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Seat assignment recommendation in airlines purchase flow to increase ancillary revenue considering weight and balance constraints

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ABSTRACT

In the highly competitive and cost-sensitive realm of low-cost carriers, ancillary services have emerged as a pivotal revenue source, supplementing the basic fare with optional extras that enhance the passenger experience. This research propels this concept forward by introducing a sophisticated Mixed Integer Linear Programming (MILP) model specifically designed to optimise revenue from seat change fees, a key ancillary service. Our model is particularly crucial for low-cost carriers, where the natural decomposition of pricing strategies allows passengers to pay for a basic service, with the option to enhance their flying experience through additional paid services. The model introduces a novel approach to encourage seat changes, particularly for passengers booked together under the same reservation. The core strategy to promote seat changes involves maximising the seating distance between passengers who opt for the automatic seat selection feature, based on the current aircraft configuration. By intentionally allocating these passengers the furthest seats apart, the model creates a natural incentive for them to pay for seat changes, aiming to sit closer together. This approach not only generates additional revenue through seat change fees but also optimises the utilization of seat inventory by encouraging the purchase of premium seat options. To address the inherent unpredictability of seat sales, the model strategically reserves premium seats and places passengers less inclined towards seat changes in less desirable locations. This ensures an optimised allocation of seats that maximises revenue potential. Incorporating computational acceleration techniques, the model is designed for real-time application, allowing airlines to dynamically adapt to booking changes and maximise ancillary revenue opportunities. This rapid response capability empowers airlines to adapt swiftly to changing dynamics in seat bookings, thereby maximising their revenue generation potential. By offering a sophisticated tool for increasing profits from passenger accommodation services, this research bridges an essential gap in existing airline industry strategies, proposing a transformative approach to ancillary service optimisation.

1. Introduction

The airline industry faces increasing competition and pressure to improve revenue and profitability. In 2016, Latin American airlines earned around $2.15 net profit per passenger, while the industry-wide airlines' mean was over $9.13 and Latin American carriers were four times lower. Commercial aviation within Latin American countries has often been limited due to high fuel costs and political challenges. Furthermore, the currency exchange for these countries makes acquiring fuel more complex, as it is generally purchased in US dollars. Specifically, in Colombia, the charges and taxes for international passengers are excessively high, amounting to over $110 per traveller (O'Connell et al., 2020).

Research suggests that airlines are exploring ways to optimise their operations and increase revenue through ancillary services in today's competitive environment. One such service is the unbundling of flight products, where airlines sell products or services separately, such as the option to reserve a specific seat. In recent years, the revenue generated from unbundled flight products has become increasingly common among low-cost and traditional airlines.

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Nomenclature

Sets

I : Seats on the airplane

Undirected Complete Graph

Let $\mathscr{G} = (\mathscr{N}, \mathscr{A})$ a graph composed of nodes $\mathscr{N}$ and arcs $\mathscr{A}$, where $\mathscr{N} = \{i\} \in I$ and $\mathscr{A} = \{(i,j) | i \in I, j \in I, i \neq j\}$

Parameters

a_i : $\begin{cases} 1 & \text{if the seat } i \in I \text{ has not been previously assigned or blocked} \\ 0 & \text{otherwise} \end{cases}$

h_i : $\begin{cases} 1 & \text{if the seat } i \in I \text{ has not been previously assigned} \\ 0 & \text{otherwise} \end{cases}$

δ : Minimum distance between seats in the same booking

β_{ij} : $\begin{cases} 1 & \text{if seats } (i,j) \in \mathscr{G} \text{ are at least } \delta \text{ units apart} \\ 0 & \text{otherwise} \end{cases}$

c_i : Price of the seat $i \in I$.

$rank_i$: Parameter that determines how important is seat $i \in I$ relative to the others

d_{ij} : Distance of seats $(i,j) \in \mathscr{G}$

b : Extra price for the most purchased seats based on historical data

q : Number of people in the booking

s : Percentage of blocked seats

ω_1 : Weight for the cost in the objective function

ω_2 : Weight for the distance in the objective function

λ_1 : Relaxation of the weight and balance constraints

λ_2 : Relaxation of the weight and balance constraints

Variables

x_i : $\begin{cases} 1 & \text{if the seat } i \in I \text{ is assigned} \\ 0 & \text{otherwise} \end{cases}$

y_{ij} : $\begin{cases} 1 & \text{if the seats } (i,j) \in \mathscr{G} \text{ are assigned} \\ 0 & \text{otherwise} \end{cases}$

For instance, Allegiant Air, a low-cost airline, reported a significant increase of 115 percent in net income in 2009, primarily due to the revenue generated from unbundled flight products. Allegiant Air is considered a world leader in turning ancillary services into revenue, with unbundled flight products representing 30 percent of its total revenue in 2009. This indicates the potential for airlines to increase their revenue streams and profitability by leveraging ancillary services, mainly through unbundled flight products. Passengers on this low-cost airline tend to pay 5 to 25 dollars for seat assignments (O'Connell, 2011).

To be more precise, ancillary revenue defined by (IdeaWorksCompany LLC, 2022) refers to the revenue beyond tickets sales. With this in mind, most low-cost airlines based a significant percentage of their total revenue on ancillaries in 2021; examples are Viva Aerobus (44.8%), Volaris (42.9%), Wizz Air (56.0%) and Ryanair Group (44.7%), among others. Whereas non-low-cost airlines tend to depend less on ancillary revenue, such as Lufthansa Network Airlines (8.5%), Qatar Airways+ (5.2%) and Aeromexico (6.0%), among others. Showing the rapid development of ancillary revenue in low-cost airlines in recent years and the need to optimise operations. Furthermore, the rapid increase on ancillary revenue can be noticed where airlines such as GOL and Ryanair Group increased their ancillary revenue in 16.0 and 10.3 percentage points from 2019 to 2021. Also, some interesting items found in 2021 show that the airlines are coming with interesting ideas on how to increase their ancillary revenue. For instance, passengers of the airline Eurowings can book a middle seat in advance; The seat selection from advance represented an average of $7.00 per passenger for Spirits airline in 2021 (IdeaWorksCompany LLC, 2022).

As mentioned, one of the vital ancillary services is seat allocation sales, which allows airlines to generate additional revenue and improve customer satisfaction. However, optimising seat allocation sales is a challenging problem that requires considering various factors, such as passenger preferences, capacity constraints, and pricing information. According to a study by (O'Connell and Warnock-Smith, 2013), low-cost airline passengers are more likely to purchase ancillary products and services than those flying with traditional airlines. The study also found that seat reservation fees were the third most acceptable ancillary revenue service among passengers. This suggests that offering unbundled flight products, such as seat reservations, may be an effective way for airlines to increase their revenue streams and meet the growing demand for ancillary services among passengers.

The operations research applications in the airline industry are diverse. They include addressing the overbooking problem to determine the optimal number of seats to overbook, given the number of seats sold and the time before departure (Rothstein, 1971), online seat assignment (Castro and Fernando, 2020), fleet assignment (Sherali et al., 2006), simultaneous aircraft routing and crew scheduling (Cordeau et al., 2001), air traffic management using deterministic optimisation (Agustín et al., 2012) and stochastic optimisation (Agustín et al., 2012), boarding strategies (Fonseca et al., 2013), and addressing social distancing in airplane seat assignments during the COVID-19 pandemic (Salari et al., 2020).

Research indicates significant opportunities to improve airline revenue management using ancillary services. For instance, Rothstein (1971) uses dynamic programming to obtain the optimal number of seats to overbook given the mean of ticket purchases per day to develop an overbooking policy for airlines and the work in (Bertsimas and de Boer, 2005) proposed an algorithm that addresses several issues faced by revenue management models, including demand uncertainty, nesting, and the dynamic nature of the booking process. Their algorithm combines ideas from stochastic gradient and approximate dynamic programming suggesting significant revenue enhancements through simulations. However, as airlines' ancillary pricing decision-making is primarily manual, in (Kummara et al., 2021) have proposed a gradient boosting machine learning algorithm that can make automated pricing decisions for ancillaries by understanding the relations between different features such as passenger type, itinerary, aircraft type, etc. One of the benefits of this algorithm is that it can automate pricing for each customer and identify trends and patterns through its training. However, there are also some limitations to consider. For instance, the algorithm does not account for competitor information and assumes that every ancillary sold is unlimited, which may not be the case. Nonetheless, automated pricing decision-making can still significantly improve airline revenue management.

Most of the available literature on airlines deals with the above-mentioned problems, and just a few cover the seat assignment problem. However, a relevant approach is presented in (Castro and Fernando, 2020), which focuses on determining where to seat the passengers who make different online purchases. The authors proposed both deterministic and stochastic models considering several future demand scenarios. This work includes an interesting network flows-based model to retrieve the optimal seat for every passenger considering different economic groups. It is also relevant the probability that a seat is purchased and determine which factors mainly affect the purchasing behaviour. In (Mumbower et al., 2015) they model a logistic regression using real data from JetBlue to determine the relevant factors in the purchasing behaviour such as the amount of passengers traveling together, how far in advance the ticket is purchased, among others. Important insights show that airlines who block premium seats, increase seat fee revenues overall. This means airlines reduce the number of preferred seats to reserve for free and they increase their seat fee revenue. Hence, the airplane appears to be more fully reserved than it really is at online check-in, leaving the premium seats unassigned for passengers who are willing to pay for them. In this study we use a similar approach which benefits are twofold: it helps to reduce the computation time significantly and it improves revenue through the blocking of historical premium seats.

Furthermore, Given the recent COVID pandemic, airlines were interested in assigning seats as furthest as possible to prevent people from spreading the virus to each other. The problem in (Salari et al., 2020) addresses the COVID-seating issues by maximising the distance between passengers using a Euclidean approach. Another important study is presented in (Cordeau et al., 2001), where the authors use binary variables to solve the simultaneous aircraft routing and crew scheduling problem. They use Benders' decomposition and column generation algorithm, resulting in excellent computational times and results.

Although there is some existing literature on using optimisation techniques for seat allocation in low-cost flights to increase profits from the sale of ancillary services, there are still some gaps in the research. For instance, the weight and balance constraints and passenger preferences for seat locations are often not considered. Some areas that require further investigation include addressing uncertainty in the operational constraints. While optimisation techniques help determine the best seat allocation strategy. However, incorporating uncertainty in demand and operational constraints, such as aircraft capacity and crew availability, can lead to more realistic and practical solutions. Incorporation of passenger preferences: by considering customer preferences, it is possible to influence their purchasing behaviour of ancillary services, increasing customer satisfaction and airline revenue.

As described before, there has been a growing interest in using optimisation techniques to enhance seat allocation sales and other ancillary services in the airline industry. In this study, we propose a network flow-based optimisation model for flight seat allocation sales to maximise airline revenue rather than improving the customer experience. Optimisation models allow airlines to allocate seats to passengers to maximise revenue and enhance the overall customer experience.

The proposed model considers several factors that affect seat allocation, including passenger preferences, capacity and balance constraints, and pricing information. We formulate the problem as a network flow-based optimisation problem, where the objective is to maximise indirectly the ancillary revenue generated from seat allocation sales subject to various constraints, i.e., we do not include the income in the objective function. Still, we expect to increase customers' probability of buying more ancillary services. We develop an algorithm to solve the optimisation model efficiently and test it using real data from an airline's reservation system.

We have made several contributions to the field, including:

I. Introducing an innovative network-flow-based formulation, this approach strategically motivates customers to increase their purchase of bundled ancillaries, including seat assignments. It leverages the distance between passengers traveling together as the primary criterion, ingeniously encouraging the selection of ancillary bundles to enhance their travel experience.
II. A comprehensive set of linear constraints to maintain the aircraft's weight balance throughout the seat assignment process, ensuring operational safety and efficiency without compromising the strategic ancillary sales model.
III. Developing a statistical method to reserve premium seats, targeting future customers with a higher propensity to purchase these high-value ancillaries, thereby optimising revenue potential from seat selection options.
IV. Proposing an acceleration technique that blocks premium seats based on historical data. Its advantages are twofold: accelerate the model and increase revenue.

The primary goal of this paper is to demonstrate the potential benefits of network flow-based optimisation for airline seat allocation sales. We aim to show that the proposed model can help airlines increase their ancillary revenue per person by allocating seats more effectively and efficiently. We also seek to contribute to the growing literature on optimisation techniques for airline ancillary services, focusing on seat allocation sales.

The structure of this document is as follows: Section 2 introduces the overall ticket sales procedure in an airline. The optimisation problem description is presented in Section 3. The networks flow-base model, including the formulation and methodology is in Section 4. The computational experiments including case study, the Pareto front and the decision-making approach proposed are presented in Section 5. Finally, the conclusions and future work are presented in Section 6.

2. Description of the airline purchase procedure

The company we are working with is a low-cost airline founded in Colombia in the early 2000s. This airline offers a variety of national and international low-cost flights in the American continent. As one of the first low-cost airlines in Colombia and Peru, it was able to reduce 66% and 30% respectively the flight's prices of their competitors in those countries. With over 20 aircraft and 35 routes, it operates approximately 40,000 flights annually to offer its customers affordable prices.

This research focuses on automatic seat assignments during check-in, specifically for customers who have not yet purchased a seat. If a customer decides not to buy a seat at check-in, the airline must assign a seat for free. Ideally, this seat should be one of the cheapest and historically unpopular among customers. Additionally, seats should be assigned as far apart as possible for check-ins with multiple passengers to encourage customers to purchase seats in the future. The ultimate objective is to indirectly increase ancillary revenue by making customers more likely to buy seats on future flights.

The overall ticket sale and seat assignment procedure can be seen in Fig. 1. It is important to note that the airline's sales process may differ from the general case for all low-cost airlines.

Following the "no" path in Fig. 1 leads us to the stage of seat assignment for pending customers who have chosen not to purchase a

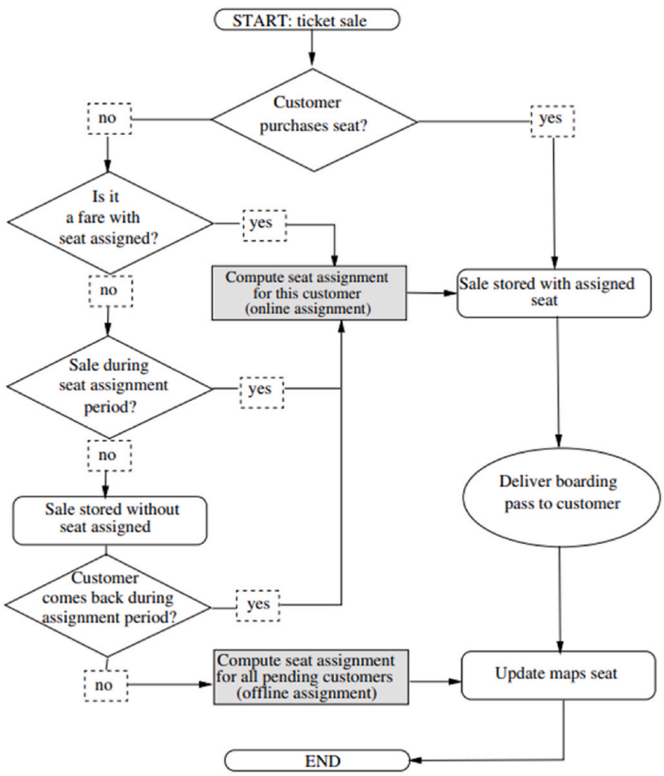

Fig. 1. Overall tickets sale and seat assignment procedure (Agustín et al., 2012).

seat. This occurs at check-in, but it is important to note that seats can also be bought at the counter desk just before flight departure or that passengers may change seats at an additional cost. Therefore, the model must ensure that preferred seats remain unassigned.

3. Optimisation problem description

In this work, we consider the seat assignment for passengers that have not paid their seat fee. To address this problem, we model the plane as a network where each node represents the seats I and the arcs connect the seats in the same booking. Let $\mathscr{G} = (\mathscr{N}, \mathscr{A})$ be a complete network where $\mathscr{N} = \{i\} \in I$ is the node set and $\mathscr{A} = \{(i,j) | i \in I, j \in I, i \neq j\}$ denotes the set of arcs. A graphical representation of a graph of a hypothetic airplane with six rows and six columns with three seats assigned in the same booking is shown in Fig. 2.

Tailored specifically for low-cost carriers, our approach recognises the unique market positioning and operational strategies of these airlines, distinguishing them from full-service counterparts that may prioritise keeping passengers of the same booking close together. Our model offers a strategic balance aimed at maximising ancillary revenue by leveraging seat assignment dynamics. It intentionally increases the seating distance between passengers of the same booking to stimulate interest in paid seat changes, thereby boosting ancillary service income. Concurrently, the model conservatively assigns passengers who initially forego the option to select their seats, with a focus on preserving premium seats for future customers willing to pay for the privilege. This dual-pronged strategy is designed to enhance ancillary revenue streams while ensuring premium seating availability, catering to the operational and financial goals of low-cost carriers. Through this approach, we provide a solution that aligns with the distinctive needs of low-cost airlines, optimising revenue opportunities and strategic seat management (Subramanian et al., 1994). For this reason, we propose a mathematical formulation that aims to maximise the distance between passengers in the same booking. We assume each seat has a fixed distance d_{ij} measured using the Manhattan

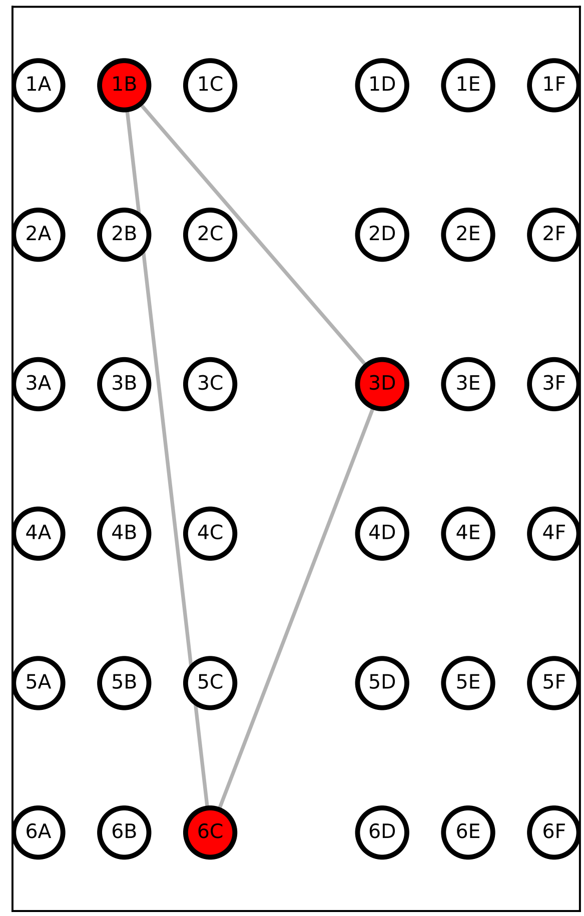

Fig. 2. Complete graph for an airplane.

approach. Additionally, if the airline's goal is to maximise, the minimum distance established is δ, characterized by β_{ij} in the mathematical formulation. To be more precise:

$$\beta_{ij} : \begin{cases} 1 & \text{if the pair of seats } (i,j) \in \mathscr{G} \text{ are at minimum } \delta \text{ units of distance} \\ 0 & \text{otherwise} \end{cases}$$

Also, the number of people q in the same booking impacts the feasibility of the model when δ and q are large, this problem is addressed by reducing δ by one unit until the model becomes feasible again. Furthermore, there are seats that are usually preferred by passengers; therefore, these seats tend to be sold quicker and airlines might not want to assign these preferred seats for free. That's why our mathematical formulation accounts the uncertainty that a seat will be bought in the future with probability distributions characterized by $rank_i$, which determines how important is seat $i \in I$ relative to the other seats. This parameter makes part of the cost structure c_i of each seat $i \in I$. Another way we address the uncertainty is by blocking a specified percentage (denoted by s) of the most important seats (i.e., seats with the higher $rank_i$).

On the other hand, our aim is to assign for free the seats with the lowest cost c_i and if there are seats already assigned, then a_i prevents the model to select seats previously sold or blocked based on the historical data. More precisely:

$$a_i : \begin{cases} 1 & \text{if the seat } i \in I \text{ has not been previously assigned or blocked} \\ 0 & \text{otherwise} \end{cases}$$

Also, to calculate the centre of mass and ensure the weight and balance in the aircraft, we use h_i that has the same function of a_i but it does not consider the artificially assigned seats (blocked seats), therefore we are able to calculate the centre of mass with the actual assigned seats:

$$h_i : \begin{cases} 1 & \text{if the seat } i \in I \text{ has not been previously assigned} \\ 0 & \text{otherwise} \end{cases}$$

Regarding both goals, when one wants to maximise distance between passengers in the same booking and assign the less preferred seats results in conflicting objectives. This must be addressed using multi-objective techniques that give optimal weights ω_1 and ω_2 that show the trade-offs the decision-maker needs to address.

Building on our strategy tailored for low-cost carriers, this work is driven by three primary objectives, designed to align with the specific operational and financial nuances of these airlines. The first objective seeks to indirectly maximise ancillary revenue through a strategic seat allocation policy that leverages the airline's preferences. By maximising the distance between passengers of the same booking, we aim to encourage them to opt for paid seat changes, thereby generating additional revenue streams for the carriers. This strategy not only increases immediate revenue but also cultivates a customer behaviour inclined towards future purchases of seat selections, enriching the airline's ancillary income over time. The second objective focuses on minimising the opportunity cost of seat assignments for passengers who have not paid for seat selection. Recognising the value attributed to certain seats—be it for their location, like window seats, those with extra legroom, or those situated in the front rows for quicker aircraft egress—this aspect of our model aims to strategically reserve these high-demand seats. This ensures they remain available for passengers willing to pay a premium, thus optimising the airline's revenue potential from seat selection fees. Lastly, we address the efficiency of our model's implementation with the third objective: to reduce the computational time required to solve the model. This focus on computational efficiency is crucial for real-time application, allowing airlines to swiftly adjust their seat allocation strategies in response to dynamic booking patterns, without compromising the quality of the solutions.

4. Methodology

To define the model, it is key that we want to maximise or minimise the distance between passengers, depending on the decision-maker and select the cheapest seats making it a multi-objective function using network flow-based model. All the concepts have been reviewed and retrieved from the Network Flows book (Orlin et al., 1993) and an interesting motivation example from Tabares et al. (2019) use all the concepts in network flows to calculate the standard network-dependent reliability indices of distribution systems by solving linear equations. This study presents an interesting perspective on how network flows can be applied to airline operations. Regarding multi-objective optimisation problems and the Pareto front, the work in (Dias de Lima et al., 2021) was used as a guide to build the Pareto front and the fuzzy function to assist decision-makers in the selection of weights in the objective function. As for accelerating the computation time, several heuristics and algorithms, such as "GRASP" and "warm start" variants, were used in (Cuellar-Usaquén et al., 2023). Their approaches are compared through benchmarking, and the computational time is analysed, illustrating the pros and cons of each approach implemented in terms of solution and time which helped us to identify parameters and techniques to apply on this study.

4.1. The network flow-based model

$$min \sum_{i\in I} \omega_1 c_i x_i + \sum_{(i,j)\in \mathscr{G}} \omega_2 d_{ij} y_{ij} \tag{1}$$

subject to:

$$\sum_{i\in I} x_i = q \tag{2}$$

$$(q-1)x_i - \sum_{j|(i,j)\in \mathscr{G}} y_{ji} = 0, \forall i \in I \tag{3}$$

$$(q-1)x_i - \sum_{j|(i,j)\in \mathscr{G}} y_{ij} = 0, \forall i \in I \tag{4}$$

$$x_i \le a_i, \forall i \in I \tag{5}$$

$$y_{ij} \le \beta_{ij}, \forall (i,j) \in \mathscr{G} \tag{6}$$

$$x_i \in \{0,1\}, \forall i \in I \tag{7}$$

$$y_{ij} \in \{0,1\}, \forall (i,j) \in \mathscr{G} \tag{8}$$

The multi-criteria function (Eq. (1)) minimises the seat assignment cost at check-in, selecting the cheapest seat available in the aircraft. Additionally, (1) maximises (or minimises) the distance between each pair of seats chosen. Constraint (2) guarantees that the number of seats selected equals the number of people in the booking. Constraints (3) and (4) connect seats in the same booking, and guarantee flow through corresponding arcs. Constraints (5) prevents using previously assigned seats. Constraints (6) ensure that seats $(i,j) \in \mathscr{G}$ are at the minimum distance δ required by the decision-maker. If the aircraft is mainly occupied, the model becomes infeasible. Therefore, the parameter δ is relaxed until the model is feasible again. Ultimately, Equations (7) and (8) represent the domain of the variables.

Given all the possible scenarios of aircraft occupancy and passengers in the same booking, it is possible that the model becomes infeasible when δ is too large and the airplane is mostly occupied, also it can happen that in the same booking there are over 20 passengers and therefore it doesn't make sense to maximise distance between them. Thus, an algorithm was designed to address all the possible scenarios while maintaining an appropriate optimality gap and a reasonable computation time. We tested our algorithm under real instances provided by an airline (with some assumptions).

In this case, we received a large dataset with relevant information including booking date, seat booking date and time (if applicable) and flight number for legs departing from several Colombian cities like Bogotá, Medellín, Cali, among others and arriving in San Andrés Island. Since we don't account with the check-in date of the bookings, we assumed the booking date was the same. Afterwards, we filtered the reservations that never booked a seat and retrieved the number of people in the booking in a chronological order and solved the model for every reservation until every booking in the flight had seats assigned.

4.2. Mono-objective static model

The future purchases of seats are highly stochastic. To capture the behaviour of the customers we calculate $rank_i$ which determines how important is seat $i \in I$ relative to the other seats. This way, we find the most valuable seats for regular airline customers and add the relative purchase frequency to the cost structure of each seat in the mathematical model. We want to allocate passengers in the cheapest and less purchasable seats.

To determine the final cost of each seat, the following formula was applied:

$$c_i = base_i + rank_i * b \quad \forall i \in I \tag{a}$$

where the cost of each seat $i \in I$ is determined by its original price (given by the airline in thousand COP) plus its relative importance ($rank_i$) times a tuneable bonus cost b. Therefore, the most important seats with $rank_i = 1$ have an additional cost of b.

We tested our algorithm minimising only the cost of the seats selected (i.e., $\omega_1 = 1$ and $\omega_2 = 0$), under different initial occupancy scenarios of the aircraft and setting $\delta = 0$, the minimum distance between seats in the same reservation.

4.3. Multi-objective static model

Following our premise, we also want passengers in the same booking to be as far as possible. Therefore, we add the second objective which maximises distance, while also preserving the weight and balance constraints in the aircraft.

To measure the distance between each seat, we use the Manhattan distance modelling the aircraft as a grid:

$$d_{ij} = |x_i - x_j| + |y_i - y_j|, \forall (i,j) \in \mathscr{G} \tag{b}$$

where x_i stands for the x-coordinate and y_i for the y-coordinate of seat $i \in I$, avoid confusions with the decision variables x_i and y_{ij}. The x and y coordinates were assigned accordingly to each seat, and one extra unit was added for the aisle.

Regarding the weight and balance constraints, it is only considered in the model when the aircraft's occupancy is between 40% and 70%. When the airplane is mostly empty, the balance depends more in other factors such as the baggage. On the contrary, when the airplane is over 70% occupied the balance is considered automatically.

We measure the centre of gravity (CG) and it is calculated using Eqs. (c) and (d), assuming that each passenger has the same mean weight $\overline{m}$, hence:

$$CG_x = \frac{\sum_{i \in I} h_i l_i \overline{m}}{\sum_{i \in I} h_i \overline{m}} = \frac{\sum_{i \in I} h_i l_i}{\sum_{i \in I} h_i} \tag{c}$$

$$CG_y = \frac{\sum_{i \in I} h_i r_i \overline{m}}{\sum_{i \in I} h_i \overline{m}} = \frac{\sum_{i \in I} h_i r_i}{\sum_{i \in I} h_i} \tag{d}$$

Assuming again that l_i stands for the x-coordinate and r_i for the y-coordinate of seat $i \in I$ to avoid confusions with the decision variables x_i and y_{ij}. The idea with this approach is to maintain the CG of both coordinates within a margin close to 0 and only account for the seats assigned h_i in the calculation of CG. Desirably we want CG_y and CG_x to be 0 or near to 0.

When the aircraft is between 40% and 70% occupied, we add the constraints $(9)-(12)$, derived from CG_x and CG_y. Since these constraints were obtained from the general expression of centre of gravity, it is desirable that its value tends to 0 to guarantee that the centre of gravity of the plane is centred. This is usually not feasible, therefore, there is a relaxation λ_1 and λ_2 that makes it feasible and let the CG be close to 0 and safe.

$$\sum_{i \in I} \left(h_i l_i + x_i l_i \right) \geq -\lambda_1 \tag{9}$$

$$\sum_{i \in I} \left(h_i l_i + x_i l_i \right) \leq \lambda_1 \tag{10}$$

$$\sum_{i \in I} \left(h_i r_i + x_i r_i \right) \geq -\lambda_2 \tag{11}$$

$$\sum_{i \in I} \left(h_i r_i + x_i r_i \right) \leq \lambda_2 \tag{12}$$

Following the same notation, l_i stands for the x-coordinate and r_i for the y-coordinate of seat $i \in I$ to avoid confusions with the decision variables x_i and y_{ij}. Also, it is important to highlight that the parameter h_i has the same function of the parameter a_i, but it does not account for the blocked seats since they are not assigned in reality and it would not make sense to calculate the CG using them. Constraints $(9)-(12)$ were linearised and are based on Eqs. (c) and (d), they aim to balance the number of assigned people at each side of the plane. The centre of the coordinate system is in the exact centre of the plane; therefore, constraints (9) and (10) admit at most λ_1 extra passengers in the left or right side of the plane. Whereas constraints (11) and (12) admit at most λ_2 extra passengers in the rear or front from the plane. If λ_1 and λ_2 take the value of 0 and the assignment is feasible, this means that the centre of gravity is centred; however, this may result infeasible, and we admit some imbalance defined by the parameters mentioned. For instance, if the airplane has 20 people assigned in the front, constraints (11) and (12) will ensure that there are at most λ_2 passengers extra in the rear part. λ_1 and λ_2 are tuneable parameters that controls the level of balance and weight in the airplane.

4.3.1. The augmented ε-constraint method

In addressing the multi-objective optimisation problem of seat allocation for low-cost carriers, we apply the augmented ε-constraint method (Mavrotas, 2009). This method effectively circumvents the issue of dominated solutions that are prevalent in the standard ε-constraint approach. Specifically, one objective function—maximising the distance between passengers in the same booking—is optimised, while the other objective is incorporated as constraints with adjustable parameter, ε, which are iteratively varied to explore the Pareto front.

Our multi-objective model initially transforms the secondary objective, the minimisation of premium seat allocation, into a constraint with an additional slack variable (SV_2). This transformation adjusts the secondary objective from a simple inequality into an equality, ensuring a balance between our objectives: minimising the number of premium seats allocated unintentionally and maximising ancillary revenue through incentivised seat changes. The adjusted objective function is represented as:

$$\min \sum_{i \in I} \omega_1 c_i x_i - \eta (SV_2 / r) \tag{13}$$

subject to: (2), (3), (4), (5), (6), (7) and (8)

$$\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij} + SV_2 = \varepsilon \tag{14}$$

The value η is chosen to be sufficiently small, ensuring the primary focus remains on $\sum_{i \in I} \omega_1 c_i x_i$, while r represents the range of $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$ values, promoting a balanced Pareto front. This fine-tuning is crucial to maintaining operational integrity while pursuing revenue optimisation.

The iterative process for constructing the Pareto front is as follows:

1. Determine a small decrement value α.
2. Set the initial value of ε to the maximum value of $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$ minus α.
3. Solve the optimisation problem to determine the values of $\sum_{i \in I} \omega_1 c_i x_i$ and $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$.
4. Adjust ε by reducing it by the amount of α or by the value obtained for $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$ in the previous iteration, whichever is smaller.
5. Continue this iterative process until ε reaches the minimum value of $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$.

Once the Pareto front is established, we employ a fuzzy decision-making approach to select the optimal compromised solution from the set of non-dominated solutions (Dias de Lima et al., 2021). This selection is guided by linear fuzzy membership functions μ_p^{cost} for $\sum_{i \in I} \omega_1 c_i x_i$ and $\mu_p^{distance}$ for $\sum_{(i,j) \in \mathscr{G}} \omega_2 d_{ij} y_{ij}$, which measure the degree to which each objective is achieved for each solution. The compromise ratio is then calculated using the set of constraints (15)–(17) to find the most balanced solution cost and distance, with importance factors ω_1 and ω_2

tuned to reflect the carrier's specific priorities, i.e., these factors are selected by the decision maker based on the company's preferences.

$$\mu_p^{cost} = \begin{cases} 1 \quad OF_p^{cost} \le \underline{OF_1} \quad \forall p \\ \dfrac{\overline{OF_1} - OF_p^{cost}}{\overline{OF_1} - \underline{OF_1}} \quad \underline{OF_1} \le OF_p^{cost} \le \overline{OF_1} \forall p \\ 0 \quad OF_p^{cost} \ge \overline{OF_1} \ \forall p \end{cases} \tag{15}$$

$$\mu_p^{distance} = \begin{cases} 1 \quad OF_p^{distance} \le \underline{OF_2} \quad \forall p \\ \dfrac{\overline{OF_2} - OF_p^{distance}}{\overline{OF_2} - \underline{OF_2}} \quad \underline{OF_2} \le OF_p^{distance} \le \overline{OF_2} \forall p \\ 0 \quad OF_p^{distance} \ge \overline{OF_2} \ \forall p \end{cases} \tag{16}$$

$$\mu_p = \frac{\omega_1 \mu_p^{cost} + \omega_2 \mu_p^{distance}}{\omega_1 + \omega_2} \quad \forall p \tag{17}$$

where OF_p^{cost} and OF_p^{cost} are the objective functions values for each point p of the Pareto front and $\overline{OF_i}, \underline{OF_i}$ represent the maximum and the minimum values of each vector, respectively.

By integrating these advanced optimisation and decision-making methodologies, our model provides low-cost carriers with a robust tool for strategic seat allocation, enhancing revenue without compromising customer experience or operational efficiency.

4.4. Blocking seats framework

In this subsection we explain how the seat blocking and the dynamic multi-objective model work together, aiming to increase the overall revenue. A common practice in the airline industry is to block premium seats so that passengers cannot book them for free. This increase the purchase probability by leaving the most historically purchasable seats unassigned and in some cases, this may lead in passenger's unsatisfaction if they get to notice that the seats were blocked, meaning that the blocking must be prudent. Anyway, this practice influences customers that usually do not purchase premium seats (Mumbower et al., 2015). The airplane appears to be more fully reserved than it really is, and this is accounted in the parameter a_i but not in h_i. The key idea is to leave premium seats unassigned for passengers willing to pay for them.

Firstly, our algorithm blocks the number of seats specified by the parameter s. Secondly, it receives the number of passengers q and depending on this parameter different stopping criteria are applied. For instance, if $q < 7$ the optimality gap is at most 5.0%; if $7 \le q < 10$ the optimality gap is at most 15.0%, if $10 \le q < 20$ the optimality gap is at most 20.0%, otherwise the booking is discarded by the model. Regarding the minimum distance δ it is initialised in 7 units so that the passengers are not located in the same row. If the aircraft is too full and the minimum distance δ cannot be possibly accomplished, the model becomes infeasible and δ is relaxed by one unit until the model is feasible again. This procedure makes the computation time longer for these iterations and when seats are finally assigned by the model, q most important seats are unblocked for the next booking and parameter a_i is updated accordingly.

To handle uncertainty and increase the purchase probability of a seat, we block premium seats with the higher $rank_i$ by setting $a_i = 1$, which are later unblocked when new seats are assigned, as explained in Fig. 3.

5. Computational experiments

All the results were run in processor Intel(R) Core (TM) i5-8250U CPU @ 1.60 GHz 1.80 GHz, RAM 8.00 GB (7.86 GB useable) in type 64-bit operating system, x64-based processor in Gurobi optimiser under version 10.0 and academic license.

In this section we will present all the relevant results. First, we will show the statistical results obtained and the relevant parameters in the mathematical model. Second, we expose the model's seat assignments and performance varying the weights in the objective function for different cases. Third, we show the multi-objective approach analysis including the Pareto front and a fuzzy function to assist decision-makers. Finally, we demonstrate the acceleration results.

5.1. Case study

From the historical data provided by the airline, we found out the 10 most purchased seats shown in Fig. 4a. We determined that every seat has been purchased at least one time and therefore the $rank_i$ of any seat is greater than 0.

Fig. 4 shows descriptive statistics for the historical seats purchased and number of people per booking. Specifically, Fig. 4a shows the results obtained for the parameter $rank_i$ for the top 10 seats, where it was determined that the most important seats in our model are 24B and 24A. Furthermore, it is essential to have an idea of how the number of people per booking behaves on regular flights to determine the stopping criteria in the model based on q. Fig. 4b shows a histogram to have an intuition of the probability distribution for the number of passengers in each booking. Statistical analysis was made with historical data from 345 flights. From Fig. 4b we can determine that the most common number of people in a booking is one passenger, followed by two and three, meaning that the model must work more efficiently for this number of passengers in the same booking. It is also interesting to see that there are bookings where the number of people is 20 or even more. This is due to charter sales, and in this particular case, where there are more than ten people or so, and this type of booking are discarded by the model.

The original cost of each seat determined by the airline in thousands COP is shown next in Table 1.

Table 1 depicts the cost provided by the airline. This is represented by $base_i$ in Eq. (a). Having these results in mind, we are now able to compute the final cost of each seat using Eq. (a). More details, like the seat numeration, can be found in the GitHub repository.[1]

5.1.1. Results for the seat assignment

It employed a color-coding scheme to elucidate the seat assignment process within our proposed model. This system is pivotal for interpreting the set of Fig. 5, which illustrate the model's assignment pattern. Specifically, red markings with an "A" denote seats that were previously assigned and thus are not allocated by the model. Green markings, accompanied by a number, indicate the sequential order of assignment by the model; seats sharing the same number belong to the same booking. This numbering commences at 1 and continues until all bookings are allocated. Notably, the green hue intensifies in correspondence with later allocations.

5.1.1.1. Mono-objective static model. In this section, we test the mathematical model using only the first objective, i.e., $\omega_1 = 1$ and $\omega_2 = 0$. Therefore, we seek to minimise the cost associated to assign seats and to leave the most probable seats to be purchased in the future without any assignation. Additionally, we run the model for different initial occupancy scenarios of the airplane starting with 30% of assigned seats, continuing with 50% and finally 80%. Assignation results are shown next.

Fig. 5 depicts how the seats were assigned in each iteration. The number in the seat represent the iteration when it was assigned; if two or more seats have the same number, it means they were selected in the

[1] Electronic companion–Seat Assignment Recommendation in Airlines Purchase Flow to Increase Ancillary Revenue Considering Weight and Balance Constraints: Data and Codes (2024) https://github.com/germanrpardo1/Airline-Seat-Assignment-MIP- [Accessed 27 Mar 2024].

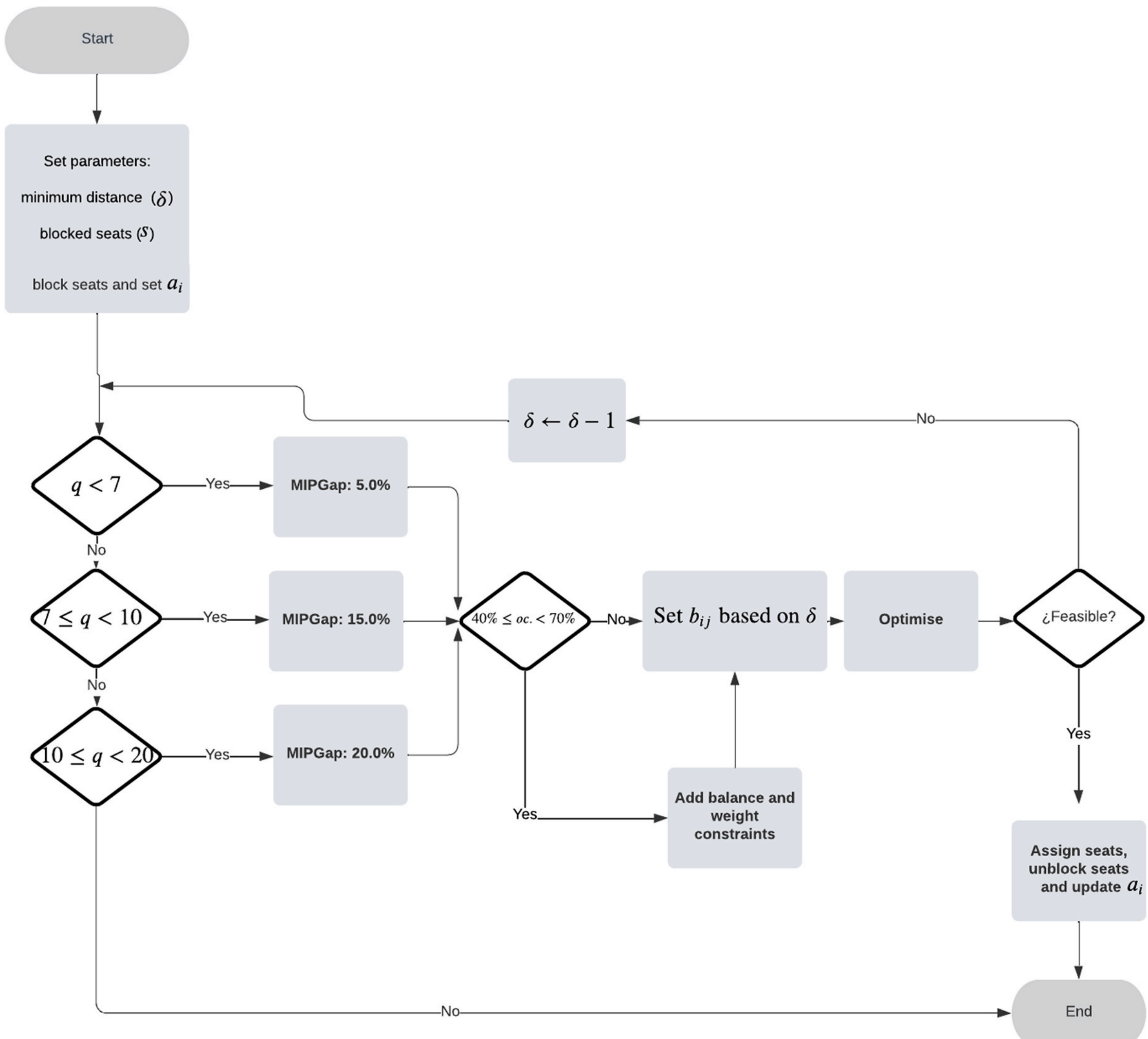

Fig. 3. Solution methodology.

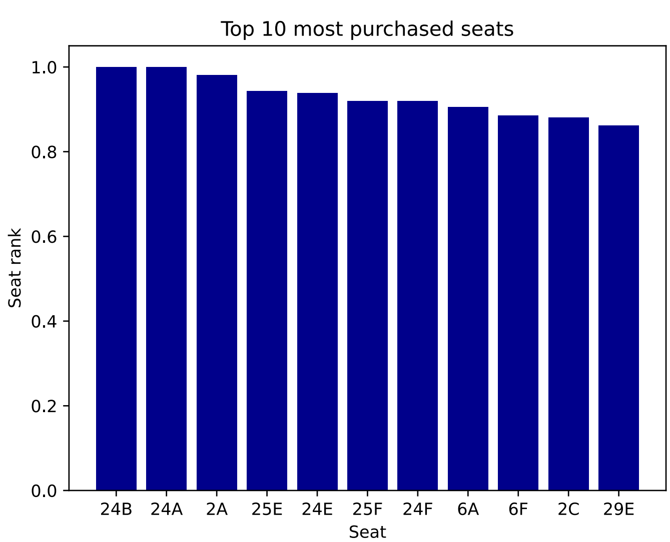

Fig. 4a. Rank for the top 10 most purchased seats.

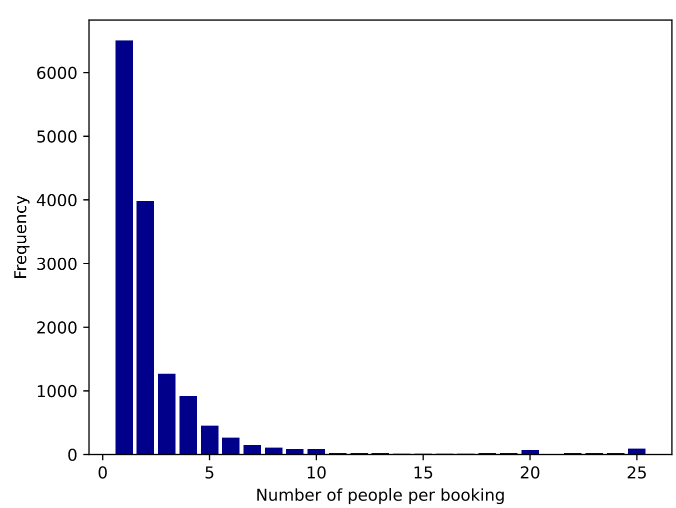

Fig. 4b. Histogram of the number of people per booking.

Table 1
Original cost in thousand COP.

Original cost [k COP]			
Row	A Window	B Middle	C Aisle
1	39	34	39
2	34	29	34
3	34	29	34
4	34	29	34
5	34	29	34
6	27	22	27
7	27	22	27
8	27	22	27
9	27	22	27
10	27	22	27
11	27	22	27
12	29	24	29
13	29	24	29
14	18	12	18
15	18	12	18
16	18	12	18
17	18	12	18
18	18	12	18
19	18	12	18
20	18	12	18
21	18	12	18
22	18	12	18
23	18	12	18
24	14	9	14
25	14	9	14
26	14	9	14
27	14	9	14
28	14	9	14
29	14	9	14
30	14	9	14
31	14	9	14
32	14	9	–

Row	A	B	C	D	E	F	Row
1		A	A	44			1
2	A	A	A				2
3		A	39	45	38		3
4		43	33	22	28	32	4
5	20	20	A	A	14	12	5
6			A	32	A		6
7		43	A	16	29	46	7
8	A	44	A	11	26	28	8
9	37	29	9	13	7	A	9
10	A	17	A	2	A	14	10
11	16	A	A	2	A	4	11
12	21	13	3	3	13	22	12
13	3	A	A	2	A	8	13
14	40	A	A	5	28	26	14
15	37	41	A	A	31	30	15
16		42	19	2	10	23	16
17	29	27	2	1	6	A	17
18	20	12	A	A	A	A	18
19	A	28	16	A	A	20	19
20	29	28	3	A	24	28	20
21	A	A	2	1	20	24	21
22	A	46	25	6	A	3	22
23	29	23	3	A	15	12	23
24			24	20			24
25	A		28	A		A	25
26	46	35	17	A	38	36	26
27	A	A	26	23	A	39	27
28	20	A	4	3	22	18	28
29	38	34	14	A		A	29
30	29	A		A			30
31	A			A			31
32		A					32

Fig. 5a. Mono-objective seat assignment with 30% of seats assigned.

Row	A	B	C	D	E	F	Row
1	A	30	A	28	29	32	1
2	37	A	A	37	29	32	2
3	33	A	A	28	28	29	3
4	A	28	24	16	A	23	4
5	13	14	A	A	10	8	5
6	36	A	27	23	A	35	6
7	A	A	21	A	A	A	7
8	A	A	13	6	20	A	8
9	26	A	5	A	A	20	9
10	14	A	2	A	3	A	10
11	A	4	A	1	3	A	11
12	14	A	A	A	9	A	12
13	A	A	A	2	2	4	13
14	A	A	22	A	A	20	14
15	26	A	A	A	A	A	15
16	A	A	A	A	6	16	16
17	A	A	2	A	3	3	17
18	A	7	A	A	A	25	18
19	19	A	12	A	A	13	19
20	23	A	A	2	20	A	20
21	A	26	2	1	A	20	21
22	29	28	20	3	3	A	22
23	22	17	A	A	12	A	23
24	A	34	18	A	A	31	24
25	29	A	A	20	A	A	25
26	A	A	A	A	28	A	26
27	A	A	A	16	24	28	27
28	A	A	A	3	15	12	28
29	A	24	11	17	29	A	29
30	22	A	40	A	A	A	30
31	38	39	41	A	A	38	31
32	38	39					32

Fig. 5b. Mono-objective seat assignment with 50% of seats assigned.

Row	A	B	C	D	E	F	Row
1	A	12	A	6	A	13	1
2	A	12	A	A	10	A	2
3	14	A	A	A	A	A	3
4	A	A	A	A	A	A	4
5	A	A	A	2	A	3	5
6	14	A	A	4	A	14	6
7	12	A	A	A	A	A	7
8	A	A	A	A	A	A	8
9	A	A	A	A	A	3	9
10	A	3	A	1	A	A	10
11	3	2	A	A	2	A	11
12	3	A	1	A	A	A	12
13	2	A	A	A	A	A	13
14	A	A	A	A	A	A	14
15	5	A	3	A	A	A	15
16	A	A	A	A	A	A	16
17	A	A	A	A	A	A	17
18	A	A	A	2	A	A	18
19	A	A	A	A	A	A	19
20	A	A	A	A	A	A	20
21	A	A	A	A	A	A	21
22	A	8	A	A	A	A	22
23	4	A	A	A	3	A	23
24	A	A	A	A	A	13	24
25	A	A	A	A	A	13	25
26	7	A	A	A	6	A	26
27	A	9	A	A	A	A	27
28	A	A	2	A	A	A	28
29	A	A	A	A	11	A	29
30	A	A	A	16	A	16	30
31	A	A	16	A	A	A	31
32	A	15					32

Fig. 5c. Mono-objective seat assignment with 80% of seats assigned.

Table 2
Weights in the objective function.

Objective function	ω_1	ω_2
Both objectives	0.55	− 0.45
Only distance	0.0	− 1.0
Only cost	1.0	0.0

Row	A	B	C	D	E	F	Row
1	A		29	A			1
2				A	A	A	2
3	A	44	46		38		3
4	A	A	24	A	20	A	4
5	3	14	13	A	12	2	5
6			28	29			6
7			A	16	43		7
8		A	A	3	37	26	8
9	A	A	17	A	A	28	9
10	A	23	A	2	13	20	10
11	20	16	1	4	14	12	11
12	3	A	A	8	24	A	12
13	2	6	10	5	A	3	13
14			A	A		A	14
15	A		31	25		A	15
16			A	7	22	28	16
17	46	A	3	2	21	A	17
18	26	20	A	18		A	18
19	34	A	27	A	A	A	19
20	A	43	9	15	35	42	20
21			4	1	29	20	21
22			36	19	17	2	22
23	29	28	11	3	23	A	23
24			A	30			24
25		A	37	32	A		25
26		38	22	26	44	39	26
27	A		23	A	A	A	27
28	A	14	12	6	16	3	28
29	A	24	2	A		46	29
30	13				A		30
31		A	A			A	31
32	A	A					32

Fig. 6a. Multi-objective seat assignment with 30% of seats assigned.

same booking. For instance, in Fig. 5a one can see that seats 17D and 21D were selected in the first iteration for the same booking. Moreover, the letter "A" means that the seat was initially assigned, and the model didn't consider that seat for the allocation. The model is always assigning the less probable purchasable seats based on historical data and, since the airplane ends almost full in most cases, the weight and balance constraints are automatically considered. One key insight to notice, is that the model tends to group the seats in the same reservation very close to each other, therefore we are not accomplishing with our objective of maximising distance between passengers.

5.1.1.2. Multi-objective static model. Following our premise, we want to allocate passengers in the same booking as far as possible. But at the same time, we want to assign the less preferable seats so that they can be bought in the future by other customers. Another important aspect to consider, is that we want to allocate passengers preserving the balance and weight constraints in the plane. This will only take place when the airplane is between 40% and 70% full, because in the rest of the cases the passenger assignment will not affect as much as other factors such as the baggage or the petrol that is constantly consumed when the airplane is flying.

The model was run using different instances and unless otherwise specified $\delta = 7$ and $b = 100$. The parameter δ starts with a value of seven so that, at least, the seats assigned in the same booking are not in the same row. As the airplane gets fuller, the problem becomes infeasible

Row	A	B	C	D	E	F	Row
1	A	29	2	A	28	20	1
2	A	37	44		32	38	2
3	42	20	A	A	A	29	3
4	28	A	A	A	13	A	4
5	A	A	12	4	2	A	5
6	45	41	A	22	A	A	6
7	38	33	3	14	20	A	7
8	A	29	16	A	17	A	8
9	A	26	A	A	A	A	9
10	A	A	6	A	11	A	10
11	A	A	A	A	3	2	11
12	14	A	A	9	A	21	12
13	A	1	A	A	A	12	13
14	28	30	A	A	25	A	14
15	27	29	A	18	A	A	15
16	35	A	19	7	A	22	16
17	20	23	3	5	15	A	17
18	16	13	8	A	26	A	18
19	A	A	A	A	24	A	19
20	A	A	A	10	20	23	20
21	29	A	A	1	A	A	21
22	34	32	A	A	14	A	22
23	22	A	2	13	A	3	23
24	A	39	A	A	38	A	24
25	A	36	17	A	37	A	25
26	3	A	A	A	A	A	26
27	20	26	A	A	A	16	27
28	A	A	4	6	12	2	28
29	A	A	A	3	24	20	29
30	2	A			A	46	30
31	A				A		31
32	A	A					32

Fig. 6b. Multi-objective seat assignment with 50% of seats assigned.

Row	A	B	C	D	E	F	Row
1	3	A	A	A	A	A	1
2	A	A	14	A	12	A	2
3	A	A	A	A	A	A	3
4	A	A	A	A	A	A	4
5	A	A	2	A	A	A	5
6	A	A	15	13	A	A	6
7	A	A	A	3	A	14	7
8	A	A	A	A	10	A	8
9	A	12	A	A	A	2	9
10	2	A	5	A	4	A	10
11	A	A	A	A	A	A	11
12	A	A	A	A	A	A	12
13	1	A	A	A	A	3	13
14	3	A	A	A	A	A	14
15	A	A	A	A	A	A	15
16	A	A	A	A	6	A	16
17	A	A	A	2	A	A	17
18	8	A	A	7	A	A	18
19	A	11	A	3	A	A	19
20	A	A	4	A	A	A	20
21	A	A	A	A	A	A	21
22	A	13	A	6	A	A	22
23	A	3	A	A	A	A	23
24	A	A	9	A	16	16	24
25	A	A	A	2	14	13	25
26	A	A	A	A	A	A	26
27	12	A	A	3	A	A	27
28	A	A	A	1	A	A	28
29	2	A	A	A	A	A	29
30	A	A	A	A	A	A	30
31	A	A	A	16	A	A	31
32	A	A					32

Fig. 6c. Multi-objective seat assignment with 80% of seats assigned.

Row	A	B	C	D	E	F	Row
1			46				1
2							2
3							3
4			43	22	28	38	4
5	13	20	16	3	14	12	5
6							6
7			28	24	44		7
8			26	23	37	29	8
9			3	31	2	28	9
10	29	32	15	6	17	20	10
11	20	22	1	4	16	3	11
12	28	23	12	11	33	39	12
13	2	10	14	7	13	24	13
14				26		45	14
15			36	34			15
16			29	2	20	38	16
17			8	5	27	3	17
18	20	3	9	21			18
19	41		35	30	40	29	19
20			2	18	42	46	20
21			4	1	32	28	21
22			44	20	25	2	22
23		28	3	19	26	14	23
24			37	29			24
25			39	43			25
26			13	28			26
27			24	23			27
28	16	17	2	3	20	12	28
29	46	38	6	22			29
30	29						30
31							31
32							32

Fig. 7a. Seat assignment considering both objectives.

Row	A	B	C	D	E	F	Row
1							1
2							2
3							3
4				28	43		4
5	24	29	20	3	23	22	5
6							6
7			41	25			7
8			27	21	39	29	8
9		46	3	24	16	38	9
10	28	26	11	2	12	20	10
11	20	17	1	4	13	14	11
12	29	23	7	6	22	28	12
13	3	2	9	5	8	3	13
14			42	18	44	37	14
15			28	20			15
16			26	2	16	31	16
17		38	4	3	17	13	17
18	20	22	6	14			18
19	29	45	23	19	32	28	19
20		46	2	12	36	44	20
21			3	1	29	35	21
22			37	16	15	2	22
23		32	10	13	20	14	23
24			33	28			24
25			43	40			25
26			24	38			26
27			39	30			27
28	20	28	3	2	29	26	28
29			12	34			29
30	46						30
31							31
32							32

Fig. 7b. Seat assignment minimising cost.

Row	A	B	C	D	E	F	Row
1			46				1
2							2
3		44	43				3
4			37	20	28	32	4
5	13	16	11	3	6	4	5
6				38			6
7			26	12	39		7
8			14	22	23	28	8
9	38	29	3	24	1	46	9
10	17	10	19		2	20	10
11	8	5	27			3	11
12	18	7	20		29	26	12
13	2	9				28	13
14	45		28				14
15	40						15
16				2	20	24	16
17	29	25				3	17
18	14	3	12				18
19	21	33	20			16	19
20	30	34	2	3	28		20
21	41	42	23			29	21
22			28	46	20	2	22
23	31	22	3	4	14	13	23
24			32	17			24
25			29	28			25
26		35	13	26	44	37	26
27			24	23	38	43	27
28	15	16	1	2	20	12	28
29	39	36	6	22			29
30	29						30
31							31
32							32

Fig. 7c. Seat assignment maximising distance.

since there is no combination of seats to satisfy the minimum distance δ, so this parameter is relaxed by one unit in such cases until the problem becomes feasible again. Table 2 shows the designated weights for each objective in the model. Specifically, the ω_i values for the multi-objective instance (second row of the table) were chosen only for illustrated purpose and can be setting to any number depending on the decision maker preferences.

Contrasting the results in Fig. 6 the key insight here is that the model now chooses the cheapest seats, but at the same time it maximises the distance between passengers in the same booking, hence persuading them to buy ancillary services.

5.1.2. *Blocking seats framework*

As explained in section 4.3, our algorithm starts by blocking a given percentage of premium seats so that they remain unassigned for customers willing to pay for them. This has several benefits such as increasing the purchase probability of premium seats and a computation time reduction in the model shown in section 5.4. Briefly explained, our algorithm initialises the parameter $a_i = 0$ for the most important seats i.e., the seats appear to be assigned when they are not really assigned. After some seats are assigned, we release the least important seats to maintain a constant percentage of blocked seats throughout the iterations and maintain premium seats blocked.

To test our multi-objective dynamic approach, we compare the seat assignment under different weights in the objective function. We use the following expressions to measure cost and distance, respectively: cost =

Table 3
Payoff table.

Optimisation	OF_1 [$ COP]	OF_2
min OF_1	107.76	48
max OF_2	1107.10	148

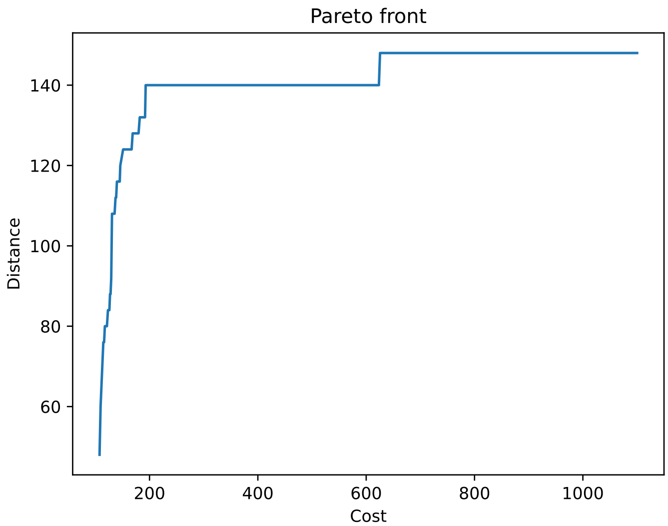

Fig. 8. Pareto front.

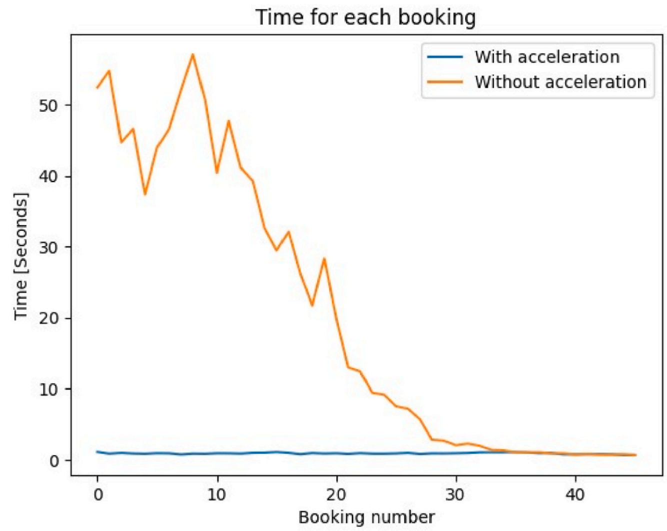

Fig. 9a. Comparison of computational time with and without acceleration.

Table 4
Results for selected solutions.

Solution	OF_1 [$ COP]	OF_2	μ_p
1	1100.19	148	0.038
25	682.52	148	2.337
50	636.52	148	2.590
75	255.29	140	4.328
100	206.52	140	4.596
107	**192.90**	**140**	**4.671**
125	160.33	124	4.131
150	116.14	76	2.214

$\sum_{i \in I} c_i x_i$ and distance $= \sum_{(i,j) \in \mathscr{G}} d_{ij} y_{ij}$ and they are multiplied by the weights ω_1 and ω_2, respectively. Additionally, Table 2 illustrates the weights used in each scenario.

Fig. 7 show how the seats were assigned in each iteration. The number in the seat represent the iteration when it was assigned and if two or more seats have the same number, it means they were selected in the same booking. For instance, in Fig. 7a one can see that seats 11C and 21D were selected in the first iteration when both expressions are considered in the objective function. Furthermore, in Fig. 7c when the cost was not considered in the objective function, the seats 9E and 28C were selected in the first iteration, i.e., when the plane was empty.

Moreover, Fig. 7 shows that the balance of the aircraft is accomplished with the constraints above mentioned. The allocation is symmetric in every case.

5.1.3. Pareto front and decision-making approach

Below are the results obtained by applying the augmented ε-constraint method for the multi-objective instance of the problem. The first step to build the Pareto front was to calculate the payoff table, i.e., each objective's maximum and minimum value as shown in Table 3. This was done by computing the model with one weight equal to 0 to find the minimum of the other part. To find the maximum, the optimisation problem was run using both weights for each objective:where $OF_1 = \sum_{i \in I} c_i x_i$ and $OF_2 = \sum_{(i,j) \in \mathscr{G}} d_{ij} y_{ij}$.

Then, applying the iterative method described in section 4.3 the following Pareto frontier is obtained:

From Fig. 8, it can be noticed how the two objectives are conflictive with each other. The more distance between seats, we obtain a higher cost. Additionally, it is evident that from around the cost of 300, the distance stops increasing; because the maximum distance was reached, and thus these are dominant points.

The results from applying equations (15) – (17) are represented in Table 4, depicting a subset of the Pareto front solutions. The selection criterion is to choose the Pareto point that maximises the membership function value, μ_p, for each solution point p. The optimal μ_p value, 4.671, is achieved at solution 107. Hence, for an optimal decision, the decision-maker should opt for the solution where $OF_1 = 192.90$ and $OF_2 = 140$, as this combination represents the most favorable trade-off between the two objectives according to the chosen decision-making criteria.

5.1.4. Computation time reduction

First, it is essential to notice how the computational time decreases when the airplane is fuller; the reason is that constraint (5) forces the variables to be 0, and the model needs to consider fewer variables. Next, it is shown a comparison of the computational time and the objective function through each iteration when there are four people per booking (i.e., $q = 4$) until the airplane is full.

At the beginning of the iterations, we set every available seat to be empty. Still, given the time vs. iteration graphs, it is noticeable that throughout the iterations, the computational time tends to a constant

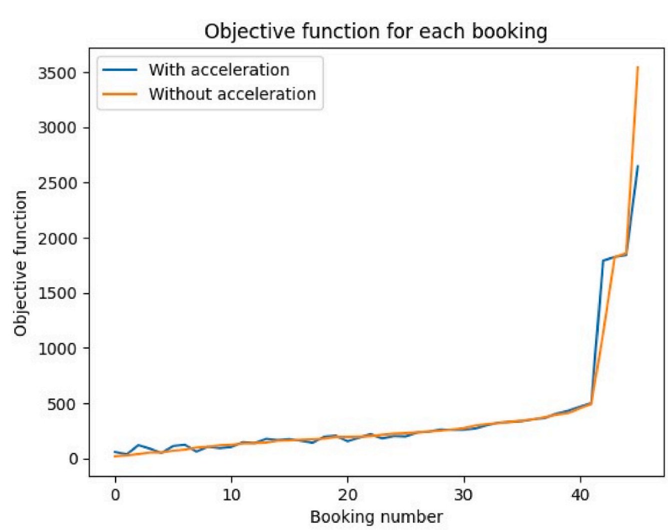

Fig. 9b. Comparison of objective function with and without acceleration.

and to be shorter. This technique consisted of blocking seats that are the most expensive with the previously defined cost for its attributes and operating cost. Also, throughout the iterations, we started to unblock the cheapest seats blocked in the first place so that the model begins to consider them again. All of these by setting parameter a_i to 1 for the most important seats. In the first iterations, we had a constant number of available/occupied seats, marking to busy (for the optimal solution in an iteration) and releasing the others marked busy without being physically busy.

As shown previously, the computational time tends to decrease when the airplane is fuller. Therefore, the blocking of seats was tested, with the same parameters and having $s = 82.5\%$ of seats blocked in the aircraft as an initial condition for the model. These seats will be released simultaneously as the model assign seats to passengers.

Comparing the graphs in Fig. 9a, we show that the computational time decreased under the same parameters, and the time is now more constant and way shorter than before. The orange objective is smoother but similar to the blue one, proving that the objective function does not change extremely in Fig. 9b. Given the blocked seats, the model performs very well in both computational time and objective function (see Fig. 9). That means the model can solve problems in real-time and can be used in real applications.

6. Conclusion

We proposed an innovative multi-criteria approach to assign seats at check-in. The two main objectives of the model were satisfied in every iteration, and the computational time started to get problematic when there were four or more people in the same booking. Anyway, this problem was addressed by blocking seats; benefits of blocking seats are twofold: improves computation time significantly and it increases seat fee revenues overall (Mumbower et al., 2015). Furthermore, the algorithm ensures balance and weight constraints in the aircraft for safety reasons, resulting in a symmetric seat assigning and not overburdening any area by using novel linear constraints derived from the centre of mass formula.

The model proposed consisted of a network flow-based formulation where every node represents one seat with a fixed cost, and every arc corresponds to the distance between seats, measured using the Manhattan approach. Additionally, we included the premium seats blocking framework to increase overall revenue, leaving premium seats unassigned for customers willing to pay for them. Finally, we proposed a decision-making approach based on Pareto front and a fuzzy function to assist decision makers prioritising each objective.

Future work includes using machine learning models to better capture the probability of a seat to be purchased. Furthermore, one goal is to model this problem using a stochastic optimisation universal framework to retrieve policies, to better address the uncertainty, solve the problem using different learning algorithms and finally set a benchmark for future possible solutions.

CRediT authorship contribution statement

Germán Pardo González: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. **Alejandra Tabares Pozos:** Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. **Camilo Quiroga:** Data curation, Validation, Writing – review & editing. **David Álvarez-Martínez:** Data curation, Methodology, Supervision.

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