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Managing other people's money: An agency theory in financial management industry

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Abstract

We build an active asset management model to study the interplay between the career concerns of a manager and prevailing market conditions. We show that fund managers overinvest in market-neutral strategies, as these have a reputational benefit. This benefit is smaller in bull markets, when investors expect more managers to use high-beta strategies, making their performance less informative about their ability than in bear markets. Consequently, fund flows that follow high-beta strategies are less responsive to the fund's performance, and flow-performance sensitivity is higher in bear markets than in bull markets.

JEL CLASSIFICATION
D82, G11, G20

1 | INTRODUCTION

The financial press is full of examples of fund managers who are stellar performers on Wall Street, beating major equity averages by considerable margins for a long time. How do they do it? By choosing investment strategies that are highly idiosyncratic and investing in a few handpicked stocks. These strategies tend to increase the managers' reputation and the flows to the corresponding funds, though they concentrate rather than spread investment risks. This highlights an old issue when it comes to managing other people's money: the "agency problem," a situation where professional managers handle other people's money in a way that is optimal for them instead of for the investors.[1] Although conflict of interest between parties is present in any industry where ownership and management are separate, it is equally—if not more—severe in the financial management industry. However, its implications for this industry have not been studied to the same extent. Unlike in traditional corporations, where managers are evaluated for their performance over a long period, fund managers are evaluated over a short period, as short as a few quarters. Moreover, the financial management industry is more complex to model and analyze because of the interplay among market indexes and proprietary funds and investors' asymmetric behavior in bull and bear markets. This could perhaps explain why the problem has been only indirectly addressed in the literature.

Our first contribution is to build a theoretical model of asset management to study this agency problem and its relation with fluctuations in market conditions. We start by modeling a manager's skill as her ability to produce idiosyncratic returns and assume that this is her own private information. The manager's market-timing ability, though superior to that of an individual investor's, is common knowledge. We then establish that a manager's career concerns lead her to skew her investment choice toward a strategy with low exposure to the market to signal her confidence in her ability to generate idiosyncratic returns. A highly skilled manager is more likely to invest in her idiosyncratic project, as this delivers superior returns on average, whereas a low-skilled manager cannot perfectly imitate this strategy, as this would be too costly for her. Investors try to infer the manager's ability, and hence they associate an idiosyncratic strategy with a competent manager; this in turn endows such a strategy with a reputational benefit. This asymmetry of information between the manager and potential investors is the main driving force behind the results of this article.

Our second contribution is to demonstrate that a manager's investment decision and her subsequent perceived reputation depend on market conditions. The manager has a dual objective: to maximize her contemporaneous return while taking into consideration her career concerns. The former increases the manager's utility directly whereas the latter affects her future income through its effect on fund flows. The better the market is (bull market), the more the manager faces a trade-off between these two objectives and the less investors penalize her for choosing high exposure to the market. Consequently, there is an interaction between career concerns and market conditions, which translates into the sensitivity of fund flows to performance being higher in a bear market.

To analyze these interactions, we consider a two-period model in which there is a continuum of investors and a single fund manager. Each investor chooses between investing his wealth through the manager or directly in the market index, and this choice is affected by an investor's specific stochastic preference shock. The manager's utility is a function of the fees she collects, which are an exogenous proportion of her fund's assets under management (AUM) at the end of each period. After investors have allocated their funds, the manager publicly chooses between a high- or low-beta investment strategy. We model the manager's ability as the ex ante expected return of her idiosyncratic strategy, which is either high or low. In each of the two periods, and before picking an investment strategy, the manager also receives a private signal on the contemporaneous profitability of her idiosyncratic project. Both her ability and this signal are her private information, and she uses them to form her final estimate of the profitability of her contemporaneous idiosyncratic strategy. As a result, a high-type manager is more likely to form a high estimate, but this is not always the case.

To model market conditions, we assume that the manager also receives a signal on the market's contemporaneous return. This signal is eventually revealed to investors, but only after they have made their own investment choice. In some sense, we allow for them to eventually understand the market conditions under which the manager acted. However, at this point it is too late for them to use this information to trade on their own. In Section 3.4, we extend our setting by allowing two managers to coexist in the market, to study how the competition is affected by market conditions. We focus on the first period because in the second the manager's investment choice is not affected by her reputational concerns. In fact, the second period is introduced to create those concerns.

[1]For instance, Catherine D. Wood, owner of Ark Innovation ETF, gained celebrity status thanks to the fund's performance in 2015–2020.

For our first result, we analyze a refinement of the perfect Bayesian equilibrium, which we call *monotonic equilibrium*, and we prove that this always exists. The only additional restriction that this refinement imposes is that the manager's reputation is nondecreasing on her performance. In addition, under mild parametric restrictions we demonstrate that the monotonic equilibrium is unique.

For our second result, we demonstrate that investing in an idiosyncratic strategy carries a reputational benefit. This is because the cutoff of the high-type manager is smaller than that of the low. In other words the high type is more receptive to the idea of adopting a low-beta strategy. Intuitively, the manager's choice is affected by two incentives. On the one hand, she wants to increase her reputation, which skews her preferences toward idiosyncratic investments. On the other hand, she cares about the realized return of her strategy because her fees depend on it. Hence, for a relatively low private signal even a high-type manager may opt to forfeit her reputational benefit because investing in the market generates higher returns and, as a result, more fees. Therefore, the investment strategy is informative but it does not fully reveal the manager's ability, which is a realistic representation of the fund industry.

For our third and most important result, we show that the reputational benefit of investing in the idiosyncratic project is decreasing in the market return. In particular, we prove that the expected sensitivity of reputation to performance is higher in bear markets than in bull markets. This is because investors understand the dual objective of managers and the fact that a manager is more likely to invest in the market when market conditions are good; thus, they update their beliefs less aggressively when this is the case. Instead, in bad times any change in a fund's performance is much more likely to be attributed to the manager's ability.

We use these results to discuss the competition between funds and its fluctuation depending on market conditions. We predict that the likelihood of changes in the ranking of the funds, measured by AUM, is hump shaped on the market return but is also higher during bear markets than during bull markets because of the higher informativeness of performance. We also find some empirical evidence supporting this prediction. This is in line with the common perception that during crises, we observe the largest rearrangements in the industry in terms of fund rankings.

Finally, as an extension to our model, we study the case where investors cannot observe the manager's investment decision. In this scenario, we assume that investors cannot observe whether the manager invested in the market or their idiosyncratic portfolio, and we conclude that under this assumption, the conditions for the existence of a monotonic equilibrium cannot be satisfied.

Academic research in financial intermediaries has focused on establishing empirical results about their structure, returns, flows, managers' skill, and many other characteristics; there have been far fewer theoretical studies. One of the seminal studies of mutual funds is from Berk and Green (2004). They construct a benchmark rational model in which the lack of persistent outperformance is not due to lack of superior skill by active managers but is instead explained by the competition between funds and reallocation of investors'capital between them.

Our article relates to many papers that study how managers' concerns about their reputation affect their investment behavior. Chen (2015) examines the risk-taking behavior of a manager who privately knows her ability and shows that investing in risky projects always makes a manager's reputation higher. Dasgupta and Prat (2008) study the reputational concerns of managers and show how these may lead to herding; they focus on the asset pricing implications of this behavior. Similarly, Guerrieri and Kondor (2012) build a general equilibrium model of delegated portfolio management and find that as investors update their beliefs about managers, these concerns lead to a reputational premium, which can change signs depending on economic conditions. Moreover, Hu et al. (2011), Malliaris and Yan (2021), and Huang et al. (2022) present theoretical models in which career concerns induce risk-taking behavior or preference for skewness; however, they do not take into account any strategic behavior by fund managers concerning their decision to be more or less exposed to the market return.

The mutual fund literature has also studied the importance of aggregate risk realizations on mutual fund investors and managers. Glode (2011) and Savov (2014) rationalize the popularity of active funds despite their apparent underperformance by emphasizing the hedging function these funds provide in bad states of the economy. Kacperczyk et al. (2014, 2016) study the effect of the economy on fund managers' decisions and find that managers are often successful in both market timing in bad times and stock picking in good times and that their choice over what information to process depends on the state of the business cycle. Closest to our article is Franzoni and Schmalz (2017), who study the relation between fund-to-performance sensitivity and an aggregate risk factor and find that this relation is hump shaped. Their focus is empirical, but they also build a model to rationalize their findings. In it, investors learn about both the manager's skills and the fund's market exposure. The second inference is noisier in extreme markets because of idiosyncratic risk and because investors cannot perfectly adjust fund returns to account for the contribution of aggregate risk realizations. As a result, it becomes harder for investors to judge the manager and this is what drives the result. Our theory differs from that of Franzoni and Schmalz (2017) because their model describes the fund's loading on aggregate risk (β) as a preset fund-specific exposure, whereas our model focuses on the ability of managers to strategically choose their investment decision. In other words, although Franzoni and Schmalz's (2017) reduced-form model assumes that α and β are jointly normal, we endogenize the manager's investment decision (and hence the distribution of β) based on her career concerns.[2]

Finally, there has been extensive empirical research on fund flows and other fund characteristics. It is well documented that mutual fund investors chase past returns. Ippolito (1992) and Warther (1995) present empirical evidence supporting our predictions. Sirri and Tufano (1998) show that the flow-performance relation is convex, and asymmetrically so on the positive side of returns, whereas Chevalier and Ellison (1997) show that managers engage in window dressing their portfolios. Our main contribution to this literature lies in the following empirical prediction stemming from our model: The fund flow to performance sensitivity depends on market conditions, and in particular, it is higher during a bear market.

2 | THE MODEL

2.1 | Setup

This is a two-period model $t \in \{1, 2\}$. There is one fund manager (she) and a continuum of investors (he) of measure one, who collectively form the market. The manager discounts the future with $\delta \in (0, 1]$.

At the beginning of period t, each investor decides how to invest a unit of wealth. At the end of period t, he consumes all the wealth this investment has generated. The investor is restricted to a binary decision. He can either opt to allocate all his wealth in an index-tracking strategy, which has the same returns as the market portfolio, given by

$$m_t \sim \mathcal{N}(\mu, \sigma_m^2), \tag{1}$$

or he can choose to invest all his wealth in the manager's fund. Our underlining intuition is that most market participants follow a rule of thumb for their investments through intermediaries; for example, they set apart 5% of their wealth and then decide whether they should invest this amount in a fund. For each unit of wealth invested with the manager, let $R_t = \exp(r_t)$ denote its value at the end of this period, where

$$r_t = (1 - \beta_t) \times a_t + \beta_t \times m_t \tag{2}$$

[2]On the technical side, the cutoff equilibrium we obtain also implies that instead of having to deal with conditional normal distributions, we need to handle truncated normal distributions, which often appear in models with voluntary disclosure of information (e.g., Banerjee et al., 2020). On the empirical side, in an earlier study, Franzoni and Schmalz (2013) empirically find that flow-performance sensitivity and between-fund flows are higher in market downturns than in upturns.

is the fund's return. This has two components: the market return m_t and

$$a_t \sim \mathcal{N}(\alpha, \sigma^2), \tag{3}$$

which represents the market-neutral component of the manager's investment strategy. For instance, we think of a long/short equity fund that invests $(1 - \beta_t)$ of its assets in a market-neutral portfolio and β_t in the S&P 500 index. In fact, our framework relies on the simple intuition that some of the returns generated by the manager stem from her ability and some from a factor loading.[3] One may ask how important it is that the idiosyncratic strategy is completely uncorrelated with the market. In fact, the results of this model are robust to an alternative specification where funds generate alpha while having positive market exposure (b), that is, where $r_t = (1 - \beta_t) \times (a_t + b \times m_t) + \beta_t \times m_t$, as long as this exposure is relatively small ($b < 1$).

Adhering to the fund industry's convention, the manager's ability to create idiosyncratic profits is called alpha and is represented by $\alpha \in \{L, H\}$ where $L < H$. The manager's ability is her private information. Investors share the public prior $\pi = \mathbb{P}(\alpha = H)$. Finally, β_t represents the fund's exposure to the market, and for simplicity we assume that $\beta_t \in \{0, 1\}$. Moreover, we assume that this is publicly chosen by the manager after the investors have allocated their wealth. Because the manager's investment choice is public, investors can use it to update their beliefs about the manager's reputation. Though the investment decision could also be inferred directly by the difference between the fund's return and the market, $r - m$, we make this assumption both because we find it realistic and because it allows us to define (in Section 2.4) an intuitive refined equilibrium, which we call monotonic equilibrium.[4] Note that the model's beta, β_t, despite its relevance to the corresponding variable of the capital asset pricing model (CAPM), is not the same variable. Rather, the former represents a deterministic investment decision, whereas the latter represents the (normalized) covariance of an asset's return with the entire market.

In addition, before making her investment decision β_t, but after the investors have allocated their wealth, the manager receives two independent signals,

$$s_t = a_t + \eta_t \text{ and } s_t^m = m_t + \eta_t^m, \tag{4}$$

where $\eta_t \sim \mathcal{N}(0, v^2)$ and $\eta_t^m \sim \mathcal{N}(0, v_m^2)$. On the one hand, s_t is private and is associated with the manager's contemporaneous confidence in her alpha. On the other hand, s_t^m is public but it becomes available only after the investors have committed their capital to the manager's fund. This market signal represents the standard piece of information that most institutional participants receive about the market's condition. We assume that this information eventually becomes available to individual investors, albeit too late to trade on it. This assumption is mainly made for tractability reasons, because otherwise investors would have to use the market return to update their beliefs about the signal s^m. However, we also find this assumption realistic because dissemination of the fund industry's research about the market to individual investors could happen through sources such as specialized media outlets or periodical reports published by investment banks.[5] Because the signal s^m corresponds to a type of market-timing ability, our assumption implies that this ability (corresponding to the precision $1/v_m^2$ of the signal) is common knowledge, so that there is no reason for investors to try to infer it based on the fund's return or its investment decision.[6]

To simplify matters, we also assume that the manager's fees are exogenously set[7] to a given percentage $f_t \in [0, 1]$ of her AUM at the end of period t (fees are collected at the end of each period so that in the second period the manager's motives are aligned with those of investors). Even though we do not allow for incentive fees,

[3]Our model is a simplified version of reality where ability is captured by the expectation of the market neutral component. In future work, one could also consider alternative measures of active portfolio managements such as the tracking error volatility, or the Active Share of Cremers and Petajisto (2009).
[4]In Section 4, we further explore the implications of a model where the manager's investment choice is unobservable.
[5]See, for example, the recent summary of the Investment Outlooks for 2023, published by various institutions (Wigglesworth, 2023).
[6]Bollen and Busse (2001) also document that fund managers possess significant market-timing skills.
[7]Endogenizing the choice of fees is left for future research. The complexity of allowing an endogenous choice is that the fees would then serve as a signaling device for the manager's ability, thus making the equilibrium much harder to find.

the plain managerial fees f_t we consider suffice to create direct incentives for the manager to perform in t, as her period income per dollar invested is $f_t R_t$.

Two other important assumptions are made. First, the manager's investment decision is binary. In particular, it allows for either investing all of the fund's assets in the manager's idiosyncratic strategy a_t, or all in the market m_t. Second, this decision is observable by the rest of the market participants. The former assumption is imposed to make the model more tractable. We speculate that altering it to allow for $\beta_t \in \{\underline{b}, \bar{b}\}$, where $\underline{b} < \bar{b}$, would not affect our results qualitatively.[8] Regarding the latter assumption, it appears to be reasonable for long investment horizons. This is because the fund's exposure to the market can be ex ante approximately inferred either by estimating a multifactor regression or by looking at its past portfolio composition, which in many cases is public.

2.2 | Payoffs and utilities

Investors are risk neutral; however, each one's decision is influenced by an exogenous preference shock, which follows an exponential distribution:

$$z_t^j \sim \exp(\lambda), \text{ where } j \in [0, 1] \tag{5}$$

stands for the shock on investor's j preferences at period t. Hence, his payoff from investing in $i \in \{1, m\}$ is

$$v(i, z_t^j) = \begin{cases} \exp(z_t^j - \bar{z}) \times (1 - f_t) \times R_t, & i = 1, \\ \exp(m_t), & i = m, \end{cases} \tag{6}$$

where $\bar{z} > 0$ is a constant we introduce to ensure that under the lowest preference shock $z_t^j = 0$, the investor opts for the market instead (this is explained in more detail in Equation (B3) in Appendix B). Under this preference, the investor is indifferent between his choices when $e^{z_t^j - \bar{z}}(1 - f_t)E[R_t] = E[e^{m_t}]$, so that expected returns are not the only determinants of an investor's decision. There is a plethora of ways to interpret this shock, one of which is to point out that each investor values specific fund characteristics, for example, the fund's classification with regard to its investment strategy, portfolio composition, leverage, and so on. An alternative explanation would be that each investor is influenced by interpersonal relationships, network effects, word of mouth, or other forms of private information. Our analysis does not address what generates this shock. However, it is worth pointing out that if preference shocks did not exist, and investors were homogeneous, the decision to invest in the fund or in the market would be deterministic, and thus a fund would either get all the money or none at all. We view this as unrealistic and therefore opt instead for the reduced-form specification.

We choose the specific utility function for the investors to reflect their choices and the fact that they are implicitly evaluating the manager relative to a benchmark (the market). This also allows us to get a tractable form for the fund's AUM. Furthermore, because R_t comes from a log-normal distribution, we could adopt a constant relative risk aversion (CRRA) utility function for the investor without altering his decision significantly. However, we opt not to do so to keep our expressions as compact as possible.

On the other hand, we assume that the manager has log preferences. In particular, if A_t stands for the fund's AUM at the beginning of t, the manager's payoff at t is $\log(A_t f_t R_t)$. Again we speculate that most of our results would not be significantly different if a more generic CRRA was used instead, but this is the most convenient functional form to work with.

[8]A possibility we exclude that is worth mentioning is that of a manager who bets against the market. In a bear market, a fund manager could choose to short the market portfolio instead of adopting a neutral strategy. This would have a significant impact on our analysis. However, because funds that systematically hold big negative positions are not that common, we omit this case.

2.3 | Timing

To sum up, the timing in our model is as follows. In each period $t \in \{1, 2\}$, first, the preference shock z_t^j, $j \in [0, 1]$, is realized and then investors decide how to allocate their wealth. Second, the manager receives the private and public signals s_t and s_t^m, respectively. Third, the investment decision β_t is made by the manager, R_t is realized, and both become public. Fourth, the fund's AUM is divided between the manager and her investors according to the fee f_t and is consumed immediately. Finally, the investors that are active in the second period observe the public variables of the first period before allocating their wealth. Importantly, they know (R_1, β_1, s_1^m) and use them to update their beliefs about the manager's ability α. Signal s_1 cannot be used because it is the manager's private information and will never be known to investors.

2.4 | Monotonic equilibrium

We call an equilibrium of our model *perfect Bayesian equilibrium* (PBE) if all market participants use Bayes' rule to update their beliefs on α, whenever possible, and choose their actions to maximize their expected discounted payoff at each point they are taking an action. There is a possibility of multiple equilibria, which is a common setback for these types of models. For this reason we refine the set of equilibria using the following definition (however, the study of additional equilibria is beyond the scope of our article):

Definition Call PBE a monotonic equilibrium if the manager's reputation, for a given choice of investment strategy, is nondecreasing in her performance.

In other words a monotonic equilibrium that satisfies $\mathbb{P}(\alpha = H \mid r, s^m, \beta)$ is increasing in r. Therefore, the only requirement that our refinement imposes is that the manager's reputation is not penalized by the fact that she delivers good returns for her investors. The definition implies that there exists φ_0 and φ_1 such that the public posterior on the manager's ability is given by

$$\begin{aligned} \varphi_0 &= \mathbb{P}(\alpha = H \mid r_1, s_1^m, \beta_1), \text{ for } \beta_1 = 0, \\ \varphi_1 &= \mathbb{P}(\alpha = H \mid r_1, s_1^m, \beta_1), \text{ for } \beta_1 = 1. \end{aligned} \tag{7}$$

We separate the posteriors that follow the two choices of β_1 because those will turn out to have different functional forms. Note that each investor conditions his belief on the fund's return r, on the market signal s^m, and on the investment choice β; the actual market return m is not used because it becomes redundant once investors observe s^m. This would not be the case if managers also had private market timing ability (e.g., a value of v^m that could be high or low), in which case the inference problem of the investor would become significantly more complicated. We refrain from adding this extra layer of uncertainty to maintain the tractability of our model.

3 | ANALYSIS

We begin our analysis by first discussing the manager's optimal investment strategy in the second period and how this affects her career concerns in the first period. Second, we characterize the monotonic equilibrium and prove its existence and uniqueness. Third, we present our results on the baseline model with the single manager. Fourth, we discuss the implications of adding a second manager.

3.1 | Investment and AUM in the second period

Here we provide a description of how we solve for the manager's investment decision in the second period and the corresponding AUM that this implies. The interested reader can find a more detailed analysis in Appendix B.

In the second period the manager faces no career concerns. Hence, the objective of her investment decision is to maximize the expected fees she collects at the end of this period. Because these fees are proportional to her fund's AUM at the end of the second period, and we assume log preferences, the manager's payoff maximization problem simplifies to

$$\max_{\beta_2 \in \{0,1\}} \mathbb{E}[\log(A_2 f_2 R_2) | \beta_2, \alpha, s_2, s_2^m].$$

When opting for her idiosyncratic strategy $\beta_2 = 0$, this expectation uses the manager's ability α and private signal s_2, whereas the index-tracking strategy $\beta_2 = 1$ depends only on the market signal s_2^m. Because we assume that the returns and corresponding signals are log-normally distributed, we can calculate the expectation for each choice in closed form. This suggests that the manager's optimal second-period strategy is to invest in her idiosyncratic project if and only if $s_2 \geq c(\alpha, s_2^m)$ where

$$c(\alpha, s_2^m) = \frac{\psi_m}{\psi} \times s_2^m + \frac{1-\psi_m}{\psi} \times \mu - \frac{1-\psi}{\psi} \times \alpha. \tag{8}$$

The constants ψ and ψ_m are the weights that the Bayesian updating gives to the signals s_2 and s_2^m, respectively, and more specifically: $\psi = \sigma^2/(\sigma^2 + v^2)$ and $\psi_m = \sigma_m^2/(\sigma_m^2 + v_m^2)$. Given the previously described cutoff strategy, we can calculate the expected terminal value of one unit of wealth that is invested by the manager. We denote the high and low types by u_2^H and u_2^L, respectively. Therefore, for given posterior reputation φ, and ignoring preference shock z, the expected payoff to an investor that opts for the manager is given by

$$[1 - f_2] \times [\varphi \times u_2^H + (1 - \varphi)u_2^L].$$

This, together with the assumed preference shock, allows us to calculate the assets of the second period in closed form. From Equation (6), we have the expected payoff to an investor who chooses to invest in a fund or in the market. He chooses the former if his expected payoff is higher. Because there is a continuum of investors with one unit of wealth, the probability of this event occurring is equal to the assets of Fund 1. Hence.

$$A_2(\varphi) = \left(e^{-\left(\mu + \bar{z} + \frac{\sigma_m^2}{2}\right)} \times [1 - f_2] \times \left[\varphi \times u_2^H + (1 - \varphi) \times u_2^L\right]\right)^{\lambda}, \tag{9}$$

which is an increasing function of the manager's reputation φ. As long as $\lambda > 1$, the AUM are a convex function of reputation φ. This is a result that has been widely documented in the empirical literature, in slightly different forms.

3.2 | Existence and uniqueness of the monotonic equilibrium

In this section we demonstrate that the monotonic equilibrium exists and is unique under mild conditions. First, we want to understand the manager's incentives in the first period. Her expected discounted payoff at this point is

$$\mathbb{E}_R[\log[R_1 f_1 A_1(\pi)] + \delta \times \log[R_2 f_2 A_2(\varphi_\beta)] \mid s^m, s, \beta, \alpha],$$

where the expectation is taken with respect to the returns of both periods. $A_1(\pi)$ is the equilibrium allocation of AUM in the first period, which has a functional form similar to that of $A_2(\varphi_\beta)$, and β is β_1.

Hereafter, our focus shifts to the interactions in the first period. As a result, to make our formulas more compact, the time subscript t is dropped whenever this does not create ambiguity. Using the properties of the natural logarithm, we simplify the manager's payoff maximization problem in period 1 to

$$\max_{\beta\in\{0,1\}} \mathbb{E}_r[r + \delta \times \lambda \times \log[\varphi_\beta(r, s^m) \times (u^H - u^L) + u^L] | s^m, s, \beta, \alpha]. \tag{10}$$

Therefore, the manager cares about both her returns in the first period r and how those returns affect her posterior reputation $\varphi_\beta(r, s^m)$. This reputation is important because it affects the amount of AUM the manager will attract at the beginning of the second period.

First, we want to offer a characterization of the monotonic equilibrium.

Lemma 1. *In any monotonic equilibrium, the high and low types invest in their idiosyncratic strategy if and only if*

$$s \geq h(s^m) \text{ and } s \geq l(s^m), \tag{11}$$

respectively, where

$$l(s^m) - h(s^m) = \frac{1-\psi}{\psi} \cdot (H - L). \tag{12}$$

Hence, the more confident the manager becomes about her alpha, the more likely she is to use her idiosyncratic strategy instead of an index-tracking strategy. In addition, the fact that the high type's cutoff is lower captures the fact that a competent manager uses her idiosyncratic investment strategy relatively more often. In particular, the existence of the interval $[h, l]$ represents the range of values of s for which it would be too costly for a low-skilled manager to imitate the strategy of a highly skilled manager.

Second, we want to calculate the manager's posterior reputation after each investment decision as a fuction of her performance.

Lemma 2. **(Posteriors)**: *In any monotonic equilibrium, the manager's posterior reputation at the beginning of the second period, if she invested in her alpha* ($\beta = 0$) *in the first period, is*

$$\varphi_0(r, s^m) = \left(1 + \frac{1-\pi}{\pi} \times \rho(r) \times \frac{\Phi\left(\frac{r - l(s^m)(1+\psi) + L\psi}{v\sqrt{1+\psi}}\right)}{\Phi\left(\frac{r - h(s^m)(1+\psi) + H\psi}{v\sqrt{1+\psi}}\right)}\right)^{-1}, \tag{13}$$

where

$$\rho(r) = \exp\left(\frac{-2(H-L)r + H^2 - L^2}{2v^2\psi(1+\psi)}\right).$$

Conversely, if she invested in the market ($\beta = 1$), *this becomes*

$$\varphi_1(s^m) = \left(1 + \frac{1-\pi}{\pi} \times \frac{\Phi\left(\frac{l(s^m) - L}{v}\right)}{\Phi\left(\frac{h(s^m) - H}{v}\right)}\right)^{-1}. \tag{14}$$

Investors form their posterior belief about the manager's ability by observing her investment decision β and the realized return r. When using her idiosyncratic investment strategy, the manager's performance r is generated by her alpha. Hence, in this case the realization r carries additional information about the manager's ability. In contrast, when using the index-tracking strategy, r is equal to the market's return m, which carries no additional information about the manager's ability. This is why φ_0 is a function of r, but φ_1 is not.

It is important to note that these posterior beliefs are consistent with our intuition about the way investors evaluate managers. That is, if a manager invests in a market-neutral strategy ($\beta = 0$), she is perceived as more skilled as her returns grow ($\varphi_0(r, s^m)$ increasing in r). However, if a manager invests in the market ($\beta = 1$), then the higher the market return, the more likely it is that the manager's reputation is high ($\varphi_1(s^m)$ increasing in s^m).[9] Using these two lemmas, we prove the main result of this part.

Proposition 1. *A monotonic equilibrium always exists. Moreover, a sufficient condition for it to be unique is that*

$$\delta \times \lambda \times (H - L) \leq \psi^2 \times v^2. \tag{15}$$

We believe that this condition is satisfied for a wide range of parametric specifications that we consider natural given the economic setting we study. This translates into two requirements: first, that the difference between the ability of the two types is not too big and, second, that the precision of the signal s is neither so small that it becomes irrelevant nor so big that the manager's ex ante ability α becomes irrelevant.

3.3 | Results

Here, we present some important properties of the unique monotonic equilibrium. We assume throughout that Equation (15) holds. To keep the notation as light as possible, we use $\varphi_0(r, s^m)$ and $\varphi_1(s^m)$ to refer to the equilibrium reputations, which are obtained after substituting the corresponding values for $h(s^m)$ and $l(s^m)$.

Proposition 2. (*Pointwise Dominance*): *There is a strict reputational benefit for the manager from investing in her alpha, that is,*

$$\varphi_0(r, s^m) > \varphi_1(s^m), \text{ for all } r, s^m \in \mathbb{R}. \tag{16}$$

We already know that in every monotonic equilibrium, conditional on a choosing a market-neutral strategy, high performance is beneficial for the manager's reputation (as also noted in Berk & Green, 2004). This propositions goes further to note that when investors can perfectly observe the strategy of the manager, the mere choice of an idiosyncratic strategy ($\beta = 0$) gives managers a reputational benefit.[10] The reason is that investors take into account the strategic choice of the manager and conclude that any idiosyncratic bad returns are more likely due to an unlucky realization of the strategy's return (a_t) than to a low skill (α). Indeed, in any monotonic equilibrium, the manager's choice to invest in the market reveals to investors that her signal about her idiosyncratic strategy is low enough; this serves as a strong signal to the investor that the fund manager is likely to be of the low type.

Proposition 2 may seem counterintuitive at first, but we view it as a proxy of how investors perceive fund managers in reality, even though it is the most challenging to empirically verify. This is because, for each fund, we never observe the counter-factual; i.e., we do not know how the fund's flows would look if it had chosen a lower (or higher) beta strategy. Moreover, the simplifying assumption $\beta \in \{0, 1\}$ makes this result stronger than what an

[9]The belief $\varphi_1(s^m)$ does not directly depend on the market return m because investors observe that the signal s^m is the signal on which managers base their investment decision. However, in expectation, we know that the higher the market return, the higher the signal s^m.

[10]It may still be the case that a manager investing in the market is perceived more favorably than one who invests in her own project and does badly, as long as the market signal s^m in the two cases is different.

alternative model, where the two possible values of beta would be closer to each other, would give. Despite that, we believe that to a certain extent, a low-beta strategy creates enough signaling value to counter the effect of low subsequent performance. Our result can also be informative in guiding the discussion on the circumstances under which managers decide to invest in the market ($\beta = 1$), other than simply investing during a bull market (when s^m is likely to be large). However, our finding depends on several assumptions: Managers choose their investment decision strategically and there are no dumb managers,[11] their idiosyncratic strategies have the same variance (but different expectations),[12] managers do not have a differential market-timing abilities (i.e., both types of managers receive the same signal about the market), and there is a monotonic equilibrium.

As a result of pointwise dominance, we can now get the following interesting proposition, which characterizes the effect of the manager's career concerns on her investment behavior.

Proposition 3. (Investment Behavior): *The equilibrium cutoffs $h(s^m)$ and $l(s^m)$ are decreasing in the discount factor δ. Moreover, there is overinvestment in the manager's idiosyncratic project, that is*

$$h(s^m) \leq c(H, s^m) \text{ and } l(s^m) \leq c(L, s^m). \tag{17}$$

This result does not necessarily require pointwise dominance. From the proof in Appendix A, one can see that it is sufficient that there is simply an expected (positive) signaling value of investing in the idiosyncratic project; in particular, we only need that

$$\mathbb{E}_r\left[\log\left[\frac{\tilde{\varphi}_0(r,h)(u^H - u^L) + u^L}{\tilde{\varphi}_1(h)(u^H - u^L) + u^L}\right] \mid s = h, H\right] > 0.$$

We use the term *overinvestment* to describe the fact that the manager invests in her idiosyncratic strategy more often than in the absence of career concerns. In other words, overinvestment exists when the manager lowers her standards with regard to her private signal; that is, she lowers the confidence level required for her to choose the idiosyncratic investment. The manager's optimal cutoff, in the absence of career concerns, corresponds to the cutoff already derived from the analysis of the second period in Equation (8). This is because it is generated by the inefficiency in the investment decision that the manager's career concerns create, which is connected to the underlying parameter δ.

Proposition 3 demonstrates there is a bias toward active management in the financial intermediation industry, which is due to its inherent informational asymmetries. To be precise, we expect managers to receive, on average, less exposure to the market than what would maximize the fund's expected return. This action is associated with competence and it is rewarded with an increase in the fund's AUM. Hence, our model provides a theoretical justification for this apparent bias. We view our results as complementary, not contradictory, to the empirical evidence found in studies of the tendency of funds to exhibit herding behavior (e.g., Cremers & Petajisto, 2009; Scharfstein & Stein, 1990; Wermers, 1999). On the one hand, when the market is performing well, managers with some timing ability are likely to invest in it, and hence they appear as if they are herding. On the other hand, although we study the incentives of managers to avoid a market-neutral strategy, it may be that the idiosyncratic strategies of managers are actually correlated. Moreover, as discussed earlier, our theory provides a better understanding of the assumptions that can lead to one phenomenon over the other.

Next, we want to see how this bias depends on the unobserved, to the econometricians, market signal s^m and the manager's prior reputation π.

[11] This is in contrast to the seminal paper of Scharfstein and Stein (1990), who assume there are managers with purely noisy signal and show that they may choose to ignore their private information and "follow the herd" when they receive correlated information.

[12] If, for instance, the variance of returns of a Low-type manager was higher, then we would imagine that a very low return would be a very bad signal for the skill of the manager and Proposition 2 would no longer hold.

Proposition 4. *The cutoffs $h(s^m)$ and $l(s^m)$ are increasing in the market signal s^m.*

In addition, there exist lower bounds $\bar{s}^m$ and $\bar{\pi}$, such that for all (s^m, π) with $s^m \geq \bar{s}^m$ and $\pi \geq \bar{\pi}$, both cutoffs $h(s^m)$ and $l(s^m)$ are also increasing in the manager's prior reputation π.

Proof. The proof of the first statement is similar to that of Proposition 3. The proof of the second statement follows from Lemma 7, which can be found in Appendix A.

The first statement is an intuitive result. The better the manager expects the market portfolio to perform, the more eager she becomes to invest in it, which translates into higher equilibrium cutoffs.

The crucial implication of the second statement in Proposition 4 is that the bias created from the signaling value, of investing in the idiosyncratic strategy, is decreasing in the manager's prior reputation. This is because the equilibrium cutoffs are bounded above by the expected return-maximizing cutoff $c(\alpha, s^m)$. Hence, the more π increases, the closer the cutoffs get to it.

A caveat of this result is that it only holds for a manager who is already relatively recognized in the market. In particular, we show in Appendix A that we need at least $\pi > 1/2$. Intuitively, the closer the prior is to either 0 or 1, the less it is affected by the actions of the manager. To make this more concrete, think of the extreme case where $\pi \to 1$, in which case it is difficult for investors to change their opinions about the manager's ability as they already know it with almost total certainty. Hence, there is a corresponding result that can be stated for managers with very low reputations. Even though in our model we allow for small funds to stay active, in reality most of them would either shut down or would not even be reported in most data sets. Therefore, we focus on funds with reputations greater than 1/2.

Another interesting feature of our specification is that it provides a better understanding of how the sensitivity of the fund's asset flows to its performance depend on market conditions. Let $\varphi^i(r^i, s^m, \beta^i)$ stand for manager i's reputation and call $d\varphi^i/dr^i$ its sensitivity with respect to her performance.

Proposition 5. *The conditional probability that a manager has invested in the market portfolio $\mathbb{P}(\beta_t^i = 1 \mid m_t)$ is increasing in its contemporaneous performance m_t. In addition, for a sufficiently reputable manager, the conditional expected sensitivity of the manager's reputation with respect to her performance, that is, $\mathbb{E}_{s^m}[d\varphi^i/dr^i \mid m_t]$, is decreasing in m_t.*

When markets are expected to perform well, the manager's direct incentives outweigh those of career concerns. Hence we know from Proposition 4 that she is more likely to give up the reputational benefit of following a low-beta strategy. However, high-beta strategies carry no information about the manager's ability. Hence, even though, as noted in Proposition 2, investing in low beta always has a reputational benefit, this benefit is less pronounced in good markets. Therefore, investors are expected to rely more on a manager's performance to update their belief about her ability when markets are bear than when they are bull.

3.4 | Competition between funds

3.4.1 | Theoretical contribution

It follows from the previous discussion that managers are judged much more strictly on their performance in bear markets than in bull markets. This in turn has some implications for the relative ranking of funds with respect to their reputations, or equivalently their AUM.

To study this, we extend our model by allowing a second manager to operate in the market. We formally define the investor's preference shock in this case and derive the corresponding AUM of the two funds in Appendix B. In fact, the whole analysis of this article and all our results remain unchanged with the addition of a second manager. The reason is that the manager's utility is such that it is only a function $\varphi_\beta(r, s^m) \times (u^H - u^L) + u^L$ and is independent of the number of managers that exist in the model. Indeed, as can be seen from Lemma 8 in Appendix B, the logarithm of the AUM is always a linear function of the previous expression; thus, even though the existence of competition leads to a different outcome in terms of the allocation of funds, from the perspective of any manager, her utility maximization problem and her investment decision remain the same.[13]

Our main aim is to study the likelihood of a change in the rank of managers in terms of investors' beliefs about their ability and relate that to market conditions. If we assume that initially $\pi^2 > \pi^1$, then this likelihood is represented by $\mathbb{P}(\varphi^1 > \varphi^2 \mid s^m)$. In what follows, we explain why this probability is not monotonic in m_t. We always condition on s^m as we know that all investors observe this market signal. For tractability, we assume that all managers observe the same market signal s^m; qualitatively, the result would not change if each manager were observing a noisy signal over the aggregate signal s^m, as long as they all have the same market-timing ability.[14] Indeed, let $\mathbb{P}(i, j \mid s^m) = \mathbb{P}(\beta_{\text{fund1}} = i, \beta_{\text{fund2}} = j \mid s^m)$, for $i, j \in \{0, 1\}$. In Appendix A we show that

$$\mathbb{P}(\varphi^1 > \varphi^2 \mid s^m) = \mathbb{P}(\varphi_0^1 > \varphi_1^2 \mid s^m)\,\mathbb{P}(0, 1 \mid s^m) + \mathbb{P}(\varphi_0^1 > \varphi_0^2 \mid s^m)\,\mathbb{P}(0, 0 \mid s^m). \tag{18}$$

What Equation (18) suggests is that the ranks of managers can change through two possible scenarios. In the first scenario, with probability $\mathbb{P}(0, 1 \mid s^m)$, one of the managers invests in her idiosyncratic portfolio and the other follows the market. This probability approaches zero for both very large and very small s^m, as then both managers invest in the market or both invest in their own project. In turn, this makes the first term on the right-hand side of Equation (18) hump shaped in s^m. Under this scenario, Manager 1 has a reputational benefit from choosing $\beta = 0$ (see Proposition 2), which then makes it possible for her ex post reputation to be higher than that of Manager 2 (despite her initial disadvantage, in terms of the priors π^1, π^2). Clearly the smaller the distance between the managers' prior reputations, $\pi^2 - \pi^1$, the larger this likelihood will be.

In the second scenario, with probability $\mathbb{P}(0, 0 \mid s^m)$ both managers invest in their own project and Manager 1 receives a much higher return than Manager 2, thus overcoming the effect of the initial prior reputations. In other words, because $\pi^1 < \pi^2$, for the posterior reputations to have the opposite order, what needs to happen is that the realized return of Manager 1 is much higher than that of Manager 2. This is clearly not possible if they both invest in the market. However, when they both invest in their idiosyncratic project, this can happen either because one is luckier than the other or because Manager 1 has high skill and Manager 2 has low skill. This scenario is less likely to occur as the market conditions get better because $\mathbb{P}(0, 0 \mid s^m)$ is decreasing in s^m, as we can see from Proposition 5. Moreover, we get the following remark:

Remark 1. The likelihood of a change in the ranks of managers is higher in a very bad market than in a very good market. That is,

$$\lim_{s^m \to -\infty} \mathbb{P}(\varphi^1 > \varphi^2 \mid s^m) > \lim_{s^m \to +\infty} \mathbb{P}(\varphi^1 > \varphi^2 \mid s^m). \tag{19}$$

The proof of this remark is simple. As the market becomes really good, the probability of a manager investing in his own project goes to zero, and hence from Equation (18) we see that the probability of a rank change will tend to

[13]In particular, Equation (A5) in Appendix A and, thus, the determination of the cutoffs l and h remain the same; technically, the assumption of log preferences is crucial for this result.

[14]If the latter were not the case, and highly skilled managers were also receiving more precise market signals, the inference of the investor would also depend on the difference $m - s^m$, as the closer to zero this is, the more precise the manager's signal would be.

zero. In contrast, for a very negative market signal, this probability is strictly positive, as $\mathbb{P}(0, 0 \mid s^m) = 1$ and $\mathbb{P}(\varphi_0^1 > \varphi_0^2 \mid s^m) > 0$.[15]

From the preceding analysis, it is clear that the overall effect does not have to be monotonic in s^m. Hence, we use simulations in Figure 1 to illustrate the properties of the probability of interest as a function of the market signal, also confirming the observation in the aforementioned remark.

On the y-axis of Figure 1, we have the probability of change in rank, and on the x-axis the corresponding market signal. As can been seen from the graph, the total effect is hump shaped in s^m: It is decreasing as the market signal becomes relatively large and it is smaller when market conditions are good compared to when they are bad.

In what follows, we find empirical evidence supporting our results. We create divisions by grouping funds in accordance with their AUM. We subsequently calculate the proportion of funds that change division from the beginning of each period to its end. Approximately, this measures the probability to which Remark 1 refers.

3.4.2 | Empirical evidence

The goal of this section is to find support for Remark 1 by demonstrating that the probability of changes in the ranking of funds, with respect to their AUM, is higher under adverse market conditions.[16] We use data from the Morningstar Center for International Securities and Derivatives Markets (CISDM) database. Our sample spans January 1994 to December 2015, and defunct funds are included in the sample. Overall, we have 415,002 fund/date observations from 4454 funds. Observations of performance or AUM, with more than 30 missing values, are deleted. Our main variable of interest is the AUM. For the market return, we consider the S&P 500; in particular, we use the corresponding Fama–French market factor obtained from the Wharton Research Data Services (WRDS) database (or from Kenneth French's website[17]). Finally, we compare only funds that were active during the whole duration of each period and consider only the US dollar universe of funds to avoid introducing noise created from fluctuations in the exchange rate.

To perform our analysis, we cluster our observations by period and rank the funds according to size. In particular, the sample is separated in 30 periods (with each period consisting of 8 months). Then, for each period, 70 clusters of funds[18] are created and funds are allocated in those clusters according to the size of their AUM at the end of each period.[19] We define a new variable, $divjumpassetUSD_t$, as the percentage of funds that changed division from the beginning to the end of period t to see whether the changes in the fund order size happen more frequently during bear markets.

Figure 2 shows a plot of our results. The y-axis shows our constructed measure ($divjumpassetUSD_t$) of changes between divisions and the x-axis shows the corresponding return of the S&P 500 (*periodmkt*), which is our proxy for the market portfolio during the same period. As it can been seen from the graph, there appears to be a negative relation between the two, which is statistically significant. We view this as preliminary evidence that in bear markets we see the largest changes in the funds' rankings, because during these times investors can better understand the abilities of the fund managers. Note that this is just an indication of the relation between fund rankings and market conditions under a simple linear regression, and thus it does not capture any second-order effects (or, for instance, a potential hump-shaped relation). A more thorough empirical investigation of this result, as well as of the other predictions in our model, concerning the characteristics of flow-performance sensitivity would be interesting, but it is outside the scope of this article.

[15] Intuitively the return of Manager 1 may be much larger than that of Manager 2 when they both invest in their own projects (either because one has high skill and the other has low skill or because one is just luckier than the other) and hence there is a positive probability there will be a change in ranks.

[16] We only want to provide some indicative evidence to support our theoretical prediction; we leave the full empirical analysis for future research.

[17] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[18] We run robustness checks with different numbers of clusters and different period lengths. The results remain qualitatively the same.

[19] Our methodology closely follows Marathe and Shawky (1999) and Nguyen-Thi-Thanh (2010), and the competition between funds is also studied by Wahal and Wang (2011).

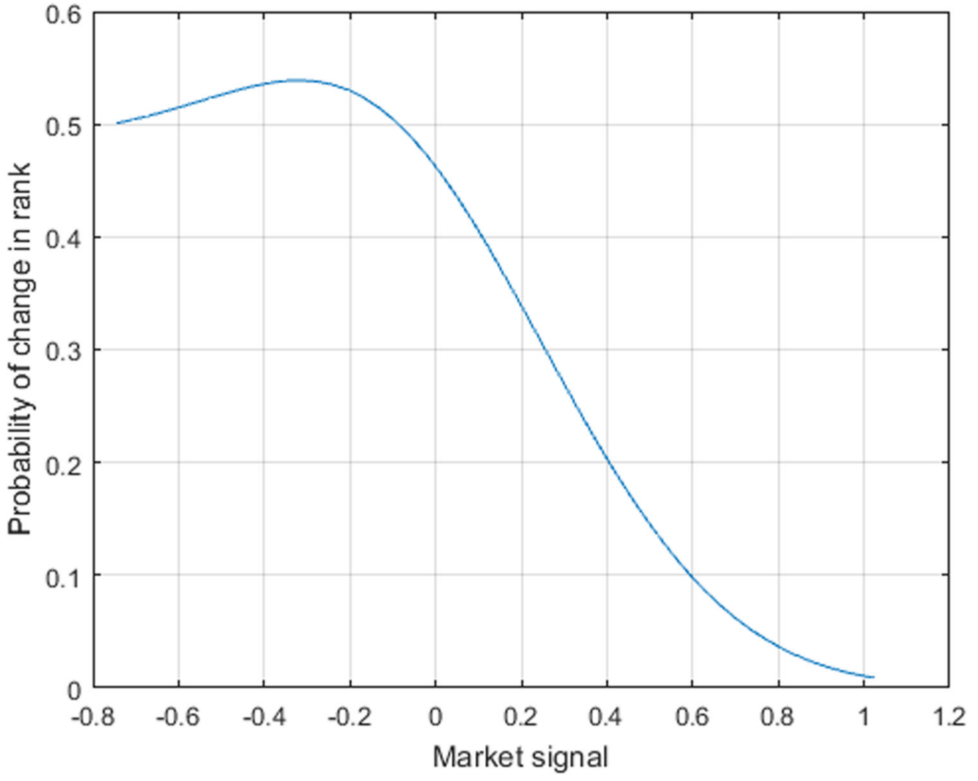

FIGURE 1 Competition between funds depending on market conditions. This graph shows how the (simulated) probability of a change in the rank (based on reputation) of two managers depends on the signal they receive on the market return. Overall, when market conditions are bad (market signal is low), the likelihood of a change in rank is significantly higher. The parameters used are: $\pi^1 = 0.6$, $\pi^2 = 0.601$, $\alpha^H = 0.16$, $\alpha^L = 0.1$, $\sigma = v = 0.35$, $f^1 = f^2 = 0.01$, $\sigma_m = v_m = 0.25$, $\lambda_1 = \lambda_2 = 0.8$, and $\delta = 0.5$. [Color figure can be viewed at wileyonlinelibrary.com]

4 | EXTENSION: UNOBSERVABLE INVESTMENT DECISION

In this section, we extend our model and investigate the equilibrium where the investment decisions of fund managers cannot be observed by investors. In this case, investors use the return of fund managers to both update their beliefs about managers' skill and understand whether they invested in their own project. In reality, investors do not know exactly a fund manager's exposure to systematic risk. Instead, they use a history of the fund return's comovement with the market return to infer the fund's statistical beta. Because the model we are examining is static, our assumption in this section is that this inference is based only on the proximity of the market return to the fund's return.

The model considers only one period and it remains largely the same as before, apart from a few changes. First, an additional error ϵ is introduced to make the manager's choice of investment unobservable by investors. (Note that without this tracking error, investors could perfectly observe the decision of managers based on whether $r = m$.) Hence, our model becomes:

$$\begin{aligned} r &= (1-\beta)\,a + \beta(m+\epsilon) \\ a &\sim \mathcal{N}(\alpha, \sigma^2) \\ m &\sim \mathcal{N}(\mu, \sigma_m^2) \\ \epsilon &\sim \mathcal{N}(0, \sigma_\epsilon^2). \end{aligned} \tag{20}$$

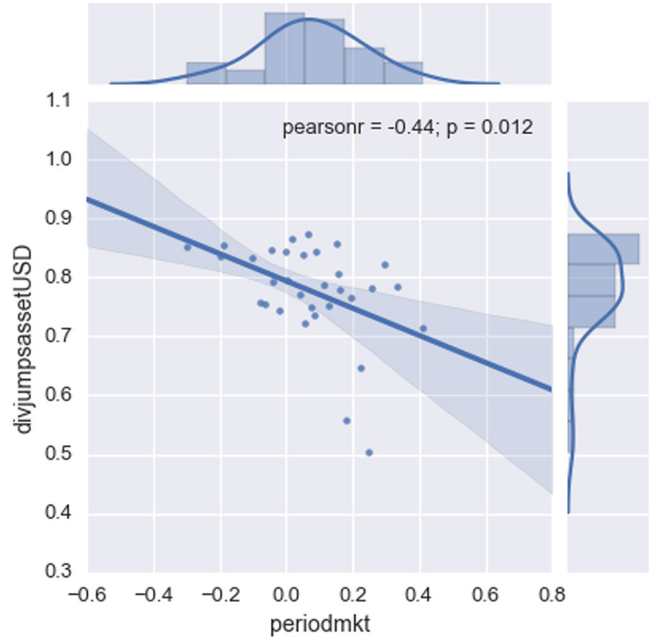

FIGURE 2 Change in funds' rankings depending on market return. This graph depicts the relation between the percentage of funds that change division in a period (where we have partitioned our sample into 70 groups ranked by their assets under management [AUM]), denoted by $divjumpassetUSD_t$, and the corresponding market return in that period, denoted by *periodmkt*. The regression line shows there is a negative relation between these two variables. [Color figure can be viewed at wileyonlinelibrary.com]

As before, we study only the simple binary case where $\beta \in \{0, 1\}$. The rest of the notation and ideas remain unchanged.

Our goal is to study whether a monotonic cutoff equilibrium (introduced in the previous sections) exists under this alternative assumption. We believe that only such an equilibrium would be interesting and realistic for further study. We find a closed-form expression for ex post reputation φ, which is given by the following lemma.

Lemma 3. *The manager's posterior reputation is given by*

$$\varphi(r, m, s^m) = \left(1 + \frac{1-\pi}{\pi}\frac{\rho(r, L, l(s^m))}{\rho(r, H, h(s^m))}\right)^{-1}, \tag{21}$$

where

$$\rho(r, \alpha, c) = \Phi\left(\frac{r - c(1+\psi) + \alpha\psi}{v\sqrt{1+\psi}}\right) \times \frac{\phi\left(\frac{r-\alpha}{v\sqrt{\psi(1+\psi)}}\right)}{v\sqrt{\psi(1+\psi)}} + \Phi\left(\frac{c-\alpha}{v}\right)\frac{\phi\left(\frac{r-m}{\sigma_\epsilon}\right)}{\sigma_\epsilon}. \tag{22}$$

Using Lemma 3, we can now see whether this model can provide us with an equilibrium where reputation $\varphi(r, m, s^m)$ is increasing in r. In fact, we get the following proposition.

Proposition 6. *A monotonic equilibrium under unobservable beta does not exist.*

What Proposition 6 shows is that reputation $\varphi(r, m, s^m)$ cannot always be increasing in r under the assumption that investors do not observe investment choices. The reason, as illustrated in the proof in Appendix C, is that in a strong bull market, the a manager's good performance may erroneously lead investors to believe that it is likely she was following the market and thus may lead to a loss of reputation (because investors cannot directly observe whether $\beta = 1$). That is, the assumption of unobservable investment choice under a static setting can lead us to counterintuitive equilibrium properties. In future research, it would be interesting to study this realistic case under a dynamic setting where the inference of beta is based on the comovement of the market return with the fund's return.

5 | CONCLUSIONS

Managing other people's money is at the core of modern capitalism, as is the agency problem, where money managers allocate funds in an optimal way for themselves rather than for the investors who give them money to manage. Although the agency problem is present in many industries, many of its implications have not been thoroughly studied in the context of the financial industry. Investors have fund managers on a short leash. Reputations can vanish far faster than they rise, leading to biases and distortions that are suboptimal for investors. Addressing this problem in the financial service industry, we develop a model that describes how the strategic investment decisions of fund managers are influenced by their career concerns.

To sum up our argument, these managers tend to overinvest in market-neutral strategies as a way to signal their ability. Moreover, we describe how a manager's reputation depends on market conditions. We find that the sensitivity of fund flows to performance is higher in bear markets than in bull markets and discuss the competition among funds, measured by the changes in their rankings, as a function of market conditions. We find empirical evidence in support of our findings. Our model includes predictions about some directly observable fund characteristics, such as size and fees, as well as some indirectly observable quantities, such as reputation and investment behavior depending on their signals. Finally, we extend our framework to include the case when the manager's investment decision is not observable by investors.

There are many ways forward with this research. The results of this model do not depend on the specific factor that funds use when they are tracking an index. Future research could apply the same logic to funds that use factors other than the market return and test the corresponding empirical predictions. In addition, using a slightly different interpretation of an investor's decision between allocating funds to a manager or to the market, future research could focus on an investor choosing between an active and a passive fund and use the closed-form solution for the fund size to see how the relative (total) size of the passive and active funds depend on market conditions.

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APPENDIX A: PROOFS

Proof of Lemma 1. Using Equation (10), it is easy to argue that both idiosyncratic and index-tracking strategies have to be chosen with positive probability. This is because the effect of reputation $\varphi_\beta(\cdot)$ on the manager's payoff is bounded, whereas that of current return r is not. However, this implies that $\varphi_0(\cdot)$ is calculated using Bayesian updating, and as a result it cannot be a function of r because in this case r provides no information on the manager's ability α.

Fix s^m; then the manager's expected payoff while investing in an index-tracking strategy $\beta = 1$ is not a function of s. Conversely, her payoff under the idiosyncratic strategy is a function of r. In particular, it follows from the definition of monotonic equilibria that this is increasing in s, which proves that the manager's equilibrium strategy is a cutoff one, as presented in Equation (11).

In addition, the indifference condition that defines $h(s^m)$ is

$$\mathbb{E}_r[a + \delta \times \lambda \times \log[\varphi_0(r, s^m) \times (u^H - u^L) + u^L] \mid s = h(s^m), \alpha = H, s^m]$$

$$= \mathbb{E}_r[m + \delta \times \lambda \times \log[\varphi_1(s^m) \times (u^H - u^L) + u^L] \mid s^m],$$

and the indifference condition that defines $l(s^m)$ is

$$\mathbb{E}_r[a + \delta \times \lambda \times \log[\varphi_0(r, s^m) \times (u^H - u^L) + u^L] \mid s = l(s^m), \alpha = L, s^m]$$

$$= \mathbb{E}_r[m + \delta \times \lambda \times \log[\varphi_1(s^m) \times (u^H - u^L) + u^L] \mid s^m].$$

However, the right-hand sides of these two equations are the same; hence, the two expressions on the left-hand sides are equal. Therefore, the two conditional normals that are used in the two left-hand sides have to be the same,[20] which implies that

$$(1 - \psi) \times H + \psi \times h(s^m) = (1 - \psi) \times L + \psi \times l(s^m),$$

from which Equation (12) follows.

Proof of Lemma 2. The time subscripts as well as the signal s^m in the cutoffs $h(s^m)$ and $l(s^m)$ are suppressed when possible. To find the posterior $\varphi_0(r)$, we calculate

$$\mathbb{P}(r, \beta = 0 \mid s^m, H) = \mathbb{P}(r \mid \beta = 0, s^m, H) \times \mathbb{P}(\beta = 0 \mid s^m, H),$$

where

$$\mathbb{P}(\beta = 0 \mid s^m, H) = \mathbb{P}(s \geq h \mid s^m, H) = \Phi\left(-\frac{h - H}{v}\right), \tag{A1}$$

and

$$\mathbb{P}(r \mid \beta = 0, s^m, H) = \int_h^\infty \phi\left(\frac{r - (1 - \psi)H - \psi s}{\sqrt{\psi}v}\right) \times \frac{1}{\sqrt{\psi}v}\phi\left(\frac{s - H}{v}\right)\frac{1/v}{\Phi\left(-\frac{h - H}{v}\right)}\,ds$$

because the left-hand side can be rewritten as $P(\alpha \mid \alpha + \eta > h)$, where $\alpha \sim N(H, \sigma^2)$. Hence, substituting gives

$$\mathbb{P}(r, \beta = 0 \mid s^m, H) = \int_h^\infty \phi\left(\frac{r - (1 - \psi)H - \psi s^i}{\sqrt{\psi}v}\right)\frac{\phi\left(\frac{s - H}{v}\right)}{\sqrt{\psi}v^2}\,ds.$$

Let $\tilde{s} = (s - H)/v$. Then the preceding equation becomes

[20] Otherwise, one random variable would stochastically dominate the other because they have the same variance. However, the expectations in the two left-hand sides could not be the same because $\varphi_0(r, s^m)$ is increasing in r.

$$\mathbb{P}(r,\beta=0\mid s^m,H) = \int_{\frac{h-H}{v}}^{\infty}\phi\left(\frac{r-H}{\sqrt{\psi}\,v}-\sqrt{\psi}\,\tilde{s}\right)\frac{\phi(\tilde{s})}{\sqrt{\psi}\,v}\,d\tilde{s}$$

$$=\frac{\phi\left(\frac{r-H}{v\sqrt{\psi(1+\psi)}}\right)}{v\sqrt{\psi(1+\psi)}}\Phi\left(\frac{r-h(1+\psi)+H\psi}{v\sqrt{1+\psi}}\right).$$

Repeat the same process to find $\mathbb{P}(r\mid\ \beta=0,s^m,L)$ and observe that it follows from Bayes' rule that

$$\varphi_0(r)=\left(1+\frac{1-\pi}{\pi}\frac{\mathbb{P}(r,\beta=0\mid s^m,L)}{\mathbb{P}(r,\beta=0\mid s^m,H)}\right)^{-1},$$

from which the provided formula follows. To derive φ_1, use Bayes' rule to get

$$\varphi_1=\left(1+\frac{1-\pi}{\pi}\frac{\mathbb{P}(\beta=1\mid s^m,L)}{\mathbb{P}(\beta=1\mid s^m,H)}\right)^{-1},$$

where $\mathbb{P}(\beta=1\mid s^m,\alpha)=1-\mathbb{P}(\beta=0\mid s^m,\alpha)$, which is derived earlier.

To prove the existence of an equilibrium, we need the following three lemmas:

Lemma 4. *If $M(\cdot)$ is the normal hazard function, then for $a\ge b$ we have,*

$$M(a)-M(b)\le a-b.$$

Proof. Because the hazard function is a continuous function, we can use the mean value theorem, which says that for any $a>b$ there exists a $\xi\in(a,b)$ such that $M(a)-M(b)=M'(\xi)(a-b)$. Therefore, it is sufficient to prove that $M'(\xi)<1$ for any ξ. To prove that, note that $M(\cdot)$ is convex, and hence $M'(\cdot)$ is increasing, so it would be sufficient to prove that $\lim_{x\to\infty}M'(x)=1$. Now we use the following inequality for the normal hazard function. We know that for $x>0$,

$$x<M(x)<x+\frac{1}{x}.$$

But this easily implies that $M(x)$ has x as its asymptote as $x\to\infty$ (i.e., $\lim_{x\to\infty}M(x)-x=0$). Finally this implies that $\lim_{x\to\infty}M'(x)=1$ and this completes the proof (note that the limit exists because $M'(\cdot)$ is increasing and bounded, as $M'(x)=M(x)(M(x)-x)<1+1/x^2<2$).

Lemma 5. *The time subscript is suppressed. A sufficient condition for $\varphi_0(r,s^m)$ to be increasing in the manager's performance r is that*

$$(H-L)\times\frac{1-\psi}{\psi}\ \ge\ l(s_1^m)-h(s_1^m).$$

Proof. Suppress inputs (r,s^m), and super/subscripts. Differentiating gives

$$\frac{d\varphi}{dr} = -\frac{\varphi(1-\varphi)}{v\sqrt{1+\psi}}\left[-\frac{H-L}{v\psi\sqrt{1+\psi}} + M\left(-\frac{r-l(1+\psi)+L\psi}{v\sqrt{1+\psi}}\right) - M\left(-\frac{r-h(1+\psi)+H\psi}{v\sqrt{1+\psi}}\right)\right]. \tag{A2}$$

Let

$$\delta^L = l(1+\psi) - L\psi$$
$$\delta^H = h(1+\psi) - H\psi,$$

and then the above is positive if and only if

$$\frac{H-L}{v\psi\sqrt{1+\psi}} \geq M\left(\frac{\delta^L - r}{v\sqrt{1+\psi}}\right) - M\left(\frac{\delta^H - r}{v\sqrt{1+\psi}}\right).$$

However, using Lemma 4, we see that the right-hand side is bounded by

$$\frac{\delta^L - \delta^H}{v\sqrt{1+\psi}} = \frac{(l-h)(1+\psi)+(H-L)\psi}{v\sqrt{1+\psi}}.$$

Hence, a sufficient condition for the inequality to hold is

$$\frac{H-L}{\psi} \geq (l-h)(1+\psi)+(H-L)\psi \Leftrightarrow (H-L)\frac{1-\psi}{\psi} \geq l-h.$$

Lemma 6. *For $c > 0$, let*

$$\mu(x) = \left(1 + c\frac{\Phi(a_0 + b\,x)}{\Phi(a_1 + b\,x)}\right)^{-1}.$$

Suppose $b > 0$, then $\mu'(x) > 0 \Leftrightarrow a_1 < a_0$, whereas $b < 0$ implies that $\mu'(x) > 0 \Leftrightarrow a_1 > a_0$.

Proof. Differentiating gives

$$\mu'(x) = -b\mu(x)[1-\mu(x)] \times [M(-a_0 - b\,x) - M(-a_1 - b\,x)].$$

Then the statement follows simple from the fact that $M(\cdot)$ is increasing.

Proof of Proposition 1. Suppress time subscript t. Also suppress the signal s^m in the cutoffs $h(s^m)$ and $l(s^m)$ and in reputations $\varphi_0(\cdot)$ and $\varphi_1(\cdot)$.

We start by proving existence. As we argue in Lemma 1, in any monotonic equilibrium the optimal strategy of a high- and low-type manager is to pick $\beta = 0$ whenever her signal s is above the cutoffs h and l, respectively. In addition, another necessary implication is that h and l satisfy Equation (12).

But then Lemma 5 together with Equation (12) give that $\varphi_0(r)$ is indeed increasing in r. Hence, the manager's best response to the functional forms of $\varphi_0(\cdot)$ and $\varphi_1(\cdot)$, as given in Lemma 2, is to use the cutoff strategies that Lemma 1 describes.

All that remains to prove existence is to show that those cutoffs always exist. To do this, the manager's payoff maximization problem when picking the first-period beta is as given in Equation (10). Let her expected payoff when picking $\beta = 0$ be denoted by

$$v_0(s,\alpha) = (1-\psi)\times\alpha + \psi\times s + \delta\times\lambda\times\mathbb{E}_r[\log(\varphi_0(r)(u^H-u^L)+u^L)\mid s,\alpha],$$

whereas for $\beta = 1$ this becomes

$$v_1 = (1-\psi_m)\times\mu + \psi_m\times s^m + \delta\times\lambda\times\log(\varphi_1(u^H-u^L)+u^L).$$

Then v_1 is bounded and $v_0(s,\alpha)\in(-\infty,+\infty)$. Hence, the manager uses both low- and high-beta strategies depending on s. Next we provide the equation that defines those cutoffs. Rewrite l as a function of h according to

$$l(h) - L = h - H + \frac{H-L}{\psi}$$

and substitute this equality in $\varphi_0(r)$ and φ_1 to obtain the following two functions, in which only h appears out of the two equilibrium cutoffs. Substituting in $\varphi_0(r)$ gives

$$\tilde{\varphi}_0(r,h) = \left(1+\frac{1-\pi}{\pi}\times\rho(r)\times\frac{\Phi\left(\frac{r-h\cdot(1+\psi)+H\psi-(H-L)/\psi}{v\sqrt{1+\psi}}\right)}{\Phi\left(\frac{r-h\cdot(1+\psi)+H\psi}{v\sqrt{1+\psi}}\right)}\right)^{-1}, \tag{A3}$$

where h is introduced as an input of the function (which now also uses the tilde sign to track this change). Similarly, substituting in φ_1 gives

$$\tilde{\varphi}_1(h) = \left(1+\frac{1-\pi}{\pi}\times\frac{\Phi\left(\frac{h-H+(H-L)/\psi}{v}\right)}{\Phi\left(\frac{h-H}{v}\right)}\right)^{-1}. \tag{A4}$$

Then the cutoff h is given by the high type's indifference condition $v_0(h,H) = v_1$, which using the preceding notation becomes

$$\delta\times\lambda\times\int\log[\tilde{\varphi}_0(r,h)(u^H-u^L)+u^L]\times\phi\left(\frac{r-(1-\psi)H-\psi h}{\sqrt{\psi}v}\right)\frac{1}{\sqrt{\psi}v}\,\mathrm{d}r$$

$$= \delta\times\lambda\times\log[\tilde{\varphi}_1(h)(u^H-u^L)+u^L] + (1-\psi_m)\times\mu+\psi_m\times s^m-(1-\psi)\times H-\psi\times h, \tag{A5}$$

where $\phi(\cdot)$ is the density of the standard normal distribution. To prove existence, we demonstrate that Equation (A5) has at least one solution. Let $LHS(h)$ denote the left-hand side of (A5), $RHS(h)$ its right-hand side, and $\Delta(h) = LHS(h) - RHS(h)$ their difference. Observe that all the terms of the equation, apart from the last one, are bounded. As a result,

$$\begin{aligned}\lim_{h\to-\infty}\Delta(h) &= -\infty\\ \lim_{h\to+\infty}\Delta(h) &= +\infty.\end{aligned} \tag{A6}$$

Then it follows from the continuity of this function that there exists at least one point where $\Delta(h) = 0$. Hence, we have proven existence of such an equilibrium.

Next we show that Equation (15) is indeed a sufficient condition for uniqueness, because as we show, it implies that $\Delta(h)$ is increasing in h. First, note that $LHS(h)$ is increasing in h because $\tilde{\varphi}_0(r,h)$ is increasing in both r and h. We already argue why this is true for r. For h, the claim is a direct implication of Lemma 6.

Hence, it suffices to identify a condition for $RHS(h)$ to be decreasing. Lemma 6 implies that $\tilde{\varphi}_1(h)$ is increasing in h. This is the opposite monotonicity; however, we can use the fact that the following expression has a relatively simple upper bound

$$\frac{d}{dh}\log[\tilde{\varphi}_1(h)(u^H - u^L) + u^L] = \frac{\tilde{\varphi}_1(h)[1 - \tilde{\varphi}_1(h)]/v}{\tilde{\varphi}_1(h) + \frac{u^L}{u^H - u^L}} \times \left[M\left(-\frac{h - H}{v}\right) - M\left(-\frac{l(h) - L}{v}\right)\right]$$

$$\leq \frac{1}{v}\left[M\left(-\frac{h - H}{v}\right) - M\left(-\frac{l(h) - L}{v}\right)\right] = \frac{1}{v}\int_{\frac{L-(1-\psi)H}{\psi}}^{H} M'\left(\frac{x - h}{v}\right)dx \leq \frac{H - L}{\psi v^2}.$$

Therefore, a sufficient condition for the right-hand side to be decreasing, which implies uniqueness, is that

$$\delta\lambda^i \frac{H - L}{\psi v^2} \leq \psi,$$

which equivalently gives inequality (15).

Proof of Proposition 2. We know that $\varphi_0(r, s^m)$ is increasing in r. Hence, it suffices to prove the conjectured result for $r \to -\infty$. The dependence on s^m is suppressed. Let $k = -h(1 + \psi) + H\psi$. To find the limit $\lim_{r\to-\infty}\varphi_0(r)$, we first need to calculate

$$\lim_{r\to-\infty} \frac{\Phi\left(\frac{r + k - (H - L)/\psi}{v\sqrt{1 + \psi}}\right)}{\Phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)\exp\left(\frac{2(H - L)r - (H^2 - L^2)}{2v^2\psi(1 + \psi)}\right)}.$$

Because both the numerator and the denominator go to zero as r goes to minus infinity, this limit becomes

$$\frac{e^{\frac{H^2 - L^2}{2v^2\psi(1+\psi)}} \lim_{r\to-\infty} e^{-\frac{(H-L)r}{v^2\psi(1+\psi)}} \times \frac{\phi\left(\frac{r + k - (H - L)/\psi}{v\sqrt{1 + \psi}}\right)}{v\sqrt{1 + \psi}}}{\lim_{r\to-\infty}\left[\Phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)\frac{H - L}{v^2\psi(1 + \psi)} + \frac{\phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)}{v\sqrt{1 + \psi}}\right]}.$$

In addition, after a few algebraic manipulations we can get

$$e^{-\frac{(H-L)r}{v^2\psi(1+\psi)}}\frac{\phi\left(\frac{r + k - (H - L)/\psi}{v\sqrt{1 + \psi}}\right)}{\phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)} = \exp\left(\frac{2k - \frac{H - L}{\psi}}{2v^2(1 + \psi)\psi/(H - L)}\right).$$

Hence, the limit becomes

$$\exp\left(\frac{2k + H + L - \frac{H - L}{\psi}}{2v^2(1 + \psi)\psi/(H - L)}\right) \times \lim_{r\to-\infty}\left(\frac{\Phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)}{\phi\left(\frac{r + k}{v\sqrt{1 + \psi}}\right)}\frac{H - L}{v\psi\sqrt{1 + \psi}} + 1\right)^{-1},$$

where

$$\lim_{r\to-\infty}\frac{\Phi\left(\frac{r+k}{v\sqrt{1+\psi}}\right)}{\phi\left(\frac{r+k}{v\sqrt{1+\psi}}\right)}=\lim_{x\to\infty}\frac{1-\Phi(x)}{\phi(x)}=0.$$

Thus, substituting k we obtain

$$\lim_{r\to-\infty}\varphi_0(r)=\left(1+\frac{1-\pi}{\pi}\exp\left[\left(H-\frac{H-L}{2\psi}-h\right)\frac{H-L}{\psi v^2}\right]\right)^{-1}.$$

Next, we want to show that the prededing is greater than $\varphi_1(r)$ for every h. This holds if and only if

$$\left(H-\frac{H-L}{2\psi}-h\right)\frac{H-L}{\psi v^2}<\log\frac{\Phi\left(\frac{h-H+(H-L)/\psi}{v}\right)}{\Phi\left(\frac{h-H}{v}\right)}.$$

Differentiating the left-hand side minus the right-hand side, with respect to h, we get

$$-\frac{H-L}{\psi v^2}+\frac{1}{v}M\left(\frac{H-h}{v}\right)-\frac{1}{v}M\left(\frac{H-h}{v}-\frac{H-L}{v\psi}\right)\leq-\frac{H-L}{\psi v^2}+\frac{H-L}{\psi v^2}=0.$$

Hence, it suffices to check that

$$\lim_{h\to-\infty}\frac{\Phi\left(\frac{h-H}{v}\right)}{\exp\left(\frac{(H-L)h}{\psi v^2}\right)\Phi\left(\frac{h-H+(H-L)/\psi}{v}\right)}\leq\exp\left[\left(\frac{H-L}{2\psi}-H\right)\frac{H-L}{\psi v^2}\right].$$

Indeed, we can get that the limit on the left-hand side is

$$\begin{aligned}\lim_{h\to-\infty}\frac{\phi\left(\frac{h-H}{v}\right)}{\exp\left(\frac{(H-L)h}{\psi v^2}\right)\phi\left(\frac{h-H+(H-L)/\psi}{v}\right)}&=\lim_{h\to-\infty}\exp\left(\frac{2(h-H)+\frac{H-L}{\psi}}{2v^2}\frac{H-L}{\psi}-\frac{(H-L)h}{\psi v^2}\right)\\&=\exp\left[\left(\frac{H-L}{2\psi}-H\right)\frac{H-L}{v^2\psi}\right].\end{aligned}\tag{A7}$$

Therefore, the inequality holds.

Proof of Proposition 3. The input s^m is suppressed. First, note that h is the solution of Equation (A5), which is the solution of $\Delta(h)=0$. Second, the optimal cutoff under no career concerns for the high type $c(H)$ is the one that corresponds to the solution of this equation for $\delta=0$, as this corresponds to the case when the next period is irrelevant. Let $h(\delta)$ denote the solution of Equation (A5) as a function of δ. Then using the implicit function theorem, we get that

$$\frac{dh(\delta)}{d\delta}=-\frac{\partial\Delta(h)/\partial\delta}{\partial\Delta(h)/\partial h}\Big|_{h=h(\delta)}.$$

However, it follows from the limits calculated in Equation (A6) that the unique monotonic equilibrium needs to have $\partial\Delta(h)/\partial h>0$. Moreover, calculating the derivative on the numerator for some generic h gives

$$\frac{\partial\Delta(h)}{\partial\delta}=\lambda\,\mathbb{E}_r[\log[\tilde{\varphi}_0(r,h)(u^H-u^L)+u^L]-\log[\tilde{\varphi}_1(h)(u^H-u^L)+u^L]\,|\,s=h,H].$$

However, it follows from Proposition 2 that this is positive because the difference inside the expectation is positive for every h. As a result, for every $\delta \geq 0$ we get that $dh(\delta)/d\delta < 0$, which through Equation (12) implies the same for the cutoff used by the low type.

Finally, note that λ and δ enter Equation (A5) in exactly the same way; hence, the same result can be stated for λ.

Lemma 7. *In the unique monotonic equilibrium, for every prior reputation $\pi > 1/2$ there exists a lower bound $\bar{s}^m(\pi)$, defined as the solution of $\varphi_1(s^m) = 1/2$, such that for every $s^m > \bar{s}^m$ we have $\varphi_1(s^m) > 1/2$, and $\bar{s}^m(\pi^i)$ is increasing in π. In addition, for every $s^m \geq \bar{s}^m(\pi)$, the cutoffs $h(s^m)$ and $l(s^m)$ are increasing in π, and the same is true for the posterior reputations $\varphi_0(r, s^m)$ and $\varphi_1(s^m)$.*

Proof. In the proof of Proposition 1 it is shown that in the unique monotonic equilibrium there exists $\tilde{\varphi}_1$ such that $\varphi_1(s^m) = \tilde{\varphi}_1[h(s^m)]$, and its functional form is given in Equation (A4). Moreover, it is an immediate implication of Lemma 6 that this is increasing in h, and it is easy to verify that

$$\lim_{h \to +\infty} \tilde{\varphi}_1(h) = \pi.$$

In addition, it follows from Equation (A5), which defines $h(s^m)$, that

$$(1 + \psi)H + \psi h(s^m) + \delta\lambda \log\left(\frac{u^H}{u^L}\right) \geq (1 - \psi_m)\mu + \psi_m s^m.$$

This provides a lower bound for $h(s^m)$, which is in an increasing function of s^m, and shows that

$$\lim_{s^m \to +\infty} h(s^m) = +\infty, \tag{A8}$$

from which the existence of the cutoffs follows. Monotonicity then also follows after using the implicit function theorem on the equation that defines it, $\tilde{\varphi}_1[\pi, h(\bar{s}(\pi))] = 1/2$, because $\tilde{\varphi}_1$ is increasing in both π and h, and $h(\cdot)$ is also an increasing function (see Proposition 4).

For the second part of Lemma 7, note that from Equation (12) it suffices to prove this for $h(s^m)$. Using the implicit function theorem on Equation (A5), we get that $dh/d\pi = -(\partial\Delta/\partial\pi)/(\partial\Delta/\partial h)$, where direct differentiation gives $\partial\Delta/\partial h = \psi > 0$, and that

$$\frac{\partial\Delta}{\partial\pi} = \frac{\delta\,\lambda}{\pi(1-\pi)}\mathbb{E}_r\left[\frac{\tilde{\varphi}_0(1-\tilde{\varphi}_0)}{\tilde{\varphi}_0 + \frac{u^L}{u^H - u^L}} - \frac{\tilde{\varphi}_1(1-\tilde{\varphi}_1)}{\tilde{\varphi}_1 + \frac{u^L}{u^H - u^L}} \mid s = h, H\right],$$

where the inputs r and s^m are suppressed. We then note that for every $\tilde{\varphi} \in [1/2, 1]$, the ratio

$$\frac{\tilde{\varphi}(1-\tilde{\varphi})}{\tilde{\varphi} + \frac{u^L}{u^H - u^L}}$$

is decreasing in $\tilde{\varphi}$. Moreover, we have from Proposition 2 that $\tilde{\varphi}_0(r, h) > \tilde{\varphi}_1(h)$ for every $r \in \mathbb{R}$. But we already show that $\tilde{\varphi}_1(h) > 1/2$ for every $s^m \geq \bar{s}^m(\pi)$. Hence, we get that $\partial\Delta/\partial\pi < 0$, which implies the second statement.

Finally, the third statement follows trivially from noting that the direct derivatives of both posteriors with respect to π are positive, and the fact that both are increasing in $h(s^m)$, as implied by Lemma 6.

Proof of Proposition 5. First, consider the investment decision of a high-type manager, for which the probability of choosing the low-beta strategy, conditional on the market signal s^m, is

$$\mathbb{P}(\beta = 0 \mid s^m) = \mathbb{P}(s \geq h(s^m) \mid s^m) = \mathbb{P}(h^{-1}(s) \geq s^m \mid s^m),$$

because it is shown in Proposition 4 that $h(\cdot)$ is increasing. Moreover, for given s^m, the distribution of m is normal and is given by

$$m \mid s^m \sim \mathcal{N}((1-\psi_m)\mu + \psi_m s^m, \psi_m v_m^2).$$

Let $\tilde{m} = [m - (1-\psi_m)\mu]/\psi_m$. Then $\tilde{m} \mid s^m \sim \mathcal{N}(s^m, v_m^2/\psi_m)$, and the ex ante distribution of s^m is $s^m \sim \mathcal{N}(\mu, \sigma_m^2 + v_m^2)$. As a result, using again the properties of Bayesian updating with normal distributions we get that

$$s^m \mid \tilde{m} \sim \mathcal{N}\left(\tilde{\psi}\mu + (1-\tilde{\psi})\tilde{m}, \frac{\tilde{\psi} v_m^2}{\psi_m}\right),$$

where $\tilde{\psi} = (\sigma_m^2 + v_m^2)/(\sigma_m^2 + v_m^2 + v_m^2/\psi_m)$. Hence, for every $\hat{m}$, m such that $\hat{m} > m$, the distribution of the corresponding normal that generates s^m conditional on $\hat{m}$ first-order stochastically dominates that of s^m conditional on m. This immediately implies that

$$\mathbb{P}(\beta = 0 \mid \hat{m}) < \mathbb{P}(\beta = 0 \mid m). \tag{A9}$$

Therefore, under better observed market conditions the manager is less likely to invest in her idiosyncratic strategy. The second statement of the proposition follows from noting that

$$\frac{d\varphi_0(r, s^m)}{dr} \geq 0 = \frac{d\varphi_1(s^m)}{dr},$$

To calculate the left derivative, it is more convenient to use the equivalent $\tilde{\varphi}_0$ function from the proof of Proposition 1. The derivative of this can be calculated in a manner similar to that used in the proof of Lemma 5 to be

$$\frac{d\tilde{\varphi}_0(r, h)}{dr} = \frac{\tilde{\varphi}_0(1-\tilde{\varphi}_0)}{v\sqrt{1+\psi}}\left[\frac{H-L}{v\psi\sqrt{1+\psi}} - \int_{\underline{x}}^{\bar{x}} M'(x + h\sqrt{1+\psi}/v)dx\right],$$

where $M(\cdot)$ is the hazard rate of the standard normal distribution,

$$\underline{x} = -\frac{r + H\psi}{v\sqrt{1+\psi}} \text{ and } \bar{x} = \underline{x} + \frac{(H-L)/\psi}{v\sqrt{1+\psi}}.$$

Next we want to show that this derivative is decreasing in s^m. This appears in $\tilde{\varphi}_0$ only indirectly through the cutoff $h(s^m)$, which has already been shown to be an increasing function. Thus,

$$\frac{d^2\tilde{\varphi}_0(r, h)}{drdh} = \frac{1-2\tilde{\varphi}_0}{\tilde{\varphi}_0(1-\tilde{\varphi}_0)}\left(\frac{d\tilde{\varphi}_0(r, h)}{dr}\right)^2 - \frac{\tilde{\varphi}_0(1-\tilde{\varphi}_0)}{v^2}\int_{\underline{x}}^{\bar{x}} M''(x + h\sqrt{1+\psi}/v)dx,$$

where the last term is always negative, as $M(\cdot)$ is a convex function. The first term of the right-hand side is also negative as long as $\tilde{\varphi}_0(r, h) > 1/2$. However, we argue in Proposition 2 that $\tilde{\varphi}_0(r, h) > \tilde{\varphi}_1(h)$ and in Lemma 7 that there exists a lower bound $\bar{s}^m(\pi)$ such that for all $s^m \geq \bar{s}^m(\pi)$ it has to be that $\tilde{\varphi}_1(h) > 1/2$. Finally, Lemma 7 gives that $\bar{s}^m(\pi)$ is an increasing function and it is easy to verify that for bounded m

$$\lim_{\pi \to 1} \mathbb{P}(\phi_1(s^m) < 1/2 \mid m) = 0.$$

Therefore, $d\varphi_0(r, s^m)/dr$ is decreasing in s^m, from which the second statement of Proposition 5 also follows.

Proof of Equation (18): We have:

$$\begin{aligned}\mathbb{P}(\varphi^1 > \varphi^2 \mid s^m) = {} & \mathbb{P}(\varphi_1^1 > \varphi_1^2 \mid s^m)\,\mathbb{P}(1, 1 \mid s^m) \\ & + \mathbb{P}(\varphi_1^1 > \varphi_0^2 \mid s^m)\,\mathbb{P}(1, 0 \mid s^m) + \mathbb{P}(\varphi_0^1 > \varphi_1^2 \mid s^m)\,\mathbb{P}(0, 1 \mid s^m) \\ & + \mathbb{P}(\varphi_0^1 > \varphi_0^2 \mid s^m)\,\mathbb{P}(0, 0 \mid s^m).\end{aligned}$$

It follows immediately from Lemma 7 that $\varphi_1^2 > \varphi_1^1$. Moreover, Proposition 2 gives that $\varphi_0^2 > \varphi_1^2$. As a result, the above becomes

$$\mathbb{P}(\varphi^1 > \varphi^2 \mid s^m) = \mathbb{P}(\varphi_0^1 > \varphi_1^2 \mid s^m)\,\mathbb{P}(0, 1 \mid s^m) + \mathbb{P}(\varphi_0^1 > \varphi_0^2 \mid s^m)\,\mathbb{P}(0, 0 \mid s^m).$$

APPENDIX B: INVESTMENT AND AUM IN THE SECOND PERIOD

First, we derive the optimal investment decision of a manager in the second period. Second, we use this to calculate her AUM as a function of her posterior reputation, which we later use to derive her continuation payoff from period 2. To avoid repetition, we consider an extended model in which there are two fund managers. In this, the investor's preferences are given by

$$v(i, z_t^{ij}) = \begin{cases} \exp(z_t^{i1} - \bar{z}) \times (1 - f_t^i) \times R_t^i, & i = 1, 2, \\ \exp(m_t), & i = m. \end{cases}$$

Hence, in this case there are two independent preference shocks, one for each fund. The results of the baseline model can be obtained by setting the fees of the second manager equal to 1, which ensures that no investor will invest in her fund.

We solve the second period backward by first considering the manager's investment decision when the funds have already been allocated. The manager's expected payoff is

$$\mathbb{E}[\log(A_2^i f_2^i R_2^i) \mid s_2^i, s_2^m, \beta_2^i, \alpha] = \log(A_2^i f_2^i) + \mathbb{E}[r_2^i \mid s_2^i, s_2^m, \beta_2^i, \alpha].$$

As a result, the manager's objective when choosing her investment strategy β_2^i in the second period is to simply maximize the expected return r_2^i. Thus, the manager invests in her alpha only if

$$\mathbb{E}[r_2^i \mid s_2^i, s_2^m, \beta_2^i = 0, \alpha] \geq \mathbb{E}[r_2^i \mid s_2^i, s_2^m, \beta_2^i = 1, \alpha]. \tag{B1}$$

Let $\psi = \sigma^2/(\sigma^2 + v^2)$ and $\psi_m = \sigma_m^2/(\sigma_m^2 + v_m^2)$. Then Equation (B1) becomes

$$(1 - \psi) \times \alpha \; + \; \psi \times s_2^i \; \geq (1 - \psi_m) \times \mu \; + \; \psi_m \times s_2^m,$$

which allows us to derive the manager's optimal investment strategy in the second period. This is a cutoff rule such that she invests in her alpha only if $s_2^i \geq c(\alpha, s_2^m)$, where

$$c(\alpha, s_2^m) = \frac{\psi_m}{\psi} \times s_2^m + \frac{1 - \psi_m}{\psi} \times \mu - \frac{1 - \psi}{\psi} \times \alpha. \tag{B2}$$

Thus, for the same market conditions, a high-type manager invests relatively more frequently in her alpha in the second period, as $c(H, s_2^m) < c(L, s_2^m)$ implies that $\mathbb{P}[s_2^i \geq c(H, s_2^m)] > \mathbb{P}[s_2^i \geq c(L, s_2^m)]$ and hence that $\mathbb{P}(\beta_2^i = 0 \mid m_2, \alpha = H) > \mathbb{P}(\beta_2^i = 1 \mid m_2, \alpha = L)$.[21]

An important point that needs to be made is that the cutoffs $c(\alpha, s_2^m)$ are not optimal for investors. This is because investors are risk neutral whereas managers are risk averse. Following the same argument as earlier, we can show that the optimal cutoff for investors is

$$c^*(\alpha, s_2^m) = c(\alpha, s_2^m) + \frac{\psi_m \sigma_m^2 - \psi \sigma^2}{2\,\psi}.$$

Thus, an investor's optimal cutoff is adjusted by a "risk-loving" factor. For example, suppose that $\psi_m \sigma_m^2 > \psi \sigma^2$; that is, investing in the market is relatively more risky conditional on the information that the manager has at her disposal when making the decision. Then an investor would require a higher level of confidence on her alpha s_2^i to agree that relying on it is preferable to "gambling" with r_2^m.

Let u_2^α denote the equilibrium payoff of an investor in the second period, conditional on investing with a type α manager, but net of her preference shock z_t^{ij} and fees f_2^i. Then this is given by

$$u_2^\alpha = \mathbb{P}[s_2^i \geq c_\alpha(s_2^m)]\,\mathbb{E}[R_2^i \mid s_2^i \geq c_\alpha(s_2^m)] + \mathbb{P}[s_2^i \leq c_\alpha(s_2^m)]\,\mathbb{E}[R_2^i \mid s_2^i \leq c_\alpha(s_2^m)],$$

which has a closed-form representation that can be derived using the formulas of the moment-generating function of the truncated normal distribution. We do not provide this representation here as it does not facilitate an understanding of the model in any meaningful way. However, it is important to point out that when the market's posterior variance $\psi_m \sigma_m^2$ is much bigger than that of the alpha-based strategy $\psi \sigma^2$, the misalignment between the manager's and the investors' preferences could be so substantial that a low-type manager would be preferable simply because she is more reluctant to use her alpha. We exclude this possibility by assuming that $u_2^H > u_2^L$ because if the parameters of the model were such that investing in an index-tracking strategy were so attractive, there would be little need for professional investors.

Let φ^i denote the public posterior belief of manager i's ability α^i at the beginning of period 2. Then the investor's expected payoff, net of fees and the preferences shock, from opting for fund i is

$$u_2^i = \varphi^i (u_2^H - u_2^L) + u_2^L,$$

and the corresponding actual payoff is $e^{z_t^{ij}}(1 - f_t^i) u_2^i$. In addition, each investor has an outside option, which is to ignore the financial intermediaries and instead invest directly in m_2, which gives expected payoff $u^m = \mathbb{E}[\exp(m_t)] = e^{\mu + \sigma_m^2/2}$.

To avoid repetition, note that similar to earlier, we can define

$$u_1^i = \pi^i (u_1^H - u_1^L) + u_1^L$$

as the expected net payoff of an investor active in the first period. However, in this case the functional form of u_1^α is completely different, as the cutoffs used by managers in the first period are influenced by their career concerns.

To ensure that when the lowest preference shocks are realized the investor would rather invest directly in the market, we assume that

[21] For any empirically relevant results, it is better to condition expectations on m_t instead of s_t^m because we do not have some measure of the latter in the data.

$$(1 - f_2^i) \times u_2^H < u^m \times e^{\bar{z}}. \tag{34}$$

We are now ready to derive the AUM of fund i at the beginning of period t, as only a function of net expected payoffs and announced fees. In the Lemma 8, we derive in closed form the AUM in the more general case when there are two fund managers who are competing for funds from investors. Note that this nests the case where there is only one fund; indeed, if we set the fees of the second fund to be equal to 1, this would imply that no investor would invest in this second fund.

Lemma 8. *In any market equilibrium, the AUM of fund i, competing against fund k, in period t is*

$$\left(\frac{(1 - f_t^i)u_t^i}{u^m}\right)^{\lambda^i}\left(1 - \frac{\lambda^i}{\lambda^i + \lambda^k}\left(\frac{(1 - f_t^k)u_t^k}{u^m}\right)^{\lambda^k}\right). \tag{B4}$$

Proof. To simplify the algebra, drop the investor superscript and time subscripts. Also let $\xi^i = \log(1 - f^i)u^i$, $i = 1, 2$, and $\xi^m = \log u^m + \bar{z}$. For an investor to prefer Fund 1 over directly investing in the market or Fund 2, it has to be that

$$\exp(z^1 - \bar{z}) \times (1 - f^1) \times u^1 \geq u^m \Leftrightarrow z^1 \geq \xi^m - \xi^1,$$

and

$$\exp(z^1)(1 - f^1)u^1 \geq \exp(z^2)(1 - f^2)u^2 \Leftrightarrow z^1 + \xi^1 - \xi^2 \geq z^2,$$

respectively. Hence, the proportion of the market that Fund 1 captures is

$$\begin{aligned}
&\mathbb{P}(z^1 \geq \xi^m - \xi^1 \cap z^1 + \xi^1 - \xi^2 \geq z_2)\\
&= \int_{\xi^m - \xi^1}^{\infty} \mathbb{P}(z^1 + \xi^1 - \xi^2 \geq z_2 \mid z^1) d\mathbb{P}(z^1)\\
&= \int_{\xi^m - \xi^1}^{\infty} (1 - e^{-\lambda^2(z^1 + \xi^1 - \xi^2)})\lambda^1 e^{-\lambda^1 z^1} dz^1\\
&= e^{-\lambda_1(\xi^m - \xi^1)} - e^{-\lambda^2(\xi^1 - \xi^2)} \frac{\lambda^1}{\lambda^1 + \lambda^2} e^{-(\lambda^1 + \lambda^2)(\xi^m - \xi^1)}\\
&= \left(\frac{(1 - f^1)u^1}{u^m \cdot e^{\bar{z}}}\right)^{\lambda^1} \cdot \left(1 - \frac{\lambda^1}{\lambda^1 + \lambda^2}\left(\frac{(1 - f^2)u^2}{u^m \cdot e^{\bar{z}}}\right)^{\lambda^2}\right)
\end{aligned}$$

The proof for Fund 2 is equivalent.

The proof calculates Equation (B4) as the probability of the intersection of two events. The first is that investor j prefers fund i to fund k. The second is that fund i is preferred to direct investment in the market. Finally, to obtain the assets when there is only one manager, we can set $f^2 = 1$ to get $AUM_i = \left(\frac{(1 - f_t^i) \times u_t^i}{u^m \times e^{\bar{z}}}\right)^{\lambda^i}$.

APPENDIX C: UNOBSERVABLE INVESTMENT DECISION

Proof of Lemma 3. First, we simplify notation by omitting the dependence on s^m both on cutoffs and on expectations. We follow the proof of Lemma 2 to find the posterior reputation of the manager $r \mid H, m$. In particular, for $\beta = 1$, we have that $r = m + \epsilon$, and hence

$$\Pr(r, \beta = 1 \mid H, m) = \phi\left(\frac{r - m}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}\Phi\left(\frac{h - H}{v}\right).$$

Moreover, we have

$$\Pr(r, \beta = 0 \mid H, m) = \frac{\phi\left(\frac{r - H}{v\sqrt{\psi(1+\psi)}}\right)}{v\sqrt{\psi(1 + \psi)}}\Phi\left(\frac{r - h(1 + \psi) + H\psi}{v\sqrt{1 + \psi}}\right).$$

The expressions for the low type are identical. Therefore, it is now trivial to use Bayesian updating to derive the posterior reputation of the manager, and complete the proof of this lemma.

Proof of Proposition 6. We investigate whether $\phi(r, m, s^m)$ can be always increasing in r. From Lemma 3, it is sufficient to check whether ρ can always be decreasing in r, where:

$$\rho = \frac{\Phi\left(\frac{r - l(1+\psi) + L\psi}{v\sqrt{1+\psi}}\right)\frac{\phi\left(\frac{r-L}{v\sqrt{\psi(1+\psi)}}\right)}{v\sqrt{\psi(1+\psi)}} + \Phi\left(\frac{l-L}{v}\right)\frac{\phi\left(\frac{r-m}{\sigma_\varepsilon}\right)}{\sigma_\varepsilon}}{\Phi\left(\frac{r - h(1+\psi) + H\psi}{v\sqrt{1+\psi}}\right)\frac{\phi\left(\frac{r-H}{v\sqrt{\psi(1+\psi)}}\right)}{v\sqrt{\psi(1+\psi)}} + \Phi\left(\frac{h-H}{v}\right)\frac{\phi\left(\frac{r-m}{\sigma_\varepsilon}\right)}{\sigma_\varepsilon}}. \tag{C1}$$

First, a necessary condition for ρ to be decreasing is $v\sqrt{\psi(1 + \psi)} = \sigma_\varepsilon$. After substituting into Equation (C1), we get:

$$\rho = \frac{\varepsilon^{A_1 r - C_1}\Phi\left(\frac{r-b_1}{v\sqrt{1+\psi}}\right) + d_1}{\varepsilon^{A_2 r - C_2}\Phi\left(\frac{r-b_2}{v\sqrt{1+\psi}}\right) + d_2},$$

where $A_1 = (L - m)/\sigma_\varepsilon^2$, $C_1 = (L^2 - m^2)/2\sigma_\varepsilon^2$, $b_1 = l(1 + \psi) - L\psi$, $d_1 = \Phi((l - L)/v)$ and similarly for A_2, C_2, b_2, d_2.

Note that $\boldsymbol{A_1} < \boldsymbol{A_2}$. Then we can take the derivative with respect to $\boldsymbol{r}$ and check whether it is negative for every $\boldsymbol{r}, \boldsymbol{m}$. We have

$$\frac{\rho'}{\epsilon^{A_1 r - C_1}\epsilon^{A_2 r - C_2}} \propto P^* + d_2\left[A_1\frac{\Phi\left(\frac{r-b_1}{v\sqrt{1+\psi}}\right)}{e^{A_2 r - C_2}} + \frac{1}{v\sqrt{1 + \psi}}\frac{\phi\left(\frac{r-b_1}{v\sqrt{1+\psi}}\right)}{e^{A_2 r - C_2}}\right] - d_1\left[A_2\frac{\Phi\left(\frac{r-b_2}{v\sqrt{1+\psi}}\right)}{e^{A_1 - C_1}}\frac{1}{v\sqrt{1 + \psi}}\frac{\phi\left(\frac{r-b_2}{v\sqrt{1+\psi}}\right)}{e^{A_1 - C_1}}\right],$$

where

$$P^* = \Phi\left(\frac{r - b_1}{v\sqrt{1 + \psi}}\right)\Phi\left(\frac{r - b_2}{v\sqrt{1 + \psi}}\right)(A_1 - A_2) + \frac{\Phi\left(\frac{r-b_1}{v\sqrt{1+\psi}}\right)\Phi\left(\frac{r-b_2}{v\sqrt{1+\psi}}\right)}{v\sqrt{1 + \psi}}\left(M\left(\frac{b_1 - r}{v\sqrt{1 + \psi}}\right) - M\left(\frac{b_2 - r}{v\sqrt{1 + \psi}}\right)\right).$$

We take any m such that $A_1, A_2 < 0$. Intuitively, we consider the case of a good realized market. Then P^* is finite (as $r \to \infty$) because $\Phi(.) \in [0, 1]$ and $M(a) - M(b) \le a - b$ for $a > b$ (Lemma 4).

We now show that as $r \to \infty$, the derivative cannot be negative. Indeed, we have that $\lim_{r\to\infty} \frac{\phi(.)}{e^{A_2 r - C}} = 0$. In addition, it is easily shown that as $r \to +\infty$,

$$\frac{d_2A_1\Phi\left(\frac{r-b_1}{v\sqrt{1+\psi}}\right)}{e^{A_2r-C_2}}-\frac{d_1A_2\Phi\left(\frac{r-b_2}{v\sqrt{1+\psi}}\right)}{e^{A_1r-C_1}}\sim\frac{d_2A_1e^{C_2}e^{r(A_1-A_2)}-d_1A_2e^{C_1}}{e^{rA_1}},$$

where $\sim$ denotes the asymptotic equivalence of the two terms. We know that $A_1-A_2<0$ so $\lim_{r\to\infty}e^{r(A_1-A_2)}=0$; therefore, in the limit the above expression is asymptotically equivalent to $-d_1A_2e^{C_1}/e^{rA_1}$. Finally, we know that $A_1<0$ so $e^{rA_1}\to 0$ and hence the whole expression tends to $+\infty$, as we also have that $A_2<0$. Therefore, we can finally conclude that ρ' cannot always be negative, or in other words, a monotonic equilibrium cannot exist.