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On large market asymptotics for spatial price competition models[☆]

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ABSTRACT

In spatial price competition models, demand factors have correlation with prices through the markup so that their identification power decreases as the number of product grows. Asymptotic results indicate lack of consistency of the estimator due to weak instruments.

1. Introduction

Economists commonly utilize instruments to solve the simultaneity problem in demand estimation. A discrete choice approach for differentiated product markets proposed by Berry et al. (1995) (hereafter, BLP) is a popular choice to overcome this issue. In BLP's framework, product characteristics are frequently used as price instruments as they correlate with prices through the markup, especially through the market share of each product. Such instruments are called BLP instruments in the literature. However, Armstrong (2016) showed that under the large market asymptotics, where share of each product decays fast enough as the number of products grows, BLP instruments may lose their identifying power and lead to inconsistent estimators.

This paper studies a weak instrument problem in demand estimation for spatial price competition models by Pinkse et al. (2002) (hereafter, PSB) where demand and cost variables, possibly including BLP instruments, are employed as price instruments. In PSB's model, consumer demands are in a product space, not in a product characteristic space, and they can consume more than one good. Since BLP takes a random coefficient discrete choice approach, the demand model of PSB is considerably different from that of BLP. However, by rewriting the markup formula induced by the Bertrand equilibrium play, one can see that this formula is a function of the demand function of each product instead of the market share in BLP. Since the market size is considered to be finite in PSB's setup, we expect that the demand function collapses to zero as the number of products grows. Therefore, the demand instruments in PSB interact with price in a similar way to BLP.

To clarify this point, we investigate PSB's semiparametric two-stage least squares estimator $\hat{\theta}$ whose estimation error $\hat{\theta} - \theta$ is characterized as

$$\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} z_i w_i'\right)^{-1}\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} z_i v_i\right) =: A_n^{-1} b_n,$$

where z_i, w_i, v_i are vectors of instruments, regressors containing series expansion terms and exogenous variables, and regression and approximation errors, respectively. Notice that we cannot apply the conventional weak instruments asymptotics in Staiger and Stock (1997) since the dimensions of A_n and b_n are growing. Our first result characterizes the stochastic orders of each element of A_n and b_n. We find that these are not degenerate, and b_n may diverge if the number of expansion terms grows at a slower rate. Our second result provides an inconsistency result of $\hat{\theta}$ given a high-level assumption on the maximum eigenvalue of $A_n' A_n$.

PSB's estimation strategy requires a more demanding dataset than BLP's one because PSB estimate the first-order condition of the Bertrand game directly, so cost variables need to be available. In the literature of applications of PSB's method, researchers often employ demand instruments in addition to cost variables to improve identification power (e.g., Pinkse and Slade, 2004; Slade, 2004; Rojas, 2008; Fell and Haynie, 2013. It should be noted that including *one* weak instrument is enough to cause the weak identification problem (Staiger and Stock, 1997). Our analysis alerts that adding demand instruments could deteriorate the identification power against the econometrician's intention.[1]

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[1] In other words, if cost variables are sufficient to identify the parameters, and demand factors are not included in the set of instruments, then the weak instruments problem does not occur.

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2. Model and estimator

Our model follows that of PSB. There are n sellers of a differentiated product. For simplicity, we assume that each firm sells one product. Let q_i and p_i be the demand and price for product $i \in \{1, \dots, n\}$, and y_i be a vector of i's product characteristics. The demand function for product i is given by

$$q_i(p, y) = a_i + \sum_{j=1}^{n}(b_{ij} p_j + c'_{ij} y_j),$$

where $p = (p_1, \dots, p_n)'$, $y = (y_1, \dots, y_n)'$, and $(\{a_i\}, \{b_{ij}\}, \{c_{ij}\})$ are parameters to be estimated. Suppose firms play the Bertrand pricing game given rival prices, i.e., firm i chooses p_i to solve

$$\max_{p_i}(p_i - \gamma MC_i) q_i(p, y) - F_i, \tag{1}$$

where MC_i and F_i are firm i's marginal and fixed costs. The best response function of firm i is

$$p_i = -\frac{1}{2\beta_{ii}}\left(a_i - b_{ii}\gamma MC_i + \sum_{j\neq i} b_{ij} p_j + \sum_{j=1}^{n} c'_{ij} y_j\right).$$

PSB estimated this best response function by employing a semiparametric approach. For example, as components of y_i, PSB chose the growth rate of GNP, deviation of regional from overall growth, city population, and/or per capita income for their empirical analysis of the spatial competition in U.S. wholesale gasoline markets. Let x_i be a d_x-vector of MC_i, finite subset of $\{y_j\}_{j\neq i}$, and other exogenous demand and cost variables. Also let $\{e_\ell(\cdot)\}$ be a sequence of basis functions, $\{d_{ij}\}$ be measures of proximity of firms i and j, and $\tilde{\psi}_{i\ell} = \sum_{j\neq i} e_\ell(d_{ij}) p_j$. Based on this notation, PSB's semiparametric model is written as

$$p_i = \sum_{\ell=1}^{\infty} \tilde{\alpha}_\ell \tilde{\psi}_{i\ell} + x'_i \beta + u_i = \psi'_i \alpha + x'_i \beta + v_i, \tag{2}$$

where $v_i = r_i + u_i$, $r_i = \sum_{\ell=L_n+1}^{\infty} \tilde{\alpha}_\ell \tilde{\psi}_{i\ell}$, $\alpha = (\tilde{\alpha}_1, \dots, \tilde{\alpha}_{L_n})'$, and

$$\psi_i = \left(\sum_{j\neq i} e_1(d_{ij}) p_j, \sum_{j\neq i} e_2(d_{ij}) p_j, \dots, \sum_{j\neq i} e_{L_n}(d_{ij}) p_j\right)'.$$

The number of endogenous regressors L_n grows with n. Letting $w_i = (\psi'_i, x'_i)'$ and $\theta = (\alpha', \beta')'$, this model can be concisely written as $p_i = w'_i \theta + v_i$.

For this model, PSB proposed to estimate θ by the (semiparametric) two-stage least squares based on K_n-dimensional vector of instruments z_i. To deal with the endogeneity of other products' prices $\{p_j\}_{j\neq i}$ in ψ_i, we employ a demand factor, say y_j^D, in x_j as an instrument for p_j.[2] To simplify the presentation, we focus on the just-identified case, so let $z_i = g(\{y_j^D\}_{j\neq i}, x_i)$ be a K_n-dimensional vector-valued function with $K_n = L_n + d_x$ given a function g.[3] For simplicity, cost shifters in x_j are not included in instruments for p_j, but the following results are valid if those are also included in z_i. Then the semiparametric instrumental variable estimator for θ is written as

$$\hat{\theta} = \left(\sum_{i=1}^{n} z_i w'_i\right)^{-1} \sum_{i=1}^{n} z_i p_i. \tag{3}$$

This paper is concerned with the limiting behaviors of $\hat{\theta}$ when the number of products n increases to infinity under suitable conditions for the price competition models. To achieve consistency of $\hat{\theta}$, it is critical to guarantee sufficiently strong correlations between p_j and y_j^D generating the instruments z_i. To understand the nature of the problem, observe that the first-order condition of (1) can be written as

$$p_j = \gamma MC_j - \frac{q_j(p, y)}{b_{jj}} + u_j. \tag{4}$$

Thus we need to guarantee sufficiently strong correlation between the instruments $z_i = g(\{y_j^D\}_{j\neq i}, x_i)$ and markup $q_j(p, y)/b_{jj}$. However, in the current setup, it is common to assume that the market size is finite, i.e., $\lim_{n\to\infty} \sum_{i=1}^{n} q_i(p, y) < \infty$, which implies that $q_j(p, y)$ decays to zero as the number of products n grows. Therefore, the markup $q_j(p, y)/b_{jj}$ may not have enough variations to yield enough correlations with the instruments $z_i = g(\{y_j^D\}_{j\neq i}, x_i)$. This phenomenon is thoroughly studied in Armstrong (2016) for the BLP model on differentiated product demands. Indeed he conjectured emergence of such a weak instruments problem in the PSB model (see, p. 1964 of Armstrong (2016)), and we verify his conjecture.

It should be noted that in contrast to BLP's setup, however, the cost variables contained in MC_j are observable in PSB's setup, and such cost variables are immune to the weak instruments problem in the above sense. Our caution on the weak instruments problem applies when the researcher includes a demand instrument y_j^D to be correlated with the markup $q_j(p, y)/b_{jj}$ in her list of instruments. Since such demand instruments are often employed in empirical analyses of the PSB models (e.g., Pinkse and Slade, 2004; Slade, 2004; Rojas, 2008; Fell and Haynie, 2013, our asymptotic analysis can be a useful guideline for empirical researchers working on spatial price competition models.

3. Large market asymptotics

We now study asymptotic properties of the instrumental variables estimator $\hat{\theta}$ in (3) under the large market asymptotics. Based on the literature, we impose the following assumptions.

Assumption Q. (i) $\text{plim}_{n\to\infty} \sum_{i=1}^{n} q_i(y, p) < \infty$. (ii) $\sqrt{n} \max_{1\le i\le n} q_i(y, p)/b_{ii}^1 \xrightarrow{p} 0$.

Assumption Q (i) says that the market size $\sum_{i=1}^{n} q_i(y, p)$ remains finite as the number of products n diverges to infinity. This assumption implies that the demand $q_i(y, p)$ for each product i decays to 0. Assumption Q (ii) requires that the decay rate of $q_i(y, p)$ normalized by b_{ii}^1 should be faster than $n^{-1/2}$ uniformly over i. An analogous assumption is employed by Armstrong (2016), Theorem 1) for the BLP model.

We also impose some regularity conditions on the series expansion in (2).

Assumption S. (i) $\sup_{1\le i\le n, \ell\in\mathbb{N}} \sum_{j\neq i} |e_\ell(d_{ij})| = O(1)$. (ii) $\max_{1\le i\le n} \sum_{j\neq i} e_\ell(d_{ij})^2 = O(1)$ for each $\ell \in \mathbb{N}$. (iii) $\sup_{\ell\in\mathbb{N}} |\tilde{\alpha}_\ell \ell^\lambda| < \infty$ for some $\lambda > 1$.

Assumptions S (i) and (iii) are also employed by PSB. Assumptions S (i) and (ii) are on the basis functions. When the supports of $\{e_\ell(d)\}_{\ell\in\mathbb{N}}$ are finite, these assumptions require that the number of non-zero elements of $e_\ell(d_{ij})$ for $i, j = 1, \dots, n$ should be finite. If the supports of $\{e_\ell(d)\}_{\ell\in\mathbb{N}}$ are infinite, Assumptions S (i) and (ii) require that $e_\ell(d)$ should decay fast enough as $d \to \infty$. Assumption S (iii) can be understood as a smoothness condition for the function to be approximated by the series expansion. Intuitively, larger λ is associated with a smoother function.

From (2) and (3), the estimation error of $\hat{\theta}$ can be written as

$$\hat{\theta} - \theta = \left(\frac{1}{\sqrt{n}} \sum_{i=1}^{n} z_i w'_i\right)^{-1} \left(\frac{1}{\sqrt{n}} \sum_{i=1}^{n} z_i v_i\right) =: A_n^{-1} b_n. \tag{5}$$

There are two notable features in this expression. First, the matrix A_n is normalized by $n^{-1/2}$, instead of n^{-1} for the case of the conventional instrumental variable regression with strong instruments. Such

[2] PSB used population and income as price instruments for their empirical application, which are considered to be demand factors.

[3] For example, since $\sum_{j\neq i} e_\ell(d_{ij}) p_j$, $\ell = 1, \dots, L_n$ need to be instrumented, a $(L_n + d_x) \times 1$ vector

$$\left(\sum_{j\neq i} e_1(d_{ij}) y_j^D, \dots, \sum_{j\neq i} e_{L_n}(d_{ij}) y_j^D, x_i\right)'$$

is a natural candidate for z_i.

normalization is employed by Staiger and Stock (1997) for the weak instruments asymptotics. As indicated in the last section, in our setup, the markup $q_i(p,y)/b_{ii}$ (and thus w_i) may not have enough correlations with the instruments z_i, and hence we adopt the analogous normalization. Second, in contrast to the conventional or weak instruments asymptotic analysis in Staiger and Stock (1997), A_n is a $K_n \times K_n$ matrix and b_n is a $K_n \times 1$ vector so that both components have growing dimensions. In other words, we need to deal with the problem of weak instruments for semiparametric or series estimators, where not only the number of instruments K_n but also the number of endogenous regressors L_n grow with n. Such an analysis is a substantial challenge in the econometrics literature (see, e.g., Freyberger, 2017; Han, 2020).

Although full development of the asymptotic theory for (5) by extending random matrix theory is beyond the scope of this paper, we present two results to indicate lack of consistency of $\hat{\theta}$. The first proposition characterizes the stochastic orders of the elements of (A_n, b_n).

Proposition 1. *Suppose $\{p_i, x_i, z_i\}_{i=1}^n$ is an i.i.d. triangular array, where each element has the finite fourth moments, and Assumptions Q and S hold true. Then each element of A_n is of order $O_p(1)$, and each element of b_n is of order $O_p(\max\{1, \sqrt{n}L_n^{1-\lambda}\})$.*

This proposition says the elements in (A_n, b_n) do not degenerate, and b_n may even diverge when the L_n (and thus K_n) grows at a slower rate. Although this result is not enough to characterize the stochastic order of $\hat{\theta} - \theta = A_n^{-1} b_n$, we can observe analogous behaviors of the corresponding terms of (A_n, b_n) for the case of the weak instruments asymptotics in Staiger and Stock (1997).

Additionally we provide a lack of consistency result in terms of the Euclidean norm $\|\hat{\theta} - \theta\|$ under some high level assumption on A_n. Let $\lambda_{\max}(A)$ be the maximum eigenvalue of a matrix A.

Proposition 2. *Suppose $\{p_i, x_i, z_i\}_{i=1}^n$ is an i.i.d. triangular array, where each element has the finite fourth moments, and Assumptions Q and S hold true. If $\lambda_{\max}(A_n A_n') \leq C_n$ with probability approaching one, and $nL_n^{2-2\lambda}/C_n \to 0$ for some C_n, then $\|\hat{\theta} - \theta\| \xrightarrow{p} +\infty$.*

This proposition provides sufficient conditions to induce inconsistency of the estimator $\hat{\theta}$. The additional condition $nL_n^{2-2\lambda}/C_n \to 0$ is analogous to Assumption (viii) in PSB (which requires $nL_n^{2-2\lambda}/\zeta_n \to 0$ for a sequence $\{\zeta_n\}$ associated with the minimum eigenvalue of $\sum_{i=1}^n z_i w_i'$). In our setup, it is beyond the scope of this paper to characterize the upper bound C_n for the maximum eigenvalue of the product matrix $A_n A_n'$ with growing dimension, which requires further developments of the random matrix theory.

To illustrate this point, suppose A_n is a $K_n \times K_n$ matrix of independent $N(0,1)$ variables. Then Johnstone (2001), Theorem 1.1) showed

$$\frac{\lambda_{\max}(A_n A_n') - \mu_n}{\sigma_n} \xrightarrow{d} \text{Tracy–Widom law of order 1},$$

where $\mu_n = k_n^2$ and $\sigma_n = k_n\{(K_n - 1)^{-1/2} + K_n^{-1/2}\}^{1/3}$ for $k_n = (K_n - 1)^{1/2} + K_n^{1/2}$. Thus, in this case, the upper bound C_n can be set as K_n. By $K_n = L_n + d_x$, the additional condition in Proposition 2 will be $nL_n^{1-2\lambda} \to 0$, which is satisfied when L_n grows fast enough and/or λ is large enough.

Data availability

No data was used for the research described in the article.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2023.111468.

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