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The Minimum Wage and Inequality – The Effects of Education and Technology

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Abstract
While there has been intense debate in the empirical literature about the effects of minimum wages on inequality in the US, its general equilibrium effects have been given little attention. In order to quantify the full effects of a decreasing minimum wage on inequality, I build a dynamic general equilibrium model, based on a two-sector growth model where the supply of high-skilled workers and the direction of technical change are endogenous. I find that a permanent reduction in the minimum wage leads to an expansion of low-skilled employment, which increases the incentives to acquire skills, thus changing the composition and size of high-skilled employment. These permanent changes in the supply of labour alter the investment flow into R&D, thereby decreasing the skill-bias of technology. The reduction in the minimum wage has spill-over effects on the entire distribution, affecting upper-tail inequality. Through a calibration exercise, I find that a 30 percent reduction in the real value of the minimum wage, as in the early 1980s, accounts for 15 percent of the subsequent rise in the skill premium, 18.5 percent of the increase in overall inequality, 45 percent of the increase in inequality in the bottom half, and 7 percent of the rise in inequality at the top half of the wage distribution.

JEL-Classification: E24, E65, J31
Keywords: Minimum wage, education, technology, wage inequality

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1 Introduction

It is well documented that income inequality has drastically increased in the United States over the past 30 years along several dimensions.[1] Inequality increased between workers with different educational levels: the college premium increased by 18 percent from 1981 to 2006. The distribution of wages also widened: the gaps between different percentiles of the wage distribution increased drastically. For example, in 2006 a worker at the 90th percentile of the wage distribution earned 283 percent more than a worker at the 10th percentile, whereas this figure was 190 percent in 1981.[2] These trends are illustrated in Figure 1.

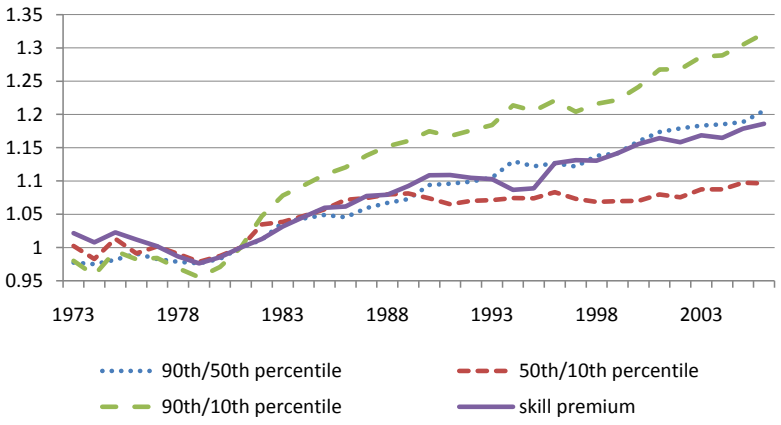

Figure 1: Wage inequality

Notes: Wages are calculated from CPS May Extracts and MORG supplements. Wages are the exponent of residuals from regressing log hourly wages on age, age squared, sex and race. The skill premium is the ratio of the average high-skilled wage to the average low-skilled wage. High school drop outs and high school graduates are low-skilled, everyone else is high-skilled.

The changes in the structure of wages fuelled an extensive debate on the forces driving them. One explanation focuses on changes in labour market institutions, and particularly, on a 30 percent decline in the real minimum wage that took place in the 1980s, since the biggest changes in wage inequality took place during this period (DiNardo, Fortin, and Lemieux (1996), Lee (1999), Card and DiNardo (2002)).

Despite the popularity of this hypothesis, there are, to my knowledge, no attempts

[1] See for example Eckstein and Nagypál (2004), Goldin and Katz (2007), and Goldin and Katz (2008).

[2] Calculations from Current Populations Survey (CPS) Merged Outgoing Rotation Group (MORG) data for years 1981 and 2006.

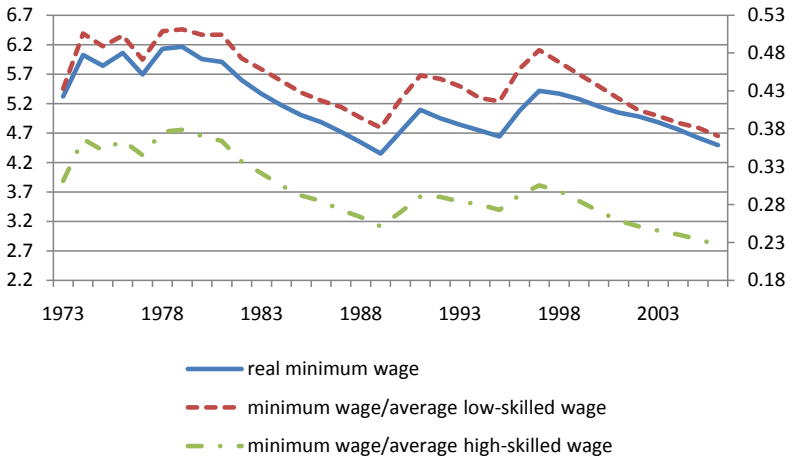

Figure 2: The decline in the real minimum wage

Notes: The real hourly minimum wage is the federal minimum wage in 2000$, calculated using the consumer price index (CPI) from the U.S. Bureau of Economic Analysis (BEA). Average wages are calculated using CPS MORG data. Wages and education categories are as in Figure 1. The left hand scale is the real value of the minimum wage, while the right hand scale is the ratio of the minimum wage compared to the average high- and low-skilled wage.

in the literature to quantitatively assess the potential significance of falling minimum wages for wage inequality in the context of a general equilibrium model. People base their educational decisions on their potential job opportunities and earnings in different occupations. Hence, in general equilibrium, changes in the minimum wage could change the educational composition of the labour force at the aggregate level. Furthermore, the change in the educational composition of the labour force affects the profitability of R&D differentially across sectors. Therefore, the change in the educational composition of the labour force affects the choices firms make about which sectors to focus their R&D activity on, and this determines the direction of technical change. Thus, through educational decisions, the minimum wage influences the direction of technical change. Due to the links between minimum wages, education, and technological change, the quantitative general equilibrium effects of changes in the minimum wage on inequality could be quite different from what simple partial equilibrium reasoning may suggest.

In this paper, I analyse the general equilibrium impact that lower minimum wages have on inequality. I consider two channels jointly: educational choices and the skill-

bias of technology. I find that lower minimum wages increase wage inequality. This overall increase is the result of two opposing forces. On the one hand, the educational and ability composition of the labour force changes, leading to an increase in inequality. On the other hand, the relative supply of high-skilled labour decreases, which reduces the skill-bias of technology, and hence inequality.

By building a general equilibrium model with endogenous education and technology, and a binding minimum wage, this paper bridges two of the most prominent explanations for increasing inequality in the literature.[3] Most of the theoretical literature on skill-biased technical change (SBTC) treats either technology or labour supply as exogenous. I contribute to this literature by allowing both technology and relative labour supply to adjust endogenously. I contribute to the literature on labour market institutions, by proposing a general equilibrium model – with endogenous education and technology – that allows the full quantitative analysis of the effects of falling minimum wages.

To do this I build on and extend the two sector model of endogenous growth in Acemoglu (1998) by adding a binding minimum wage and allowing the supply of college graduates to be endogenous. As in Acemoglu (1998), the production side is a two sector Schumpeterian model of endogenous growth, with more R&D spending going towards technologies that are complementary with the more abundant factor. I explicitly model the labour supply side: workers, who are heterogeneous in their ability and time cost of education, make educational decisions optimally. I solve for the balanced growth path and calibrate the model to the US economy in 1981 in order to compare the transitional dynamics with the observed patterns of wages in the US over the subsequent thirty years.

I find that a decrease in the minimum wage increases the observed skill premium and the wage gaps between different percentiles of the wage distribution. According to the model, the 30 percent decline in the minimum wage accounts for about 15 percent of the observed increase in the skill premium in the US from 1981 to 2006. The fall in the minimum wage also explains almost one fifth of the observed increase in the

[3] Another prominent explanation for the increasing inequality – that my paper does not relate to – is the increasing openness to trade, Goldberg and Pavcnik (2004) provide an extensive review of this literature.

90/10 wage differential, and accounts for about one half of the increase in the 50/10 wage gap. In my model, the minimum wage also has some spill-over effects to the top end of the wage distribution, explaining 7 percent of the increase in the 90/50 wage gap.

The minimum wage affects inequality through several channels: through changes in the skill composition, in the ability composition and in directed technology.

The skill composition of the employed changes. As the minimum wage decreases, low ability workers flow into the low-skilled labour market.[4] This increases the skill premium in the short-run, thus increasing the incentives for acquiring education for higher ability workers. However, a lower minimum wage also makes it easier to find employment, reducing the role of education in avoiding unemployment. Educational attainment decreases at the lower end of the ability distribution and increases at the top end.

The ability composition of the labour aggregates changes, due to both the inflow from unemployment and the changing decision structure of skill acquisition. As the minimum wage decreases, lower ability workers flow into employment, thereby widening the range of abilities present among the employed. As both labour aggregates expand, the average ability in both sectors decrease. Since more low-ability individuals enter the low-skilled labour force, the average ability in the low-skilled sector decreases more. This composition effect reinforces the initial increase in the observed skill premium.

Finally, the direction of technology reacts to changes in the size of the low- and high-skilled labour aggregate. The direct effect of the minimum wage – the expansion of the low-skilled labour force – dominates, decreasing the relative supply of high-skilled labour. This implies that technology becomes less skill biased in the long run.

[4]The effects of minimum wages on unemployment are debated in the empirical literature. Brown, Gilroy, and Kohen (1982), Wellington (1991), Neumark and Wascher (1992) found negative employment effects, while Card (1992), Card and Krueger (1994) and Machin and Manning (1994) found no or small positive effects. These latter studies are controversial, see exchange between Neumark and Wascher (2000) and Card and Krueger (2000). Baker, Benjamin, and Stanger (1999) find a negative employment effect analyzing Canadian data.

2 Related Literature

The underlying causes of increasing inequality are highly debated among labour economists. There are two leading explanations, skill-biased technical change (SBTC) and labour market institutions. Many empirical studies concluded that SBTC is the driving force behind widening earnings inequality (Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993), Krueger (1993), Berman, Bound, and Griliches (1994), Autor, Katz, and Krueger (1998)). This literature stems from the observation that the relative supply of high-skilled workers and the skill premium can only increase together if the relative demand for high-skilled workers also increases. [5]

Other authors have argued that the unprecedented increase in wage inequality during the 1980s cannot be explained by skill-biased technical change alone. DiNardo, Fortin, and Lemieux (1996) find that changes in labour market institutions – namely de-unionization and declining minimum wages – are as important as supply and demand factors in explaining increasing inequality. Lee (1999) uses regional variation in federal minimum wages to identify their impact on inequality, and finds that minimum wages can explain much of the increase in the dispersion at the lower end of the wage distribution. However, he also finds that the reduction in minimum wages is correlated with rising inequality at the top end of the wage distribution. This is seen by many as a sign that the correlation between declining minimum wages and increasing inequality is mostly coincidental (Autor, Katz, and Kearney (2008)). Card and DiNardo (2002) revise evidence for the claim that SBTC caused the rise in wage inequality and find that this view has difficulties accommodating the stabilization of wage inequality that occurred in the 1990s.

In the model presented here, the correlation between minimum wages and upper tail inequality is not coincidental: I provide a theoretical channel through which changes in minimum wages can affect inequality along the entire wage distribution. I find that minimum wages affect the bottom end of the wage distribution more, their impact on the top end is significant as well.

[5] Beaudry and Green (2005) find little support for ongoing skill-biased technological progress; in contrast, they show that changes in the ratio of human capital to physical capital conform to a model of technological adoption following a major change in technological opportunities.

In my model, compositional effects play an important role in increasing inequality, as has been documented in the empirical literature. Lemieux (2006) finds that the compositional effects of the secular increase in education and experience explain a large fraction of the increased residual inequality. The study shows that increases in residual inequality and the skill premium do not coincide, implying that there must be other forces at play besides rising demand for high-skilled workers. Autor, Katz, and Kearney (2005) argue that even though compositional effects have had a positive impact on wage inequality, they mainly affect the lower tail, while the increase in upper tail inequality is mainly due to increasing wage differentials by education. Autor, Manning, and Smith (2009) assess the effects of minimum wages on inequality and find that minimum wages reduce inequality, but to a smaller extent, and that minimum wages also generate spill-over effects to parts of the wage distribution that are not directly affected by them.

In this study, minimum wages increase educational attainment at the low end of the ability distribution, while reducing educational attainment everywhere else through spill-over effects. In line with these findings, the empirical evidence on the effects of minimum wages on educational attainment is mixed. Neumark and Wascher (2003) and Neumark and Nizalova (2007) find that higher minimum wages reduce educational attainment among the young, and that individuals exposed to higher minimum wages work and earn less than their peers. Sutch (2010) finds that minimum wages induce more human capital formation.[6]

Theoretical explanations either rely on exogenous skill-biased technical change or on exogenously increasing relative supply of high-skilled workers; to my knowledge this is the first paper where both the bias of technology and skill formation are endogenous.[7] Caselli (1999), Galor and Moav (2000) and Ábrahám (2008) allow for endogenous skill formation and explore the effects of exogenous skill-biased technical change. Heckman, Lochner, and Taber (1998) develop a general equilibrium model

[6] A related debate is on the effects of minimum wages on formal on-the-job training; see, for example, Acemoglu and Pischke (2003), Acemoglu (2003), Pischke (2005) and Neumark and Wascher (2001).

[7] My paper more generally connects to the literature on the effects of labour market institutions on investments, which mainly focus on the differences in the European and American patterns (Beaudry and Green (2003), Alesina and Zeira (2006), Koeniger and Leonardi (2007)). Another strand of literature that relates to my paper analyses the effects of labour market distortions on growth and educational attainment, for example Cahuc and Michel (1996) and Ravn and Sorensen (1999).

with endogenous skill formation, physical capital accumulation, and heterogeneous human capital to explain rising wage inequality. In this framework they find that skill-biased technical change explains the patterns of skill premium and overall inequality rather well. Explanations for the skill-bias of technology rely on exogenous shifts in the relative labour supplies. Acemoglu (1998) and Kiley (1999) use the market size effect in research and development, while Krusell, Ohanian, Ríos-Rull, and Violante (2000) rely on capital-skill complementarity and an increasing supply of high-skilled labour to account for the path of the skill premium.

3 The Model

I begin by describing the model's production technologies, the R&D sector, the demographic structure and educational choices. Next I define the decentralized equilibrium, and finally, I analyse the balanced growth path and the transitional dynamics.

3.1 Overview

Time is infinite and discrete, indexed by $t = 0, 1, 2...$ The economy is populated by a continuum of individuals who survive from one period to the next with probability λ, and in every period a new generation of measure $1 - \lambda$ is born. Individuals are heterogeneous in two aspects: in their time cost of acquiring education and in their innate ability.

In the first period of his life every individual has to decide whether to acquire education or not, with the time to complete education varying across individuals. Those who acquire education become high-skilled. In my calibration I identify the high-skilled as having attended college. Those who opt out from education remain low-skilled. Workers with high and low skills perform different tasks, are employed in different occupations, and produce different goods. The high-skilled sector includes skill-intensive occupations and production using high-skilled labour, while the low-skilled sector includes labour-intensive occupations and production using low-skilled labour. In equilibrium working in the high-skilled sector provides higher wages and

greater protection from unemployment.

The government imposes a minimum wage in every period, and those who would receive a lower wage – depending on their skill and innate ability – cannot work and become unemployed. As soon as the minimum wage falls below their marginal productivity, they immediately become employed in the sector relevant to their skill.

There is a unique final good in this economy, which is used for consumption, the production of machines, and as an investment in R&D. It is produced by combining the two types of intermediate goods: one produced by the low- and the other by the high-skilled workers. Intermediate goods are produced in a perfectly competitive environment by the relevant labour and the machines developed for them.

Technological progress takes the form of quality improvements of machines that complement a specific type of labour, either high- or low-skilled. R&D firms can invest in developing new, higher quality machines. Innovators own a patent for machines and enjoy monopoly profits until it is replaced by a higher quality machine. There is free entry into the R&D sector, and more investment will be allocated to developing machines that are complementary with the more abundant labour type.

The economy is in a decentralized equilibrium at all times: all firms maximize their profits – either in perfect competition or as a monopoly – and individuals make educational decisions to maximize their lifetime income. I analyse how a permanent unexpected drop in the minimum wage affects the steady state and the transitional dynamics within this equilibrium framework.

3.2 Production

The production side of the model is a discrete time version of Acemoglu (1998). It is a two-sector endogenous growth model, where technological advances feature a *market size* effect, by which more R&D investment is allocated to develop machines complementary to the more abundant factor.

3.2.1 Final and intermediate goods

The unique final good is produced in perfect competition by combining the two

intermediate goods:

$$Y = \left((Y^l)^\rho + \gamma(Y^h)^\rho\right)^{\frac{1}{\rho}},$$

where Y_l is the intermediate good produced by the low-skilled workers and Y_h is the intermediate good produced by high-skilled workers. The elasticity of substitution between the two intermediates is $1/(1-\rho)$, with $\rho \leq 1$. Perfect competition implies that the relative price of the two intermediate goods is:

$$p \equiv \frac{p^h}{p^l} = \gamma \left(\frac{Y^l}{Y^h}\right)^{1-\rho}. \tag{1}$$

Normalizing the price of the final good to one implies that the price of intermediate goods can be expressed as:

$$p^l = \left(1 + \gamma p^{\frac{\rho}{\rho-1}}\right)^{\frac{1-\rho}{\rho}}, \tag{2}$$

$$p^h = \left(p^{\frac{\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}. \tag{3}$$

Intermediate good production is also perfectly competitive in both sectors $s \in \{l, h\}$. I simplify notation by allowing a representative firm:

$$Y^s = A^s (N^s)^\beta \quad \text{for} \quad s = \{l, h\}, \tag{4}$$

where $\beta \in (0, 1)$, N^s is the amount of effective labour employed and A^s is the technology level in sector s.[8] Productivity of labour is endogenous and depends on the quantity and quality of machines used. There is a continuum $j \in [0, 1]$ of machines used in sector s. High- and low-skilled workers use different technologies in the sense that they use a different set of machines. Firms decide the quantity, $x^{s,j}$ of a machine with quality $q^{s,j}$ to use. The productivity in sector s is given by:

$$A^s = \tfrac{1}{1-\beta} \int_0^1 q^{s,j} (x^{s,j})^{1-\beta} dj \quad \text{for} \quad s \in \{l, h\}.$$

Notice that even in the short run, productivity is not completely rigid. Produc-

[8] See labour supply section for exact definition of N^s.

tivity, A^s depends on the quality of machines and the *quantity* of each machine used. Producers of intermediate goods choose the quantity of machines ($x^{s,j}$) depending on the price and on the supply of effective labour it complements (N^s).

Since intermediate good production is perfectly competitive, industry demand for machine line j of quality $q^{s,j}$ and price $\chi^{s,j}$ is:

$$X^{s,j} = \left(\frac{p^s q^{s,j}}{\chi^{s,j}}\right)^{\frac{1}{\beta}} N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{5}$$

3.2.2 R&D firms

Technological advances are a discrete time version of Aghion and Howitt (1992). Investment in R&D produces a random sequence of innovations. Each innovation improves the quality of an existing line of machine by a fixed factor, $\overline{q} > 1$. The Poisson arrival rate of innovations for a firm k that invested z_k^j on line j is ηz_k^j. Denoting the total investments on line j by $\overline{z}^j \equiv \sum_k z_k^j$, the economy wide arrival rate of innovations in line j is $\eta \overline{z}^j$. Hence the probability that the quality of line j improves in one period is $(1 - e^{-\eta \overline{z}^j})$. In Section A.1 of the Appendix I show that the probability that the innovation is performed by firm k is $(1 - e^{-\eta \overline{z}^j}) z_k^j / \overline{z}^j$. The cost of investing z_k^j units in R&D is $Bq z_k^j$ in terms of final good. There are two key features to note: one is that the probability of success is increasing and concave in total investment, $\overline{z}^j$, the other is that the cost of investment is increasing in the quality of the machine line. The first feature guarantees the existence of an interior solution, while the second guarantees the existence of a steady state.

Notice that the probability of success for any single firm depends not only on their own R&D expenditure, but also on the total expenditure of other firms. There are many R&D firms, each of them small enough to take the total R&D spending as given when deciding how much to invest. There is free entry into the R&D sector: anyone can invest in innovation.

R&D firms with a successful invention have perpetual monopoly rights over the machine they patented. In Section A.2 of the Appendix I show that if quality improvements are sufficiently large, then even if the second highest quality machine were sold at marginal cost, firms would prefer to buy the best quality machine, the leading vin-

tage at the monopoly price. I assume that this condition applies, therefore the price of the leading vintage in line j and sector s with quality q is:

$$\chi^{s,j} = \frac{q}{1-\beta} \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1].$$

Hence, if quality improvements are large enough, then each machine's productive life is limited. Once a higher quality machine is invented producers of intermediate goods switch to using the highest quality machine.

Monopoly pricing and industry demand (5) yield the following per period profit for the owner of the leading vintage in line j and sector s:

$$\pi^{s,j} = q^{s,j}\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^s)^{\frac{1}{\beta}}N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{6}$$

The per period profit depends on the price of the intermediate good that the machine produces, and on the efficiency units of labour that can use the machine. A higher price of the intermediate good and a higher supply of effective labour, generates a greater demand for the machine. The second component drives the *scale effect* in R&D. A higher per period profit means a higher lifetime value from owning a patent, which implies more investment into improving that machine.

The value of owning the leading vintage is the expected discounted value of all future profits. This in turn depends on the per period profit and the probability that this quality remains the leading vintage in the following periods.

The value of owning the leading vintage of quality q in line j and sector s can be expressed as:

$$V_t^{j,s}(q) = \pi_t^{j,s}(q) + \frac{1}{1+r}(e^{-\eta\overline{z}_t^{j,s}(q)})V_{t+1}^{j,s}(q) \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{7}$$

Total R&D spending on line j in sector s of current quality q at time t is $\overline{z}_t^{j,s}(q)$, hence $e^{-\eta\overline{z}_t^{j,s}(q)}$ is the probability that quality q remains the leading vintage in line j in period $t+1$. The present value of owning the leading vintage of quality q in line j and sector s in period $t+1$ is $\frac{1}{1+r}V_{t+1}^{j,s}(q)$.

The value of owning a leading vintage is increasing in current period profit and in

the continuation value of owning this vintage. It is decreasing in the amount of R&D spending targeted at improving quality in this line of machines.

Free entry into the R&D sector implies that all profit opportunities are exhausted. The expected return from R&D investment has to equal its cost for each firm.

$$\frac{E_t(V_{t+1}^{s,j}(q_t^{s,j}))}{1+r}(1-e^{-\eta\overline{z}_t^{j,s}(q_t^{s,j})})\frac{z_k^{j,s}}{\overline{z}_t^{j,s}(q_t^{s,j})} = Bq_t^{s,j}z_k^{j,s} \quad \text{for} \quad s=\{l,h\} \quad \text{and} \quad j\in[0,1] \tag{8}$$

The left hand side is the expected return of investing $z_k^{j,s}$ in R&D, while the right hand side is the cost. The expected return depends on the discounted value of owning the leading vintage, and on the probability that firm k makes a successful innovation. Notice that both the expected return and the costs are proportional to the R&D investment of firm k. Hence, in equilibrium, only the total amount of R&D spending targeted at improving line j in sector s is determined.

3.2.3 Technology and Prices

Given monopoly pricing the equilibrium production of intermediate goods is:

$$Y_t^s = (1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1-\beta}{\beta}}N_t^sQ_t^s \quad \text{for} \quad s=\{l,h\}. \tag{9}$$

Where $Q_t^s = \int_0^1 q_t^{j,s}dj$ is the average quality of the leading vintages in sector s. The average quality evolves according to the R&D targeted at improving the machines:

$$Q_{t+1}^s = \int_0^1 q_t^{j,s}\left((1-e^{-\eta\overline{z}_t^{j,s}(q_t^{j,s})})\overline{q}+\left(e^{-\eta\overline{z}_t^{j,s}(q_t^{j,s})}\right)\right)dj \quad \text{for} \quad s=\{l,h\}. \tag{10}$$

The growth rate of average quality in sector s is:

$$g_{t+1}^s = \frac{Q_{t+1}^s}{Q_t^s} \quad \text{for} \quad s=\{l,h\}.$$

Let $Q_t \equiv \frac{Q_t^h}{Q_t^l}$ denote the relative average quality or relative technology. This evolves according to:

$$Q_{t+1} = \frac{g_{t+1}^h}{g_{t+1}^l}\frac{Q_t^h}{Q_t^l} = \frac{g_{t+1}^h}{g_{t+1}^l}Q_t. \tag{11}$$

Combining (9) with the relative price equation (1) gives:

$$p_t = \gamma^{\frac{\beta}{(1-(1-\beta)\rho)}} \left(\frac{Q_t^h}{Q_t^l} \frac{N_t^h}{N_t^l} \right)^{-\frac{(1-\rho)\beta}{(1-(1-\beta)\rho)}}. \tag{12}$$

Note that the relative price – the price of the intermediate produced by the high-skilled compared to the one produced by the low-skilled – is decreasing in the relative supply of high-skilled labour and in the relative quality of the machines used by high-skilled workers. If the relative share of the high-skilled or the relative quality of the machines that complement them increases, then their production increases compared to the production of the low-skilled labour. This leads to a fall in the relative price of the intermediate produced by the high-skilled.

3.3 Labour supply

In this section I describe the labour supply side of the model. I assume that the only reason for unemployment is productivity below the minimum wage. I further assume that the only incentive for acquiring education is the higher lifetime earnings it provides. Education increases earnings potentially through two channels: a higher wage in periods of employment, and better employment opportunities for high- than for low-skilled individuals. These incentives and the minimum wage determine the optimal education decision of people, depending on their cost and return to education.

Individuals are heterogeneous in two aspects: in their cost of acquiring education, c and in their innate ability, a. Let $f(c, a)$ be the joint time invariant distribution of abilities and education costs at birth.[9] The demographic structure is as in Blanchard (1985): every period a new generation of mass $1 - \lambda$ is born, while the probability of surviving from period t to $t + 1$ is λ. These assumptions imply that both the size of the population and the joint distribution of costs and abilities are constant over time.

Each individual has to decide whether to acquire education in the first period of his life. Only those born in period t can enrol to study in period t. Completing education takes a fraction c_i of the first period of individual i's life, and during this time, he

[9] I explain why I introduce heterogeneous time cost in Section 5.2.

cannot participate in the labour market.[10] The time cost of education is idiosyncratic and is determined at birth. An individual who completes education becomes high-skilled and has the option of working in the high-skilled sector for life. High-skilled workers with ability a earn wage $w_t^h(a)$ in period t. Those who choose not to acquire education, remain low-skilled and can start working in the period they are born as low-skilled. The wage in period t for a low-skilled worker with ability a is $w_t^l(a)$.

I model innate ability as a factor that increases individual productivity. Each worker supplies one unit of raw labour inelastically, which translates to a units of efficiency labour for someone with ability a.

Using monopoly pricing and the implied demand for machines, the wage can be expressed in terms of the average quality of machines:

$$w_t^s(a) = a\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s \quad \text{for} \quad s = \{l, h\}. \tag{13}$$

Since ability is equivalent to efficiency units of labour, it can be separated from other factors determining the wage. Let $w_t^s \equiv \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s$ denote the wage per efficiency unit of labour in sector s in period t.

The government imposes a minimum wage $\underline{w}_t$ in every period. Nobody is allowed to earn less than the minimum wage, hence those with marginal product below the minimum wage in period t are unemployed in period t. People only remain unemployed while their marginal productivity is below the minimum wage.

This implies that for both skill levels, there is a cutoff ability in every period below which people become unemployed. This threshold is:

$$\underline{a}_t^s \equiv \frac{\underline{w}_t}{w_t^s} \quad \text{for} \quad s = \{l, h\} \tag{14}$$

Workers with innate ability $a \geq \underline{a}_t^s$ work in sector s in period t.[11]

Individuals choose their education level to maximize the present value of their

[10] In the calibration exercise I set the length of a period to be five years.

[11] If the wage per efficiency unit for the high- and the low-skilled were equal, than some high skilled could work in the low-skilled sector. However, I later show that in equilibrium $w_t^h > w_t^l$ for all t.

expected lifetime utility from consuming the unique final good:

$$E_t \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^j u_{t+j}$$

where u_{t+j} is their consumption of the final good, λ is the probability of staying alive until the next period, r is the discount rate, which is also the interest rate due to linear utility.

Consider the decision of an individual with ability a and cost c born in period t. Denote the expected present value of lifetime income by $\mathbf{W}_t^h(a,c)$ if high-skilled, and by $\mathbf{W}_t^l(a,c)$ if low-skilled; periods of zero income account for the possibility of unemployment. The optimal decision is then summarized by:

$$e(a,c)_t = \begin{cases} 1 & \text{if} \quad \mathbf{W}_t^h(a,c) \geq \mathbf{W}_t^l(a,c) \\ 0 & \text{if} \quad \mathbf{W}_t^h(a,c) < \mathbf{W}_t^l(a,c) \end{cases} \tag{15}$$

where $e(a,c)_t = 1$ if the individual acquires education and $e(a,c)_t = 0$ otherwise.

Let $d(a)_t^s$ be an indicator that takes the value one if an individual with skill s and ability a has marginal product higher than the minimum wage in period t, and zero otherwise. The lifetime earnings of an educated individual can be expressed as:

$$\mathbf{W}_t^h(a,c) = a \sum_{s=1}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^h d(a)_{t+s}^h + a(1-c) w_t^h d(a)_t^h \tag{16}$$

Acquiring education takes a fraction, c, of the first period of an individual's life, implying that he can only work in the remaining fraction, $1-c$, of the first period. The lifetime earnings of a high-skilled individual are decreasing in c, the time acquiring education takes him. The more time he spends acquiring education, the less time he has to earn money.

The lifetime earnings of a low-skilled individual are:

$$\mathbf{W}_t^l(a,c) = a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^l d(a)_{t+s}^l \tag{17}$$

Notice that the lifetime earnings of a low-skilled worker do not depend on c, while the

earnings of a high-skilled worker are decreasing in c. This gives rise to a cutoff rule in c for acquiring education.

Education is worth the investment for an individual with ability a and cost c if $W_t^h(a,c) > W_t^l(a,c)$. As described earlier, there are two channels through which education can increase lifetime earnings: either the wage per efficiency unit is higher for high-skilled than for low-skilled workers, or being high-skilled offers greater protection against unemployment. The second case arises when a is such that $aw_t^l < \underline{w}_t < aw_t^h$, which also requires that $w_t^l < w_t^h$. Hence the following remark:

Remark 1. *To have high-skilled individuals in a generation born in period t, there has to be at least one period $s \geq t$, such that the wage per efficiency unit of labour is higher for the high-skilled than for the low-skilled: $w_s^l < w_s^h$.*

This implies that the only reason for acquiring skills is the *skill premium*, a higher wage per efficiency unit in the high- than the low-skilled sector. Using the relative price of intermediates, (12) and the wage per efficiency unit, (13), the skill premium can be expressed as:

$$\frac{w^h(a)}{w^l(a)} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{Q^h}{Q^l}\right)^{1-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{N^h}{N^l}\right)^{-\frac{1-\rho}{1-(1-\beta)\rho}}.$$

The above equation shows the ways in which education increases workers' wages. The first, represented by γ, arises because goods produced by high- and low-skilled workers are not weighed equally in final good production. If $\gamma > 1$, the high-skilled intermediate contributes more to the final good, and the overall productivity of the high-skilled, measured in units of final good is greater. The second source is the different quality machines: Q^h is the average quality in the high-skilled, and Q^l is the average quality in the low-skilled sector. If technology for the high-skilled is more advanced, then teaching workers to use these more advanced technologies makes workers more productive. The final source is decreasing returns in production: if the share of high-skilled workers is very low, their relative marginal productivity becomes very high.

The labour supply aggregates N_t^h and N_t^l are the total amount of high- and low-

skilled efficiency units of labour available in period t:

$$N_t^l = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j \int_{a_t^l}^{\infty}\int_c f(a,c)(1-e(a,c)_{(t-j)})d(a)_t^l dcda \tag{18}$$

$$\begin{aligned} N_t^h = \quad & (1-\lambda)\textstyle\int_{a_t^h}^{\infty}\int_c f(a,c)(1-c)e(a,c)_{(t-j)}d(a)_t^h dcda \\ & +(1-\lambda)\textstyle\sum_{j=1}^{\infty}\lambda^j\int_{a_t^h}^{\infty}\int_c f(a,c)e(a,c)_{(t-j)}d(a)_t^h dcda \end{aligned} \tag{19}$$

Recall that high-skilled workers born in period t only work for a fraction $(1-c)$ of period t, since they spend a fraction c studying.

4 Equilibrium

In this section I define the equilibrium of the economy and show that the steady state is fully characterized by two unemployment thresholds and a cutoff time cost for acquiring education. I also show that a lower minimum wage implies a shift in all three thresholds. These shifts lead to steady state changes in both the observed skill premium and the overall wage inequality. Inequality is affected mostly through composition: the *ability composition* in both skill groups and the *skill composition* at all percentiles along the wage distribution are altered.

The economy is in a decentralized equilibrium at all times; that is, all firms maximize profits and all individuals maximize their lifetime utility given a sequence of minimum wages.

Definition 1. *A **decentralized equilibrium** is a sequence of optimal education decisions* $\{e(a,c)_t\}_{t=0}^{\infty}$*, cutoff ability levels* $\{\underline{a}_t^h, \underline{a}_t^l\}_{t=0}^{\infty}$*, effective labour supplies* $\{N_t^h, N_t^l\}_{t=0}^{\infty}$*, discounted present values of expected lifetime income* $\{\mathbf{W}_t^h, \mathbf{W}_t^l\}_{t=0}^{\infty}$*, intermediate good prices* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$*, average qualities* $\{Q_t^h, Q_t^l\}_{t=0}^{\infty}$*, investments into R&D* $\{\overline{z}_t^{j,h}, \overline{z}_t^{j,l}\}_{t=0}^{\infty}$ *and values of owning the leading vintage* $\{V_t^{j,h}, V_t^{j,l}\}_{t=0}^{\infty}$ *for all lines* $j \in [0,1]$*, where* $\{Q_0^h, Q_0^l, N_0^h, N_0^l\}$ *and* $\{\underline{w}_t\}_{t=0}^{\infty}$ *are given, such that the following conditions are satisfied:*

1. *the effective labour supplies satisfy* (19) *and* (18)

2. *lifetime earnings are as in* (16) *and* (17)

3. *the average quality in sector* s *evolves according to* (10)

4. *total R&D investment* $\overline{z}_t^{j,s}$ *satisfies* (8) *for all* $t \geq 0$ *and all* $j \in [0,1]$

5. *the sequence* $\{V_t^{j,s}\}_{t=0}^{\infty}$ *satisfies* (7)

6. *the price sequence* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$ *satisfies* (2) *and the relative price,* p_t *satisfies* (12)

7. *the optimal education decisions,* $\{e(a,c)_t\}_{t=0}^{\infty}$ *are as in* (15)

8. *the cutoff abilities for unemployment,* $\{\underline{a}_t^h, \underline{a}_t^l\}_{t=0}^{\infty}$ *satisfy* (14)

4.1 Steady State

As is standard in the literature, in this section I focus on steady states or balanced growth paths (BGP), which are decentralized equilibria, where all variables are constant or grow at a constant rate. In Section B of the Appendix I solve for the BGP in detail, here I present a more informal discussion.

In the BGP the total R&D spending on all lines within a sector are equal, $\overline{z}^{j,s*} = \overline{z}^{s*}$ for $j \in [0,1]$ and $\overline{z}^{s*}$ is given by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{s*}\frac{(1+r-e^{-\eta\overline{z}^{s*}})}{1-e^{-\eta\overline{z}^{s*}}} \quad \text{for} \quad s = \{l,h\}. \tag{20}$$

The above equation shows that R&D effort in a sector is increasing in the period profit from machine sales. These profits are higher if the price of the intermediate produced by it, p^{s*}, is higher, or if more effective labour, N^{s*}, uses this technology.

Along the BGP, relative quality in the two sectors, Q^*, has to be constant, which requires equal R&D spending in the two sectors: $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$. From (20) R&D spending in the two sectors is equal if:

$$p^* = \frac{p^{h*}}{p^{l*}} = \left(\frac{N^{h*}}{N^{l*}}\right)^{-\beta}. \tag{21}$$

Combining the relative price (1),(21) with the intermediate output (9) gives:

$$Q^* = \frac{Q^{h*}}{Q^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}} \right)^{\frac{\beta\rho}{1-\rho}}. \tag{22}$$

The above two equations are the key to understanding the dynamics of the skill premium. The skill premium, which is the ratio of the high- to low-skilled wage per efficiency unit, depends on the relative price of the intermediates and the relative quality in the two sectors. Since both of these ratios depend on the relative supply of skills, their interaction determines the effect of relative skill supply on the skill premium.

Equation (21) shows that the relative price of the two intermediates depends negatively on the relative supply of high-skilled workers. If there are more high-skilled workers, high-skilled intermediate production is greater, other things being equal. The technology effect reinforces this, since more R&D is directed towards the larger sector (from (22)), implying a higher relative quality, Q^*. Intuitively, having more high-skilled workers and better technologies, leads to more high-skilled intermediate production, and lowers the relative price of the intermediate.

Equation (22) shows that the relative quality level depends on the relative abundance of the two types of labour along the balanced growth path. The average quality in the high-skilled sector relative to the low-skilled sector depends positively on the relative supply of high-skilled workers. With more high-skilled workers, an innovation in the high-skilled sector is more profitable. Hence technology is more skill-biased – Q^* is greater, – if the relative supply of skills is higher.

Note that along the steady state, technological *change* is not biased towards either sector, the skill-bias of technology is constant, since both sectors are growing at the same rate. As pointed out earlier, total R&D investment in the two sectors is equal, hence the relative quality of the two sectors is constant along the balanced growth path.

The skill premium per efficiency unit of labour, using (13), is:

$$\frac{w_t^{h*}}{w_t^{l*}} = \left(\frac{p^{h*}}{p^{l*}} \right)^{\frac{1}{\beta}} \frac{Q_t^{h*}}{Q_t^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}} \right)^{\frac{\beta\rho}{1-\rho}-1}. \tag{23}$$

The wage per efficiency unit of labour depends on two components: the price of the intermediate good and the average quality of machines in that sector. Since the relative price depends negatively, while the relative quality depends positively on the relative supply of skilled workers, the net effect depends on which influences the wages more.

This ultimately depends on the elasticity of substitution between the two intermediates. If the two intermediates are highly substitutable, ρ is higher, and relative output affects relative price less; hence the price effect is smaller. On the other hand, if they are not substitutable and ρ is low, the price effect is stronger than the quality effect. If $(\beta\rho)/(1-\rho)-1>0$, then the skill premium per efficiency unit of labour is an increasing function of the relative supply of skills. In this case, the increase in relative quality more than compensates for the decrease in relative price. Hence, an increase in the relative supply of skills increases the skill premium, implying that technology is *strongly biased*. If $(\beta\rho)/(1-\rho)-1<0$ then the skill premium per efficiency unit of labour is decreasing in the relative supply, and technology is *weakly biased*: the technology effect does not compensate for the price effect.

The skill premium per efficiency unit of labour is not the same as the empirically observed skill premium. The observed skill premium is the ratio of the average wages:

$$\frac{\overline{w}_t^{h*}}{\overline{w}_t^{l*}}=\frac{w_t^{h*}}{w_t^{l*}}\frac{\overline{a}^{h*}}{\overline{a}^{l*}},$$

where $\overline{a}^{h*}$ is the average ability among the high-skilled and $\overline{a}^{l*}$ is the average ability among the low-skilled.

The skill premium per efficiency unit is constant from (23). From Remark 1, the skill premium has to be greater than one in at least one period. This implies that $w_t^{h*}>w_t^{l*}$ for all $t\geq 0$.

The threshold ability of unemployment for the low-skilled is defined in (14). Combining this with steady state wages yields:

$$\underline{w}_t=\underline{a}^{l*}w_t^{l*}=\underline{a}^{l*}\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^{l*})^{\frac{1}{\beta}}Q_t^{l*}. \tag{24}$$

Note that for the existence of a BGP, it is required that the minimum wage grows at the

same rate as the low-skilled wage per efficiency unit, g^*. Since the growth in average quality is driving wage growth, let $\widetilde{w}_t \equiv \frac{w_t}{Q_t^l}$ denote the *normalized minimum wage*, which has to be constant for a steady state.

Given $\underline{a}^{l*}$, the cutoff ability for the high-skilled is given by:

$$\underline{a}^{h*} = \underline{a}^{l*} \frac{w_t^l}{w_t^h}. \tag{25}$$

As pointed out earlier, the skill premium is greater than one, implying that the threshold ability for unemployment for the low-skilled is higher than the threshold ability for the high-skilled: $\underline{a}^{h*} < \underline{a}^{l*}$. Acquiring skills through education, for instance learning how to use different machines, increases workers' productivity and protects them from unemployment. Acquiring skills allows people with low ability to increase their marginal productivity above the minimum wage, and to find employment.

In the steady state everyone has a constant employment status: they are either unemployed or employed in the low- or high-skilled sector. Moreover, depending on their innate ability, a, everyone falls into one of the following categories: $a < \underline{a}^{h*}$, $a \in [\underline{a}^{h*}, \underline{a}^{l*})$ or $a \geq \underline{a}^{l*}$.

Consider an individual with ability $a < \underline{a}^{h*}$. He does not acquire education in equilibrium because he would be unemployed regardless of his skills.

Now consider an individual with ability $a \in [\underline{a}^{h*}, \underline{a}^{l*})$. If he does not acquire education, he becomes unemployed and earns zero income in every period. On the other hand, by completing his studies he earns the high-skilled wage. Since the opportunity cost of education is zero in this case, acquiring education to become high-skilled is the optimal decision.

Finally, consider an individual with ability $a \geq \underline{a}^{l*}$, who is always employed regardless of his skill level. Such an individual acquires education if the present value of his earnings as high-skilled (16) exceed his present value earnings as low-skilled (17).

Result 1. *Every individual with ability $a \geq \underline{a}^{l*}$ born in period t acquires education if his cost*

$c < c^*$, *where* c^* *is the* cutoff time cost *implicitly defined by:*

$$c^* = \frac{1 - \frac{w_t^{l*}}{w_t^{h*}}}{1 - \frac{g^*\lambda}{1+r}} \tag{26}$$

Proof. Combining (15) with (16) and (17) and using that in equilibrium $d_{t+k}^s(a) = 1$ for all $k \geq 0$, for $s = l, h$, and $a \geq \underline{a}^{l*}$, implies that the condition for acquiring education is:

$$a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^{h*} - a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^{l*} \geq a w_t^{h*} c.$$

This shows that the optimal education decision is equivalent to a threshold time cost, c_t^*. Using the fact that wages in both sectors grow at a constant rate g^*, and that the skill premium, w_t^{h*}/w_t^{l*} is constant, $c_t^* = c^*$ is constant and given by (26). □

The threshold time cost for acquiring education and consequently the fraction of high-skilled workers depends positively on the skill premium and on the growth rate of the average qualities. The threshold is increasing in the skill premium, since a higher skill premium implies a greater per period gain from working as high-skilled. The growth rate of wages also increases the threshold time cost; if wages grow at a higher rate, then for a given skill premium, future gains are greater.

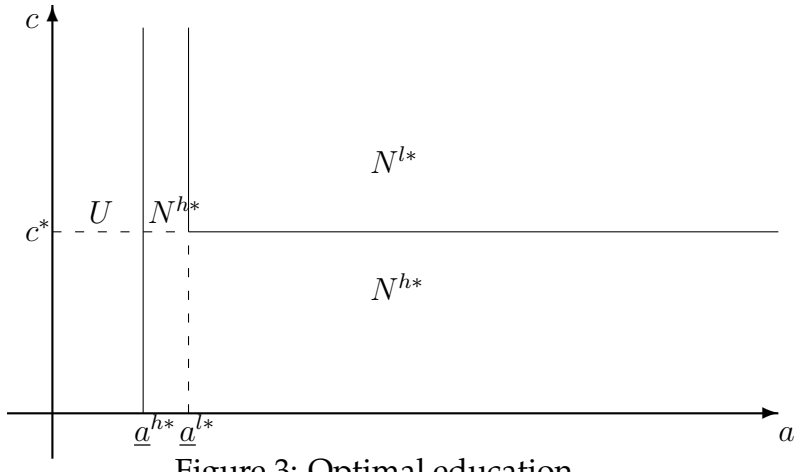

Figure 3: Optimal education

Notes: The horizontal axis represents the support of the ability distribution, and the vertical axis represents the support of the cost distribution.

Figure 3 depicts educational choices in the steady state. Individuals with ability

lower than $\underline{a}^{h*}$ are unemployed and do not acquire education (U). Between the two thresholds, $\underline{a}^{h*} \leq a < \underline{a}^{l*}$, everyone acquires education and becomes high-skilled to avoid unemployment. Finally individuals with ability above $\underline{a}^{l*}$ acquire skills if their time cost is below c^*.

The three cutoff values determine the effective labour supplies, N^{h*} and N^{l*}. In turn, the effective labour supplies determine every other variable in the economy in steady state. Therefore, the steady state of the economy is characterized by the three thresholds $\underline{a}^{h*}, \underline{a}^{l*}$ and c^*. Furthermore, the three thresholds are also connected through the equilibrium condition (25). This condition relates the two cutoff values of unemployment through the skill premium.

Lemma 1. *The pair $(\underline{a}^{l*}, c^*)$ uniquely defines $\underline{a}^{h*}$.*

Proof. See Appendix B.4. □

The balanced growth path is defined by two key equations: the equilibrium c^* given the threshold for low-skilled unemployment (26) and the equilibrium $\underline{a}^{l*}$ given the cutoff time cost for acquiring education (24). Figure 4 graphs these two equations.

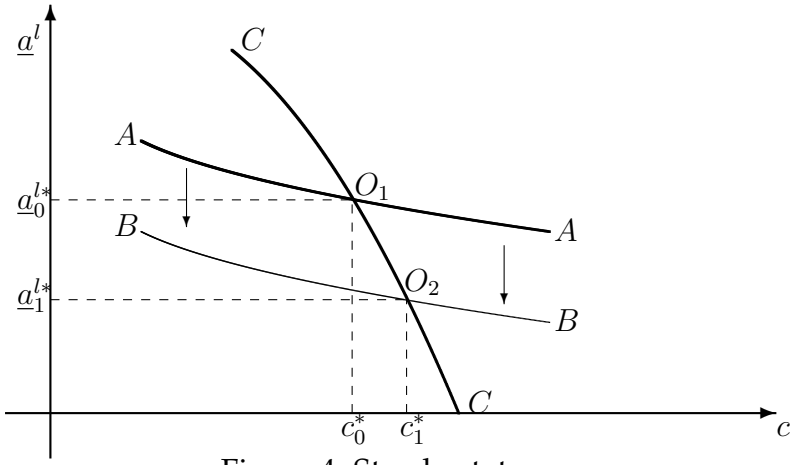

Figure 4: Steady state

The curve CC represents the equilibrium c^* for different values of $\underline{a}^l$ (26). The threshold ability for low-skilled unemployment affects c^* through two channels. The first is the growth rate: a higher $\underline{a}^l$ decreases the total amount of effective labour in the

economy. Due to scale effects in R&D, this reduces the growth rate of the economy.[12] A lower growth rate implies a lower lifetime gain from being high-skilled, hence a lower c^*.

The second channel is the skill premium. A higher $\underline{a}^l$ reduces N^l and increases N^h, so the relative supply of high-skilled workers increases. A weak technology bias reduces the skill premium, and the gain from acquiring education; thus, a higher $\underline{a}^l$ reduces c^* both through its affect on growth and on the skill premium, so the curve represented by CC is downward sloping.
On the other hand if technology is strongly biased, then an increase in N^h/N^l increases the skill premium. The decreasing growth rate pushes c^* down, while the increasing skill premium pushes c^* up. The overall effect on the gain from education can be ambiguous if technology is strongly biased. For the range of values that are of interest, the overall effect is small and negative.

The curve AA represents the equilibrium unemployment threshold $\underline{a}^{l*}$ for different values of c (24). If c is higher, there are more high-skilled workers, and their production increases. This, in turn, depresses the price of their intermediate, p^h, while the price of the low-skilled intermediate increases. A higher p^l allows workers with both lower ability and skills to participate in the market. Hence the threshold for unemployment for the low-skilled is a decreasing function of c, implying the downward sloping AA curve in Figure 4.

4.2 Lowering the minimum wage

To analyse the effects of minimum wage on inequality, I consider an unanticipated permanent decrease in the normalized minimum wage. A lower minimum wage excludes fewer people from the labour market, by lowering the unemployment threshold for both the high- and the low-skilled. Moreover, through endogenous R&D, the increase in the supply of effective labour raises the growth rate of the economy, thus increasing the incentives to acquire education, resulting in a higher cutoff cost for acquiring education. The shift of these three thresholds changes the *ability composition*

[12] See Appendix section B.3 for the exact dependence of the growth rate on the supply of high- and low-skilled effective labour.

in both sectors and the *skill composition* along the ability distribution. Average ability in both sectors decreases, with high-skilled average ability decreasing less. The fraction of high-skilled workers changes at every percentile in the wage distribution, increasing at the top end and decreasing at the bottom end, thereby increasing overall inequality.

The normalized minimum wage shifts curve AA and leaves curve CC unaffected. From (24) a lower $\widetilde{w}$ implies that a lower $\underline{a}^{l*}$ satisfies the equation for any c. Therefore, a higher normalized minimum wage shifts the curve up, and a lower value shifts the curve down.

Curve BB in Figure 4 represents the equilibrium unemployment threshold $\underline{a}^{l*}$ for any cutoff time cost of education for a lower $\widetilde{w}$. The steady state moves from O_1 to O_2. The new steady state features a lower threshold for unemployment, $\underline{a}_1^{l*}$ and a higher threshold for the time cost of education, c_1^*. The effect of these changes on the supply of high- and low-skilled effective labour are depicted in Figure 5.

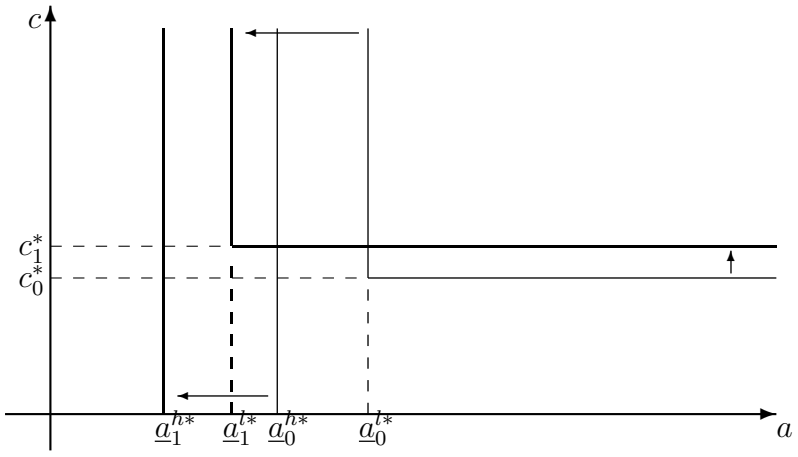

Figure 5: Change in the optimal education and labour market participation

Notes: In the graph I represent a case where $[\underline{a}_1^{h*}, \underline{a}_1^{l*}]$ and $[\underline{a}_0^{h*}, \underline{a}_0^{l*}]$ do not overlap. I chose to show such a case, since this is what I find in the calibration exercise.

The direct effect of an increase in c^* is to decrease N^{l*} and increase N^{h*}. A higher c^* implies that more people acquire education for higher wages. The fraction of low-skilled workers decreases while the fraction of high-skilled increases among those with ability greater than $\underline{a}_0^{l*}$.

A lower $\underline{a}^{l*}$ entails that fewer people acquire education to avoid unemployment.

While previously everyone with ability, $a \in [\underline{a}_0^{h*}, \underline{a}_0^{l*})$ became high-skilled to avoid unemployment, now they would be employed regardless of their skill level. Only those with cost lower than c_1^* acquire education. This increases N^{l*} partly by reducing N^{h*} and partly by reducing unemployment.

A decrease in $\underline{a}^{l*}$ also implies a lower $\underline{a}^{h*}$, which increases N^h* by reducing unemployment. A lower unemployment cutoff for the high-skilled shifts down the range of abilities for which people acquire education to avoid unemployment.

The overall effect of a decrease in the minimum wage on the relative supply of skills depends on the elasticity of a^{l*} relative to the elasticity of c^*. The change in the supply of high and low skills governs the change in the skill premium as well.

In general, the effect of minimum wages on the supply of skills is ambiguous. However, numerical results suggest that a lower minimum wage increases the supply of high-skilled less than it increases the supply of low-skilled effective labour, leading to a decrease in the relative supply of skills. The calibration exercise presented in Section 5 yields that technology is strongly biased; hence, a reduction in the supply of skills decreases the skill premium per efficiency unit of labour.

Overall inequality in the economy, measured by the wage gap between different percentiles of the wage distribution, increases. With a lower minimum wage the range of abilities in the labour market widens, and the fraction of high-skilled increases at the top end of the ability distribution, and decreases at the bottom end. These forces both push towards greater inequality.

5 Calibration

I first present estimates of the parameters set outside the model. I then present maximum likelihood estimates of the ability and time cost of education distributions, based on the equilibrium conditions of the model. Finally, I calibrate the remaining parameters by globally minimizing the distance between data moments and steady state moments of the model.

5.1 Interest rate, lifespan and production technology

Three parameters, namely, the share of labour in the production function, β, the interest rate, r, and the survival probability, λ, can be set outside the model.

The intermediate good is produced by labour and machines, and the exponent on labour is β. This implies a wage bill of βY in the aggregate economy. Since the wage bill has been roughly constant at $\frac{2}{3}$ over long run US history, I set $\beta = \frac{2}{3}$.

The interest rate and the probability of survival depend on the length of a period in the model. Since people can spend only a fraction of their first period studying in the model, I set one period in the model to correspond to five years.[13]

Based on the real interest rate in the US, which has been about five percent annually, I set the interest rate for five years to be $r = 1.05^5 - 1$.

On average, since people spend 45 years working and studying, the rate of survival can be set to give an expected 9 periods of work, including the period of study.[14] This gives the value $\lambda = 1 - \frac{1}{9}$.

5.2 Ability and cost distribution

Estimating the distribution of abilities and costs is a crucial part of the calibration exercise. Since ability and the cost of education are not directly observable, I combine equilibrium conditions of the model with observable characteristics such as wages, education levels and age to estimate these distributions.

Figure 6, which represents the hourly wages of high- and low-skilled individuals, offers a good starting point for identifying the ability and cost of education distributions. A striking feature in the figure is the significant overlap between the wages of the two educational groups. An appropriate distribution, therefore, must reproduce this pattern.

[13] A longer model period would also allow for completing education in one period. However, shorter periods provide richer transitional dynamics.

[14] The expected lifespan of someone who has a per period survival probability of λ is

$$E(j) = \sum_{j=1}^{\infty} j\lambda^{j-1}(1-\lambda) = \frac{1}{1-\lambda}$$

Solving for $E(j) = 9$ gives $\lambda = 1 - \frac{1}{9}$.

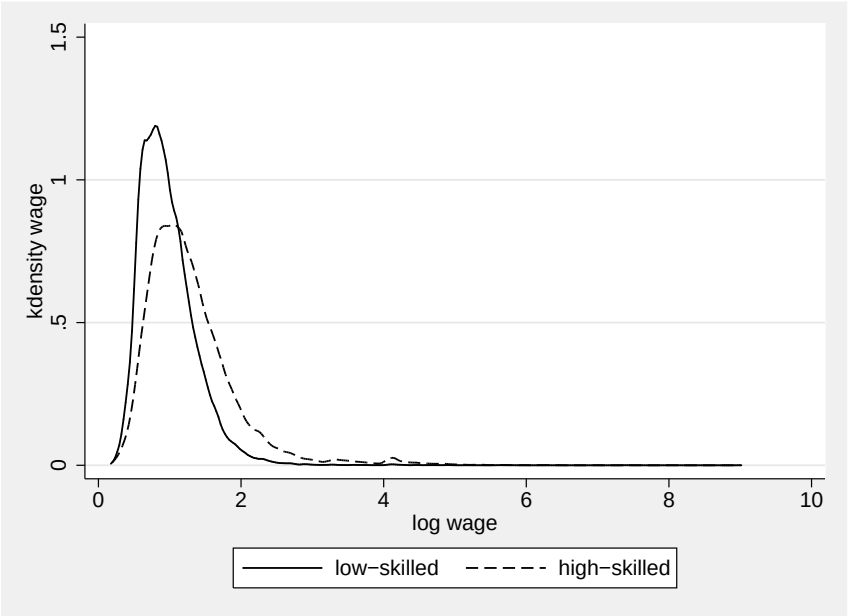

Figure 6: Hourly wages of the high- and low-skilled in 1981

Notes: Wages are calculated from the CPS MORG supplements. Wages are the exponent of the residuals from regressing log hourly wage on age, age square, sex and race. Those who attended college are high-skilled, everyone else is low-skilled. The lines represent the kernel density estimate produced by Stata.

In general there are two components to the cost of education: a time cost and a consumption cost. The time cost arises because a person can work part-time at most while studying. The consumption cost is due to tuition fees and other expenses. Both these costs could be thought of as homogeneous or heterogeneous across individuals. For example, a model with credit constraints and differential endowments would yield a heterogeneous education cost in reduced form. I consider three cases—a homogeneous cost, a distribution of consumption costs and finally a distribution of time costs—and show that only heterogeneous time costs of education can reproduce the overlapping wages.[15] Therefore in the calibration and in the numerical results I assume that the cost of education is purely an idiosyncratic time cost.

First, consider the case with a homogeneous consumption cost of acquiring education. In this case, the returns to education are increasing in ability, while the cost is fixed. In equilibrium there is a cutoff ability above which people acquire educa-

[15]For sake of brevity in the discussion of the various cases I only consider the decision of those individuals, who acquire education for higher wages and not to avoid unemployment. In all cases, there would be a range of abilities at the very bottom end of the ability distribution, where some people would acquire education to avoid unemployment, while the rest would be unemployed.

tion, and below which they do not. Since both ability and wage per efficiency unit are higher for high-skilled individuals, equilibrium choices imply higher wages for high-skilled individuals. Wage distributions in this setup would not overlap, contradicting the empirically observed pattern.[16]

Second, assuming a distribution of consumption costs does not fit the empirical pattern of overlapping wage distributions either. A distribution of consumption costs implies a cutoff cost for every ability level in equilibrium. Given the cutoff for an ability level, those with the respective ability and lower cost of education acquire education, while those with cost higher than the cutoff do not. The equilibrium cutoff cost is increasing in ability: people with higher ability, have higher returns from education and are willing to pay a higher consumption cost for education. This implies that the fraction of high-skilled is increasing in the ability level, implying a higher average ability among the high-skilled. As in the previous case, high-skilled individuals have higher wages due to a higher unit wage and higher average abilities, contradicting the overlapping wage distribution pattern.[17]

Third, assuming instead, that the cost of education is a time cost, the equilibrium cutoff cost for acquiring education is independent of ability. If the ability and cost distributions are independent, then the high-skilled have higher wages only because of higher unit wages, since the average ability in the two sectors are equal. The distribution of wages for the high-skilled is a shifted and compressed version of the distribution of wages for the low-skilled. Hence, in this case predictions on the distribution of wages in the high- and low-skilled sector match well with the pattern observed in Figure 6. Therefore in the calibration and in the numerical results I assume that the cost of education is purely an idiosyncratic time cost.

For simplicity I assume that ability and education costs are independently distributed. I assume a uniform time cost distribution on $[0, \bar{c}]$, with $\bar{c} \leq 1$, allowing

[16] If the homogeneous cost was a time cost, everyone would need to be indifferent between acquiring education or not. Since both the cost and the returns to education are linearly increasing in ability, if people were not indifferent then either everyone would acquire education or nobody would. An equilibrium based on indifference cannot be estimated from the data, since the ability, and therefore the wages of high- and low-skilled individuals are indeterminate in equilibrium.

[17] This holds even when the ability and cost distributions are independent. With a negative correlation between ability and the consumption cost of education, the two wage distributions would overlap even less.

a maximum of five years for studies if $\overline{c} = 1$. The probability density function is $g(c) = 1/\overline{c}$. I assume that ability is lognormally distributed, with probability density function $f(a) = \frac{1}{a\sigma}\phi(\frac{\ln(a)-\mu}{\sigma})$, where ϕ is the pdf of the standard normal distribution.

Since all variables of interest in the steady state calibration and in the quantitative assessment of the transition are invariant to the mean of the ability distribution, I normalize this mean to be one.[18]

In the model, the wage of an individual with ability a_i and education s is given by $w^s(a_i) = a_i w^s$, while the average wage in sector s is $\overline{w}^s = \overline{a}^s w^s$, where $\overline{a}^s$ is the average ability among those with education s, and w^s is the wage per efficiency unit in sector s. Based on this:

$$\frac{a_i}{\overline{a}^s} = \frac{w^s(a_i)}{\overline{w}^s} \equiv \widetilde{a}_i^s.$$

An individual's ability relative to the average ability in his education group is equal to his wage relative to the average wage in that sector. Since the education and wages of every respondent in the sample are recorded, I can infer *relative ability*, $\widetilde{a}_i^s$, from the data.

If the distribution of time costs and abilities is known, cutoff values for unemployment, $\underline{a}^{h*}, \underline{a}^{l*}$ and time cost c^* can be found by matching the fractions of unemployed, low- and high-skilled workers. The thresholds $\underline{a}^{h*}, \underline{a}^{l*}$ and c^*, and the parameters of the ability and cost distributions are sufficient to calculate the average ability in both education groups, $\overline{a}^h, \overline{a}^l$ (see Figure 3 and Appendix C.1).

Multiplying the relative ability of a person by the average ability in his education group gives his ability level:

$$a_i = \frac{a_i}{\overline{a}^s}\overline{a}^s = \frac{w^s(a_i)}{\overline{w}^s}\overline{a}^s.$$

According to the model, if a high-skilled individual i's wage is lower than a low-

[18]This normalization is equivalent to:

$$E(a) = e^{\mu+\frac{1}{2}\sigma^2} = 1 \quad \Leftrightarrow \quad \mu = -\tfrac{1}{2}\sigma^2$$

Furthermore, in any model, where agents are heterogeneous in ability, the mean of the ability distribution and the technology level are not separable along any observable measure. Since this setup does not require the absolute level of technology, or the mean of the ability distribution for any quantity of interest, this normalization is without loss of generality.

skilled individual's wage, and since the skill premium is greater than one, it follows that his ability has to be lower as well. This implies the following:

$$k_i \equiv \arg\min_{j|w_i^h<w_j^l} w_j^l \quad a_i^h \leq a_{k_i}^l.$$

Similarly, the ability of any low-skilled individual has to be higher than the ability of all high-skilled individuals with a lower wage:

$$k_i \equiv \arg\max_{j|w_i^l>w_j^h} w_j^h \quad a_i^l \geq a_{k_i}^h.$$

A high-skilled individual has wage w_i^h if his ability is $a_i^h = \frac{w_i^h}{\overline{w}^h}\overline{a}^h$, and he acquired education either to avoid unemployment, or because his time cost is lower than the threshold, $c_i \leq c^*$. If he is in the first period of his life, his time cost of education must be lower than the maximum amount of time he could have spent studying. The probability of observing a high-skilled individual with wage w_i^h at age d is:

$$P(w_i^h, h, d) = \begin{cases} P(a = a_i^h) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad d \geq 23 \\ P(a = a_i^h)P(c \leq \frac{d-18}{5}) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad d < 23 \\ P(a = a_i^h)P(c_i < c^*) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad d \geq 23 \\ P(a = a_i^h)P(c_i < \min\{c^*, \frac{d-18}{5}\}) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad d < 23 \end{cases}$$

Since there is an upper bound on the ability a high-skilled individual can have, the likelihood of observing a given wage, w_i^h for a high-skilled person can be written as:

$$\mathfrak{L}(w_i^h, d; \sigma, \overline{c}) = \begin{cases} 0 & \text{if} \quad a_i^h < \underline{a}^{h*} \quad \text{or} \quad a_i^h > a_{k_i}^l \\ f(a_i^h) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d \geq 23 \\ f(a_i^h)G(\frac{d-18}{5}) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d < 23 \\ f(a_i^h)G(c^*) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d \geq 23 \\ f(a_i^h)G(\min\{c^*, \frac{d-18}{5}\}) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d < 23 \end{cases} \tag{27}$$

Similarly, a low-skilled individual earning wage w_i^l must have $a_i^l = \frac{w_i^l}{\overline{w}^l}\overline{a}^l$, and cost exceeding the cutoff time cost; $a_i^l \geq a_{k_i}^h$ must also hold. The probability of observing

w_i^l is then:

$$P(w_i^l, l) = P\left(a = a_i^l\right) P(c_i \geq c^*).$$

The likelihood of observing wage w_i^l for a low-skilled individual is:

$$\mathfrak{L}(w_i^l; \sigma, \bar{c}) = \begin{cases} 0 & \text{if} \quad a_i^l < \underline{a}^{l*} \quad \text{or} \quad a_i^l < a_{k_i}^h \\ f(a_i^l)(1 - G(c^*)) & \text{if} \quad a_i^l \geq \underline{a}^{l*} \quad \& \quad a_i^l \geq a_{k_i}^h \end{cases} \tag{28}$$

I calculate the likelihood of observing the sample of wage and education pairs using (27) and (28). I maximise the likelihood by choosing parameters σ and $\bar{c}$.

I use the May and Outgoing Rotation Group supplements of the Current Population Survey for 1981. I choose 1981 as the initial steady state because from 1982 onwards, the minimum wage was not adjusted by inflation, and its real value started declining. I divide the population into high- and low-skilled based on college education: those who attended college are high-skilled, those who did not are low-skilled. I calculate the fraction of unemployed, low-skilled and high-skilled workers using the education and the employment status categories .[19] In order to capture only the effects of education and underlying ability, I use a cleaned measure of wage. This measure is the exponent of the residuals generated from regressing log hourly wages on age, age square, sex and race.

The maximum likelihood yields $\sigma = 0.73$ and $\bar{c} = 0.82$, which corresponds to about four years.

5.3 Final good production and R&D

I calibrate the remaining parameters to minimize the distance between moments of the initial steady state and the same moments from the data. It is common in calibration exercises to match n moments exactly by choosing n parameters, and use the remaining moments to test the goodness of fit of the model. In this method the parameters chosen depend heavily on which moments are matched, and the choice of these moments are rather arbitrary. The method I use, which is similar to a method of

[19] In the calibration I do not make a distinction in the educational attainment of the unemployed. In the steady state, only those who will be employed in the future should acquire education. In the data, half of the unemployed have some college education.

Table 1: Moments

Moment	Data	Model
L^u	0.0693	0.1023
L^l	0.5338	0.4923
L^h	0.3554	0.3964
g	0.0800	0.0798
$\overline{w}^h/\overline{w}^l$	1.3344	1.0518
$\overline{w}/w_{50}$	1.1072	1.2942
w_{90}/w_{50}	1.7060	2.4252
w_{50}/w_{10}	1.7006	2.0778
$\overline{w}^h/\overline{w}$	1.1796	1.0280

moments estimation, is to choose the values of 6 parameters to minimize the weighted distance from 9 moments of the data. The weight of the ith moment, is the estimated standard deviation of the ith moment in the data. I run a grid search over the set of parameter values and find the set that globally minimizes the distance from the moments.

I chose three types of moments: moments that describe the skill-composition and fraction of unemployed in the economy, those that describe the wage distribution, and those that reflect the R&D process. Moments of the first type are important to match, as most of the movement in the model comes from changes in these aggregates. The second type is also crucial, since I analyse the effects of minimum wages on inequality. Finally, matching the growth rate, which is governed by the R&D process, determines the responsiveness of technology. The moments and the fit of the model are summarized in the Table 1.

I globally minimize the distance from the data moments by choosing $\rho, \gamma, \eta, \overline{q}, B$ and $\widetilde{w}$. The calibrated parameters are summarized in Table 2. Parameters η and B control the profitability of R&D activity, while $\overline{q}$, η and B together determine the growth rates. Parameter η determines how much R&D spending increases the Poisson arrival rate of innovations, while parameter B determines how costly R&D investments are in terms of the final good. The value of $\overline{q}$ determines the size of the improvement between two quality levels over a five year period. The weight of the high-skilled

intermediate in the production of the final good is given by γ.

Table 2: Calibrated parameters

ρ	γ	η	$\overline{q}$	B	$\widetilde{w}$	β	λ	r	$\overline{c}$	σ
0.9	1.15	0.25	2.08	0.15	0.4	2/3	8/9	1.05^5	0.82	0.73

Parameter ρ controls the elasticity of substitution between the *intermediate goods* produced by the high- and low-skilled. This elasticity, $1/(1-\rho)$ cannot be estimated directly from the data. Note that the elasticity of substitution between the intermediate goods is not the same as the elasticity of substitution between high- and low-skilled workers, which has been estimated by several authors. However, their estimates are not comparable to ρ, since technology is usually modelled as exogenous, while in my model it is endogenous.[20]

6 Transitional Dynamics

In this section I discuss the transitional dynamics following a reduction in the minimum wage. The transition takes relatively long as new generations have to replace older ones, as the new steady state features a different educational composition. During the transition, the average skill premium and the wage gaps between different percentiles in the wage distribution all increase. The increase is the most pronounced in the period of the announcement, due to the entry of previously unemployed workers into the labour force. Inequality measured by the skill premium and wage gaps continues to increase throughout the transition, as both the skill composition of the labour force and the ability composition of the two skill groups change.

Initially the economy is in steady state. The minimum wage grows at the same rate as the wages and the quality in both sectors. The government unexpectedly announces a permanent decrease in the value of the normalized minimum wage. The normalized minimum wage drops to its new lower value in the period of the announcement, and

[20]See Section C.2 of the Appendix for further details.

stays there forever. Individuals and R&D firms have perfect foresight over the future sequence of the minimum wage, and form correct expectations about the future path of the average quality levels of machines and education acquisition of future generations. The economy is in a decentralized equilibrium along the transitional path from the initial BGP to the new one.

I use a second order approximation of the equations that have to hold throughout the transition to produce the transitional dynamics (see Appendix section D for details).[21]

Figure 2 shows that the real value of the minimum wage decreased by about 30 percent until the late 1980s, while the minimum wage compared to the average high- and low-skilled wage decreased by about 20 percent. In the transitional dynamics I mimic this pattern by a one-time 20 percent drop in the value of the normalized minimum wage. Since in the steady state the real minimum wage is not stationary, it is not possible to simulate a shock by changing its value while using perturbation methods. The change in the normalized minimum wage is not necessarily the same as the change in the minimum wage compared to the average wage, but the transition shows that it is sufficiently close.[22]

Figure 7 shows the transitional path from the original steady state to the new one, which features a 20 percent lower normalized minimum wage. The horizontal axis denotes the year, with the drop in the normalized minimum wage occurring in 1981.

The top two panels in Figure 7 show the path of the unemployment thresholds. At the moment of the announcement, both $\underline{a}^{h*}$ and $\underline{a}^{l*}$ drop almost to their new steady state value. It is not visible on the graphs, but the threshold ability for low-skilled unemployment initially stays above its steady state value and gradually falls towards

[21] I use the code designed in Schmitt-Grohe and Uribe (2004) to produce the transitional dynamics.

[22] Using the normalized minimum wage implies:

$$\widetilde{w}_1 \equiv \frac{\underline{w}_t}{Q_t^l} = \underline{a}_t^l \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^l)^{\frac{1}{\beta}},$$

while using the minimum wage compared to the average low-skilled wage implies:

$$\widetilde{w}_2 \equiv \frac{\underline{w}_t}{\overline{w}_t^l} = \underline{a}_t^l \overline{a}_t^l.$$

These clearly do not imply the same dynamics for $\underline{a}_t^l$, but since the magnitude of the change in both p_t^l and $\overline{a}_t^l$ is small, their effect will be dominated by the drop in $\widetilde{w}$ throughout the transition.

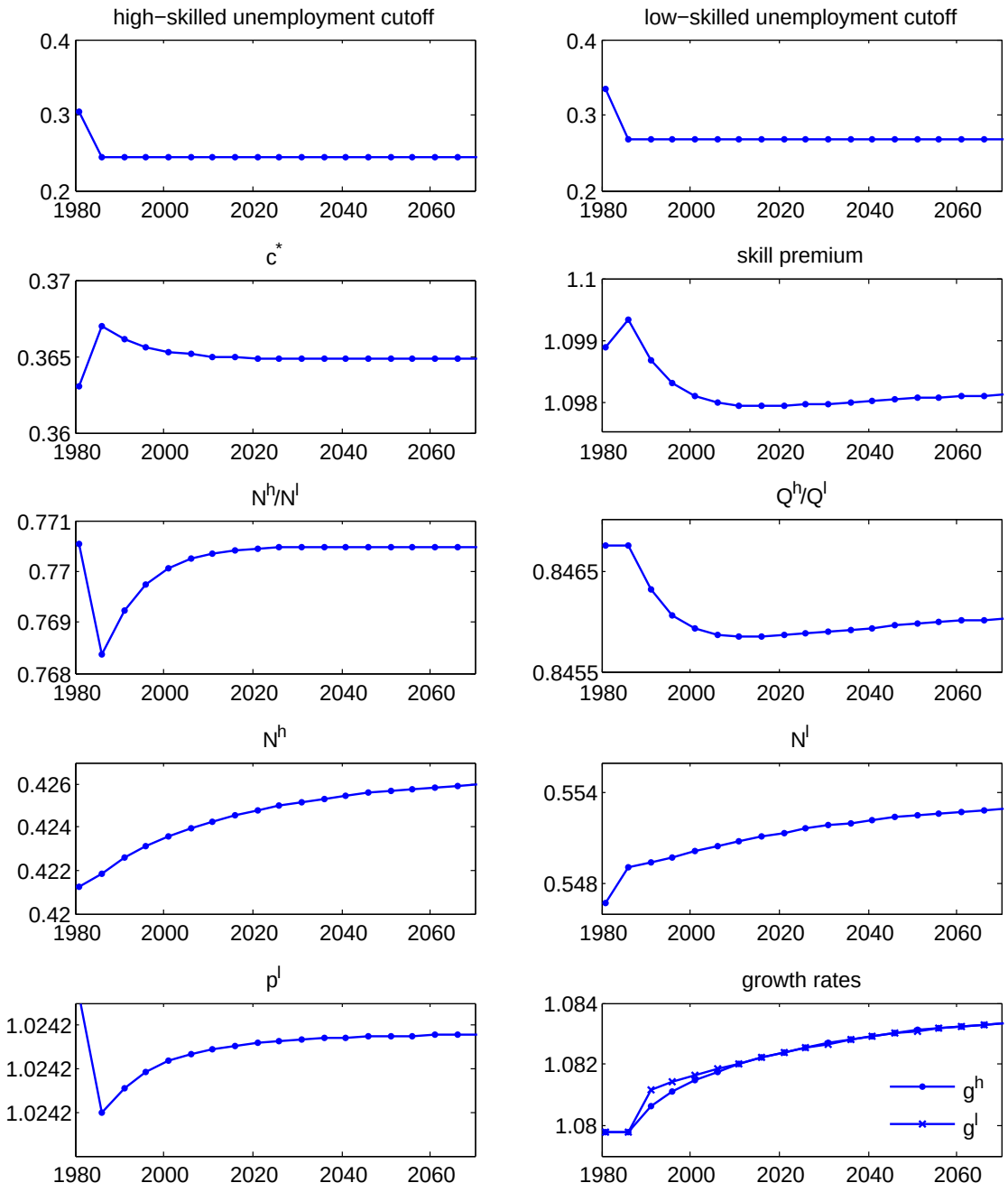

Figure 7: Transition of the main variables

it, while the threshold for high-skilled unemployment drops slightly below, then increases to its new steady state value. Equation (24) shows that only the price of the low-skilled intermediate affects the path of $\underline{a}^l$. As the bottom left panel of Figure 7 shows, the change in the steady state price is very small, which explains the seemingly immediate jump of $\underline{a}^l$ to its new steady state value. The movement of $\underline{a}^h$ can be understood from (25): $\underline{a}^h$ follows $\underline{a}^l w_t^l / w_t^h$, therefore the initial overshooting of the skill premium (second row, right panel in Figure 7) explains the undershooting of $\underline{a}^h$. The thresholds for unemployment do not change much after the initial drop because intermediate prices and the skill premium do not change much either.

Note that the new value of $\underline{a}^{l*}$ is lower than the initial $\underline{a}^{h*}$; this suggests that those who acquire education in order to avoid unemployment in the new steady state and during the transition have lower ability than those who did the same in the previous steady state.

The path of the cutoff time cost for acquiring education is shown in the left panel in the second row of Figure 7. This threshold c^* initially overshoots and then decreases monotonically towards its new steady state value, which is higher than the original one. This pattern can be understood by looking at the path of the skill premium (second row, right panel) and the path of the growth rates (bottom right panel). The initial jump in the skill premium drives the overshooting of c^*, then as the skill premium decreases, so does c^*. The monotone increase in the growth rate increases the present value gain of being high-skilled for a given skill premium, which keeps the new steady state value of c^* above the initial one.

Taking the path of the three cutoffs $\underline{a}^{h*}, \underline{a}^{l*}$ and c^* as given, the paths of the effective supply of high- and low-skilled labour (depicted in row 4 of Figure 7) can be understood. Figure 8 plots the effect of changes in the cutoffs on the high- and low-skilled effective labour supply and on the labour market participation of individuals. The initial steady state thresholds are denoted by $\underline{a}_0^{l*}, \underline{a}_0^{h*}, c_0^*$, while the new steady state values are denoted by $\underline{a}_1^{l*}, \underline{a}_1^{h*}, c_1^*$. The maximum value of c^*, which is reached in the period of the announcement is denoted by c_{max}^*.

The shift in the cutoffs lead to two types of changes: in the education decisions and in the labour force participation of individuals. These mostly affect the *new generations*:

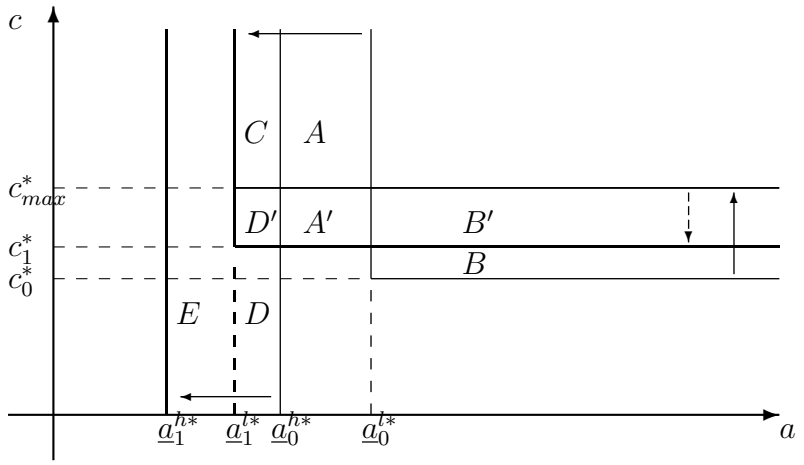

Figure 8: Change in the optimal education and labour market participation

Notes: Ability is on the horizontal axis, the time cost is on the vertical axis. The maximum of the time cost $\overline{c}$ is omitted from the graph, to make it less cluttered. The initial steady state cutoffs are: $\underline{a}^{h*}_0, \underline{a}^{l*}_0, c^*_0$, while the new ones are: $\underline{a}^{h*}_1, \underline{a}^{l*}_1, c^*_1$. I denote the maximum threshold time cost that is reached in the period of the announcement by c^*_{max}.

those born in the period of the announcement, and in subsequent generations. This is because the option of acquiring education is only available at birth, and individuals are not allowed to retrain themselves in later periods. Thus, the labour supplies adjust gradually, as new generations replace old ones, lengthening the transition period.

The only case where this is not true is for members of previous generations (for example person C, D or D')with ability between $\underline{a}^{l*}_1$ and $\underline{a}^{h*}_0$. They are low-skilled and have been unemployed until now, but in the period of the announcement they can immediately start working as low-skilled workers. Their *entry* into the workforce instantaneously increases the supply of low-skilled workers, which is reflected by the jump in N^l.

Members of the new generation with ability between $\underline{a}^{l*}_1$ and $\underline{a}^{h*}_0$ either start working as low-skilled, as C, or enrol in education at birth, as person D or D'. People with the same time cost as D' will only become high-skilled if they belong to generations born close to the initial shock, whereas people with time cost as D become high-skilled regardless of the generation they are born in. This implies that the initial increase in low-skilled labour supply will be diminished to some extent in future periods, as individuals similar to D become high-skilled instead of working as low-skilled. They replace some members of the older generations who went from unemployment into

the low-skilled workforce. The *education* of individuals like D increases the supply of high-skilled workers while decreasing the supply of low-skilled workers gradually. This is reflected by the gradual increase in N^h.

Consider person E from one of the new generations. He would have been unemployed under the previous regime, but now can avoid unemployment by becoming high-skilled. This is true for all members with ability in $[\underline{a}_1^{h*}, \underline{a}_1^{l*})$ in the new generations. The *entry* of these individuals leads to a gradual increase in N^h.

Individuals similar to A and A' would have been high-skilled with the original, higher minimum wage in order to avoid unemployment. Under the new, lower minimum wage, they can work without acquiring education. Initially only individuals with time cost as high as A remain low-skilled. Gradually as c^* decreases to c_1^* individuals with time cost as A' also opt out from education. The change in the *education* of individuals with ability in $[\underline{a}_0^{h*}, \underline{a}_0^{l*})$ and high enough education time cost gradually increases the supply of low-skilled workers at the expense of the high-skilled workforce, reflected in the gradual increase in N^l.

Since the cutoff time cost initially overshoots and then decreases monotonically to its new steady state value, in generations closer to the announcement, more individuals become high-skilled among those with ability greater than $\underline{a}_1^{l*}$. Consider individual B'. If born in the period of the announcement, he acquires education. In the long run, however, it will only be individuals with time cost as B whose education choice is different from the choice of generations born before the change in the minimum wage. This implies that initially individuals with higher time cost acquire education than the new steady state implies. The *education* of these individuals gradually increases the supply of the high-skilled workforce.

The left panel in row 3 of Figure 7 shows the overall effect of these changes on the relative supply of skills, N^h/N^l: the relative supply of skills decreases on impact. This is the result of two forces. First there is mass entry from unemployment into the low-skilled labour force at the time of the announcement.The effect of this can be seen on the right panel in row 4 as N^l jumps up. Second, there is entry from unemployment into the high-skilled labour force, but the effect of this is offset to some extent by the exit of some ability levels, N^h initially increases only slightly (left panel in row 4).

As time passes the effect of the initial increase in the supply of low-skilled workers is diminished, the relative supply of high-skilled workers starts increasing more, and growth in the supply of low-skilled workers decreases. Row 4 of Figure 7 shows that both supplies increase gradually, and both measures rise above their initial level in the long run.

The skill premium per efficiency unit depends on two factors along the transitions: the relative supply of high-skilled workers and the relative technology available. The interaction of the two is shown in the right panel of row 2 in Figure 7: on impact the skill premium increases. The initial decline in the relative supply of skills increases the skill premium. This supply effect is not offset by technology, as depicted in the right panel in row 3. Even though technology becomes less biased towards the high-skilled workers in the long run, in the short run it does not have sufficient time to react to these changes. As the change in the relative supply is unanticipated, technology can only adjust from the next period onwards. This explains the initial increase in both the skill premium per efficiency unit of labour (right panel in row 2), and the average skill premium (top panel of Figure 9).

From the second period on, technology adapts according to the change in relative supplies, shown on the right panel in row 3. It reacts with a lag to the initial decline in relative supply by undershooting, and then gradually increasing to its new steady state value, which is slightly below the original steady state. As technology starts to react to the change in relative supply, the skill premium drops as well, undershooting its final steady state value. In the long run the skill premium converges to its new steady state value, which is slightly lower than its initial value.

The variables with empirically observable counterparts are the relative supply of high- and low-skilled raw labour, L^h/L^l, and the average skill premium, $\overline{w}^h/\overline{w}^l$. The relative raw labour supply is shown in the bottom panel of Figure 9. Its path is very similar to that of the effective labour supply, but the magnitude of change is quite different. This difference in magnitude is due to the difference in ability between those who join the low-skilled and the high-skilled labour market. The measure of people joining the low-skilled workforce is much larger than the measure of those joining the high-skilled workforce, reflected in the significant overall decline in the relative supply

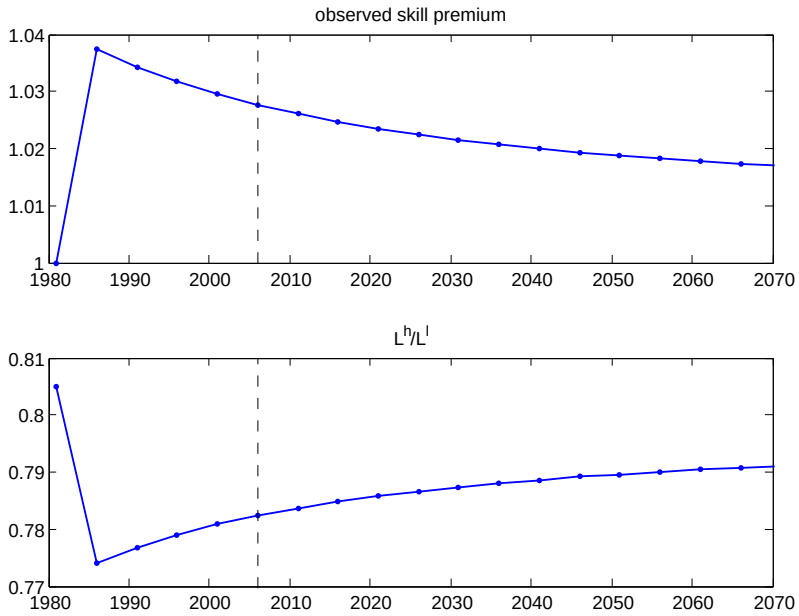

Figure 9: Average skill premium and relative raw labour supply

Notes: The vertical dashed line represents 2006, the year to which I am comparing the results to. The top panel represents the change in the observed skill premium compared to its initial value, while the bottom panel shows the path of the relative supply of raw high-skilled labour.

of raw high-skilled labour. On the other hand, the average ability of those joining the high-skilled workforce is higher than the average of those joining the low-skilled. This is demonstrated by the only slight long run decline in the relative supply of high-skilled effective labour. This implies that *compositional* changes play an important role in both the high-skilled and the low-skilled workforce. The average ability in both sectors decreases, but it decreases relatively more among the low-skilled than among the high-skilled workers.

The top panel in Figure 9 represents the change in the observed skill premium compared to its initial value. The observed skill premium increases on impact and then decreases gradually, as does the skill premium per efficiency unit of labour. However, unlike the skill premium per efficiency unit, the average skill premium converges to a value higher than its initial value in the long-run. This is due to *compositional* effects: since the average ability in the low-skilled labour force decreases more than in the high-skilled labour force, the average skill premium increases relative to its initial value.

Between 1981 and 2006 the average skill premium increased by 18 percent (see Figure 1). In the model, twenty five years after the decline in the minimum wage (at the dashed vertical line), the increase is 2.7 percent, implying that the minimum wage accounts for 15 percent of the increase in the observed skill premium.

The widening wage inequality is well captured by the increasing gap between the wages of workers in the 90th, 50th and 10th percentile. Figure 10 shows the change in these measures during the transition. The dashed vertical line represents the year 2006.

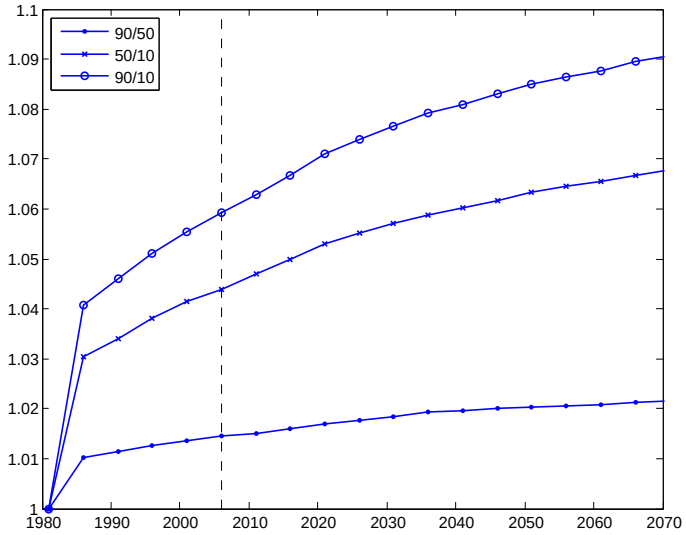

Figure 10: Wage gaps during the transition

Notes: The vertical dashed line represents 2006, the year to which I am comparing the results to.

These wage gaps increase due to two factors: changes in the skill premium per efficiency unit, and compositional effects.

Changes in the skill premium only increase inequality in the period of the announcement; from the third period onwards these changes compress the wage distribution (see Figure 7 second row right panel).

Compositional forces always put an upward pressure on inequality. One component is the widening range of abilities present on the labour market. As the normalized minimum wage drops, the threshold abilities for unemployment decrease, increasing the range of abilities present on the labour market. As the range of abilities widens,

the gap between the ability level at the 90th percentile gets further away from the ability level at the 50th percentile, which gets further from the 10th percentile. The second component is the changing ratio of high- to low-skilled workers at every percentile in the wage distribution. The fraction of high-skilled workers among the top 10 percent of earners increases, while their ratio at the bottom 10 percent decreases.

All three wage gaps increase the most in the period of the announcement, since the skill premium and the compositional effects both put an upward pressure on them in this period. After the first period, the wage gaps widen further, but at a slower rate. The 90/10 wage differential increases the most, while the 90/50 increases the least. This is expected, since most of the compositional changes affect the lower end of the wage distribution.

Note, however, that the the change in the minimum wage causes the top end of the wage distribution to widen as well. This is mostly due to the compositional changes both in ability and in skill levels, which affect the position of the 90th percentile and the 50th percentile eraner differentially.

The 90/10 wage gap increased by 32 percent between 1981 and 2006, the 90/50 wage gap increased by 21 percent, and the 50/10 wage gap increased by 10 percent (see Figure 1). The model is most successful at predicting the 50/10 wage gap - it explains about 45 percent of the observed increase, while it explains about 18.5 percent and 7 percent of the increase in the 90/10 and 90/50 wage gaps, respectively.

7 Decomposition

I consider three simplified versions of the model, in order to better understand the contributions of changing technology and education to the effects of minimum wages on the patterns of wage inequality. The first version is one where both educational attainment and technology are fixed. In the second version, the skill composition is endogenous, but technology is fixed. The third version features fixed educational attainment and endogenously directed technical change.

Comparing the transitional dynamics of the four models quantitatively shows that most of the initial effects are due to the inflow from unemployment into the labour

market. The decomposition shows that in the case of endogenous education, compositional effects play an important role, and that the change in technology does not have a quantitatively big impact on overall inequality.

7.1 Exogenous education, exogenous technology

Consider a model, where the production side is as in the model, but technology and education are fixed. Technology in the low- and the high-skilled sector is growing at the same rate. There are high and low-skilled individuals, but the choice of acquiring education is fixed in other words, nobody can acquire additional education and nobody can opt out from education. I assume that the education and employment structure in the initial steady state is as in the full model.

If both education and technology are fixed, then lowering the minimum wage affects the wage distribution only through an expansion of low-skilled employment. A lower minimum wage allows people who have been previously unemployed, and are hence low-skilled, to enter the low-skilled labour market (see section E.1 of the Appendix). With constant technology, this decreases the wage per unit of efficiency for the low-skilled, thereby increasing the skill premium. However, since education is fixed, this does not translate into an increase in the supply of high-skilled labour. The average ability in the low-skilled sector decreases, hence the observed skill premium increases more than the skill premium per efficiency unit.

In this setup there are no transitional dynamics, as low-skilled employment expands in the period of the announcement, the skill-premium responds, and there are no further adjustments. As a consequence of a fall in the minimum wage, the supply of low-skilled labour increases, the skill premium increases and wage gaps between different percentiles of the distribution also increase.

7.2 Endogenous education, exogenous technology

Now consider a model where educational choices are made optimally, but technology is fixed. As in the previous model, quality in the high- and the low-skilled sector is growing at the same rate. Since education changes endogenously, I model the

labour market side exactly as in the full model. The key difference is that since growth is exogenous, there is no feedback from the effective labour supplies to the direction and rate of technological improvements. Therefore, the relative supply of skills only affects the skill premium through the price effect, as the market size effect is removed. Hence, in this setup, the skill premium per efficiency unit is always decreasing in the relative supply of skills:

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} \left(\frac{Q^h}{Q^l}\right)^{\frac{\beta\rho}{1-\rho+\beta\rho}}.$$

The unemployment cutoffs and the threshold for acquiring education are determined exactly as in the full model (see Appendix section E.2 for details). The only differences are that the skill premium is always decreasing in the relative supply (see equation above) and the growth rate is exogenous and independent of the relative supply of skills.

In the Section E.2 of the Appendix, I show that the system can be reduced to two thresholds, $\underline{a}^{l*}$ and c^*, as in the full model, and the two equations defining the steady state are as in Figure 5. This also implies that as in the full model, a reduction in the minimum wage reduces the unemployment threshold in both sectors, and increases the threshold cost of acquiring education.

In the long-run, the supply of high- and low-skilled effective labour increases, with the relative supply of skills decreasing. This implies an increase in the skill premium per efficiency unit, unlike in the full model. Moreover, the average ability in the low skilled sector decreases more, which implies that the observed skill premium increases more that the skill premium per efficiency unit. The wage gaps between different percentiles also increase.

The transition takes a long time, as in the full model, since complete educational adjustment takes several generations.

7.3 Exogenous education, endogenous technology

Finally, consider an economy where education is fixed, but technology changes

endogenously. In such a setup, a lower minimum wage increases the supply of low-skilled labour, thus increasing the skill premium. This does not lead to an increase in the supply of skills, as educational choices are fixed. The average ability in the low-skilled sector decreases, implying that the observed skill premium increases more than the skill premium per efficiency unit.

Transition takes time, as technology needs to adapt to the new relative labour supplies. In the long-run, technology becomes less skill-biased and the skill premium per efficiency unit falls below its original value.

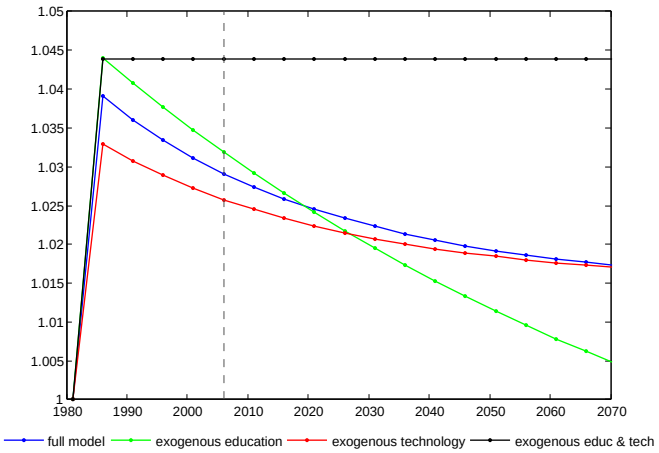

Figure 11: The role of education and technology in the average skill premium

Notes: The vertical dashed line represents 2006, which is the final year, to which I am comparing the results. The colours represent: blue – full model, red – exogenous technology, endogenous education, green – endogenous technology, exogenous education, black – exogenous technology, exogenous education.

7.4 Decomposition results

Figures 11 and 12 show the path of the observed skill premium and wage gaps between different percentiles in the distribution. The observed skill premium increases the most in the case of fixed education and technology, both in the short- and the long-run. This is not true for the wage gaps: the wage gaps in the short-run increase the most in the case of fixed education and technology, but in the long-run, the effects are bigger when education is endogenous. The different pattern of the skill premium and the wage gaps suggest that the increase in the observed skill premium is driven by the

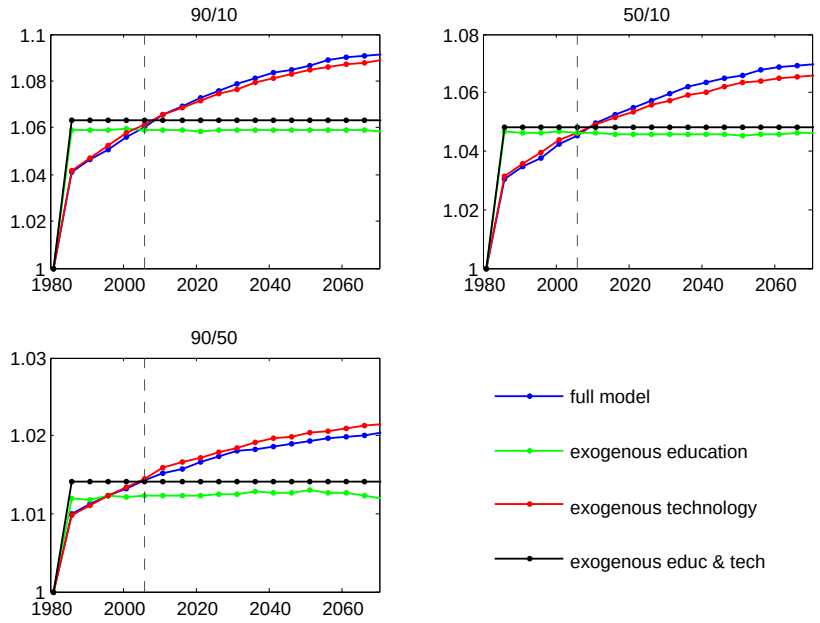

Figure 12: The role of education and technology in the wage gaps

Notes: The vertical dashed line represents 2006, the final year of data. The colours represent: blue – full model, red – exogenous technology, endogenous education, green – endogenous technology, exogenous education, black – exogenous technology, exogenous education.

expanding employment of the low-skilled.

The observed skill premium increases the most in the case of exogenous technology and exogenous education (see Figure 11), as the increase in the supply of low-skilled labour is the largest. With endogenous technology, the initial impact is the same, but is diminished in the long-run as technologies become less skill-biased. When education is endogenous, the initial impact of lowering the minimum wage is smaller. This is due to an expansion of high-skilled employment. As low-skilled workers enter the labour market and the skill premium increases, the incentives for acquiring education increase, leading to an expansion of the high-skilled labour force, thus diminishing the initial increase in the skill premium. The initial increase in the skill premium is larger when technology is endogenous, due to the higher growth rate of the economy. An expansion of the labour force leads to a higher growth rate in case of endogenous technology, which implies a higher lifetime gain from working in the high-skilled sector. Therefore, if technology is endogenous, the cutoff time cost for education increases more, leading to a larger change in average abilities and larger compositional effects.

Figure 12 shows the patterns of wage gaps. In all three graphs, the biggest initial impact is in the case of exogenous education, implying that most of the initial increase is due to the inflow of previously unemployed workers into the low-skilled labour market. In the long-run, the wage gaps increase the most in the case of endogenous education, suggesting that compositional effects play a significant role in the widening dispersion of wages.

8 Concluding Remarks

There has been much debate about the contribution of the falling minimum wage to the widening wage inequality in the US. The real value of the minimum wage eroded over the 1980s, losing 30 percent of its initial value. At the same time - in the early 1980s - there was an unprecedented surge in inequality. The wage gap widened between any two points in the wage distribution, and the college premium increased sharply. However, to my knowledge, there are no attempts in the literature to assess the quantitative significance of falling minimum wages for wage inequality in the context of a general equilibrium model.

In this paper I propose a general equilibrium model to analyse the effects of a permanent decrease in the value of the minimum wage on inequality. This model incorporates minimum wages, endogenous educational choices and endogenous technological progress. All these components are relevant in their own right: minimum wages affect the educational decisions of individuals through their effect on job and earning opportunities; educational decisions shape the skill composition of the labour force and the ability composition of different skill groups; the supply of high- and low-skilled labour affects the direction of technological change and the direction of technological change affects the educational decision of individuals.

The analysis in general equilibrium reveals that a reduction in the minimum wage affects overall inequality through three channels. First, a reduction in the minimum wage widens the range of abilities present on the labour market, thereby increasing the difference between any two percentiles in the distribution. Second, it differentially affects the shares of high- and low-skilled workers at every percentile in the wage

distribution, thus increasing overall inequality. A third channel is the reduction in the skill premium per efficiency unit, which reduces inequality. Therefore, a reduction in the minimum wage affects inequality at the top end of the wage distribution, even if only to a smaller extent.

The full effects of minimum wage reductions are only realized in the long run. Minimum wages affect the educational decisions of individuals in successive cohorts. New cohorts have to replace old ones for the new equilibrium to be reached. Through considering three simplified models, I show that the initial and highest increase in all measures of inequality is due to the inflow from unemployment in the period of the announcement. After this period, the observed skill premium contracts, while the widening of the wage distribution continues due to compositional changes in both ability and skills.

In this model, a reduction in the minimum wage reduces the skill-bias of technology, since the inflow from unemployment is mainly into the low-skilled sector. In future research I plan to test the robustness of the results to different labour market structures. More specifically the low-skilled sector should feature either monopsony or search frictions. In these scenarios the reduction of the minimum wage does not affect unemployment to the same extent, but it still triggers an expansion of the high-skilled labour force through the increase in the skill premium.

References

ÁBRAHÁM, Á. (2008): "Earnings Inequality and Skill-Biased Technological Change with Endogenous Choice of Education," *Journal of the European Economic Association*, 6(2-3), 695–704.

ACEMOGLU, D. (1998): "Why Do new Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, 113(4), 1055–1089.

——— (2003): "Cross-country Inequality Trends," *Economic Journal*, 113, 121149.

ACEMOGLU, D., AND J.-S. PISCHKE (2003): "Minimum Wages and On-the-job Training," in *Research in Labor Economics, Volume 22*, pp. 159–202. Emerald Group Publishing Limited.

AGHION, P., AND P. HOWITT (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60(2), 323–351.

ALESINA, A., AND J. ZEIRA (2006): "Technology and Labor Regulations," NBER Working Papers 12581, National Bureau of Economic Research, Inc.

AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2005): "Rising Wage Inequality: The Role of Composition and Prices," Harvard Institute of Economic Research Working Papers 2096, Harvard - Institute of Economic Research.

——— (2008): "Trends in U.S. Wage Inequality: Revising the Revisionists," *Review of Economics and Statistics*, 90(2), 300–323.

AUTOR, D. H., L. F. KATZ, AND A. B. KRUEGER (1998): "Computing Inequality: Have Computers Changed the Labor Market?," *Quarterly Journal of Economics*, 113(4), 1169–1213.

AUTOR, D. H., A. MANNING, AND C. L. SMITH (2009): "The Role of the Minimum Wage in the Evolution of U.S. Wage Inequality over Three Decades: A Modest Re-Assessment," *MIT Working Paper*.

BAKER, M., D. BENJAMIN, AND S. STANGER (1999): "The Highs and Lows of the Minimum Wage Effect: A Time-Series Cross-Section Study of the Canadian Law," *Journal of Labor Economics*, 17(2), 318–350.

BEAUDRY, P., AND D. A. GREEN (2003): "Wages and Employment in the United States and Germany: What Explains the Differences?," *American Economic Review*, 93, 573–602.

——— (2005): "Changes in U.S. Wages, 1976-2000: Ongoing Skill Bias or Major Technological Change?," *Journal of Labor Economics*, 23(3), 609–648.

BERMAN, E., J. BOUND, AND Z. GRILICHES (1994): "Changes in the Demand for Skilled Labor within U.S. Manufacturing: Evidence from the Annual Survey of Manufactures," *Quarterly Journal of Economics*, 109(2), 367–97.

BLANCHARD, O. J. (1985): "Debt, Deficits and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.

BROWN, C., C. GILROY, AND A. KOHEN (1982): "The Effect of the Minimum Wage on Employment and Unemployment," *Journal of Economic Literature*, 20(2), pp. 487–528.

CAHUC, P., AND P. MICHEL (1996): "Minimum Wage Unemployment and Growth," *European Economic Review*, 40(7), 1463–1482.

CARD, D. (1992): "Using Regional Variation in Wages to Measure the Effects of the Federal Minimum Wage," *Industrial and Labor Relations Review*, 46(1), pp. 22–37.

CARD, D., AND J. E. DINARDO (2002): "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles," *Journal of Labor Economics*, 20(4), 733–783.

CARD, D., AND A. B. KRUEGER (1994): "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review*, 84(4), 772–93.

CARD, D., AND A. B. KRUEGER (2000): "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Reply," *American Economic Review*, 90(5), 1397–1420.

CASELLI, F. (1999): "Technological Revolutions," *American Economic Review*, 89(1), 78–102.

DINARDO, J., N. M. FORTIN, AND T. LEMIEUX (1996): "Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach," *Econometrica*, 64(5), 1001–1044.

ECKSTEIN, Z., AND É. NAGYPÁL (2004): "The Evolution of U.S. Earnings Inequality: 1961-2002," *Quarterly Review*, 28(2), 10–29.

GALOR, O., AND O. MOAV (2000): "Ability-Biased Technological Transition, Wage Inequality, and Economic Growth," *Quarterly Journal of Economics*, 115(2), 469–497.

GOLDBERG, P. K., AND N. PAVCNIK (2004): "Trade, Inequality, and Poverty: What Do We Know? Evidence from Recent Trade Liberalization Episodes in Developing Countries," NBER Working Papers 10593, National Bureau of Economic Research.

GOLDIN, C., AND L. F. KATZ (2007): "Long-Run Changes in the Wage Structure: Narrowing, Widening, Polarizing," *Brooking Papers on Economic Activity*, 2, 135–165.

——— (2008): *The Race Between Education and Technology*. The Belknap Press of Harvard University Press, Cambridge, MA.

HECKMAN, J., L. LOCHNER, AND C. TABER (1998): "Explaining Rising Wage Inequality: Explanations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents," *Review of Economic Dynamics*, 1(1), 1–58.

JUHN, C., K. M. MURPHY, AND B. PIERCE (1993): "Wage Inequality and the Rise in Returns to Skill," *Journal of Political Economy*, 101(3), 410–442.

KATZ, L. F., AND K. M. MURPHY (1992): "Changes in the Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, 107(1), 35–78.

KILEY, M. T. (1999): "The Supply of Skilled Labour and Skill-Biased Technological Progress," *Economic Journal*, 109(458), 708–24.

KOENIGER, W., AND M. LEONARDI (2007): "Capital Deepening and Wage Differentials: Germany versus US," *Economic Policy*, 22.

KRUEGER, A. B. (1993): "How Computers Have Changed the Wage Structure: Evidence from Microdata, 1984-1989," *Quarterly Journal of Economics*, 108(1), 33–60.

KRUSELL, P., L. E. OHANIAN, J.-V. RÍOS-RULL, AND G. L. VIOLANTE (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68(5), 1029–1054.

LEE, D. S. (1999): "Wage Inequality in the United States During the 1980s: Rising Dispersion or Falling Minimum Wage?," *Quarterly Journal of Economics*, 114(3), 977–1023.

LEMIEUX, T. (2006): "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?," *American Economic Review*, 96(3), 461–498.

MACHIN, S., AND A. MANNING (1994): "The Effects of Minimum Wages on Wage Dispersion and Employment: Evidence from the U.K. Wages Councils," *Industrial and Labor Relations Review*, 47(2), pp. 319–329.

NEUMARK, D., AND O. NIZALOVA (2007): "Minimum Wage Effects in the Longer Run," *Journal of Human Resources*, (2), 435–452.

NEUMARK, D., AND W. WASCHER (1992): "Employment Effects of Minimum and Subminimum Wages: Panel Data on State Minimum Wage Laws," *Industrial and Labor Relations Review*, 46(1), pp. 55–81.

——— (2000): "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania: Comment," *American Economic Review*, 90(5), pp. 1362–1396.

——— (2001): "Minimum Wages and Training Revisited," *Journal of Labor Economics*, 19(3), pp. 563–595.

——— (2003): "Minimum Wages and Skill Acquisition: Another Look at Schooling Effects," *Economics of Education Review*, 22, 1–10.

PISCHKE, J.-S. (2005): "Labor Market Institutions, Wages, and Investment: Review and Implications," *CESifo Economic Studies*, 51(1), 47–75.

RAVN, M. O., AND J. R. SORENSEN (1999): "Schooling, Training, Growth and Minimum Wages," *Scandinavian Journal of Economics*, 101(3), 441–57.

SCHMITT-GROHE, S., AND M. URIBE (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28(1), 755–775.

SUTCH, R. (2010): "The Unexpected Long-Run Impact of the Minimum Wage: An Educational Cascade," NBER Working Papers 16355, National Bureau of Economic Research.

WELLINGTON, A. J. (1991): "Effects of the Minimum Wage on the Employment Status of Youths: An Update," *Journal of Human Resources*, 26(1), 27–46.

A R&D

A.1 Probability of successful innovation for a given R&D firm

The Poisson arrival rate of innovation for all firms indexed by $k = 1, 2, ...$ when spending z_k units on R&D is ηz_k. Since Poisson processes are additive, the economy wide arrival rate of innovation is $\eta \sum_{k=1}^{\infty} z_k$ if $\sum_{k=1}^{\infty} z_k \equiv \overline{z} < \infty$. In this case the probability that there is at least one innovation until the end of the period is:

$$\int_0^1 -\eta \overline{z} e^{-\eta \overline{z} t} dt = 1 - e^{-\eta \overline{z}}$$

I assume that once a firm has a successful innovation, that firm receives the patent and innovation on that line is finished for that period. Then the probability that matters is the probability that a given firm has the first innovation. The probability that firm k has the first innovation at time t is:

$$-\eta z_k e^{-\eta z_k t}(e^{-\eta(\overline{z}-z_k)t}) = -\eta z_k e^{-\eta \overline{z} t}$$

The probability that firm k has the first successful innovation until the end of the period is just:

$$\int_0^1 \lambda z_k e^{-\eta \overline{z} t} dt = \frac{z_k}{\overline{z}}(1 - e^{-\eta \overline{z}})$$

Which is what I wanted to show.

A.2 Monopoly pricing

Lemma 2. *If $\overline{q} > (1-\beta)^{-\frac{1-\beta}{\beta}}$ then at any moment in time only the best quality of any machine will be bought at its monopoly price.*

Proof. When the marginal cost of producing a machine of quality q_1 is q_1, then given the demand in (5) the monopoly price of this machine is $\chi_1 = \frac{q_1}{1-\beta}$. If an intermediate good producing firm uses this machine his profit is:

$$\pi_1 = (p^s)^{\frac{1}{\beta}} N^s q_1 \left((1-\beta)^{\frac{1-\beta}{\beta}-1} - (1-\beta)^{\frac{1-\beta}{\beta}} \right)$$

If the firm instead uses a lower $q_2 = \frac{q_1}{\overline{q}^k}$ quality machine at the price of its marginal cost $\chi_2 = q_2$, then his profit is:

$$\pi_2 = (p^s)^{\frac{1}{\beta}} N^s q_1 \left(\frac{1}{\overline{q}^k}(1-\beta)^{-1} - \frac{1}{\overline{q}^k} \right)$$

If $\pi_1 > \pi_2$ for all $k > 0$ integers, then only the best quality of any machine will be bought in equilibrium at its monopoly price.
The $\pi_1 > \pi_2$ condition is equivalent to:

$$(1-\beta)^{\frac{1-\beta}{\beta}-1} - (1-\beta)^{\frac{1-\beta}{\beta}} > \frac{1}{\overline{q}^k}(1-\beta)^{-1} - \frac{1}{\overline{q}^k}$$

With some algebra we get that this is equivalent to:

$$\overline{q}^k > (1-\beta)^{-\frac{1-\beta}{\beta}}$$

Since the above holds for $k = 1$ and $\overline{q} > 1$, it holds for all $k \geq 1$. □

B Steady State

B.1

Since the total size of the population is constant, both N^{h*} and N^{l*} are constant along the BGP. The supply of effective labour, N^{h*} and N^{l*} can only be constant if the threshold abilities for unemployment, $\underline{a}^l, \underline{a}^h$ and the optimal education decision $e(a, c)$ for all a and c are constant. The cutoff abilities for unemployment are defined by:

$$\underline{w}_t = \underline{a}^l \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^{l*})^{\frac{1}{\beta}} Q_t^{l*}$$
$$\underline{w}_t = \underline{a}^h \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^{h*})^{\frac{1}{\beta}} Q_t^{h*}$$

Hence along the steady state where both $\underline{a}^h$ and $\underline{a}^l$ are constant

$$\frac{\underline{a}^l}{\underline{a}^h} \frac{p_t^{h*}}{p_t^{l*}}^{-\frac{1}{\beta}} = \frac{Q_t^{h*}}{Q_t^{l*}},$$

The relative price of the intermediate goods depends on the relative quality and the relative labour supply in the two groups. Combining the above with (12) gives:

$$\frac{Q_t^{h*}}{Q_t^{l*}}^{1-\frac{(1-\rho)}{(1-(1-\beta)\rho)}} = \frac{\underline{a}^l}{\underline{a}^h}\gamma^{-\frac{1}{(1-(1-\beta)\rho)}}\left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{(1-\rho)}{(1-(1-\beta)\rho)}} \tag{29}$$

Since $\beta \neq 0$ the relative quality, $Q_t^* = Q_t^{h*}/Q_t^{l*}$ is constant in the steady state. This also immediately implies that the relative price of the intermediates, $p^* = p_t^{h*}/p_t^{l*}$ is constant in the steady state. Since the price of the final good is normalized to one, this also implies that p^{h*} and p^{l*} are constant.

If prices of intermediate goods are constant, and the supply of both types of effective labour is constant, then from (6), the per period profit from owning a leading vintage of quality q is constant as well. In the next section I show that constant period profits imply that steady state R&D investments on a line j in sector s are independent of the quality of the leading vintage in that line.

B.2 R&D spending

Using that the steady state profits in sector s are constant:

Lemma 3. *The total R&D spending on any line for a given quality is constant along the BGP:* $\overline{z}_t^{j,s*}(q) = \overline{z}_{t+T}^{j,s*}(q) = \overline{z}^{j,s*}(q)$ *for all* $t, T \geq 0$.

Proof. The R&D spending on each line has to be either constant or growing at a constant rate along the balanced growth path. This implies that the equilibrium total R&D spending on line j in sector s can be written as: $\overline{z}_{t+T}^{j,s*}(q) = \gamma^T \overline{z}_t^{j,s*}(q)$. Where $\gamma > 0$ is the growth rate of the R&D spending on line j in sector s for a given quality q. In what follows I denote $\overline{z}_t^{j,s*}(q)$ by z_t. Conditional on quality q, the per period profit is constant, $\pi^{s*}q$, since both N^{s*} and p^{s*} are constant along the BGP. Iterating forward (7), the value of owning the leading vintage on line j with quality q at time $t+T$ can be written as:

$$V_{t+T}(q) = q\pi^{s*}\sum_{\tau=0}^{\infty}\frac{e^{-\eta z_t \gamma^T \frac{\gamma^\tau - 1}{\gamma - 1}}}{(1+r)^\tau}.$$

Given $V_{t+T}(q)$ the equilibrium level of R&D spending is z_{t+T} if (8) is satisfied:

$$\frac{1}{1+r}\frac{(V_{t+T}(q))}{z_{t+T}}(1-e^{-\eta z_{t+T}})=q.$$

This has to hold for all $T>0$, implying that

$$\begin{aligned}&\textstyle\sum_{k=0}^{\infty}\frac{e^{-\eta z_t}\frac{\gamma^k-1}{\gamma-1}}{(1+r)^k}(1-e^{-\eta z_t})=\\&\textstyle\frac{1}{\gamma}\sum_{k=0}^{\infty}\frac{e^{-\eta z_t\gamma}\frac{\gamma^k-1}{\gamma-1}}{(1+r)^k}(1-e^{-\eta z_t\gamma})=...\\&\textstyle\frac{1}{\gamma^T}\sum_{k=0}^{\infty}\frac{e^{-\eta z_t\gamma^T}\frac{\gamma^k-1}{\gamma-1}}{(1+r)^k}(1-e^{-\eta z_t\gamma^T}).\end{aligned}$$

To simplify notation denote $a_k\equiv\frac{\gamma^k-1}{\gamma-1}$ and $\eta z_t\equiv b$. Since the above should hold for any $T>0$, this implies that the difference between two consecutive terms should be zero. Taking logarithm and derivative with respect to T yields the following condition:

$$0=\ln\gamma\left(-1+\left(\frac{b\gamma^Te^{-b\gamma^T}}{1-e^{-b\gamma^T}}-\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}\right)\right). \tag{30}$$

This has to hold for all $T>0$, even as $T\to\infty$. There are three cases: $\gamma>1$, $\gamma<1$ and $\gamma=1$. For $\gamma=1$ the above trivially holds for all $T>0$.

For $\gamma>1$ taking the limes yields:

$$\begin{aligned}&\lim_{T\to\infty}\left(\frac{b\gamma^Te^{-b\gamma^T}}{1-e^{-b\gamma^T}}-\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}\right)=\\&0-\lim_{T\to\infty}\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}<0\end{aligned}$$

Where the second term is non-negative, implying a negative value as T grows very large. Hence, for $\gamma>1$ (30) does not hold for all $T>0$.

For $\gamma<1$ I will show that the second term in the brackets is strictly smaller than 1, except in the limit. Denote $x\equiv b\gamma^T$, then as $T\to\infty$, $x\to0$. The first term is smaller than 1 for any $x>0$:

$$\frac{xe^{-x}}{1-e^{-x}}<1\Leftrightarrow e^{-x}(1+x)<1$$

For $x = 0$, $e^{-x}(1+x) = 1$. The derivative of the left hand side is $-e^{-x}x$, which is negative for all $x > 0$, implying that for any $x > 0$ the above inequality strictly holds. The second term in the brackets is strictly positive for all $T > 0$ and finite. This implies that the term in the brackets is strictly smaller than 1 for any finite T. Hence (30) does not hold for any $T > 0$. □

Therefore in the steady state $\overline{z}^{j,s*}$ is constant for a line with quality q. This also implies that the value of owning the leading vintage with quality q in line j and sector s is constant in the steady state. Its value can be expressed from iterating (7) forward and using the above lemma as:

$$V_t^{j,s*}(q) = \frac{q\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*}}{1-\frac{e^{-\eta\overline{z}^{j,s*}(q)}}{1+r}}.$$

Note that the value of owning a leading vintage is proportional to its quality level. This observation leads to the following corollary:

Corollary 1. *In the steady state the total R&D spending on each line within a sector is constant and equal:* $\overline{z}_t^{j,s*} = \overline{z}_{t+v}^{k,s*} = \overline{z}^{s*}$ *for all* $j, k \in s$ *and all* $v \geq 0$.

Proof. Using (8) and the steady state value of owning a leading vintage, the total amount of R&D spending on line j in sector s with quality q is implicitly defined by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{j,s*}(q)\frac{(1+r-e^{-\eta\overline{z}^{j,s*}(q)})}{1-e^{-\eta\overline{z}^{j,s*}(q)}}.$$

The left hand side only depends on sector specific variables, hence the total amount of R&D spending on improving line j in sector s is independent of the current highest quality, q on that line. Since it is only the quality level that distinguishes the lines from each other within a sector the corollary follows. □

B.3

Therefore, the total amount of R&D spending on each line within a sector is equal and constant over time. This equilibrium R&D spending is given by (20). In the steady state $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$ and the growth rate is $g^* = 1 + (\overline{q} - 1)(1 - e^{-\eta\overline{z}^*})$.

The price of the intermediates can be expressed from substituting the steady state relative price (21) into the intermediate good prices (2):

$$p^{l*} = \left(1 + \gamma \left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}} \tag{31}$$

$$p^{h*} = \left(\left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{\beta\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}} \tag{32}$$

Using the steady state relative price and the steady state R&D investment:

$$B\overline{z}^* \frac{(1 + r - e^{-\eta\overline{z}^*})}{1 - e^{-\eta\overline{z}^*}} = \beta(1-\beta)^{\frac{1-\beta}{\beta}} \left(\gamma N^{h*\frac{\beta\rho}{1-\rho}} + N^{l*\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\beta\rho}} \tag{33}$$

The right hand side is the steady state per period profit from owning the leading vintage normalized by the quality of the vintage. This profit is increasing in both N^{h*} and N^{l*}. If the labour supply increases, then any unit of investment into R&D has a higher expected return, since there are more people who are able to use it. This implies that the steady state R&D spending and the steady state growth rate is increasing in the effective labour supplies.

B.4 Proof of Lemma 2

Proof. To see that $\underline{a}^{l*}$ and c^* uniquely define $\underline{a}^{h*}$ consider equation (25), making use of (23):

$$\underline{a}^{h*} = \underline{a}^{l*}\gamma^{-\frac{1}{1-\rho}} \left(\frac{N^{l*}}{N^{h*}}\right)^{\frac{\beta\rho}{1-\rho}-1}.$$

N^{h*} is decreasing in $\underline{a}^{h*}$. If $\frac{\beta\rho}{1-\rho} - 1 < 0$ then the right hand side is decreasing in $\underline{a}^{h*}$, while the left hand side is increasing, hence there is a unique $\underline{a}^{h*}$ that satisfies the equation.

If $\frac{\beta\rho}{1-\rho} - 1 > 0$, then both the right and the left hand side is increasing in $\underline{a}^{h*}$. The derivative of the left hand side is 1, while the derivative of the right hand side is:

$$\frac{\partial \underline{a}^{l*}\gamma^{-\frac{1}{1-\rho}} \left(\frac{N^{l*}}{N^{h*}}\right)^{\frac{\beta\rho}{1-\rho}-1}}{\partial \underline{a}^{h*}} = \underline{a}^{h*} \left(\frac{\beta\rho}{1-\rho} - 1\right) \frac{\left((1-\lambda)\int_{\underline{c}}^{\overline{c}}(1-c)g(c)dc + \lambda\right)\underline{a}^{h*}f(\underline{a}^{h*})}{N^{h*}}.$$

The second two terms are smaller than one, and the first term is also smaller than one for any $\underline{a}^{h*}$ that gives a sensible unemployment rate. This implies that in the region of interest there is a unique solution. □

C Calibration

C.1 Ability and Cost Distribution

Given the assumptions on the distribution of a and c, and the thresholds $\underline{a}^{l*}, \underline{a}^{h*}$ and c^* the high- and low-skilled effective labour supplies are:

$$\begin{aligned} N^{h*} = \quad & \left((1-\lambda)\int_0^{\bar{c}}(1-c)g(c)dc+\lambda\right)\int_{\underline{a}^{h*}}^{\underline{a}^{l*}} af(a)da + \\ & \left((1-\lambda)\int_0^{c^*}(1-c)g(c)dc+\lambda G(c^*)\right)\int_{\underline{a}^{l*}}^{\infty} af(a)da \end{aligned} \tag{34}$$

$$N^{l*} = (1-G(c^*))\int_{\underline{a}^{l*}}^{\infty} af(a)da \tag{35}$$

Where $f(\cdot)$ is the probability density function of the ability distribution and $G(\cdot)$ is the cumulative distribution function of the cost distribution. The above expressions account for the fact that those members of the new generation who choose to acquire education only work $1-c$ fraction of the first period of their life.

Note that the effective supply of labour is not equivalent to the measure of high- and low-skilled individuals, the difference being that the former counts the total *ability* available, while the latter counts the number of people. The measure of high-skilled, low-skilled and unemployed is given by:

$$\begin{aligned} L^{h*} = \quad & \left((1-\lambda)\int_0^{\bar{c}}(1-c)g(c)dc+\lambda\right)\int_{\underline{a}^{h*}}^{\underline{a}^{l*}} f(a)da + \\ & \left((1-\lambda)\int_0^{c^*}(1-c)g(c)dc+\lambda G(c^*)\right)G(c^*)\int_{\underline{a}^{l*}}^{\infty} f(a)da, \\ L^{l*} = \quad & (1-G(c^*))\int_{\underline{a}^{l*}}^{\infty} f(a)da, \\ L^{u*} = \quad & \int_0^{\underline{a}^{l*}} f(a)da. \end{aligned}$$

The cutoff ability of unemployment for the low-skilled is found by matching the

fraction of unemployed:

$$U = \int_0^{\underline{a}^{h*}} f(a)da \quad \Leftrightarrow \quad \underline{a}^{h*} = e^{(\sigma\Phi^{-1}(U)+\mu)} \tag{36}$$

The cutoff time cost is found by matching the fraction of low-skilled:

$$L^l = (1 - G(c^*)) \int_{\underline{a}^{l*}}^{\infty} f(a)da, \tag{37}$$

where $\underline{a}^{l*}$ satisfies (using (25)):

$$\underline{a}^{l*} = \underline{a}^{h*} \frac{w^{h*}}{w^{l*}} = \underline{a}^{h*} \frac{\overline{w}^h}{\overline{w}^l} \frac{\overline{a}^l}{\overline{a}^h},$$

and $\overline{a}^h, \overline{a}^l$ are the average abilities and $\overline{w}^h, \overline{w}^l$ are the average wages in the two education groups. The average ability in a sector is the ratio of the supply of efficiency units of labour to the supply of raw labour in that sector: $\overline{a}^s = N^s/L^s$. The supply of high- and low-skilled raw labour, L^h and L^l are observed from the data, but N^h and N^l have to be calculated using (34).

This way for any cost and ability distribution $\underline{a}^{l*}, \underline{a}^{h*}$ and c^* is given as a function of the fraction of unemployed and low-skilled workers. Finally note that the three thresholds and the parameters of the ability and cost distribution are sufficient to calculate the average ability in both education groups.

C.2 Elasticity of Substitution

The consensus value is around 1.4 based on the paper by Katz and Murphy (1992). This original estimate was based on 25 data points, and Goldin and Katz (2008) updated this estimate by including more years and found an elasticity of 1.64. The estimating equation is:

$$\log \frac{w^h}{w^l} = \alpha_1 + \alpha_2 \log \frac{H}{L}. \tag{38}$$

These estimates typically adjust for productivity differentials within a skill-group, but do not adjust for differentials between skill groups. Hence the labour aggregates H and L are between the measure of effective labour and raw labour. The parameter

estimate $\widehat{\alpha}_2$ is interpreted as the inverse of the elasticity of substitution between the two types of labour. I cannot use these estimates directly for several reasons.

First of all, the interpretation of $\widehat{\alpha}_2$ is different depending on the assumptions. To see this note that the skill premium per efficiency unit can be expressed as

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho}} \frac{N^h}{N^l}^{\frac{\beta\rho}{1-\rho}-1},$$

along the balanced growth path, while it can be measured as

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{N^h}{N^l}\right)^{-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{Q^h}{Q^l}\right)^{\frac{\beta\rho}{1-(1-\beta)\rho}}.$$

in the transition. Thereby, the interpretation along the BGP is $\widehat{\alpha}_2 = \beta\rho/(1-\rho) - 1$, while along the transition it is $\widehat{\alpha}_2 = -(1-\rho)/(1-(1-\beta)\rho)$. However, the estimate of $\widehat{\alpha}_2$ in the transition will be biased due to the lack of a good measure of average quality in the two sectors. Second, as noted before, the measure of labour supply aggregates used inKatz and Murphy (1992) are not the effective supply of labour, which in the model determines wages. Moreover, the measure of skill premium is not the skill premium per efficiency unit w^h/w^l of the model, it is probably closer to the average skill premium. Due to these reasons, reinterpreting the implications of the value of $\widehat{\alpha}_2$ for ρ is not sufficient to use these estimates in my calibration.

D Transitional Dynamics

To use the Schmitt-Grohe and Uribe algorithm, all equations have to be defined in terms of variables that are stationary in the steady state. Let v_t^s denote the normalized value of owning the leading vintage in sector s at time t:

$$v_t^h = \frac{V_t^h(q)}{q} \quad v_t^l = \frac{V_t^l(q)}{q}$$

Let Δ_t denote the normalized present value gain per unit of effective labour from acquiring education conditional on being employed in every future period (normalized

by the current quality in the low-skilled sector):

$$\Delta_t = \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^j \frac{w^h_{t+j} - w^l_{t+j}}{Q^l_t}$$

The equations that hold throughout the transition in terms of these normalized variables are:

$$\begin{aligned}
v^s_{t+1} = &\quad B\tfrac{(1+r)\overline{z}^s}{1-e^{-\eta\overline{z}^s_t}} && s = l, h\\
v^s_t = &\quad \beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^s_t)^{\frac{1}{\beta}} N^s_t - \tfrac{e^{-\eta\overline{z}^s_t}}{1+r} v^s_{t+1} && s = l, h\\
g^s_{t+1} = &\quad 1 + (\overline{q}-1)(1-e^{-\eta\overline{z}^s_t}) && s = l, h\\
p^h_t = &\quad \left(\gamma + \gamma^{\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N^h_t}{N^l_t}\right)^{-\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}}\\
p^l_t = &\quad \left(1 + \gamma^{1-\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N^h_t}{N^l_t}\right)^{\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}}\\
\widetilde{w} = &\quad \underline{a}^l_t \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^l_t)^{\frac{1}{\beta}}\\
\widetilde{w} = &\quad \underline{a}^h_t \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^h_t)^{\frac{1}{\beta}} Q_t\\
Q_{t+1} = &\quad \tfrac{g^h_{t+1}}{g^l_{t+1}} Q_t\\
\Delta_t = &\quad c^*_t \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^h_t)^{\frac{1}{\beta}} Q_t\\
\Delta_t = &\quad \beta(1-\beta)^{\frac{1-2\beta}{\beta}}\left((p^h_t)^{\frac{1}{\beta}} Q_t - (p^l_t)^{\frac{1}{\beta}}\right) + \tfrac{\lambda}{1+r}\tfrac{1}{g^l_{t+1}}\Delta_{t+1}\\
N^h_t = &\quad \lambda N^h_{t-1}\\
&\quad -\lambda(1-\lambda)^{\frac{\overline{c}-\frac{\overline{c}^2}{2}}{\overline{c}}} \textstyle\int_{\underline{a}^h_{t-1}}^{\underline{a}^l_{t-1}} a f(a) da\\
&\quad +\lambda(1-\lambda) \textstyle\int_{\underline{a}^h_{t-1}}^{\underline{a}^l_{t-1}} a f(a) da\\
&\quad -\lambda(1-\lambda)^{\frac{c^*_{t-1}-\frac{{c^*_{t-1}}^2}{2}}{\overline{c}}} \textstyle\int_{\underline{a}^l_{t-1}}^{\infty} a f(a) da\\
&\quad +\lambda(1-\lambda)^{\frac{c^*_{t-1}}{\overline{c}}} \textstyle\int_{\underline{a}^l_{t-1}}^{\infty} a f(a) da\\
&\quad +\lambda^{\frac{c^*_t-\frac{{c^*_t}^2}{2}}{\overline{c}}} \textstyle\int_{\underline{a}^l_t}^{\infty} a f(a) da\\
&\quad +\lambda^{\frac{\overline{c}-\frac{\overline{c}^2}{2}}{\overline{c}}} \textstyle\int_{\underline{a}^h_t}^{\underline{a}^l_t} a f(a) da\\
&\quad +\lambda(1-\lambda) \textstyle\int_{\underline{a}^h_t}^{\underline{a}^h_{t-1}} a f(a) da\\
N^l_t = &\quad \lambda N^l_{t-1} + (1-\lambda)^{\frac{\overline{c}-c^*_t}{\overline{c}}} \textstyle\int_{\underline{a}^l_t}^{\infty} a f(a) da
\end{aligned}$$

E Decomposition

I denote the initial steady state by a subscript 0 and the new steady state by a

subscript 1.

E.1 Exogenous education, exogenous technology

Since the total supply of high-skilled effective and raw labour is constant $N_0^{h*} = N_1^{l*} = N^{h*}$.

The equations that define the new steady state are:

$$N_1^{l*} = \int_{\underline{a}_1^{l*}}^{\underline{a}_0^{h*}} af(a)da + N_0^{l*}$$

Note that this adjustment only takes place if $\underline{a}_1^{l*} < \underline{a}_0^{h*}$, that is if the decrease in $\widetilde{w}$ is large enough. When the change in the minimum wage is small, then the decline only implies that some people should not get educated, because they would be productive enough even without acquiring skills. However, since education is fixed, this would imply no adjustments in the economy.

$$\underline{a}_1^{l*}(p_1^{l*})^{\frac{1}{\beta}} = \widetilde{w}_1$$

$$p_1^{l*} = \left(1 + \gamma^{1-\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N^{h*}}{N_1^{l*}}\right)^{\frac{\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}}\right)^{\frac{1-\rho}{\rho}}$$

where $Q = Q^h/Q^l$ and $Q^s = \frac{1}{\beta}\int_0^1 (q^{s,j})^{\frac{1}{\beta}} (\chi^{s,j})^{\frac{1-\beta}{\beta}} dj$. I do not explicitly model the pricing of the machines, I denote the price of a machine with quality q^s in line j by $\chi^{s,j}$. The assumption that technology is exogenous boils down to having Q^h and Q^l growing at the same constant rate. If the pricing of machines would follow monopoly pricing or competitive pricing, then this would be equivalent to a constant growth rate in the quality of each line.

Since education and technology are fixed, the new steady state is reached in the moment of the announcement. The lower bound of unemployment for the low-skilled, which implies the adjustment in the size of the low skilled labour force. The new skill premium is:

$$\frac{w_1^{h*}}{w_1^{l*}} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N_1^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}},$$

which is higher than before.

E.2 Endogenous education, exogenous technology

The supply of high- and low-skilled workers in the new steady state are as in (34) and (35), while through the transition they are governed by the same equations as in section D of the Appendix. The threshold for low- and high-skilled unemployment are given exactly as in (25) and (24) (again the transition is as in section D of the Appendix, except for $Q_t = Q$ here, since technology is exogenous). The cutoff time cost for acquiring education is given by:

$$c^* = \frac{1 - \frac{w_t^{l*}}{w_t^{h*}}}{1 - \frac{g\lambda}{1+r}}$$

where g is the exogenous growth rate of the economy. The skill premium is given by:

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}}.$$

The price of intermediates is given by:

$$p_t^{h*} = \left(\gamma^{\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N_t^{h*}}{N_t^{l*}}\right)^{\frac{-\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{-\beta\rho}{1-\rho+\beta\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}$$

$$p_t^{l*} = \left(1 + \gamma^{1-\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N_t^{h*}}{N_t^{l*}}\right)^{\frac{\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}}\right)^{\frac{1-\rho}{\rho}}$$

It is straightforward that Lemma 1 applies in this setup as well. The only thing left to show is that the two curves are both downward sloping, with the curve which gives $\underline{a}^{l*}$ for different values of c being flatter. This curve is downward sloping as before: a higher c implies an increase in the fraction of high skilled and a decrease in the fraction of low-skilled, implying an increase in p_t^{l*}. This from (24) implies a lower $\underline{a}^{l*}$. The other curve, which defines the optimal c^* for any value of $\underline{a}^l$ is also downward sloping. To see this, consider an increase in $\underline{a}^l$, which increases the relative supply of skills, as $\underline{a}^l$ shifts up, the population between $\underline{a}^h$ and $\underline{a}^l$ get a bigger weight in the relative supply

of skills. An increase in the relative supply decreases the skill premium, which in turn decreases c^*.

E.3 Exogenous education, endogenous technology

The supply of high and low skilled workers evolves the same way as in section E.1 of the Appendix. The main difference is that the intermediate price in the new steady state is given by:

$$p_1^{l*} = \left(1 + \gamma \left(\frac{N^{h*}}{N_1^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}$$

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