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Bargaining under the Illusion of Transparency†

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This paper studies bargaining with noncommon priors where the buyer projects and exaggerates the probability that her private information may leak to the seller. Letting the buyer name her price first, raises the seller's payoff above his payoff from posting a price. In seller-offer bargaining, projection implies a partial reversal of classic Coasian comparative static results. Weakening price commitment can benefit the seller and, as long as the relative speed at which imaginary information versus offers arrive does not converge to zero too quickly, frictionless bargaining converges to a fast haggling process which allows the seller to extract all surplus from trade. Bargaining under common prior transparency is instead slow and becomes equivalent to simply waiting. The comparative static predictions are consistent with experimental evidence. (*JEL* C78, D82)

> *Was it possible they heard not? Almighty God!—no, no! They heard!—they suspected!—they knew!—they were making a mockery of my horror!—* [...] *I could bear those hypocritical smiles no longer!* [...] *"Villains!" I shrieked, "dissemble no more! I admit the deed!—tear up the planks! here, here!—It is the beating of his hideous heart!"*
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> —Poe, "The Tell-Tale Heart" (1843)

Haggling as a mode of price formation has been of considerable interest to economists for long. In the classic problem, underlying monopoly theory, a seller with a known cost offers an object to an interested, but privately informed buyer: a local dealer wants to sell a new car to a client but is uncertain how much the client is willing to pay; a firm wishes to hire an expert to complete a task of fixed value, but does not know the expert's reservation wage. Is it more beneficial for the seller—the uninformed party—to haggle, that is, to establish a price via frictionless bargaining instead of committing to a price or a price schedule over time, or adopting a no haggling policy?

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The standard assumption in bargaining theory is that the buyer correctly understands that her information is private vis-à-vis the seller. The key result under this assumption is a no-haggling result. The optimal way for the uninformed seller to sell the object requires ex ante commitment to a price or price sequence without flexibility (posted prices), and typically corresponds to the seller making a single take-it-or-leave-it offer, e.g., Myerson (1981), Riley and Zeckhauser (1983), Skreta (2006).[1]

The fundamental Coase conjecture provides a powerful illustration of the value of price commitment. Even if the seller has the power to make all offers when bargaining, a buyer who fully appreciates the uncertainty the seller faces about her valuation can take advantage of the fact that she knows more. Since at any given price, a higher buyer type is more likely than a lower one to buy, the uninformed seller becomes more pessimistic every time the buyer rejects the offer. In the absence of nonnegotiable prices, the buyer anticipating such future pessimism limits the seller's ability to charge higher prices in the present and hurts his ability to maintain his full-commitment payoff. As friction between offers goes to zero, bargaining without commitment forces the seller to sell immediately at the lowest possible reservation value of the buyer, e.g., Fudenberg, Levine, and Tirole (1985); Gul, Sonnenschein, and Wilson (1986).

The Coasian logic of bargaining, and the no-haggling results more generally, are key insights of modern microeconomics, and provide a strategic rationale for the use of nonnegotiable (posted) prices. Empirical observation suggests, however, that in important cases, well approximated by the classic setup, sellers choose to haggle even if they could just post a price; such as in the sale of new cars. This is true despite the fact that survey evidence indicates that most buyers would prefer not to haggle (Auto Shopper Influence Study 2016).[2] Furthermore, the resulting price variation need not be explained by the readily observable buyer characteristics, e.g., Goldberg (1996).

In contrast to the standard assumption, evidence from the field and the lab (e.g., Samuelson and Bazerman 1984) suggests that people, also when bargaining, act as if they failed to fully appreciate informational differences. Instead, the typical person projects information, and too often acts as if others are also be able to act on her information. When bargaining, a privately informed buyer in effect thinks that she is more transparent than she truly is. This illusion then possibly shapes bargaining, as bargaining outcomes may depend critically on just how much information the buyer *believes* her actions actually conceal.

This paper incorporates information projection into sequential bargaining and explores the robustness of the above results to its presence. I show that this mistake provides a powerful channel through which bargaining, relative to posting a price may greatly benefit the uninformed seller. Haggling not only hurts the seller's monopoly profit less, but becomes a process that allows him to improve upon what

[1] The optimal auction literature, e.g., Myerson (1981), studies the optimal selling mechanism and shows, that in the case of a single buyer, the optimal auction is a posted price.

[2] The Auto Shopper Influence Study (2016) finds that 64 percent of those surveyed would prefer not to haggle. Similarly, DealerSocket's Dealership Action Report (2016), using Google Consumer Surveys, finds not only that 81 percent of the surveyed buyers do not enjoy the car buying process, but that 57 percent would prefer to be given the "best price" offer instead of negotiating over the price.

he can achieve through any single-offer scheme or posted price (sequence), thus achieve a payoff which exceeds that from adopting the unbiased Bayesian optimal mechanism.

The paper also considers the possibility that the buyer's private information may actually leak to the seller. I show that common priors about such transparency, or, the buyer's underestimation of her transparency, the illusion of privacy, lead to realistic, but sharply different forms of behavior and conclusions. In the absence of the buyer's exaggeration of her transparency, a generalized Coasian conjecture holds, that is, the seller's revenue from bargaining converges to the value of the seller's outside option. The seller prefers commitment to a simple no-haggling policy over frictionless bargaining. Instead, if the buyer exaggerates her transparency, the seller may realize a strictly higher revenue when bargaining without commitment than with any no-haggling policy or bargaining with friction. The results shed novel light on empirical phenomena, the value of price commitment, and have implications for a number of applications.

Section I describes the setup. The privately informed buyer thinks that whenever she interacts with the seller, he might come to condition his strategy on her valuation. Her perception is potentially exaggerated. The seller is sophisticated: he has a correct perception about such a leakage process, but understands the buyer's perception. The players then bargain given such heterogeneous priors.

Section II applies the model to a simple setting with two periods. The unbiased Bayesian no-haggling result implies a silence and a Coasian property of bargaining: the seller's dynamic revenue—whether the parties alternate or the seller makes all offers—is bounded from above by making a single take-it-or-leave-it (TIOLI) offer. Section II then considers the presence of an ex ante chance that, before the bargaining starts, the salesman may privately infer the buyer's valuation based on some observable characteristic. As long as the buyer is well calibrated about this chance, the seller's ex ante expected revenue is still highest given a TIOLI offer following the parties' initial encounter. In contrast, both of these properties are violated if the buyer suffers from the illusion of transparency and exaggerates this leakage probability.

Consider now an alternating-offer protocol in the above context. For the seller to gain from bargaining over making a nonnegotiable offer, the buyer needs to reveal her private information. In equilibrium, a high type only reveals herself if she believes that this way she secures a surplus greater than the return from pretending to be a lower type. The former is given by the buyer's bargaining rent, that is, the difference between the lowest offer that the seller would still accept today, as opposed to holding the buyer to her valuation after some delay in the next round. The latter is given by the buyer's (perceived) information rent, that is, the price difference between what the two types offer to pay multiplied by the (perceived) probability that the seller is uninformed. Thus, if the buyer is well calibrated about this probability, she reveals herself only if her bargaining rent is higher than her information rent. In turn, the seller's expected payoff must be lower than his payoff from a nonnegotiable offer upon meeting the buyer. In short, it is not beneficial for the seller to let the buyer name her own price.

In contrast, a biased buyer exaggerates the probability that the seller will be informed. Hence, a high type is willing to reveal herself in exchange for a smaller bargaining rent than in the unbiased case. As a consequence, relative to making a nonnegotiable offer, the seller is able to learn the buyer's type at a cost that is smaller than the value of the information gained. Although there still needs to be sufficient delay between offers so that haggling is nominally costly for the seller, if a counteroffer may occur only after sufficient, but not too much delay, the seller can capitalize on the buyer's mistaken beliefs. Here, letting the buyer speak is a successful sale tactic.

Sections III and IV then consider classic seller-offer bargaining with a continuum of types. I first describe the consequence of the illusion of transparency per se, then allow for to the buyer's private information to actually leak to the seller and consider general heterogeneous priors about such a leakage process.

Section III considers the case in which the buyer's perception of leakage is purely an illusion in the classic infinite-horizon context. Given unbiased expectations, as friction between offers decreases, the seller drops the set of prices he offers and gradually loses his ability to extract revenue. In contrast, given a stationary acceptance strategy of the buyer, I show that if the buyer mistakenly believes that there is any nonzero chance with which the seller figures out her type in each bargaining round, the opposite happens. Dynamic bargaining improves the seller's revenue, and not only allows the uninformed seller to improve upon his monopoly payoff but as the seller loses commitment, his revenue smoothly converges to the full surplus from trade.

Information projection introduces a countervailing force that alters not only the unbiased Coasian limit result, but comparative static predictions well away from the limit as well. This is true despite the fact that leakage is entirely fictional and the seller is fully aware of this. In particular, the interaction of the force due to information projection and the classic Coasian force leads to a U-shaped comparative static result.

To provide intuition, consider the unbiased case. Under sequentially rational price discrimination, the uninformed seller knows that he will become more pessimistic after each rejection and will want to reduce the price. This negative selection leads to a temptation problem. Given a positive interest rate, the price reductions of the seller's future acting selves reduce the seller's current acting self's equilibrium incentive to hold a higher price. As the frequency of offers increases, the seller cannot appropriately limit the price gradient, that is, the extent to which he drops the price from one round to the next. Specifically, as offers become more frequent, the overall extent to which the seller drops the price over a fixed amount of interest-bearing delay, i.e., the speed of bargaining, increases. As the cost of delay between offers vanishes, this speed converges to an unbounded amount. This is the Coasian force. In equilibrium, the unbiased buyer optimally hides behind her private information. In turn, as offers become more frequent, the seller needs to transfer more and more rent to the buyer to be able to sell. This eventually forces the seller's continuation value to zero and he must sell immediately at the lowest possible valuation of the buyer. Commitment to a price ex ante, the durability of prices over time, solves this problem.

The reversal of this classic result under projection holds despite the fact that the buyer's fictional belief in leakage leaves the above Coasian force fully intact. The uninformed seller understands that he will never become informed and the return on simply waiting is null. He still becomes more pessimistic after each rejection and is subject to the same temptation problem as in the unbiased case. In turn, the speed of bargaining is unchanged. As friction decreases, this speed continues to accelerate, still tending to an unbounded amount. The buyer understands that the uninformed seller acts this way. Yet, at the same time, she slightly underestimates his sequential pessimism. Since a higher type has more to lose from leakage than a lower one, this represents a new force the seller may take advantage of.

Crucially, both the fully intact Coasian force and the new force due to the buyer's projection get stronger as friction decreases. As friction vanishes, the speed of bargaining tends to an unbounded amount; that is, the seller still tends to run down any initial price to null over a unit of interest-losing delay. Simultaneously, the buyer's illusory perception of the probability with which the seller eventually learns her type over a unit of interest-losing delay tends to one. Neither force is globally dominant and whether one or the other accelerates more as the frequency of offers increases depends on the frequency of offers itself. Their complex interaction leads to a nonmonotone comparative static result. Initially, the Coasian force gets stronger faster. As the frequency of offers decreases, the level of the seller's prices drops and his revenue decreases. Nevertheless, no matter how tiny projection is, there is always some strictly positive level of friction where the relative strength of these two accelerating forces switches. Past this point, while the uninformed seller still cannot reduce the acceleration of the speed of bargaining, he now starts to smoothly increase his initial price offer as the frequency of offers increases further. His revenue also starts to increase. While, as the seller loses commitment, the Coasian force still gets unbounded, by haggling smooth and fast, that is in a way that the expected interest-loss before an agreement is reached tends to zero, his revenue converges to the full surplus from trade, V_F.

In the main specification described above, the buyer's illusory perception was linked to strategic interactions; that is, the relative speed at which imaginary information versus offers arrived was constant in the bargaining friction. This is consistent with the buyer's mistaken fear that there is some nonzero chance that the seller may read her mind, or infer her private information from the way she acts or responds, each time she and the seller interact.

Section IIIA then considers the alternative specification where the buyer's illusory perception is linked to the amount of costly delay that passes; e.g., she may mistakenly fear that the seller is doing some background research on her, and the likelihood that this succeeds is purely a matter of how much costly delay passes. Here, the speed at which imaginary information arrives is now independent of the speed at which offers arrive, that is, from how often the parties actually negotiate. The impact of projection is now strongest when offers are rare, and vanishes as offers become arbitrarily frequent. The comparative static is reversed. Initially, the force due to projection outweighs the Coasian force. Weakening price commitment first raises the seller's revenue above the one attainable by any posted price sequence. Once offers become sufficiently frequent, however, the Coasian force takes over; the seller's revenue starts to decrease, and the classic limit follows.

Figure 1 below illustrates the impact of decreasing friction, weakening price commitment, on the seller's revenue (i) in the unbiased case (*blue line*), (ii) when the buyer's fictional belief in leakage is linked to strategic interactions (*red line*), and (iii) when the buyer's fictional belief is linked purely to the amount of costly

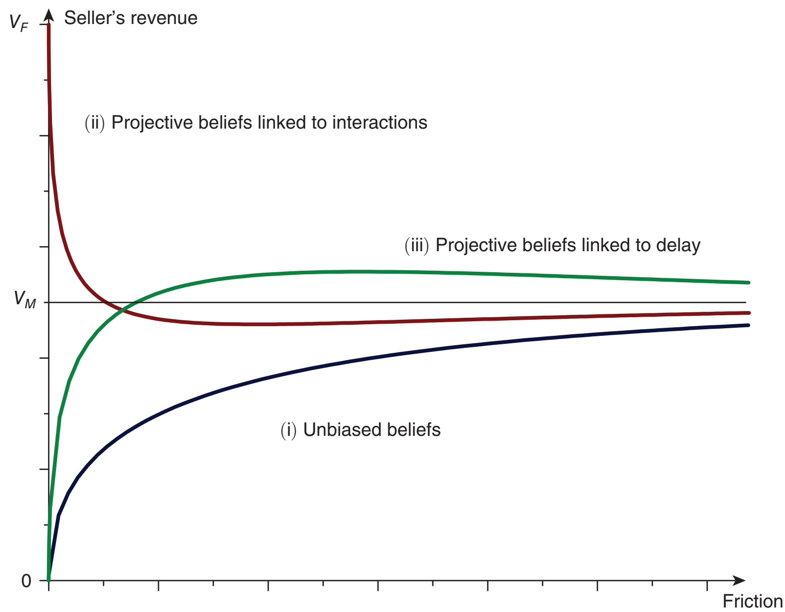

FIGURE 1

delay that passes (*green line*). In the infinite friction limit, full price commitment, these all converge to the revenue from the seller-optimal posted price, the static monopoly rent V_M.[3]

Section IIIB compares the predictions to relevant evidence. The data show that while the speed of bargaining matches the unbiased predictions, crucially, the two key comparative statics are the reverse of what is predicted given unbiased expectations. These findings are instead consistent with the predictions of the model where players' beliefs are shaped by the illusion of transparency.

Section IV then considers general heterogeneous priors about an actual leakage process. I first show that true transparency operates through a fundamentally and observably different channel than the buyer's fear of transparency per se. The latter, impacts only the level of the seller's offers. True transparency instead changes the speed of bargaining. The seller's belief in a benefit from waiting relaxes his dynamic temptation problem and leads to a slowdown in bargaining. The buyer's fear of leakage and true leakage now have opposite comparative static implications: the former increases, the latter decreases the speed of bargaining, but the overall effect is always negative. Furthermore, even as friction vanishes, the speed of bargaining remains bounded, and in the limit the seller effectively just waits and makes a last-minute concession when leakage arrives.

Finally, Section IV also considers frictionless bargaining with heterogeneous and vanishing priors; that is, the case where the perceived and true probabilities of leakage from one offer to the next may vanish as friction, between offers vanishes. I

[3] In line (ii), the buyer's illusory perception of the probability with which the seller figures out her type from one round to the next is 0.24. In line (iii), the buyer's illusory perception of the probability with which the seller figures out her type over a unit of interest-losing delay is 0.87.

allow for different speeds at which this (perceived or true) probability converges to zero. These different speeds correspond to different extents to which (perceived) leakage is linked to strategic interactions versus the passing of costly delay. I show that in the absence of the illusion of transparency, irrespective of how fast this probability converges to zero, the generalized Coasian conjecture holds; the seller's revenue corresponds to the value of effectively just waiting. In turn, the seller prefers ex ante commitment to a simple no-haggling policy over bargaining without commitment. In the presence of the illusion of transparency, this no longer holds. In particular, I establish a discontinuity. If the buyer believes that this probability converges to zero faster than the square-root of friction, the generalized Coasian result continues to follow. If she believes that it converges to zero slower than that, bargaining without commitment continues to correspond to a gradual but fast haggling process that dominates any no-haggling policy, bargaining with friction, or posted price schedule.

Haggling, despite being associated with a host of transaction costs, was not only common historically, but is present in important modern day settings. The model is able to account for empirically relevant forms of bargaining behavior, such as an inflexible seller who makes a last-minute concession before an agreement is reached, or a seller who haggles gradually but fast. It sheds light on when, ceteris paribus, one may expect to see one or the other. Furthermore, it shows that information projection provides a structured channel through which a sales person, even if he could just post a price or adopt a no-haggling policy, may greatly benefit from haggling with a privately informed buyer.

A. *Evidence*

Evidence for information projection comes from a variety of domains. Key manifestations include the curse of knowledge, e.g., Camerer, Loewenstein, and Weber (1989); Newton (1990); Birch and Bloom (2007); or the illusion of transparency and the spotlight effect, e.g., Gilovich, Savitsky, and Medvec (1998, 2000), who show that the average person exaggerates the probability with which others can read her preferences and detect her lies.

In the domain of strategic bargaining, the classic bargaining study of Samuelson and Bazerman (1984)—henceforth, SB—provides strong evidence for information projection. They study bilateral bargaining with one-sided private information and common-values, the canonical acquiring-a-company game. The informed party (the target) values the firm at q and the uninformed party (the acquiring company) values it at $q + 30$. Only the target knows the realization of q drawn from a commonly known uniform distribution over $[0, 100]$. All of this is publicly announced.

SB consider the case in which the target makes a TIOLI price offer. The most common strategy corresponds to the fully revealing strategy of $p(q) = q + 30$. The uninformed party accepts all prices with essentially the same probability, except for very high values of p. In turn, the target provides zero ex post rent to the acquirer, but also significantly underbids (underbluffs) relative to what his payoff maximizing strategy would be given the uninformed acquirer's actual acceptance behavior. As SB (1984, p.19) write, "this naive behavior would be appropriate if the [uninformed] acquirer had the same information as the target—which the targets knew was not the case. In short, a target made price offers as if the acquirer had identical information as itself." The pattern above clearly contradicts the implications of a perfect Bayesian equilibrium (PBE) since neither systematic underbidding nor uniform separation can hold. However, this pattern is consistent with the presence of significant information projection.[4] The other preference specifications considered by SB provide further support for information projection.

Keysar, Ginzel, and Bazerman (1995) provide further direct evidence. They study the same setting as above, except the acquirer's ex post valuation is now $1.5q$. In their Experiment 2, they elicit incentivized predictions from external observers, Kellogg MBA students, about the uninformed acquirer's behavior. These external observers were informed about the state q and were told the structure of the game including very clearly that the acquirer was uninformed.

The external observer then had to estimate whether or not a given TIOLI price offer made by the target's agent $(p = 85)$, submitted in writing with no communication between the players, would be accepted by the uninformed acquirer. In the first treatment, accepting this offer was ex post strictly beneficial $(q = 80)$ to the uninformed acquirer. In the second treatment, it was ex post strictly detrimental $(q = 20)$ to the uninformed acquirer. Although the uninformed acquirer faced identical information across the two treatments, 58 percent of the external observers predicted acceptance in the first, while only 30 percent predicted acceptance in the second treatment.

Keysar, Ginzel, and Bazerman additionally study a third treatment with contradictory information. Specifically, the external observer is told that the target knows that the true value is low (20), but that the target's agent actually engaging in bargaining with the acquirer is told that the true value is in fact high (80). Given such contradictory information, this setting cannot be directly mapped into bargaining with one-sided private information. The authors find that the external observer predicts acceptance very similarly to that in the first treatment (59 percent). This remains inconsistent with the unbiased case, the offer is always the same. Yet at the same time, it is consistent with the parsimonious assumption that the external observer always adopts the perspective of the (mis)informed target's agent actually engaging in bargaining, and projects that onto the uninformed acquirer.[5]

Finally, by exogenously varying whether players are symmetrically or asymmetrically informed, Danz, Madarász, and Wang (2015) provide further support for the assumptions employed in this paper. By directly eliciting first- and second-order beliefs about the beliefs of others, they find not only that better-informed subjects

[4] Madarász (2016), introducing the notion of projection equilibrium for a general class of Bayesian games, provides a formal equilibrium account of these statements.

[5] An alternative moral explanation, also entertained by Keysar, Ginzel, and Bazerman (1995), is in terms of a "belief in a just world," whereby the external observer falsely believes that the uninformed acquirer is more likely to reject an offer by the target's agent if it is a "lie" rather than made in "good faith"—*given* the target's agent information. A plausible variant of this alternative, however, would again need to invoke information projection. First, here what is "just" would need to be insensitive to, by the observer known, the payoff consequences; acceptance is beneficial in the first, and equally harmful in the second and third treatments for the acquirer. Thus, what is just would need to relate instead to the "intentions" of the target's agent irrespective of payoffs. This concern aside, invoking such an explanation, without also invoking information projection, would need to mean that the external observer thinks that the acquirer's hand is guided by some external force of which the acquirer, since he is always uninformed, is unaware of. A more plausible variant of this moral explanation, as also discussed by the authors, is that the external observer falsely believes that whether the target's agent's offer is made in "good faith" or is a "lie," somehow becomes "transparent" to the uninformed acquirer; i.e., this alternative again relies on information projection.

heavily project their private information onto lesser-informed others, but that lesser-informed subjects anticipate a great deal of such projection onto them.[6]

B. *Related Literature*

Among those already mentioned, Riley and Zeckhauser (1983) characterize the optimal selling mechanism under commitment in a dynamic setting and show that it corresponds to making a single TIOLI offer. Skreta (2006) extends this to the case where the seller can commit only within a period. She shows that the optimal mechanism is a seller-offer protocol with a revenue bounded by the static monopoly profit.[7]

A large literature investigates the robustness of the Coasian logic of bargaining without commitment: e.g., Bond and Samuelson (1984) consider good depreciation; Sobel (1991) and Fuchs and Skrzypacz (2010) consider the stochastic arrival of new buyers. Fuchs and Skrzypacz (2010), similarly in a discrete time setting, study the atomless stationary frictionless limit and show that a generalized Coasian conjecture holds. Given a public and common prior Poisson arrival process, they show that "the seller payoff [from bargaining] equals what he can achieve by simply awaiting an arrival," that is, the value of the seller's outside option. In Section IV, I return to a discussion of the link between common prior action-independent true (as opposed to fictional) leakage and their results. Ausubel and Deneckere (1989) show that in nonstationary equilibria, the seller can attain any revenue bounded from above by the static monopoly payoff. Board and Pycia (2014) further show that, given a privately known outside option for the buyer, the uniqueness of the no-haggling equilibrium where the seller sells only at the static monopoly price. In contrast to these settings, frictionless bargaining under projection becomes a smooth and fast haggling process which boosts the seller's revenue above both the value of his outside option and his static monopoly payoff.

Feinberg and Skrzypacz (2006) study an infinite horizon seller-offer game with two buyer types. In their setting, the buyer also faces initial uncertainty about whether or not the seller is informed. They maintain common knowledge of the information structure and show that such uncertainty produces delay even in the limit. Bargaining, however, is still a process that leads to a lower ex ante expected revenue than a TIOLI offer following the seller's meeting with the buyer.[8] Finally, Yildiz (2003), given perfect information about payoffs, studies alternating-offer bargaining with heterogeneous (and, overall, too optimistic) priors about the distribution of proposing rights. He provides conditions where, despite such overoptimism, immediate agreement follows, but he also describes settings with inefficient delay. This paper differs in that proposing rights are always common knowledge, but is similar in that it also employs the standard equilibrium approach with noncommon priors about the distribution of nature's moves.

[6] Less structured but also supportive evidence for information projection in the context of negotiations comes from, e.g., Vorauer and Ross (1999); Van Boven, Gilovich, and Medvec (2003).

[7] See also, e.g., Stokey (1979), Hart and Tirole (1988).

[8] Admati and Perry (1987) consider a buyer who has the power to delay a sale. Under the uniqueness of their separating equilibrium, the seller still achieves a revenue that is bounded from above by the optimal TIOLI offer.

I. Setup

The seller has an object that he values at a (normalized) amount of 0. The buyer values it at some weakly positive amount θ. Time is discrete and bargaining happens sequentially over periods t. In each round, one of the parties makes a price offer that the other party can accept or reject. Bargaining lasts until an offer is accepted or until some final date T—potentially at infinity—arrives. As standard, friction, denoted by $\Delta \in \mathbb{R}^+$, corresponds to some noise in the process or the delay between action rounds. Let $e^{-\Delta}$ denote the probability that bargaining breaks down from one round to the next. Equivalently, Δ can be interpreted as the cost of delay between offers given discounting with a normalized interest rate of one. As standard, the terminology that bargaining becomes frictionless, the cost of delay between bargaining rounds vanishes, refers to $\Delta \to 0$.

True Information: At $t = 0$, the buyer privately learns θ, drawn from a commonly known prior distribution. Nature makes no further moves and bargaining begins.

Information Projection: Information projection by the buyer corresponds to an exaggerated perception in her mind that whenever the seller *acts*, he may come to do so on the basis of her information. I capture such illusory beliefs by the buyer thinking that, at the beginning of each round t, her private information may leak to the seller with some probability $\rho \in [0,1)$.

I also consider extensions of the classic assumptions about the true informational environment. Formally, let $\Gamma = \{H, I_k, v_k, f_{N,h}\}$ be the bargaining game above with observable moves by the players: H the set of histories, I_k player k's information partition over histories with generic element ι_k, v_k her payoff function. Nature's moves, $f_{N,h}$, are such that she first picks θ, and consequently, at the beginning of each round t, she plays a history-independent binary leakage lottery $\epsilon_t \in \{1,0\}$. This lottery determines whether or not θ is leaked to the seller in round t, its realization is observed only by the seller. The seller initially believes that $\Pr(\epsilon_t = 1) = \alpha \in [0,1)$, the buyer that $\Pr(\epsilon_t = 1) = \rho$. I assume that if $\alpha > 0$, then $\rho > 0$. In solving the model, I adopt perfect Bayesian equilibrium with the players now having heterogeneous priors about the distribution of information in the game.

Let σ_k be player k's strategy and μ_k her system of beliefs including those about the distribution of Nature's moves. If $\alpha = 0$ and $\rho > 0$, there are information sets that the seller believes he could never reach. Nevertheless, I require the seller's strategy to also be defined over such information sets. Since these sets are singletons, the seller must have correct degenerate beliefs there. Finally, let $V_k(\sigma, \mu_k | \iota_k)$ denote k's expected utility of the lottery induced over terminal nodes at information set ι_k.

DEFINITION 1: *A pair* $\mu = \{\mu_B, \mu_S\}$ *and* $\sigma^* = \{\sigma^*_B, \sigma^*_S\}$ *forms a perfect equilibrium if*

(*i*) $V_B(\sigma^*_B, \sigma^*_S, \mu_B | \iota_B) \geq V_B(\sigma_B, \sigma^*_S, \mu_B | \iota_B)$ *for any* σ_B *at any* $\iota_B \in I_B$;

(*ii*) $V_S(\sigma^*_B, \sigma^*_S, \mu_S | \iota_S) \geq V_S(\sigma^*_B, \sigma_S, \mu_S | \iota_S)$ *for any* σ_S *at any* $\iota_S \in I_S$;

(*iii*) μ_B *is derived from Bayes' rule* (*whenever possible*) *given* σ^* *and* $\Pr(\epsilon_t = 1) = \rho$;

(*iv*) μ_S *is derived from Bayes' rule* (*whenever possible*) *given* σ^* *and* $\Pr(\epsilon_t = 1) = \alpha$.

Discussion: Note that given the above heterogeneous priors, whatever happens in equilibrium is consistent with what players think can happen. It is simply that the buyer attaches positive probability to histories occurring that might never occur, or occur with a different probability than perceived.[9] By projecting information, the buyer too often thinks that whenever the seller *acts*, he conditions his behavior on the same information she does.

Projection is a feature of the buyer's beliefs about the seller's beliefs and characterizes the buyer's thinking per se. It can be interpreted as a biased form of thinking whereby the informed party is too anchored to the information she has when imagining the information the other party shall act upon at each of his decision nodes. Such a mistake occurs in a stationary way, irrespective of the past. Alternatively, it can be interpreted as an exaggerated fear that her information may leak whenever she interacts with the seller; e.g., a fear that the way she responds to the seller may reveal her preferences, or that the seller may read her mind during each interaction. In turn, the buyer need not stop projecting at some arbitrary round *t*, and projection is naturally formulated as a persistent mistake in prospective perspective taking or fear of gradual leakage.

In line with the above motivations, since each bargaining round corresponds to a separate interaction, I characterize the buyer's perception of the probability of leakage by some $\rho = (1 - \hat{\rho})\alpha + \hat{\rho}$ where $\hat{\rho} \in [0,1)$ corresponds to her degree of projection. In the infinite horizon context, I will also consider alternative specifications where the buyer's perception (and that of actual leakage) depends on the bargaining friction itself. For results other than the comparative statics with respect to friction in the infinite-horizon context, and thus the frictionless limit therein, such potential dependence considered is qualitatively inconsequential.

Notation: Below, by V_F I denote the ex ante expected revenue corresponding to the full surplus from trade. By V_M I denote the seller's true ex ante expected revenue (revenue, henceforth) given the optimal price-commitment made at $t = 1$. I will also refer to this as the (static) monopoly payoff.

II. Two Periods

Consider a two-period setting where $\theta \in \{l, h\}$, and let the prior on the high type be q. I first restate the fact that, under rational expectations and the classic assumption of no true leakage, the seller's bargaining revenue is always lower than the optimal (static) monopoly payoff (e.g., Skreta 2006). Let $V_H(\Delta)$ denote the seller's

[9] At the same time, unlike the bounded rationality model of analogy-based expectation equilibrium (Jehiel 2005), or cursed equilibrium where people have correct average expectations about others' strategies, here, such expectations may well be wrong *on average*.

maximal ex ante expected revenue (revenue, henceforth) from bargaining, that is, the highest payoff achievable over all perfect equilibria that could arise over all fixed sequential bargaining protocols given friction Δ.

REMARK 1: *Suppose that* $\rho = \alpha = 0$, *then* $V_H(\Delta) \leq V_M$ *for all* Δ.

Two consequences are worth highlighting. The first concerns the seller's ex ante expected revenue from an alternating-offer protocol. The second, from the seller-offer protocol. Specifically, relative to the optimal use of nonnegotiable prices (a TIOLI offer), it follows that

1. **Silence property,** letting the buyer name her own price, does not improve the seller's revenue.

2. **Coasian property,** allowing for a positive chance with which the seller can make a second offer, in case his first offer is rejected, does not improve the seller's revenue either.

For the remainder of Section II, consider the extension of the classic setup and suppose that the seller may privately infer the buyer's valuation, but suppose *only* at the beginning of $t = 1$. Formally, if α_t (ρ_t) denotes the true (perceived) probability of leakage arriving by the beginning period t, then suppose that $\alpha_2 = 0$. The buyer now faces uncertainty at the beginning of the game as to whether or not the seller is informed. The next result shows that as long as the buyer is well calibrated about such (second-order) uncertainty—which is, thus, common knowledge—the above properties extend.

PROPOSITION 1: *Given common priors*, $V_H(\Delta) \leq V_M$ *for all* Δ.

To strengthen the results below, and to provide a better comparison with the above, in the remainder of this section I also restrict the buyer's projection to the beginning of round $t = 1$. With a slight abuse of notation, in this section only, I then also denote α_1 (ρ_1) by α (ρ). Here, a buyer who projects to degree $\hat{\rho}$ only exaggerates the probability of facing an informed seller at the beginning of the bargaining process. All revenue results hold a fortiori when allowing for projection in the second round as well.[10]

A. *Alternating Offers*

Suppose, when selling a new car, a salesperson asks the buyer to name her own price first. If the buyer refuses to do so, by saying that she would rather have the seller name a price, or if the price she names is not acceptable, the salesman will have to consult his manager.[11] The manager might be away that day, and the

[10] Under the alternating-offer protocol, the case with only initial leakage and gradual leakage are isomorphic.

[11] Note that by making a nonserious offer, e.g., a price of zero, the buyer can also refuse to name a price and just wait for the seller to make a serious offer.

salesman may be able to return with a counteroffer only after some delay. Does such a practice allow for a higher revenue than if the seller makes a nonnegotiable price offer after meeting the buyer?

Consider the maximal perfect equilibrium revenue that the seller can achieve in an alternating-offer protocol given friction (delay) Δ between the offers and a ρ-biased buyer. Let's denote this by $V_A^\rho(\Delta)$. The next proposition shows that this revenue exceeds V_M whenever there is enough, but not too much, friction in the bargaining process and the buyer is sufficiently biased.

PROPOSITION 2: *There exist bounded* $\tilde{\Delta}^\rho_{\min} < \tilde{\Delta}^\rho_{\max}$ *and* $\widehat{\rho}_A$, *such that* $V_A^\rho(\Delta) > V_M$ *whenever* $\widehat{\rho} > \widehat{\rho}_A$ *and* $\Delta \in (\tilde{\Delta}^\rho_{\min}, \tilde{\Delta}^\rho_{\max})$.

Consider first, the classic case where $\alpha = 0$. For the seller to gain from bargaining, he must learn information about the buyer's type from her offer. It is thus sufficient to consider protocols where the buyer makes the first offer. Since an unbiased buyer understands that the seller can never distinguish between a "lie" and the "truth," no fully revealing equilibrium exists. Since a semirevealing equilibrium cannot generate excess revenue either, positive delay between the offers can only hurt the seller. Suppose instead that the buyer projects information. She now mistakenly believes that the seller knows her type with positive probability. For a fully revealing equilibrium to exist, the high type buyer's surplus conditional on revelation must exceed her perceived return from pretending to be a low type. Formally, suppose the high type names a price of $p_h = e^{-\Delta}h$, and the low type some lower feasible price $p_l \in \left[e^{-\Delta}l, l\right]$. For such self-revelation to be (perceived) incentive compatible, the following must hold, $h - p_h \geq (1 - \rho)(h - p_l)$, which implies that friction must exceed:

$$(1) \qquad \Delta \geq -\ln\left(\rho + (1 - \rho)\frac{p_l}{h}\right).$$

Note that this *lower* bound on Δ is decreasing in ρ, and it goes to zero, as projection becomes full. Intuitively, as projection becomes full, the buyer effectively thinks to herself that the seller almost surely knows her type, and will almost surely not lower the price, so waiting is unlikely to result in a better deal for her.

For such an equilibrium to generate excess revenue, there also needs to be an upper bound on the delay between offers. Since $p_l \geq e^{-\Delta}l$ must hold, if $\Delta \leq \ln(V_F) - \ln(V_M)$, the seller obtains a revenue in excess of V_M. Because the lower bound on friction is decreasing in ρ, this is always possible if ρ is sufficiently large.

More generally, suppose $\alpha > 0$. A fully revealing equilibrium can now exist even if the buyer is unbiased. Revelation, however, requires that the high type receives a surplus that is greater than the return on pretending to be a low type, $(1 - \alpha)(h - p_l)$. Consequently, relative to a TIOLI offer in $t = 1$, the bargaining rent transferred to the buyer allowing for revelation exceeds the ex ante expected value of the information revealed. Proposition 1 shows that the same holds when considering potential semirevealing equilibria. In contrast, a buyer who projects information reveals herself in exchange for a smaller surplus, $(1 - \rho)(h - p_l)$. For the same reason as before, if $\hat{\rho}$ is sufficiently large, and the buyer has enough, but not too much, bargaining power, the seller obtains excess revenue. The above logic, which applies to mixed equilibria as well, implies the following corollary.

COROLLARY 1: *For all* $\hat{\rho} > \hat{\rho}_A$, $V^{\rho}_A(\Delta)$ *is maximal given strictly positive but bounded* Δ.

If $\hat{\rho} = 0$, the seller's maximal bargaining revenue is achieved under no friction or delay, $\Delta = 0$. This minimizes the buyer's bargaining rent. Under sufficient projection, the revenue from bargaining may decrease both when delay is shorter and also when it is greater than a certain boundedly positive amount. If delay is too short, the buyer still refuses to reveal herself because she would receive too little bargaining rent in exchange. Transferring bargaining power to the buyer is a pure cost for the seller. If delay is longer than the lower bound described above, the buyer is willing to reveal herself. However, if delay is too long, the seller's gain from the buyer's self-revelation is smaller than his loss due to the loss of his bargaining power. In sum, positive delay between offers, which is nominally costly for the seller, in conjunction with the lack of commitment to future prices, allows the seller to capitalize on the buyer's mistaken beliefs by asking her to name her own price first.[12,13]

B. *Seller Offers*

Let me turn to the case where the seller makes both offers. Consider the maximal perfect equilibrium revenue that the seller can achieve in a seller-offer protocol, given friction Δ and a $\hat{\rho}$-biased buyer. Let us denote this by $V^{\rho}_{So}(\Delta)$. The next proposition establishes an analogue to the previous result.

PROPOSITION 3: *There exist* $\hat{\Delta}^{\rho}_{\min} < \hat{\Delta}^{\rho}_{\max}$, $\hat{\rho}_{So} > 0$ *such that* $V^{\rho}_{So}(\Delta) > V_M$ *whenever* $\hat{\rho} > \hat{\rho}_{So}$ *and* $\Delta \in (\hat{\Delta}^{\rho}_{\min}, \hat{\Delta}^{\rho}_{\max})$.

For the seller to gain from dynamic bargaining, two facts need to be true in equilibrium. First, different buyer types must both buy, and at different prices. Second, the uninformed seller and the (fictional) informed seller—conditional on the buyer being a high type—must initially pool. Such pooling ensures that the high-type buyer does not learn whether or not the seller is informed. In its absence, the seller's revenue is separable across the seller's informational types.

[12] Additional points are worth noting. The quantity $\tilde{\Delta}^{\rho}_{\min}$ might be determined by a semirevealing equilibrium. Here the high-type buyer mixes in the first period, and trade takes place with positive probability in both periods. The same would be true if considering a continuum of buyer types.

[13] Note also that while in this setting only initial projection was sufficient to generate excess revenue, in an infinite horizon setting with a continuum of types, such as is considered in Section III, repeated projection would be necessary.

Suppose the (fictional) informed seller type, conditional on $\theta = h$, and the uninformed seller type pool on price $p_{1,h}$ in $t = 1$. For this pooling price, two conditions need to hold.

(i) The high type's surplus must exceed her perceived continuation value from a rejection,

$$h - p_{1,h} \geq e^{-\Delta}(1 - \rho)(h - l);$$

(ii) It must also be credible that $p_{1,h}$ could be coming from the (fictional) informed seller,

$$p_{1,h} \geq e^{-\Delta} h, \tag{2}$$

ensuring that it is too costly for the informed seller to wait rather than pool. Combining the two constraints above implies a positive *lower* bound on Δ. This lower bound is decreasing in $\hat{\rho}$ and goes to zero as projection becomes full. In the second period, the uninformed seller names a price of l and sells to the low type. The informed seller, conditional on $\theta = l$, may separate and sell immediately at l.

In the unbiased case, incentive compatibility implies, even if $\alpha > 0$, that the benefit of price discrimination can never outweigh the associated incentive cost. A high type demands a rent proportional to $(1 - \alpha)$. This is always greater than the ex ante expected value of price discrimination. The Coasian property holds given any boundedly positive friction. In contrast, a biased buyer, underestimates the probability with which the seller becomes pessimistic after a rejection and demands a rent that is proportional only to $(1 - \rho)$. As a result, the price at which the seller can sell to the high type in $t = 1$ increases in $\hat{\rho}$. If $\hat{\rho}$ is sufficiently high, the seller can effectively pretend to be more informed than he truly is. The benefits of bargaining can then outweigh the incentive costs. For bargaining to generate excess revenue, friction may also need to be bounded from above. This logic again implies a straightforward corollary.

COROLLARY 2: *For all $\hat{\rho} > \hat{\rho}_{So}$, $V^{\rho}_{So}(\Delta)$ is maximal given strictly positive but bounded Δ.*

In the unbiased case, no friction or infinite friction is always optimal. Under information projection, positive but bounded friction, leading to bargaining over real time, is characteristic of the optimal selling protocol. Again positive delay, which must be costly for the seller, in conjunction with no commitment to future prices, allows the seller to pretend to be informed more often than he truly is. Capitalizing on the buyer's mistaken belief in this fashion, the seller can boost his revenue above what he could achieve by nonnegotiable prices.

III. Infinite Horizon

In the previous section, I considered the basic two-period setting. Bargaining helped raise the seller's revenue even if projection was constrained to happen only at the beginning of the initial round. I now return to the setup of Section I and consider the model's predictions to classic infinite-horizon seller-offer bargaining. The seller faces a continuum of buyer types whose valuations are distributed uniformly on $[0,1]$, thus the general no-gap assumption holds. Bargaining lasts as long as the parties do not agree on an offer. To isolate the key mechanisms at play, I proceed gradually. In this section I focus on the case of pure projection, i.e., $\alpha = 0$. In Section IV, I then allow for real leakage as well, $\alpha > 0$, and consider general heterogeneous priors over it.

Below, I describe both the imaginary equilibrium path—which only the buyer thinks happens with positive probability—and the real equilibrium path. As standard in the Coasian bargaining literature, I focus on equilibria where the buyer uses a stationary acceptance strategy (e.g., Fudenberg, Levine, and Tirole 1985; Gul, Sonnenschein, and Wilson 1986—GSW, henceforth). The players' strategies will then be parametrized by ρ and Δ. Specifically, unless the buyer believes that she faces an informed seller, she accepts price p if and only if her valuation θ is greater than $\lambda(\rho,\Delta)p$. Since the skimming property will continue to hold, the seller then always faces a left truncation of the prior. In turn, his strategy on the real path is Markov with respect to the relevant state variable, the highest possible buyer type who has not bought yet. The seller names an initial price of $\gamma(\rho,\Delta)$ and follows a pricing rule which, in equilibrium, is linear in each round t in the highest type who has not bought yet. On the imaginary path, that is, the path where the seller is actually informed, the informed seller holds the buyer to her valuation.[14] On this path, the buyer plans to accept immediately. This is supported by the fact that the fictional informed seller and the uninformed seller always separate in equilibrium.

The above strategies imply that, on the real equilibrium path, there will be a decreasing sequence of prices, p_t for rounds $t \in \{1,2,\ldots\}$, such that

$$p_t = \gamma(\rho,\Delta)^t \lambda(\rho,\Delta)^{t-1}, \tag{3}$$

where the initial price is $\gamma(\rho,\Delta)$, and the price gradient, that is, the ratio of the price in round $t+1$ and the price in round t, is $\gamma(\rho,\Delta)\lambda(\rho,\Delta)$. Let $V_S^{\rho}(\Delta)$ be the seller's true ex ante expected equilibrium revenue given a ρ-biased buyer and friction (delay) Δ.

In the unbiased case, GSW (Theorem 3) establish the Coasian result: any perfect equilibrium, given a stationary strategy by the buyer, satisfies the Coase conjecture; as friction vanishes, the seller's initial price converges to the buyer's lowest possible valuation and the seller is unable to extract any rent. Following GSW, below I restate the unique perfect equilibrium in which the buyer's stationary acceptance strategy is also analytic in her type.

[14] This holds except for a measure-zero set of types where the buyer's valuation corresponds to a price on the real path. For such valuations, to ensure a fully separating equilibrium in the seller's type, the imaginary informed seller names a price slightly below the informed buyer's valuation. If leakage is perceived to be verifiable, such no-rent separation under perfect equilibrium holds exactly for all types.

PROPOSITION 4: *Let* $\rho = 0$. *The seller's initial price* $\gamma(0,\Delta)$ *increases and* $\lambda(0,\Delta)$ *decreases in* Δ. *Furthermore,* $V_S^0(\Delta)$ *smoothly increases in* Δ, *and* $\lim_{\Delta \to 0} V_S^0(\Delta) = 0$.

I now turn to the case with information projection. The next result nests the above case and describes how the buyer's mistaken beliefs impact this classic result.[15]

PROPOSITION 5: *A class of perfect equilibria in the stationary strategies above exists. Furthermore,*

(*i*) $\lambda(\rho,\Delta)$ *smoothly decreases in* ρ;

(*ii*) *the price gradient* $p_{t+1}/p_t = e^{\Delta}\left(1 - \sqrt{1 - e^{-\Delta}}\right)$ *is independent of* $\rho \geq 0$ *for any given* $\Delta > 0$;

(*iii*) *for any* $\rho > 0$, *there exists* $\bar{\Delta}^{\rho} > 0$ *such that* $\gamma(\rho,\Delta)$ *increases in* Δ *if* $\Delta > \bar{\Delta}^{\rho}$, *and decreases in* Δ *if* $\Delta < \bar{\Delta}^{\rho}$, *where* $\bar{\Delta}^{\rho}$ *is increasing in* ρ;

(*iv*) *for any* $\rho > 0$, *and any* $\tau > 0$, *there exists* $\Delta^{\rho}(\tau) > 0$ *such that if* $\Delta \in [0, \Delta^{\rho}(\tau)]$, *then* $V_S^{\rho}(\Delta)$ *smoothly decreases in* Δ *and* $\left|V_S^{\rho}(\Delta) - V_F\right| \leq \tau$.

Let me describe the predictions. First, the buyer's demand withholding smoothly decreases in her degree of projection; second, the seller's price gradient, the extent to which the seller drops the price from one round to the next, is independent of the buyer's mistake; third, the comparative static is nonmonotone. Initially, if the frequency of offers increases, the seller's initial price and revenue may decrease. However, for any $\rho > 0$, there is a switch point $\bar{\Delta}^{\rho}$ after which a further increase in the frequency of offers leads to the exact opposite; the seller's initial price, as well as his entire decreasing price sequence, now increases. The seller's revenue eventually surpasses the static monopoly profit. As bargaining becomes smooth, his initial price converges to the highest possible type of the buyer and his ex ante expected revenue to the full surplus from trade. The comparative static with respect to friction is smooth everywhere.

Proposition 5 also describes the reversal of the unbiased comparative static well away from the limit for any $\rho > 0$. This is true despite the fact that leakage is a pure *illusion* and the seller knows this.[16]

To describe the logic, note that for price discrimination to be sequentially rational, the seller's cost from reducing his price in any given round, as opposed to only in the next one, must be greater than the associated gain thereof. The gain is the forgone interest loss on the seller's continuation value. The cost is the intertemporal price difference multiplied by the probability that the buyer buys at a higher price now, as opposed to only at a lower price in the next round.

[15] For any given $\Delta > 0$, it can be shown that the corresponding perfect equilibrium characterized by Proposition 5 is the essentially unique limit, unique in terms of prices and payoffs, of the sequence of perfect equilibria of the finite T horizon games as this horizon $T \to \infty$.

[16] Note also that for any fixed friction, the mechanical perceived utility consequence of leakage goes to zero as projection goes to zero; for any $\Delta > 0$, $\lim_{\rho \to 0} \sum_{t=1}^{\infty} e^{-\Delta(t-1)} \rho (1-\rho)^{t-1} = 0$.

Under sequentially rational price discrimination, the uninformed seller becomes more pessimistic after each rejection and must reduce the price. Given a positive interest rate, this negative selection leads to a severe temptation problem; the price reductions of the seller's future acting selves reduce the seller's current acting self's equilibrium incentive to offer appropriately more limited price reductions. As the frequency of offers increases, this problem becomes more severe. The ratio of the seller's price in round t versus in round $t+1$, the inverse of the price gradient, $p_t/p_{t+1} = \big(\lambda(\Delta,\rho)\gamma(\Delta,\rho)\big)^{-1}$, decreases as Δ decreases. The extent to which the seller drops the price from one round to the next *decreases*. Crucially, however, the speed of bargaining, that is, the ratio of the seller's price in round t versus in round $t+1/\Delta$, $p_t/p_{t+1/\Delta} \simeq \big(\lambda(\Delta,\rho)\gamma(\Delta,\rho)\big)^{-(1/\Delta)}$, continuously increases as Δ decreases.[17] The extent to which the seller drops the price over a unit of interest-losing delay *increases*. As offers become arbitrarily frequent, this speed tends to an unbounded amount, $\lim_{\Delta\to 0}\big(\lambda(\Delta,\rho)\gamma(\Delta,\rho)\big)^{-(1/\Delta)} = \infty$. This is the classic Coasian force. In equilibrium, the unbiased buyer optimally hides behind her private information. As the frequency of offers increases, the seller needs to reduce the level of his offers in order to sell. This then eventually forces the seller's continuation value to zero, and he must sell immediately at the lowest possible valuation of the buyer. Ex ante commitment to a nonnegotiable offer shuts down this temptation problem.

Key to the logic of Proposition 5 is that the Coasian force remains fully intact. The uninformed seller knows that leakage is purely an illusion and that there is no return on simply waiting. His future acting selves impose the same negative impact on his current acting self's equilibrium incentives as in the unbiased case. The seller's relevant intertemporal trade-offs determining the speed of bargaining are unchanged. This implies the second point above: the price gradient and thus the speed of bargaining is independent of the buyer's mistake. As the frequency of offers increases, this speed continuously accelerates and still tends to an unbounded amount.[18]

The biased buyer in equilibrium understands that the uninformed seller thinks this way and has fully correct expectations about his price gradient. At the same time, there is now a countervailing force coming from the buyer's mistaken second-order beliefs. The fear that play moves to an imaginary path hurts higher types more than lower ones. This implies the first point above. The buyer's willingness to withhold demand smoothly decreases in ρ.

The logic is then based on the fact that both the classic force, due to the seller's temptation problem, and the force due to projection become stronger as friction decreases. Neither of them is globally dominant and whether one or the other accelerates faster depends on the frequency of offers itself. Their complex interaction leads to a nonmonotone comparative static. Initially, the classic Coasian force accelerates more. The speed of bargaining increases, while the level of the seller's offers decreases. For any positive degree of projection, however, there exists a switch point $\bar{\Delta}^\rho$ where the relative strength of these countervailing forces is reversed. While the speed of bargaining still accelerates, the seller's initial price now starts

[17] A unit of interest-losing delay involves $\lfloor 1/\Delta \rfloor$ number of distinct rounds. This is approximately $1/\Delta$ whenever Δ is sufficiently small.

[18] Formally, $\lim_{\Delta\to 0} 1/\big(\lambda(\Delta,\rho)\gamma(\Delta,\rho)\big)^{\frac{1}{\Delta}} = \infty$ for all $\rho \geq 0$.

to smoothly increase as the frequency of offers increases further. First, the seller fully recovers his earlier losses relative to the static monopoly profit. His initial price and revenue continues to smoothly increases. The binding incentive constraint that forced the seller's continuation value to zero in the unbiased case is now slack. Given any positive degree of projection, bargaining fast and smoothly, that is, in a way that the ex ante expected interest loss suffered before an agreement is reached also vanishes, the uninformed seller extracts the full surplus from trade.[19]

A. *Delay-Based Projective Beliefs*

Above, the buyer's illusory perception was linked to strategic interactions. The buyer feared that there was some nonzero chance that her private information leaked in each negotiating round. There, the relative speed at which information versus offers were feared to arrive was constant in bargaining friction. To highlight the role of this assumption for the comparative static prediction, I now consider the alternative specification in which the buyer's perception is instead linked to the amount of interest-losing delay that passes. Here, it is the speed at which information is feared to arrive which is constant in friction. The buyer's illusory perception of the seller becoming informed over a unit of interest-losing delay is now independent from the frequency of offers therein.[20]

To illustrate, consider moving from weekly to daily negotiations. When the buyer's perception is linked to strategic interactions, her perception of the probability that the seller becomes informed from one round to the next one, is the same whether they negotiate daily or weekly. For example, she may think that the seller can read her mind whenever he meets her. She fears that the seller may infer her preferences from how she blinks, sweats when she lies, how her voice quivers when talking to him over the phone, or how she responds to an e-mail. When instead, the buyer's perception is linked purely to the amount of costly delay that passes, her perception of the probability that the seller becomes informed over a week is the same whether they negotiate daily or only once a week. For example, the buyer may think that the seller is doing some research on her in the background, possibly piercing the veil of her private information. She then thinks that the chance that this research succeeds is simply proportional to the amount of time that passes.

Suppose then that the probability of perceived leakage over a unit of costly delay is given by some constant $l \in [0,1)$ independent of Δ. Maintaining the memoryless

[19] In this context, one may further endogenize projective beliefs. Specifically, one may assume that the informed party's perception of the probability of leakage depends on the *kind* of action she takes. First, note that any asymmetry with respect to the buyer rejecting or accepting an offer is inconsequential since only the former matters. Second, one may consider an extension to a setting where the buyer, in each round, has a third action, she can decide to abstain in that round. If she abstains, the seller has no action to take in that round. Yet, the buyer may believe that by abstaining, she can reduce or completely eliminate the probability of leakage in that round. Such an extension shall not upset the equilibrium play described. Fixing this, assuming no abstentions, the seller's off-path beliefs about the buyer's type and his continuation strategy can remain unchanged following a deviation to abstention; given the seller's stationary strategy, if in some round any surviving type deviated, the continuation game would then remain the same. This implies that abstention would only postpone a sale which is costly to *each* surviving and potentially deviating type.

[20] In Section IV, when describing frictionless bargaining, I also consider all cases between these two polar ones with respect to how the relative speed at which imaginary information versus offers arrive changes as the speed at which offers arrive changes.

property, the per-period illusory leakage probability is then $\rho_l(\Delta) = 1 - (1-l)^{\Delta}$. Let $V_S^l(\Delta)$ denote the seller's revenue.

PROPOSITION 6: *For any fixed* $l \in (0,1)$, *there exists* $\Delta_l^* \in (0,\infty)$, *such that*

(*i*) *the seller's initial price* $\gamma(l,\Delta)$ *is strictly decreasing in* Δ *if* $\Delta > \Delta_l^*$ *and strictly increasing in* Δ *if* $\Delta < \Delta_l^*$ *and the same holds for* $V_S^l(\Delta)$;

(*ii*) *there exists* $\Delta_l^{\top} < \Delta_l^*$ *where for all* $\Delta \in (\Delta_l^{\top},\infty)$, *it follows that* $V_S^l(\Delta) > V_M$.

Proposition 6 shows a single-peaked comparative static for any given $l \in (0,1)$. This is the reverse of that implied by Proposition 5. It is inverse U-shaped. As friction decreases, starting from the infinite friction limit, the seller now first smoothly raises, as opposed to drops, his initial price and his revenue smoothly increases. Once friction drops to some threshold level Δ_l^*, this process smoothly reverses and the fully intact Coasian force takes over. The seller's prices and revenue start and continue to smoothly decrease and vanish as friction decreases and vanishes.

To provide intuition, note first that all other aspects of the equilibrium remain as described by Proposition 5. Consider now a decrease in friction. Holding the buyer's acceptance strategy constant, more frequent offers allow the seller to engage in more profitable price discrimination. More frequent offers, however, increase the speed of bargaining. This is the Coasian force. In turn, an unbiased buyer's demand for rent increases to an extent which more than offsets the mechanical benefit from quicker screening. In the biased case, the buyer underestimates the seller's sequential pessimism. In this delay-based specification, this underestimation from one offer to the next is the largest when decreasing friction from the infinite friction limit. In this instance, for any $l > 0$, the buyer thinks that by the next offer the seller will almost surely be informed; hence, the biased buyer's acceptance strategy effectively stays intact which allows the seller to increase his revenue.[21]

Before, both the Coasian force and the force due to projection got stronger as the frequency of offers increased. Now, the Coasian force gets stronger, but the force due to projection gets weaker. The speed of bargaining, $(\lambda(\Delta,\rho)\gamma(\Delta,\rho))^{-(1/\Delta)}$, accelerates in Δ just as before. The buyer's perception of the probability that the seller gets informed over a unit of interest-losing delay is, however, now l, which is constant in Δ. In turn, the relative speed at which information versus new offers are feared to arrive, $(\rho_l(\Delta)/\Delta)/(1/\Delta) = \rho_l(\Delta)$, now decreases in Δ. Furthermore, it does so in proportion to the frequency of offers, $\rho_l(\Delta) \sim -\Delta \ln(1-l)$, for Δ sufficiently small. As the speed of bargaining converges to an unbounded amount, the buyer's perception of the overall probability that the seller becomes informed before running down the price converges null. In turn, by continuity, there exists a single threshold such that if friction drops below this threshold, the Coasian force overtakes the force due to projection, and a further increase in the frequency of offers leads the buyer to demand more rent to be willing to buy. The seller's revenue

[21] $\lim_{\Delta\to\infty} \rho_l(\Delta) = 1$ for any $l > 0$.

starts to smoothly decrease. As the seller loses commitment, the classic limit follows. It is now bargaining with bounded but sufficient friction which dominates the use of posted prices.[22]

B. *Comparative Static Evidence*

Let me now turn to the available experimental evidence. I first describe the comparative static evidence with respect to the friction in the bargaining process and then the comparative static evidence with respect to the durability of the bargaining process. The evidence is inconsistent with unbiased expectations, but, though not a direct test of projective beliefs per se, it is consistent with the presence of information projection linked to strategic interactions as described above.

Friction: The fact that Proposition 5 implies a reversal of the comparative static prediction with respect to how friction impacts the price schedule means that the model is readily testable. In particular, Rapoport, Erev, and Zwick (1995) study bargaining with payoff discounting in a setup that in effect corresponds to the one studied above. The buyer has a privately known valuation θ drawn uniformly from $(0, 100]$. The seller makes all price offers sequentially. The setting is publicly announced.[23] The treatment variable is the common per-period payoff discount factor $\delta = e^{-\Delta}$.

The authors implement three treatments varying δ over the values of $\{1/3, 2/3, 9/10\}$. The main findings are summarized in Figure 2, from Rapoport, Erev, and Zwick (1995), which describes the data in the three treatments from sessions with experience.[24]

Rapoport, Erev, and Zwick (1995, p. 385) find that (i) the seller's strategy corresponds to a decreasing price-path and the price gradient roughly matches the PBE predictions; (ii) the buyers withhold demand, but accept offers too soon; and crucially, (iii) sellers' initial prices increase rather than decrease in the treatment variable $\delta(\Delta^{-1})$. The treatment effect is the opposite of the Bayesian prediction. As the authors conclude, even "after considerable experience with the game, all […] subjects […] exhibited an ordering of the 3 conditions which is *diametrically opposed* to the equilibrium ordering."

[22] Note that this delay-based specification of projective beliefs implies a formal link with the players facing heterogeneous interest rates, with the buyer facing a higher one, $r_b > r_s$. Given such heterogeneous interest rates, the standard Coasian result follows, e.g., Fudenberg, Levine, and Tirole (1985). Formally, such a link holds however *only if* $\rho(\Delta)$ is such that $\Delta^{-1}\ln(1 - \rho(\Delta))$ is constant in Δ; i.e., the delay-based specification where the buyer's perception over a unit of delay is independent of the frequency of offers therein. It breaks down as soon as the buyer's misperception over a unit of interest-losing delay does depend on how frequently the players interact therein. Such cases are discussed more generally in Section IV.

The economic consequences of heterogeneous interest rates and even this pure delay-based idea of illusory leakage still differ. Under the former, given any fixed Δ, the seller's optimal revenue is still achieved by committing ex ante to a fixed price schedule, a posted price sequence. This eliminates the Coasian force and allows the seller to exploit the benefit from intertemporal price discrimination given different interest rates. Instead, under projection, even here, given any $l > 0$, since it is the buyer's miscalibrated uncertainty about the seller's future offers, posted prices would eliminate such uncertainty and bargaining with sufficient friction which again dominates posted prices.

[23] The game is terminated either when the buyer accepts the seller's offer or—to deal with finite experimental time—when the highest possible discounted profit becomes smaller than the smallest unit of the currency used.

[24] The findings from the initial sessions without experience are very similar and exhibit even more pronounced comparative statics in the same direction and even higher prices in the high discount-factor treatments.

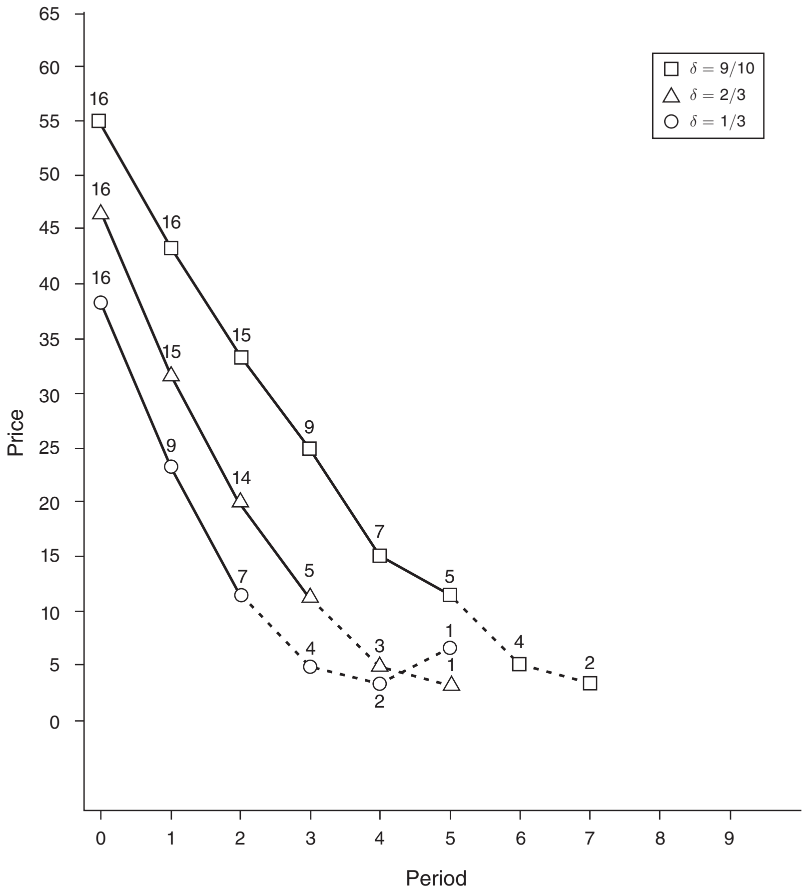

FIGURE 2

These findings, the correct unbiased price gradient and the reversal of the unbiased level effect are directly consistent with Proposition 5. This reversal over all implemented treatments holds for any $\rho > \rho^* = 0.32$. For the higher treatment variables, $\delta \in \{2/3, 9/10\}$ only, it holds for any $\rho > \rho^* = 0.17$. Basic calibrations show that the initial prices across the higher discount-factor treatments are also consistent with a value of ρ under which the model predicts the observed non-Coasian comparative statics.[25] Crucially, again consistent with the predictions under projection, the authors note that if $\delta = 0.9$, the seller's payoff is significantly higher than what is predicted by PBE. In addition, the sellers' initial prices are above the static monopoly price and their payoffs exceed the full-commitment unbiased Bayesian optimum.

[25] Rapoport, Erev, and Zwick (1995) report average initial prices from the sessions without experience (Iterations 1–3) and from the sessions with experience (Iterations 7–9) (see p. 384). For the lowest discount factor used in Iterations 7–9, neither the unbiased nor the biased model may rationalize the data, as the average initial price is too low. For the other discount factors, the model can calibrate the data with significant information projection both for (Iterations 1–3) and (Iterations 7–9). Specifically, in sessions *without* experience (Iterations 1–3), assuming a $\rho \in [0.43, 0.44]$ matches the average initial prices reported for both discount factors—i.e., $\delta \in \{2/3, 0.9\}$. In sessions *with* experience (Iterations 7–9), assuming a $\rho \in [0.27, 0.32]$ matches the average initial prices reported for both discount factors—i.e., $\delta \in \{2/3, 0.9\}$.

Durability: Reynolds (2000) empirically studies the other, for the general Coasian logic, key comparative static. In a finite T horizon seller-offer game, he considers the impact of changing the durability of the process; that is, varying the horizon T corresponding to the maximal number of distinct bargaining rounds in the game. Reynolds implements three treatments, exogenously varying T over $\{1,2,6\}$ while holding all other aspects of the game, including the per-period cost of delay, $\delta = 0.8$, constant. The buyer's privately known valuation is drawn uniformly from $[0,12]$. The setting is again publicly announced.

The unique prediction of PBE here implies that the higher T is, the lower the seller's initial price is and the higher the buyer's surplus. Instead, the data in Reynolds reveal an opposing pattern. As the author summarizes, "Observed initial prices were higher in multiperiod experiments than in single-period experiments, in contrast to equilibrium predictions" (Reynolds 2000, p. 375) While there was nontrivial demand withholding, initial prices were increasing while the buyer's surplus was decreasing in T over the three treatments. It is straightforward to show that these observed comparative static findings are again consistent with the predictions of the model.[26]

Finally, in these settings bargaining friction is implemented by the common shrinking of the pie between action rounds rather than by any significant temporal delay between rounds. These results are thus consistent with the main specification whereby the buyer's illusory perception is linked to strategic interactions. The setup is harder to interpret in light of the alternative specification in which the buyer's perception is linked to the amount of temporal delay per se. The results would not follow if such a misperception was only initial or if projection required significant temporal delay to have a significant impact.

IV. Actual and Perceived Leakage

In the previous section, I considered classic bargaining between an uninformed seller and a privately informed buyer. Consistent with the classic assumption, the seller did not exogenously become informed, e.g., could not read the buyer's mind. I now turn to the general case where, consistent with the setup in Section I, the buyer's private information may truly leak to the seller. I consider general heterogeneous priors about such a real leakage process; that is, I allow the buyer's beliefs to be exaggerated, correct, or understated about the likelihood of leakage. The analysis describes how bargaining given true leakage differs from classic bargaining, or bargaining given purely projected leakage. It also shows how the consequences of such true leakage differ given the absence or presence of the illusion of transparency.

Consider the same setup as before, but now let $\alpha \in [0,1)$. When leakage is real, the buyer can both over- and underestimate the true probability α. It is then useful to distinguish between three scenarios: (i) common priors, $\alpha = \rho$; (ii) information projection, $\rho > \alpha$; and (iii) the opposite, $\alpha > \rho$, whereby the buyer is too confident

[26] As a supplement to this paper, Madarász (2017) replicates the design of Reynolds (2000) with the three treatments now being $T \in \{1,2,3\}$. The data reveal the same reversal of the unbiased comparative static predictions as observed by Reynolds, both with respect to the three initial prices and with respect to the two second prices in treatments $T = 2$ and $T = 3$. All the observed comparative statics are consistent with the predictions of the model given any $\rho > 0.31$.

that her valuation stays private. I refer to this last case, e.g., where the buyer may exaggerate the probability that she can keep a poker face, as the *illusion of privacy*. Consistent with the focus of the paper, given a continuum of types, I focus on equilibria in which, conditional on leakage, the informed seller optimally holds the buyer to her reservation value except possibly for a measure zero set.[27]

A. *Finite Horizon*

To first show the implications of true leakage in the absence versus in the presence of the illusion of transparency, consider finite-horizon bargaining. Since the skimming property will continue to hold, let θ_t denote the highest buyer type who, along the equilibrium path, has not bought by the beginning of round t, conditional on no leakage until (inclusive) round t. Let me then distinguish between two opposing forms of equilibrium behavior.

DEFINITION 2: *The seller bargains in each round if* $\theta_t > \theta_{t+1}$ *for all* $t < T$. *The seller never bargains if* $\theta_t = \theta_{t+1}$ *for all* $t < T$.

If the seller bargains in each round, he makes a distinct serious offer, and keeps the store open in each round until the buyer buys; the price changes gradually. If the seller never bargains, he instead refuses to make a serious offer, and he keeps the store closed, in all except for a single round in which the item is either exchanged or the players depart.

PROPOSITION 7: *Let T be finite. The following must hold in equilibrium.*

(*i*) *If* $\alpha \geq \rho$, *there exists* $L^*(\alpha,\rho,T) \geq 0$ *such that the seller never bargains if and only if* $\Delta \leq L^*(\alpha,\rho,T)$. *Furthermore,* $L^*(\alpha,\rho,T)$ *is increasing in* α, *decreasing in* ρ, *and decreasing in T.*

(*ii*) *If* $\rho > \alpha$, *the seller bargains in each round.*

Under true dynamic leakage, the seller has an outside option: simply waiting from one round to the next without effectively engaging in bargaining. His incentive to effectively bargain while waiting is determined by the wedge between the true and perceived frequency of leakage. If delay per se is not costly, then in any finite horizon game, the seller bargains if and only if the buyer projects information. In the absence of the illusion of transparency, he makes only a single serious offer along the equilibrium path which the buyer either accepts or the players depart without trade.

[27] This is ensured for all types if true leakage is assumed to be verifiable. Note that verifiable leakage in a seller-offer game with a continuum of types is a very weak assumption. The informed seller can always essentially prove himself and separate by naming the buyer's valuation, which is also his most preferred price, because the uninformed seller has a zero chance of mimicking this.

If the buyer suffers from the illusion of privacy, equilibrium involves no bargaining even if delay is costly, as long as this cost is lower than a strictly positive threshold. This threshold $L^*(\alpha, \rho, T)$ is strictly increasing in α but strictly decreasing in ρ. In turn, true transparency increases, whereas the buyer's perception thereof decreases, the set of environments where the seller just waits without bargaining. Finally, if the cost of delay is larger than $L^*(\alpha, \rho, T)$, the seller may bargain in some rounds, though not necessarily all rounds.

To provide intuition, consider the case where delay per se is not costly. If the uninformed seller were to drop the price from one round to the next, the marginal buying type, $\hat{\theta}$, would have to believe that she will be facing the same expected price in the next round as today. Formally, $\hat{\theta} - p_t = (1-\rho)(\hat{\theta} - p_{t+1})$, equivalently, $p_t = \rho\hat{\theta} + (1-\rho)p_{t+1}$, would need to hold. If $\alpha \geq \rho$, the buyer's belief about the next round is either truthful or too optimistic. In turn, the seller cannot gain from selling to $\hat{\theta}$ today, as opposed to only in the next round. As long as $\alpha \geq \rho$, for all $\theta > \hat{\theta}$, the inequality $p_t < \alpha\theta + (1-\alpha)p_{t+1}$ would also need to follow. Hence, the seller would need to expect a strictly lower profit from selling to all inframarginal types today, as opposed to only in the next round. In turn, the uninformed seller has no incentive to drop the price. In contrast, if $\rho > \alpha$, the marginal buying type is too pessimistic. Hence, the uninformed seller can realize a strictly larger profit from selling to type $\hat{\theta}$ today than only in the next round. The same holds for a positive measure of inframarginal types. The logic under costly delay follows from the above.

B. *Infinite Horizon*

I now return to the classic infinite-horizon setting—the domain of the Coase conjecture—and extend Proposition 5 to the general setup. The buyer will again use a stationary acceptance strategy and, as long as she believes that she faces the uninformed seller, the buyer accepts the seller's offer p if and only if her type θ is greater than $\lambda(\rho, \alpha, \Delta)p$. In turn, the uninformed seller faces a left truncation of the prior in each round on the equilibrium path, and his strategy is again Markov with respect to the highest point in the support of his posterior; his initial price is denoted by $\gamma(\rho, \alpha, \Delta)$. Finally, $V_S^{\alpha,\rho}(\Delta)$ denotes the seller's equilibrium revenue.[28]

PROPOSITION 8: *A class of perfect equilibria in the stationary strategies above exists,* $\gamma(\rho, \alpha, \Delta)$ *and* $\lambda(\rho, \alpha, \Delta)$ *are continuous in* α *and* ρ, *and* $\lim_{\Delta \to 0} V_S^{\alpha,\rho}(\Delta) = V_F$. *For any given* $\Delta > 0$ *the seller's stationary price gradient* p_t/p_{t+1}, *conditional on no leakage until* $t+1$,

(*i*) *strictly decreases in* α;

(*ii*) *strictly increases in* ρ;

[28] For any given $\Delta > 0$ and $\rho \geq \alpha \geq 0$, it can again be shown that the corresponding perfect equilibrium characterized by Proposition 9 is the essentially unique limit, unique in terms of prices and payoffs of the sequence of perfect equilibria of the finite T horizon games as $T \to \infty$. See the online Appendix.

(*iii*) *under common priors* $(\rho = \alpha)$, *strictly decreases in* α;

(*iv*) $\lim_{\Delta \to 0} 1/\big(\lambda(\Delta, \rho, \alpha)\gamma(\Delta, \rho, \alpha)\big)^{\frac{1}{\Delta}} = e^{\rho/\alpha}$.

The result above shows that, for any given level of friction, true transparency impacts bargaining through an observably different channel than the buyer's mistaken second-order beliefs, per se.

Recall that pure projection impacted only the *level* of the seller's offers. It left the price gradient, hence, the speed of bargaining unaffected. The Coasian force was intact. This speed increased in the frequency of offers and converged to an unbounded amount. Frictionless bargaining was fast; the price moved gradually and the ex ante expected interest loss that the parties suffered before an agreement was reached vanished as friction vanished.

True leakage instead operates through a negative *gradient* effect. The uninformed seller's belief in leakage implies a belief in the benefit of waiting. Not simply does the seller now actually become informed, and is thus able to condition the price on such information, but the fact that he believes this is key. Believing that his future acting selves have a limited incentive to drop the price mitigates the temptation problem of his currently acting uninformed self. This eliminates the unbounded Coasian force and allows the seller to respond by slowing down the speed of bargaining. For any given Δ, conditional on no leakage until then, each buyer type now buys later than before.

An increase in the frequency of true versus perceived leakage have opposite comparative static implications. The former decreases, the latter increases the speed of bargaining. These opposing effects, however, never cancel out. The overall effect is always negative. Irrespective of the buyer's perception, the dominant force is waiting and true leakage leads to slower bargaining. The dynamics of how the speed of bargaining changes with friction is now complex and it need no longer increase as friction decreases. Yet, even as friction vanishes, it always remains bounded. It converges to $e^{\rho/\alpha}$. Frictionless bargaining per se is *slow*. Conditional on no leakage arriving the ex ante expected interest-loss the parties suffer before an agreement is reached converges to a distinctly positive amount. Bargaining induces delay.

Let me then describe the sharply different forms of trade that arise given pure projection versus true leakage under bargaining without commitment. Let $R(\rho, \alpha, \Delta)$ be the ex ante probability that the seller sells only once leakage has occurred.

COROLLARY 3: *For any given* $\Delta > 0$, $R(\rho, \alpha, \Delta)$ *increases in* α *and decreases in* ρ. *Furthermore*, $\lim_{\Delta \to 0} R(\rho, \alpha, \Delta) = 1$ *if* $\alpha > 0$ *while* $\lim_{\Delta \to 0} R(\rho, \alpha, \Delta) = 0$ *if* $\alpha = 0$.

When leakage is a pure illusion, trade always follows a gradual decrease in the price. As friction vanishes, trade takes the form of a fast and smooth haggling process. Instead, when leakage is real, the seller effectively just holds a high price (makes a series of, with probability one, nonserious offers), and simply waits for leakage to occur. Once it occurs, he makes a single concession and the buyer accepts. While trade is fast, the seller simply waits and in effect does not bargain in equilibrium.

To further highlight that under true leakage bargaining per se is slow, suppose that the buyer's and the seller's perceptions were still given by some positive α and ρ, but in reality, leakage did not occur. As friction vanishes, while the seller's initial price still tends to the highest possible valuation of the buyer, the seller's expected revenue no longer tends to V_F. Instead, it tends only to $V_F \rho/((1/2)\alpha + \rho)$ given the interest-loss suffered from bargaining being slow. When the seller believes in leakage, the dominant strategic force is waiting and the speed of bargaining remains bounded; conditional on no leakage, selling involves nonvanishing delay. This demonstrates the distinctly different mechanism present under true leakage versus under Proposition 5.

C. *Frictionless Bargaining*

Finally, consider not only heterogeneous but also vanishing priors, that is, when the probability that the seller becomes informed from one offer to the next vanishes as the cost of delay between these offers. Such potential dependence of true and perceived leakage on the bargaining friction matters only for the comparative static with respect to friction and thus for frictionless bargaining (bargaining, henceforth). The results below unpack the points above and will further highlight the difference in the economic implications of bargaining in the presence versus in the absence of the illusion of transparency.

Let $\alpha = \xi \Delta^{\phi}$ and $\rho = \beta \Delta^{\kappa}$, for sufficiently small Δ, with $\beta, \xi \in [0, \infty)$ and $\phi, \kappa \geq 0$. These reflect the relative speeds at which information versus offers arrive, or believed to arrive, over a unit of interest-losing delay, as friction vanishes. The parameters could then be interpreted as the "elasticities" of the (perceived) probability of leakage from one offer to the next with respect to the cost of delay between offers as this cost vanishes.

(i) If $\phi = 0$ or $\kappa = 0$, the specification corresponds to leakage being linked to negotiating rounds, strategic interactions. The relative speed at which offers versus information arrives (is thought to arrive) is constant in friction. Returning to the earlier example of moving from weekly to daily negotiations, the (perceived) probability that the seller learns the buyer's type from one offer to the next is the same whether the parties negotiate daily or weekly.

(ii) If $\phi = 1$ or $\kappa = 1$, the specification corresponds to leakage being linked purely to costly delay. Here, the speed at which information arrives (is thought to arrive) is constant in friction, the Poisson case.[29] The (perceived) probability that the seller learns the buyer's type over a week, is the same irrespective of whether the parties negotiate daily, or only once a week. Hence, in contrast to when $\phi, \kappa < 1$, even as the speed at which offers arrive, $1/\Delta$, tends to an unbounded amount, the (perceived) probability that the seller learns the buyer's type over a unit of interest-losing delay stays constant. As mentioned,

[29] This is the limit of the delay-based specification with $\beta = -\ln(1 - l)$.

given common priors, such an assumption has been considered in the literature before.

(iii) If $\kappa, \phi \in (0,1)$, the speed at which information arrives (is thought to arrive) still increases in the speed at which new offers arrive. Yet, their relative speed again decreases and vanishes as friction decreases and vanishes. Here, the (perceived) probability that the seller learns the buyer's type over a week is higher when they negotiate daily as opposed to only once a week. Meanwhile, the (perceived) probability that the seller becomes informed from one offer to the next is smaller with daily than with weakly negotiations. This is consistent with the interpretation that the likelihood of leakage over interest-losing delay is sensitive to the frequency of offers therein—e.g., leakage occurs in meetings, while the likelihood of leakage from one round to the next is also sensitive to the amount of costly delay that passes between rounds—e.g., the amount of costly time the seller has to reflect on what happened in the previous round before making a new offer.

D. *Pure Projection*

Consider first purely illusory leakage. Here, the two competing forces are the classic Coasian one and the force due to projection. The next result generalizes Proposition 5 and provides the link to Proposition 6.

PROPOSITION 9: *Let* $\xi = 0$. *For all* $\beta > 0$:

If $\kappa > 0.5$, $\lim_{\Delta\to 0} V^{\rho(\Delta)}(\Delta) = 0$.

If $\kappa = 0.5$, $\lim_{\Delta\to 0} V_S^{\varrho(\Delta)}(\Delta) = V_F\beta/(1+\beta)$.

If $\kappa < 0.5$, $\lim_{\Delta\to 0} V_S^{\varrho(\Delta)}(\Delta) = V_F$.

The above establishes how bargaining depends on the interaction of two unbounded forces in the limit. If the perceived probability of leakage vanishes at most half as fast as friction does, bargaining tends to a smooth and fast haggling process with the seller's revenue converging to full rent extraction. If it vanishes faster than that, the classic result follows. In the knife-edge case of $\kappa = 1/2$, bargaining remains a fast and smooth haggling process and the distribution of the surplus depends on the scaling parameter β, with the seller obtaining $\beta/(1+\beta)$ fraction of the surplus.

The logic of the result above highlights the strategic nature of Proposition 5. In particular, note that the buyer's limiting fictional belief of the total probability of leakage over a unit of interest-losing delay is independent of κ for any $\kappa < 1$. For $\kappa = 1$, it converges to $1 - \lim_{\Delta\to 0}(1-\beta\Delta)^{1/\Delta} = 1 - e^{-\beta}$, but for all $\kappa < 1$ it converges to one. In other words, the outcome is not based on a continuity argument with respect to this imaginary quantity—the value of the seller's imaginary outside option. This reflects the intact presence of the unbounded Coasian force, the speed of bargaining converging to an unbounded amount. Crucially, however, it is also not necessary that the buyer's perception remains nonvanishing. The buyer's perception that the seller learns her type between two offers can vanish as friction vanishes. Instead, what matters is how fast the relative speed at which imaginary information versus offers arrive goes to zero.

To illustrate, consider the buyer's decision whether to reject a given offer and wait for the next one. The benefit is the rate at which the uninformed seller drops the price from one offer to the next. The cost is the one due to delay and the one due to the buyer's fear that the seller will be informed when making the next offer. The cost due to delay vanishes asymptotically linearly in the period length. The benefit, as expressed by the price gradient, instead vanishes asymptotically only like the square root of the period length. This is the Coasian force whereby the seller cannot appropriately limit the rate at which he drops the price from one offer to the next. What matters then is how fast the buyer's perception of the chance that the seller figures out her private information from one offer to the next vanishes. If it vanishes faster than the price gradient, the seller needs to provide greater and greater rent to the buyer to able to sell. If it vanishes slower, the benefit of rejecting an offer diminishes faster than the cost thereof, and the buyer is willing to buy in exchange for a smaller rent. Here, even if the buyer's perception of the relative speed at which information versus offers arrive vanishes, bargaining still corresponds to a fast and gradual haggling process with the seller's revenue smoothly converging to the full (proportional if $\kappa = 1/2$) surplus from trade.

E. *General Case*

Let me now turn to the general case given real leakage. There are now three forces at play: the classic Coasian force, the force due to the buyer's possibly illusory perception, and the force due to real leakage. To simplify notation, below I say that $\alpha \geq \rho$ whenever this holds for all $\Delta > 0$ sufficiently small, and analogously for $\rho > \alpha$.

PROPOSITION 10: *For all*, $\xi, \beta > 0$:

(*i*) *If* $\phi < 1$, *then* $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = V_F$.

(*ii*) *If* $\phi = 1$, *then* $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = \frac{\xi V_F}{\xi + 1}$ *if* $\kappa > 0.5$, *and* $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = V_F$ *if* $\kappa < 0.5$.

(*iii*) *If* $\phi > 1$, *then* $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = 0$ *if* $\kappa > 0.5$, $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = \frac{\beta V_F}{\beta + 1}$ *if* $\kappa = 0.5$, *and* $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta) = V_F$ *if* $\kappa < 0.5$.

The interaction of these three forces is complex, but it leads to a clear pattern.

Consider first the absence of projection, $\rho \leq \alpha$. Let me first note that, as mentioned, the case under common prior, true, Poisson leakage, $\phi = \kappa = 1$ and $\beta = \xi$, links formally to the setup of Fuchs and Skrzypacz (2010). As mentioned, they characterize the frictionless stationary bargaining limit under the public Poisson arrival of a new trader. In this case, as mentioned above, the probability that the seller learns the buyer's type over a unit of interest-losing delay is $1 - e^{-\xi}$, independent of the bargaining process, i.e., even as friction vanishes. They find that "there is *delay* (emphasis added) in equilibrium, and the seller slowly screens out buyers with higher valuations" and that the "seller's payoff equals what he can achieve by simply awaiting arrival." Trade is now slow, the ex ante expected interest loss suffered before an agreement is reached does not vanish as friction vanishes; in other words, there is delay. Selling follows either a very slow dropping of the price, or a sharp discontinuous drop of the price. Furthermore, bargaining generates the same payoff for the seller as simply waiting without bargaining and "this generalization of the Coasian conjecture [means] that as the seller loses commitment, he cannot earn more than his outside option."[30]

In the absence of the illusion of transparency, the result above shows that this generalization of the Coasian conjecture holds more broadly. Irrespective of the relative speed at which information arrives, or is thought to arrive, as long as $\alpha \geq \rho$, the seller's payoff from bargaining is bounded by the value of his outside option. The seller cannot earn more than if he simply waited for leakage without effectively engaging in bargaining. This equivalence is also manifest in behavior. As mentioned, true leakage leads to a gradient effect. The seller's best response to true leakage is to slow down the speed of bargaining. When information arrives fast, trade also tends to a fast process, but it is because there is no effective bargaining in equilibrium, the seller just waits and makes a last-minute concession. When information arrives slowly, trade is also slow; it involves costly delay. Gradual bargaining does happen in equilibrium, even if it may get interrupted by leakage, but it is very slow inducing delay, as just described above. In short, in the absence of the illusion of transparency, bargaining and waiting are equivalent, and selling does not follow a fast and gradual haggling process.

In the presence of the illusion of transparency, these conclusions no longer hold. The seller's revenue from bargaining can now significantly exceed that from effectively simply waiting. Furthermore, whenever this is the case, selling follows a fast and gradual haggling process. It is this directional wedge between the seller's belief of his own future acting selves' beliefs and the buyer's belief thereof which implies that bargaining without commitment generates excess value and takes the form of a gradual and fast haggling process.

A corollary follows. To describe this, I define the appropriate extension of a TIOLI offer to this context which I shall refer to as the single-offer scheme. Under this simple single-offer scheme, the seller commits ex ante to making only a single serious offer over the underlying real time τ. The seller need not commit to a particular moment by which he quotes this offer, he adopts flexibility about this, but he commits that once he decides to quote a serious price it is final and nonnegotiable. Equivalently, the dealer adopts a no-haggling policy; upon meeting the buyer he may just wait and make no serious offers for some time, but his first offer is also his "best offer," one he is committed never to improve. By dynamic consistency, the seller's optimal revenue under such a no-haggling policy is then the maximum of the static monopoly profit (without leakage), quoting the monopoly price immediately, and the value of the seller's outside option, i.e., the true present value of just waiting for leakage without effectively engaging in bargaining.

[30] Fuchs and Skrzypacz (2010, pp. 802 and 814).

COROLLARY 4:

(*i*) *If* $\rho \leq \alpha$, $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta)$ *is smaller, and often strictly so, than the seller's revenue from the best single-offer scheme.*

(*ii*) *If* $\rho > \alpha$ *and* $\kappa < 0.5$, $\lim_{\Delta \to 0} V_S^{\varrho,\alpha}(\Delta)$ *is larger, and often strictly so, than the seller's revenue from any single-offer scheme.*

In the absence of projection, the no-haggling policy either generates the same or strictly higher revenue than bargaining. The relationship is strict when $\phi > 1$ and also when $\phi = 1$ and the arrival rate ξ is smaller than the normalized interest rate. In this case, quoting the monopoly price initially is best. In sum, the seller prefers the no-haggling policy to bargaining without commitment.

In the presence of the illusion of transparency, this no longer holds. The seller's revenue from bargaining may now well exceed his revenue from any no-haggling policy. This is true not only in the absence of true leakage, but also in the presence of true leakage (as long as $\phi \geq 1$). Bargaining now strictly dominates any single-offer scheme, ex ante commitment to a price or price schedule, or bargaining with friction. It is the dealer facing a projecting buyer who escapes the generalized Coasian result here and, because it allows him to take advantage of the buyer's illusion, gains substantially from selling via bargaining without commitment.

V. Alternative Psychological Factors

A number of other psychological factors have been discussed to shape bargaining behavior. Some impact preferences directly and possibly imply that sellers are able to inflate buyers' valuations when interacting with them directly. For example, Akerlof and Shiller (2015) emphasize the role of high-pressure sales tactics. One-on-one interactions may more easily expose buyers to visceral influences and lead to temporal inflations in valuations (Loewenstein 1996). They may also cause buyers to want to please the seller, akin to a form of pressure from being "asked," as documented by Andreoni, Rao, and Trachtman (2017) in a different context; deplete their self control, e.g., Baumeister (2002); or allow the seller to better take advantage of the sunk-cost fallacy, e.g., Arkes and Blumer (1985).

A key factor that also affects the buyer's beliefs is overconfidence. Neale and Bazerman (1992) emphasize overconfidence and the kind of limited informational perspective taking consistent with information projection as the key forces impacting negotiations. Such overconfidence about the players' outside options has been studied extensively. It serves as an explanation of inefficient bargaining impasse even in the absence of systematic private information about the size of the pie.

In a classic experiment, Babcock et al. (1995) compare two treatments. In treatment *A*, subjects initially processed symmetric information about an actual tort case without knowing what side (plaintiff or defendant) they will take in the ensuing negotiation which, if failed, led to an uncertain outside option. In treatment *B*, subjects were told their roles before being presented with the case. While 94 percent reached a settlement in treatment *A*, only 72 percent did in treatment *B*. Furthermore, in the latter treatment the average person's estimate of her outside options was systematically biased in a self-serving fashion. By considering players with self-image concerns who prefer not to see themselves as losers, Bénabou and Tirole (2009) provide an equilibrium explanation of such, eventually harmful, behavior. Such self-serving beliefs should also affect players' initial claims and willingness to settle more generally.[31]

While a buyer's overconfidence about her outside option provides a key explanation of bargaining impasse and delay, it is less likely to provide a strict bargaining rationale for the seller. Specifically, in the context of the classic bargaining problem considered in this paper, suppose that the buyer was overly optimistic about her outside option, i.e., the value she could realize upon irrevocably departing from the seller. Suppose also that the distribution of her belief about this was common knowledge between the seller and the buyer. Since conditional on the buyer exiting, it does not matter whether her outside option was fact or fiction, the analysis of Board and Pycia (2014) suggests that while such overconfidence does introduce positive selection whereby the seller can avoid the Coasian outcome, the seller still does not haggle, but simply holds a price. Furthermore, his revenue from bargaining is still bounded from above by the revenue from posting a price, a TIOLI offer at $t = 1$.[32]

VI. Discussion and Conclusion

This paper incorporates information projection, and heterogeneous priors about informational leakage more generally into classic bargaining. It shows that the buyer's exaggerated perception of leakage greatly impacts bargaining behavior. This is true both in a two-period setting with only initial leakage and projection and in an infinite-horizon setting with prospective but possibly vanishing projection. The comparative static predictions of the model, that are the reverse of the unbiased predictions, match the existing evidence. A corollary is that individual price negotiations, as opposed to nonnegotiable price offers, may be preferred by an uninformed monopolist when facing privately informed but biased buyers. By haggling flexibly over time, an uninformed salesman can capitalize on the buyer's mistaken beliefs and surpass his maximal revenue from any no-haggling policy.

Relative to posting a price or a price schedule, adopting dynamic negotiations are typically associated with additional transaction costs for the seller. They may be more time consuming; require the seller to incur additional training, compensation, and agency costs when involving third-party sales people; or be subject to additional administrative costs when registering sales prices. Such transaction costs are unlikely to be fully sensitive to the object for sale. The model then implies that the potential benefit of bargaining is more likely to outweigh these transaction costs the greater is the absolute wedge between the full surplus and the seller's static monopoly payoff, i.e., $V_F - V_M$. This wedge depends on the elasticity of the demand and on the level of the value distribution for the object. All else equal, negotiations are

[31] Studying bargaining between teachers and school boards, Babcock, Wang, and Loewenstein (1996) also provide field evidence consistent with the hypothesis that opposing sides select comparison groups in a self-serving fashion. They find that the resulting discrepancy between the allocations the two sides propose helps predict impasse.

[32] Yildiz (2003) considers an alternative form of optimism regarding players' beliefs about their prospective proposing rights in a setting with common knowledge about the size of the pie. Such overconfidence by the buyer leads to potentially inflated demand for bargaining rents. It shall then only hurt the seller's revenue in the seller-offer game.

then more likely to be adopted for objects that are associated with an overall higher value *per* transaction—houses or cars as opposed to smaller value household electronics. The mechanism suggests that the average price, per transaction, of items sold through fixed prices should be lower than the average price, per transaction, of items sold through haggling. Since the wedge above depends on the buyer having private information, in the absence of private information $V_M = V_F$, bargaining may also be more common the greater is the truly unobservable heterogeneity in customers' willingness-to-pay, the less transparent the buyer truly is. Similar comparative statics follow with respect to the extent to which bargaining involves comparatively higher transaction costs than a fixed price and the extent to which demand elasticity impacts the $V_F - V_M$ wedge.

Although by no means an explicit test of the model, evidence from dealerships selling new cars in the United States suggests that sellers predominantly opt to haggle and allow considerable room for individual price negotiations. This practice leads to significant variation in dealer margins across customers—controlling for car characteristics and dealer fixed effects. Using Consumer Expenditure Survey data, Goldberg (1996) finds that readily observable socioeconomic variables provide an insignificant explanation of this variation. Although this provides no direct evidence for projection in bargaining, it suggests that behavioral factors may play a crucial role. The prediction that sellers prefer to haggle to exploit buyers' transparency illusions and achieve successful price discrimination based on truly unobservable characteristics is consistent with this observation. More broadly, Scott-Morton, Silva-Risso, and Zettelmeyer (2011) find that an increase in a variable that describes a buyer's fear of being taken advantage of by the seller when bargaining does predict a significant decrease in dealer discounts.[33]

More broadly, historical accounts suggest that haggling was the dominant mode of sale in retail at various points in history. These accounts link the more widespread use of posted prices to the rise of large department stores in the nineteenth century, e.g., Phillips (2012), Koehn (2001).[34] Authors attribute this shift to sellers' desire to reduce transaction costs by taking advantage of the economies of scale when selling many identical items, e.g., Phillips (2012).[35]

Future research can extend the results in several ways. It would be interesting to consider the presence of a less experienced seller who does not fully appreciate the extent to which the buyer mistakenly projects. It would also be interesting to extend Section II and consider infinite-horizon alternating-offer bargaining with projection. Similarly, it may be useful to consider optimal dynamic contracting in the presence of information projection more generally, the privately informed agent dynamically underestimating her information rent. Optimal contracts are

[33] This is true even after controlling for the extent to which a buyer was happy about the price negotiated and a variety of demographic characteristics and search activities. The average dealer margin in this dataset from 2002 is $1,565.

[34] In the context of the local retail selling practices of the time, John Wanamaker, founder of the Wanamaker's store of Philadelphia writes, "[prior to 1861] there was no selling price for goods—there was an asking price, and the most persistent haggler bought the goods far below the unwary." (Wanamaker 1911, p. 27).

[35] In particular, Phillips (2012, p. 33–34) notes that "in the small shops that preceded the advent of modern department stores, sales clerks performed long apprenticeships before they could be trusted with full responsibility. List prices massively simplified the job. It enabled Wanamaker, Macy, and Boucicaut to recruit thousands of workers for their new stores who could become productive sales clerks only after minimal training."

shaped by the trade-off the principal faces between allocative efficiency and the costly information rent needed to be offered to the agent. Thus the information gap between the principal and the agent has fundamental implications for contract design, and a wedge between the actual versus the perceived gap may then significantly affect such trade-offs and the relative performance of contracts. When incorporating information projection into dynamic games, one could then also consider more complex dependence between projection and the kind of action taken by a privately informed party.

Appendix

Proofs omitted from below are in the online Appendix.

PROOF OF PROPOSITION 5:

Part I: To also nest Proposition 9, let $\rho, \alpha \geq 0$. The skimming property must hold on the equilibrium path until leakage occurs (except possibly for a measure zero set of types) because the frequency of leakage is independent of θ and leads to zero surplus to the buyer. Hence, on the path until leakage the seller must offer a (weakly) decreasing sequence of prices and at the beginning of each period t; if leakage has not occurred until then, he faces a left truncation of the type distribution: $[0, \theta_t]$. The variable θ_t is the relevant state variable. In round t, given the buyer's stationary strategy, the seller offers $p_t = \gamma(\rho, \alpha, \Delta)\theta_t$. To simplify notation, when this is without confusion, I suppress the arguments of $\lambda(\rho, \alpha, \Delta)$ and $\gamma(\rho, \alpha, \Delta)$. In each round t, given state variable θ_t, the seller's dynamic optimization problem is

$$V_s(\theta_t) = \max_{p_t}\left\{(\theta_t - \lambda p_t)p_t + e^{-\Delta}\left(\alpha\lambda p_t \frac{\lambda p_t}{2} + (1-\alpha)V_s(\lambda p_t)\right\}\right..$$

Taking the first-order condition with respect to p_t, one obtains that $\theta_t - 2\lambda p_t + e^{-\Delta}\left(\alpha\lambda^2 p_t + (1-\alpha)\lambda V_s'(\lambda p_t)\right) = 0$. Suppose, as will be verified, that the equilibrium value function $V_s(\theta)$ is of the form $V_s(\theta) = \varphi(\rho, \alpha, \Delta)\theta^2$ for some equilibrium value $\varphi(\rho, \alpha, \Delta)$. From the envelope theorem, given the buyer's strategy, it then follows that

$$\theta_t - 2\lambda p_t + e^{-\Delta}\left(\alpha\lambda^2 p_t + 2(1-\alpha)\lambda^2\varphi p_t\right) = 0. \tag{A1}$$

Consider now the buyer's strategy. In any round t where the buyer learns that no leakage has occurred yet, the buyer will be indifferent between accepting the current price versus rejecting it and buying in the next round if her value is $\theta_{t+1} = \lambda p_t$. Hence,

$$\lambda p_t - p_t = e^{-\Delta}(1-\rho)(\lambda p_t - \gamma\lambda p_t) \tag{A2}$$

must hold, since the buyer believes that with probability ρ she might be detected in the next round and receive no surplus. Finally, it must be true, by the definition of the value function, that

$$\varphi = (\gamma - \lambda\gamma^2) + e^{-\Delta}\big(0.5\alpha(\lambda\gamma)^2 + (1-\alpha)\varphi(\lambda\gamma)^2\big).$$

There are then three equations for the three parameters. After some straightforward but tedious algebra, one obtains that the unique solution is

$$\text{(A3)} \qquad \lambda(\rho,\alpha,\Delta) = \frac{1 - 2\rho + \alpha - e^{-\Delta}(1-\rho)^2}{\alpha - \rho + (1-\rho)\big(e^{-\Delta}(\rho - 0.5\alpha) + Z(\rho,\alpha,\Delta)\big)};$$

$$\gamma(\rho,\alpha,\Delta) = \frac{\big(1-(1-\rho)e^{-\Delta}\big)^2 - 0.5\alpha e^{-\Delta} - Z(\rho,\alpha,\Delta)}{e^{-2\Delta}(1-\rho)^2 - e^{-\Delta}(1-2\rho) - \alpha e^{-\Delta}},$$

where $Z(\rho,\alpha,\Delta) \equiv \sqrt{(1-e^{-\Delta})\big(1-e^{-\Delta}(1-\rho)\big)^2 + 0.25\alpha^2 e^{-2\Delta}}$ and, finally, $\varphi(\rho,\alpha,\Delta) = \gamma(\rho,\alpha,\Delta)/2$.[36] Below, I also use the short form Z to donate the function $Z(\rho,\alpha,\Delta)$.

Let me turn to the second-order condition: $-2\lambda + e^{-\Delta}\lambda\big(\alpha\lambda + (1-\alpha)\lambda\gamma\big) < 0$. Using equation (A2), this is equivalent to $(\lambda - 1)\big(e^{-\Delta} - (1-\alpha)/(1-\rho)\big) - 2 + e^{-\Delta} < 0$, since $\lambda \geq 1$. If $\rho \geq \alpha$, this is immediately satisfied. Consider $\alpha > \rho$. One can rewrite $(\lambda-1)\big(e^{-\Delta} - (1-\alpha)/(1-\rho)\big) - 2 + e^{-\Delta}$ as

$$\frac{(1+\alpha-2\rho)(2Z + 2 - e^{-\Delta}\alpha) - 2(1-\rho)^2 e^{-\Delta}(2 - e^{-\Delta})}{-2Z(1-\rho) - (2\alpha - 2\rho)\big(1 - e^{-\Delta}(1-\rho)\big) - \alpha e^{-\Delta}(1-\rho)}.$$

The denominator is always negative if $\alpha > \rho$. The numerator is bounded from below by $(1+\alpha-2\rho)2 - 2(1-\rho)^2 e^{-\Delta}(2-e^{-\Delta}) > 2\alpha - 2\rho^2 > 0$, since $Z(\rho,\alpha,\Delta) \geq 0.5e^{-\Delta}\alpha$ and $2(1-\rho)^2 e^{-\Delta}(2-e^{-\Delta}) \leq 2(1-\rho)^2$.

Finally, consider the informed seller's separation. In case leakage is verifiable, this is immediate. Otherwise, the informed seller always names a price of θ except for the measure zero set of types whose valuations equal a price on the equilibrium path without leakage. For such values the informed seller's first price is arbitrarily smaller than θ, but greater than $e^{-\Delta}\theta$ and not equal to any p_t on the path without leakage. Since the seller's price schedule has countably many points, for any $\Delta > 0$, and given that $\lambda(\rho,\alpha,\Delta) > 1$, such a price exists and the set of types concerned is measure zero. The informed seller's price in any round after the separating one is $p = \theta$ for all types. Off-equilibrium path beliefs of the buyer following any deviation before separation assign full weight to these originating from the uninformed seller. It is thus optimal for the buyer to follow her cut-off strategy. Finally, the uninformed seller would never want to deviate to a price of the informed seller

[36] Note the equation (A1) can be written as $0.5\gamma - \lambda\gamma^2 + e^{-\Delta}\big(0.5\alpha(\lambda\gamma)^2 + (1-\alpha)(\lambda\gamma)^2\varphi\big) = 0$.

since; given equilibrium beliefs with positive support over a truncation of the prior, the probability of positive profit upon deviation is zero.

Part II: Fix $\alpha = 0$ and suppress α from the notation. One obtains that

$$\lambda(\rho, \Delta) = \frac{\rho\left(1 - \sqrt{1 - e^{-\Delta}}\right) + \sqrt{1 - e^{-\Delta}}}{1 - e^{-\Delta}(1 - \rho)},$$

$$\gamma(\rho, \Delta) = \left(1 - e^{-\Delta}(1 - \rho)\right)\frac{1 - e^{-\Delta}(1 - \rho) - \sqrt{1 - e^{-\Delta}}}{e^{-2\Delta}(1 - \rho)^2 - e^{-\Delta}(1 - 2\rho)}.$$

To show that $\lambda(\rho, \Delta)$ is decreasing in ρ, note that

$$\lambda_\rho(\rho, \Delta) = -\frac{1}{\left(e^{-\Delta}(\rho - 1) + 1\right)^2}\left(\sqrt{1 - e^{-\Delta}} - \left(1 - e^{-\Delta}\right)\right) < 0.$$

Furthermore, $\lambda(\rho, \Delta)\gamma(\rho, \Delta) = e^{\Delta}\left(1 - \sqrt{1 - e^{-\Delta}}\right)$ is independent of ρ for any Δ. It then follows that $\gamma_\rho(\rho, \Delta) > 0$.

For ease of exposition only, let me introduce an equivalent notation whereby I denote $e^{-\Delta}$ by δ. Below, I will switch between the notation using Δ or δ to economize on exposition. In turn, when considering the limit where $\Delta \rightarrow 0$, I will consider the equivalent limit where $\delta \rightarrow 1$.

Consider then $\gamma_\delta(\rho, \delta)$:

$$\overbrace{\frac{\delta(1 - \rho) + 2\rho - 1}{\left(2\sqrt{1 - \delta}\right)\delta^2\left(\delta(1 - \rho)^2 + 2\rho - 1\right)^2}}^{\mathrm{I}}$$
$$\times \overbrace{\left(\delta^2(1 - \rho)^2 + 2(1 - \delta + \delta\rho)\left(1 - \sqrt{1 - \delta}\right) - \delta\right)}^{\mathrm{II}}.$$

Term II can be rewritten as $\left(\sqrt{1 - \delta} - 1\right)^2\left(\sqrt{1 - \delta}(\rho - 1) + \rho\right)^2 \geq 0$, which is always strictly positive—except at the two roots where it is zero.[37] Hence, the sign of $\gamma_\delta(\rho, \delta)$ is determined by the sign of term I. Term I is positive if and only if $\delta \geq \max\left\{(1 - 2\rho)/(1 - \rho), 0\right\}$. It follows that if $\delta \geq \bar{\delta}^\rho$ with $\bar{\delta}^\rho = \max\left\{(1 - 2\rho)/(1 - \rho), 0\right\}$, then $\gamma_\delta(\rho, \delta) > 0$. Otherwise $\gamma_\delta(\rho, \delta) \leq 0$. It is easy to see that $\gamma_\delta(0, \delta) < 0$ and $\lim_{\delta \rightarrow 1}\gamma_\delta(\rho, \delta) > 0$ if and only if $\rho > 0$.

Note that $\lim_{\Delta \rightarrow 0}\gamma(\rho, \Delta) = 1$ for any $\rho > 0$. Given that $\lim_{\Delta \rightarrow 0}\lambda(\rho, \Delta)\gamma(\rho, \Delta) \rightarrow 1$, it follows from the continuity of $V_S^\varrho(\Delta)$ in Δ that for any $\rho > 0$, and any $\tau > 0$, there exists $\hat{\Delta}^\rho(\tau) > 0$, such that, if $\Delta \leq \hat{\Delta}^\rho(\tau)$, then $\left|V_S^\varrho(\Delta) - V_F\right| \leq \tau$. Formally, since $\varphi = \gamma(\rho, \Delta)/2$ for any $\rho > 0$, $\lim_{\Delta \rightarrow 0}V_S^\varrho(\Delta) = V_F$. Simple algebra shows, that in the case where $\rho = 0$, since $\gamma(0, \Delta)$ is increasing in Δ, $V_S^0(\Delta)$ is increasing in Δ as well. ∎

[37] Note that by substituting $x = \sqrt{1 - \delta}$ and $y = \rho - 1$, we can rewrite term II as $(-1 + x)^2(1 + y + yx)^2 = \left(-1 + \sqrt{1 - \delta}\right)^2\left(\sqrt{1 - \delta}(\rho - 1) + \rho\right)^2 \geq 0$.

PROOF OF PROPOSITION 8:

The first part of the proof is contained in the proof of Proposition 5. Consider $p_t/p_{t+1} = (\lambda\gamma)^{-1}$. Note that

$$\frac{\delta^2(1-\rho)^2 - \delta(1-2\rho) - \alpha\delta}{1 - 2\rho + \alpha - \delta(1-\rho)^2} = -\delta.$$

Hence, $(\lambda\gamma)^{-1}$ equals

$$-\delta\left[\frac{\alpha - \rho + (1-\rho)\big(\delta(\rho - 0.5\alpha) + Z(\rho,\alpha,\Delta)\big)}{\big(1-(1-\rho)\delta\big)^2 - 0.5\alpha\delta - Z(\rho,\alpha,\Delta)}\right]. \tag{A4}$$

The derivative of the numerator of the expression inside the bracket of equation (A4) with respect to α is $\big(1 - 0.5\delta(1-\rho)\big) + 0.25\alpha\delta^2(1-\rho)\big(Z(\rho,\alpha,\Delta)\big)^{-1} > 0$. The denominator's is $-\Big(0.5\delta + 0.25\alpha\,\delta^2\big(Z(\rho,\alpha,\Delta)\big)^{-1}\Big) < 0$. Since $-\delta < 0$, $(\lambda\gamma)^{-1}$ is strictly decreasing in α.

It can be verified, by expressing Z explicitly, that equation (A4) can be rewritten as

$$\frac{2Z(\rho,\alpha,\Delta) - \delta\big(2(1-\rho) + \alpha\big) + 2}{2\delta\rho - 2\delta + 2}. \tag{A5}$$

[38]By differentiating equation (A5) with respect to ρ, after some rearrangements, one obtains

$$-\frac{\alpha\delta}{(2\delta\rho - 2\delta + 2)^2}\left(2 - \frac{\alpha\delta}{Z(\rho,\alpha,\Delta)}\right) < 0,$$

where to see why the inequality holds, note that $\big(Z(\rho,\alpha,\Delta)/\alpha\delta\big)^2 > 1/4$ since $Z > 0.5\delta\alpha$. Since $-\delta < 0$, it then follows that $(\lambda\gamma)^{-1}$ is strictly increasing in ρ as long as $\alpha > 0$. The case where $\alpha \equiv \rho$, follows from the above and the fact that under $\alpha = 0$, $\lambda\gamma$ is independent of ρ.

Finally, consider, for any $\omega > 0$,

$$\lim_{\Delta\to 0}(1/\lambda\gamma)^{\frac{\omega}{\Delta}} = \lim_{\Delta\to 0}\left(\frac{2Z(\rho,\alpha,\Delta) - 2e^{-\Delta} - \alpha e^{-\Delta} + 2\rho e^{-\Delta} + 2}{2e^{-\Delta}\rho - 2e^{-\Delta} + 2}\right)^{\frac{\omega}{\Delta}}.$$

This can be rewritten as $\exp\Big[\lim_{\Delta\to 0}\Big((\omega/\Delta)\Big(\ln\big(2Z(\rho,\alpha,\Delta) - 2e^{-\Delta} - \alpha e^{-\Delta} + 2\rho e^{-\Delta} + 2\big) - \ln\big(2e^{-\Delta}\rho - 2e^{-\Delta} + 2\big)\Big)\Big)\Big]$. After substituting in for $Z(\rho,\alpha,\Delta)$, one can apply L'Hôpital's rule to the term inside the square bracket. Note that

[38]Note that $\Big(-\delta\big(\alpha - \rho + (1-\rho)(\delta(\rho - 0.5\alpha) + Z)\big)\Big)(2\delta\rho - 2\delta + 2) - \Big(\big(1-(1-\rho)\delta\big)^2 - 0.5\alpha\delta - Z\Big) \times\big(2Z - \delta(2(1-\rho) + \alpha) + 2\big) = 2Z^2 - 2\big((1-\delta)(1-\delta(1-\rho))^2 + 0.25\alpha^2\delta^2\big) = 0.$

$\lim_{\Delta\to 0} d\ln\left(2e^{-\Delta}\rho - 2e^{-\Delta} + 2\right)/d\Delta = (1-\rho)/\rho$. Note also that, after some tedious algebra, $\lim_{\Delta\to 0} d\ln\left(2Z - 2e^{-\Delta} - \alpha e^{-\Delta} + 2\rho e^{-\Delta} + 2\right)/d\Delta$ equals

$$\lim_{\Delta\to 0}\frac{-0.5\,\alpha^2 + \rho^2 + 2Z + \alpha Z - 2\rho Z}{Z(2Z - \alpha + 2\rho)} = \frac{\rho^2 + \alpha - \rho\alpha}{\alpha\rho},$$

where, for the last equality, I used the fact that $\lim_{\Delta\to 0} Z = 0.5\alpha$. Hence, one obtains $\exp\left[\lim_{\Delta\to 0}\left(\omega\left((\rho^2 + \alpha - \rho\alpha)/(\rho\alpha) - (1-\rho)/\rho\right)\right)\right] = e^{\omega\rho/\alpha}$. ∎

PROOF OF PROPOSITION 9:

Note that, from Proposition 5, $\gamma\left(\rho(\Delta),\Delta\right)$ can be rewritten as

$$\gamma\left(\rho(\Delta),\Delta\right) = \overbrace{\frac{\left(1-(1-\rho)\delta\right)^2}{\delta^2(1-\rho)^2 - \delta(1-\rho) + \rho\delta}}^{\text{III}} - \overbrace{\frac{\left(\sqrt{1-\delta}\right)(\delta\rho - \delta + 1)}{\delta^2(1-\rho)^2 - \delta(1-\rho) + \rho\delta}}^{\text{IV}}.$$

When considering $\delta \to 1$, term IV always converges to zero independent of ρ. See also Lemma 2. By substituting $\rho = \beta\left(-\ln\delta\right)^{\kappa}$, given that $-\Delta = \ln\delta$, and setting $\beta = 1$, and rearranging terms in term III, we get that

$$\lim_{\delta\to 1}\frac{1}{\delta\left(\delta - 2\delta\ln^{\kappa}\frac{1}{\delta} + \delta\ln^{2\kappa}\frac{1}{\delta} + 2\kappa\left(\ln^{\kappa-1}\frac{1}{\delta}\right)(\delta - 1) - 2\kappa\delta\ln^{2\kappa-1}\frac{1}{\delta}\right)}.$$

The terms inside the bracket of the denominator in this expression, except for the first and the last ones, converge to zero. The last term goes to zero if $\kappa > 0.5$, and becomes unbounded if $\kappa < 0.5$. In the former case, we have $\lim_{\Delta\to 0}\gamma\left(\rho(\Delta),\Delta\right) = 0$. In the latter case, $\lim_{\Delta\to 0}\gamma\left(\rho(\Delta),\Delta\right) = 1$. In these cases the results hold for any $\beta > 0$, as it does not impact the limit. In the case where $\kappa = 0.5$, it is easy to show that $\lim_{\Delta\to 0} V_S^{\rho}\left(\rho(\Delta),\Delta\right)$ converges to $V_F\beta\left(1+\beta\right)^{-1}$. ∎

REFERENCES

Admati, Anat R., and Motty Perry. 1987. "Strategic Delay in Bargaining." *Review of Economic Studies* 54 (3): 345–64.

Akerlof, George, and Robert Shiller. 2015. *Phishing for Phools*. Princeton, NJ: Princeton University Press.

Andreoni, James, Justin M. Rao, and Hannah Trachtman. 2017. "Avoiding the Ask: A Field Experiment on Altruism, Empathy, and Charitable Giving." *Journal of Political Economy* 125 (3): 625–53.

Arkes, Hal R., and Catherine Blumer. 1985. "The Psychology of Sunk Cost." *Organizational Behavior and Human Decision Processes* 35 (1): 124–40.

Ausubel, Lawrence M., and Raymond J. Deneckere. 1989. "Reputation in Bargaining and Durable Goods Monopoly." *Econometrica* 57 (3): 511–31.

Auto Shopper Influence Study. 2016. "PCG Companies." http://www.pcgresearch.com/auto-shopper-influence-study/.

Babcock, Linda, George Loewenstein, Samuel Issacharoff, and Colin Camerer. 1995. "Biased Judgments of Fairness in Bargaining." *American Economic Review* 85 (5): 1337–43.

Babcock, Linda, Xianghong Wang, and George Loewenstein. 1996. "Choosing the Wrong Pond: Social Comparisons in Negotiations That Reflect a Self-Serving Bias." *Quarterly Journal of Economics* 111 (1): 1–19.

Baumeister, Roy. 2002. "Yielding to Temptation: Self-Control Failure, Impulsive Purchasing, and Consumer Behavior." *Journal of Consumer Research* 28 (4): 670–76.
Bénabou, Roland, and Jean Tirole. 2009. "Over My Dead Body: Bargaining and the Price of Dignity." *American Economic Review* 99 (2): 459–65.
Birch, Susan A.J., and Paul Bloom. 2007. "The Curse of Knowledge in Reasoning about False Belief." *Psychological Scienc*e 18 (5): 382-86.
Board, Simon, and Marek Pycia. 2014. "Outside Options and the Failure of the Coase Conjecture." *American Economic Review* 104 (2): 656–71.
Camerer, Colin, George Loewenstein, and Martin Weber. 1989. "The Curse of Knowledge in Economic Settings: An Experimental Analysis." *Journal of Political Economy* 97 (5): 1232–54.
Danz, David, Kristóf Madarász, and Stephanie Wang. 2015. "The Biases of Others." Unpublished.
DealerSocket. *DealerSocket Dealership Action Report*. https://dealersocket.com/downloads/dar2016.pdf.
Feinberg, Yossi, and Andrzej Skrzypacz. 2005. "Uncertainty about Uncertainty and Delay in Bargaining." *Econometrica* 73 (1): 69–91.
Fuchs, William, and Andrzej Skrzypacz. 2010. "Bargaining with Arrival of New Traders." *American Economic Review* 100 (3): 802–36.
Fudenberg, Drew, David K. Levine, and Jean Tirole. 1985. "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information." In *Game-Theoretic Models of Bargaining*, edited by Alvin E. Roth, 73–98. Cambridge: Cambridge University Press.
Gilovich, Thomas, Victoria Medvec, and Kenneth Savitsky. 2000. "The Spotlight Effect in Social Judgment: An Egocentric Bias in Estimates of the Salience of One's Own Actions and Appearance." *Journal of Personality and Social Psychology* 78 (2): 211–22.
Gilovich, Thomas, Kenneth Savitsky, and Victoria Medvec. 1998. "The Illusion of Transparency: Biased Assessments of Others' Ability to Read Our Emotional States." *Journal of Personality and Social Psychology* 75 (2): 332–46.
Goldberg, Pinelopi Koujianou. 1996. "Dealer Price Discrimination in New Car Purchases: Evidence from the Consumer Expenditure Survey." *Journal of Political Economy* 104 (3): 622–54.
Gul, Faruk, Hugo Sonnenschein, and Robert Wilson. 1986. "Foundations of Dynamic Monopoly and the Coase Conjecture." *Journal of Economic Theory* 39 (1): 155–90.
Hart, Oliver D., and Jean Tirole. 1988. "Contract Renegotiation and Coasian Dynamics." *Review of Economic Studies* 55 (4): 509–40.
Jehiel, Philippe. 2005. "Analogy-Based Expectation Equilibrium." *Journal of Economic Theory* 123 (2): 81–104.
Keysar, Boaz, Linda E. Ginzel, and Max H. Bazerman. 1995. "States of Affairs and States of Mind: The Effect of Knowledge of Beliefs." *Organizational Behavior and Human Decision Processes* 64 (3): 283–93.
Koehn, Nancy. 2001. "Marshall Field and the Rise of the Department Store." Harvard Business School Case Study 801-349.
Loewenstein, George. 1996. "Out of Control: Visceral Influences on Behavior." *Organizational Behavior and Human Decision Processes* 65 (3): 272–92.
Madarász, Kristóf. 2016. "Projection Equilibrium: Definition and Applications to Social Investment and Persuasion." Unpublished.
Madarász, Kristóf. 2017. "Technical Report: An Experiment Following Reynolds (2000)." Unpublished.
Myerson, Roger. 1981. "Optimal Auction Design." *Mathematics of Operations Research* 6 (1): 58–73.
Neale, Margaret A., and Max H. Bazerman. 1992. "Negotiator Cognition and Rationality: A Behavioral Decision Theory Perspective." *Organizational Behavior and Human Decision Processes* 51 (2): 157–75.
Newton, Elizabeth L. 1990. The Rocky Road from Actions to Intentions. Ph.D. thesis, Stanford University.
Phillips, Robert. 2012. "Why Are Prices Set the Way They Are?" In *The Oxford Handbook of Pricing Management*, edited by Özalp Özer and Robert Phillips, 13–44. Oxford: Oxford University Press.
Rapoport, Amnon, Ido Erev, and Rami Zwick. 1995. "An Experimental Study of Buyer-Seller Negotiation with One-Sided Incomplete Information and Time Discounting." *Management Science* 41 (3): 377–94.
Reynolds, Stanley S. 2000. "Durable-Goods Monopoly: Laboratory Market and Bargaining Experiments." *RAND Journal of Economics* 31 (2): 375–94.
Riley, John, and Richard Zeckhauser. 1983. "Optimal Selling Strategies: When to Haggle, When to Hold Firm." *Quarterly Journal of Economics* 98 (2): 267–89.

Samuelson, William, and Max Bazerman. 1984. "The Winner's Curse in Bilateral Negotiations." Alfred P. Sloan School of Management Working Paper 1513-84.

Scott-Morton, Fiona, Jorge Silva-Risso, and Florian Zettelmeyer. 2011. "What Matters in a Price Negotiation: Evidence from the U.S. Auto Retailing Industry." *Quantitative Marketing and Economics* 9 (4): 365–402.

Skreta, Vasiliki. 2006. "Sequentially Optimal Mechanisms." *Review of Economic Studies* 73 (4): 1085–1111.

Sobel, Joel. 1991. "Durable Goods Monopoly with Entry of New Consumers." *Econometrica* 59 (5): 1455–85.

Stokey, Nancy L. 1979. "Intertemporal Price Discrimination." *Quarterly Journal of Economics* 93 (3): 355–71.

Yildiz, Muhamet. 2003. "Bargaining without a Common Prior--An Immediate Agreement Theorem." *Econometrica* 71 (3): 793–811.

Van Boven, Leaf, Thomas Gilovich, and Victoria Husted Medvec. 2003. "The Illusion of Transparency in Negotiations." *Negotiation Journal* 19 (2): 117–31.

Vorauer, Jacquie D., and Michael Ross. 1999. "Self-awareness and Feeling Transparent: Failing to Suppress One's Self." *Journal of Experimental Social Psychology* 35 (5): 415–40.

Wanamaker, John. 1911. *The Golden Book of the Wanamaker Stores*. Privately published.