Understanding Scientific Representation

Roman Frigg

London School of Economics

1. Introduction

Models matter. Scientists spend substantial amounts of time building, testing and revising models, and significant parts of many journal articles are concerned with exploring their features.[1] At least in part, models' importance is owed to the fact that many models are representations: they stand for a selected part or aspect of the world. This raises the question what it means for a model to represent a target system. We begin our discussion by distinguishing between three different kinds of models (Section 2). We then introduce the DEKI account of representation and illustrate how it works with examples (Section 3). We end by highlighting seven open questions for DEKI which can be the focus of future research (Section 4).

2. Setting the Scene: Three Kinds of Models

The term "model" has countless meanings. Some of them are obviously irrelevant in the current context. For instance, when the teacher says that "Atsuko is a model student" or when a newspaper article discusses the "new model of the MacBook Air", the term is used in ways that are irrelevant to the concerns of science. But even if we set non-scientific uses aside, we must distinguish between three different uses of "model", leading to three different kinds of models.

The first kind of model is a *representational model*. Fundamentally, a representational model is an item that represents something else. Some representational models are miniature replicas of the original thing, for instance of a ship or an airplane. Others are objects that represent their target systems in more abstract ways. The Phillips-Newlyn machine is case in point: it is a water-pipe system that represents an economy (we will return to this example later). Other representational models are abstract. An imaginary string of beads connected by springs represent a polymer, and the so-called logistic equation represents the growth of a population. It is common to refer to the part or aspect of the world that is represented by a model as the model's *target system*.

The second kind of model is a *logical model*. A logical model is a collection of objects which have properties and stand in certain relations to each other that make a formal sentence true if the terms of the sentence are interpreted as referring to these objects along with their properties and relations. In the idiom of formal logic, the model *satisfies* the sentence. Hence, being a logical model is a relational attribute that something has with respect to a formal sentence and an interpretation.

[1] This a written version of the keynote lecture of the conference 'Science and Model', which took place at Hokkaido University on 22 February 2023. The lecture covers material from Chapters 1, 2, 3, 9, and 14 of (Frigg 2023) and Chapters 8 and 9 of (Frigg and Nguyen 2020).

Being a logical model and being a representational model are not intrinsic properties of objects; they are functions that objects perform in a certain context. Sometimes objects function as an interpretation of a formal sentence; sometimes objects function as a representation of something else; sometimes objects do both; and sometimes objects do neither one nor the other. The third option is crucial here because this means that being logical model and being a representational model are compatible with each other. In fact, as Hesse (1967, 354) points out, many models in science are at once logical and representational models: they are interpretations of a formal apparatus while also being representations of a target system. Newton's model of the motion of a planet illustrates this point. The model is an imagined object that consists of two perfect spheres with a homogenous mass distribution in empty space that attract with each other with a $1/r^2$ force law. The formal apparatus of Newtonian mechanics is applied to this model and so the model ends up being a logical model of the formalism. At the same time, the model represents the sun-earth-system and hence it is also a representational model.

The third kind of model is a *data model*. In many scientific disciplines making observations amounts to performing measurements with measurement-devices, and these measurements produce numerical data as outputs. A simple example is the measurement of temperature with a thermometer. There are different stages in the production and the processing of data. The immediate and unprocessed outputs of the measurement device are the raw data. These data are then processed and put into an orderly form in which they are more useful, a process known as data reduction. The end-result of this procedure is a data model. So we can say that that a data model is a processed, corrected, rectified, regimented, and in many instances idealised, summary of the raw data we gain from immediate observation. It is one of the tasks of statistics to describe and summarise a body of data in a compact and useful way, and so the construction of a data model is the province of statistics.

All three types of models are important in the practice of science, but their differ in their need for conceptual elucidation. Logical models have been extensively discussed in logic, and the construction of data models is studied in descriptive statistics. For this reason, we set these aside and focus on representational models.[2] The core question when it comes to representational models is: what does takes for a something to be a model and to represent a target system.[3] A number of accounts of representation are available: conventionalist accounts, similarity accounts, isomorphism accounts, inferentialism, direct representation accounts, and representation-as accounts. We refer the reader to (Frigg and Nguyen 2020) for an extensive review and discussion of these and here focus on the DEKI account, which belongs to the family of representation-as accounts.

3. The DEKI Account of Representation

[2] For a discussion of logical models and data model see, respectively, Chapters 2 and 3 of my (2023) and references therein.
[3] In fact, scientific representation raises a number of interrelated questions. For a discussion of these see (Nguyen and Frigg 2022, Ch. 1).

In this section I introduce the DEKI account of representation, which gets its name from its four principal components: denotation, exemplification, keying-up, and imputation.[4] But before we can delve into the details of this account, a remark about the ontology of models is in place. Some representational models are ordinary material objects. Oval blocks of wood are used as models of ships, the Kendrew model of the myoglobin is a plasticine sausage, and the so-called Phillips-Newlyn machine, which is system of waterpipes, reservoirs and pumps, models an economy.[5] Following Thomson-Jones (2012), I call models of this kind *concrete models*. But not all models are concrete. As we have seen, when Newton modelled the solar system, he invited his readers to imagine two perfect spheres located in empty space, one large and one small, where both have a homogenous mass distribution and attract each other with a $1/r^2$ force law. This is not object that you can put on your laboratory table. Hacking memorably described such models as things that "you hold in your head rather than your hands" (1983, 216). I call such models *non-concrete models*. The negative characterisation is deliberate because there is an open question about the ontology of these models: what, if any, sort of object are they? This is an interesting problem in ontology, but one that we cannot get into at this point.[6] To boost intuitions, readers can think about models of this kind as ontologically on par with the characters and places we encounter in works of fiction.

In what follows I introduce the DEKI account in the context of concrete models, mostly using the Phillips-Newlyn machine as my example. This is a pedagogical decision. As we will see later, the account equally applies to non-concrete models, but some notions that we will appeal to are more naturally introduced in the context of concrete models.

For a model to be a *representation of* a target, it must denote the target. This is Goodman's point when he notes that denotation is the core of representation (1976, 5). The Phillips-Newlyn machine, for instance, has been used to represent the economy of Guatemala, and to do so it has to denote the economy of Guatemala. To distinguish between something being a representation of something else and other forms of representation we introduce the locution "representation-of".

Not every representation is a representation-of. A picture showing a unicorn is not a representation-of a unicorn because things that don't exist can't be denoted (and I take it that there no unicorns). Yet there is a sense in which such a picture is a representation – after all it *shows* a unicorn. Goodman and Elgin's solution to this conundrum is to distinguish between being a representation-of something and being a something-representation (Goodman 1976, pp. 21-26; Elgin 2010, pp. 1-2). A picture showing a unicorn is a unicorn-representation, but it is not a representation-of a unicorn. Generally, something is a *Z*-representation if it portrays a *Z*, and it is a representation-of a *Z* if it denotes a *Z*. Crucially, something being a *Z*-representation does not imply it also being representation-of a *Z*, and *vice versa*. The word "Japan" denotes a country, namely Japan, and hence is representation-of Japan. But it is not a country-representation. Vice versa, a 17$^{\text{th}}$ century drawing may be an Atlantis-representation, but it cannot be a representation-of Atlantis because Atlantis does not exist. This does not preclude the notions to go hand in hand in some cases: a portrait of Haruki Murakami is both a man-representation and representation of a man. The point here is that there is no necessary

[4] For a general statement of the DEKI account see (Frigg and Nguyen 2018) and (Frigg and Nguyen 2020), and for a discussion of how the account develops out of Goodman and Elgin's account of representation-as see (Frigg and Nguyen 2017b). For discussion of how mathematics is used in models see (Nguyen and Frigg 2021).

[5] For a discussion of these models see, respectively, (Leggett 2013), (de Chadarevian 2004), and (Barr 2000).

[6] For discussion of different options see Chapter 14 of my (2023).

connection between being a Z-representation and being a representation of a Z, nor are the two notions coextensive for some other reason.

This raises the question of what turns something into a Z-representation. What, for instance, turns a canvas covered with pigments into unicorn-representation? In the context of pictures this is a much-discussed question. Perceptual accounts argue that a picture is a Z-representation if, under normal conditions, an observer sees a Z in the picture (see, for instance, Lopes 2004). Goodman and Elgin analyse Z-representation in terms a picture belonging to a certain genre (Elgin 2010, 2-3; Goodman 1976, 23). Whatever the merits of these accounts in the context of visual representation, neither of them is helpful when dealing with scientific models. The DEKI account submits that what turns a model into a Z-representation is an act of interpretation by a model user: we *interpret* the objects that constitute the model in terms of Z.

“Interpretation” means different things in different context. In the context of DEKI, an interpretation correlates properties of the model object with properties of a domain of interest. Let X be the object that serves as the model. In our example X is the Phillips-Newlyn machine, a system of waterpipes and reservoirs. Let Z be the domain of interest. In our example that is an economy. An interpretation then pairs up sortal properties of X with sortal properties of Z, for instance by saying that having a reservoir on the right of the central column corresponds to having a banking sector. Mass terms are also endowed with a mass correlation function, which, in our example, correlates the amount of water with the amount of money (for instance by saying that one litre of water corresponds to a million Pound Sterling). Under such an interpretation the Phillips-Newlyn machine becomes an economy-representation.

We are now in position to present an account of models. Let Λ be an interpretation in the sense just introduced. As we have just seen, it is Λ that turns the object X into a Z-representation. The crucial point to realise is that in constructing a Z-representation we have at once constructed a model. The Phillips-Newlyn machine endowed with an economy representation is a model, or more precisely and economy-model. This can be encapsulated in the equation $M = \langle X, \Lambda \rangle$, where the angular brackets indicate an ordered pair and “M” stands for “model”. In other words, a model is just the ordered pair of an object and its interpretation.

This has the immediate consequence that models as such need not have targets: having a target is not part of the concept of a model. While this may seem counterintuitive, it is the correct conclusion. A number of important models have no targets, think for instance of the four-sex population models in biology, the ϕ^4 model in quantum field theory and the Kac ring model in statistical mechanics.[7] And our own example is no exception. Even though many central banks had a Phillips-Newlyn machine, only the Guatemalan central bank seems to have used it as representation of their country’s economy.[8] If the Guatemalans had not bought a machine, or if they hadn’t used it predictively, then the Phillips-Newlyn machine would never have been a representation-of any economy. And yet it still would have been an economy-representation, and indeed one that was widely used in classrooms.

[7] For in-depth discussions of these models see, respectively, (Weisberg 2013), (Hartmann 1995), and (Jebeile 2020).

[8] See (Frigg and Nguyen 2020, Ch. 8) and references therein.

The "E" in "DEKI" stands for exemplification. Something exemplifies a property if it at once instantiates the property and refers to it. As Goodman puts it: "Exemplification is possession plus reference. To have without symbolising is merely to possess, while to symbolise without having is to refer in some other way than by exemplifying" (Goodman 1976, p. 53). Samples are canonical examples of items that represent by exemplification. The chip of paint on a manufacturer's sample card instantiates a certain colour, and at the same time refers to that colour (Elgin 1983, p. 71). Exemplification is selective in that not every property that is instantiated by an object is exemplified by it. The chip of paint, for example, does not exemplify its shape. In order to exemplify a property, an object must both instantiate the property and the property itself must be made salient. How saliency is established will be determined on a case-by-case basis, depending on the context and the epistemic interest of the users of a representation.

At this point some philosophical house-keeping is in order. If exemplification is possession plus reference, it follows that a model can only exemplify X-properties because X only instantiates X-properties. In our example, the Phillips-Newlyn machine instantiates waterpipe-properties. But if X is used as a model, then one would like it to be able to instantiate properties of the domain of interest. That is, we want the Phillips-Newlyn machine to instantiate economy-properties. To make this possible, we introduce the notion of *instantiation under an interpretation* Λ (or Λ-instantiation, for short). If we have an interpretation that says that certain tank is a bank and that a litre of water is a million Pound Sterling, then the fact that litre of water sits in that tank means that the model, under the chosen interpretation, instantiates a there being a million in the bank. With the notion of an instantiation under an interpretation we can now also define the notion of *exemplification under an interpretation*, which is exactly like exemplification except that it requires instantiation under an interpretation instead of exemplification simpliciter.

We can now introduce the notion of representation-as. We encounter this mode of representation in caricatures, where we see, for instance, Margaret Thatcher represented as boxer or Winston Churchill as a bulldog. The grammar of the concept is that an object X represents a subject or target T as being Z. With the tools developed so far we can define representation-as in the following way: X represents T as Z if, and only if, (i) X denotes T (i.e. X is a representation-of T) , (ii) X is a Z-representation exemplifying certain properties associated with Z, and (iii) X imputes these properties, or related ones, to T (Elgin 2010, p. 10).

Scientific representation is representation-as, and the "I" in "DEKI" stands for "imputation", as requested in (iii). But to make the basic idea tick in the context of scientific representation, a fourth element needs to be added. Condition (iii) of the definition of representation-as says that X imputes certain properties, *or related* ones, to T. The reason for adding this qualification is that the properties exemplified by a scientific model and the properties imputed to its target system need not be identical. In fact, few models portray their targets as exhibiting *exactly* the properties of the model itself.

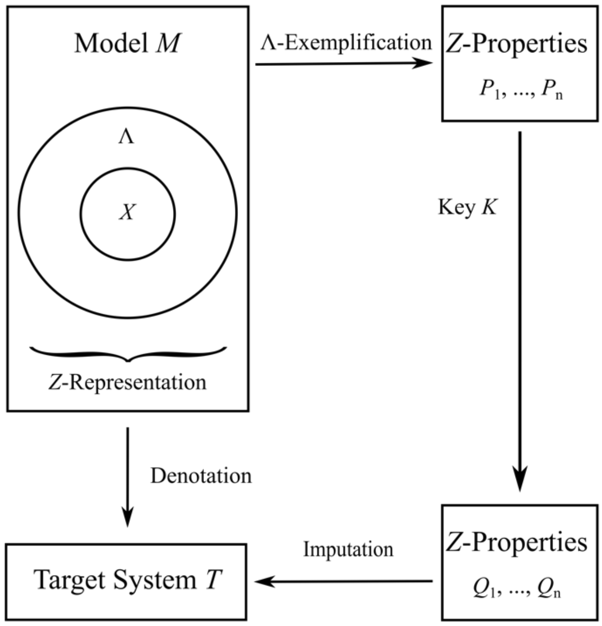
Precision can be added to the account by building a specification of the relationship between model properties and target properties directly into an account of representation. Such a specification is given by a *key*, which is the "K" in "DEKI". A key in effect translates one set of properties (the ones exemplified by the model under an interpretation) into another set of properties (the ones imputed to the target). A key can, but need not, be identity; any rule that associates a set of properties ready to be imputed to the target with the properties exemplified by the model can do the job. The relevant clause in the definition of representation-as then becomes: X exemplifies one set of properties and imputes another set of properties to T where the two sets of properties are connected to each other by a key.

Maps provide an intuitive example. The exemplified map-property is the measured distance on the map between the point labelled "Rome" and the point labelled "London"; the property to be imputed is the distance between Rome and London; and the key is the scale of the map. The key allows us to translate the property of the map (the distance between the two dots being 18cm) into a property of the world (the distance between Rome and London being 1800km). The keys used in scientific models are often more complicated, but they perform the same function.

As an example from science, consider again the Phillips-Newlyn machine. The circulation period is the time it takes for an active balance to circulate once around an economic system. What does the machine tell us about the circulation time? To answer this question, we first observe how long it takes for particular bit of water to move around the machine, which can be done, for instance, by inserting a coloured blob of water into it. It turns out that this time is about 17 seconds. But obviously we must not take this to imply that the machine represents the Guatemalan economy as being such that a balance moves around the economy in 17 seconds. We need to take into account that there is relation between model-time and real-economy-time that makes 1 minute of running time in the machine equivalent to 1 year in the economy. This relation is a part of the interpretation, and so applying this relation we find that the machine exemplifies the economy-property that it takes active balances about 3.4 months to circulate around the economy. However, economists will not impute this precise figure to the Guatemalan economy. Phillips discussed this issue with care and concluded that the circulation time in a real economy would be "somewhere between 3 and 4 months" (Phillips 1950, p. 291). So we must transform "3.4 months" to "between 3 and 4 months" to get an imputable feature, and this is what the key does. A mapping from a precise value to an interval is a relatively simple key. More complex keys can involve various forms of idealisations and analogies.[9]

Pulling together the different elements we have introduced so far furnishes the sough-after analysis of representation. Consider a scientist (or scientific community). The scientist chooses an object and turns it into a Z-representation by adopting an interpretation Λ. The model M is the package of the object together with the interpretation that turns it into a Z-representation: $M = \langle X, \Lambda \rangle$. The model M then represents target T iff (i) M denotes T (and, possibly, parts of M denote parts of T); (ii) M is a Z-representation exemplifying certain Z-properties; (iii) M comes with a key K specifying how the Z-properties exemplified in the model translates into other properties, and (iv) M imputes at least one of these other properties to T. This is the DEKI account of representation, which is also visualised in the figure below.

[9] For discussion of different options see Chapters 10-12 of my (2023).



At the beginning of this section, we distinguished between concrete and non-concrete models, and we introduced the DEKI account with a concrete model. Let us now return to the latter and say how DEKI applies to non-concrete models. As an example of such a model, consider Newton's model of the sun-earth system, which we have introduced earlier: two perfect spheres, located in empty space, with a homogenous mass distribution that attract with each other with a $1/r^2$ force law. What turns this assemblage into a solar-system-representation is that scientists working with the model interpret the large sphere as the sun, the small sphere as the earth and the $1/r^2$ force law as gravity. *Per se*, the two spheres are just a fictional object. It becomes a sun-and-earth-system-representation only when we interpret it. The above interpretation is common, but by no means forced on us by the object itself. We can just as well interpret the large sphere as a proton, the small sphere as an electron, and the $1/r^2$ force as electrostatic attraction. Under this interpretation, our two spheres are helium-atom-representation, and which of course the Bohr model of hydrogen atom.

Once endowed with an interpretation, the model exemplifies the planet moving in perfectly elliptical orbit. But it does not impute a perfectly elliptical orbit to planet earth because we know that a number of idealising assumptions have been made: real planets are not perfectly spherical, they are not in empty space, etc. The net effect of these assumptions is that we should expect planets to move in an approximately elliptical orbit. So the key transforms "perfectly elliptical" into "approximately elliptical", and this is the property that the model imputes to the target.

Let us now add a few corollaries to our definition of representation. First, we call a representation "faithful" if the target actually possesses the properties that the representation imputes to it. It is crucial that faithfulness is not part and parcel of being a representation. Indeed, representations can be misrepresentations, and that the representation be faithful is not part of the notion of a representation.[10] Second, DEKI provides an explanation of how learning from models takes place: we investigate the model and figure out what properties it exemplifies, then key them up with other properties, and finally impute these to the target.

[10] For a discussion of this point see (Millson and Risjord 2022) and (Frigg and Nguyen 2022).

This generates a claim about the model that can then be tested empirically. Third, what we have formulated so far is the general form of an account of representation. On each occasion of application of DEKI, an explanation must be given of how denotation is established, what properties are exemplified and why, and what key is used. Doing so will provide scientists with an understanding of how their representation works.

4. Open Questions for DEKI

The DEKI account is not a closed book. There are a number of open questions for the account, and this section we review seven of them.

The first question concerns keys. As we have seen, keys are crucial because they ultimately determine what properties the model imputes to the target. This raises that question: what keys are there and how can they be classified? We have seen a few simple keys at work, for instance when we converted an exact number for the circulation time for an active balance into an interval. But not all keys are as simple as that. Mechanical models often use keys that involve complex limit relations (Nguyen and Frigg 2020). But these would seem to be typical of models in physics, and even in physics not all keys may be limit keys. It also leaves open the question of how models in other domains such as biology or psychology work. Future work on DEKI should therefore focus on the nature of keys and investigate across the full spectrum of science what keys there are and how they work.[11]

The second question concerns the issue of justification. In the previous section we emphasised that being veridical is not part and parcel of a representation, and whether a claim that is generated by a model is correct has to be established through independent means. Typically, these means are experimental tests. But not all model-results can be subjected to empirical test. A climate model may tell us what level of global warming we are facing by 2050, and virological model may tell us what consequences a certain policy intervention has on the spread of a disease. Neither of these claims may be empirically testable now, the former because the relevant time is still several decades into the future and the latter because certain interventions may be unethical. How, then, can we know in such cases whether a model-result is correct? Are there kinds of representations that come with an "in-built" justification that make results trustworthy even in the absence of empirical test?

The third question concerns the identification of target systems. In our discussion we took it for granted that we know what the target system is and that we can establish a denotation relation between the model and the target. In some cases this is plausible, for instance when targets are macroscopic objects we can point to. But there are cases where things are less straightforward. A target system may be too far away for us to have any direct contact with; it may be too small to see; or it may be so entangled with its surroundings that it that it is hard to isolate. In these cases, there is a question about how we identify a target and establish reference to it.[12]

The fourth question is how model objects are selected. The account says that a model is an object *X* endowed with an interpretation, and it says nothing about what the object is. Implicit in this treatment of models is the view that anything can be a model, and that anything can

[11] For a discussion of keys for model organisms, see (Ankeny and Leonelli 2021).
[12] For a discussion of this issues in the case of biology, see (Elliott-Graves 2020).

represent anything else. This view has explicitly been advocated by a number of authors.[13] But from the fact that anything can in principle represent anything else, it does not follow that in actual practice anything can serve as a representation. This raises the question whether there are constraints on what kind of objects can be models, and whether certain objects can be ruled out on principled grounds. My current thinking is that there are no such constraints and that what can be a model is a question that has to be answered on case-by-case basis. But this issue deserves further thought.

The fifth question concerns the role of theories. As discussed here, representation is relation between a model and its target, and nothing has been said about the involvement of theories. Indeed, there is a more general question in the background: what is the relation between models and theories? Different answers to this question have been proposed,[14] but no systematic account of the relation between models and theories, and of the role of theories in representation, is available.

The sixth question concerns representations other than models. While models are an important class of representations in the sciences, they are not the only kind of representations that scientists produce. For instance, images of various kinds play important roles in different disciplines. This raises the question of whether the workings of scientific images can also be explained through DEKI. If so, how exactly does DEKI account for the workings of scientific images? If not, what does an alternative account of representation that captures how images work look like and how does it differ from DEKI? My current thinking is that DEKI is a universal account that covers models, images, and other representations, but this claim needs to be substantiated through a detailed analysis of such representations.

Finally, the seventh question concerns the relation between representation in science and representation in non-scientific domains, in particular the arts. The basic question here is the same as in the previous paragraph: does DEKI cover these cases too? And again my answer would be that my current thinking is that it does, but this would have to be substantiated through an analysis of various non-scientific forms of representation.[15]

The DEKI account explains how models represent, and it raises interesting questions. I hope that it will be the focus of critical discussion in the years to come.

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[13] See (Frigg and Nguyen 2020, pp. 27-28) and references therein.

[14] For a review of options, see (Frigg forthcoming).

[15] For a beginning of such an investigation, see (Frigg and Nguyen 2017a).

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