Information Spillovers in Experience Goods Competition

Online Appendix

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Contents

A Negative information spillovers, $\rho \in [-1, 0)$

When ρ is negative, information spillovers between new goods allow consumers to learn from a bad experience that they are likely to have a high valuation for the untried product. Since our primary interest is in competition between similar products and an outside option, this case is less relevant. Nonetheless, it is still useful to understand the equilibrium behavior in the negative correlation setting because the case of no information spillovers is a special case of the equilibrium with $\rho \in [-1, 0)$. Moreover, because Theorem 1 is shown to apply over this range of ρ, this appendix demonstrates the theorem's generality.

While Theorem 1 in the paper is shown to also apply over this range of ρ, some interesting contrasts emerge in comparing equilibria. Whereas when $\rho \in (0, 1)$, the mass market led to price discrimination in the second period, and created the possibility of first-period profits, the mass market equilibrium when $\rho \in [-1, 0]$ generalizes the findings for $\rho = 0$ that the prices to repeat and switching consumers are equal, the market is split equally between these consumer segments, profits are equal for each segment, and first-period price is always equal to marginal cost. That is, there is Bertrand competition in the first period, and no dynamic pricing interdependencies, whenever there is a mass market equilibrium.

It is in the niche and semi-niche equilibria when $\rho \in [-1, 0]$ that firms price discriminate and make profit from switching consumers. The relative profitability of repeat and switching consumers determines the intensity of first-period price competition in ways that are similar to the mass market equilibrium outcomes for $\rho \in (0, 1)$.

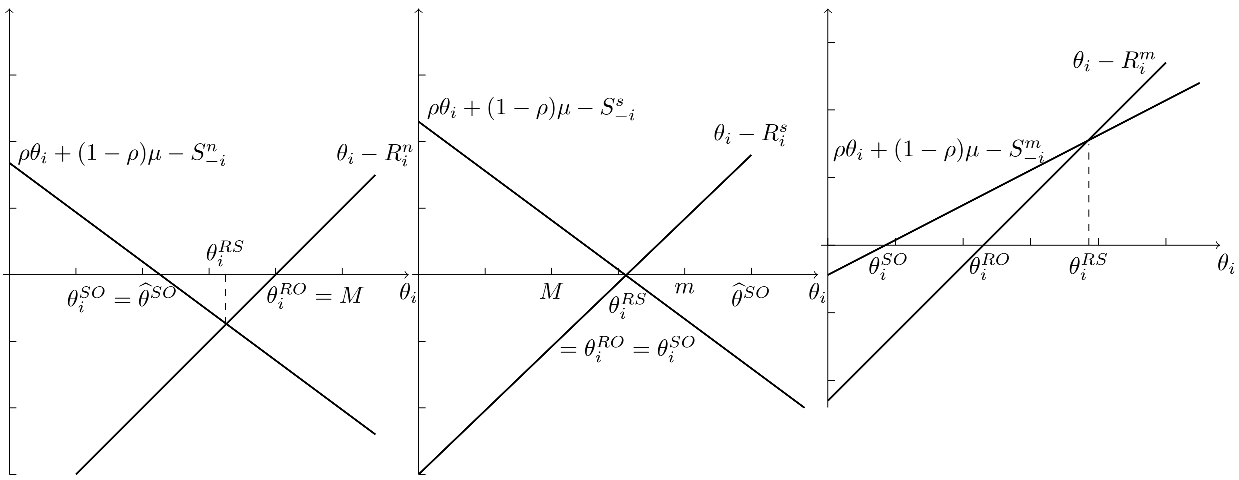
We use backward induction to characterize the sub-game perfect equilibrium of the game for this case as well, and start by characterizing the second-period equilibria.

A.1 Second-period equilibria

In the second period, there are again three forms of equilibria, illustrated in Figure 1: a niche market equilibrium, in panel a, where some consumers leave the market; a mass market equilibrium, in panel c, where no consumers leave the market; and a semi-niche equilibrium, in panel b, where the marginal repeat consumer is indifferent between repeat buying, switching, or leaving the market. In each panel of Figure 1, observe that the expected value of consuming $-i$ in the second period is negatively sloped. Note that the new market is fully covered in the mass and semi-niche market equilibria when $\rho \in [-1, 0]$ in contrast to these equilibria when $\rho \in (0, 1)$.

Figure 1: Second-period equilibria when $\rho \in [-1, 0)$; consumer valuations after learning θ_i.

(a) Niche market equilibrium, where $R_i^n = M$ and $S_{-i}^n = \widehat{S}$, given by equation (3) | (b) Semi-niche market equilibrium, where R_i^s, S_{-i}^s satisfy equations (4) and (5) | (c) Mass market equilibrium, where $R_i^m = S_{-i}^m = (1-\rho)/\left[2f(\mu)\right]+c$



Accordingly, firm i's second-period profit can be written:

$$\underbrace{(R_i - c)\lambda_i \int_{\max(\theta_i^{RS}, \theta_i^{RO})}^{+\infty} dF(\theta_i)}_{\text{profit from repeat consumers}} + \underbrace{(S_i - c)\lambda_{-i} \int_{-\infty}^{\min(\theta_{-i}^{RS}, \theta_{-i}^{SO})} dF(\theta_{-i})}_{\text{profit from switching consumers}}. \tag{1}$$

In a niche market equilibrium, we have $\max(\theta_i^{RS}, \theta_i^{RO}) = \theta_i^{RO}$ and $\min(\theta_{-i}^{RS}, \theta_{-i}^{SO}) = \theta_{-i}^{SO}$ in expression (1). In a semi-niche market equilibrium, $\theta_i^{RO} = \theta_i^{RS} = \theta_i^{SO}$ in expression (1). In a mass market equilibrium, we have $\max(\theta_i^{RS}, \theta_i^{RO}) = \theta_i^{RS}$ and $\min(\theta_{-i}^{RS}, \theta_{-i}^{SO}) = \theta_{-i}^{RS}$ in expression (1).

The prices in the niche market equilibrium can be characterized by the first order approach:

$$R_i^n = M \equiv \frac{1 - F(M)}{f(M)} + c, \tag{2}$$

$$S_i^n = \widehat{S} \equiv -\frac{\rho F\left(\widehat{\theta}^{SO}\right)}{f\left(\widehat{\theta}^{SO}\right)} + c. \tag{3}$$

where $\widehat{\theta}^{SO} = \frac{\widehat{S} - (1-\rho)\mu}{\rho}$. Note that we drop the subscript in equation (3) as the price and the cutoff type are independent of i and that $\widehat{S} > c$ as $\rho < 0$. The equilibrium prices in the mass market equilibrium can also be characterized by first order conditions with respect to prices. The semi-niche market equilibria are characterized by construction.

Lemma A.1. *Suppose $\rho \in [-1, 0)$. Then, in the second period:*

- *if and only if* $M \geq \widehat{\theta}^{SO}$, *there exists a niche market equilibrium with prices given by equations (2) and (3).*
- *if and only if* $m \geq \mu$ *and* $M \leq \widehat{\theta}^{SO}$, *there exists a continuum of semi-niche market equilibria with prices satisfying*

$$S^s_{-i} = \rho R^s_i + (1-\rho)\mu, \tag{4}$$

and

$$\max(M, \mu) \leq R^s_i \leq \min(m, \widehat{\theta}^{SO}); \tag{5}$$

- *if and only if* $m \leq \mu$, *there exists a mass market equilibrium with prices*

$$R^m_i = S^m_i = \frac{1-\rho}{2f(\mu)} + c; \tag{6}$$

In the niche market equilibrium characterized in Lemma A.1, firms charge the monopoly price M to their repeat consumers and make the monopoly profit $\pi(M)$. This is the same as in the $\rho = 0$ setting, set out in Appendix C of the main text.

Corollary A.1. *Suppose* $\rho \in [-1, 0)$*. In the second period,*

- *in the niche market equilibrium, the prices satisfy* $R^n_i = M \geq S^n_i = \widehat{S}$*, where the equality is true only when* $\rho = -1$*;*
- *in the semi-niche market equilibrium, the prices satisfy* $R^s_i \geq S^s_i$*.*

Lemma A.1 also shows that in the mass market equilibrium, as when $\rho = 0$, firms charge the same price to repeat and switching consumers despite having the option to price discriminate based on purchasing history. By equation (1) in the main text of the paper, this gives $\theta^{RS}_i = \mu$. These observations are perfectly consistent with the mass market equilibrium with $\rho = 0$. In fact, the equilibrium prices given in Lemma A.1 and the second-period profit of a firm in the mass market equilibrium, $\frac{1-\rho}{4f(\mu)}$, are equal to the corresponding terms in Appendix C in the paper by setting $\rho = 0$.

A.2 Sub-game perfect equilibrium of the two-stage pricing game

Consumers purchase good i in the first period if and only if the relevant sum of the surplus in the first period and the anticipated surplus in the second period is greater than the sum of surpluses from choosing $-i$ and zero. The consumer surplus from purchasing good i in the first period is thus given by:

$$\begin{aligned}\underbrace{(\mu - p_i)}_{\text{consumer surplus from trying } i} &+ \delta \underbrace{\int_{\max\left(\theta_i^{RO},\theta_i^{RS}\right)}^{\infty} (x - R_i^*)\, dF(x)}_{\text{consumer surplus from sticking with } i} \\ &+ \delta \underbrace{\int_{-\infty}^{\min\left(\theta_i^{SO},\theta_i^{RS}\right)} \left[E(\theta_{-i}|x) - S_{-i}^*\right] dF(x),}_{\text{consumer surplus from switching to } -i \text{ or to the outside option}} \end{aligned} \tag{7}$$

where $R_i^* \in \{R_i^m, R_i^s, R_i^n\}$, and $S_i^* \in \{S_i^m, S_i^s, S_i^n\}$.

Since, according to Lemma A.1, firms make the same second-period profit from repeat and switching consumers in the mass market equilibrium, they compete in the first period as in Bertrand competition. Hence, their first-period prices are equal to the marginal cost. This observation is similar to the positive correlation setting with the mass market equilibrium in the second period.

However, firms also make positive profits in the semi-niche and the niche market equilibria in the second period when consumer values are negatively correlated, whereas they make zero profit in these equilibria when values are positively correlated. This is again due to the observation that firms' market power arises from the ability to poach the competitor's consumers and the fact that, according to Lemma A.1, firms always make a positive profit from switching consumers in the second period when the correlation is negative.

Proposition A.1. *When $\rho < 0$:*

- *if and only if $M \geq \widehat{\theta}^{SO}$ and $\mu \in [c - \delta(SS^n - 2\pi_i^{Sn}) + \max(0, \delta(\mu - \widehat{S})), \infty)$, there exists an equilibrium where firm i makes a profit of $\delta\pi_i^{Sn} > 0$ by charging $p_i^n = c - \delta(\pi(M) - \pi_i^{Sn})$ in the first period and R_i^n, S_i^n as given by equations (2) and (3) in the second-period niche market equilibrium, where*

$$\begin{aligned} SS^n &= \int_M^{\infty} (x - c)\, dF(x) + \int_{-\infty}^{\widehat{\theta}^{SO}} [E(\theta_{-i}|x) - c]\, dF(x), \\ \pi_i^{Sn} &= -\frac{\rho F\left(\widehat{\theta}^{SO}\right)^2}{f\left(\widehat{\theta}^{SO}\right)}. \end{aligned}$$

- *if and only if $M \leq \widehat{\theta}^{SO}$ and $\mu \in [c - \delta(SS^s - 2\pi_i^{Ss}) + \max(0, \delta(\mu - S_i^s)), m]$, there exists a continuum of equilibria where firm i makes a profit of $\delta\pi_i^{Ss} > 0$ by charging $p_i^s = c - \delta[\pi(R_i^s) - \pi_i^{Ss}]$ in the first period and S_i^s, R_i^s satisfying equation (4) in the second-*

period semi-niche market equilibria, where

$$\begin{aligned}
SS^s &= \int_{R_i^s}^{\infty}(x-c)dF(x) + \int_{-\infty}^{R_i^s}\left[E(\theta_{-i}|x)-c\right]dF(x),\\
\pi_i^{Ss} &= \left[\rho R_i^s + (1-\rho)\mu - c\right]F(R_i^s),\\
\max(M,\mu) &\le R_i^s \le \min\left(m, \widehat{\theta}^{SO}\right);
\end{aligned}$$

- *if and only if $\mu \in [m, \infty)$, there exists an equilibrium where firm i makes a profit of $\frac{\delta(1-\rho)}{4f(\mu)} > 0$ by charging $p_i^m = c$ in the first period and R_i^m, S_i^m as given by equation (6) in the second-period mass market equilibrium.*

According to Theorem 1, firms make positive profits in the two-stage pricing game whenever there are switchers in the second period. With negative information spillovers, consumers do not leave the market after a bad experience as they expect to have a high valuation for the untried product. Hence, they switch to that product. Those who choose to leave the market in the second period, if any, are the consumers with intermediate valuations for the good they have tried. Therefore, since a positive mass of consumers switch products in the second period in all equilibria with $\rho \in [-1, 0)$, firms always make positive profits in this case.

As before, the relative profitability of switching consumers determines how intensely firms compete for market share in the first period. In the mass market case here, as when $\rho = 0$, switching and repeat customers are equally profitable. This breaks the link between first and second period competition and firms compete à la Bertrand in the first period, setting price equal to marginal cost.

A.3 Proofs for the negative information spillovers setting

Proof of Lemma A.1. We prove the lemma in three steps.

Step 1: The niche market equilibrium

We now prove that condition $M \ge \widehat{\theta}^{SO}$ is necessary and sufficient for the existence of a niche market equilibrium.

We start from sufficiency. We first show that when $M \ge \widehat{\theta}^{SO}$ holds, we have $M \ge \mu$. Suppose $M < \mu$, then $\mu - \frac{1}{2f(\mu)} - c > 0$. Given that $M \ge \widehat{\theta}^{SO}$ we also have $\widehat{\theta}^{SO} < \mu$, which implies that $\widehat{S} > \mu$, because $\widehat{\theta}^{SO} = \frac{\widehat{S}-(1-\rho)\mu}{\rho}$, and that, according to equation (3),

$$\widehat{S} < \frac{-\rho}{2f(\mu)} + c < \frac{1}{2f(\mu)} + c.$$

But then this implies $\mu < \frac{1}{2f(\mu)} + c$. A contradiction to $M < \mu$. Hence, we have $M \ge \mu$.

To show that the prices given in the lemma form an equilibrium, we need to rule out two sorts of profitable deviations. First, we show that among all prices such that $\theta^{RO} > \theta^{SO}$ holds, i.e., repeat and switching consumers' surplus curves cross at a point below the horizontal axis—similar to the niche market equilibrium—firms can do no better than charging M to repeat consumers and $\widehat{S}$ to switching consumers. Given that the other firm charges the equilibrium price, a firm makes the following profit from repeat consumers if it charges a price that satisfies $R_i > \widehat{\theta}^{SO}$, i.e., the surplus curves cross below the horizontal axis:

$$\lambda_i(R_i - c)\int_{R_i}^{\infty} dF(\theta). \tag{8}$$

From the first order condition of profit function (8) w.r.t R_i, any local deviations by firm i such that the two surplus curves still cross at a point below the horizontal axis must not be more profitable than M, because otherwise the first order derivative w.r.t R_i is not equal to zero. Hence, the optimal choice is that $R_i = M$ because the firm behaves as a monopoly in its share of the market. Alternatively, it makes the following profit from switching consumers if it charges a price that satisfies $S_i > \rho M + (1-\rho)\mu$, or put it differently, $\theta^{SO}_{-i} = \frac{S_i-(1-\rho)\mu}{\rho} < M = \theta^{RO}_{-i}$, i.e., S_i maintains a situation similar to the niche market equilibrium:

$$\lambda_{-i}(S_i - c)\int_{-\infty}^{\frac{S_i-(1-\rho)\mu}{\rho}} dF(\theta).$$

The optimal price can be found by the first order approach and is, in fact, given by $\widehat{S}$. Therefore, M and $\widehat{S}$ are the optimal prices to repeat and switching consumers among all prices where consumers' surplus curves cross below the horizontal axis.

Second, we show that prices that let consumer surplus curves cross above the horizontal axis are not profitable deviations. Suppose firm i deviates the price to repeat consumers to a $R_i < \widehat{\theta}^{SO}$ such that the two surplus curves cross above the horizontal axis, given that the rival firm chooses equilibrium prices. Then, the profit from repeat customers is given by

$$\lambda_i(R_i - c)\int_{\mu+\frac{R_i-\widehat{S}}{1-\rho}}^{\infty} dF(\theta), \tag{9}$$

then, by the first order approach, the optimal price must satisfy the following condition:

$$R_i^* = \frac{(1-\rho)\left[1 - F\left(\mu + \frac{R_i^*-\widehat{S}}{1-\rho}\right)\right]}{f\left(\mu + \frac{R_i^*-\widehat{S}}{1-\rho}\right)} + c.$$

Such an optimal price is unique, as the right side of the above equation is a decreasing function of R_i^* due to the MHR property. According to $R_i^* < \widehat{\theta}^{SO}$, we have

$$R_i^* > \frac{(1-\rho)[1-F(\widehat{\theta}^{SO})]}{f(\widehat{\theta}^{SO})} + c > \frac{(1-\rho)[1-F(m)]}{f(m)} + c = m > M \geq \widehat{\theta}^{SO},$$

which contradicts $R_i^* < \widehat{\theta}^{SO}$. Therefore, $R_i^* \geq \widehat{\theta}^{SO}$ and any price $R_i < \widehat{\theta}^{SO}$ is not a profitable deviation, because the first order derivative of the profit function (9) must be positive for any such price R_i. In other words, the firm would always prefer to increase the price whenever $R_i < \widehat{\theta}^{SO}$.

Now suppose firm i deviates to a price $S_i < \rho M + (1-\rho)\mu$ when the rival firm chooses the equilibrium prices, then the two surplus curves cross at a point above the horizontal axis. The profit from doing so is given by

$$\lambda_{-i}(S_i - c)\int_{-\infty}^{\mu + \frac{M-S_i}{1-\rho}} dF(\theta),$$

and hence, the optimal deviation price must satisfy

$$S_i^* = \frac{(1-\rho)F\left(\mu + \frac{M-S_i^*}{1-\rho}\right)}{f\left(\mu + \frac{M-S_i^*}{1-\rho}\right)} + c.$$

Since $S_i^* < \rho M + (1-\rho)\mu$ and $M \geq \mu$, we have

$$S_i^* > \frac{(1-\rho)F(M)}{f(M)} + c \geq \frac{(1-\rho)[1-F(M)]}{f(M)} + c > \frac{1-F(M)}{f(M)} + c = M \geq \mu.$$

But by $S_i^* < \rho M + (1-\rho)\mu$ and $M \geq \mu$, we have $S_i^* \leq \mu$. A contradiction. Hence, $S_i^* \geq \rho M + (1-\rho)\mu$, or, to put it differently, $\theta_{-i}^{SO} = \frac{S_i^* - (1-\rho)\mu}{\rho} \leq M = \theta_{-i}^{RO}$, must hold. Therefore, any price $S_i < \rho M + (1-\rho)\mu$ is not a profitable deviation, because it is less than the optimal price S_i^*. Hence, the firm would rather choose a price that satisfies $S_i \geq \rho M + (1-\rho)\mu$. We have completed the proof of sufficiency.

Now we turn to necessity. Suppose the prices in equations (2) and (3) form an equilibrium. Then it must be true that $M \geq \widehat{\theta}^{SO}$ because, otherwise, the two surplus curves would cross at a point above the horizontal axis.

Step 2: The semi-niche market equilibria

We start by proving sufficiency. Note that the conditions imply that

$$\widehat{\theta}^{SO} \equiv \frac{\widehat{S} - (1-\rho)\mu}{\rho} > R_i^s > \max(\mu, M). \tag{10}$$

Given that the other firm chooses the equilibrium prices, and suppose firm i charges a price $R_i > R_i^s$. The repeat consumers' surplus curve moves to the right and crosses the switching consumers' surplus curve at a point below the horizontal axis. In this case, the profit from repeat consumers is again given by equation (8) which is maximized at M. Since expression (10) requires that $R_i^s > \mu$ and the profit function above is inversely U-shaped due to our monotone hazard rate assumption, the first order derivative of firm i's profit from repeat consumers at $R_i > R_i^s > M$ must be negative, which implies that R_i is not a profitable deviation.

Suppose, instead, firm i deviates to a price $R_i' < R_i^s$. Then the two surplus curves cross at a point above the horizontal axis, and the profit from repeat consumers is given by

$$\lambda_i(R_i' - c)\int_{\mu + \frac{R_i' - S_{-i}^s}{1-\rho}}^{\infty} dF(\theta).$$

By the first order condition w.r.t R_i', the implicit best response function of firm i is given by

$$R_i'^* = \frac{(1-\rho)\left[1 - F\left(\mu + \frac{R_i'^* - S_{-i}^s}{1-\rho}\right)\right]}{f\left(\mu + \frac{R_i'^* - S_{-i}^s}{1-\rho}\right)}.$$

Note that the MHR assumption guarantees that the solution of the first order condition is indeed the optimal. Suppose $R_i'^* < R_i^s$, then according to the above best response function,

$$R_i'^* > \frac{(1-\rho)[1 - F\left(\mu + \frac{R_i^s - S_i^s}{1-\rho}\right)]}{f\left(\mu + \frac{R_i^s - S_i^s}{1-\rho}\right)} + c = \frac{(1-\rho)[1-F(R_i^s)]}{f(R_i^s)} + c > \frac{(1-\rho)[1-F(m)]}{f(m)} + c = m > R_i^s.$$

A contradiction. Thus, $R_i'^* \geq R_i^s$. Therefore, deviating to any price $R_i' < R_i^s$ is not profitable because $R_i' < R_i'^*$, which implies that the first order derivative of the profit w.r.t R_i' is positive.

Now, suppose firm i charges a price $S_i > S_i^s$ to attract firm $-i$'s consumers. Then, these switching consumers' surplus curve moves left and the two surplus curves cross at a point below the horizontal axis. Firm i's profit from switching consumers is then given by

$$\lambda_{-i}(S_i - c)\int_{-\infty}^{\frac{S_i - (1-\rho)\mu}{\rho}} dF(\theta)$$

and the optimal price is in fact $\widehat{S}$. According to equations (10) and (4), which imply that

$$\widehat{S} < \rho R^s_{-i} + (1-\rho)\mu = S^s_i < \mu,$$

we have $S_i < S^s_i < \widehat{S}$, and hence, the first order derivative of firm i's profit w.r.t S_i must be positive. Thus, any $S_i < S^s_i$ is not a profitable deviation.

Suppose instead, firm $-i$ deviates to a price $q'_{-i} < S^s_{-i}$. The switching consumers' surplus curve must now cross the repeat consumers' surplus curve at a point above the horizontal axis. The profit from the switching consumers is then given by

$$(S'_i - c)\int_{-\infty}^{\mu + \frac{R^s_{-i} - S'_i}{1-\rho}} dF(\theta).$$

By the first order approach, firm $-i$'s implicit best response function is given by

$$S'^*_i = \frac{(1-\rho)F\left(\mu + \frac{R^s_{-i} - S'^*_i}{1-\rho}\right)}{f\left(\mu + \frac{R^s_{-i} - S'^*_i}{1-\rho}\right)} + c.$$

Suppose $S'^*_i < \widehat{S}$, then

$$S'^*_i > \frac{(1-\rho)F(R^s_{-i})}{f(R^s_{-i})} + c > \frac{1-\rho}{2f(\mu)} + c \geq \mu.$$

However, equations (10) and (4) imply that $\widehat{S} < \mu$. A contradiction. Hence, $S'^*_i \geq \widehat{S}$ and it is not profitable to deviate to $S'_i < S^s_i$ since the first order derivative of firm i's profit from switching consumers w.r.t S'_i must be positive.

Step 3: The mass market equilibrium

We can solve for the equilibrium prices by taking first order conditions w.r.t R_i and S_i of the profit function of firm i in mass market equilibrium given below

$$\lambda_i(R_i - c)\int_{\theta_i^{RS}}^{+\infty} dF(\theta_i) + \lambda_{-i}(S_i - c)\int_{-\infty}^{\theta_{-i}^{RS}} dF(\theta_{-i}). \tag{11}$$

The profit function (11) is obtained by letting $\max(\theta_i^{RS}, \theta_i^{RO}) = \theta_i^{RS}$ and $\min(\theta_{-i}^{RS}, \theta_{-i}^{SO}) = \theta_{-i}^{RS}$ in (1). We then obtain the following necessary conditions for equilibrium prices:

$$R_i^m = \frac{(1-\rho)[1 - F(\theta_i^{RS})]}{f(\theta_i^{RS})} + c \quad \text{and} \quad S_i^m = \frac{(1-\rho)F(\theta_{-i}^{RS})}{f(\theta_{-i}^{RS})} + c. \tag{12}$$

We can find the cutoff types for any λ_i:

$$(1-\rho)(\theta_i^{RS}-\mu) = R_i^m - S_{-i}^m = \frac{(1-\rho)[1-2F(\theta_i^{RS})]}{f(\theta_i^{RS})}$$

hence,

$$\theta_i^{RS} \quad = \quad \mu + \frac{1-2F(\theta_i^{RS})}{f(\theta_i^{RS})} \tag{13}$$

Suppose $\theta_i^{RS} < \mu$, then $\frac{1-2F(\theta_i^{RS})}{f(\theta_i^{RS})} > 0$ and hence the RHS of equation (13) must be greater than μ, which is a contradiction. Suppose $\theta_i^{RS} > \mu$, then $\frac{1-2F(\theta_i^{RS})}{f(\theta_i^{RS})} < 0$ and hence the RHS of equation (13) must be less than μ. This means that it must be true that $\theta_i^{RS} = \mu$, where $i \in \{A, B\}$. Plugging $\theta^{RS} = \mu$ back into the two equations for R_i^m and S_i^m, respectively, we obtain the mass market equilibrium prices.

The above analysis is valid if and only if the prices indeed form a mass market equilibrium, i.e., $\theta^{RS} - R_i^m \geq 0$ is true. Hence, the necessary and sufficient condition for the existence of the above mass market equilibrium is:

$$\mu - \frac{1-\rho}{2f(\mu)} - c \geq 0, \tag{14}$$

or equivalently, $m \leq \mu$.

□

Proof of Corollary A.1. We prove the corollary in two steps.

Step 1. The fact that the niche market equilibrium exists implies that $M \geq \widehat{\theta}^{SO}$ and $M \geq \mu$.

Claim 1: When $\rho = -1$, $M = \widehat{S} \geq \mu$ and $\pi(M) = \pi_i^{Sn}$.

First, note that when $\rho = -1$, it is true that:

$$R_i^n \quad = \quad M = \frac{1-F(M)}{f(M)} + c, \tag{15}$$

$$S_i^n \quad = \quad \widehat{S} = \frac{F(2\mu - \widehat{S})}{f(2\mu - \widehat{S})} + c. \tag{16}$$

We can rewrite $\widehat{S}$ as

$$\widehat{S} = \frac{F(2\mu - \widehat{S})}{f(2\mu - \widehat{S})} + c = \frac{F(\mu - (\widehat{S} - \mu))}{f(\mu - (\widehat{S} - \mu))} + c = \frac{1 - F(\mu + (\widehat{S} - \mu))}{f(\mu + (\widehat{S} - \mu))} + c = \frac{1 - F(\widehat{S})}{f(\widehat{S})} + c$$

This implies that $R_i^n = \widehat{S} = M \geq \mu$ and $\pi(M) = \pi_i^{Sn}$ when $\rho = -1$.

Claim 2: When $\rho \in [-1, 0)$, $\widehat{S} \leq M$.

Take the first order derivative of equation (3)'s RHS w.r.t ρ:

$$\frac{d\widehat{S}}{d\rho} = -\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})} - \frac{d\left(\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})}\right)}{d\widehat{\theta}^{SO}} \frac{d\widehat{S}}{d\rho} - \frac{d\left(\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})}\right)}{d\widehat{\theta}^{SO}} \frac{\mu - \widehat{S}}{\rho},$$

and rearrange

$$\left[1 + \underbrace{\frac{d\left(\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})}\right)}{d\widehat{\theta}^{SO}}}_{>0}\right] \frac{d\widehat{S}}{d\rho} = -\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})} - \frac{d\left(\frac{F(\widehat{\theta}^{SO})}{f(\widehat{\theta}^{SO})}\right)}{d\widehat{\theta}^{SO}} \frac{\mu - \widehat{S}}{\rho}.$$

Hence, $\widehat{S}$ decreases in ρ as long as $\widehat{S} \geq \mu$. Since $\widehat{S} = M$ when $\rho = -1$ according to Claim 1, there exists a $\rho' > -1$ such that $\mu \leq \widehat{S} \leq M$ is true for any $\rho \in [-1, \rho']$.

For $\rho \in (\rho', 0)$, $\frac{d\widehat{S}}{d\rho} < 0$ is not necessarily true, as $\widehat{S} < \mu$ may hold. Denote by $\rho'' \equiv \sup\{\rho | \widehat{S} \geq \mu, \rho \geq \rho'\}$ the smallest ρ such that $\widehat{S} \geq \mu$. Then for any $\rho \in (\rho'', 0)$, we have $\widehat{S} \leq \mu$, because whenever $\widehat{S}$ increases to a level sufficiently close to μ, $\frac{d\widehat{S}}{d\rho} < 0$ holds and $\widehat{S}$ starts to decrease. Therefore, $\widehat{S} \leq M$ holds for all $\rho \in [-1, 0)$. See the following figure for this part of the proof.

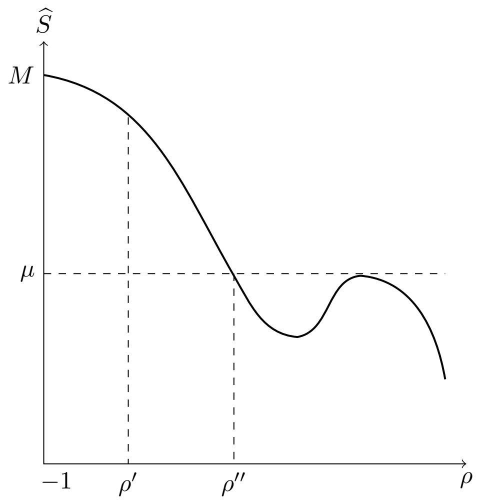


Figure 2: $\widehat{S} \leq M$ holds for all $\rho \in [-1, 0)$.

Step 2. According to Lemma A.1, the necessary and sufficient condition for the semi-niche market implies $R_i^s \geq \mu$. Since $S_i^s = \rho R_i^s + (1 - \rho)\mu$ and $\rho < 0$, it is true that $S_i^s \leq \mu$ and hence, $S_i^s \leq R_i^s$. □

Proof of Proposition A.1. This proof has three steps.

Step 1: The equilibrium when the second-period sub-game has the mass market equilibrium

Since the second-period prices and the profits are the same across repeat and switching consumers, firms are indifferent across any market share in the second period. Hence, they compete in the first period only for the first-period profit. This implies that they behave as if they are in Bertrand competition by charging the marginal cost c. In this case, consumer surplus must be non-negative since $\mu - c \geq m - c \geq S_i^m - c = R_i^m - c = \frac{1-\rho}{2f(\mu)} > 0$, i.e., consumers' first-period surplus is positive according to the condition for existence of the mass market equilibrium, and their second-period surplus must be non-negative. Furthermore, their first period surplus $\mu - c$ is greater than the surplus from purchasing only in the second period, given by $\mu - S_i^m = \mu - \frac{1-\rho}{2f(\mu)} - c$. Therefore, the necessary and sufficient condition is equivalent to the condition for existence of the mass market equilibrium in the second period.

Note that for each firm, selling only in the second period is weakly dominated by selling in both periods. To see why, consider firm i sells only in the second period while firm $-i$ sells in both periods. Then, it must hold that $\lambda_i = 0$ and firm i's profit is given by $(S_i^m - c) \cdot F(\mu) = \frac{\delta(1-\rho)}{4f(\mu)}$, which is equivalent to the equilibrium profit. On the other hand, consider firm i sells only in the second period while firm $-i$ does the same. Then, both firms make zero profit as they end up in Bertrand competition. Therefore, selling only in the second period is weakly dominated.

Step 2: The equilibrium when the second-period sub-game has the semi-niche market equilibria

Given any equilibrium in a semi-niche market, we can find the profit firm i makes from repeat consumers, $(R_i^s - c)[1 - F(R_i^s)]$, and from switching consumers, $[\rho R_i^s + (1-\rho)\mu - c]F(R_i^s)$, with $\max(M, \mu) \leq R_i^s \leq \min(m, \widehat{\theta}^{SO})$.

In Theorem 1 we have shown that each firm's profit in the whole game is given by $\delta\pi_i^{S*}$. The following derivation illustrates how this holds in the current context. When $\pi(R_i^s) \geq \pi_i^{Ss}$ firms make greater profit from repeat consumers than from switchers. Firms then compete for market share in the first period by lowering price to the point where $c - p_i^s = \delta(\pi(R_i^s) - \pi_i^{Ss})$.

Suppose firm i deviates to a higher price in the first period, then it avoids the loss $\lambda_i(c - p_i^s)$ in the first period, but its profit in the second period also decreases by $\delta\lambda_i(\pi(R_i^s) - \pi_i^{Ss})$. Suppose it deviates to a slightly lower price in the first period, then it obtains a gain of $\delta\lambda_{-i}(\pi(R_i^s) - \pi_i^{Ss})$ but also incurs an additional loss of $\lambda_{-i}(c - p_i^s)$ in the second period. Hence, neither deviating upwards or downwards is profitable.

Alternatively, when $\pi(R_i^s) < \pi_i^{Ss}$ firms make greater profit from switchers than from repeat consumers. Firms then compete for lower market share by increasing price up to $p_i^s - c = \delta(\pi_i^{Ss} - \pi(R_i^s))$. Suppose firm i deviates to a higher price in the first period, then it loses profits $\lambda_i(p_i^s - c)$ in the first period, but its profits in the second period increase by $\delta\lambda_i(\pi_i^{Ss} - \pi(R_i^s))$.

Suppose it deviates to a slightly lower price in the first period, then it makes an additional profit of $\lambda_{-i}(p_i^s - c)$, but loses $\delta\lambda_{-i}(\pi_i^{Ss} - \pi(R_i^s))$ in the second period. Hence, neither deviating upwards or downwards is profitable. Firm i's total discounted profit over the two periods is thus $\lambda_i(p_i^s - c) + \delta\left(\lambda_i\pi(R_i^s) + \lambda_{-i}\pi_i^{Ss}\right) = \delta\pi_i^{Ss}$.

Following a similar procedure as in step 1, it can be shown that selling only in the second period is weakly dominated for each firm.

Consumer surplus from purchasing good i in the first period is given by

$$\begin{aligned}
& (\mu - p_i^s) + \delta\left[\int_{R_i^s}^{\infty}(x - R_i^s)dF(x) + \int_{-\infty}^{R_i^s}\left[E(\theta_{-i}|x) - S_{-i}^s\right]dF(x)\right] \\
= \;& (\mu - c) + \delta(\pi(R_i^s) - \pi_i^{Ss}) + \delta\left[\int_{R_i^s}^{\infty}(x - c)dF(x) - \pi(R_i^s) + \int_{-\infty}^{R_i^s}\left[E(\theta_{-i}|x) - c\right]dF(x) - \pi_i^{Ss}\right] \\
= \;& (\mu - c) + \delta\left[\underbrace{\int_{R_i^s}^{\infty}(x - c)dF(x) + \int_{-\infty}^{R_i^s}\left[E(\theta_{-i}|x) - c\right]dF(x)}_{SS^s} - 2\pi_i^{Ss}\right]
\end{aligned}$$

Hence, the above consumer surplus is non-negative if and only if $\mu \geq c - \delta \cdot (SS^s - 2\pi_i^{Ss})$ and is greater than the surplus from purchasing only in the second period if and only if $\mu \geq c - \delta \cdot (SS^s - 2\pi_i^{Ss}) + \delta(\mu - S_i^s)$. Combining the two inequalities yields the lower bound of the necessary and sufficient condition given in the proposition.

Step 3: The equilibrium when the second-period sub-game is the niche market equilibrium

Suppose firm i deviates to a price higher than p_i^n. Then, profits in the first period increase by $\lambda_i(c - p_i^n)$ to 0. The discounted profits in the second period decrease by $\lambda_i\pi(M) - \lambda_i\pi_i^{Sn}$ to π_i^{Sn}. Since the gain is the same as the loss, it is not a profitable deviation. Suppose firm i deviates to a slightly lower price $p_i^n - \epsilon$, then the profits in the first period decrease by $\lambda_{-i}(c - p_i^n)$ to $c - p_i^n$. The discounted profits in the second period increase by $\lambda_{-i}\pi(M) - \lambda_{-i}\pi_i^{Sn}$. Thus, the gain is the same as the loss and it is not a profitable deviation.

Following a similar procedure as in Step 1, it can be shown that selling only in the second period is weakly dominated for each firm.

The last question to answer is whether consumers are willing to buy one of the products in the market in the first period given the equilibrium prices. Consumer surplus can be calculated by accounting for the equilibrium prices:

$$
\begin{aligned}
& (\mu - p_i^n) + \delta \left[\int_M^{\infty} (x - M) dF(x) + \int_{-\infty}^{\widehat{\theta}^{SO}} \left[E(\theta_{-i}|x) - \widehat{S} \right] dF(x) \right] \\
= \;& (\mu - c) + \delta(\pi(M) - \pi_i^{Sn}) + \delta \left[\int_M^{\infty} (x - c)\, dF(x) - \pi(M) + \int_{-\infty}^{\widehat{\theta}^{SO}} \left[E(\theta_{-i}|x) - c \right] dF(x) - \pi_i^{Sn} \right] \\
= \;& (\mu - c) + \delta \left[\underbrace{\int_M^{\infty} (x - c)\, dF(x) + \int_{-\infty}^{\widehat{\theta}^{SO}} \left[E(\theta_{-i}|x) - c \right] dF(x)}_{SS^n} - 2\pi_i^{Sn} \right]
\end{aligned}
$$

Hence, the above consumer surplus to is non-negative if and only if $\mu \geq c - \delta(SS^n - 2\pi_i^{Sn})$ and is greater than the surplus from purchasing only in the second period if and only if $\mu \geq c - \delta(SS^n - 2\pi_i^{Sn}) + \delta \cdot (\mu - \widehat{S})$. Combining the two inequalities yields the lower bound of the necessary and sufficient condition given in the proposition. □

B Competitive pricing of normal goods: A benchmark

In our main analysis, we provide comparative statics regarding how firm profits and consumer surplus vary with the degree of information spillovers between experience goods. In this appendix, we provide a benchmark analysis of two-period price competition in a normal goods market, where each consumer knows their value for each good at the start of the game.

Consumers know their willingness to pay for both normal goods, whereas firms only know the joint (bi-normal) distribution of the willingness to pay, denoted by $f(\theta_A, \theta_B; \rho)$. This model is equivalent to our main model except that consumers do not have to learn about their WTPs—they are fully aware of them from the start of the first period.

Denote the single-period price of firms by p_A, p_B. Then, the probability that a consumer purchases from firm A is

$$
\Pr\left(\theta_A - \theta_B \geq p_A - p_B, \theta_A - p_A \geq 0\right).
$$

Hence, the single-period profit of firm A is

$$
\begin{aligned}
& (p_A - c) \int_{p_A}^{+\infty} \int_{-\infty}^{\theta_A - p_A + p_B} f(\theta_A, \theta_B; \rho) d\theta_B d\theta_A \\
= \;& \left[\int_{p_B}^{+\infty} \int_{\theta_B - p_B + p_A}^{+\infty} f(\theta_A, \theta_B; \rho) d\theta_A d\theta_B + \int_{p_A}^{+\infty} \int_{-\infty}^{p_B} f(\theta_A, \theta_B; \rho) d\theta_B d\theta_A \right] (p_A - c)
\end{aligned}
$$

By taking the first order derivative, we find that the price in a symmetric equilibrium must satisfy:

$$p^* = \frac{\int_{p^*}^{+\infty}\int_{\theta_B}^{+\infty} f(\theta_A, \theta_B; \rho)d\theta_A d\theta_B + \int_{p^*}^{+\infty}\int_{-\infty}^{p^*} f(\theta_A, \theta_B; \rho)d\theta_A d\theta_B}{\int_{p^*}^{+\infty} f(\theta_B, \theta_B; \rho)d\theta_B + \int_{-\infty}^{p^*} f(p^*, \theta_B; \rho)d\theta_B} + c. \tag{17}$$

Denote the firm's single-period profit in the symmetric equilibrium by π^*. Note that in the sub-game perfect equilibrium of the two-period normal goods pricing game, firms play the single-period Nash equilibrium in each period. Thus, each firm makes a profit that equals $(1+\delta)\pi^*$ in the two-stage pricing game. So, for sufficiently small δ, firms in the current setting make a greater profit than in the main model, where profits are $\delta\pi^{S*}$. The intuition is as follows: In the main model, firms compete in the first period for differential profits in the second period. When firms are sufficiently impatient, they are less concerned about future differential profits, and focus on the first period profit, which is, hence, competed down to close to zero. In the normal goods benchmark, firms are already differentiated in the first period, which is what allows them to earn a profit in that period.

C Uniform pricing

Now we turn to uniform pricing, where firms cannot price discriminate between consumers based on their purchasing history. Denote by U_i^k with $k \in \{m, n\}$ the uniform price of firm i in a mass ($k = m$) or a niche ($k = n$) market equilibrium. Note that under uniform pricing, we have $R_i = S_i = U_i$ in the profit function of each firm.

C.1 Without information spillovers, i.e., $\rho = 0$

For the mass market equilibrium, firm i's second-period profit function from charging U_i is given by:

$$\begin{aligned} & (U_i - c) \cdot \left[\lambda_i \int_{\theta_i^{RS}}^{+\infty} dF(\theta_i) + \lambda_{-i} \int_{-\infty}^{\theta_{-i}^{RS}} dF(\theta_{-i})\right] \\ = \quad & (U_i - c) \cdot \left\{\lambda_i[1 - F(\mu + U_i - U_{-i})] + \lambda_{-i} F(\mu + U_{-i} - U_i)\right\}. \end{aligned}$$

When $\lambda_i = 1$ holds, the equilibrium prices are determined by

$$\begin{aligned} U_i - c &= \frac{1 - F(\mu + U_i - U_{-i})}{f(\mu + U_i - U_{-i})} \\ U_{-i} - c &= \frac{F(\mu + U_i - U_{-i})}{f(\mu + U_i - U_{-i})} \end{aligned}$$

Note that $U_i = U_{-i} = \frac{1}{2f(\mu)} + c$ satisfies the conditions. This is the unique solution because of the monotonic hazard rate property.

When $\lambda_i = \frac{1}{2}$ holds, the first order condition yields

$$U_i - c = \frac{\lambda_i \cdot [1 - F(\mu + U_i - U_{-i})] + \lambda_{-i} \cdot F(\mu + U_{-i} - U_i)}{\lambda_i \cdot f(\mu + U_i - U_{-i}) + \lambda_{-i} \cdot f(\mu + U_{-i} - U_i)},$$

and the second order condition holds whenever $\lambda_i = \lambda_{-i} = \frac{1}{2}$. The equilibrium prices for $\lambda_i = \frac{1}{2}$ must satisfy $U_i = U_{-i}$, hence,

$$U_i - c = \frac{1}{2f(\mu)}.$$

On the other hand, in a niche market equilibrium, firms make the monopoly profit in the second period. The second-period equilibrium is characterized below:

Proposition C.1. *Suppose $\rho = 0$ and only uniform pricing is allowed, in the second period there exists:*

- *a niche market equilibrium where $U_i^n = M$ if and only if $\mu < M$;*
- *a mass market equilibrium where $U_i^m = \frac{1}{2f(\mu)} + c \geq M$ if and only if $\mu \geq M$.*

An interesting observation we can make from Proposition C.1 is that the semi-niche market equilibrium no longer exists. This suggests that it is behavior-based price discrimination that enables the existence of the semi-niche market equilibrium.

Now we turn to the first period. From our analysis in the main text, it is clear that when firms anticipate a niche market equilibrium in the second period, they compete intensely for first-period market share, resulting in zero overall profit. Since obtaining any market share $\lambda_i \in (0,1)$ yields firm i a second-period profit of $\lambda_i \pi^M$, firms drop first-period prices for greater market share until the first-period loss equals the discounted second-period profit.

On the other hand, in the mass market equilibrium, if $\lambda_i = 1$, then the profits of firm i and its rival are given by

$$\pi_i^u = \frac{1}{2f(\mu)} \cdot [1 - F(\mu)] = \frac{1}{4f(\mu)} \text{ and } \pi_{-i}^u = \frac{1}{2f(\mu)} \cdot F(\mu) = \frac{1}{4f(\mu)}.$$

If, instead, $\lambda_i = \frac{1}{2}$, each firm's profit is again $\frac{1}{4f(\mu)}$. Since a firm obtains the same profit, $\frac{1}{4f(\mu)}$, regardless of whether it has zero market share, half the market, or the entire market, the first-period competition replicates a Bertrand competition and the equilibrium price equals marginal cost $p = c$.

Proposition C.2 illustrates these findings. The proof of the proposition is omitted.

Proposition C.2. *Suppose $\rho = 0$ and only uniform pricing is allowed. In the two-stage pricing game, there exist:*

- *an equilibrium in which each firm charges $p^* = c - \delta\pi^M$ in the first period and $U_i^n = M$ in the second period, and yields a zero profit over the two periods if and only if $\mu < M$;*
- *an equilibrium in which each firm charges $p^* = c$ in the first period and $U_i^m = \frac{1}{2f(\mu)} + c$ in the second period, and yields a profit of $\frac{\delta}{4f(\mu)}$ over the two periods if and only if $\mu \geq M$.*

C.2 With information spillovers, i.e., $\rho \in (0, 1)$

C.2.1 Second-period equilibria

Given that $\rho > 0$ and that firms cannot price discriminate based on purchase history, the second-period profit function of firm i is given by

$$\pi_i = (U_i - c)\left[\lambda_i \int_{\max\left(\mu + \frac{U_i - U_{-i}}{1-\rho}, U_i\right)}^{+\infty} dF(\theta_i) + \max\left(0, \lambda_{-i} \int_{\frac{U_i - (1-\rho)\mu}{\rho}}^{\mu + \frac{U_{-i} - U_i}{1-\rho}} dF(\theta_{-i})\right)\right] \tag{18}$$

where U_i is the uniform price of firm i.

In a mass market equilibrium, the first maximization takes $\mu + \frac{U_i - U_{-i}}{1-\rho}$ and the second maximization takes the nonzero term. Take the first order derivative with respect to U_i, we have

$$U_i - c = \frac{\lambda_i \left[1 - F\left(\mu + \frac{U_i - U_{-i}}{1-\rho}\right)\right] + \lambda_{-i}\left[F\left(\mu + \frac{U_{-i} - U_i}{1-\rho}\right) - F\left(\frac{U_i - (1-\rho)\mu}{\rho}\right)\right]}{\frac{\lambda_i}{1-\rho} f\left(\mu + \frac{U_i - U_{-i}}{1-\rho}\right) + \frac{\lambda_{-i}}{1-\rho} f\left(\mu + \frac{U_{-i} - U_i}{1-\rho}\right) + \frac{\lambda_{-i}}{\rho} f\left(\frac{U_i - (1-\rho)\mu}{\rho}\right)} \tag{19}$$

The second order condition holds for $\lambda_i = \frac{1}{2}$ because of the monotonic hazard rate property.

Proposition C.3 illustrates the symmetric mass market equilibrium in the second period under uniform pricing. As in the $\rho = 0$ setting illustrated by Proposition C.1, there no longer exists any semi-niche market equilibrium in the uniform pricing setting when $\rho \in (0, 1)$.

Proposition C.3. *Suppose $\rho \in (0, 1)$, $\lambda_i = \frac{1}{2}$, and only uniform pricing is allowed. In the second period, there exist:*

- *a niche market equilibrium where firm i charges $U_i^n = M$ if and only if $\mu \leq M$;*
- *a mass market equilibrium where firm i charges*

$$U_i^m = \frac{\rho(1-\rho)[1-F(\theta_{-i}^{SO})]}{2\rho f(\mu) + (1-\rho)f(\theta_{-i}^{SO})} + c, \quad \textit{where } \theta_{-i}^{SO} = \frac{U_i^m - (1-\rho)\mu}{\rho},$$

if and only if

$$\mu - \frac{1-\rho}{2f(\mu)}\frac{\rho}{1+\rho} - c \geq 0. \tag{20}$$

Note that $\mu > M$ implies that condition (20) holds, suggesting that the mass market equilibrium exists whenever the niche market equilibrium does not exist. Correspondingly, violation of expression (20) implies that $\mu \leq M$, suggesting the existence of the niche market equilibrium whenever the mass market equilibrium does not exist. When μ satisfies condition (20) and $\mu \leq M$, both the mass and the niche market equilibrium exist.

The implication of Proposition C.3 is that the necessary and sufficient condition for the mass market equilibrium in the uniform pricing setting, i.e., condition (20), is more restrictive than the condition for the mass market equilibrium in the price discrimination setting given by Proposition 1 in the main text. To see why, note that (20) implies

$$\mu \geq \frac{1-\rho}{2f(\mu)}\frac{\rho}{1+\rho} + c. \tag{21}$$

Since the right hand side of the inequality is greater than c, condition (20) implies $\mu > c$ whenever $\rho > 0$. On the other hand, the condition given in Proposition 1 is $\mu \geq \frac{c-\rho m}{1-\rho}$, the right hand side equals $c - \frac{\rho(m-c)}{1-\rho}$, which is less than the marginal cost c when $\rho > 0$. Therefore, since firms make a positive second-period profit only in the mass market equilibrium, behavior based price discrimination allows firms to make a positive second-period profit from some minor innovations, whereas uniform pricing only allows firms to make a positive profit from major innovations.

Another implication of Proposition C.3 is that given that firms cannot price discriminate, information spillovers expand the range of innovation from which firms make positive profits. According to Proposition C.1, when $\rho = 0$ a mass market equilibrium exists if and only if $\mu \geq M$, whereas condition (20) reveals that when $\rho > 0$ a mass market equilibrium can exist even if $\mu < M$.

C.2.2 Sub-game perfect equilibrium of the two-stage pricing game

Now we characterize the first-period prices in a symmetric equilibrium in which the firms split the market equally. We show that a continuum of equilibria exists, when firms are in the mass market equilibrium in the second period.

Denote by $\lambda_i \cdot \pi_i^R(\lambda_i)$ firm i's profit function from repeat customers given that its first-period market share is λ_i and firms cannot price discriminate. We can write $\pi_i^R(\lambda_i)$ as

$$\pi_i^R(\lambda_i) = (U_i - c)\left[1 - F\left(\mu + \frac{U_i - U_{-i}}{1-\rho}\right)\right],$$

where U_i and U_{-i} are determined by equation (19), and hence, are functions of λ_i. Similarly, denote by $\lambda_{-i}\pi_i^S(\lambda_i)$ firm i's profit from switching customers given that its first-period market share is λ_i and firms cannot price discriminate. Thus,

$$\pi_i^S(\lambda_i) = (U_i - c)\left[F\left(\mu + \frac{U_{-i} - U_i}{1-\rho}\right) - F\left(\frac{U_i - (1-\rho)\mu}{\rho}\right)\right]$$

Denote by p^* the first-period price in the symmetric equilibrium. Firm i yields the following overall profit over the two periods in the symmetric equilibrium:

$$\frac{1}{2}(p^* - c) + \delta\left[\frac{1}{2}\pi_i^R(\frac{1}{2}) + \frac{1}{2}\pi_i^S(\frac{1}{2})\right].$$

Deviating to a price higher than p^* loses the first-period market share and a profit of $\frac{1}{2}(p^* - c)$ and loses the second-period profit of $\frac{1}{2}\left[\pi_i^R(\frac{1}{2}) + \pi_i^S(\frac{1}{2})\right]$, but yields a payoff of $\pi_i^S(0)$ in the second period. In a symmetric equilibrium, this must not be profitable: $\frac{1}{2}(p^* - c) + \delta\left[\frac{1}{2}\pi_i^R(\frac{1}{2}) + \frac{1}{2}\pi_i^S(\frac{1}{2})\right] \geq \delta\pi_i^S(0)$, implying

$$p^* \geq \underline{p} \equiv c - \delta\left[\pi_i^R(\frac{1}{2}) - \pi_i^S(0) + \pi_i^S(\frac{1}{2}) - \pi_i^S(0)\right].$$

Deviating to a price slightly lower than p^* yields the entire market for firm i and an additional profit of $\frac{1}{2}(p^* - c)$ in the first period and a profit of $\pi_i^R(1)$ in the second period, while losing $\frac{1}{2}\pi_i^R(\frac{1}{2}) + \frac{1}{2}\pi_i^S(\frac{1}{2})$ in the second period. In the symmetric equilibrium, this again must not be profitable, hence, $\frac{1}{2}(p^* - c) + \delta\pi_i^R(1) \leq \delta\left[\frac{1}{2}\pi_i^R(\frac{1}{2}) + \frac{1}{2}\pi_i^S(\frac{1}{2})\right]$, implying

$$p^* \leq \overline{p} \equiv c - \delta\left[\pi_i^R(1) - \pi_i^R(\frac{1}{2}) + \pi_i^R(1) - \pi_i^S(\frac{1}{2})\right].$$

Therefore, the necessary condition for the existence of a symmetric equilibrium is $\overline{p} \geq \underline{p}$, i.e.,

$$\pi_i^R(1) - \pi_i^R(\frac{1}{2}) \leq \pi_i^S(\frac{1}{2}) - \pi_i^S(0). \tag{22}$$

This is not the sufficient condition because consumers may not be willing to purchase in the first period given equilibrium prices. Proposition C.4 characterizes the first-period equilibrium.

Proposition C.4. *Suppose $\rho \in (0,1)$ and only uniform pricing is allowed. In the two-stage pricing game, there exist:*

- *an equilibrium in which each firm charges $p^* = c - \delta\pi^M$ in the first period and $U_i^n = M$ in the second period, and makes a zero overall profit, if and only if $\mu \leq M$;*
- *an equilibrium in which each firm charges $p^* \in [\underline{p}, \overline{p}]$, where*

$$\begin{aligned} \underline{p} &= c - \delta\left[\pi_i^R(\frac{1}{2}) - \pi_i^S(0) + \pi_i^S(\frac{1}{2}) - \pi_i^S(0)\right], \\ \overline{p} &= c - \delta\left[\pi_i^R(1) - \pi_i^R(\frac{1}{2}) + \pi_i^R(1) - \pi_i^S(\frac{1}{2})\right], \end{aligned}$$

and makes a profit no less than $\delta\pi_i^S(0)$, if and only if conditions (20), (22), and the following condition holds:

$$\mu \geq c - (SS^u - 2\pi_i^S(\frac{1}{2})) + \max[0, \delta(\mu - U_i^m)], \tag{23}$$

where

$$SS^u \equiv \int_{\mu}^{+\infty}(x-c)dF(\theta_i) + \int_{\frac{U_i^m-(1-\rho)\mu}{\rho}}^{\mu}[\rho x + (1-\rho)\mu - c]dF(\theta_{-i}).$$

Suppose firms are in the mass market equilibrium in the second period and the equilibrium first-period price equals the lower bound, $\underline{p}$. Then, each firm's overall profit equals $\delta\pi_i^S(0)$, which is consistent with Theorem 1. If the equilibrium price equals the upper bound, then each firm's overall profit equals $\delta\left[\pi_i^R(\frac{1}{2}) + \pi_i^S(\frac{1}{2}) - \pi_i^R(1)\right]$ which is no less than $\delta\pi_i^S(0)$ according to condition (22). The overall profit of each firm must be between the two extremes because it is monotonically increasing in p^*.

Uniform pricing results in higher prices to switching and lower prices to repeat customers than price discrimination based on purchase history. The effect of uniform pricing on consumer welfare is ambiguous since repeat customers are better off, whereas switching customers are worse off.

C.3 Proofs of uniform pricing

Proof of Proposition C.1. We consider each equilibrium in turn:

Step 1: Niche market equilibrium

In the niche market equilibrium, there are no switching customers. Each firm finds it optimal to charge the monopoly price M to its repeat consumers.

When the type $\theta_i = M$ switches to $-i$, she has a surplus of $\mu - M$. Then, a sufficient condition for the existence of the niche market equilibrium is that $\mu < M$, as type $\theta_i = M$ and all types with higher valuations must find it unprofitable to switch. Suppose firm i deviates to a lower price U. If $U \geq \mu$ holds, then no consumers switch. If instead $U < \mu$, then firm i's profit function now becomes

$$(U - c)\left[\lambda_{-i}\int_{-\infty}^{\mu-U+M} dF(\theta_{-i}) + \lambda_i \int_{U}^{+\infty} dF(\theta_i)\right].$$

The first order derivative w.r.t U yields

$$\begin{aligned}
& \lambda_{-i}F(\mu - U + M)\left[1 - \frac{(U-c)f(\mu - U + M)}{F(\mu - U + M)}\right] + \lambda_i[1 - F(U)]\left[1 - \frac{(U-c)f(U)}{1 - F(U)}\right] \\
> \quad & \{\lambda_{-i}F(\mu - U + M) + \lambda_i[1 - F(U)]\}[1 - 2(U-c)f(\mu)] \\
> \quad & 0
\end{aligned}$$

where the first inequality is due to $U < \mu$ and $U < M$ and the second inequality is due to $U < \mu < \frac{1}{2f(\mu)} + c$.

Now we turn to necessity. Suppose the niche market equilibrium exists, then switching must yield negative surplus, i.e., $\mu - M < 0$.

Step 2: Mass market equilibrium

For the mass market equilibrium, firm i's profit function from charging U_i is given by:

$$\begin{aligned}
& (U_i - c)\cdot\left[\lambda_i \int_{\theta_i^{RS}}^{+\infty} dF(\theta_i) + \lambda_{-i}\int_{-\infty}^{\theta_{-i}^{RS}} dF(\theta_{-i})\right] \\
= \quad & (U_i - c)\cdot\{\lambda_i[1 - F(\mu + U_i - U_{-i})] + \lambda_{-i}F(\mu + U_{-i} - U_i)\}.
\end{aligned}$$

Taking the first order derivative w.r.t U_i, we have

$$\begin{aligned}
& \lambda_i \cdot [1 - F(\mu + U_i - U_{-i})] + \lambda_{-i}\cdot F(\mu + U_{-i} - U_i) \\
& - U_i - c\,[\lambda_i \cdot f(\mu + U_i - U_{-i}) + \lambda_{-i}\cdot f(\mu + U_{-i} - U_i)] = 0 \qquad (24)
\end{aligned}$$

We look for symmetric equilibrium where $U_i = U_{-i}$, then the first order condition (24) implies that $U_i^m = \frac{1}{2f(\mu)} + c$.

Since the marginal consumer has a WTP of μ, the mass market equilibrium exists if and only if $\mu - U_i^m \geq 0$, i.e.,

$$\mu - \frac{1}{2f(\mu)} - c \geq 0,$$

which is equivalent of $\mu \geq M$. □

Proof of Proposition C.3. In a niche market equilibrium, it must be true that the marginal consumer leaves the market, i.e., $\theta_i^{RS} - U_i^n < 0$. Since no consumer switches in the niche market equilibrium, each firm must charge the monopoly price M. No consumer wishes to switch if and only if the surplus of the consumer with $\theta_i = M$ from switching is non-positive: $\rho M + (1-\rho)\mu - M \leq 0$, i.e., $\mu \leq M$.

In a mass market equilibrium, it must hold that the marginal consumer switches instead of leaving the market. To prove sufficiency, we show that when the condition given in the proposition holds, it must be true that $\theta_i^{RS} - U_i^m \geq 0$. First, note that the equilibrium is symmetric, hence $\theta_i^{RS} = \mu$. Thus,

$$\begin{aligned}
\theta_i^{RS} - U_i^m &= \mu - U_i^m \\
&= \mu - \frac{\rho(1-\rho)[1 - F(\theta_{-i}^{SO})]}{2\rho f(\mu) + (1-\rho) f(\theta_{-i}^{SO})} - c \\
&\geq \mu - \frac{\rho(1-\rho)}{1+\rho} \frac{1 - F(\theta_{-i}^{SO})}{f(\theta_{-i}^{SO})} - c, \qquad (25)
\end{aligned}$$

where the inequality is due to $f(\theta_{-i}^{SO}) \leq f(\mu)$. If $\theta_{-i}^{SO} > \mu$, then $\frac{1-F(\theta_{-i}^{SO})}{f(\theta_{-i}^{SO})} \leq \frac{1}{2f(\mu)}$, which then implies that (25) satisfies

$$\mu - \frac{\rho(1-\rho)}{1+\rho} \frac{1 - F(\theta_{-i}^{SO})}{f(\theta_{-i}^{SO})} - c \geq \mu - \frac{1-\rho}{2f(\mu)} \frac{\rho}{1+\rho} - c \geq 0.$$

This implies that $\mu \geq U_i^m$. However, given that $\theta_{-i}^{SO} > \mu$, we have $U_i^m \equiv \rho\theta_{-i}^{SO} + (1-\rho)\mu > \mu$. A contradiction. Hence, $\mu \geq \theta_{-i}^{SO}$ must hold, which implies that $U_i^m \equiv \rho\theta_{-i}^{SO} + (1-\rho)\mu \leq \mu$.

Now we turn to necessity. Given that $\mu - U_i^m \geq 0$,

$$\begin{aligned}
\mu - \frac{1-\rho}{2f(\mu)} \frac{\rho}{1+\rho} - c \geq \mu - \frac{\frac{1}{2}\rho(1-\rho)}{2f(\mu)} \frac{\rho}{1-\rho} - c &\geq \mu - \frac{\rho(1-\rho)[1 - F(\theta_{-i}^{SO})]}{2\rho f(\mu) + (1-\rho) f(\theta_{-i}^{SO})} - c \\
&\geq \mu - U_i^m \geq 0 \qquad (26)
\end{aligned}$$

holds. Hence, we have proved the second part of the proposition.

Note that whenever $\mu > M$ holds such that the niche market does not exist, we have

$$\mu - \frac{1}{2f(\mu)} - c > 0 \;\Rightarrow\; \mu - \frac{1}{2f(\mu)}\frac{\rho}{1+\rho} - c > 0 \tag{27}$$
$$\Rightarrow\; \mu - \frac{1}{2f(\mu)}\frac{\rho}{1+\rho} + \frac{\rho^2}{1+\rho} - c > 0, \tag{28}$$

which implies condition (20) holds, i.e., a mass market equilibrium exists. Alternatively, when condition (20) is violated, $\mu < M$ must be true. □

Proof of Proposition C.4. For the first part, suppose firm i deviates to a price above $p^* = c - \delta\pi^M$. Then it loses all its market share, and both first- and second-period profit are zero. Alternatively, suppose the firm deviates to a slightly lower price, then it obtains the entire market, incurs a loss of $\delta\pi^M$ in the first period, and obtains a profit of the same amount in the second period. Hence, neither deviation is profitable. Since the firms make zero profit in the equilibrium, consumers must obtain non-negative surplus, which equals the social surplus. Hence, consumers are willing to purchase in the first period.

For the second part, note that condition (22) guarantees that deviating away from p^* is not profitable. Here, we verify that consumers are willing to purchase in the first period. Consumer surplus, given the equilibrium prices, is:

$$\begin{aligned}
& (\mu - p^*) + \delta\left[\int_\mu^\infty (x - U_i^m)\,dF(x) + \int_{\frac{U_i^m-(1-\rho)\mu}{\rho}}^{\mu} \left[E(\theta_{-i}|x) - U_{-i}^m\right] dF(x)\right] \\
= \; & (\mu - c) + \delta\left[\int_\mu^\infty (x - U_i^m)\,dF(x) + \pi_i^R(\frac{1}{2}) - \pi_i^S(\frac{1}{2}) + \int_{\frac{U_i^m-(1-\rho)\mu}{\rho}}^{\mu} \left[E(\theta_{-i}|x) - U_{-i}^m\right] dF(x)\right] \\
= \; & (\mu - c) + \delta\left[\underbrace{\int_\mu^\infty (x - c)\,dF(x) + \int_{\frac{U_i^m-(1-\rho)}{\rho}}^{\mu} \left[E(\theta_{-i}|x) - c\right] dF(x)}_{SS^u} - 2\cdot\pi_i^S(\frac{1}{2})\right].
\end{aligned}$$

In equilibrium, the above consumer surplus must be no less than the maximum of zero and the consumer surplus from purchasing only in the second period, i.e., $\delta(\mu - U_i^m)$. Hence, the necessary and sufficient condition for the consumer to be willing to follow the equilibrium and purchase one of the products is given by:

$$\mu - c \geq -\delta\cdot\left(SS^u - 2\cdot\pi_i^S(\frac{1}{2})\right) + \max\left[0, \delta(\mu - U_i^m)\right]$$

where $SS^u = \int_{\mu}^{\infty} (x-c)\, dF(x) + \int_{\frac{U_i^m-(1-\rho)\mu}{\rho}}^{\mu} [E(\theta_{-i}|x)-c]\, dF(x)$. $\square$

D Price elasticities

D.1 When $\rho \in (0,1)$

Denote by D_i^R the demand from repeat consumers of firm i, and by D_i^S the demand from switching consumers of firm i. In general,

$$\begin{aligned} D_i^R &= \lambda_i \int_{\max\left(\theta_i^{RS},\theta_i^{RO}\right)}^{+\infty} dF(\theta_i) \\ D_i^S &= \max\left(0, \lambda_{-i}\int_{\theta_{-i}^{SO}}^{\theta_{-i}^{RS}} dF(\theta_{-i})\right). \end{aligned}$$

In a mass market equilibrium, $D_i^R = \lambda_i \int_{\theta_i^{RS}}^{+\infty} dF(\theta_i)$ and $D_i^S = \lambda_{-i}\int_{\theta_{-i}^{SO}}^{\theta_{-i}^{RS}} dF(\theta_{-i})$. Note that D_i^R is a function of R_i^m and S_{-i}^m as θ_i^{RS} is determined according to $\mu + (R_i^m - S_{-i}^m)/(1-\rho)$, and that D_i^S is a function of R_{-i}^m and S_i^m as θ_{-i}^{RS} is determined by $\mu + (R_{-i}^m - S_i^m)/(1-\rho)$ and θ_{-i}^{SO} is determined by $(S_i^m - (1-\rho)\mu)/\rho$.

The relevant elasticities are then given by

$$E_{R_i^m}^{D_i^R} = \frac{dD_i^R}{dR_i^m}\frac{R_i^m}{D_i^R} = -\frac{\lambda_i f(\theta_i^{RS})}{1-\rho}\cdot\frac{R_i^m}{\lambda_i[1-F(\theta_i^{RS})]} = -\frac{R_i^m}{R_i^m - c} \tag{29}$$

$$E_{S_{-i}^m}^{D_i^R} = \frac{dD_i^R}{dS_{-i}^m}\frac{S_{-i}^m}{D_i^R} = \frac{\lambda_i f(\theta_i^{RS})}{1-\rho}\cdot\frac{S_{-i}^m}{\lambda_i[1-F(\theta_i^{RS})]} = \frac{S_{-i}^m}{R_i^m - c} \tag{30}$$

$$E_{R_{-i}^m}^{D_i^S} = \frac{dD_i^S}{dR_{-i}^m}\frac{R_{-i}^m}{D_i^S} = \frac{\lambda_i f(\theta_{-i}^{RS})}{1-\rho}\cdot\frac{R_{-i}^m}{\lambda_i[F(\theta_{-i}^{RS})-F(\theta_{-i}^{SO})]} = \frac{f(\theta_{-i}^{RS})R_{-i}^m}{(1-\rho)[F(\theta_{-i}^{RS})-F(\theta_{-i}^{SO})]} \tag{31}$$

$$E_{S_i^m}^{D_i^S} = \frac{dD_i^S}{dS_i^m}\frac{S_i^m}{D_i^S} = -\lambda_{-i}\left[\frac{f(\theta_{-i}^{RS})}{1-\rho} + \frac{f(\theta_{-i}^{SO})}{\rho}\right]\cdot\frac{S_i^m}{\lambda_{-i}[F(\theta_{-i}^{RS})-F(\theta_{-i}^{SO})]} = -\frac{S_i^m}{S_i^m - c}. \tag{32}$$

In a niche market equilibrium, $D_i^S = 0$ and hence, the relevant elasticities are zero: $E_{R_{-i}^n}^{D_i^S} = E_{S_i^n}^{D_i^S} = 0$. We also have $D_i^R = \lambda_i[1-F(\theta_i^{RO})]$ and hence,

$$E_{R_i^n}^{D_i^R} = \frac{dD_i^R}{dR_i^n}\frac{R_i^n}{D_i^R} = -\frac{\lambda_i f(R_i^n)R_i^n}{\lambda_i[1-F(R_i^n)]} = -\frac{M}{M-c} \tag{33}$$

$$E_{S_{-i}^n}^{D_i^R} = 0. \tag{34}$$

In a semi-niche market equilibrium, the demand curves are kinked:

$$D_i^R = \begin{cases} \lambda_i[1-F(R_i)], \text{ if } R_i \le R_i^s \\ \lambda_i[1-F(\theta_i^{RS})], \text{ if } R_i > R_i^s \end{cases} = \begin{cases} \lambda_i[1-F(R_i^S)], \text{ if } S_{-i} \ge S_{-i}^s \\ \lambda_i[1-F(\theta_i^{RS})], \text{ if } S_{-i} < S_{-i}^s \end{cases} \tag{35}$$

$$D_i^S = \begin{cases} 0, \text{ if } R_{-i} \leq R_{-i}^s \\ \lambda_{-i}[F(\theta_{-i}^{RS}) - F(\theta_{-i}^{SO})], \text{ if } R_{-i} > R_{-i}^s \end{cases} = \begin{cases} 0, \text{ if } S_i \geq S_i^s \\ \lambda_{-i}[F(\theta_{-i}^{RS}) - F(\theta_{-i}^{SO})], \text{ if } S_i < S_i^s \end{cases} \tag{36}$$

Hence, the corresponding elasticities are given by

$$E_{R_i}^{D_i^R} = \frac{dD_i^R}{dR_i}\frac{R_i}{D_i^R} = \begin{cases} -\frac{f(R_i)R_i}{1-F(R_i)}, \text{ if } R_i \leq R_i^s \\ -\frac{f(\theta_i^{RS})R_i}{(1-\rho)[1-F(\theta_i^{RS})]}, \text{ if } R_i > R_i^s \end{cases} \tag{37}$$

$$E_{S_{-i}}^{D_i^R} = \frac{dD_i^R}{dS_{-i}}\frac{S_{-i}}{D_i^R} = \begin{cases} 0, \text{ if } S_{-i} \geq S_{-i}^s \\ \frac{f(\theta_i^{RS})S_{-i}}{(1-\rho)[1-F(\theta_i^{RS})]}, \text{ if } S_{-i} < S_{-i}^s \end{cases} \tag{38}$$

$$E_{R_{-i}}^{D_i^S} = \frac{dD_i^S}{dR_{-i}}\frac{R_{-i}}{D_i^S} = \begin{cases} 0 \text{ if } R_{-i} \leq R_{-i}^s \\ \frac{f(\theta_{-i}^{RS})R_{-i}}{(1-\rho)[F(\theta_{-i}^{RS})-F(\theta_{-i}^{SO})]}, \text{ if } R_{-i} > R_{-i}^s \end{cases} \tag{39}$$

$$E_{S_i}^{D_i^S} = \frac{dD_i^S}{dS_i}\frac{S_i}{D_i^S} = \begin{cases} 0, \text{ if } S_i \geq S_i^s \\ -\frac{\left[\frac{f(\theta_{-i}^{RS})}{1-\rho}+\frac{f(\theta_{-i}^{SO})}{\rho}\right]S_i}{F(\theta_{-i}^{RS})-F(\theta_{-i}^{SO})} \text{ if } S_i < S_i^s \end{cases} \tag{40}$$

D.2 When $\rho \in (-1, 0]$

The demand function of firm i consists of two parts, the demand from repeat and from switching consumers:

$$D_i^R = \lambda_i \int_{\max(\theta_i^{RS}, \theta_i^{RO})}^{+\infty} dF(\theta_i) \tag{41}$$

$$D_i^S = \lambda_{-i} \int_{\infty}^{\max(\theta_{-i}^{RS}, \theta_{-i}^{SO})} dF(\theta_{-i}). \tag{42}$$

In the mass market equilibrium, we have $D_i^R = \lambda_i \int_{\theta_i^{RS}}^{+\infty} dF(\theta_i)$ and $D_i^S = \lambda_{-i} \int_{-\infty}^{\theta_{-i}^{RS}} dF(\theta_i)$. Then, the price elasticities are:

$$\begin{aligned} E_{R_i^m}^{D_i^R} &= \frac{dD_i^R}{dR_i^m}\frac{R_i^m}{D_i^R} = -\frac{\lambda_i f(\theta_i^{RS})}{1-\rho}\frac{R_i^m}{\lambda_i[1-F(\theta_i^{RS})]} = -\frac{R_i^m}{R_i^m - c} \\ E_{S_{-i}^m}^{D_i^R} &= \frac{dD_i^R}{dS_{-i}^m}\frac{S_{-i}^m}{D_i^R} = \frac{\lambda_i f(\theta_i^{RS})}{1-\rho}\frac{S_{-i}^m}{\lambda_i[1-F(\theta_i^{RS})]} = \frac{S_{-i}^m}{R_i^m - c} \\ E_{S_i^m}^{D_i^S} &= \frac{dD_i^S}{dS_i^m}\frac{S_i^m}{D_i^S} = -\frac{\lambda_{-i} f(\theta_{-i}^{RS})}{1-\rho}\frac{S_i^m}{\lambda_{-i}F(\theta_{-i}^{RS})} = -\frac{S_i^m}{S_i^m - c} \\ E_{R_{-i}^m}^{D_i^S} &= \frac{dD_i^S}{dR_{-i}^m}\frac{R_{-i}^m}{D_i^S} = \frac{\lambda_{-i} f(\theta_{-i}^{RS})}{1-\rho}\frac{R_{-i}^m}{\lambda_{-i}F(\theta_{-i}^{RS})} = \frac{R_{-i}^m}{S_i^m - c} \end{aligned}$$

We have shown that in equilibrium $R_i^m = R_{-i}^m = S_i^m = S_{-i}^m$, thus, $E_{R_i^m}^{D_i^R} = E_{S_i^m}^{D_i^S} = -E_{S_{-i}^m}^{D_i^R} =$

$-E^{D_i^S}_{R^m_{-i}}$.

Alternatively, in the niche market equilibrium, $D_i^R = \lambda_i \int_{\theta_i^{RO}}^{+\infty} dF(\theta_i)$ and $D_i^S = \lambda_{-i} \int_{-\infty}^{\theta_{-i}^{SO}} dF(\theta_i)$. The elasticities are given by:

$$
\begin{aligned}
E^{D_i^R}_{R_i^n} &= \frac{dD_i^R}{dR_i^n}\frac{R_i^n}{D_i^R} = -\lambda_i f(\theta_i^{RO})\frac{R_i^n}{\lambda_i[1-F(\theta_i^{RO})]} = -\frac{M}{M-c} \\
E^{D_i^R}_{S^n_{-i}} &= \frac{dD_i^R}{dS^n_{-i}}\frac{S^n_{-i}}{D_i^R} = 0 \\
E^{D_i^S}_{S_i^n} &= \frac{dD_i^S}{dS_i^n}\frac{S_i^n}{D_i^S} = \frac{\lambda_i f(\theta_{-i}^{SO})}{\rho}\frac{S_i^n}{\lambda_{-i}F(\theta_{-i}^{SO})} = -\frac{S_i^n}{S_i^n-c} \\
E^{D_i^S}_{R^n_{-i}} &= \frac{dD_i^S}{dR^n_{-i}}\frac{R^n_{-i}}{D_i^S} = 0.
\end{aligned}
$$

In a semi-niche equilibrium, the demand curves are kinked:

$$
D_i^R = \begin{cases} \lambda_i[1-F(R_i)], \text{ if } R_i \geq R_i^s \\ \lambda_i[1-F(\theta_i^{RS})], \text{ if } R_i < R_i^s \end{cases} = \begin{cases} \lambda_i[1-F(R_i^s)], \text{ if } S_{-i} \geq S_{-i}^s \\ \lambda_i[1-F(\theta_i^{RS})], \text{ if } S_{-i} < S_{-i}^s \end{cases} \tag{43}
$$

$$
D_i^S = \begin{cases} \lambda_{-i}F\left(\frac{S_i^n-(1-\rho)\mu}{\rho}\right), \text{ if } R_{-i} \geq R_{-i}^s \\ \lambda_{-i}F(\theta_{-i}^{RS}), \text{ if } R_{-i} < R_{-i}^s \end{cases} = \begin{cases} \lambda_{-i}F\left(\frac{S_i-(1-\rho)\mu}{\rho}\right), \text{ if } S_i \geq S_i^s \\ \lambda_{-i}F(\theta_{-i}^{RS}), \text{ if } S_i < S_i^s \end{cases} \tag{44}
$$

Hence, the corresponding elasticities are given by

$$
E^{D_i^R}_{R_i} = \frac{dD_i^R}{dR_i}\frac{R_i}{D_i^R} = \begin{cases} -\frac{f(R_i)R_i}{1-F(R_i)}, \text{ if } R_i \geq R_i^s \\ -\frac{f(\theta_i^{RS})R_i}{(1-\rho)[1-F(\theta_i^{RS})]}, \text{ if } R_i < R_i^s \end{cases} \tag{45}
$$

$$
E^{D_i^R}_{S_{-i}} = \frac{dD_i^R}{dS_{-i}}\frac{S_{-i}}{D_i^R} = \begin{cases} 0, \text{ if } S_{-i} \geq S_{-i}^s \\ \frac{f(\theta_i^{RS})S_{-i}}{(1-\rho)[1-F(\theta_i^{RS})]}, \text{ if } S_{-i} < S_{-i}^s \end{cases} \tag{46}
$$

$$
E^{D_i^S}_{R_{-i}} = \frac{dD_i^S}{dR_{-i}}\frac{R_{-i}}{D_i^S} = \begin{cases} 0, \text{ if } R_{-i} \geq R_{-i}^s \\ \frac{f(\theta_{-i}^{RS})R_{-i}}{(1-\rho)F(\theta_{-i}^{RS})}, \text{ if } R_{-i} < R_{-i}^s \end{cases} \tag{47}
$$

$$
E^{D_i^S}_{S_i} = \frac{dD_i^S}{dS_i}\frac{S_i}{D_i^S} = \begin{cases} \frac{f\left(\frac{S_i-(1-\rho)\mu}{\rho}\right)S_i}{\rho F\left(\frac{S_i-(1-\rho)\mu}{\rho}\right)}, \text{ if } S_i \geq S_i^s \\ -\frac{f(\theta_{-i}^{SO})S_i}{(1-\rho)F(\theta_{-i}^{RS})}, \text{ if } S_i < S_i^s \end{cases} \tag{48}
$$