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Pari-Mutuel Betting Markets: Racetracks and Lotteries Revisited

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Abstract

This survey discusses the state of the art in research in racetrack and lottery investment markets. Market efficiency and the pricing of various wagers are studied along with new developments since the Thaler & Ziemba (1988) review. The weak form inefficient market pricing approach using stochastic programming optimization models changed racetrack betting from handicapping to a financial market allowing professional syndicates to operate as hedge funds. Topics discussed include arbitrage and risk arbitrage, syndicates, betting exchange rebates, behavioral biases, and fundamental and mispricing information in racetrack and lottery markets. Similar models can be used to successfully trade stock market anomalies. **Supplemental Materials** are included online.

1. INTRODUCTION

Racetracks and lotteries are important financial markets. These investments have negative expected value for most investors. But they have wide appeal for entertainment, the possibility of small and large gains, the intellectual aspects of their study, and valuable insights in other financial markets. There are economic and behavioral effects as well as an extensive literature concerning biases, strategies, regulations, and successes. In a well-cited paper, Thaler & Ziemba (1988) analyzed these markets including their market efficiency and rationality. They argued that

> wagering markets have a better chance of being efficient because the conditions (quick, repeated feedback) are those which usually facilitate learning. (Thaler & Ziemba 1988)

In this review, I focus on racetracks and lotteries. I begin with market efficiency and then cover other key topics such as arbitrage, risk arbitrage, locks, rebates, the importance of prediction, racetrack pool guarantees, the favorite-longshot bias, winning strategies based on optimization of various types of wagers (including pick 6 and rainbow pick 6), professional syndicates, effects of breeding, and similarities with other financial markets in sports and conventional markets. Information on efficiency in the US National Football League (NFL) and other sports markets is discussed in Ziemba & MacLean (2018), with further references in the **Supplemental Materials**.

Supplemental Material >

Table 1 lists typical racetrack bets in the United States and their characteristics. These bets vary from high-probability/low-payoff wagers like betting win, place, and show on a single horse to low-probability/high-payoff wagers like the pick 3–6.

1.1. Efficiency of Financial Markets

There is a large body of literature on the efficiency of financial markets. The standard material includes the work by Kendall (1953), Fama (1970, 1991), and Roll (1977). For later work on the stock market, see the **Supplemental Materials**.

1.2. Efficiency of Sports Betting Markets

Sports betting markets are well suited for testing market efficiency and bettor rationality due to the vast amounts of available data, including prices for devising technical systems and other information for devising fundamental systems (Vaughan Williams 2005). Each bet has a specified termination point when its final asset value is determined. For rationality tests, markets with this

Table 1 Typical racetrack wagers in the United States

Wager type	Definition
Win	Horse must come in first.
Place	Horse must come first or second.
Show	Horse must be first, second, or third.
Quinella	Two horses must be first and second, in either order.
Exacta	Two horses must be first and second, in exact order.
Trifecta	Three horses must be first, second, and third, in exact order.
Superfecta	Four horses must be first, second, third, and fourth, in exact order.
Hi five	Five horses must be first, second, third, fourth, and fifth, in exact order.
Double	Two horses, one winning one race and the other winning the second race.
Pick 3–6	Three, four, five, or six horses that win each of their races.
Place pick all, N races	Selected horses must place first or second in the selected (N) number of races.

latter property offer an advantage over markets, like securities markets, where the current price depends upon future events and current expectations of future values. Also, some wagering markets have characteristics that reduce the problematic nature of the joint hypothesis test. Dana & Knetter (1994) showed that point spread bets on NFL games have identical risk and return characteristics as well as similar horizons. This allows a test of efficiency without specifying the bettors' utility functions.

The special properties of sports betting and lotteries lead one to speculate that they are even more efficient than financial markets. However, another aspect of these markets confounds the notion of rationality: For the market to work and bets to be offered, the average bettor must lose. Indeed, given the transactions costs involved in these markets (e.g., approximately 13–30% for horseracing and approximately 50% for lotteries), the average losses are large. This has not stopped the search for profitable wagering systems, and there are notable successes. Thorp (1962) demonstrated that card counters can win playing blackjack. Beyond the academic work surveyed here, evidence abounds of individuals who have successfully beaten the odds. See, for example, Akst (1989), Benter (1994, 2008), Ziemba (2005, 2012, 2017, 2019a), Gergaud & Ziemba (2012), MacLean & Ziemba (2022), and Ziemba & MacLean (2018).

The continued success of these winning systems is related to complicating factors in their development or execution. For instance, the system may involve short odds and complex probability estimation (e.g., place and show wagering at the racetrack), may rely on syndicates of bettors (e.g., cross-track horserace betting), or could require extremely long time horizons (e.g., lotteries) or extensive data collection and statistical work (e.g., fundamental handicapping systems for horseracing). The winning systems described are a subset of the winning systems used in practice. The incentives to disclose details of a winning system may not be sufficient in some cases, given that such an action typically reduces the system's profitability as others employ it. Throughout the paper, I discuss optimal betting strategies for exploiting inefficiencies when they are present.

Sports betting and lotteries involve substantial transactions costs. Because it directly affects prices, the take—what the gambling establishment keeps for its operation—is properly accounted for in all of the analyses discussed in this survey. Another cost is information (e.g., tip sheets at the racetrack). Costly information requires a redefinition of efficiency, one where prices are said to reflect information to the point where the cost of additional information equals the benefit of acting on that information. As they are difficult to measure, this survey ignores information costs (and other transactions costs beyond the take). The findings of efficiency gain further support with the introduction of these costs. In racing, the advent of rebate bettors has reduced the take from 13–30% for various bets to approximately 11–13% for large bettors. The rebate bettors and the large bettors share a discount given by the track for the signals (outcomes).

These discounted bets are blended with other bets. Rebates at a lower level are available to small bettors. For example, a win bet might have a track take of 15%. The rebate shop might take 5%, the bettor gets 5% off, and the track gets only 5% for the signal, namely the results. So, the rebate bettor pays effectively 10%, and the track gets its 15% by charging the non-rebate wagerers more than 15%. For exotic wagers, such as exactas (getting the first two finishers in exact order) and other wagers, the take is in the 20–25% area, so the rebate is larger. The largest volume racetrack betting market is in Hong Kong. There, the standard rebate gives 12% back on losing bets of more than $10,000 Hong Kong dollars.

1.3. Racetrack Betting Markets

At a typical racetrack, the market accepts wagers for approximately 20–60 minutes. These wagers come from those at the track and from simulcasts that feed money into the betting pools from all over the United States and the world. Historically, there were separate pools for each race and type of wager with different prices that could be arbitraged, as discussed by Hausch & Ziemba (1990a,b). Now all the money goes into the same pools from the host track and many other betting venues.

Some of the exotic bets involve multiple horses in a given race, while others involve multiple races. The wagers are basically of two types: high probability of winning/low-payoff bets and low probability of winning/high-payoff bets. The latter can return a million dollars or more. **Table 1** describes typical bets available in the United States. Hong Kong bets are similar, but there are more of them, including 1–2–3 trifecta, 1–2–3–4 superfecta in exact or any order, and double and triple trio, which to win you must get 1–2–3 in exact order in two or three races. The latter bet has 48 million combinations with 14 horses per race, so it is similar to a lottery. They also have other high-payoff/low-probability wagers such as modified pick 6, where the first prize is first or second in all six races and the second prize is the more difficult ordinary pick 6.

The arbitrages called locks in the gaming industry occur because the racetrack minimum payoff is 5% or 10% profit after the pari-mutuel payout. So with a super horse that is highly favored, you can construct a large bet on the super horse plus small bets on the other horses, so that a profit is guaranteed without risk for place or show wagers. Hausch & Ziemba (1990b) developed this strategy, including deriving the conditions for the arbitrage to exist and showing how to calculate the bet sizes. If the bets on the non-super horses are assumed to be equal, approximately 5%, then a lock exists when $K > 1 - \frac{Q(n-1)}{21(n-3)}$, where $K =$ the fraction of the show bet on the super horse, S is the total show pool, and $\frac{(1-K)S}{(n-1)}$ are the bets on the other horses. A linear program can calculate the wagers for unequal bets. In the United States, such a lock occurs approximately 10 times per year with a gain of approximately 2%. Ziemba & Hausch (1987) pointed out a flaw in the United Kingdom and Ireland betting rules. They have a minimum payout like the United States, which is to get your money back. However, the pool was split differently so that the payoffs for the losing bets do not pay for the winning bets, because the net pool is shared equally after the take. This can lead to a minus pool, where the house has a loss and an arbitrage exists for the players. Jackson & Waldron (2003) fully analyzed this and successfully exploited it in 1998 in the United Kingdom and Ireland, winning approximately £50,000 until the tracks eliminated the flaw in 1999 in the United Kingdom and in 2000 in Ireland.

Racetrack betting is an application of portfolio theory. The racetrack offers many bets that involve the results of 1 to approximately 20 horses. Each race is a special financial market with betting over a short horizon then a race that takes 1 or a few minutes. Unlike the financial markets, one cannot stop the race when one is ahead or have the market going almost 24/7. There is a well-defined end point. As in portfolio theory (Chopra & Ziemba 1993), a key issue is to get the means right—that is, the probabilities of various *ijkl* finishes for a superfecta or *ijk* for place and show bets—and to size the bets appropriately. For the latter, the Kelly capital growth criterion is widely used and maximizes the expected logarithm of final wealth almost surely. Transaction and price pressure odds changes fit well into the stochastic programming models discussed below.

Professional racetrack betting syndicates have been as successful as hedge funds, with gains approaching US$1 billion over several years for the most successful. This is not easy, as the markets are quite efficient (see, e.g., Ali 1977, 1979; Hausch, Lo & Ziemba 1994, 2008; Hausch & Ziemba 2008). Since I had coproduced key books in racetrack betting, I was well-known and was invited to consult with high-level practitioners. Besides the advantages of rebates for the syndicates, over half the betting is not recorded in the pools until the race is being run. Monies are bet near the start of the race and come from many off-track sites and betting exchanges through aggregators, which are combined with the on-track bets into the track pool. This takes time. So, estimates of future prices are crucial. Betting exchanges such as Betfair in London allow short as well as long bets. This allows for more arbitrage and the ability to take advantage of known biases.

Investing in traditional financial markets has many parallels with racetrack and lottery betting, and much of the analysis is similar. Behavioral anomalies such as the favorite-longshot bias exist and can be exploited in the S&P 500 and FTSE 100 futures and equity puts and calls options markets. Biases favor buying high-probability favorites and selling low-probability longshots just like the high-probability/low-payoff racing wagers. For data and calculations, see Hodges, Tompkins & Ziemba (2004), Tompkins, Ziemba & Hodges (2008), and Ziegler & Ziemba (2015). But in complex low-probability/high-payoff exotic wagers such as the pick 6, the bias reverses to overbetting the favorite, so one must include other value wagers in the betting program to balance these risks.

Another major difference is that racing markets have many more types of bets, and some are essentially lotteries and betting exchanges, such as Betfair in London and elsewhere, that allow short as well as long wagers to hedge, eliminate, or increase the investment positions. These are run in continuous time, which allows for mean-reversion risk arbitrage during the race as the horses are running well or poorly.

2. BREEDING

Fundamental information such as breeding is important and is especially useful for the Kentucky Derby and Belmont Stakes races, which have distances horses have not typically encountered before. The idea is that horses with more stamina in the sire's lineage can help estimate winners of the race. Since the horses have never run this distance before, a forecast of how they might do from their breeding is helpful. The key idea is that some stallions, called chef-de-race, impart consistent speed and stamina in their offspring. These stallions are identified and classified, and it is they who essentially form the racetrack breed. Speed versus stamina is measured by a dosage index using five categories—brilliant, intermediate, classic, solid, and professional—with more stamina and less speed in the latter categories. For more information, readers are referred to the work by Hausch, Bain & Ziemba (2006) and Gramm & Ziemba (2008), who study this by merging the odds (prices) with expert opinion (breeding measured by dosage), and the comprehensive work by Roman (2016). (For details on how to calculate the dosage index and its use, especially in the Kentucky Derby and Belmont Stakes, see the **Supplemental Materials**.)

Supplemental Material >

3. PICK 6 AND VARIANTS

Popular racetrack wagers are the pick 3, pick 5, and pick 6 in the United States, Hong Kong, and elsewhere. Payoffs are frequently large and even larger with carryover, but the probability of winning is low because it is difficult to pick all six winners or even five of six for a smaller consolation prize. At $2 per combination, the cost of the wager is high to have a reasonable chance of winning. Bettors with small bankrolls typically bet the top horses in each of the six races. They have a small chance of winning and, if they win, the payoff will usually be small, as there are likely to be many winners sharing the pari-mutuel pool. Large syndicates can have more combinations and cover more horses. Ziemba (2019a) discussed the pick 3 and pick 4.

3.1. The Rainbow Pick 6

The rainbow pick 6 (called the empire six in New York state and other names at other racetracks) has become the substitute to the ordinary pick 6, which has lost favor as it is too expensive for most bettors to have a reasonable chance of winning (Ziemba 2019a,b). The idea is to create large payoffs from a small wager, which is a lottery idea mixed with fundamental handicapping skill. The rainbow pick 6 tickets are 10¢ or 20¢. The bet has two prizes. The jackpot is awarded to the holder of the unique pick 6 ticket. If there is no unique pick 6 winner but multiple pick 6 winners, then, after a 20% take, the net pool is given 60% to be divided by the multiple pick 6 winners to share and 40% goes to the carryover. If no one wins the pick 6, then the entire net pool goes to the carryover. So the expected value (*EV*) of a bet has three parts, namely:

$$EV = \text{(Prob you win the unique pick 6)} \times \text{(Value of unique pick 6, which is the carryover plus today's entire net pool)} + \text{(Prob you win a nonunique pick 6)} \times \text{(Value of each share of the pick 6)} + \text{Rebate.}$$

There are no pick 5/6 consolation prizes, but a shared pick 6 second prize is awarded if there are two or more pick 6 winners. So, with the cheap 10¢-tickets, $300 will give the bettor $6,000 worth of $2 pick 6 action, which frequently wins the pick 6. So, almost for sure, the pick 6 will have multiple winners on most days. There have been very large payoffs, as in lotteries. (For some examples, see the **Supplemental Materials**.)

Supplemental Material >

3.2. Pool Guarantee Insurance in the Pick 6

Another new feature of racetracks is pool guarantee sports insurance companies, including SCA Gaming of Dallas, Texas, run by bridge champion Robert Hamman. These companies guarantee the $1 million and other jackpots for the pick 6 and other exotic wagers. In Ziemba (2019a), I discussed bets such as those made with colleague Cary Fotias, including the pick 6 at the 2001 Breeders Cup at the Belmont Racetrack. The insurance was the $2 to $3 million part, to guarantee a pool of at least $3 million. For example, if only $2.15 million was bet, the insurance company would be liable for $850,000. We studied and proposed to bet a random amount if needed.

The idea is to bet enough to get to $3 million and return the insurance company's money by winning pick 6s and 5/6s. It was risky, as September 11th had just occurred, and the Arab owners such as Sheikh Mohammed of Dubai were not in attendance. Their horses and trainers were, though. It turned out to be a glorious day, so the crowd sent the pick 6 pool well over $4 million. Our client said you two can just play about $25,000–30,000 of tickets. So we had a $2,000 ticket twice and what we call a gorilla ticket for $28,000. We had some 5/6s and got most of the money back. The pick 6 paid approximately $250,000. The race we lost was the sprint. Squirtle Squirt, which my handicapping colleague did not like at 9–1, beat the front-running filly, Xtra Heat, at 14–1, even though she had been in the lead all the way until the finish. So, if she had won, we would have had three approximately $450,000 pick 6s plus more 5/6s. Squirtle Squirt had run at Belmont and had the top jockey, Jerry Bailey, on board and was trained by the recently deceased legendary trainer Broadway Bobby Frankel. But, the next week, we won a similar race at Santa Anita, while guaranteeing a $1 million pick 6, collecting $240,000 for the client and a bonus for us. There are pool guarantees in other betting markets such as the pick 4.

4. BETTING STRATEGIES

Kelly and fractional Kelly betting are used extensively in modeling racetrack betting. A full Kelly model is the maximization of the expected logarithm of final wealth subject to constraints (Kelly 1956). That is an expected utility approach with $u(w) = \log w$, for wealth w. Log with very low Arrow-Pratt risk aversion $-u''(w)/u'(w) = 1/w \cong 0$ is very risky in the short term despite wonderful long-term growth properties. MacLean, Thorp & Ziemba (2010, 2011) provided an extensive treatment of the key ideas and major papers on this topic. MacLean et al. (2011) provided simulations of typical behavior of these optimization models.

Fractional Kelly is simply the idea to blend cash with the Kelly strategy, similar to blending of cash and the market index in portfolio theory. Under a lognormal asset assumption, this amounts to

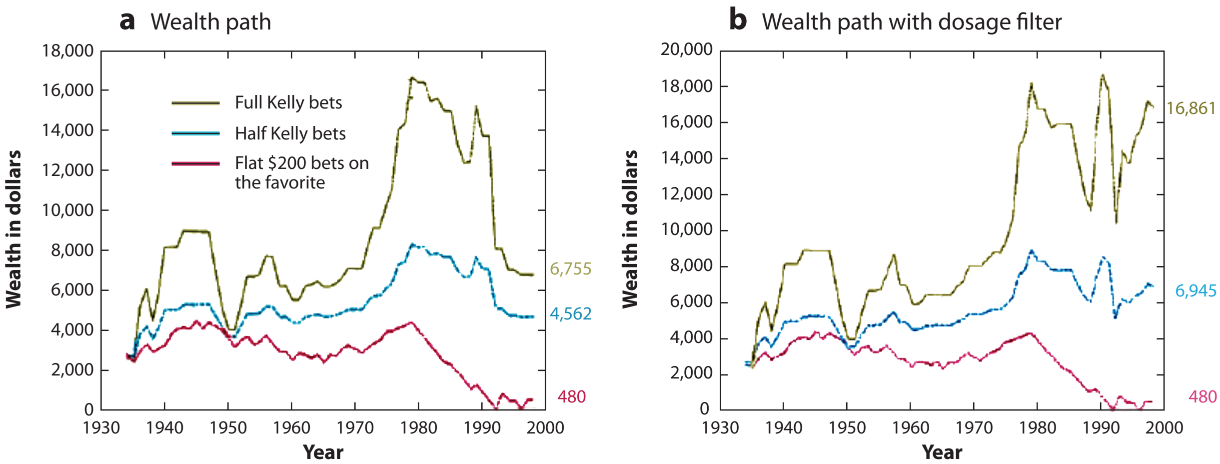

Figure 1

Wealth paths for the Kentucky Derby 1934–2000 with different betting strategies. Both assume beginning with $2,500 wealth. Panel *a* shows full Kelly, half Kelly, and betting on the favorite with no dosage filter. Panel *b* shows the same information with a dosage filter to eliminate horses that cannot run 1¼ miles on the first Saturday in May of their 3-year-old careers. Figure adapted from Hausch, Bain & Ziemba (2006).

a less-risky negative power utility function rather than log, which is the most-risky utility function one would ever want to use. Fractional Kelly leads usually to less growth and more security and a less violent wealth path. Half Kelly is a frequently used strategy. It has 75% of the full Kelly growth but the security, measured by the probability of breaking even, rises from 87% with full Kelly to 95.4% with half Kelly. For lognormal assets this is the negative power utility function $u(w) = -1/w$, and this is approximate for other return distributions. The wealthy history of bets for the Kentucky Derby from 1934 to 2005 is shown in **Figure 1*a***, and the same history with a dosage filter to eliminate horses that cannot run 1¼ miles on the first Saturday in May of their 3-year-old careers is shown in **Figure 1(*b*)**. These use the place and show system originally devised by Hausch, Ziemba & Rubinstein (1981). A system that bets on the favorite turns $2,500 into $480 so is a loser, while the full and half Kelly systems have gains.

Full Kelly usually provides the most money but has a violent wealth path. Fractional Kelly has a smoother wealth path but provides less final wealth. For graphs of various fractions, see the work by MacLean et al. (2011).

In all cases, the strategy to win is the same as in the financial markets:

1. Get the mean right using accurate probabilities of various outcomes.
2. Use the odds and a betting model such as the Kelly criterion to optimize the bet sizes.

For situations with not many wagers, either the Kelly capital growth maximize expected logarithm or its safer version, fractional Kelly, is useful as a decision tool, especially with many repeated bets. Then one has a stochastic program to maximize the expected utility using a logarithmic utility function of final wealth subject to various constraints. The Kelly strategy bets more on the attractive situations. In wagers where one makes hundreds of bets that are combinations of several individual bets, it is often better to use a tree approach where many of the bets are of equal value. Aside from making it more convenient to make these multiple bets, this approach gets around integer problems, as the wagers will be integers that can easily be bet, whereas the Kelly optimization strategy needs modifications to produce integer wagers. This approach can be computerized to print out the tickets, and the higher probability wagers can be bet more to approximate a Kelly strategy. (For more information, see the **Supplemental Materials**.)

Supplemental Material >

The late MIT Professor Paul Samuelson sent me three letters where he discussed his concerns about full Kelly investing. I responded to his four points in Ziemba (2015). Paul's critique was so influential that Fidelity Investments in Boston did not use Kelly strategies in any of their investment portfolios. They simply said if Samuelson is worried we won't use it. That was a major reason that Kelly investing, despite its desirable mathematical properties, is not taught in many finance departments and is mostly used by sports bettors and hedge fund managers like those of the Renaissance Medallion Fund. Fidelity had me give a talk to explain what it was that Samuelson was worried about. To make a long story short, it is mostly the risk of overbetting and risky short-term behavior that worried Samuelson, because the Arrow-Pratt risk aversion is almost zero so the wealth path is very jagged but long-run wealth is usually high. Ziemba (2005) found that Warren Buffett, George Soros, and John Maynard Keynes were Kelly investors and had losses in approximately one-third of the months but had high final wealth.

The Kelly and fractional Kelly ideas are very useful if used carefully, but they do concentrate the wagers into a few good assets so do not diversify. Samuelson said "the data are consistent with half Kelly," so I assume he found that strategy appropriate. For this reason some of the Hong Kong racing syndicates use probability weighting that invests in all assets with expected values above an estimation error cutoff. These are weighted best to worst.

4.1. The Importance of Accurate Mean Estimates

The **Supplemental Materials** include a table and figure showing that getting the mean right is the most important aspect of any portfolio decision problem. Chopra & Ziemba (1993) discussed that and estimated the effect of errors in means, variances, and covariances using the cash equivalent of the approximate versus optimal solutions. It is in the ratio 20:2:1 for errors in means, variances, and covariances in terms of error impact on certainty equivalent value. We measure risk aversion by the Arrow-Pratt risk aversion index $R_A(w) = -u''(w)/u'(w)$, where primes denote differentiation of the utility of wealth function u. In low risk aversion utility functions, such as log with $R_A = 1/w \cong 0$, the effect of the errors is more like 100:3:1, so getting the mean right is even more important. In horse racing, that is the probabilities for horses coming first, second, third, etc.

4.2. The Favorite-Longshot Bias

The favorite-longshot bias is the tendency in horseracing, sports betting, and financial options for the most likely outcome to be underbet and the less likely outcomes overbet. This bias has been well-known to Irish and other bookmakers, who actually create the bias with the bets they have offered for the last 100+ years. Griffith (1949), McGlothin (1956), and Fabricand (1965) are early references.

Figure 2*a* shows the basic effect over time for horseracing. The top curve for 1986 and the more recent curve on the bottom show that the longshots are still overbet and, most significantly, the favored horses now have lower expected value and extreme favorites no longer have an advantage. In particular after 1997, favorites were no longer underbet enough to turn a profit betting them. **Figure 2*b*** shows the effective track payback less breakage (rounding down) for various odds levels in California. The curve changes slightly with different track takes. These data reflect more than 300,000 races over various years and tracks, in 1986 and in recent data. Historically, there was a small profit, approximately 3%, in betting horses to win at US odds of 3–10 (UK odds of

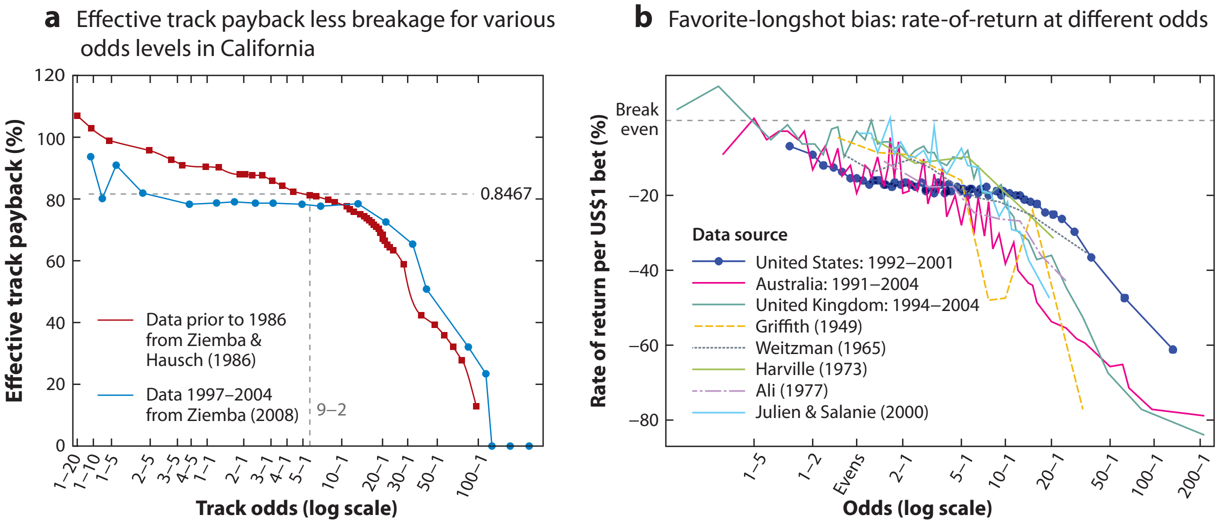

Figure 2

Effective track payback less breakage for various odds levels in California. Panel *b* adapted from an independent study by Snowberg & Wolfers (2008, 2010).

1.30 or less), and at odds of 100–1, the fair odds were approximately 700–1, making those bets worth only approximately 13.7¢ per $1 bet.

The bias curve is often different for different types of races. Higher quality races like the Kentucky Derby have flatter biases. For the 1903–1986 graph, see Ziemba & Hausch (1987). There are also flatter biases in matched betting exchanges like Betfair in the United Kingdom. Smith, Paton & Vaughan Williams (2009) showed that in highly liquid markets the betting exchange odds predict better than the bookmaker odds. Tompkins, Ziemba & Hodges (2008) demonstrated similar biases in the S&P 500 and FTSE 100 index futures options, which is consistent with option pricing theory with positive risk premiums.

Also, the curve is flat across medium odds horses. However, investors can still short longshots on betting exchanges like Betfair and make a profit. Smith & Vaughan Williams (2010) show that the bias in the United Kingdom has also declined.

Some markets have a reverse bias—these include Asian racing markets for Hong Kong (Busche & Hall 1988; Busche 1994; Benter 1994, 2008) and Japan (Busche & Hall 1988) as well as those for baseball (Woodland & Woodland 1994). The main reason for this is that bettors do not have risk preferences like most favorite-longshot bias markets, but they are risk averse. The Asian preference might also be related to the favoritism for and avoidance of certain numbers as well as inside information.

Thaler & Ziemba (1988) included the idea that there are more bragging rights from picking longshots than from favorites: 50–1, while 2–5 is an easy pick. Transaction costs are another factor: Betting $50 to win $10 is hardly worth the effort. The bias is consistent with the Tversky & Kahneman (1979) prospect theory of actual human behavior and behavioral finance, see Barberis & Thaler (2003). Low-probability gains are risk seeking and overestimated, and high-probability gains are risk averse, so they are underestimated. Mental accounting (Kahneman & Tversky 1984) may be involved where the bettor is risk seeking in one area and risk averse in another. Additional references on the favorite-longshot bias are in the **Supplemental Materials**.

Supplemental Material >

Snowberg & Wolfers (2008, 2010) focused on providing evidence for the competing hypotheses of risk love and misperception, with the latter the favored cause. Ottaviani & Sorensen (2008, 2015) surveyed the main explanations: misinformation of probabilities, market power of informed bettors, risk preference, heterogenous beliefs, market power of uninformed book makers, limited arbitrage, simultaneous betting by insiders, and the timing of bets. For further information, see Ottaviani & Sorensen (2015).

This bias is part of much human behavior and appears in other markets such as lotteries. Lottery management wants well-selling games that exploit the risk-seeking low-probability high-payoff bias. With the help of University of British Columbia colleagues Shelby Brumelle and Sandra Schwartz, I designed a bingo game with a $100 million first prize, a $10 million second prize, down to a $10 lowest prize. The game is based on patterns of chosen and not-chosen bingo numbers. The edge for the house was still more than 50% after selling the risk of the two top prizes to insurance providers, such as Warren Buffett's Berkshire Hathaway or Lloyd's of London. These insurers get bets at very favorable odds for the huge payoffs, and then the game has very limited risk for the management since they are not responsible for these huge prizes.

4.3. Place and Show and Exotic Optimization with Transactions Costs

The Dr Z system, codeveloped with Donald Hausch with some early help from the late Mark Rubinstein, presented a winning method for betting on underpriced wagers by pricing the bets. The idea of the system is to use the data from the simpler win market to fairly price bets in the more complex markets, such as place and show. For example, with 10 horses, there are 720 possible finishes for show. Then, one searches for mispriced place and show opportunities. This is a weak form violation of the efficient market hypothesis based solely on prices. How much to bet depends on how much the wager is out of whack, and it is a good application of the Kelly betting system. The formulation below shows such an optimization. There are a lot of data here on all the horses and not much time at the track. So a simplified approach is suggested. Don and I solved thousands of such models with real data and estimated approximation regression equations that only involve four numbers, namely, the amounts bet to win in the total pool and the horse under consideration for a bet, plus the total place or show pool and the place or show bet on the horse under consideration.

These equations appear below. In the books by Ziemba & Hausch (1984, 1986, 1987) and in the papers by Hausch, Ziemba & Rubinstein (1981) and Hausch & Ziemba (1985), we study this in various ways, including different track takes, multiple bets for place and show on the same horse, and how many can play the system before the edge is gone. This system revolutionized the way racetrack betting was perceived, viewing it as a financial market not just a race to handicap. This approach led to pricing of wagers and the explosion of successful betting by syndicates in the United States, Hong Kong, and elsewhere using some of these ideas as discussed by Hausch, Lo, & Ziemba (1994, 2008), Hausch & Ziemba (2008), and Ziemba (2017, 2019a).

The effect of transactions costs, which is called slippage in commodity trading, is illustrated with the following place/show horseracing optimization formulation (see Hausch, Ziemba & Rubinstein 1981). Here, q_i is the probability that i wins, and the Harville probability of an ij finish is $\frac{q_i q_j}{1-q_i}$, etc. That is, $q_j/(1-q_i)$ is the probability that j wins a race that does not contain i, that is, comes second to i. Q, the track payback, is approximately 0.82 (but is approximately 0.88 with professional rebates). The players' bets are to place p_j and show s_k for each of the approximately 10 horses in the race out of the players' wealth, w_0. The bets by the crowd are P_i with $\sum_{i=1}^{n} P_i = P$ and S_k with $\sum_{k=1}^{n} S_k = S$. The payoffs are computed so that for place, the first two finishers (say, i and j) in either order share the net pool profits once each P_i and p_i bets costs are returned. The show payoffs are computed similarly. The maximum expected utility Kelly model is

$$\max_{[p_i]_{i=1}^n,[s_i]_{1=i}^n} \sum_{i=1}^{n}\sum_{\substack{j=i\\ j\neq i}}^{n}\sum_{\substack{k=i\\ k\neq i,j}}^{n} \frac{q_i q_j q_k}{(1-q_i)(1-q_i-q_j)} \log \left[\begin{array}{l} \frac{Q\left(P+\sum_{l=1}^{n} p_l\right)-(p_i+p_j+P_{ij})}{2} \\ \quad\times\left[\frac{p_i}{p_i+P_i}+\frac{p_j}{p_j+P_j}\right] \\ \quad+\frac{Q\left(S+\sum_{l=1}^{n} s_l\right)-(s_i+s_j+s_k+S_{ijk})}{3} \\ \quad\times\left[\frac{s_i}{s_i+S_i}+\frac{s_j}{s_j+S_j}+\frac{s_k}{s_k+S_k}\right] \\ \quad+w_0-\sum_{\substack{l=i\\ l\neq i,j,k}}^{n} s_l-\sum_{\substack{l=i\\ l\neq i,j}}^{n} p_l \end{array}\right]$$

$$\text{s.t.} \sum_{i=1}^{n}(p_i+s_i)\leqslant w_0, \quad p_i\geqslant 0, \quad s_i\geqslant 0, \quad i=1,\ldots,n,.$$

Here, the sums are over an ijk finish for all i, j, k, the expression before the log is the ijk probability using the Harville formulas, and inside the large bracket is the net payoff betting on p_i and s_i for all i after the track take plus the initial wealth w_0.

While the Harville (1973) formulas make sense, the data indicate that they are biased. Savage (1957), Henery (1981), and Dansie (1983) discuss this Bayesian-type formula more. For place and show, the win favorite-longshot bias and the second- and third-finish bias tend to cancel, so the corrected Harville formulas are not needed here. For other exotic bets to correct for this, professional bettors adjust the Harville formulas, using, for example, discounted Harville formulas, to lower the place and show probabilities for favorites and raise them for the longshots; for more details, see papers such as those by Benter (1994, 2008), Henery, Stern, and Lo in Hausch, Lo & Ziemba (1994, 2008), and Lo and Bacon-Shone and others in Hausch & Ziemba (2008). Rebate is added to final wealth inside the large brackets by adding the rebate rate times all the bets, winners and losers.

The discounted probabilities are

$$q_i^* = \frac{q_i^\alpha}{\sum_i^n q_i^\alpha}$$

for α approximately 0.81, then one uses the q_i^* in the second place position. For third, one uses α^2 approximately 0.64, and for fourth place α^3. These empirical numbers vary over time and by track. This is important for exacta, trifecta, and superfecta pricing.

This is a nonconcave program, but it seems to converge when nonlinear programming (NLP) algorithms are used to solve such problems. But a simpler way is via expected value regression approximation equations using thousands of sample calculations of the NLP model from Ziemba & Hausch (1984, 1987). These are

$$\text{Ex Place}_i = 0.319 + 0.559\left(\frac{w_i/W}{p_i/P}\right),$$

$$\text{Ex Show}_i = 0.543 + 0.369\left(\frac{w_i/W}{s_i/S}\right).$$

These expected values are functions of only four numbers: the totals to win (W) and place (P) for the horse in question and the total pool for these bet types. (Further details on the optimal wagers are described by Ziemba & Hausch 1984, 1987). These equations approximate the full optimized betting model. For more on this and additional features, see Hausch & Ziemba (1985) (used in calculators). For a discussion of use at the first Breeders' Cup in 1994 with Ed Thorp

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to test the system, see Ziemba & Hausch (1986) and Ziemba (2019a). An example of the 1983 Kentucky Derby is shown in the **Supplemental Materials**.

4.4. Pricing Exotic Wagers

The basic idea from Hausch, Ziemba & Rubinstein (1981) was to price the bets and then make the good ones. All the wagers in the United States, Hong Kong, and elsewhere can be modeled this way, and some of this is done by the syndicates that are successful.

A less commonly used and known bet called the place pick all is available at Santa Anita Park. The idea is to create a ticket with I_l horses in $l = 1, \ldots, L$ races, where you have either the winner or the second place horse in each race. The number of races I_l varies from approximately 7 to 12.

The probability that a ticket with i_l, the chosen horse in race l, is first or second is the sum of the two terms, namely

$$\hat{p}_i^l = p_i^l + \sum_{j=1,\ldots,I_l} \frac{q_j^l}{1-q_i^l} \quad \text{for} \quad i \neq j,$$

using the discounted Harville probability for second, where $\alpha \cong 0.81$ is track dependent. For more on these discounted Harville formulas, see the papers in Hausch, Lo & Ziemba (1994, 2008) and Hausch & Ziemba (2008). The chance that a given ticket with i_l, where $l = 1, \ldots, I$ is a winner of the place pick all, is

$$\prod_{l=1,\ldots,L} \hat{p}_i^l.$$

4.5. Some Stochastic Programming Optimization Formulations

There are basically three strategies for the optimization of racetrack bets by the syndicates: full and fractional Kelly expected log optimization; probability weighting, which is betting on all positive expectation wagers above a cutoff; and the tree tickets approach. The probability weighting and tree tickets strategies approximate the Kelly strategies via the weighting of the wagers.

The expected log problems are typically solved using a NLP code such as CONOPT, which has produced good results even though the problems are nonconcave. The bets must be computed very quickly, as the odds are changing. While this may not be general but because of an epsilon optimality convergence criteria, the Minos-Stanford code may converge to a nonoptimal strategy. Hence, CONOPT is safer.

In general, given modern computing, the Kelly expected lot optimization can be used for essentially all the bets, even possibly Hong Kong's triple trio, namely, getting the 1–2–3 finish in any order in three races with 14 horses in each race, of which many horses are 200–1 but can still finish third. This has 48 million combinations. I focus here on the US bets and expected log Kelly optimization for high-probability low-payoff bets and the tickets approach for the low-probability high-payoff events, which can closely approximate the Kelly strategy and yields easily implemented tickets that are integers.

The simplest bet is the exacta. To win, you must get the winner plus the second place finisher in exact order. This uses elements of the place pick all formulation, except it is just for one race and it is not first *or* second but first *and* second. First is just p_i the probability that i wins. Second uses the discounted Harville formula, so it is $\frac{q_i}{1-q_i}$, where $q_i = \frac{p_i^\alpha}{\sum p_i^\alpha}$, where $\alpha \cong 0.81$. The probability of an ij finish is $\frac{p_i q_j}{1-q_i}$. Let x_{ij} be our bet on an ij finish. The Kelly optimization problem is then

$$\max_{x\in K} E\log W = \sum_{i=1}^{I}\sum_{j\neq i}^{J} p_i \frac{q_j}{1-q_i}\log\left\{w_0 + r\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij} + Q\frac{\left(B+\sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right)}{B_{ij}+x_{ij}}\left(\frac{x_{ij}}{x_{ij}+B_{ij}}\right) - \sum_{i=1}^{I}\sum_{j\neq i}^{J} x_{ij}\right\},$$

where r is the rebate percent payable on all bets, losers, and winners, B is the total exacta bet by the crowd, with B_{ij} their bets on ij, Q is the track payback, and $I = J$ is the number of horses in the race. The constraints $x \in K$ can include a maximum or minimum bet on any of our x_{ij} bets as well as on each i and on the total bet, where x is the matrix of the x_{ij}. The final wealth includes rebate plus profits minus the bets, and E is the expected value. Like all the formulations, the objective of the optimization is to maximize the expected log of final wealth to obtain the optimal betting weights subject to the constraints K. Inside the bracket is initial wealth plus rebate and gain from the bets, namely the xs, less the costs of these bets, adjusting for the track payback Q. Other high-probability low-payoff bets have similar formulations. (For the expected log model for the place pick all, see the **Supplemental Materials**.)

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Other bets, such as the trifecta and superfecta that involve three or four horses in one race or the pick 2 (the double), pick 3, pick 4, pick 5, pick 6, etc. that involve a single winner in multiple races, have similar formulations. The basic elements are the overall probability and the payoffs for the weights of the various bets. For example, in a trifecta for an exact 1–2–3 finish, one uses the discounted Harville probabilities, which are

$$p_{ijk} = \frac{p_i q_j r_k}{(1-q_i)(1-r_i-r_j)},$$

where $q_j = \frac{p_i^{\alpha}}{\sum_{i=1}^{I} p_i}$, $r_k = \frac{p_i^{\alpha^2}}{\sum_{i=1}^{I} p_i}$, and $\alpha = 0.81$ for typical data.

For the superfecta, we use one more probability term, namely s_m, with α^3 in the formulas. In Hong Kong, they offer 1–2–3 and 1–2–3–4 in exacta order or any order as separate bets. For exacta order, the formulas are similar to the above, but for any order there are the six terms: 123, 213, 132, 231, 312, and 321.

For the double and pick 3,4,5,6, one simply multiplies the individual probabilities to get an overall probability, and the probability in each race is determined from the individual horse probabilities of winning. So the expression is basically the same idea with just more terms.

The probability weighting approach is frequently used instead of Kelly optimization, which has very few large bets and can have a violent wealth path. There, one finds the bets with expected value edges and weights them. In Hong Kong, this approach is popular in wagers with low-probability high payoffs to diversify the overall wager better.

The ticket formulation breaks the picks (or horses), the js, into categories I, II, and III for each race i (**Table 2**). Category I picks are high value, which have high probability of winning. Category II picks are major contenders, and category III picks are longer odds horses who could upset the favorites. Suppose there are eight races, so $L = 8$, where n_{ij}^{l} is the number of horses in category ij in race l.

The score 8 tickets have all category I picks, and these tickets should be allocated the most money. There are $\prod_{i=1}^{8} N_{iI}$ of these. The score 9 tickets have seven category I picks and one category II pick. There are eight such tickets with smaller sized bets. The score 10 tickets have six

Table 2. Sample ticket for pick 8

	1	2	3	4	5	6	7	8
I	N_{1I}	N_{2I}						
II	N_{1II}	N_{2II}						
III	N_{1III}	N_{2III}						

N_{1I} is the number of horses in each category in each race.

category I picks and two category II picks or seven category I picks and one category III pick with even lower bets. There are $\binom{8}{2}$, and eight of these with the lowest bets. One might go to score 11 and have bet sizes to approximate a Kelly strategy. The number of tickets gets very large here, as does the cost. A computer program can be used to generate these tickets. A sample printout of all the bets made in an interesting pick 6 using this tree approach is in the **Supplemental Materials**.

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4.6. Professional Racetrack Betting Syndicates

I consulted for several of the major syndicate hedge fund teams using the approaches discussed in the *Beat the Racetrack* (Ziemba & Hausch 1984, 1986, 1987) and *Efficiency of Racetrack Betting Markets* (Hausch, Lo & Ziemba 1994, 2008) books and other contacts. Hausch and I both talked to Bill Benter, the top racetrack syndicate organizer, early in his Hong Kong career. He had started betting but had not put together a successful syndicate yet. So he quizzed us on the Dr Z place and show system and other ideas in phone calls. We did help him a bit, but he said that "we were academics spreading knowledge and he was a businessman so could not pay us" (personal communication). He did have other paid consultants on factor models. Later, he recommended me as an expert witness in a court case in Hong Kong where a spinoff group from the Benter group had a model they paid for stolen. My job was to prove that indeed it had been stolen.

Benter successfully pioneered the use of two types of 80+ factor models that could do the following:

1. predict the fair odds probabilities of various horses outcomes and compare these to the public's odds or
2. include the track odds as one of the variables to get even better probability estimates (see Sung & Johnson 2008).

They use multinominal probit or logistic regressions to estimate probabilities that are close to zero or one. One also needs a second model to estimate final track odds, given that there is much close-to-post-time betting that usually, but not always, drops the final odds on the favorites. Then, they bet with the Kelly criterion, probability weighting, or the tree method.

I do not know if Benter picked up the Kelly betting approach from Ziemba & Vickson (1975) or from Ed Thorp's blackjack writings. Benter had been a blackjack player, and Thorp introduced Kelly betting there as *Fortune's Formula*, so that may be where he learned it. A key early paper by Thorp (1975) is in Ziemba & Vickson (1975).

Benter pioneered the use of such factor models. I had a bit of a hand in there, as the major paper on this was published while I was the *Management Science* departmental editor for finance and I processed and accepted it for publication. That paper by Bolton & Chapman (1986), along with the only paper Benter published, is reprinted in Hausch, Lo & Ziemba (1994, 2008). Another paper by Chapman (1994), which uses Hong Kong data, is also in the book.

I met Benter in 1993 at the spring *Informs* meeting in Phoenix, where I organized the finance sessions and helped on the racing sessions. I recall correcting Benter's (1994, 2008) paper in

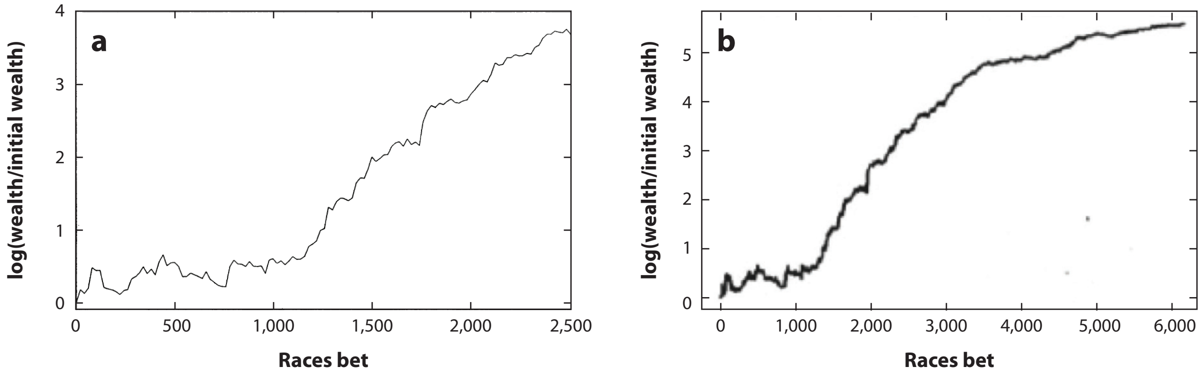

Figure 3

Benter's Hong Kong racing syndicate returns. (*a*) Hong Kong racing syndicate to 1994 (see Benter 1994). (*b*) Hong Kong racing syndicate to 2001. Figure adapted from Ziemba (2005).

Efficiency of Racetrack Betting Markets, which had one good new development. In the Dr Z method, the biases to win and being second and third tend to cancel, so we did not need to make any changes except that, because of approximations, bets should not be made to place or show unless the expected value was significantly above break even. We suggested 1.10 for the best races at the best tracks and 1.14 and 1.18 for lesser races. This worked well for US place and show betting. Benter and others found that the Dr Z system did not work well in Hong Kong, as the biases there were different. He discovered how to correct the second, third, etc. biases through the discounted Harville formulations discussed above.

Victor Lo did his PhD thesis in Hong Kong, directed by statistician John Bacon-Shone on this problem. Much of his research is in Hausch, Lo & Ziemba (1994, 2008) along with related papers by others. Bacon-Shone wrote a joint paper in Hausch & Ziemba (2008) with the late Alan Woods, who had his own betting team in the Philippines after he left Benter's racing syndicate.

Benter's real contribution is shown in **Figure 3**. Namely, he made the systems work and in the process became a very rich man with total profits in the US$1 billion area. His paper in Hausch, Lo & Ziemba (1994) plus the other papers made our book a cult item, with originals selling for $2,000 up to $12,000 on eBay and Amazon. Originals are still trading at high prices, approximately $600. I sold one for $1,400 to one of the copycat syndicates in Australia for whom I was consulting in Sydney. Another syndicate wanted to buy up all the copies and burn them, keeping one for their research. A second edition was published with a new preface in 2008 (Hausch, Lo & Ziemba 2008), along with the sports and lottery handbook (Hausch & Ziemba 2008).

The gains in Hong Kong by Benter's team and others were in a market without rebates and high commissions. But they utilized several advantages:

1. Hong Kong Chinese bettors favor and dislike certain numbers, which makes horses with these numbers differ from the true odds.
2. There are fewer horses, as mostly Australian geldings running in almost all the races on just two racetracks, so prediction was easier than in the United States.
3. Data feeds were every 12 seconds and later every minute, giving access to pool odds that could be successfully used.
4. The market was deep with huge betting, so the price impact of individual bets was low.
5. Lastly, they could bet electronically into the pools.

Since China's takeover of Hong Kong in 1997, there have been some changes. But the syndicates continue and trade in many markets as of this writing in 2022, such as Japan and Korea, as well as in the United States, Canada, and Europe. My personal experience consulting extensively for two other syndicates is that the setup cost for the research and computer implementation is a major time and financial undertaking, requiring 1–3 years and approximately $1 million. Like most markets, it was easier in years past to gain an edge and much more difficult now. Syndicates with many workers (up to 300 for the leading one in Australia) and good experience have an edge on new ones.

4.7. Conclusion on Racetrack Research

Racetrack betting remains a very active set of markets. The basic betting problems are various versions of portfolio management. The problems are usually one-period stochastic programs with nonconcave objective functions because of the fractional functions inside the objective function that are needed; thus, the effect of the syndicates' wagers on the odds is considered. But the problems are easily solved, and for many situations, there are simplified strategies. The objective is usually the full or fractional Kelly expected log criterion. However, in cases of low-probability high-payoff bets, there can be hundreds or thousands of separate tickets and the bets must be integers. So a ticket network tree (with equal bets on different combinations) or a probability weighting approach that takes all positive expectation bets above a cutoff is useful, and the Kelly strategy to bet more on the higher probability outcomes can be approximated.

The racetrack market is smaller than the markets for bonds, currency, and stock markets but there are enough racetracks for approximately ten syndicates in the United States, Australia, Hong Kong, and elsewhere to make US$50–100 million or more per year. It is not an easy market to enter at a high level, as the setup costs are high (approximately $1 million), the competition is fierce, and prediction is difficult. Many syndicates fail, and some exit when their models no longer make gains with rebate. Consider a grass race at a mile with seven horses: One horse has not run in a year but did well then, one has only run on dirt, one has never run past six furlongs (three-quarters of a mile), one was racing in France long distances (1½ miles plus on grass) but losing consistently, and the others have run similar distances on grass but not on this racetrack. Add in jockey and trainer changes, and you see why "it is a supreme intellectual challenge" as argued by Andy Beyer, a noted racetrack writer (Beyer 2013). The models bypass this difficulty with probabilities through prediction models and optimization. Much research on the economics of gambling is in the work by Vaughan Williams & Siegel (2013).

5. LOTTERIES

I was a consultant to the BC Lottery Corporation, the Canadian government, Singapore Pools, and the Mansion Lottery website in Gibraltar on various aspects of game design. Some of this experience is discussed here.

Figure 4 provides a taxonomy of games people can invest in. Games are classified by (*a*) whether the chance of winning is purely luck or can be influenced with skill and (*b*) whether the payoff upon winning is predetermined or can be improved with skill. Luck-luck games allow no possibility of discovering a profitable strategy, and so as markets, they are trivially efficient. On the other hand, there need not be a guarantee of efficiency for luck-skill games such as lotto (where we discuss a strategy of betting unpopular numbers, which does not affect the probability of winning but does affect the payoff upon winning) and skill-luck games (which are relatively uncommon). Blackjack and horseracing, skill-skill games, have profitable strategies.

Unlike most financial securities markets, the average lottery and sports betting participant must lose. We may, indeed, choose to differentiate gambling and investing by their expected returns, using the terms "gambling" when the expected profit is negative and "investing" when the expected

		Chance of winning	
		Complete luck	Skill involved
Payoff	Complete luck	Scratch lottery games with fixed payment	Example: Pay $1 for a chance to pick all winners of hockey games on a particular day. From those who have all correct selections, one name is randomly drawn and awarded $100,000.
	Skill involved	Lotto, such as 6/49, with some or all pari-mutuel payoffs	• Sports pool games • Horseracing • Blackjack • Sports betting

Figure 4

Taxonomy of games. Figure adapted from Ziemba et al. (1986).

profit, including all transactions costs and risk adjustments, is positive. Obviously, a willingness to assume risk in the face of negative expected returns is inconsistent with the traditional assumptions that (*a*) individuals maximize the expected utility of wealth and (*b*) utility functions are concave (i.e., risk aversion). Instead of the second assumption, Friedman & Savage (1948) assumed a utility function that is convex in a neighborhood of the individual's present wealth but concave over higher and lower wealth levels. Given their different payoff distributions, simultaneously purchasing lottery tickets and insurance can be consistent with this form of a utility function. Markowitz (1952) offered a functional form that eliminates some behavior admitted by Friedman and Savage's form that is not generally observed. He also pointed to the possibility of a utility function that recognizes the "fun" of gambling. Conlisk (1993) formalized this notion and found his model to be largely consistent with actual risk-taking behavior. Lane & Ziemba (2008) studied hedging strategies for Jai Alai. Haigh (2008) discussed the statistics of lotteries, and Vaughan Williams (2012) discussed national lotteries and other gambling games. (For more on lotteries, see the **Supplemental Materials**.)

Supplemental Material >

5.1. Sports Lotteries

Besides Las Vegas and other legalized gambling locales, there are extensive sports lottery games, mostly run by individual states. My own experience is largely as a consultant to the BC Lottery Commission, Singapore Pools, Mansion in Gibraltar, and the Canadian National Sports Pool.

The idea behind sports lotteries is to design games that players like and bet on and feel that with skill they can make profits. So the goal of management is to have the game look winnable with skill when in reality it is designed to be close to random, with a negative expected value to bettors after the house take. A typical lottery has approximately 15 games, and the players must correctly pick a home win, loss, or tie in each game. The first prize is 15/15, second 14/15, and third 13/15.

Most sports have a home advantage. For example, in hockey, baseball, and other sports, the home team is designed for the stadium. The simple model for a favorable betting system is to combine the home and away win-loss record with the home bias. One can use this to establish market efficient odds for the management. For the purposes of the lottery, a tie must be defined. In baseball, a tie is a one-run game. In the NFL and basketball, a tie is ± 3 points. So, just like in regular lotto games with unpopular numbers where you can rearrange the ticket numbers so the popular numbers are in unpopular positions, in sports lotto games you can design the tickets so the chances of win, loss, or tie are 1/3 each or such that the game is unwinnable in reality but looks winnable with skill by the bettors. However, sophisticated computer models, similar to those used by professional racetrack models, can be used to beat sports pools. I did this in a court case for the federal government of Canada in a dispute with Quebec, which had a hockey lottery game in violation of the Canadian criminal code forbidding games of skill. The model was able to beat the Quebec game (see Ziemba 2017). Our computer simulation won \$73,000 in ten plays of the Quebec lottery using a model that Shelby Brumelle, another University of British Columbia professor, and I made to bet up to 4,000 tickets per play. Despite this, the federal government lost the court case, as there was a precedent in the Trudeau government that had allowed a skill game in Manitoba.

5.2. Buying the Pot in Lotto Games

The carryover feature of lotto games, when the top prize is not won, builds up the jackpot pool on the next draw. Similar to the racing rainbow pick 6, with some skill, lotto games can have a way to create positive expected value bets. It is these large jackpot pools that generate sales for the hope of players to become rich. You can guarantee to win the jackpot and many other lesser prizes by buying all the numbers. The question is, when is this a good idea and how do you do it? Ziemba et al. (1986) showed that two conditions must be met: (*a*) a large carryover, so that the expected value of a random ticket is positive, and (*b*) not many tickets sold. Situations exist where these conditions are met. Lotto BC, a 5 of 40 game in British Columbia and Rhode Island, had a design feature that led to this—a 5/5 jackpot paying 91% of the net pool and little in the 4/5 and other prize. So, only the jackpot was paid. Racing is similar, with its pick 3, 4, and 5 wagers that only pay for the jackpot. As the jackpot carryover grew in Lotto BC, the ticket sales—instead of rising with a larger jackpot—actually fell, because the public thought that the game was not winnable. With only 658,008 combinations, the buying-the-pot strategy was very doable.

Moffitt & Ziemba (2019a,b) further analyze the buying-the-pot strategy and use one important fact: No matter how many ticket combinations are played by players picking their chosen or randomly selected numbers, there are bound to be some combinations that are not covered. Indeed, even in lotto games with millions of combinations, it is typical that 20–40% of the combinations are not covered. They show that buying the pot has a positive expected return if

$$a + (t + c)(1 - x)E\left[\frac{1}{1 + X}\right] - t > 0,$$

where t is the total number of tickets in the lotto game, $a \geqslant 0$ is the ticket equivalent of the carryover, $0 \leqslant x < 1$ is the lotto take of the betting pool, c is the number of tickets bought by the crowd, and X is the random number of winning tickets held by the crowd.

The jackpot $a + (t + c)(1 - x)$ is shared equally by the syndicate, which has one winning ticket, and any other players with the winning jackpot combination. Typical expected returns are in the 10–25% area for the syndicate. Steven Moffitt did this successfully for Jai Alai carryovers on mandatory payout days, while consulting for Susquehanna. The optimal strategy for the crowd is $q = \frac{1}{t}$, that is, buy all unique tickets with no doubles. But when the crowd bets not equally, as they do in all lotto games, with doubles and misses, then the expected syndicate edge is $E(q(\frac{1}{1+X}))$. Moffitt & Ziemba (2019b) discussed general lotto games, and Moffitt & Ziemba (2019a) discussed the 6/49 with many prize levels played in Canada and other countries. These papers describe the previous literature. Major syndicates are doing this in lotto games, pick 6, and other racing wagers, and they are making millions.

The papers by Moffitt & Ziemba (2019a,b) provide the conditions for the optimal circumstances to buy the pot, but how do you buy all the tickets needed? In essentially all lottery games and in 6/49, combination tickets as well as individual tickets are offered. For example, you can buy all the tickets that have five specific numbers on all of them, plus one more number. There are 44 such tickets. Also, you can buy combination tickets that contain all the possible six numbers out of seven, eight, or nine given numbers. These contain 7, 28, and 84 distinct combinations, respectively. Euler & Ziemba (2022) provide suggestions of when it is optimal to use these combination tickets to cover the board and purchase all the possible tickets. Their results develop the cheapest way to do this. However, some numbers will have to be doubles.

6. CONCLUSIONS

It is now a well established idea that racetracks and lotteries can be viewed as financial markets, and my research helped broker this. The racing industry continues to expand with many bets and top races all the time, and rebates, racetrack guarantees, and syndicates are now well established. While the racing market is weak form efficient, optimization models and various handicapping ideas can be used to create winning strategies.

FUTURE ISSUES

1. Analyze high-frequency betting exchange data, especially in-play data: There might be things we can learn as either bettors or researchers from studying the now-easily-available exchange betting data.
2. Uncover more suggestive behavioral biases in betting markets, including racetracks and lotteries, other than only the favorite-longshot bias, which can be exploited.
3. Explore new sources of information and edge, such as the wisdom of the crowd.
4. Analyze the Asian soccer in-play markets and their effect on worldwide prices.
5. Analyze the US National Football League fantasy market, where participants pick teams of players from different teams and are subject to a budget.

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