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A Simple Theory of Managerial Talent, Pay Contracts and Wage Distribution

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Abstract
This paper develops a simple theory of pay structures and pay levels across heterogeneous agents by bringing together optimal contracts inside the firm and competitive resource allocation in the market. The central idea is that more talented people tend to create greater value but face larger conflicts of interest in their employment relationship, and different pay contracts are optimally designed to mitigate different levels of agency problems. Sorted by their talent, people are stratified into production workers, self-employed, salaried managers with low-powered performance pay, and CEOs with high-powered equity-based pay. In a general equilibrium framework, I show that the sorting of managerial talent into pay contracts is tied to firm size. The theory highlights that high-powered incentive pay and large scales of operations cause the disproportionately large wage earnings at the top, and are the main source of income inequality. Market forces that reallocate resources from smaller to larger firms tend to increase the threshold talent for becoming a manager, increase the prevalence of high-powered incentive pay, raise the top earnings, and spread out the wage distribution.

Keywords: Managerial Talent, Limited Liability, Provision of Incentives, Pay Structure, CEO Pay, Wage Distribution
JEL Classifications: D2, J3, L1, L2, M5

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1 Introduction

Why is it so unusual for a construction worker or a low-ranked manager to receive a stock option tied to the profits of the company they work for? Why are people running a small grocery store usually self-employed? Why is a CEO in large companies seldom paid by a flat salary? It seems that there exists some rule that assigns people to different pay structures.

The role of pay structures has caught more and more attention in the debate about the recent trends in wage distribution. A number of studies have attributed the surging top income to the prevalence of the equity-based pay scheme among CEOs and other senior managers in large companies (Hall and Liebman 1998; Murphy 1999; Piketty and Saez 2003, 2006; Autor et al 2006; Kaplan and Rauh 2010). Based on a broad cross-section of the workforce in the US, Lemieux et al (2009) show that performance pay accounts for a significant fraction of the growth in the variance of male wages, and for most of the increase in wage inequality at the top percentiles. A better understanding of the relationship between pay structures and pay levels is called for to address the issue of wage distribution.

The purpose of the current paper is to discover the rule that determines the coincidence between pay structures and pay levels, trace its causes, and explore its economic consequences. Traditional labor economics that equates workers' wages to their marginal products cannot explain workers' pay structures. On the other hand, agency theory answers the question "how to pay" inside the firm by referring to information asymmetries and moral hazard, but says little about "how much to pay", and is silent on the distribution of pay level in the aggregate economy. My approach, therefore, is to bring together agency problems inside the firm and competitive resource allocation in the market.

Starting with an exogenous distribution of managerial talent in the population, I endogenize the distribution of firm size, the level of agency problems inside the firms, the optimal pay structures and managerial incentives, and ultimately the distribution of wage earnings in a structural general equilibrium model. The central idea is that more talented people tend to create greater value, which leads to larger interest conflicts in a principal-agent relationship. Confronted with contractual frictions, firms have to design different pay contracts to mitigate different levels of agency problems. Sorted by their talent, people are stratified into production workers, self-employed, salaried managers with low-powered performance pay, and CEOs with high-powered equity-based pay tied to the market value of firms. Although there exists a monotonic relationship between talent and pay level, the power of pay structure is not monotonic in people's talent.

This non-monotonicity delivers a number of interesting results. The tendency to reinforce and exaggerate the difference in generic talent varies across pay structures and across firms with different size. A large fraction of people works as production workers and has no chance to activate their managerial talent; the pay to the self-employed is high-powered, but is constrained by the size of their firms; mediocre managers employed by medium-size firms receive the same level of expected salary, although they differ in their talent; only very talented people employed by large firms have the opportunities to amplify their talent through notable market value. Under reasonable assumptions on the talent distribution and parameter values of market conditions, the model generates patterns in pay structure and wage earnings that match well-established empirical findings: the high-powered equity-based pay scheme is essentially a top manager and large firm phenomenon; the wage distribution is skewed to the right with particularly high inequality at the top. Moreover, the theory identifies the structure of the product markets as an important factor to explain the large variations in the wage distribution across sectors, between countries and over time. Market forces that reallocate resources from smaller to larger firms tend to select more talented managers into the managerial labor market, increases the fraction of firms that use high-powered incentive pay, raise the top earnings, and spread out the wage distribution.

The analysis is organized sequentially in two steps to solve the problems of "how to pay" and "how much to pay". The first step is to endogenize the structure of managerial compensation through optimal contractual choice between the owner of a firm and its manager. The owner hires a manager whose management affects the productivity of the firm. Owing to asymmetric information on unobservable managerial effort, a pay contract is designed to induce desirable managerial incentives. Under the restriction of limited liability, the firm's ability to punish bad management is limited by the manager's wealth. This wealth constraint is not binding for a manager with low talent, as the surplus created by his effort is so small that he is able to purchase the surplus with his future income. The optimal pay contract is to "sell the store" to the manager, who then becomes self-employed. However the surplus that a sufficiently talented manager created is too large and beyond the manager's future income in a bad state. Then the wealth constraint binds, and the manager's incentive is distorted. Provided that the difference between good and bad management increases in the managerial talent, the consequence of this distortion is more severe for more talented managers. For a mediocre manager, the value created by his effort is not large enough, and the firm is not willing to sacrifice rents to solve a mild level of interest conflicts, and pay its manager by a "fixed-bonus" wage scheme. Only if a manager is talented enough does his effort have sufficient value for the firm. Then the owner is willing to offer an equity-based contract to share residual claims with the manager. Therefore, the heterogeneity in talent leads to the heterogenous value of managerial effort, which in turn determines different levels of agency problems and different pay structures.

After solving the problem of "how to pay", I integrate the limited liability firms into a standard monopolistic competition framework with Dixit-Stiglitz preferences to determine the level of managerial pay. The monopolistic competition model possesses two key features in the superstar literature pioneered by Rosen (1981): imperfect product substitution on the demand side and economies of scale on the supply side. These two features permit the amplification of managerial talent through resources controlled by a manager. The neat properties of the Dixit-Stiglitz preferences allow me to establish a monotone relationship between talent, the market value of managerial efforts, and firm size. Endogenously, the conflicts of interest between the owner and the manager inside a firm increase in firm size. The sorting of people with heterogeneous managerial talent into pay contracts is tied to firm size: small firms are run by the self-employed who are of low talent; medium firms hire mediocre managers who in expectation receive a fixed salary; large firms hire the most talented managers and adopt a high-powered equity-based incentive scheme. The model is closed by a market entry condition which divides people into two broad occupations: production workers and managers.

If people's generic talent is drawn from a Pareto distribution, the market equilibrium displays several properties that fit empirical regularities on pay structure and wage distribution. Firstly, most of the people are employed as production workers, self-employed and salaried managers; only a small fraction of highly talented people manage large firms and receive equity-based incentive pay. Secondly, the wage distribution is skewed to the right, being relatively even at the bottom and highly skewed at the top.

In the market equilibrium, both pay structures and pay levels are correlated with firm size. The market conditions that affect the distribution of firm size will impact on the wage distribution. For example, in a market where products are more homogeneous or the demand elasticity is larger, resources are more disproportionately allocated towards the large firms, and the surviving conditions for small firms are worsened. As a result, the threshold for becoming a manager is higher, the proportion of the self-employed decreases, and a larger fraction of firms is willing to adopt the high-powered pay scheme. The adjustments in pay structure and corresponding managerial efforts lead to an increase in the average pay level at the top distribution, a greater skewness of the wage earnings among the people who receive high-powered performance pay, and a more dispersed wage distribution in the whole population.

My analysis can be seen through the lens of an assignment model, in which people with different talent are assigned to firms that have a different size and adopt different pay contracts.[1] The equilibrium wage distribution of the most talented people (CEOs) in my paper resembles results derived from a competitive assignment between heterogenous CEO talent and heterogeneous firms (Tervio 2008; Gabaix and Landier 2008). My paper goes beyond the assignment literature in two aspects. Firstly, I endogenize the distribution of firm size, which emerges as an equilibrium outcome instead of exogenously given. In this sense, my paper is closer to the seminal work by Lucas (1978) and Rosen (1982). Secondly and more importantly, the matching between managers and firms in my model is bonded by optimal pay contracts to solve different levels of agency problems. This allows me to trace the whole wage distribution and address the substantial differences between the bottom and the top in a coherent framework.

[1] Sattinger (1993) provides an excellent survey of the literature on assignment models and income distribution.

My paper contributes to the literature that tries to embed a concrete structure inside the firm in a market equilibrium to understand the returns to managers and skilled workers. Along the line of Calvo and Wellisz (1979) and Rosen (1982), Garicano and Rossi-Hansberg (2006) build knowledge-hierarchies in a market equilibrium to analyze the earning distributions across firms and within firms. In contrast with their emphasis on the coordination aspect of the firm, my paper focuses on the incentive aspect of the firm, which is a key to understanding firm behavior and managerial pay contracts. Edmans et al (2009) and Bandiera et al (2009) also model an incentive problem with CEOs in a talent assignment market equilibrium, but their models do not endogenize the firm size and relate the pay structure to market structure.

Recent research on wage distribution has stressed the variations in income inequality, particularly the surge of CEO pay and top income, over time (Piketty and Saez 2003, 2006; Frydman and Sakes 2010; Atkinson et al 2011), between countries (Kaplan 1994; Abowd and Bognanno 1995; Atkinson and Piketty 2010), and across sectors (Gordon and Dew-Becker 2007; Kaplan and Rauh 2010). These observed heterogeneities challenge the traditional explanations such as supply and demand of human capital and skill biased technological change (Katz and Murphy 1992, Murphy and Welch 2001; Autor et al 2003). Several studies have drawn attention to increasing returns to general, rather than specific, human capital (Murphy and Zabojnik 2004; Frydman 2007), loose discipline on managerial power (Bebchuk and Fried 2004), and institutional factors (Piketty and Saez 2006; Levy and Temin 2007). My paper offers a new explanation to highlight the role of the product market in affecting the distribution of firm size and thus the wage distribution. This explanation opens an avenue for empirical research to examine the impact of market conditions such as demand elasticity and fixed costs on the patterns of pay structure and wage distribution.

The rest of the paper is organized as follows. Section 2 sets up the model, specifying the firm structure and the market structure. Section 3 establishes the market equilibrium and characterizes the equilibrium distribution of pay structure and wage level. Section 4 employs a Pareto distribution of managerial talent to illustrate the main insights of the analysis. Section 5 is devoted to comparative statics analysis. I conclude with a discussion of potential extension of the paper. Proofs of propositions and lemmas are relegated to the appendix.

2 The Model

The economy has a continuum of people with a mass normalized to one. Each person is endowed with one unit of homogenous raw labor and heterogenous managerial talent indexed with a. I assume that managerial talent is single-dimensional, drawn from a continuous distribution $G(a)$ over $(0, \infty)$ with a well-defined probability density function $g(a)$. In this paper, managerial talent can be interpreted as entrepreneurship and general human capital, which is not industry specific and can be adapted to any technology.

There exists a sufficiently large number of technologies in the economy. Each technology produces one variety of goods, and has identical productivity ex ante. In order to activate and operate a technology, the owner needs to hire a manager and production workers. For simplicity, I assume that labor, either worker or manager, is the only input in production. A technology can be thought of as a stock of capital. In this section, I will first model a modern managerial firm, in which ownership and control rights are separate and pay contracts play a key role in inducing managerial incentives. Then I will specify the structure of the market in which the firms compete and the division of labor is determined.

2.1 The Firm

The firm in this model departs from the neoclassical firm in two aspects. Firstly, a manager is indispensable for a firm to organize production. In addition to the assumption that managerial talent determines the initial productivity of a firm following Lucas (1978), I assume that a manager can adjust his effort to improve management that affects the productivity of his firm. Secondly, I admit the separation of ownership and control rights: a firm is owned by a collective of shareholders (the feminine principal), but is run by a manager (the masculine agent). Potential agency problems such as moral hazard arise.

2.1.1 Hiring in The Labor Market

Before a firm is formed, a principal who owns a production technology needs to hire a manager from the labor market. I assume an extremely simple managerial labor market: no mobility, that is, after being hired by a firm, a manager cannot leave unless the firm shuts down. Implicitly, I assume that the searching costs in the labor market are extremely high.[2] This assumption pushes down the outside option of the heterogenous managers to the same minimum level, and allows the firms to retain positive rent.[3]

Specifically, the principal of a technology posts a managerial vacancy in the labor market. After meeting a manager, the principal observes the manager's talent and makes a take-it-or-leave-it offer to the manager. If the offer is rejected, the principal will exit the market, and the manager will choose the alternative occupation as a production worker. If the offer is accepted, a firm is set up and starts to employ production workers, who will supply their raw labor inelastically and receive a salary normalized to a unity.

[2] Alternatively, we can think of this as a random matching model, in which firm productivity is match specific.

[3] In this model, the managerial talent is scarce while technology is abundant. If searching is costless, all the rent will shift to the managers. As long as searching is imperfect, the qualitative results about pay structures in my model will remain unchanged, although the split of surplus will depend on a potentially complex bargaining process.

2.1.2 Contracting: A Limited Liability Model

I introduce inside the firm a classic agency problem between the owner and the manager: moral hazard with limited liability.[4] Two key assumptions are imposed. Firstly, managerial effort is unobservable, and the principal and the manager can only contract on observable profits of the firm. Secondly, both parties are risk neutral, but the pay contract is subject to limited liability: the owner's ability to reward (punish) the manager is constrained by the latter's wealth.

A firm that hires a manager with talent a has initial productivity a. After exerting an effort e, the manager can improve the firm's productivity to φa, $\varphi > 1$, with probability e . This specification captures two features of a manager's function: 1) the local public good property of his service, which shifts up the firm's total factor productivity; 2) the uncertain aspect of his service. The cost function of e takes a quadratic form $\frac{1}{2k}e^2$ with $k > 0$, satisfying the regular convexity conditions. Note that I model the cost of managerial effort in terms of the value of raw labor instead of disutility, following a broad interpretation of managerial efforts as investment in human capital.

The owner of the firm maximizes her expected value by offering a wage profile $\{b(\varphi a), b(a), s(a)\}$, where $b(\varphi a)$ is the contingent payment (bonus) to the manager in a good managerial state when the firm's observed productivity is φa, $b(a)$ is the contingent payment in a bad managerial state when the productivity remains at the initial level a, and $s(a)$ is a non-contingent transfer (flat salary) from the owner to the manager. For the moment, I assume that the value of the firm with productivity a takes a reduced form $\pi(a, Z, \Phi)$, in which Z is a set of endogenous variables such as price, quantity and employment of labor that the firm will optimally choose, and Φ is a set of exogenous variables that the firm will take as given. For notational simplicity, I will write $\pi(a, Z, \Phi)$ as $\pi(a)$ whenever no confusion occurs.

Formally, the owner faces the following constrained optimization problem:

$$\max_{b(.),\, s(.)} e[\pi(\varphi a) - b(\varphi a)] + (1 - e)[\pi(a) - b(a)] - s(a) \tag{1}$$

subject to

$$\begin{aligned}
(PC) &: \quad eb(\varphi a) + (1 - e)b(a) - \frac{1}{2k}e^2 + s(a) \geq 1, \\
(IC) &: \quad e \in \arg\max_{e'} e'b(\varphi a) + (1 - e')b(a) - \frac{1}{2k}e'^2 + s(a), \\
(WC) &: \quad \min\{b(\varphi a) + s(a), b(a) + s(a)\} \geq \underline{w}.
\end{aligned}$$

Here PC is the participation constraint, meaning that the net return to the manager by working for the firm should be no less than his outside option as a production worker whose wage is normalized to one; IC is the incentive compatibility constraint, as a rational manager

[4] See Laffont and Martimort (2002) for a textbook treatment.

will maximize his expected payoffs; WC is the wealth constraint or limited liability constraint, saying that regardless of the managerial state, the owner cannot pay the manager less than $\underline{w}$, which is exogenously imposed by legal institutions or social norms. I will restrict the wealth constraint $\underline{w} < 1$ to make the problem more interesting.[5] I assume the existence of an interior solution for the managerial effort $e \in (0,1)$ to capture the idea that no matter how smart and diligent a manager is, the manager cannot ensure a hundred percent of success in a complex business world. This interior solution is guaranteed by the assumption that k is sufficiently small.

2.1.3 Optimal Pay Contract

The solution to the constrained optimization problem (1) critically depends on whether or not the wealth constraint and the participation constraint are binding.

Lemma 1 *At least one of the wealth constraint and the participation constraint has to bind at optimum.*

Proof. See the Appendix **A1**. ■

When the wealth constraint is relaxed, the first best level of effort can be implemented even if the managerial effort is not observable, because the contracting parties are both risk neutral.

Lemma 2 *If and only if* $\pi(\varphi a) - \pi(a) < \sqrt{\frac{2(1-\underline{w})}{k}}$*, the first best effort is implemented* $e^{FB}(a) = k[\pi(\varphi a) - \pi(a)]$.

Proof. See the Appendix **A2**. ■

I call this type of contract "sell-the-store", since the first best effort is achieved by transferring ownership from the principal to the manager. Being a residual claimant, the manager does not distort his incentive. The condition $\pi(\varphi a) - \pi(a) < \sqrt{\frac{2(1-\underline{w})}{k}}$ ensures that the manager can pay for the "store" ex post in both managerial states even if he has no wealth ex ante. [6] The biting restriction is that a manager's earnings at the low state must be large enough to pay the price that the principal has set to extract all the surplus between good and bad management created by the manager.

Lemma 3 *Suppose the wealth constraint is binding.*
1) When the participation constraint is binding (indicated by BP), the optimal contract takes the form: $\{s(a) = \underline{w},\ b(a) = 0,\ b^{BP}(\varphi a) = \sqrt{\frac{2(1-\underline{w})}{k}}\}$*; the manager will exert a fixed level of effort* $e^{BP}(a) = \sqrt{2k(1-\underline{w})}$.

[5] When $\underline{w} \geq 1$, the participation constraint is always relaxed, and there will be only one type of optimal pay contract as will be seen in the following analysis.

[6] This implies that the manager can use his future income on the job as collateral to purchase the ownership ex ante. I rule out the possibility that people as share holders can collateralize their dividen income to purchase the firm.

2) When the participation constraint is relaxed (indicated by RP), the optimal contract is $\{s(a) = \underline{w}, b(a) = 0, b^{RP}(\varphi a) = \frac{1}{2}[\pi(\varphi a) - \pi(a)]\}$*; the managerial effort is* $e^{RP}(a) = \frac{k}{2}[\pi(\varphi a) - \pi(a)]$.

Proof. See the Appendix **A3**. ■

This lemma is intuitive. The principal tries to use a contingent performance pay scheme to induce managerial incentives by rewarding his good management and punishing his bad management. However, the binding wealth constraint limits the principal's ability to punish bad management. Thus the principal has to rely more on rewards to induce desirable managerial incentives. A high payment for good management induces a higher managerial effort, but leaves a positive rent to the manager over his outside option. This trade-off between inducing managerial effort and giving up the limited liability rent depends on whether or not the participation constraint binds. A binding participation constraint implies that it is not worthwhile giving up the rent. Then the principal pays a fixed amount based on whether the productivity is improved, which will be referred to as a fixed-bonus contract. If the participation constraint is relaxed, the manager's pay is tied to the value of the firm, which I refer to as an equity-based contract. Both types of pay scheme reflect the contractual frictions due to hidden actions, and only yield second best managerial efforts.

2.1.4 Sorting

The above setup describes a situation, in which a principal, under the limited liability constraint, offers a menu of three types of contracts: "sell-the-store", fixed-bonus, and equity-based. A manager who is aware of his own talent and the resulting firm value, participates in the managerial labor market. After the match, the managerial talent is revealed, and a particular pay scheme from the menu is agreed upon.

The following assumption ensures positive sorting of talent into pay contracts.

Assumption 1 $\Delta\pi(a) = \pi(\varphi a) - \pi(a)$ *is monotonically increases in* a.

The assumption implies that a more talented manager creates a larger differential in the firm value between good and bad management, regardless of the endogenous variables Z and the exogenous parameters Φ. The monotonicity can be derived from the the scale of operations through the market or from the leverage of managerial talent inside the firm. In the next subsection, I will show that the assumption holds in a natural economic environment.

Together with the following technical assumption, the managers are sorted into three types of pay contract.

Assumption 2 $\Delta\pi(a)$ *is continuous in* a*;* $\lim_{a\to 0}\Delta\pi(a) = 0$ *and* $\lim_{a\to\infty}\Delta\pi(a) = \infty$.

Proposition 1 *Under Assumption 1 and 2, the optimal contract is "sell-the-store" for a manager with talent* $0 < a \leq a^*$*, fixed-bonus for* $a^* < a \leq a^{**}$*, and equity-based for* $a > a^{**}$*,* *where* a^* *and* a^{**} *are two threshold values of managerial talent such that* $\Delta\pi(a^*)^2 = \frac{2}{k}(1-\underline{w})$ *and* $\Delta\pi(a^{**})^2 = \frac{8}{k}(1-\underline{w})$.

Proof. See the Appendix **A4**. ■

From the perspective of managers, the three types of contracts can be interpreted as debt, bond and stock. Managers with a different level of talent invest their human capital in the financial market in a different way. The least talented managers collateralize their future income to borrow money to set up small firms. This debt contract allows the managers to become self-employed and residual claimants. The self-employed can be thought of as those people who run a grocery store, open a hairdressing salon, or manage other small family business.[7] The mediocre managers invest a fixed amount of their human capital to buy a bond that promises a fixed high pay in the good state and a fixed low pay in the bad state. These managers bear very little risk, shielding themselves from the shocks in the market. This type of contract is prevalent among managers in small firms and in the low ranked managerial profession. Examples are a division manager or a head of administrative staff in a company, junior lawyers, investment bankers and consultants in professional service firms. The most talented managers, who expect to control abundant resources, purchase the stocks of their firms and share the overall risks together with the owners. This is a typical pay scheme for CEOs and other senior managers in large companies.

I have defined three managerial classifications according to a manager's pay contract: the self-employed, the salaried managers with low-powered incentive pay (salaried managers for simplicity) and the CEOs with high-powered incentive pay, corresponding to the three contractual forms: "sell-the-store", fixed-bonus and equity-based respectively. The pay contract determines how to share the surplus between the principal and the agent. The levels of the joint surplus and managerial pay need to be pinned down in the market.

2.2 The Market

This section specifies the market structure in which I will embed the reduced-form firms described above. Following the spirit of Rosen (1981), I admit imperfect substitution of goods in people's consumption bundles and increasing returns to scale in production. In particular, the economic environment is monopolistic competition with Dixit-Stiglitz preferences. The monopolistic competition framework is highly tractable, and is suitable to analyze economy-wide or broad-sector-wide phenomena, which are the focus of this paper.

[7] In this static model, I do not consider those start-ups that are run initially by self-employed entrepreneurs and then grow up rapidly.

2.2.1 Preferences

A representative individual, independent of her ability and occupation, derives utility from a CES preference:

$$U = (\int_{\omega \in \Omega} q_{\omega}^{\gamma} d\omega)^{\frac{1}{\gamma}} \tag{2}$$

where q_{ω} is the consumption of one variety of differentiated goods ω from an endogenized continuum of bundles Ω. The parameter $\gamma \in [0, 1]$ measures the degree of substitutability between any pair of differentiated varieties. The corresponding elasticity of substitution is $\sigma = \frac{1}{1-\gamma} > 1$. A larger γ or σ means that the varieties are more substitutable or less differentiated.

It is well known that this Dixit-Stiglitz type of preference yields the demand for each variety $\omega : q_{\omega} = Q(\frac{p_{\omega}}{P})^{-\sigma}$, where $Q = (\int_{\omega \in \Omega} q_{\omega}^{\gamma} d\omega)^{\frac{1}{\gamma}}$ can be regarded as an aggregate good and $P = (\int_{\omega \in \Omega} p_{\omega}^{1-\sigma} d\omega)^{\frac{1}{1-\sigma}}$ is an aggregate price index. Define aggregate spending $R \equiv PQ$. Then the expenditure (revenue) on a single variety can be expressed as $r_{\omega} = q_{\omega} p_{\omega} = R(\frac{p_{\omega}}{P})^{1-\sigma}$.

2.2.2 Production

The production technology features increasing returns to scale. A firm with productivity a produces q units of final products at costs $c(q) = \frac{q}{a} + F$ in terms of the homogenous raw labor. Here $\frac{1}{a}$ is the marginal cost, and F is the irreversible fixed costs such as overhead costs and distribution costs. Within a firm, there are no decreasing returns to scale in the managerial talent. The span of managerial control is disciplined by the specificity of the management to a particular firm, whose output is constrained by market demand.

2.2.3 Market Structure

The market structure is monopolistic competition. A firm obtains a certain degree of monopolistic power to cover fixed costs. Given the above preferences and production technology, the pricing rule for each variety is a constant mark-up over the marginal cost $p(a) = \frac{1}{\gamma a} = \frac{\sigma}{\sigma - 1}\frac{1}{a}$. Therefore, the demand and revenue for a variety produced with productivity a are respectively

$$q(a) = Q(a\gamma P)^{\sigma} \text{ and } r(a) = R(a\gamma P)^{\sigma-1}.$$

The relative sales and relative revenues of two firms with different productivity can be explicitly expressed even without knowing the aggregates:

$$\frac{q(a_i)}{q(a_j)} = (\frac{a_i}{a_j})^{\sigma} \text{ and } \frac{r(a_i)}{r(a_j)} = (\frac{a_i}{a_j})^{\sigma-1}, \text{ for all } i, \ j \in \Omega. \tag{3}$$

These relations show that more productive firms produce more and have a larger size. In other words, more talented managers tend to control more resources and generate more surplus, since the firm productivity is derived from the managerial talent.

The profit (net of fixed costs) of a firm with productivity a is

$$\pi(a) = \frac{r(a)}{\sigma} - F = \frac{R}{\sigma}(a\gamma P)^{\sigma-1} - F.$$

Corresponding to the earlier notation of a firm's value function, the endogenous variables in $\mathbf{Z}$ can be written as a function of a and the exogenous variables $\Phi = \{R, P, \sigma, F\}$. Hence $\pi(a, Z, \Phi)$ is reduced to $\pi(a, \Phi)$, and $\Delta\pi(a) = (\varphi^{\sigma-1} - 1)\frac{R}{\sigma}(a\gamma P)^{\sigma-1}$. Assumption 1 and 2 are satisfied.

2.2.4 Contracts across Firm Size

The above parsimonious economic environment establishes a one-to-one mapping between the surplus created by managerial efforts and firm size. The latter is measured by its revenue when the firm operates at its initial productivity. A direct extension of Proposition 1 yields the following result.

Corollary 1 *A firm will "sell-the-store" to the manager when its size is small:* $r(a) \in (0, \frac{\sigma}{\varphi^{\sigma-1}-1}\sqrt{\frac{2(1-\underline{w})}{k}}]$ *; a firm will adopt a fixed-bonus pay contract when its size is medium:* $r(a) \in (\frac{\sigma}{\varphi^{\sigma-1}-1}\sqrt{\frac{2(1-\underline{w})}{k}}, \frac{2\sigma}{\varphi^{\sigma-1}-1}\sqrt{\frac{2(1-\underline{w})}{k}}]$ *; a firm will adopt an equity-based pay contract when its size is large:* $r(a) \in [\frac{2\sigma}{\varphi^{\sigma-1}-1}\sqrt{\frac{2(1-\underline{w})}{k}}, \infty)$.

This corollary captures the relationship between firm size and the level of agency problems. Potentially the conflicts of interest between the principal and the agent increase in firm size, because the marginal value of managerial effort is greater in larger firms. The presence of optimal contracts changes this monotonic relationship. When a firm is small, the manager is able to purchase the store and becomes a residual claimant. The agency problem is eliminated. When operating on a large scale, a firm is willing to use an equity-based contract to induce managerial efforts, and the agency problem is mitigated. The medium firms suffer most from the agency problem, as a high-powered incentive scheme is too costly.

The result that the power of incentives increases in managerial talent and firm size among the top managers (CEOs) is consistent with recent empirical findings. Based on a detailed survey on senior managers in Italian firms, Bandiera et al (2009) find that more talented managers are matched with firms that have a larger size and offer deeper contracts, and work harder. Using CEO pay data of large companies in the US, Gayle and Miller (2010) find a positive relationship between higher equity incentives and firm size.

The optimal pay contracts determine the optimal managerial efforts and pay levels. For the self-employed who run small firms, and the mediocre managers who run medium firms, their pay, net of the compensation to their efforts, equals their outside option, since their participation constraint binds. Only in the large pay-by-equity firms does the net managerial compensation, $V_m(a) = \underline{w} + \frac{k}{8}(\varphi^{\sigma-1} - 1)^2[\frac{r(a)}{\sigma}]^2$, increase in firm size.

The expected value of a firm with initial productivity a can be obtained by substituting (3) and the optimal managerial efforts into (1):

$$\begin{aligned} V_f^{Sell}(a) &= \frac{k}{2}(\varphi^{\sigma-1}-1)^2[\frac{r(a)}{\sigma}]^2 + \frac{r(a)}{\sigma} - (1+F); \\ V_f^{Bonus}(a) &= [\sqrt{2k(1-\underline{w})}(\varphi^{\sigma-1}-1)+1]\frac{r(a)}{\sigma} - (2-\underline{w}+F); \\ V_f^{Equity}(a) &= \frac{k}{4}(\varphi^{\sigma-1}-1)^2[\frac{r(a)}{\sigma}]^2 + \frac{r(a)}{\sigma} - (\underline{w}+F), \end{aligned} \tag{4}$$

where the superscript $Sell$ indicates the "sell-the-store" contract, $Bonus$ indicates the fixed-bonus contract, and $Equity$ indicates the equity-based contract. Figure 1 depicts the value function of the firm V_f and the net managerial pay V_m. The former is strictly increasing in firm size/talent, and the latter is weakly increasing in firm size/talent.

3 Equilibrium

I have solved the problem of "how to pay", taking the market conditions as given. The level of pay ought to be pinned down in a market equilibrium, in which the following conditions are satisfied: 1) all manager-firm matches are stable; 2) pay contracts are optimally designed, and managers exert optimal efforts; 3) all the people optimally choose their occupations; 4) a firm is active if and only if it receives a non-negative expected payoff; 5) both the labor market and the product market clear.

3.1 Market Entry and Stable Matching

The matching between firms and managers is bonded by different types of pay contract. Given the contracting environment and market conditions, the pay contracts are optimally designed by the firms, and the managers choose their optimal efforts accordingly. The stability of the matching is constructed by assumption, and is stable in the sense that given the contractual constraints, no manager/firm pair wishes to rematch with another firm/manager. This can be seen from Figure 1. Both the value of firms and the pay to managers increase in managerial talent. The joint surplus created by a match strictly increases in a single factor representing both managerial talent and firms' initial productivity. The result is (second best) efficient given the limited liability constraint.

The managerial jobs are created by the firms. A firm will enter the market if and only if its expected value is non-negative, that is, the expected profits net of the pay to the manager and workers should be large enough to cover the fixed costs, F. Therefore the marginal firm/manager will be pinned down by the zero firm value condition: $V_f(\underline{a}) = 0$.[8] The size of fixed costs will select different types of firm, pay contract, and managers into the market. For

[8] A unique $\underline{a}$ always exists since $V_f(a)$ is continuously increasing in the domain $(0, \infty)$ with $\lim_{a\to 0} V_f(a) < 0$ and $\lim_{a\to\infty} V_f(a) = \infty$.

example, if fixed costs are too large, small firms cannot survive, the "sell-the-store" contract is not feasible, and the class of the self-employed vanishes. Unless otherwise specified, I make the following assumption on the size of the fixed costs to activate all the three types of pay contracts.

Assumption 3 *The fixed costs F are sufficiently small so that some self-employed firms can survive in the market.* [9]

3.2 Market Clearing

Clearing the labor market requires that every person in the economy is employed, either as a worker or as a manager. All the production workers are fully absorbed by all the active firms. Denote the number of employed managers $M \equiv \int_{\underline{a}}^{\infty} g(a)da$, which is also the number of active firms. The supply of raw labor is simply $1 - M$. The demand for raw labor is $M \cdot F + \int_{\underline{a}}^{\infty} \overline{\frac{q(a)}{a}} g(a)da$. The first term captures the labor required for each firm to bear the fixed costs; the second term captures the labor to bear the variable cost. $\overline{\frac{q(a)}{a}} = e(a)\frac{q(\varphi a)}{\varphi a} + [1 - e(a)]\frac{q(a)}{a}$ is an integrated term, indicating the expected variable raw labor demanded by a firm that hires a manager with talent a, who exerts effort and has a probability $e(a)$ to improve the firm's productivity to φa. I assume that all managers work independently. Then by the law of large number, $e(a)$ can be regarded as the proportion of firms that successfully improve their productivity among the firms with initial productivity a.[10] In equilibrium, labor demand equals labor supply:

$$M \cdot F + \int_{\underline{a}}^{\infty} \overline{\frac{q(a)}{a}} g(a)da = 1 - M. \tag{5}$$

The aggregate income of the population includes the total wages to all production workers

[9] This assumption requires $F < \frac{k}{4}(\varphi^{\sigma-1} - 1)^2[\frac{r(a^*)}{\sigma}]^2 + \frac{r(a^*)}{\sigma} - \underline{w}$, where $r(a^*) = \frac{\sigma}{\varphi^{\sigma-1}-1}\sqrt{\frac{2(1-\underline{w})}{k}}$ is the revenue of the largest firm that adopts the "sell-the-store" contract.

[10] Alternatively, we can integrate the labor demand (or other variables) with an ex post productivity distribution:

$$\mu(a) = e(\tfrac{a}{\varphi})\tfrac{1}{\varphi}g(\tfrac{a}{\varphi}) + [1 - e(a)]g(a) \quad \text{if } a \geq \varphi\underline{a};$$

$$= [1 - e(a)]g(a) \qquad\qquad \text{if } \underline{a} \leq a \leq \varphi\underline{a}.$$

The ex post productivity of the ex ante least productive firm in a good managerial state is $\varphi\underline{a}$. So the firms with ex post productivity between $[\underline{a}, \varphi\underline{a}]$ are those whose managers have talent in this domain and do not succeed in improving productivity. The firms with ex post productivity $a > \varphi\underline{a}$ may come from two sources:1) the firms run by managers with talents $a > \varphi\underline{a}$ but fails to improve productivity and 2) those run by managers with talent $\frac{a}{\varphi}$ but in a good managerial state. This ex post approach will give the same result as the ex ante approach that I adopt in this paper.

and managers and the dividend income for all share holders.[11] Clearing the product market requires that the total expenditure (the total income) equals the total market value of output (the total revenues):

$$1 - M + \int_{\underline{a}}^{\infty} [V_m(a) + \frac{e(a)^2}{2k}] g(a) da + \int_{\underline{a}}^{\infty} V_f(a) g(a) da = R = \int_{\underline{a}}^{\infty} \overline{r}(a) g(a) da$$

where $\overline{r}(a) = e(a) r(\varphi a) + [1 - e(a)] r(a)$ is the expected revenue of a firm that employs a manager with talent a. The overall revenue R is exhausted by the total payment to production workers, the total managerial pay, and the total dividend payment to shareholders. Market clearing in the labor market and the product market boils down to the same condition, as the price of goods is determined by the wage of production workers: a constant mark-up pricing in this model.

Substituting $q(a) = \frac{r(a)}{p(a)}$ and $r(\varphi a) = \varphi^{\sigma-1} r(a)$, I rewrite (5) as

$$\int_{\underline{a}}^{\infty} [e(a)(\varphi^{\sigma-1} - 1) + 1] (\frac{\sigma - 1}{\sigma}) r(a) g(a) da = 1 - M(1 + F). \tag{6}$$

In general, $r(a)$ and $e(a)$ are functions of the unknown aggregates R and P. Fortunately, by (3), $r(a) = (\frac{a}{\underline{a}})^{\sigma-1} r(\underline{a})$, where $r(\underline{a})$ can be computed from the zero firm value condition $V_f(\underline{a}) = 0$. Moreover $e(a)$ can be expressed in terms of a, $\underline{a}$ and the parameters, using the relative relationships between managerial effort and talent according to the optimal contract. Therefore, the market clearing condition (6) pins down the equilibrium cutoff value $\underline{a}$, which in turn determines the equilibrium threshold values a^* and a^{**}. With these threshold values, the equilibrium distribution of the choice of managerial contracts, managerial efforts, firm productivity and firm size (revenues), together with the equilibrium aggregates (M, R, P, Q), can be computed accordingly. Finally the distribution of managerial pay can be characterized.

3.3 Existence and Uniqueness of Equilibrium

In this subsection, I show that there always exists a unique equilibrium $(\underline{a}, a^*, a^{**}, M, R, P, Q)$ in the economy. The cutoff firm productivity and the marginal managerial talent in the market is determined by $V_f^{Sell}(\underline{a}) = 0$. This pins down the revenue of the marginal firm and the effort of the marginal manager:

$$\begin{aligned} r(\underline{a}) &= \sigma \frac{-1 + \sqrt{1 + 2k(\varphi^{\sigma-1} - 1)^2 (F + 1)}}{k(\varphi^{\sigma-1} - 1)^2}; \\ e(\underline{a}) &= \frac{-1 + \sqrt{1 + 2k(\varphi^{\sigma-1} - 1)^2 (F + 1)}}{\varphi^{\sigma-1} - 1}, \end{aligned} \tag{7}$$

[11] For simplicity, I assume that all the firms' profits are redistributed to their shareholders, and the shares held by a person is proportional to his or her wage income. So the wage inequality is an index of income inequality as well.

which only depend on parameters. Without causing confusion, I denote $\underline{r} = r(\underline{a})$ and $\underline{e} = e(\underline{a})$.

Combining notations in Proposition 1 and Corollary 1, I define $r(a^*) = \frac{\sigma}{(\varphi^{\sigma-1}-1)}\sqrt{\frac{2(1-\underline{w})}{k}}$ and $r(a^{**}) = \frac{2\sigma}{(\varphi^{\sigma-1}-1)}\sqrt{\frac{2(1-\underline{w})}{k}}$, where a^* identifies the manager/firm that is indifferent between the "sell-the-store" contract and the fixed-bonus contract, and a^{**} identifies the pair that is indifferent between the fixed-bonus contract and the equity-based contract. Through the relationship between relative firm revenues and managerial talents (3), I obtain

$$\frac{a^{**}}{a^*} = 2^{\frac{1}{\sigma-1}} \; and \; \frac{a^*}{\underline{a}} = [\frac{\sqrt{2k(1-\underline{w})}}{\underline{e}}]^{\frac{1}{\sigma-1}}. \tag{8}$$

Then, it is straightforward to establish the relationship between managerial effort and managerial talent in any firm relative to the marginal firm:

$$\begin{aligned} e(a) &= (\frac{a}{\underline{a}})^{\sigma-1}\underline{e} \qquad \text{for } a \in [\underline{a}, a^*]; \\ e(a) &= \sqrt{2k(1-\underline{w})} \quad \text{for } a \in [a^*, a^{**}]; \\ e(a) &= (\frac{a}{\underline{a}})^{\sigma-1}\frac{\underline{e}}{2} \qquad \text{for } a \in [a^{**}, \infty). \end{aligned} \tag{9}$$

Substituting these equations in the market clearing condition (6), the equilibrium marginal managerial talent and the division between the working class and the managerial class are determined.

Proposition 2 *There exists a unique cutoff managerial talent $\underline{a}$ such that people with talent above $\underline{a}$ become managers, and people with talent below $\underline{a}$ are production workers. This equilibrium cutoff value is determined by the market clearing condition:*

$$\underbrace{[M \cdot F + (\sigma-1)\int_{\underline{a}}^{\infty}\frac{r(a)}{\sigma}g(a)da]}_{extensive\ margin} + (\sigma-1)\int_{\underline{a}}^{\infty}\underbrace{(\varphi^{\sigma-1}-1)\cdot e(a)\cdot\frac{r(a)}{\sigma}}_{intensive\ margin}g(a)da = 1-M. \tag{10}$$

Proof. See Appendix A. ■

The left hand side of Equation (10) is labor demand, which is downward sloping in $\underline{a}$. An increase in the cutoff managerial talent will generate two effects on labor demand. Firstly, the number of operating firms is reduced and thus the economy demands fewer production workers to bear the fixed and the variable costs. This is the adjustment at the extensive margin, captured by the term in the bracket. Secondly, the managers who remain in the market adjust their efforts. The managers adapt their efforts relative to that of the marginal manager, which absorbs the general equilibrium effect and is fixed by market conditions. By (9), all managers, except for those who exert constant amount of efforts, work less hard since their comparative advantages relative to the marginal manager are reduced. As a result, a smaller proportion of the surviving firms improve their productivity, and the demand for labor decreases. This adjustment at the intensive margin is captured by the second term. The right hand side of Equation (10) is the supply of raw labor, which increases in $\underline{a}$, as fewer managers/firms release more production workers into the economy. The two curves intersect at a single point, which pins down the equilibrium marginal managerial talent/ firm productivity.

3.4 Occupational Stratification and Wage Distribution

In equilibrium, the cutoff value $\underline{a}$ divides the population into two broad occupational classes: production workers and managers. Then the threshold values a^* and a^{**} refine the managerial occupation further into three classifications, identified by their pay structures. In particular, the least talented people with $a \in (0, \underline{a})$ choose their occupation as workers and receive a flat salary; people with low talent $a \in [\underline{a}, a^*)$ become self-employed and residual claimants; mediocre people with talent $a \in [a^*, a^{**})$ are salaried managers who earn a fixed-bonus that yields a constant expected pay level; the most talented people with $a \in [a^{**}, \infty)$ are CEOs and paid by an equity-based scheme.

We can construct measures of the relative prevalence of each type of pay scheme and the fraction of each type of manager among the managerial occupation:

$$\underline{\theta} = \frac{\int_{\underline{a}}^{a^*} g(a)da}{\int_{\underline{a}}^{\infty} g(a)da}; \quad \theta^* = \frac{\int_{a^*}^{a^{**}} g(a)da}{\int_{\underline{a}}^{\infty} g(a)da}; \ \theta^{**} = \frac{\int_{a^{**}}^{\infty} g(a)da}{\int_{\underline{a}}^{\infty} g(a)da}. \tag{11}$$

The wage distribution of the whole population follows immediately. A person's expected wage $W(a) = V_m(a) + \frac{e(a)^2}{2k}$ consists of the net value of expected managerial pay and a compensation for managerial efforts. I write $W(a)$ as a function of the firm's revenue $r(a)$, which in turn is a monotone function of managerial talent:

$$\begin{aligned}
W^{PW}(a) &= 1 && \text{for } a \in (0, \underline{a}) \ ; \\
W^{SE}(a) &= 1 + \frac{k}{2}\frac{(\varphi^{\sigma-1}-1)^2}{\sigma^2} r(a)^2 && \text{for } a \in [\underline{a}, a^*]; \\
W^{SM}(a) &= 2 - \underline{w} && \text{for } a \in [a^*, a^{**}]; \\
W^{CEO}(a) &= \underline{w} + \frac{k}{4}\frac{(\varphi^{\sigma-1}-1)^2}{\sigma^2} r(a)^2 && \text{for } a \in [a^{**}, \infty),
\end{aligned} \tag{12}$$

where the superscripts PW, SE, SM and CEO indicate worker, self-employed, salaried manager and CEO respectively. Figure 2 depicts the wage curve in terms of managerial talent, combined with the stratification of occupations. The workers simply earn the unity wage. The self-employed earn their outside option as a worker and a variable part to compensate their efforts. The salaried managers receive a constant wage to compensate their outside option and the fixed amount of effort. They earn the same amount of wage because of their pay structure in spite of the heterogeneity in their generic talent. Finally, the earnings of the CEOs increase rapidly in their talent, as they share the profits of the firms.[12] This rich picture of wage distribution generated from the theoretical model can be used to address a range of current debates about managerial compensation and wage inequality.

Managerial Wage Premium The population is divided into two classes. The managerial class earns a premium over the working class because they activate their managerial talent, exert efforts (invest in human capital), and are rewarded through a different pay structure. I define the average managerial wage as

$$\widetilde{W}^M = \frac{\int_{\underline{a}}^{\infty} W(a)g(a)da}{1 - G(\underline{a})}.$$

This is also a measure of the managerial premium, given the normalization of the workers' wage.

Wage Distribution among Managers One main prediction of the model is about the wage distribution within the managerial class. Being a manager does not necessarily yield a high wage income. The self-employed don't suffer from any agency problem, but their pay is constrained by their small market value. The pay to the salaried managers is constrained by their pay contract. Only when a manager is talented enough to become a CEO will his pay be amplified by a large scale of market value.

I define the average pay to the self employed, the salaried managers and the CEOs as

$$\widetilde{W}^{SE} = \frac{\int_{\underline{a}}^{a^*} W(a)g(a)da}{G(a^*) - G(\underline{a})}; \; \widetilde{W}^{SM} = \frac{\int_{a^*}^{a^{**}} W(a)g(a)da}{G(a^{**}) - G(a^*)}; \; \widetilde{W}^{CEO} = \frac{\int_{a^{**}}^{\infty} W(a)g(a)da}{1 - G(a^{**})}.$$

Then the average managerial pay can be decomposed as

$$\widetilde{W}^M = \underline{\theta} \cdot \widetilde{W}^{SE} + \theta^* \cdot \widetilde{W}^{SM} + \theta^{**} \cdot \widetilde{W}^{CEO}.$$

CEO Pay It is of particular interest to look at the wage distribution of the CEOs, who are identified by the equity-based pay structure in this model. The wage function of the CEOs $W^{CEO}(a)$ is proportional to $a^{2(\sigma-1)}$, provided that $\underline{w}$ is very small. A sufficiently large σ will yield a convex relation between managerial pay and managerial talent, leading to a superstar phenomenon as in Rosen (1981).[13] This convexity creates a large wage dispersion among

[12] From the wage function for the CEOs, the elasticity of CEO pay with respect to firm size is a constant 2, which is much larger than the empirical constant 0.3 discussed by Rosen (1982), Gabaix and Landier (2008) and Frydman and Saks (2010) . This can be reconciled by revising the quadratic cost function of managerial effort to a higher power function.

[13] There is no consensus of the value of the parameter σ. But a value between 3 and 10 is used in the literature.

the CEOs. Take two managers i and j with talents $a_i, a_j > a^{**}$, and $\frac{W^{CEO}(a_i)}{W^{CEO}(a_j)} \simeq (\frac{a_i}{a_j})^{2(\sigma-1)}$. Suppose $\sigma = 8$. This implies the price is about 14% higher than the marginal cost. If manager i is 10% more talented than manager j, the pay to the smarter manager will be about 280% higher than the pay to the less smart one.[14]

Wage Inequality Concerning both the inequality between the workers and the managers and the inequality within the managerial class, one can use the Theil index to measure the wage inequality among the whole population. The Theil index is constructed as

$$T = \int_0^\infty [\frac{W(a)}{\widetilde{W}} \ln \frac{W(a)}{\widetilde{W}}] dG(a),$$

where $\widetilde{W} = \int_0^\infty W(a)g(a)da$ is the average wage of the whole population.

4 Pareto Distribution

In order to generate sharper empirical implications, I specify the generic talent distribution as a Pareto distribution. The use of the Pareto distribution is not only for its technical convenience, but for its empirical relevance. Pareto distribution has been used to characterize income distribution and inequality (Arnold 1983), the distribution of firm size (Axtell 2001), and firm productivity (Helpman et al 2010). Gabaix and Landier (2008) show that a Pareto type distribution of top talents can be obtained through random draws from a large family of extreme value distributions. A large body of recent empirical literature use the Pareto distribution to interpolate the top wage distribution across different countries (see Atkinson et al 2011 for a survey).

Suppose the talent distribution $G(a) = 1 - a^{-\lambda}$ over $(0, \infty)$, where λ governs the shape of the distribution and the measure of talent inequality.[15] A larger λ means a flatter distribution and a smaller inequality. When $\lambda \to \infty$, the distribution collapses at a single point and stands for extreme equality. To guarantee the existence of a meaningful solution to the economic system in the previous sections, I impose the restriction $\lambda > 2(\sigma - 1)$.

After some manipulation, the market clearing condition (10) can be written as

$$\begin{aligned} \frac{\underline{a}^\lambda - (1+F)}{\lambda(\sigma-1)} &= \frac{(1+F)}{\lambda-(\sigma-1)} + \frac{\underline{e}^2}{2k} \frac{\lambda}{[\lambda-2(\sigma-1)][\lambda-(\sigma-1)]} \\ &\quad - \frac{2(1-\underline{w})(\sigma-1)}{[\lambda-2(\sigma-1)][\lambda-(\sigma-1)]} (\frac{a^*}{\underline{a}})^{-\lambda} (1 - 2^{\frac{-\lambda}{\sigma-1}+1}), \end{aligned} \quad (13)$$

[14] Gabaix and Landier (2008) and Tervio (2008) generate a larger pay differential for CEOs with a smaller talent differential. The amplification effect can be enhanced in my model by introducing capital in the production function or by allowing a team of managers whose talent is complementary inside the firm.

[15] With a Pareto distribution of the form $G(x) = 1 - (\frac{b}{x})^\lambda$ where $b > 0, \lambda > 1, x \geq b$, the regular Gini index can be calculated as $(2\lambda - 1)^{-1}$.

which determines the unique equilibrium triple of threshold values $(\underline{a}, a^*, a^{**})$.

The relative prevalence of each type of pay contracts and the relative employment of each type of manager defined in (11) can be computed as follows:

$$\begin{aligned}\underline{\theta} &= 1-(\frac{a^*}{\underline{a}})^{-\lambda}; \\ \theta^* &= (\frac{a^*}{\underline{a}})^{-\lambda}(1-2^{\frac{-\lambda}{\sigma-1}}); \\ \theta^{**} &= 2^{\frac{-\lambda}{\sigma-1}}(\frac{a^*}{\underline{a}})^{-\lambda}.\end{aligned} \quad (14)$$

Note that $\frac{\theta^{**}}{\theta^*} = \frac{2^{\frac{-\lambda}{\sigma-1}}}{1-2^{\frac{-\lambda}{\sigma-1}}}$ is bounded from above by $\frac{1}{3}$ when $2(\sigma-1)$ approaches towards λ. The ratio shows that the equity-based contract is less common than the fixed-bonus contract, or CEOs account for a smaller portion than salaried managers within the managerial occupation.

Under the Pareto specification, we can easily characterize the wage distribution in the economy. On the left side of the distribution, a share of the population, $1-\underline{a}^{-\lambda}$, are production workers who receive a unity wage, despite that they differ in their managerial talent. The wage of the self-employed consists of two parts, a fixed compensation for their outside option and a variable compensation for their efforts. The second component is drawn from a Pareto distribution with a shape parameter $\frac{\lambda}{\lambda-2(\sigma-1)}$ and being truncated by $\frac{e^2}{2k}$ from below and by $1-\underline{w}$ from above. Then appear a mass of mediocre managers who in expectation earn a fixed wage $2-\underline{w}$. Finally the CEOs on the right tail of the distribution receive a constant $\underline{w}$, and a variable component drawn from a Pareto distribution with a shape parameter $\frac{\lambda}{2(\sigma-1)}$ and with a minimum value $2(1-\underline{w})$.

This equilibrium wage distribution matches two well-established regularities in the distribution of labor earnings. Firstly, the empirical earning distribution tends to be skewed to the right and display long right tails. The right skewness of the wage distribution in my model inherits the property of the Pareto distribution of talent. But the presence of the two mass points (production workers and the salaried managers) makes the distribution incline toward a log-normal distribution, which fits better the empirical patterns. Secondly, the top income distribution is highly skewed, and the top percentiles of earnings account for a disproportionate share of total earnings. This can be seen from the distribution of CEO pay in the model. The shape parameter of the generic talent distribution λ is scaled down by a factor $\frac{1}{2(\sigma-1)}$. For a reasonable parameter value of σ, this market rescaling effect can transform a fairly even talent distribution into a substantially skewed wage distribution in favor of the top earnings.

By the properties of Pareto distribution, the average pay to the self-employed and to the

CEOs are

$$
\begin{aligned}
\widetilde{W}^{SE} &= 1+\frac{\underline{e}^2}{2k}\frac{\lambda}{\lambda-2(\sigma-1)}\frac{1-(\frac{a^*}{\underline{a}})^{2(\sigma-1)-\lambda}}{1-(\frac{a^*}{\underline{a}})^{-\lambda}}; \qquad (15)\\
\widetilde{W}^{CEO} &= \underline{w}+2(1-\underline{w})\frac{\lambda}{\lambda-2(\sigma-1)}.
\end{aligned}
$$

Substituting the share of each type of firm and the average wage of each type of manager, the average managerial pay can be written as

$$
\widetilde{W}^{M} = 1+\frac{\underline{e}^2}{2k}\frac{\lambda}{\lambda-2(\sigma-1)}-(1-\underline{w})\frac{2(\sigma-1)}{\lambda-2(\sigma-1)}(1-2^{\frac{-\lambda}{\sigma-1}+1})(\frac{a^*}{\underline{a}})^{-\lambda}. \qquad (16)
$$

The average wage in the economy is

$$
\begin{aligned}
\widetilde{W} &= G(\underline{a})+[1-G(\underline{a})]\widetilde{W}^{M}\\
&= 1+\underline{a}^{-\lambda}\frac{\underline{e}^2}{2k}\frac{\lambda}{\lambda-2(\sigma-1)}-(a^*)^{-\lambda}(1-\underline{w})\frac{2(\sigma-1)}{\lambda-2(\sigma-1)}(1-2^{\frac{-\lambda}{\sigma-1}+1}).
\end{aligned}
$$

Then the Theil index can be computed accordingly.

5 Comparative Statics

One influential explanation for the recent pattern of wage distribution highlights the role of firm size distribution in amplifying CEO pay (Baker and Hall 2004; Gabaix and Landier 2008). However, the distribution of firm size and the wage distribution are a joint equilibrium outcome, as established in my theory. Firm size provides an intermediate channel to shape the wage distribution, but is not an ultimate source or an exogenous shock that causes the wage distribution. In this section, I identify a number of factors that have a potential impact on the patterns of firm size. I conduct comparative statics on the structural parameters in the model to derive empirically testable predictions. To facilitate the analysis, I maintain the assumption that managerial talent is Pareto distributed.

5.1 Demand Elasticity

In Rosen (1981), a key factor that generates the superstar phenomenon is the disproportionate demand for high quality against low quality. For example, an excellent singer attracts more than double the audience than two mediocre singers. Technological advances such as television or the Internet that increase the accessibility of goods tend to enhance the demand for superstars and reduce the demand for mediocrity. In the Dixit-Stiglitz preference (2), the parameter σ governs the substitutability between goods and the demand elasticity. Although not able to capture precisely the quality-quantity trade-off in Rosen (1981), this parameter reflects the idea that demand elasticity varies across consumers with inherently different preferences or across markets with different levels of market access and searching costs. Intuitively when goods are treated as more substitutable or less differentiated, consumers shift their demand more easily, and firms gain more from improvements in productivity. Therefore a larger demand elasticity increases the marginal value of managerial effort, which creates the surplus between good and bad management. Benefiting more from an increase in the demand elasticity, larger firms are more willing to use the equity-based incentive scheme. This response triggers a series of responses in the market.

Proposition 3 *Greater substitutability between goods or a larger demand elasticity in a market leads to the following results:*
1) The talent threshold for becoming a manager increases, and the employment of managers decreases;
2) The fraction of the self-employed decreases; the fraction of CEOs among the managerial class increases;
3) Both the average CEO pay and the wage inequality among CEOs increase.

Proof. See Appendix **A6**. ■

The first result is the selection effect. A larger demand elasticity enables larger firms to grab a larger market share, which in turn worsens the surviving environment of small firms and drives out the marginal ones. This selection at the entry margin reinforces the allocation of resources from smaller to larger firms. As a result, the fraction of the self-employed decreases while the fraction of the CEOs who receive high-powered incentives increases. The impact on the fraction of the salaried managers is ambiguous, because they erode the market of the self-employed, but suffer from the pressure from the CEOs. These results hold for a general class of talent distribution other than the Pareto distribution.

The effects of an increase in the demand elasticity on wage distribution are more complex. The CEOs always gain from the resource reallocation towards larger firms. Because of the Pareto distribution, it is easy to show that the average CEO pay increases; within the CEOs, the wage distribution becomes more skewed since the superstars control more resources and work harder. However, the average pay to the whole managerial class does not necessarily increase, because the average pay to the self-employed may decrease. Only when the fraction of the self-employed is small does the ambiguity disappear. For example, suppose $\frac{a^*}{\underline{a}} = 1$ so that the whole segment of the self-employed vanishes. Then upon an increase in the demand elasticity, the proportion of CEOs expands at the cost of the salaried managers, leading to a larger inequality within the managers and a higher managerial wage premium over the working class.

The class of the self-employed plays an interesting role in buffering the impact of the changes in demand elasticity on the wage inequality among the whole population. When the average pay to the self-employed decreases, the inequality within the managerial class increases, but the inequality between managers and production workers is dampened. In contrast, if the average pay to the self-employed increases, this between-group inequality always increases while the within-manager inequality may be reduced.

In the monopolistic competition model with Dixit-Stiglitz preference, the demand elasticity σ is negatively related to the price-cost margin of a firm. Therefore a larger σ implies more intense market competition in the sense of more aggressive interactions between firms. This notion of market competition is also aligned with a new indicator of competition in a market with heterogenous firms, using relative profit differences between firms (Boone 2008). The above discussion is thus related to the literature on the impact of market competition on managerial incentives (e.g. Hermalin 1992; Schmidt 1997; Raith 2003; Vives 2008). In their homogenous firm setting, they emphasize a trade-off between market enlargement (business stealing) and higher competitive pressure (business stolen); the effect of tougher market competition on managerial incentives is in general ambiguous.[16] In my model with heterogenous firms, I emphasize the uneven impact of market competition across firms; competition on average improves managerial incentives and compensation, largely because of firms' adjustment at the extensive margin. This provides a potential explanation for the empirical findings that more intense market competition tends to improve managerial incentives and increase the power of incentive pay for executives (Cunat and Guadalupe 2005, 2009). But the heterogenous effects of competition across firms with different productivity calls for further examination.

5.2 Fixed Costs

Fixed costs are an important determining factor of market structure. Firms need to be able to recover fixed costs to survive market competition. The magnitude of fixed costs determines the size of the marginal firm and the number of firms in the market. In an economic sector with heterogenous firms, an increase in fixed costs tends to select better firms, which induces resources reallocated from smaller to larger firms. In the current model, this selection leads to a general equilibrium effect that triggers a series of changes inside the firms, and then affects the aggregate levels of the employment of managers and the distribution of their wages.

Proposition 4 *A larger fixed cost F leads to the following results:*
1) The talent threshold for becoming a manager increases, and the employment of managers decreases;
2) Among the managers, the fraction of the self-employed declines, but the fractions of the salaried managers and the CEOs increase;
3) The managerial wage premium increases.

Proof. See the Appendix **A7**. ■

[16] The exception is Raith (2003), which shows that competition(e.g. greater substitutablity between goods) unambiguously increases managerial incentives in a product differentiation model when market entry is endogenized.

These results are intuitive. Larger fixed costs require a larger expected firm value to induce market entry. The marginal firm needs to hire a more talented manager. As a result, more resources (raw labor) are available for the surviving firms. Since larger firms have a larger demand and their managers work harder, they will absorb more resources, and worsen the operating conditions for the self-employed whose activities are of a small scale. In equilibrium, the range of talent between the self-employed and the salaried manager is reduced ($\frac{a^*}{\underline{a}}$ decreases). Given the Pareto distribution of talent, people with higher talent are more scarce, and a smaller fraction of the self-employed implies a reduction in their employment. On the contrary, larger firms are less affected, and the shares of the salaried managers and the CEOs among the surviving managers increase.

Unlike the demand elasticity, fixed costs have no direct impact on a firm's behavior. An increase in fixed costs changes the wage distribution only through the composition of different types of managers. Larger fixed costs can increase or decrease the average pay to the self-employed, depending on the response of the boundary conditions of a and a^*. The average pay to the salaried managers is independent of fixed costs because of the nature of their pay contracts. The average pay to the CEOs is not affected as the lower bound of the distribution of their wages is not affected by fixed costs.[17] Under the Pareto distribution, larger fixed costs increase the average managerial wage premium, because of a larger share of the salaried managers and CEOs who earn a higher wage than the self-employed.

In the above analysis, I maintain the assumption that fixed costs are sufficiently small so that the three types of pay structures coexist in the market. If fixed costs are large, not all the types of pay contracts can be active in the market. Collecting the notations in (4) and (8), we have the following result.

Corollary 2 *If the fixed cost F is large enough so that the cutoff managerial talent is determined by $V_f^{Equity}(\underline{a}) = 0$, all surviving firms adopt the equity-based contract; if the fixed cost F is intermediate so that the cutoff managerial talent is determined by $V_f^{Bonus}(\underline{a}) = 0$, firms with productivity between $\underline{a}$ and a^{**} will choose the fixed-bonus contract, and firms with productivity above a^{**} choose the equity-based contract.*

Proposition 4 and Corollary 2 convey the idea that fixed costs act as a selection mechanism of the contractual forms. They make empirically testable predictions about the composition of talent and managerial pay structures across sectors with different fixed costs. A fragmented market, where fixed costs are small (e.g. local service), accommodates a wider range of talent and more diversity of pay structure. The self-employed earn a niche, and the equity-based pay scheme is less prevalent. In contrast, in a condensed market where fixed costs are large (e.g. modern manufacturing), only highly productive firms can survive, only talented people can take the occupation as a manager, and the high-powered equity-based of pay scheme becomes more popular.

[17] This result depends on the special property of Pareto distribution, but the idea that fixed costs only affect boundary conditions holds for general distributions .

5.3 Talent Distribution

In the model, the absolute level of labor supply plays no role in affecting the patterns of pay structure and wage distribution. This is because with the Dixit-Stiglitz preferences, a change in the absolute supply of labor will translate into the number of firms in a fixed proportion, leaving the distribution of firm size constant. What plays an important role for the patterns of firm size and managerial pay is the parameter λ, which governs the shape of the Pareto distribution of talent.

Consider an extreme example such that $\lambda \rightarrow \infty$: people have equally low managerial talent. Then some people will be randomly selected to operate a technology. Given the scale of their operations is small, the "sell-the-store" contractual arrangement is optimal. The economy will be preoccupied by the small self employed firms. The wage inequality is low, because none of the people has the opportunity to amplify their talent through a large scale of operations. When λ decreases, the talent distribution has a longer right tail and becomes less equal. This supports the existence of large firms that face the principal-agent problem and have to adopt second best pay contracts. For a sufficiently large λ, the type of large firms that adopt the equity-based incentive scheme emerges. The presence of this pay structure makes the wage distribution more sensitive to market conditions and talent distribution. A further reduction in λ increases the average CEO pay and the wage dispersion among CEOs.

The implication from this simple exercise is that the supply of human capital does matter for wage distribution, but in a way other than the traditional labor supply story. The talent composition of human capital will affect the distribution of firm size, the demand for managerial efforts, and the pay structure and pay level. For example, more imbalance in managerial ability among people due to MBA education or the migration of top talent can drive up the top income inequality. From an international perspective, the talent distribution of the American labor forces is arguably more skewed than that of the European and Japanese labor forces (Grossman and Maggi 2000). My analysis implies that the top wage income inequality would be more pronounced in the US than in Europe and Japan holding other structural parameters constant, which seems consistent with the empirical findings.

5.4 Limited Liability Constraint

In the limited liability model, the parameter $\underline{w}$ indicates the minimum wealth that a person needs to have to shield away from a negative shock. In an economy where people are not endowed with wealth as I have assumed, $\underline{w}$ is the wage that the firm must pay its manager for bad management. It reflects the level of labor rigidity or contractual frictions in the principal-agent relation. A lower $\underline{w}$ means that the managerial labor market is more flexible, and the institutional environment allows the principal to share more risk with the agent. Changing $\underline{w}$ has a direct impact on the self-employed. A relaxation in $\underline{w}$ increases the range of the firms that can sell their "stores" to the manager, and thus enlarges the fraction of the self-employed in the economy. On the other hand, the fraction of either the salaried managers or the CEOs is reduced. Upon a reduction in $\underline{w}$, the salaried managers will work harder and receive higher compensation. The average CEO pay increases accordingly. The responses of the whole managerial class worsen the surviving condition for small firms, push up the talent threshold for becoming a manager, and increases the average managerial pay.

In an extreme case, $\underline{w} \to -\infty$. The wealth constraint is always relaxed. Then all firms will adopt the "sell-the-store" contract and all managers will exert the first best level of efforts. The economy will achieve first best efficiency but feature a higher level of inequality. This is in the line with the traditional argument about the trade-off between efficiency and inequality.

6 CONCLUSION

In this paper, I have developed a general equilibrium model to analyze the patterns of pay structure and wage distribution among people with different managerial talent and in firms with different size. Ex ante more talented people will control more resources ex post. This scale of operations effect on the one hand amplifies the market value of people's talent, but on the other hand creates potential conflicts of interest in the principal-agent relation. Different pay contracts are optimally designed to mitigate agency problems inside the firms, resulting in substantial heterogeneities in pay structure and in the extent of the scale of operations. People with low talent work as a residual income recipient in self-employment. They don't suffer from agency problems, but their wages are limited by the small scale of their activities. Managers with intermediate talent run medium firms, and receive a salary with low-powered incentives. They have the most serious agency problem, and their pay is not at all amplified by the market. Only highly talented people will manage large firms and share the market value of their firms. High-powered incentive pay and large scales of operations are the cause of disproportionately large wage earnings at the top distribution of talent, and are the main source of income inequality in an economy.

Without much speculation on parameter values, the model generates predictions that fit several empirical regularities. Moreover, within the model, I identify factors such as increases in demand elasticity and fixed costs, which lead to a tighter market structure (fewer but larger firms/managers in the market), as the explanation of the surging top income and the increasing inequality between the working rich and the working poor. Another factor along this line is international trade, which induces resources reallocated from smaller domestic firms towards larger exporting firms within the same industries (Melitz 2003). In a separate paper, I extend the current model to an open economy to examine the impact of intra-industry international trade on pay structure and wage inequality (Wu 2011).

In the current paper, I focus on the incentive aspect of the firm without considering the hierarchical structure inside the firm. Managers are classified only by their pay structures. I do not distinguish a manager of a medium firm from a division manager in a large firm if they receive the same type of pay contract. Future research will be devoted to refining the classification of managers through integrating the incentive aspect and other aspects of the firm, as pointed out by Rosen (1992). This extension will address the substantial observed heterogeneities in the pay structure and pay level across managers both within and across firms. Another extension is to relax the assumption of non-mobility in the managerial labor market. Mobility and turnovers of managers will bring about the discussion of general versus specific human capital and career concerns, which are important for the determination of managerial compensation.

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7 APPENDIX

Proof of Propositions and Lemmas

A1. Lemma 1

Proof. In the limited liability model, at least one of the participation constraint (PC) and the wealth constraint (WC) is binding, because $s(a)$ is a lump-sum transfer that does not distort incentives. Suppose that the PC is relaxed. If $b(a) + s(a) > \underline{w}$, the principal can increase payoffs by reducing a small amount of $s(a)$. On the other hand, suppose the WC is relaxed. If the PC is relaxed, the principal can be better off by reducing $s(a)$ slightly so that the manager is still willing to participate. Hence at least one of the PC or WC constraints should bind at optimum. ■

A2. Lemma 2

Proof. Since the WC is relaxed, the PC has to bind by Lemma 1. The principal "sells the store" to the manager at the price $s^{FB}(a) = 1 - \{e^{FB}(a)\pi(\varphi a) + [1 - e^{FB}(a)]\pi(a) - \frac{(e^{FB}(a))^2}{2k}\}$, where $e^{FB}(a) = k[\pi(\varphi a) - \pi(a)]$ is the first best effort. The condition for attaining this is that WC is relaxed at the low state: $\pi(a) + s^{FB}(a) > \underline{w}$ or $a < a^*$ where $\frac{k}{2}[\pi(\varphi a^*) - \pi(a^*)]^2 = 1 - \underline{w}$. ■

A3. Lemma 3

Proof. Since the incentive compatibility constraint (IC) is a concave function in e, we replace it with the first order condition $e = k[b(\varphi a) - b(a)]$ (IC'). Substitute this into the principal's objective to obtain her value function: $V_f(a) = e[\pi(\varphi a) - b(\varphi a)] + (1 - e)[\pi(a) - b(a)] - s(a)$, which is decreasing in $b(a)$. Hence it is always optimal to set $b(a) = 0$. Then $s(a) = \underline{w}$ by the binding WC.

Case 1: If the PC is binding, I denote all the solutions with a superscript BP. The net payoff accrued to the manager is $V_m^{BP}(a) = kb(\varphi a)^2 - \frac{1}{2k}k^2 b(\varphi a)^2 + \underline{w} = 1$. Then we solves $b^{BP}(\varphi a) = \sqrt{\frac{2(1-\underline{w})}{k}}$ and $e^{BP} = \sqrt{2k(1-\underline{w})}$ under the restriction $1 - \frac{1}{2k} < \underline{w} < 1$.

Case 2: If PC is relaxed, I denote all the solutions with a superscript RP. Substituting IC', $b(a) = 0$, and $s(a) = \underline{w}$ into the objective function, we obtain $e^{RP}(a) = \frac{k[\pi(\varphi a)-\pi(a)]}{2}$ and $b^{RP}(\varphi a) = \frac{\pi(\varphi a)-\pi(a)}{2}$. ■

A4. Proposition 1.

Proof. Let $\Delta\pi(a) = \pi(\varphi a) - \pi(a)$. Denote the value function of the firm as V_f^{FB}, V_f^{BP}, and V_f^{RP} respectively for the three cases in which 1) the first best is achieved; 2) both the wealth constraint and the participation constraint are binding; 3) the wealth constraint is binding, but the participation constraint is relaxed. Substitute the optimal managerial efforts and pay in Lemma 2 and 3 into the firm's objective function:

$$\begin{aligned} V_f^{FB}(a) &= \frac{k}{2}\Delta\pi(a)^2 + \pi(a) - 1, \\ V_f^{BP}(a) &= \sqrt{2k(1-\underline{w})}\Delta\pi(a) + \pi(a) - (2-\underline{w}), \\ V_f^{RP}(a) &= \frac{k}{4}\Delta\pi(a)^2 + \pi(a) - \underline{w}. \end{aligned}$$

Define a^* as in Lemma 2 and a^{**} such that $e^{RP} = e^{BP}$ or $\frac{\sqrt{k}}{2}\Delta\pi(a^{**}) = \sqrt{2(1-\underline{w})}$.

Case 1: $a < a^*$. Since a is so small that the first best effort is achievable. Obviously $V_f^{FB}(a) > V_f^{RP}(a)$ and $V_f^{FB}(a) - V_f^{BP}(a) = [\sqrt{\frac{k}{2}}\Delta\pi(a) - (1-\underline{w})]^2 \geq 0$ with equality at a^*. So the principal will choose the sell-the-store contract.

Case 2: $a^* \leq a < a^{**}$. Now the first best effort is not feasible. If the manager accepts the equity-based pay, his net payoff would be $V_m^{RP}(a) = \frac{k}{8}\Delta\pi(a)^2 + \underline{w}$, which is smaller than his outside option when $\pi(a) < \pi(a^{**})$. Therefore both the wealth constraint and the participation constraint are binding. The principal can only choose the fixed–bonus contract.

Case 3: $a \geq a^{**}$. The wealth constraint is binding, and the participation constraint is relaxed. $V_f^{RP}(a) - V_f^{BP}(a) = [\frac{\sqrt{k}}{2}[\pi(\varphi a) - \pi(a)] - \sqrt{2(1-\underline{w})}]^2 \geq 0$ with equality at a^{**}. Hence the principal will choose the equity-based contract. ■

A5. Proposition 2.

Proof. Rewrite the market clearing condition as

$$\int_{\underline{a}}^{\infty} r(a)g(a)da + (\varphi^{\sigma-1} - 1)\int_{\underline{a}}^{\infty} e(a)r(a)g(a)da = \frac{\sigma}{\sigma-1}[1 - M(1+F)]. \tag{A1}$$

Substituting (3) and (9), the left hand side becomes

$$\underline{r}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da+(\varphi^{\sigma-1}-1)\{\underline{e}\cdot\underline{r}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da$$

$$+\sqrt{2k(1-\underline{w})}\underline{r}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da+\frac{\underline{e}\cdot\underline{r}}{2}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da\}.$$

The first term is decreasing in $\underline{a}$ since $\frac{d\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da}{d\underline{a}}=-g(\underline{a})-\frac{\sigma-1}{\underline{a}}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma}g(a)da<0.$

Denote the terms in curly bracket $\Phi(\underline{a})$. Then $\Phi'(\underline{a})=$

$$\underline{e}\cdot\underline{r}[\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{2\sigma-2}g(a^*)-g(\underline{a})-\frac{2\sigma-2}{\underline{a}}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da]$$

$$+\sqrt{2k(1-\underline{w})}\underline{r}[\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\sigma-1}g(a^{**})-\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\sigma-1}g(a^*)-\frac{\sigma-1}{\underline{a}}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da]$$

$$+\frac{\underline{e}\cdot\underline{r}}{2}[-\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{2\sigma-2}g(a^{**})-\frac{2\sigma-2}{\underline{a}}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2\sigma-2}g(a)da].$$

Since $\frac{a^*}{\underline{a}}=[\frac{\sqrt{2k(1-\underline{w})}}{\underline{e}}]^{\frac{1}{\sigma-1}}$, the terms

$$\begin{aligned}&\underline{e}\cdot\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{2\sigma-2}g(a^*)-\sqrt{2k(1-\underline{w})}\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\sigma-1}g(a^*)\\ =\;&\underline{r}\frac{da^*}{d\underline{a}}(\frac{a^*}{\underline{a}})^{\sigma-1}g(a^*)[\underline{e}(\frac{a^*}{\underline{a}})^{\sigma-1}-\sqrt{2k(1-\underline{w})}]=0.\end{aligned}$$

Since $\frac{a^{**}}{\underline{a}}=[\frac{2\sqrt{2k(1-\underline{w})}}{\underline{e}}]^{\frac{1}{\sigma-1}}$, the terms

$$\begin{aligned}&\sqrt{2k(1-\underline{w})}\underline{r}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\sigma-1}g(a^{**})-\frac{\underline{e}\cdot\underline{r}}{2}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{2\sigma-2}g(a^{**})\\ =\;&\underline{r}\frac{da^{**}}{d\underline{a}}(\frac{a^{**}}{\underline{a}})^{\sigma-1}g(a^{**})[\sqrt{2k(1-\underline{w})}-\frac{\underline{e}}{2}(\frac{a^{**}}{\underline{a}})^{\sigma-1}]=0.\end{aligned}$$

All the other terms are negative. Hence $\Phi'(\underline{a})<0$ and the demand curve is downward sloping. Obviously $M=\int_{\underline{a}}^{\infty}g(a)da$ decreases in $\underline{a}$. Then the right hand side of (A1) increases in $\underline{a}$. Moreover, the difference between the left hand side and the right hand side is positive when $\underline{a}\to 0$, and the difference is negative when $\underline{a}\to\infty$. Therefore by the intermediate value theorem, the two sides intersect at a single interior point $\underline{a} \in (0, \infty)$. ■

A6. Proposition 3.

Proof.

Part 1). I prove the first result in a general case without specifying the Pareto distribution of talent. From Equation (A1) and using the relation between $\underline{e}$ and $\underline{r}$, define an implicit function in $(\underline{a}, \sigma)$:

$$\begin{aligned}\Omega(\underline{a},\sigma) &= \frac{\underline{e}}{k(\varphi^{\sigma-1}-1)}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da + \frac{\underline{e}^2}{k}\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{2(\sigma-1)}g(a)da \\ &+\sqrt{2k(1-\underline{w})}\frac{\underline{e}}{k}\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da + \frac{\underline{e}^2}{2k}\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{2(\sigma-1)}g(a)da \\ &-\frac{\sigma}{\sigma-1}[1-(1+F)\int_{\underline{a}}^{\infty}g(a)da].\end{aligned}$$

We add a term $\frac{\underline{e}^2}{2k}\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da$ and subtract the same value $\frac{\underline{e}^2}{2k}[\int_{\underline{a}}^{a^*}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da+\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da+\int_{a^{**}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da]$. Rearranging terms and using the market entry condition: $V_f^{Sell}(a) = 0$ or $\frac{\underline{e}^2}{2k} + \frac{\underline{e}}{k(\varphi^{\sigma-1}-1)} = 1 + F$, $\Omega(\underline{a}, \sigma)$ can be written as:

$$\begin{aligned}&= (1+F)\int_{\underline{a}}^{\infty}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da + \frac{\underline{e}^2}{k}\int_{\underline{a}}^{a^*}[(\frac{a}{\underline{a}})^{2(\sigma-1)} - \frac{1}{2}(\frac{a}{\underline{a}})^{\sigma-1}]g(a)da \\ &+[\sqrt{2k(1-\underline{w})}\frac{\underline{e}}{k} - \frac{\underline{e}^2}{2k}]\int_{a^*}^{a^{**}}(\frac{a}{\underline{a}})^{\sigma-1}g(a)da + \frac{\underline{e}^2}{2k}\int_{a^{**}}^{\infty}[(\frac{a}{\underline{a}})^{2(\sigma-1)} - (\frac{a}{\underline{a}})^{\sigma-1}]g(a)da \\ &-\frac{\sigma}{\sigma-1}[1-(1+F)\int_{\underline{a}}^{\infty}g(a)da].\end{aligned}$$

Holding $\underline{a}$ constant, differentiate $\Omega(\underline{a}, \sigma)$ with respect to σ. It is straightforward to show that $\Omega(\underline{a}, \sigma)$ increases in σ if a^* and a^{**} are held as constant. The key is to check the terms involving the derivatives of the boundary values of the integral (a^* and a^{**}), which are collected as:

$$= \underbrace{[\frac{\underline{e}^2}{k}(\frac{a^*}{\underline{a}})^{\sigma-1} - \frac{\underline{e}^2}{2k} - \frac{2\sqrt{2k(1-\underline{w})}\underline{e} - \underline{e}^2}{2k}]}_{=0}(\frac{a^*}{\underline{a}})^{\sigma-1}g(a^*)\frac{da^*}{d\sigma}$$
$$+\underbrace{[\frac{2\sqrt{2k(1-\underline{w})}\underline{e} - \underline{e}^2}{2k} + \frac{\underline{e}^2}{2k} - \frac{\underline{e}^2}{2k}(\frac{a^{**}}{\underline{a}})^{\sigma-1}]}_{=0}(\frac{a^{**}}{\underline{a}})^{\sigma-1}g(a^{**})\frac{da^{**}}{d\sigma} = 0.$$

The effects through the threshold values cancel out. Hence $\frac{\partial\Omega(\underline{a},\sigma)}{\partial\sigma} > 0$. From the proof of Proposition 2, we know $\frac{\partial\Omega(\underline{a},\sigma)}{\partial\underline{a}} < 0$. Therefore $\frac{d\underline{a}}{d\sigma} = -\frac{\frac{\partial\Omega(\underline{a},\sigma)}{\partial\sigma}}{\frac{\partial\Omega(\underline{a},\sigma)}{\partial\underline{a}}} > 0$.

Part 2)-4). I prove the rest of the proposition under specification of the Pareto distribution of talent. By (8) and (14), $\underline{\theta} = 1 - [\frac{\sqrt{2k(1-\underline{w})}}{\underline{e}}]^{\frac{-\lambda}{\sigma-1}}$ and $\theta^{**} = 2^{\frac{-\lambda}{\sigma-1}}[\frac{\sqrt{2k(1-\underline{w})}}{\underline{e}}]^{\frac{-\lambda}{\sigma-1}}$. Since both $\frac{d\underline{e}}{d\sigma} > 0$ and $\frac{d\frac{-\lambda}{\sigma-1}}{d\sigma} > 0$, $\frac{d\underline{\theta}}{d\sigma} < 0$ and $\frac{d\theta^{**}}{d\sigma} > 0$.

The average CEO pay is given in (15). It is straightforward to show that $\frac{d\widetilde{W}^{CEO}}{d\sigma} > 0$. The skewness of the wage distribution of the CEOs is fully characterized by $\frac{\lambda}{2(\sigma-1)}$, which decreases in σ. Hence an increase in σ increases the skewness of the distribution of CEO pay.

■

A7. Proposition 4

Proof.

Part 1). The first result holds for general talent distributions. The proof is similar to that for Proposition 3, and is thus neglected. Under the specification of the Pareto distribution of talent, it is easy to show that holding $\underline{a}$ constant, an increase in F pushes downwards the left hand side of Equation (A1) but pushes upwards the right hand side. Therefore, the new equilibrium shifts to the right, resulting in a larger $\underline{a}$.

Part 2)-3). It is straightforward to show $\frac{d\underline{\theta}}{dF} < 0$, $\frac{d\theta^*}{dF} > 0$ and $\frac{d\theta^{**}}{dF} > 0$ since $\frac{d\underline{e}}{dF} > 0$ and $\frac{d(\frac{a^*}{\underline{a}})^{-\lambda}}{dF} > 0$.

Part 4). For the last result, rewrite (16) as

$$\widetilde{W}^M = 1 + \frac{\underline{e}^2}{2k}\frac{\lambda}{\lambda-2(\sigma-1)} - (1-\underline{w})\frac{2(\sigma-1)}{\lambda-2(\sigma-1)}(1-2^{\frac{-\lambda}{\sigma-1}+1})[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}}.$$

It can be shown that $\frac{d[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}}}{d\underline{e}} = \frac{\lambda}{\sigma-1}[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}-2}\frac{\underline{e}}{2k(1-\underline{w})}$. Then

$$\frac{d\widetilde{W}^M}{d\underline{e}} = \frac{\underline{e}}{k}\frac{\lambda}{\lambda-2(\sigma-1)} - \frac{\lambda}{\lambda-2(\sigma-1)}\frac{\underline{e}}{k}(1-2^{\frac{-\lambda}{\sigma-1}+1})[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}-2}$$
$$= \frac{\underline{e}}{k}\frac{\lambda}{\lambda-2(\sigma-1)}\{1-(1-2^{\frac{-\lambda}{\sigma-1}+1})[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}-2}\} > 0,$$

since $2^{\frac{-\lambda}{\sigma-1}+1} < 1$ and $[\frac{\underline{e}}{\sqrt{2k(1-\underline{w})}}]^{\frac{\lambda}{\sigma-1}-2} < 1$. Hence $\frac{d\widetilde{W}^M}{dF} = \frac{d\widetilde{W}^M}{d\underline{e}}\frac{d\underline{e}}{dF} > 0$ as $\frac{d\underline{e}}{dF} > 0$. ■

Figure 1. Managerial Talent and Optimal Choice of Pay Contracts

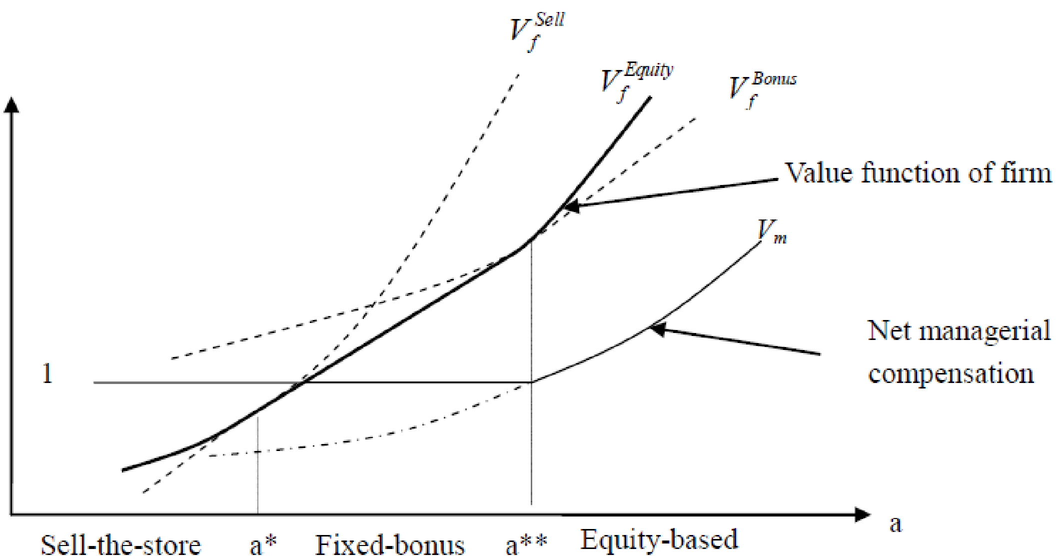
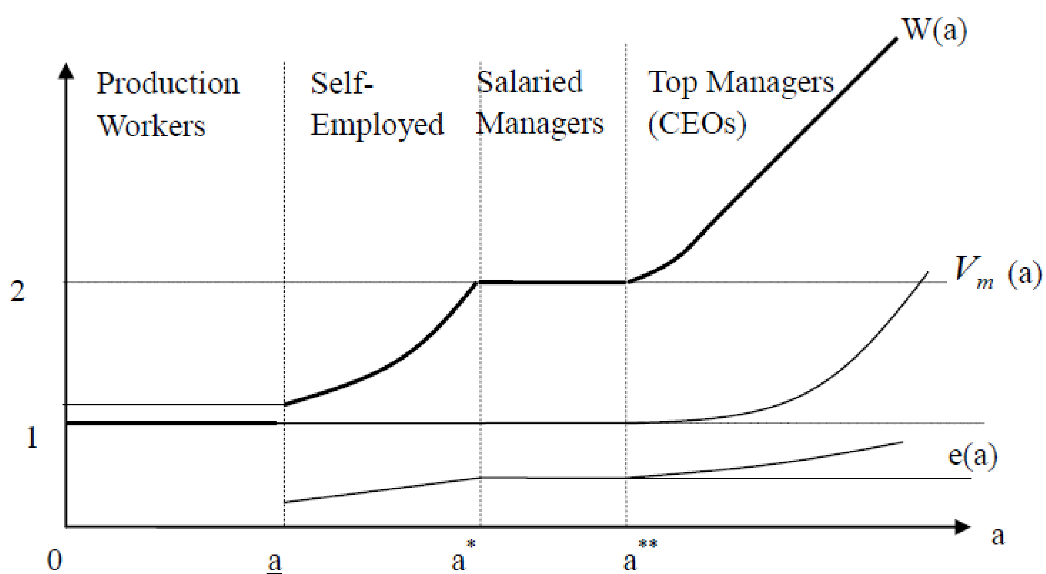


Figure 2. Distributions of Occupations and Wages in the Market Equilibrium



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