas the African Rand was the most appreciated. The evaluation of model features revealed that many regime-specific coefficients differed significantly from their common mean. A forecasting exercise was then performed for the out-of-sample period to assess the model's performance. A significant improvement was observed over the basic random walk (RW) model and the linear Bayesian vector autoregression (BVAR) model.

KEYWORDS
BRICS, cointegration, exchange rate forecasting, Markov switching, time-varying parameters

1 | INTRODUCTION

Given the pivotal role of forecasting in economic policy formulation and business decision-making (Wieland & Wolters, 2013), the exchange rate stands out as a key macroeconomic variable that demands accurate prediction. To highlight, the exchange rate market is the biggest market in the financial world and crucial for open economies. The currency exchange rate of an economy remains one of the key parameters in assessing its economic and financial stability. There is a vast literature that discusses exchange rate linkages to major macro variables like interest rates, commodity prices, monetary policy formulation, economic growth, business cycle, and international trade (Calvo & Reinhart, 2002; Goldberg & Tille, 2008; Monacelli & Galí, 2005; Morana, 2017; Rogoff et al., 2003). The significance of exchange rates extends profoundly into financial markets, influencing capital flows, investment decisions, and market returns (Combes et al., 2012; Froot & Stein, 1991; Katechos, 2011). Given its immense importance, exchange rate forecasting exercise remains one of the most challenging and pivotal research topics in international finance.

The existing exchange rate literature spans various economic theories that start with the Uncovered Interest rate parity or UIRP, introduced by Fisher (1896) which initially used interest rates differentials for modeling purpose. Frankel (1976) further refined the model by incorporating the concept of Purchasing Power Parity (PPP) along with UIRP, leading to the inclusion of other important macroeconomic differentials. Another major strand of exchange rate literature is based on the Taylor Rule fundamentals given by Taylor (1993). Although UIRP links exchange rate movements to interest rate differentials between countries, the Taylor Rule connects exchange rate forecasts to monetary policy responses based on inflation and output gaps. Subsequent theories have also emphasized the inclusion of financial variables and indicators, including portfolio returns, commodity prices, foreign asset holdings, stock returns, and oil prices (Y.-C. Chen et al., 2010; Ferraro et al., 2015; Gourinchas & Rey, 2007; Salisu et al., 2021). Apart from evolution in theoretical part, a substantial improvement in methodological aspect of exchange rate modeling has already been in place simultaneously spanning a wide array of linear and nonlinear setups. Rossi (2013) conducts extensive research on the existing literature for exchange rate prediction. Nonetheless, despite having a plethora of forecasting models, understanding the dynamics of the exchange rate model and the accuracy of prediction compared to the basic Random Walk (RW) model remains one of the main challenges for econometricians (Meese & Rogoff, 1983). Addressing this gap, our study utilizes a nonlinear Bayesian regime-switching technique for modeling the exchange rates.

Our motivation for employing this particular modeling relies on the three characteristics shown by the economic variables. The first characteristic is the property of showing dynamic patterns over the evolution of time. The existing literature has significant models exploring this dynamic behavior of the economic variables. Chow (1960) was the first study investigating the time-varying characteristics of regression parameters through the development of the Chow test. Although this was a breakthrough innovation, the technique was too rigid in its assumptions, asserting only a fixed number of structural changes. This major drawback allowed the development of sophisticated set of regime-switching models. These not only gave the flexibility of having multiple structural changes but also allowed multiple regimes to be incorporated into the model. The economic interpretation of regimes can be given as the different economic scenarios under which the variables have distinct behaviors. One such leading model among the class of regime-switching models is the Markov Switching (MS) model developed by Hamilton (1989). Engel and Hamilton (1990) were the first to employ a Markov switching model in the exchange rate context to study the persistence in movements of the US dollar. Their model was able to generate a superior forecast over the basic RW model. In another study, Engel (1994) performed exchange rate modeling for 18 countries using a two-regime Markov switching model. He found that though the forecasting performance was not that superior, they did have more economic information vis-à-vis the RW model. After this, a plethora of studies were conducted that used Markov switching techniques for modeling the exchange rate. The majority of them concluded that these models had a better fit and generated superior forecast compared to the basic RW model (see Clarida et al., 2001; Engel & Hakkio, 1996; Engel & Kim, 1999).

The second characteristic corresponds to the phenomenon of cointegration. Cointegration happens when the linear combination of the variables happens to be stationary. Cointegration models, also known as Error Correction Models (ECM) or Vector Error Correction Models (VECM), enable us to comprehend the long-run dynamics in addition to the short-run dynamics of the model. Engle and Granger (1987), in their pioneering study, found that consumption and income of the US economy were cointegrated. After that, many studies were conducted investigating the exchange rate dynamics in the presence of cointegration (Baillie & Selover, 1987; G Kilian, 1999; Y. Kim, 2008; Sarantis & Stewart, 1995; Taylor, 1988). Hoque and Latif (1993) found that including error-correction term increased the forecasting power against the simple vector autoregression (VAR). Mark (1995) performed out-of-sample exchange rate forecasts using the error correction method of four currencies vis-à-vis the US dollar. He found a significant improvement over the RW model. Sarno et al. (2004) used the Markov switching mechanism and VECM modeling approach to study the US dollar's dynamic relationship with its fundamentals. The results supported the usage of regime-switching for exchange rate modeling.

As the literature for exchange rate forecasts using VAR models expanded, a wide adaptation for the Bayesian method also started taking place. The Bayesian approach help mitigate computational complexities which are apparent in VAR modeling. Canova (1993), Hoque and Latif (1993), and Liu et al. (1994) were some of the early studies which employed Bayesian techniques using various forms of VAR for exchange rate modeling. They all used the revolutionary Litterman prior for estimation. McCrae et al. (2002) employed the concept of cointegration among various Asian currencies, and the study revealed that the out-of-sample forecasts were notably superior compared to those derived from univariate models. Chen and Leung (2003) employed the Bayesian approach for forecasting exchange rates using the cointegration technique. They found that the Bayesian VECM (BVECM) model showed huge improvements from the competing Bayesian VAR and the RW model. Crespo Cuaresma et al. (2018) explore the performance of currency portfolios in the context of exchange rate forecasting using a wide array of Bayesian as well as error correction models. Huber and Zörner (2019) used a regime-switching Bayesian threshold cointegration model approach to model the exchange rates of five currencies against the US dollar. This model had robust regime identification and showed improved forecasting performance of the currencies compared to the RW model, the linear BVAR, and the BVECM models.

The third and the final characteristics is the concept of time-varying nature of parameters. In a significant discovery, Cox (1972) introduced the concept of time-varying regression coefficients in his famous proportional hazard or PH regression model. This concept is well adopted in economics as TVP (Time-Varying Parameter) concept. TVP models are used widely in economic modeling because they can identify structural differences in relationships between various macroeconomic fundamentals. The existing literature on TVP shows that these models are better equipped in capturing the turning points of the variables and hence have superior estimates compared to fixed ones. Wolff (1987) and Schinasi and Swamy (1989) applied the TVP approach for modeling exchange rates. They found that the out-of-sample forecasts showed improvement over the RW when allowed for time-varying nature. Canova (1993) used a Bayesian setup with Litterman prior under time-varying parameters pretext and observed similar forecasting improvements. In the context of Markov switching, Filardo (1994) argued that the assumption of fixed transition probabilities is too restrictive and that the introduction of time-varying parameters into models helps better analyze the system's dynamic behavior. Lee (1991), in his study, concluded that allowing time-varying transition probabilities not only helped better understand the exchange rate dynamics but also had higher forecasting power compared to the RW model with fixed transition probability.

This study utilizes the Markov switching, error correction, and TVP in the Time-Varying Transition Probability-Markov Switching Vector Error Correction Model or TMV model to study the exchange rate dynamics for five sets of currencies against the US dollar. This study adds to the exchange rate literature in two significant ways. First, using fully Bayesian methods, it employs a time-varying parameter approach within the MS-VECM framework. This represents a considerable methodological improvement over simpler models. Integrating the Markov Switching mechanism with the VECM framework not only facilitates the detection of regime shifts but also understands how these shifts relate to long-term equilibrium relationships among key economic indicators. The model's time-varying nature enhances its capacity to reflect the evolving economic relationships accurately. Second, the Bayesian framework effectively manages nonlinearities in the data, incorporating adequate regularization simultaneously. For the Bayesian estimation, this study adopts a mix of two shrinkage priors: the normal-gamma (NG) prior introduced by Griffin and Brown (2010) and the Minnesota prior by Litterman (1979). Hauzenberger et al. (2021) and Huber and Zörner (2019) demonstrate that these shrinkage priors provide the necessary flexibility for regime-specific coefficient identification.

Our second contribution is using the TMV model for studying exchange rate dynamics for Brazil, Russia, India, China, and South Africa (BRICS) nations' currencies. Collectively, The BRICS nations together form a significant chunk of the global economy amounting to near 23% of the global GDP, 42% of the world's population, and nearly 18% of the total world's trade.[1] Their diverse economic structures, ranging from commodity-driven economies like Brazil and Russia to the service and manufacturing-driven economies like India and China, present unique challenges in exchange rate forecasting. When viewed from the lens of emerging economies, the exchange rate is crucial given that most of their trades, credits, and debts are pegged to the US dollar. Frankel and Saravelos (2012) in their study demonstrate how exchange rate acts as an important and critical early warning indicator for emerging economies like BRICS. Furthermore, in contrast to the exchange rates of developed nations, emerging economies like the BRICS exhibit significantly higher volatility. This, coupled with the diverse range of exchange rate policies implemented by their respective central banks, adds a layer of complexity to the modeling of exchange rates in these emerging markets (Jiang, 2019). Nonetheless, only a limited number of studies exist that explore the predictability of exchange rates for BRICS nations. Salisu et al. (2021) predict the exchange rates using stock returns based on Uncovered Equity parity or UIP model. Salisu et al. (2022) in their study utilize the Taylor Rule along with the TVP technique in forecasting the exchange rate for BRICS nations. The comprehensive Bayesian TVM model in this study enhances the extensive literature on the dynamics of exchange rate forecasting, particularly within the context of BRICS nations.

Overall, the TMV model reported significant differences among the regime-specific coefficients from their common mean. The model also was able to distinguish well between the two regimes, that is, appreciation and depreciation states across all currency pairs. The model identified the Russian ruble as the most depreciated currency when compared to the rest of the BRICS nations for the sample period considered. Also, the Russian ruble seemed to show stickiness for the state of being depreciated. In the forecasting exercise conducted for the out-of-sample period, the TMV model defeated the RW model for all currencies for the point forecasts. While for density forecasts, the TMV model was able to defeat the RW for Brazil, Russia, and India and failed to outperform the RW model for China and South Africa. Information encompassing test found that the TMV model for all the exchange rates forecasts contained significant additional economic information compared to the RW model's forecast.

The rest of the paper is structured as follows: Section 2 lays down the econometric framework used. Section 3 discusses the data and the model features. Section 4 gives the forecasting results, and finally, we end the discussion with a conclusion.

2 | ECONOMETRIC MODEL SETUP

Let $\{\boldsymbol{y}_t\}_{t=0}^{T}$ be a K-dimensional vector consisting of a set of endogenous variables which has a unit root, that is, $\boldsymbol{y}_t \sim I(1)$, and there exists a $K \times r$ matrix $\boldsymbol{\beta}$ such that $\boldsymbol{\beta}'\boldsymbol{y}_t \sim I(0)$ then the variables are said to be cointegrated with r number of cointegration relations. The econometric model used for analysis is developed using the Markov regime-switching model given by Goldfeld and Quandt (1973) and the error correction model provided by Engle and Granger (1987),

$$\Delta \boldsymbol{y}_t = \boldsymbol{\alpha}_{s_t}\boldsymbol{\beta}'\boldsymbol{y}_{t-1} + \boldsymbol{B}_{s_t1}\Delta \boldsymbol{y}_{t-1} + \ldots\ldots\ldots\ldots + \boldsymbol{B}_{s_tp}\Delta \boldsymbol{y}_{t-p} + \boldsymbol{L}_{s_t}\boldsymbol{\eta}_t \tag{1}$$

Here s_t denotes the Markov states or regimes that take discreet values 0, …, N and follow a first-order Markov process. Here the regimes represent different economic scenarios. In our study, we will consider s_t to be following two specific states, that is, $s_t = 0,1$. Here $s_t = 0$ belongs to a state of being appreciated, and $s_t = 1$ belongs to the state of being depreciated. $\boldsymbol{\alpha}_{s_t}$ is a state-specific $K \times r$ matrix consisting of short-run adjustment coefficients and $\boldsymbol{B}_{s,j}$ ($j = 1$ … …, p), also a state-specific $K \times K$ matrix comprising of coefficients determining the short-run dynamics across each state. $\boldsymbol{L}_{s_t}$ is the lower Cholesky decomposition of the state-specific variance–covariance matrix Σ_{s_t} and $\boldsymbol{\eta}_t \sim N(\boldsymbol{0}, \boldsymbol{I}_K)$. Note that among all the parameters to be estimated, only $\boldsymbol{\beta}$ has been kept state independent because $\boldsymbol{\beta}$ denotes the long-run relation and is generally not supposed to change rapidly. Although there is literature that allows $\boldsymbol{\beta}$ to be state dependent, like in work performed by Jochmann and Koop (2015). We define $\boldsymbol{z}_t(\boldsymbol{\beta}) = \boldsymbol{\beta}'\boldsymbol{y}_{t-1}$, hence $\boldsymbol{z}_t$ is an r dimensional vector of error correction terms and is a function of $\boldsymbol{\beta}$. Also, observe that α and β appear in the equation as a product; hence, they are not identified. Based on Huber and Zörner (2019), we put an identification condition of $\beta = (I_r, \xi')'$ because we will be providing sufficient prior information to the elements of β. This particular choice of identification condition renders the arrangement of elements in y_t to be important but at the same time makes the model exactly identified.

The state variable s_t follows a Markov chain of the first order. The transition probabilities $p_{mn,t}$ defined as the probability of occurrence of state n given that now it is at stage m is denoted as $P(s_t = n | s_t = m, \delta, z_t)$. Notice that the transition probability is time varying and is additionally conditioned on the latent variable z_t and δ apart from the current state it is in. Here δ consists of a set of regression coefficients over z_t. Also, if the number of states are 1, … … s, then

$$\sum\nolimits_{\boldsymbol{n}=1}^{s} \boldsymbol{p}_{\boldsymbol{mn},\boldsymbol{t}} = \boldsymbol{1} \tag{2}$$

As described by Amisano and Fagan (2013), the Markov regime-switching mechanism is governed by the early warning indicator variable $\boldsymbol{z}_t$ and its probit parametrization is given by

$$p_{mn,t} = \psi(c_{0m} + \delta' z_t) \tag{3}$$

$$\psi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}t^2\right\} dt \tag{4}$$

Here the m^{th} element of δ measures the sensitivity of $p_{mn,t}$ with respect to latent variable z_t. From the above specification, it can be observed that both the intercept and δ are time invariant, with the intercept term being state dependent. The probit model for the latent variable is given by

$$z_t' = c_{0m} + \delta' z_t + \epsilon_t, \quad \epsilon_t \sim N(0,1) \tag{5}$$

2.1 | Prior specification

As discussed, because the number of parameters to be calculated get proliferated substantially with an increase in the number of regimes, we use Bayesian techniques for calculating parameters. Based on Hauzenberger et al. (2021), we first stack the parameters to be estimated in the vector form $\boldsymbol{B}_{S_t} = \left(\boldsymbol{\alpha}_{st}, \boldsymbol{B}_{S_t1}, \boldsymbol{B}_{S_t2}, \ldots\ldots\ldots, \boldsymbol{B}_{S_tp}\right)$, which is of dimension $K \times M$ where $M = r + Kp$ and then vectorize it as $\boldsymbol{b}_{s_t} = Vec(\boldsymbol{B}_{s_t})$. We also define $\Lambda = \mathrm{diag}(\sigma_1, \sigma_2, \ldots\ldots, \sigma_j)$ for $j = MK$ as a variance–covariance matrix of variances σ_j. Then based on Griffin and Brown (2010), we have an NG prior defined on elements of $\boldsymbol{b}_{s_t}$ as

$$\boldsymbol{b}_{s_t} \sim N(\boldsymbol{b}, \boldsymbol{\Lambda}) \tag{6}$$

$$\sigma_j = Ga(g_0, g_1) \tag{7}$$

Here σ_j is the scaling factor for the prior, which follows a gamma distribution with hyperparameters g_0 and g_1. As explained by Griffin and Brown (2010), g_0 is the parameter that controls for kurtosis or tails thickness, whereas g_1 controls for the shrinkage. The above setup helps determine whether the coefficients differ across regimes s_t and simultaneously help assess which of them are homogenous over different currency regimes. We set the value of g_0 and g_1 to 0.01 to induce shrinkage on $\boldsymbol{b}_{s_t}$ based on Huber and Zörner (2019). Also, we define prior for the mean $\boldsymbol{b}$ as

$$\boldsymbol{b} \sim N\left(\tilde{b}, \tilde{\Lambda}\right) \tag{8}$$

Here $\tilde{b}$ is the prior mean, and we set this value to zero. The variance–covariance matrix $\tilde{\Lambda}$ has its prior setup carried out based on Litterman (1979), where coefficients for the autoregressive terms of immediate lag are given more weights, compared to other coefficients and higher lag terms. We have the Litterman prior defined on the variances as

$$\tilde{\Lambda}(\alpha) = \lambda_0 \mathbf{I}_{\mathrm{r}} \tag{9}$$

$$diag\left(\tilde{\Lambda}\left([\mathrm{B}_l]_{ij}\right)\right) = \begin{cases} \dfrac{\lambda_1}{\left(l^d\right)^2} \text{ for } i = j \text{ and } l = 1, \ldots.., p \\ \dfrac{\lambda_2 \hat{\tau}_i^2}{\left(\mathrm{l}^d\right)^2 \hat{\tau}_j^2} \text{ for } i \neq j \text{ and } l = 1, \ldots., p \end{cases} \tag{10}$$

Here $\tilde{\Lambda}(\alpha)$ represents the variance–covariance matrix for the long-run adjustment coefficients α and $\tilde{\Lambda}\left([\mathrm{B}_l]_{ij}\right)$ represent the respective i,j elements in the B_l matrix for the short-run coefficients. $\lambda_0, \lambda_1, \lambda_2$, and d are the tightness information on the priors. Here λ_0 defines the overall tightness for the prior defined in Equation 9; similarly, λ_1 sets the overall tightness for the prior defined in Equation 10 while λ_2 defines tightness for the lagged values, and d controls the speed at which the lag values reach zero. We set the values for $d, \lambda_0, \lambda_1, \text{and} \lambda_2$ as 1, 100, 0.04, and 0.01. These values make the prior fairly uninformative and reasonably good for estimating highly parameterized models (Hauzenberger et al., 2021; Huber & Zörner, 2019). The label switching issue described by Frühwirth-Schnatter (2006) is addressed using an identification condition embedded in the iteration process by assuming that the conditional mean of the short-run interest rate equation is higher in a scenario when the currency is depreciated and is implemented as a rejection step in the MCMC algorithm.

We then proceed by setting $\Psi = (c_{0m}, \delta')^{`}$, and we define the prior for Ψ as

$$\Psi \sim N(0, U); U = 10 \times I_{r+1} \tag{11}$$

The value of U, the scaling factor, is set in such a way so that we do not infuse heavy information via the prior, but at the same time having a larger variance will induce fat tails allowing a larger range.

We now impose a hierarchal Wishart prior for $\Sigma_{s_t}^{-1}$ and the priors hyperparameters based on Frühwirth-Schnatter (2006), Malsiner-Walli et al. (2016), and Huber and Zörner (2019).

$$\Sigma_{s_t}^{-1} \sim W(V, \nu) \tag{12}$$

$$V \sim W(S, s) \tag{13}$$

where

$$\nu = 2.5 + \frac{K-1}{2}$$
$$s = 0.5 + \frac{K-1}{2}$$
$$S = \frac{100s}{\nu} \Sigma_{0.}$$

Here $\Sigma_0 = diag\left(\hat{\rho}_1^2, \hat{\rho}_2^2, \ldots\ldots, \hat{\rho}_k^2\right)$ and $\hat{\rho}_i^2$ is the ordinary least squares (OLS) variance of the AR(1) estimated models.

Finally, for $e = Vec(\xi)$, we again use a Gaussian prior as

$$e \sim N(0, \theta \times \mathrm{I}_{\mathrm{v}}) \tag{14}$$

Here $v = (k - r)r$, which denotes the free elements of β. We set this hyperparameter $\theta = 1$ based on Huber and Zörner (2019).

2.2 | Posterior distribution

Employing the Bayes theorem on the priors defined in Equations 6 to 14 with suitable likelihoods leads to well-defined conditional posterior distributions for all the parameters. We use the Gibbs sampling method to obtain full conditional posterior distribution for all the parameters. The algorithm samples conditional posterior distributions for $\boldsymbol{b}_{s_t}$, $\tilde{\boldsymbol{b}}$, $\tilde{\boldsymbol{\Lambda}}$, $\boldsymbol{\Psi}$, $\boldsymbol{\Sigma}_{s_t}^{-1}$ and $\boldsymbol{e}$ and takes well-defined form (see Albert & Chib, 1993; Amisano & Fagan, 2013; Hauzenberger et al., 2021; Huber & Zörner, 2019; Zellner, 1996). Following are the MCMC algorithm steps that the model follows in brief:

1. Obtain draws for b_{s_t} conditioned on the rest of the parameters. Here b_{s_t} refers to two parameters in the two states ($s_t = 0, 1$).
2. Sample e from the Gaussian distribution with zero mean and $\theta \times \mathrm{I_v}$ variance.
3. Draws the common mean $\boldsymbol{b} \mid b_{s_t}, \Lambda \sim N\left(\tilde{b}, \Lambda \odot \tilde{\Lambda}\right)$.
4. Draws $\sigma_j \mid \boldsymbol{b}, b_{s_t} \sim GIG\left(g_0 - 1 \sum_{s_t=0}^{1} (b_{s_t} - \boldsymbol{b}) 2g_1\right)$.
5. Draws $\sum_j^{-1} \mid \boldsymbol{b},\ V^T, V,\ D \sim W(V + \frac{1}{2}\sum_{s_t}\left(\Delta y_t - \mathrm{B}_{s_t} x_t'\right)$ $\left(\Delta y_t - \mathrm{B}_{s_t} x_t'\right)'\ v + \frac{n_{s_t}}{2})$; n_{s_t} equals the number of observations in the specific state s_t.
6. Draws $V \mid b_{s_t} \sim W\left(S + \sum_{s_t=0}^{1} \sum_j^{-1}, s + 2v\right)$.
7. The algorithm then simulates the full history of V^T, $p_{mn,t}$, z_t', and Ψ using the methods outlined by Albert and Chib (1993), Amisano and Fagan (2013), and Kim and Nelson (2017).

The MCMC algorithm iterates for 60,000[2] times where the initial 10,000 draws are discarded as burn-in. Among the rest of the 50,000 draws, we thin our posterior draws by selecting every 50th draw to remove any possible autocorrelation among them. The convergence diagnostics generated favorable results across the countries and showed rapid convergence. Trace plots and Autocorrelation plots diagnostics were also accurate.

3 | DATA AND MODEL FEATURES

The selection of variables for the exchange rate prediction is based on the works performed by Dornbusch (1976), Frankel (1976), and Meese and Rogoff (1983). The quasi-reduced form of the exchange rate equation is given as

$$y_t = \beta_1\left(i_t - i_t^*\right) + \beta_2\left(\pi_t - \pi_t^*\right) + \beta_3\left(q_t - q_t^*\right) + \beta_4\left(M_t - M_t^*\right) + \epsilon_t \tag{15}$$

Here y_t equals the spot exchange rate, i_t is the short-run interest rate of the country proxied by 3 months T-bills, π_t denotes the inflation, here proxied by CPI of the nation, q_t denotes the output of the nation, here proxied by the production index because of the fact that GDP is not calculated monthly and the production index is the next best proxy monthly variable for it, and lastly M_t is the M3 money supply of the nation. The terms i_t^*, π_t^*, q_t^*, and M_t^* denote the variables for the foreign nation. Barring i_t and i_t^*, all the variables were taken in their natural logs. As pointed out in the earlier section that the arrangement of variables is crucial because of the identification criterion, we have put for β. Hence, we have the following setup:

$$S_t = (\mathrm{SR_t}, \mathrm{CPI_t}, \mathrm{IP_t}, \mathrm{M3_t})$$

Here SR represents the differential of short-run interest rates, that is, $i_t - i_t^*$. Similarly, we have CPI, IP, and M3 for $\pi_t - \pi_t^*$, $q_t - q_t^*$, and $M_t - M_t^*$. Our data span monthly observation from 2000: M1 to 2021:M12. We choose a lag of $p = 2$ based on the fact that we are using shrinkage priors for our analysis which tend to push coefficients of higher lags towards zero. All the data for the analyses have been sourced from the respective central banks' websites of the individual nations.

Figure 1 shows the time series plot of the exchange rates of the BRICS nations from 2000 to 2021. It can be seen that apart from China, all other member nations' exchange rates have depreciated vis-à-vis US dollar in time. China pegged its currency until 2005; even after that, China's central bank heavily intervenes in the exchange rate market, unlike other member nations that follow a floating exchange rate regime. Also, observe a typical spike in exchange rate plots during the global recession in BRICS nations except for China. Table 1 shows the summary statistics of the exchange rate variable. The Russian ruble tends to show the highest volatility, as evidenced by a standard deviation of 17.392, implying considerable fluctuations. This fact can also be observed by its broad interquartile range. Conversely, the Chinese yuan represents the most stable currency within the group, with the lowest standard deviation of 0.801, reflecting China's more regulated exchange rate regime. The Indian rupee, although less volatile than the ruble, still displays significant movement within its range, indicating moderate variability. The Brazilian real and the South African rand exhibit comparable volatility, as reflected in their respective interquartile ranges. However, the rand consistently registers higher rates, suggesting a weaker valuation against the dollar throughout the observed period.

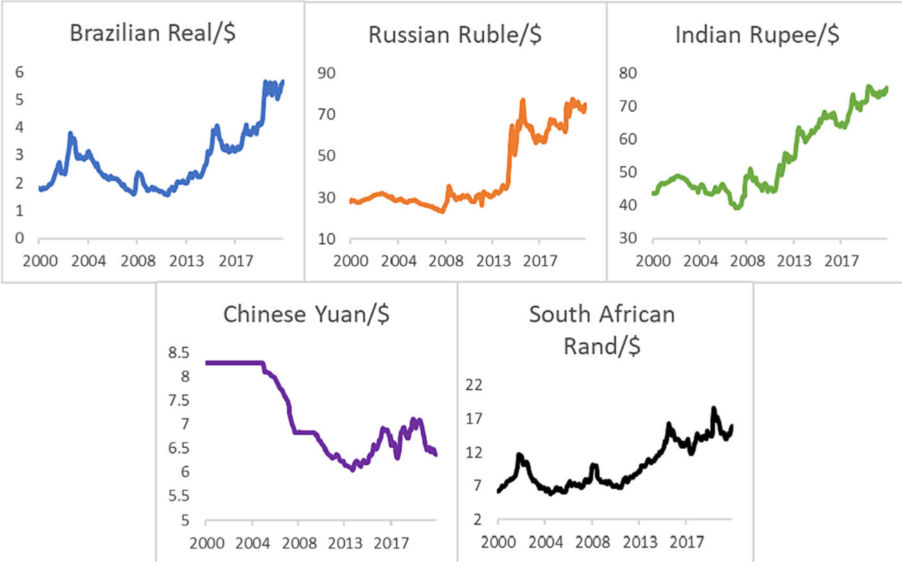

FIGURE 1 Time series plot showing exchange rates of BRICS nations' currency vis-à-vis the US dollar. Note: The time series plot here spans from the time period 2000:M1 to 2021: M12. The *x*-axis here plots the time axis whereas the *y*-axis plots the nominal value of the local currency/$. BRICS, Brazil, Russia, India, China, and South Africa.

TABLE 1 Summary statistics for the BRICS nations exchange rates.

Exchange rate	*N*	Mean	Std. dev.	Minimum	Maximum	25th Pctl.	75th Pctl.
Brazilian real/$	264	2.798	1.052	1.563	5.655	1.983	3.295
Russian ruble/$	264	41.191	17.392	23.35	77.589	28.59	59.74
Indian rupee/$	264	54.692	11.098	39.268	76.168	45.491	65.029
Chinese yuan/$	264	7.152	0.801	6.051	8.28	6.464	8.239
African rand/$	264	9.995	3.259	5.723	18.565	7.18	13.165

Note: Ruble turns out to be the most volatile currency, whereas yuan turns out to be the most stable currency. BRICS, Brazil, Russia, India, China, and South Africa.

3.1 | Model features

3.1.1 | Regime allocation

We start assessing the model's performance with the regime identification part. The plot shown in Figure 2 represents the regime identification for all the BRICS nations. The gray-shaded region shows the period of depreciation, that is, they represent $s_t = 1$ while the white portion represents appreciated periods. The blue and red lines denote time-varying transition probabilities. The blue line represents transition probability $P(S_t = 1 | S_{t-1} = 0)$, that is, probability of going into a depreciated state given it was appreciated in the previous period. Similarly, the red line represents $P(S_t = 0 | S_{t-1} = 1)$, that is, probability of going into an appreciated state given it was depreciated in the previous period. The model primarily identifies the major economic crises of the respective nations.

The model predominantly identifies the Brazilian real as being appreciated in relation to the US dollar. Exceptions to this trend were observed during specific periods of economic instability in the nation. The early 2000s, for instance, saw a depreciation of the real, coinciding with the South American economic crisis of 2002 that had a profound impact on Brazil. Further depreciations were noted during the Global Financial Crisis and the national economic crisis (2014). More recently, significant depreciation occurred during the COVID-19 pandemic. Throughout the majority of the sample time period, the blue line surpasses the red line, except during the aforementioned crises in 2002, the global financial crisis, and the COVID-19 period. This suggests that the Brazilian real, for a significant portion of the time, had a higher probability of entering a depreciated state in the subsequent month, given it was appreciated in the preceding month.

Next, the Russian ruble appears to be highly depreciated for a considerable duration. This depreciation exhibits a certain degree of persistence, implying that once the ruble enters a state of depreciation, it tends to remain in that state for a significant period, especially when compared to periods of appreciation. The ruble experienced a phase of appreciation in the early 2000s, a period characterized by a booming Russian economy because of the implementation of rigorous economic reforms. These reforms were introduced following the

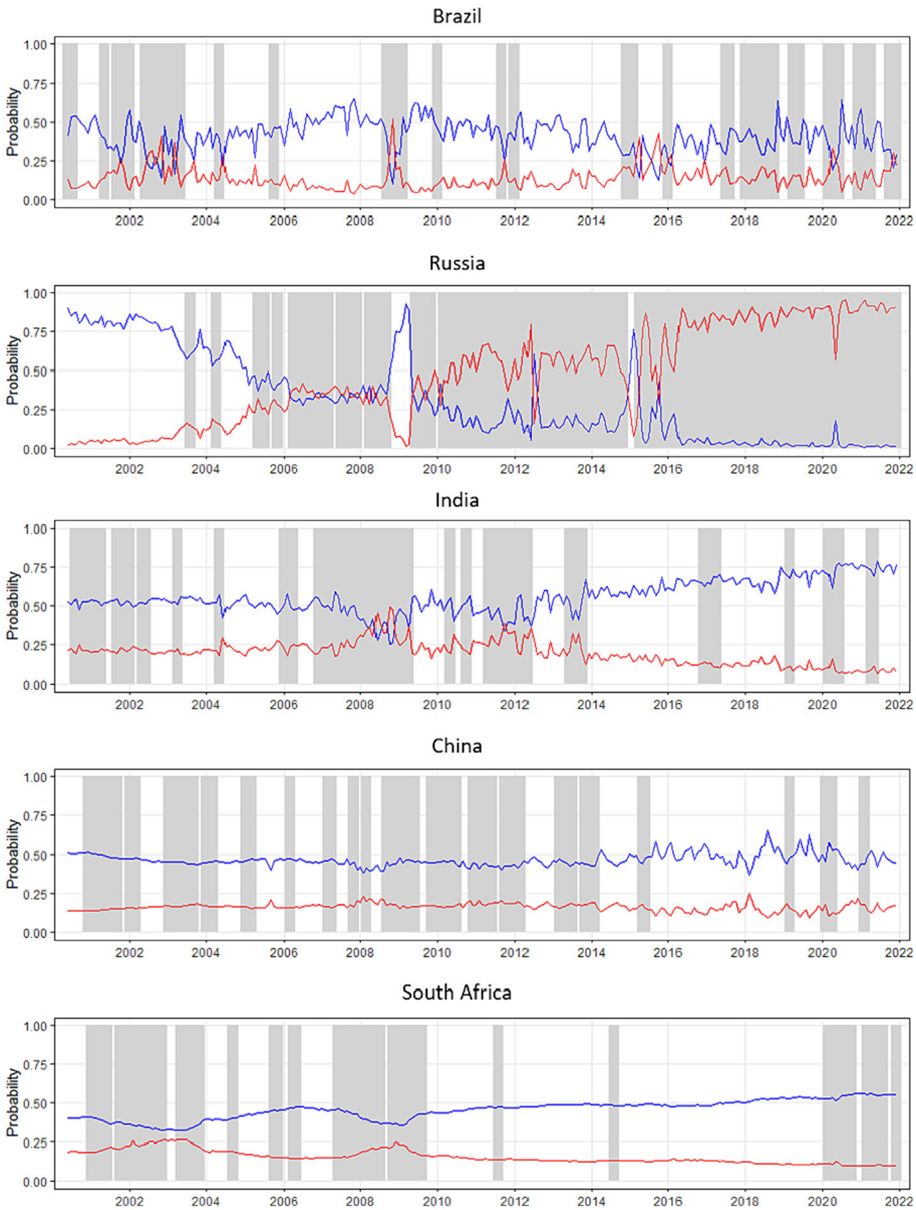

FIGURE 2 Regime identification for BRICS nations' currencies. *Note*: The *y*-axis plots the probabilities whereas the *x*-axis plots the time horizon. The gray shaded region shows the period of depreciation, that is, they represent $s_t = 1$, whereas the white portion represents appreciated periods. The blue and red lines denote time-varying transition probabilities. The blue line represents transition probability $P(S_t = 1 \mid S_{t-1} = 0)$, that is, probability of going into a depreciated state given it was appreciated in the previous period. Similarly, red line represents $P(S_t = 0 \mid S_{t-1} = 1)$, that is, probability of going into an appreciated state given it was depreciated in the previous period. BRICS, Brazil, Russia, India, China, and South Africa.

1998 Russian economic crisis. During this time, the red line largely surpasses the blue line, with exceptions occurring in the early 2000s and during the global financial crisis, where a noticeable spike in the blue line is evident. Two significant events are clearly visible in the transition probabilities: the Global Recession and the period following the Crimea incident, which resulted in heavy economic sanctions imposed on Russia by Western countries. During the Global Recession, the Russian ruble appreciated, with $P(S_t = 1 \mid S_{t-1} = 0)$ nearing 1. The second event is marked by a consistent peak in the red line, reaching its zenith post the Crimean crisis, with a minor dip during the initial wave of the COVID-19 pandemic. This suggests that the Russian ruble experienced significant depreciation post-2014, with a high probability of appreciation in the subsequent period, as indicated by the large values of $P(S_t = 0 \mid S_{t-1} = 1)$.

The model mainly identifies the Indian rupee as depreciated during the early 2000s, followed by a period of appreciation from 2002 to 2006. This appreciation could be attributed to the substantial foreign direct investments India attracted during this phase, which likely led to the rupee's appreciation. The model subsequently indicates a depreciation of the rupee from 2007 until the Global Recession. Further depreciation was observed from 2011 to mid-2012, a period characterized by fiscal instability and high inflation within the country. The 2013 Indian rupee crisis is also identified by the model as a period of depreciation. The rupee was further depreciated during the implementation of significant economic reforms, such as the demonetization of the Indian rupee and the introduction of the Goods and Services Tax (GST). Finally, two phases of depreciation are observed, coinciding with the onset of the COVID-19 pandemic and its second wave in the country. The blue line surpasses the red line for the majority of the sample periods, with the exception of the Global Recession. A consistent increase in the transition probability $P(S_t = 1 | S_{t-1} = 0)$, represented by the blue line, is observed post-2014, coinciding with a change in government and the adoption of different economic principles.

The model primarily identifies the Chinese yuan as depreciated until 2015, followed by a period of appreciation post-2015. This trend aligns with the fact that China, unlike other nations that adhere to floating exchange rate regimes, frequently intervenes in the exchange rate market through its central bank. This intervention often results in the consistent devaluation of the yuan to maintain the viability of exports. Throughout the sample, the blue line, representing the transition probabilities consistently dominates the red line.

Lastly, as for the South African rand, the model predominantly identifies it as depreciated during three major time periods. The first period spans from late 2000 to 2003, coinciding with the rand currency crisis. The second period of depreciation occurred during the Global Recession, and the third during the COVID-19 pandemic, which began in early 2020 and continued until the end of the sample. Similar to the Chinese yuan, the blue line representing the transition probabilities $P(S_t = 1 | S_{t-1} = 0)$ dominates the red line throughout the sample. Table 2 summarizes the overall regime identification part by our model.

3.2 | Assessing differences in parameters across the states from their common mean

3.2.1 | Short-run adjustment coefficients α_{s_t}

Figure 3 depicts a radar plot showing the differences in the posterior mean of α_{s_t} also called the short-run adjustment coefficient associated with the three cointegrated error terms from their common mean, that is, whether $\alpha_{s_t=0}$ and $\alpha_{s_t=1}$ differ from their common mean α. The short-run adjustment term within the VECM framework represents the speed at which the variables return to equilibrium in the long run after a short-term shock. The difference is denoted in log terms; hence, the more negative the value is, the more it resembles the common mean, whereas a value closer to zero implies a more significant difference from their respective common means. China mostly shows large negative values signifying state-specific short-run adjustment parameters coinciding with the expected common mean, especially for the second cointegration term. Conversely, Russia's long-run adjustment state-specific parameters differ significantly from the standard mean, especially for the first cointegration error term. Also, parameters differ from their common mean in the first cointegration term for all countries, followed by the second and third cointegration terms.

3.2.2 | Autoregressive coefficients $B_{s_t p}$

Figure 4 shows differences in the posterior mean of the state-specific $B_{s_t p}$ from their common mean for the exchange rate equation for all the BRICS nations. Again, the values are depicted in log terms, meaning the difference from the common mean increase as the value reaches closer to zero. It is observed that the differences from the common mean are almost homogenous for coefficients of the variables $\mathrm{CPI}_{t-1}, \mathrm{CPI}_{t-2}, \mathrm{IP}_{t-1}, \mathrm{IP}_{t-2}$, $\mathrm{M3}_{t-1}$, and $\mathrm{M3}_{t-2}$ across nations with values close to zero, signifying a significant difference of state-specific coefficients from their respective common means. However, there is vast heterogeneity among the coefficients for the autoregressive terms $\mathrm{XR}_{t-1}, \mathrm{XR}_{t-2}$ and $\mathrm{SR}_{t-1}, \mathrm{SR}_{t-2}$. For example, the coefficient for XR_{t-2} for Russia is significantly different from its common mean, whereas for South Africa, it is mostly similar to its common mean.

TABLE 2 Summary of regime identification by the TMV model.

Currency	Most observed state	Transition probabilities trend	Volatility
Brazilian real	Appreciated state	Probability of going into depreciation dominates that of appreciation	Moderately volatile
Russian ruble	Depreciated state	Probability of going into appreciation dominates that of depreciation	Highly volatile
Indian rupee	Depreciated state	Probability of going into depreciation dominates that of appreciation	Moderately volatile
Chinese yuan	Depreciated state	Probability of going into depreciation dominates that of appreciation	Less volatile
South African rand	Appreciated state	Probability of going into depreciation dominates that of appreciation	Less volatile

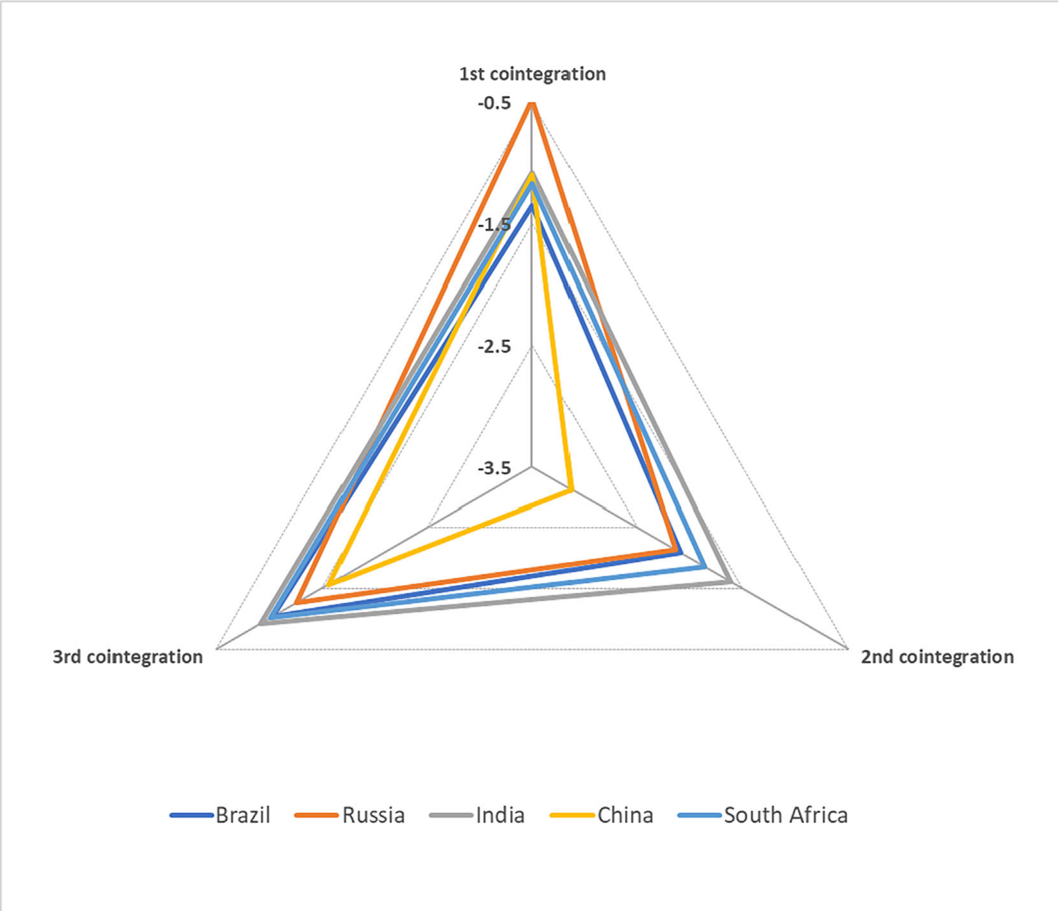

FIGURE 3 Differences in the posterior mean of the state-specific α_{s_t} from their common mean. *Note*: α_{s_t} represents state-specific short-run adjustment coefficient plotted for different number of cointegration relations in the radar plot. The axis in the plot represents $\log(\sigma_j)$. A value closer to zero signify larger difference in the state-specific α_{s_t} from its common mean.

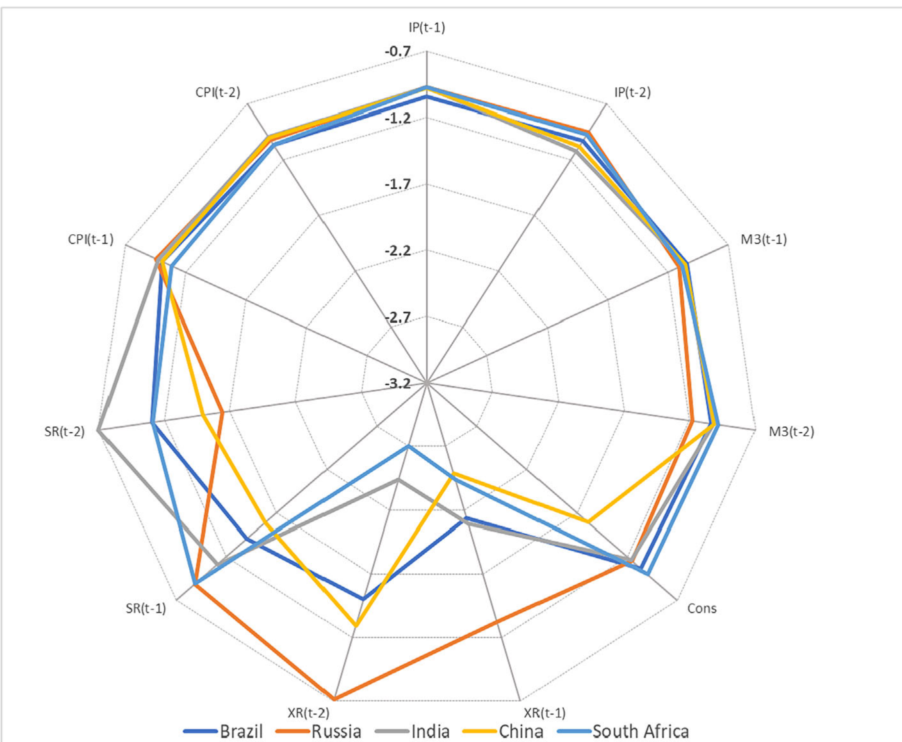

FIGURE 4 Differences in the posterior mean of the state-specific $B_{s_t p}$ from their common mean. *Note*: $B_{s_t p}$ represents the state-specific autoregressive coefficients associated with the p^{th} lag. The axis here in the radar plot represents $\log(\sigma_j)$.

The state-specific XR_{t-1} coefficients appear different for Russia, whereas not much difference is observed for the rest of the nations. For SR_{t-2}, India shows the maximum difference from its common mean, followed by Brazil, South Africa, China, and Russia, whereas for SR_{t-1}, Russia and South Africa show almost the same magnitude of differences in coefficients from their common mean, followed by India, Brazil, and China.

3.2.3 | Interstate differences

Figure 5 shows the heatmap for differences in the magnitude of coefficients from their common mean in each specific state. Observe that the left side of the heatmap is more saturated, meaning the coefficients for $s_t = 0$ happen to be more different from their respective common means compared to coefficients calculated for $s_t = 1$. We observed that in Figure 4, coefficients for XR_{t-2} for Russia were significantly different from its common mean, and Figure 5 points out that this vast difference was because of the significant negative deviation from the coefficients corresponding to $s_t = 0$. Similarly, large deviation from the common mean for SR_{t-1} was mainly contributed by large positive deviation from the coefficient corresponding to $s_t = 0$.

3.2.4 | Error variances

Figure 6 shows boxplots for the marginal posterior distributions of error variance matrices (in logarithms) for each exchange rate equation of respective nations for different states. We find a relatively lower error variance for

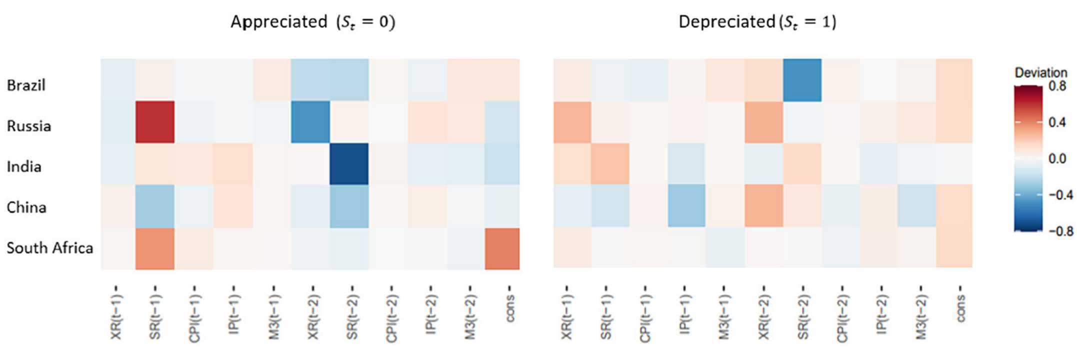

FIGURE 5 Heatmap showing deviation of state-specific coefficients from their common mean.

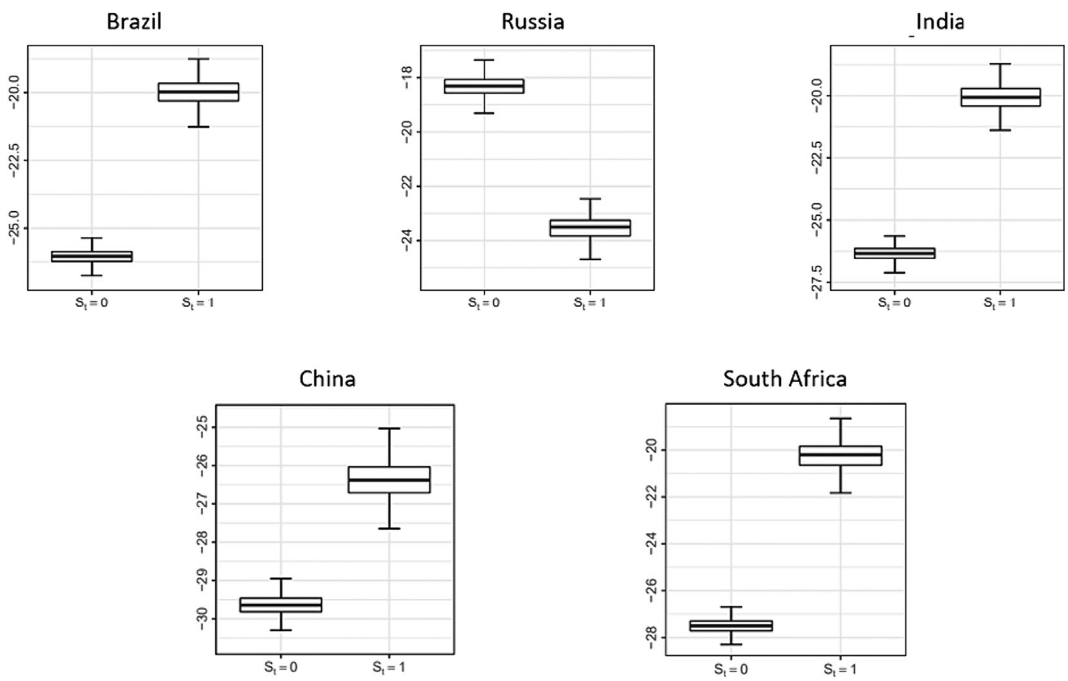

FIGURE 6 Box plots showing posterior distribution of the error variance across BRICS nations. BRICS, Brazil, Russia, India, China, and South Africa.

$s_t = 0$, that is, state of appreciation against the case of $s_t = 1$, that is, state of depreciation for all nations except Russia. Russia shows low error variances for $s_t = 1$ as compared to $s_t = 0$. This might stem from the fact that depreciation scenarios have occurred consistently in large amount compared to other BRICS nations. The largest difference among values of error differences between each state was observed in South Africa and the least observed in the case of China. This can indeed be attributed to the nature of their exchange rate regimes. South Africa operates under a flexible exchange rate regime, which allows its currency value to fluctuate according to market forces, leading to higher volatility and thus a larger error variance. However, China maintains a more controlled, fixed exchange rate regime, resulting in lower volatility and a smaller error variance.

The assessments in the previous two sections for α_{s_t}, $B_{s_t p}$, and Σ_{s_t} show that the regime-specific coefficients differ significantly from their common mean across all BRICS nations.

4 | FORECASTING PERFORMANCE

To perform the forecasting exercise, we have split the data into two parts: The in-sample estimation period ranges from 2000:M1 to 2012:M12, and the out-of-sample forecasting period ranges from 2013:M1 to 2021:M12. The forecasting exercise starts using the initial 156 observations to estimate the model parameters used to get the first 1-month ahead forecast, that is, the forecast for the period 2013:M1. In the next step, the data for 2013:M1 is included in the estimation period, that is, we use 157 data points, and model parameters are re-estimated, which then are used to perform 1-month ahead forecast for the period 2013:M2. Again, the estimation sample window is expanded by one more observation, which is repeated until we reach the end of the out-of-sample period. Our contending models are TVM models for $r = 1, 2$, and 3, along with the BVAR model, which uses the same prior setup. Our benchmark model is the basic RW model. We assess our forecasting performance for out-of-sample periods using Theil U statistics for point forecasts and log-predictive scores or LPS for the density forecasts. Lastly, we perform Fair and Shiller (1990) test to assess the quantity of information in forecasts obtained by individual models over others.

We assess the point forecasts of the model using the Theil U statistics, which is defined as follows:

$$RMSE = \left\{ \sum_{i=1}^{N} \left[y_{t+s}^{*} - y_{t+s} \right]^2 / N \right\}^{0.5}$$

$$Theil\ U = \frac{RMSE(\text{contending model})}{RMSE(\text{random walk model})}$$

Here y_{t+s}^{*} denotes s-step ahead forecast obtained from the model and y_{t+s} denotes the actual value. Theil U statistics is defined as the ratio of the RMSE (root mean square error) of the contending model over the RMSE of the benchmark model (here the RW model). Hence, a value of less than 1 shows a superior performance relative to the benchmark model, whereas a value of greater than 1 indicates an inferior performance compared to the benchmark RW model. Table 3 lists different Theil U values for different BRICS nations for all the competing models. For Brazil, India, and South Africa, all our competing models performed better than the RW model with the TMV ($r = 2$), TMV ($r = 2$), and TMV ($r = 3$) scoring the highest for Brazil, India, and South Africa, respectively. Also, the BVAR model ends up mostly with the lowest scores (though performing better than the benchmark model). In the case of Russia, only TMV ($r = 3$) can outperform the RW model, with the other two TMV models being just as good as the RW model, whereas the

TABLE 3 Theil U indicator for all competing exchange rate models.

Theil U					
Competing model	**Brazil**	**Russia**	**China**	**India**	**South Africa**
TMV ($r = 3$)	0.9331	0.945406	0.901762	0.8364	0.815208
TMV ($r = 2$)	0.8001	1.000038	0.952909	0.8244	0.829923
TMV ($r = 1$)	0.8883	1.003183	0.929611	0.8011	0.830431
BVAR	0.8839	1.132469	1.14664	0.9115	0.923132

Note: Theil U indicator index is calculated here as the ratio of the RMSE value of the competing model forecasts for the out-of-sample period upon RMSE value of the random walk model forecasts for the out-of-sample period. A value less than 1 signifies a superior performance compared to the random walk. According to the point forecasts accuracy metrics, the TMV models clearly outperform the linear model and RW with the best performance appearing in South Africa and worst in Russia. RMSE, root mean square error; RW, random walk; TMV, Time-Varying Transition Probability-Markov Switching Vector Error Correction Model.

BVAR model performs poorer than the benchmark model. In the case of China, TMV ($r = 3$) ends up with the highest score, whereas the BVAR model fails to outperform the RW model.

The density forecast performance of our model is assessed using the famous Log Predictive Scores or LPS. Table 4 depicts the LPS scores for all our competing models for the BRICS nations with respect to the RW model. For Brazil, we can see that apart from TMV ($r = 3$) all other models perform better than the RW model with the linear BVAR being the best performer. In Russia, TMV ($r = 3$) is the best performer, whereas TMV ($r = 1$) is the worst performer. For China, all the TMV models perform superior compared to the linear BVAR and the RW model. Coming to India, all TMV models perform better than the RW model apart from the BVAR model with TMV ($r = 2$) being the highest scorer. Finally, in the case of South Africa, all models outperform the RW model with TMV ($r = 1$) being the overall winner. Figure 7 shows the evolution of LPS for the BRCIS

TABLE 4 Log predictive scores for different exchange rate models with respect to the benchmark random walk model.

LPS					
Competing model	**Brazil**	**Russia**	**China**	**India**	**South Africa**
TMV ($r = 3$)	−13.9655	37.1939	8.6222	17.55042	11.0115
TMV ($r = 2$)	13.1013	−31.55914	4.49867	36.34827	17.895
TMV ($r = 1$)	6.0343	−20.70791	0.34261	39.85249	18.3747
BVAR	22.2166	21.7989	−32.03978	−29.2078	14.6622

Note: The values in the table represents the total sum of the Log predictive score of each forecast for the out-of-sample period, that is, it represents the $\sum_t \left(LPS_{(\text{competing model})_t} - LPS_{(\text{RW})_t}\right)$, where t is the out-of-sample time period. A value greater than zero signifies a superior performance over the random walk. Density forecasts metrics turns out best for India where all the versions of the TMV model beat the linear and the RW model while China scores inferior. LPS, Log Predictive Scores; RW, random walk; TMV, Time-Varying Transition Probability-Markov Switching Vector Error Correction Model.

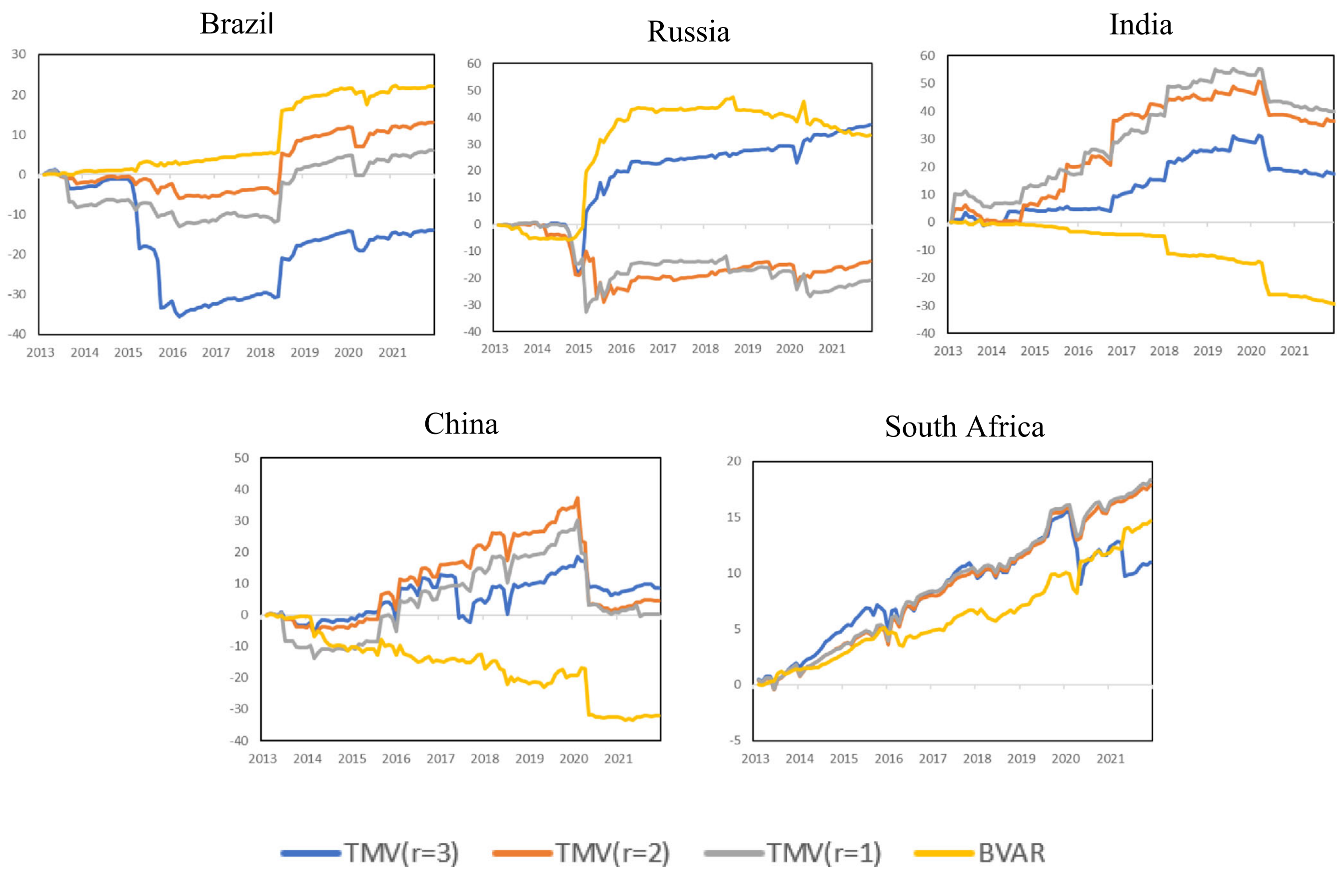

FIGURE 7 Evolution of log predictive scores for all competing exchange rate models of BRICS nations' currencies with respect to the RW model. BRICS, Brazil, Russia, India, China, and South Africa; RW, random walk.

TABLE 5 Regression results for the Fair and Shiller information test of the competing exchange rate models.

Brazilian real/$					
Constant	**TMV ($r = 3$)**	**TMV ($r = 2$)**	**TMV ($r = 1$)**	**BVAR**	R^2
2.74733	−0.07295	1.22769***			0.3889
(1.677)	(−0.393)	(6.150)			
2.2312	0.3312		0.7113***		0.2563
(1.376)	(1.181)		(3.566)		
2.6774	0.1121			0.6010**	0.3017
(1.695)	(0.571)			(4.499)	
2.69442		1.20068***	−0.03456		0.3881
(1.831)		(5.116)	(−0.146)		
2.76358		1.11059***		0.4641	0.3884
(1.875)		(3.869)		(0.255)	
2.7945			0.1615	0.5658**	0.3017
(1.772)			(0.566)	(3.191)	
Russian ruble/$					
Constant	**TMV ($r = 3$)**	**TMV ($r = 2$)**	**TMV ($r = 1$)**	**BVAR**	R^2
0.7698	0.7235***	−0.1631			0.1836
(0.491)	(3.482)	(−0.674)			
0.7313	0.7873***		−0.257		0.1868
(0.469)	(3.462)		(−0.926)		
0.5060	0.5079***			0.1692	0.2001
(0.327)	(3.543)			(1.607)	
0.2381		0.2323	0.2817		0.0976
(0.145)		(0.792)	(1.075)		
0.0137		0.33467*		0.2548*	0.2541
(0.009)		(1.984)		(2.394)	
0.1496			0.3185	0.2425*	0.1329
(0.093)			(1.997)	(2.20)	
Indian rupee/$					
Constant	**TMV ($r = 3$)**	**TMV ($r = 2$)**	**TMV ($r = 1$)**	**BVAR**	R^2
0.2309	0.4312	0.6463*			0.3456
(0.377)	(1.625)	(2.484)			
0.5208	0.3217		0.8212**		0.379
(0.857)	(1.359)		(3.469)		
0.04399	0.82899***			0.4334	0.3254
(0.071)	(4.825)			(1.703)	
0.5970		0.4066†	0.7421**		0.3855
(1.033)		(0.7421)	(3.083)		
0.3018		0.8516***		0.4656†	0.3522
(0.498)		(5.339)		(1.928)	
0.6233			0.9832***	0.2526	0.3738
(1.036)			(5.749)	(0.995)	

TABLE 5 (Continued)

Chinese yuan/$					
Constant	**TMV ($r = 3$)**	**TMV ($r = 2$)**	**TMV ($r = 1$)**	**BVAR**	R^2
0.3014	1.3086**	−0.5362			0.2213
(0.850)	(3.355)	(−1.369)			
0.3188	0.86532*		−0.04762*		0.2072
(0.891)	(2.084)		(2.084)		
0.3213	0.86179***			−0.067	0.2091
(0.900)	(4.484)			(−0.515)	
0.2919		−0.2300	−0.531*		0.176
(0.800)		(−0.531)	(2.231)		
0.2610		0.6337**		0.063	0.1382
(0.700)		(3.610)		(0.489)	
0.29789			0.7366**	−0.03184	0.1742
(0.816)			(4.254)	(0.812)	
South African rand/$					
Constant	**TMV ($r = 3$)**	**TMV ($r = 2$)**	**TMV ($r = 1$)**	**BVAR**	R^2
0.6675	0.6453*	0.3579			0.344
(0.478)	(2.074)	(1.109)			
0.6376	0.6553*		0.3542		0.3436
(0.458)	(2.123)		(1.085)		
0.3540	1.1202***			−0.2893	0.343
(0.259)	(5.481)			(−1.039)	
1.2028		0.5186	0.5015		0.3313
(0.866)		(1.588)	(1.505)		
1.2321		1.0242***		−0.1082	0.3177
(0.875)		(5.010)		(−0.403)	
1.2314			1.1138***	−0.2232	0.319
(0.875)			(5.034)	(−0.784)	

Note: Each row in the table represents the Fair and Shiller test performed for two models at a time. The values in parentheses correspond to the t-values associated with the null hypothesis of $H_0 : \beta = 0$ and $\gamma = 0$ for the equation, $y_{t+s} - y_t = \alpha + \beta\left(y^*_{1t,t+s} - y_t\right) + \gamma\left(y^*_{2t,t+s} - y_t\right) + \epsilon_t$, where $y^*_{1t,t+s}$ and $y^*_{2t,t+s}$ are predicted values obtained using model 1 and 2 for time period t + s which are to be tested against the random walk model. Clearly, the TMV model outperforms the linear and RW model in terms of information content most of the time. RW, random walk; TMV, Time-Varying Transition Probability-Markov Switching Vector Error Correction Model.
***Statistically significant at the 0.1% level.
**Statistically significant at the 1% level.
*Statistically significant at the 5% level.
†Statistically significant at the 10% level.

nations. Observe how in the case of India there is a consistent increase in the performance for the density forecasts over time. In the case of Russia, a gradual increase in LPS of the TMV model over the RW can be observed during the last time period. Lastly, a stagnation in the density forecast performance of the model over RW model can be observed during the corona pandemic.

We use the information content test, also called the information encompassing test designed by Fair and Shiller (1990), to assess whether forecasts generated by a given model contain additional, meaningful information when compared to the benchmark RW model. This approach allows us to extend our analysis beyond mere forecast accuracy. The test offers two key benefits. First, it helps determine which model provides additional information when models yield similar Theil U scores, addressing scenarios where conventional accuracy measures may fall short. Second, it is possible for a model to perform worse than the RW in terms of forecast scores, yet still contain more information and be more economically meaningful. This aspect is especially relevant when modeling the complex dynamics of exchange rate movements for emerging economies like BRICS. The test involves the regression

$$y_{t+s}-y_t=\alpha+\beta\left(y^{*}_{1t,t+s}-y_t\right)+\gamma\left(y^{*}_{2t,t+s}-y_t\right)+\epsilon_t$$

Here y_{t+s} and y_t denotes the actual observed values at $t+s$ and t time period, respectively, whereas $y^{*}_{1t,t+s}$ and $y^{*}_{2t,t+s}$ are predicted values obtained using models 1 and 2 for time-period $t+s$. The test involves a null hypothesis, $H_0:\beta=0$ and $\gamma=0$ against the alternative that at least one of them is not zero. We have three defined outcomes. First, $\beta\neq 0$ and $\gamma\neq 0$, this implies that both models' forecasts contain information more than the RW model forecasts and the information contained is independent of each other; in other words, both of them encompass the RW model. Second, either of them does not equal 0. In this case, the one which is not zero only encompasses the RW. Third, $\beta=0$ and $\gamma=0$; in this case, none of the models encompass the RW model. The regression results for the test are shown in Table 5 for all currencies. Note that each regression involves two comparing models; hence, for each currency, six regression were run, and each row shows a regression for unique pairs of our competing models. We observe that the coefficients for the regression are positive in sign, implying the information associated with the forecasts of all the models is positively correlated with the actual change in exchange rates. The TMV model was able to encompass the basic RW model in all cases, signifying a higher information containment in the forecasts obtained using them. They were also able to defeat the basic BVAR in most of the cases. TMV($r=3$) seems to be containing more information than its peers, except in Brazil where TMV ($r=2$) is seen as encompassing all. We note that although in Russia, TMV for $r=2,3$ performs almost similar to the RW in Theil's U score, still they appear to contain information more than the RW with high significance.

5 | CONCLUSION

To conclude, this study employed a two-regime Markov switching model with transition probabilities that used the vector error correction mechanism to model the exchange rate for BRICS nations. The cointegration error is the latent variable responsible for the regime-switching mechanism. A fully Bayesian approach was used for parameter estimation. The Bayesian process employed two hierarchical shrinkage priors, namely the NG prior and the Litterman prior, inducing a more flexible shrinkage effect among the regime-specific parameters. The parameters are then estimated using the MCMC algorithm.

The Russian ruble was identified to be mostly depreciated, whereas the South African rand was identified to be mostly appreciated among the BRICS nations' currencies for the sample period investigated in the regime identification part. The model's identification of the states as appreciated or depreciated mostly coincided with the economic events in the respective nations during those periods. Significant differences among the regime-specific coefficients and cointegration error terms were observed from their common mean, with the larger deviation observed for the variables corresponding to the state of depreciation. Error variances appeared to be smaller in the case of appreciation for most of the BRICS nations except Russia, which exhibited lower error variance for the state of depreciation.

Forecast exercises concerning point forecasts and density forecasts were then conducted to assess the performance of our exchange rate model for the BRICS nations. For this, the data were split into two parts: the in-sample period, which spanned 2000:M1 to 2012:M12 and the out-of-sample period, spanning 2013:M1 to 2021:M12. The competing models included TMV for $r=1$, 2, and 3, and a linear BVAR against the basic RW model. Under Theil U, the TMV model was able to defeat the basic RW model and linear Bayesian VAR model for all the BRICS nations, whereas for the density forecasts, our model was able to outperform the benchmark RW model and linear BVAR model for Russia, China, India, and South Africa. The evolution of the LPS showed an increasing forecast performance over time. An information-encompassing test based on Fair and Shiller was then conducted to check if the forecasts obtained had essential economic information, and it was observed that the TMV model had significant additional information compared to the benchmark RW model and the linear BVAR model.

To conclude, our study presents a methodological advancement in exchange rate forecasting literature utilizing the TMV model. These sophisticated approaches provide a deeper comprehension of exchange rate dynamics, particularly in the context of emerging economies like BRICS. The model effectively identified the regimes correctly and showed significant improvement in forecasting over the coveted RW model. We also found that most of the currencies' states had an overlap mostly during a common global economic condition, showing a persisting level of heterogeneity among the exchange rates of the BRICS nations. The results are also important from the viewpoint of investors as any foreign investment is pivotal to the exchange rate prices as well as its dynamics. Policymakers may also take note of the connectedness among the BRICS nations' currencies to better formulate trade policies and may also help assess economic contagions. Our model future work may include the inclusion of more emerging countries in the basket. Different sets of hierarchal shrinkage priors can also be used to see if more informative results can be obtained. One can also see if the inclusion of more number of regimes helps in improvement in the forecasting performance. One can also perform forecasting exercises for longer horizons. Overall, methodological frameworks incorporating models like the TMV are adaptable and can offer more thorough analysis, making them useful for a wide range of applications, including both emerging and developed economies. Using these frameworks for future research on larger and more diverse datasets could help analyze global macro and financial trends. This could provide important insights for international economic policy and improve our understanding of the intricate dynamics in the globalized world.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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ENDNOTES

[1] https://www.dw.com/en/have-the-brics-hit-a-wall/a-51182058

[2] A sufficiently large number of draws was strategically made to ensure sufficient convergence of the Markov chains.

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