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The social discount rate and the cost of public funds: a search for more consistency and better practice

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ABSTRACT

Three, mutually inconsistent approaches to social discounting and the cost of public funds have persisted for fifty years. All have distinguished academic support. All present problems of concept and/or application. This leads to a global loss of cost effectiveness in public spending and regulation. This essay reviews the issues and offers a practitioner's view on what limited but realistic potential there may be for more consistency and better practice.

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1. Introduction

The past fifty years have seen impressive, continuing advances in microeconomic analysis of government spending and regulation in fields such as modelling, data collection, regulatory analysis and ever more challenging monetary valuation. There have been important advances too in social discounting, including achievement of some global consensus on discounting the very long term.

However the history of social discounting has left a stable but inefficient legacy. Three main, mutually inconsistent approaches emerged in the 1960s and their relative prominence in developed economies today has barely changed since the 1970s.¹ All have distinguished academic support. All, despite refinements, have significant problems of their own. The consequent efficiency loss is hidden within complex political structures. However it merits concern, given the scale of spending and regulation influenced by social discounting regimes.

This essay examines the three approaches, their problems and mutual inconsistencies, and why these persist. It then suggests steps that many institutions might consider towards better practice.

These issues are not specific to environmental policy. Environmental public spending and regulation are however now a leading field of microeconomic policy analysis. Environmental concerns are also now central to global debate on social discounting.

The essay is written near the end of the author's practitioner lifetime with these issues since the late 1960s.² This included periods of overconfidence, but brought in recent decades deeper understanding of the differences in beliefs and framing, and of the challenges of applying analytically robust conventions.

Section 2 summaries history and current practice.

It first outlines the early years of social discounting from which today's main approaches emerged. The social opportunity cost (SOC) approach, led largely by Arnold Harberger, framed the cost of public funding as a rate return, used also as a time discount rate. The social time

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preference (STP) approach, promoted by Kenneth Arrow, discounted at an STP rate for consumption, with the cost of public funding handled separately. The financial economics approach, then promoted mainly by Jack Hirshleifer, costed public funding as if it were financing by a competitive financial market.

The still widely cited paper by Arrow and Lind (1970) was written to challenge Hirshleifer, but without the authors knowing that developments within financial economics had already shown that their paper did not effectively refute Hirshleifer's argument. Literature promoting the financial economics approach duly revived from the 1980s.

In 1985 it was observed that equity market risk premiums are typically an order of magnitude higher than would be estimated by a neoclassical index of risk aversion. This generated an extensive literature on potential explanations.

Academia and practitioners mostly settled from the 1970s on the SOC or the STP approach. Subsequent literature has been on developments within each approach or on promotion or defence of one or the other, but with no convergence of view, except on the very long term. Global debate from the 1990s led to a wide but not uncontested consensus that the social discount rate should decline over future decades.

Current practice includes, widely in Europe and partially in the US, STP regimes with discount rates around 3.5% and diverse approaches to the cost of public funds. SOC regimes are applied rather more widely, with rates typically around 7%. The financial economists approach is very rarely applied, but there is today a slowly growing adoption of equity market risk premium additions to STP rates.

Section 3 discusses framing and quantification of the cost of public funds. It argues that whether marginal public spending is funded by taxation or by borrowing is in this context unimportant. It then argues that the cost of public funding can be usefully framed as an absolute cost (≥ 1) relative to consumption, but not as a rate of return. It follows that in appraising public funding of 'choices of technique', such as engineering design, the cost of public funds is generally unimportant. This is recognised within STP discounting regimes, but incomprehensible if the cost of public funding is framed as a rate of return.

Section 3 then discusses quantification of a shadow price for public spending, stressing the wide ranging impacts of taxation, extending far beyond those for which there are useful quantitative data. Shadow prices often reflect the fine work of Bev Dahlby, but that work does not measure the tax-burden cost. This cost may be impossible to quantify with useful accuracy.

Section 3 concludes with a brief discussion of the qualitatively very different social costs of public funding and private financing. It concludes that accurate comparisons are generally not possible, but that broad judgements can be made about where public funding or private financing is likely to be more cost-effective.

Section 4 discusses incorporation of the cost of public funds into STP regimes without its explicit valuation, by estimating value for money within politically constrained budgets. The principle is well recognised. The approach has recent academic support and is successfully applied, but it is rarely used. Its advantages and challenges are described, including reference to a working example.

Section 5 examines the background of the financial economics view that equity market risk premiums apply to public funding. It then examines the arguments, drawing on the literature on why equity market risk premiums are so much higher than would be derived from neoclassical analysis. It notes the differences between taxation and equity financing. Equity markets bring strong and costly fluctuations; public funding brings tax-burden costs. It concludes that no analytical case has been made for equity market risk being materially relevant to the social cost of tax-funding.

Section 6 discusses why differences are so entrenched, why they matter, and potential for better practice within existing STP and SOC regimes. It suggests that the analytical and operational strengths and weaknesses of all three approaches are not political or philosophical. They are a product of history, maintained by many factors. The consequent inefficiencies are invisible. Tax raising

and private financing are qualitatively so different that many arguments in this field are counter-intuitively right or (more often) wrong. The issues draw on many fields of expertise and experience which make it vulnerable to framings becoming set in an institutional or disciplinary culture. They then become a starting point to be explained and defended, but impossible to question.

The section discusses the problems posed by each of the main regimes. In STP regimes the cost of public funds is often overlooked or ascribed shadow prices that may be serious undervaluations. Within SOC regimes, using a much higher discount rate for ‘choice of technique’ analysis biases choices towards higher than optimal operating and maintenance costs. This is followed by discussion of what steps might be feasible to promote better practices or wider understanding.

Section 7 sets out summary conclusions and recommendations. The recommendations are for consideration of possible changes within some STP and some SOC regimes, that STP rates should not include adjustments for income-correlated risk derived from equity market risk premiums, and for students to be encouraged to see the continuing wide, global inconsistencies of practice as an issue for concern.

The essay’s scope is limited to reviewing the fundamentals of social discounting regimes in developed economies. It does not review issues such as declining discount rates or lower rates for environmental or other impacts that increase in real value over time. Nor does it address the integration of monetised analysis into decision making processes.

The paper is addressed equally to academics and practitioners.

All references to rates of return and discount rates are in real terms.

2. History and current practice

2.1. History

During the 1960s many governments, following the example of major industries, adopted ‘discounted cash flow’ analysis for major public investments. Contemporary literature and practice mostly saw the government’s social discount rate (SDR) as closely analogous to a commercial enterprise’s cost of capital.

Many governments favoured a ‘pragmatic’ social opportunity cost (SOC) approach, often setting the SDR equal to the (average) before-tax real rate of return on private sector investment. However Harberger derived a weighted average ‘cost of capital’ specifically for public funding (Harberger 1969) and this established the basis of today’s main SOC approach.

Meanwhile Arrow approached the issue as an optimisation problem constrained by the government’s limited role (as a provider of public services) and showed that the optimal time preference rate was the social time preference rate for consumption. This established the basis of today’s STP approach (Arrow 1966; Feldstein 1964; Layard 1972, 27–51). This however required, for comparisons of public spending with consumption benefits, a shadow price for public spending. This received little attention at that time and no widely accepted reliable method has emerged for its valuation.³

Harberger continued to promote the SOC approach.⁴ Arrow developed his work (Arrow and Kurz 1973), but did not respond to Harberger’s criticisms. However Feldstein (1973) explained clearly the limitations of the SOC approach.

Feldstein (1970) also highlighted the distinction between cost benefit analysis (CBA) – comparing public spending with consumption benefits – and cost effective analysis (CEA) or ‘choice of technique’ – comparing public spending profiles to achieve a given output. In CEA the shadow price of public funds applies equally to costs and cost savings and so is generally unimportant. Social discounting in the 1960s and 1970s focused on CEA, such as comparisons of hydro and coal fired power stations, and this contributed to a neglect of the cost of public funds. This was unfortunate and Feldstein (1997, 197) was decades later deploring the lack of progress on its valuation.

Arrow and Lind (1970) challenged the view, then argued by Hirshleifer (1966), that public funding should be costed as if it were sourced from a competitive financial market. Arrow and Lind was however unfortunately timed. The authors did not then know that developments within financial economics had already shown that their predominant focus on idiosyncratic risk was mistargeted.

From the 1980s financial economists revived the argument that *the discount rate for governments' projects equals the expected return on comparable investments in the capital markets* (Brealey, Cooper, and Habib 1997). Recent financial economics comment has focused more specifically on equity risk premiums, either case-specific or as a pragmatic, fixed addition to STP discount rates.

Mehra and Prescott (1985) showed that overall equity market risk premiums are typically an order of magnitude higher than would be estimated, for the same variance, by a neoclassical index of relative risk aversion. This 'equity premium puzzle' generated a large literature, comprehensively reviewed twenty years later by Mehra (2006).

From the 1990s climate change and other issues generated academic interest in social discounting over the very long term. In this case, STP discounting became a widespread global convention (Drupp et al. 2018; Hänsel et al. 2020), although this is contested (Burgess, 2018; Harberger [and Just], 2012, 21/22; Nordhaus 2019). Progress on more routine discounting however has been within rather than between the STP and SOC approaches.

The STP, SOC and financial economics approaches are compared in the table below.

Approaches to social discounting, the cost of public funds and systematic risk.

	Social discount rate (SDR)	Cost of public funds	Systematic (income-correlated) risk in public service outputs
STP ('social time preference' regimes)	Social time preference for consumption.	Shadow price for public spending dollars. or CBA ranking by ratio of consumption benefit to spending from a constrained budget.	Cost normally estimated from neoclassical index of risk aversion. Usually small.
SOC ('social opportunity cost' regimes)	Equal to cost of funds as defined in next column	Weighted average rate of return to foreign borrowing and displaced private sector activity. or Pragmatic average pre-tax return on private sector investment.	Not explicitly addressed, but discount rate includes an equity market risk premium.
Financial economics approach	Risk free rate plus case-specific multiple of equity market risk premium. or Pragmatic, fixed equity risk premium in STP rate.	Tax-burden costs not considered. Implicitly included as a shadow price for public spending.	Equity market based risk premium in discount rate.

2.2. Current practice

SOC discounting is applied in Federal Canada, in Federal and State governments in Australia, in New Zealand and widely in developing economies in Central and South America. The European Commission and several European countries apply STP discount rates, some with a shadow price for public spending. The UK adopts an STP rate and often incorporates the cost of public funds by ranking CBA options by their ratios of consumption benefits to spending from the agency's budget. France adds an adjustment for systematic risk to an STP rate (France Stratégie 2017). The Netherlands adopts a debt and equity financing rate. With the very low real risk-free rates of many recent years this has led to SDRs similar to STP rates.

The US Environmental Protection Agency (EPA 2010, 6–19) follows the guidance Circulars of the Office of Management and Budget (OMB 1992 on spending and OMB 2003 on regulation). These guides read as compromises between economists from different schools. They specify a

pragmatic form of SOC discounting for CBA, but STP (or sometimes questionable proxies) for ‘choice of technique’ and some other cases. Draft revisions of both Circulars were published in April 2023 for public consultation.⁵

An equity risk premium of 0.6% has been proposed for very long-term discounting of climate change impacts (Dietz, Gollier, and Kessler 2018).

Groom et al. (2022, 470, 481–484) includes a comprehensive account of recent practice across 21 nations and international institutions.

3. The social cost of public funds

This section discusses the framing and quantification of the social cost of public funds, using the following acronyms.

MCPF: Marginal cost of public funds. This is the present value of the stream of lost consumption or consumption-equivalent per marginal dollar of general taxation, discounted at the STP rate for consumption.

METB (= MCPF – 1): Marginal excess tax-burden.

CBA: Cost–benefit analysis = comparison of public spending with consumption benefits.

CEA: Cost-effectiveness analysis = ‘choice of technique’ = comparison of public spending profiles for *given* consumption benefits.

“Cost saving analysis” describes comparisons of options that immediately reduce public spending.

3.1. Is marginal public spending funded by taxation or by borrowing?

Moore, Boardman, and Vining (2013) records that *the STP approach contends that a marginal government project will be tax-financed [whereas] the SOC belief is that government project will be deficit-financed*. In practice the level of public borrowing is often dominated by macroeconomic concerns, but governments do sometimes link net borrowing to net public investment.

However a competent government will always seek to equalise the marginal social cost of borrowing and taxation. The social costs of a marginal change in either are complex. It appears that neither can be explicitly valued with the accuracy needed for microeconomic policy or project analysis. Public borrowing is a commitment to future public revenue to service the debt. Literature on the consumption-equivalent cost of public spending focuses on tax-funding.

For all these reasons marginal public spending is assumed here to be funded from general taxation. This is not necessary, but it avoids distracting and unnecessary detail.

3.2. Framing the cost of public funds: a rate of return or present value of lost consumption?

Government revenue is generally paid into some form of consolidated fund, from which payments are drawn by spending agencies according to previously negotiated capital and current budgets. Capital and current spending are distinguished in expenditure planning and budgeting, but there is no distinctive ‘cost of capital’. One cost of taxation is a loss of private investment that would have produced an expected rate of return higher than the STP rate. This loss however, whatever its internal rate of return, cannot indefinitely compound faster than the national economy. It has a finite present value when discounted at a plausible STP rate.⁶ Tax-funded dollars cost more than consumption dollars, but this tax-burden cost cannot be usefully expressed as a rate of return.⁷

Harberger (2007) acknowledged this framing for public revenue or spending dollars. That paper compared expansion of a state-owned power plant with improvement of an untolled road, with

identical costs. Net revenue from higher electricity sales (in public revenue dollars) was numerically the same as the value of the road improvement (in consumption dollars). Harberger concluded that

The cleanest, most straightforward way to take tax financing and the excess burden associated with it into account is to apply an extra charge or benefit of λ to each and every cash outflow or cash inflow from and to the public treasury.

Harberger's λ is the marginal excess tax burden (METB). This was a bold step, but it was not taken up by SOC discounting regimes, perhaps because it became clear that handling the METB in this way requires an STP discount rate.⁸

Cost-effectiveness analysis (CEA), or 'choice of technique', illustrates strikingly how different tax-funding is from private financing. CEA compares alternative profiles of public spending, as in engineering design, where there is no comparison of public spending with consumption. A privately financed enterprise would generally discount such cash flows at no less than its cost of capital. However in CEA with public funding the cost of funds shadow price applies equally to costs and cost savings and is therefore usually unimportant. The lowest cost option is revealed directly by discounting at an STP rate. To many economists this is 'obvious' (e.g. Boardman et al. 2020, 5), but it is incomprehensible if the cost of public funds is framed as a rate of return.^{9,10} It is sometimes suggested that the argument is wrong because the costs could be invested in financial markets to earn more than the STP rate. However, even if this were institutionally possible,¹¹ it would be irrelevant, because the cost savings could be invested in the same way. All that matters here is that the tax-burden cost (Harberger's λ) is broadly constant over time.

This difference between CBA and CEA justifies the US OMB default guidance of an SOC discount rate for CBA but a lower rate for CEA (OMB 1992, section 8c (1)).

The challenge of conceptualising separation of the cost of public funds from time preference is illustrated by Harberger's comment, when under pressure on very long-term discounting, that: *where we are justifying the extraction of money over time, which has a demonstrable cost of 6%, 7%, 8%, or 10%, we should not then discount the resulting future benefit back at 2%* (Harberger and Just 2012, 21/22). However, as Harberger had acknowledged five years earlier, the cost of a marginal dollar of public funds is not an annual percentage rate, but $(1 + \lambda)$ consumption dollars. And this 2% is a time preference rate but not a cost of money.¹²

3.3. Quantifying the cost of public funds

The social costs of a marginal increase in taxation include collection and enforcement costs, direct effects of price distortions (where marginal cost will often far exceed average cost), and distortions of individual and corporate decisions on investment, location, corporate reward structures, choices of education and jobs, and pension decisions. Higher taxes also increase tax avoidance services and illegal tax evasion. These impacts extend far beyond those for which there are useful quantitative data.

Boardman et al. (2020, 1–7) records eleven METB estimates published over the previous 25 years, for the US, Canada, Australia and the UK. When adjusted to MCPFs (= METB + 1) these range from 1.12 to 1.43.^{13,14} Barrios, Pycroft, and Saveyn (2013) estimate MCPFs for many countries of close to 1.0 for green taxes, but close to 2.0 for labour taxes. Feldstein (1997) proposes an MCPF of 'more than 2'.¹⁵ A widely quoted source is the extensive work of Bev Dahlby, such as Dahlby and Ferede (2011) which derives a 'marginal cost of funds' (MCF) of 1.11 for Canadian federal sales taxes and 1.7 for corporate income tax. However that excellent and useful work does not estimate the MCPF as defined above.¹⁶

The US OMB in 2017 faced an Executive Order (later rescinded) requiring that new regulation should not increase the total cost of regulation. This prompted a public consultation (OMB 2019) noting that the current guidance (OMB 1992) specified an METB of 0.25, and said that research now suggested 0.4–0.5.¹⁷ The consultation attracted little response.

Having regard to the literature and observation of public investment decisions, the MCPF for aggregate public spending in developed economies is in the author's view often much closer to 2.0 than to 1.0. However any such view is debatable. Resolving the problem is discussed later in Section 4.

3.4. The social costs of privately financed and publicly funded investment spending

The social cost in consumption dollars of *private* financing is the present value, at the STP rate, of the after-tax financing costs.¹⁸ The cost of *public* funding is the present value of the spending at the STP rate, multiplied by the MCPF. There are limited case-specific data on the former and the MCPF appears not to be reliably quantifiable.

This does not however prevent useful conclusions on when public funding or private financing is more appropriate for public services. In a developed market economy most public sector procurement is from private sector enterprises and privately financed. However for major projects it may often be more efficient for the government to save public spending on private financing costs by direct public funding of the capital investment, on completion of successive stages.

4. Incorporating the cost of public funds into STP regimes

Section 3.3 suggested that public spending dollars cannot be reliably converted directly into consumption dollars. This may often not matter for CEA, but it does matter for the comparison of public spending with consumption benefits in CBA. The difficulty can be resolved by prioritising CBA options by their net consumption benefits per dollar of net spending from the politically constrained budget.

This principle is well documented (Marglin 1963, 278; Minken 2016) and its practical application is promoted with historical precedent by Finkelstein and Hendren (2020). It has long been applied in government, but not widely.

The approach has important advantages over a pragmatic, imposed MCPF. It reinforces analysis that is needed anyway for financial management of an agency's budget. It incorporates the MCPF automatically rather than addressing it (or overlooking it) as a separate item. It incorporates the implicit MCPF for the specific spending programme.¹⁹ Some finance ministries oppose an explicit MCPF but accept prioritising by value for money. It is an analytically preferred approach.²⁰ It does however present challenges.

It highlights issues of budget attribution. As costs and benefits are not defined by their sign there may be 'negative benefits' (reductions in consumption) and 'negative costs' (public revenues or cost savings). Also the procedure works differently for CBA, for CEA and for cost saving analysis.

An example of the method's application is outlined in Department for Transport (2017, 25–26, Boxes 5.1 and 5.2), where the categorisations in both tables are pragmatic but serve satisfactorily in practice. Benefit cost ratios are already widely used in government but, as illustrated for transport in Mackie and Worsley (2013), the denominator is not always defined as budget-constrained spending.

5. Equity market risk and public funding

5.1. Background

Fluctuations in the value of even a well diversified equity portfolio are severe. Eugene Fama comments, with respect to long term fluctuations, that *[historical data suggest] that getting a positive equity premium (of any size) is highly likely only for holding periods of 35 years (an investment life-time) or more. Given this result the historical equity premium does not seem too high* (Fama and French 2009). For active investors shorter term fluctuations are also costly.

The 1960s financial economics revolution established that the premium for a given stock is determined mainly by the systematic variation of the stock's return with the overall equity market return. The best known model, the Capital Asset Pricing Model (CAPM), is outlined in [Box 1](#). This showed that Arrow and Lind (1970) was mistaken in focusing on idiosyncratic risk, not systematic risk. A well managed equity portfolio diversifies away the cost of idiosyncratic risk.

Box 1. CAPM and CCAPM

Traditional CAPM is a static, one-period model expressed as:

$$E(R_i) = R_f + \beta_{mi}(E(R_m) - R_f)$$

where

$E(R_i)$ = the expected rate of return on asset i

R_f = the risk-free interest rate

$E(R_m)$ = the expected average market rate of return

β_{mi} (the 'market beta' for asset i) = $\text{cov}(R_i, R_m) / \text{var}R_m$

The term in curly brackets is the overall equity market risk premium.

If the return to the asset varies proportionately to the market average return $\beta_{mi} = 1$ and the estimated asset risk premium is equal to the overall equity market premium.

The late 1970s saw development of the *Consumption CAPM (CCAPM)*, which is a generalised, intertemporal model, maximising expected lifetime utility. The algebra is more complex but the coefficient on the consumption beta is still the overall equity market risk premium (Mankiw and Shapiro 1986, 454). So even if the CCAPM performed well its relevance to public funding would be dependent on the relevance to public funding of the equity market premium.

In practice the CCAPM, while an important theoretical model, does not perform well. The traditional CAPM is more widely applied.

Overall equity market premiums are typically an order of magnitude greater than would be estimated, for the same variance, from a neoclassical index of relative risk aversion.

Gollier and Hammitt (2014, 291) presents a neoclassical estimate of the cost of risk for public service outputs closely correlated with national consumption. The term CCAPM in the following extract is used to describe the application of a neoclassical index of 2.0 (high in the plausible range) and an empirically based volatility of log national consumption of 3%.

... the CCAPM ... has emerged as the common language and practice of economists over the past four decades. The CCAPM has mostly failed to explain how financial markets value risk. For example, the equity-premium puzzle (Mehra and Prescott 1985) shows that the CCAPM predicts a systematic risk premium of $2 \times (3\%)^2 = 0.18\%$... which is an order of magnitude smaller than the observed risk premium of [equity market financed] assets with $\beta = 1$. On a more normative ground, considering such a small systematic risk premium looks very counterintuitive because doing so makes the riskiness of projects nearly irrelevant to their evaluation.

The intuition in the last sentence was evidently not shared by Arrow.

Gollier (2021) promotes the pragmatic addition of a fixed systematic risk adjustment to STP rates.²¹

Lucas (2014) rebuts Arrow and Lind (1970), noting that *Arrow and Lind defended [focusing on idiosyncratic risk] with the assertion that correlated risk is likely to be insignificant for many government investments*. Arrow and Lind does however address systematic (income-correlated) risk as follows (p 373). The last sentence is important.

It is sometimes argued that the returns from public investments are highly correlated with other components of national income through the business cycle. However, if we assume that stabilization policies are successful, then this difficulty does not arise. ... Further, if there is some positive correlation between the returns of an investment and other components of national income, the question remains as to whether this correlation is so high as to invalidate the previous result.

The last sentence would be presuming, as would many economists today, a neoclassical estimation of the cost of income-correlated risk in most public service outputs. As noted by Gollier above, this makes the [systematic] riskiness of projects nearly irrelevant.

Lucas says later that *The conclusions of Arrow and Lind rest on the presumption that government investments are free of aggregate risk*. However the contentious issue today is not the existence of

such risk, but its cost. Lucas reports that *the view that market rates should be used to discount risky government investments appears to be the predominant one among present-day financial economists*. Many other economists are unpersuaded that equity market risk premiums are materially relevant to the cost of tax-funding.

5.2. The relevance of equity risk premiums to the cost of public funding

Public funding provides services to customers in ways often broadly analogous to the provision of goods and services by equity financed enterprises. The value of these services, whether publicly funded or privately financed, often varies systematically with income.

The cost of this risk to customers in private sector markets attracts little if any academic interest. Interest focuses on the feedback from sales revenues to dividends and equity values, including the percentage premium required to compensate investors for equity market risk.

With tax-funding there is no such financial feedback to taxpayers and no equity market risk. The financial economics literature nonetheless assumes that competitive financial markets reveal the social cost of financing an activity, regardless of whether it is financed by private sector debt and equity or tax-funded.

Market, or other willingness to pay prices of goods and services are very widely used in public sector analysis as measures of social cost, but there is no market price for tax-funding. Equity financing is a competitive activity supporting a dynamic market economy and incurs a significant cost of undiversifiable equity market risk. Tax-funding is a state-imposed activity providing non-marketed public services and incurs tax-burden costs.

The financial economics literature supposes that the income-correlated risk faced by public service beneficiaries should be valued as if it were financial risk falling on equity market investors, and costed as the product of its consumption beta and the overall equity market risk premium. However, although the 1960s saw remarkable new understanding of equity financing, there appears to be no analytical foundation for supposing that these insights are materially relevant to the cost of tax-funding.

The systematic risk faced by shareholders differs radically from that faced by public service beneficiaries. Fluctuations in equity values are large and perceived by active investors as financial gains and losses against a clear baseline.²² Fluctuations in publicly funded outputs may be cost savings to taxpayers or consumption benefits to public service users. Fluctuations in savings to taxpayers are imperceptible to any individual. Fluctuations in benefits to users, if perceived at all, are rarely perceived against any clear baseline. They are also generally weaker than fluctuations in equity markets. They are usually well suited to neoclassical valuation, as in Little and Mirrlees (1990).

Box 2 briefly discusses the literature on the overall equity market risk premium.

Box 2. Explaining the overall equity market risk premium.

Neoclassical estimation is suitable for costing marginal and weakly perceived income fluctuations, but inappropriate for the risk faced by active equity market investors. However this failure of neoclassical analysis has generated an extensive literature. The first two decades of that literature are comprehensively reviewed in Mehra (2006). Very few of the many proposed explanations are potentially relevant to public funding. It even appears likely from Mehra's review that a significant part of equity market risk premiums is attributable to *non-risk* characteristics of equity markets.

A much cited proposal that could in principle be relevant to public funding is Epstein and Zin (1991). This addresses the standard neoclassical assumption of the same aversion to variation of consumption across different states *at a particular time and over time*. Epstein and Zin constructs a (recursive) consumption function which does not impose this equality. However Mehra notes that the recursive model depends on unobservable variables and is difficult to assess against independent data. Where this can be done it does not perform well. It is anyway intuitively unlikely that these two aversions should materially differ for fluctuations that are marginal and weakly perceived, as is generally the case with public service benefits.

Neither conventional financial economics nor Epstein and Zin offer a quantitative explanation of the overall equity market premium. However Barberis and others (Barberis, Huang, and Santos 2001; 2016) have examined in depth the 'prospect theory' effects of active equity investors giving more weight to losses than to gains. They find that this behavioural effect may explain much of the equity risk premium.

It is sometimes proposed that taxpayers should require as high a rate of return for society from public investment as they would expect on their personal investments. However, as noted in section 3.4, social ‘value for money’ from equity financing and from tax-funding are usefully measured in quite different ways.

It is also sometimes argued that public funding could finance *private enterprise* investments to obtain a commercial return (without the costly equity market risk premium), and that this is the ‘opportunity’ being missed. But public spending of this kind is confined to special cases, because it loses the benefits of a competitive financial market and incurs tax-burden costs.

6. Discussion

6.1. Why such entrenched differences?

There will always be enduring differences of opinion within economics, because of differences in philosophy, politics, and views on how the economy will evolve on its own or following policy change. Politics and philosophy can influence economists’ quantitative judgements on STP rates and the cost of public funds, but the analytical and operational issue addressed in Sections 3–5 are not significantly political or philosophical.

The main differences are a product of history, maintained by many factors. The inevitable inefficiencies are invisible and so provides no pressure for reform. Tax raising and private financing are qualitatively so different that comparing approaches to social discounting leads into many arguments that are counterintuitively correct or counterintuitively incorrect. The difficulty of reliably valuing the cost of public funds has led to an academic and practitioner focus overwhelmingly on discounting and is a serious obstacle to the wider application of STP. Disciplinary boundaries impede mutual understanding.

Assessment of the approaches needs to draw upon, among other disciplines, public sector micro-economics, financial economics, behavioural science and knowledge of the practicalities of high level macro and micro economic planning. This makes the field unusually vulnerable to the human tendency to frame problems within a very specific professional toolkit. When such a framing becomes established in an institutional or disciplinary culture it can become a starting point to be explained and defended, but impossible to question.

6.2. Why do these differences matter?

There appears to be, unsurprisingly, no well researched evidence on the cost of the inconsistent approaches to social discounting. Whether this cost could be estimated with useful reliability is at best doubtful. However in developed economies the scale of public spending and costs of regulation influenced by social discounting amount to significant fractions of GDP. Any potentially material efficiency loss therefore merits concern. Material inefficiencies include the following.

The need for the STP approach to weight public spending dollars more highly than consumption dollars is often overlooked or probably undervalued. If \$1 of public funds costs >\$1.5 of consumption as seems likely, or >\$2 as Feldstein believed, valuing it at only \$1.25, or less, greatly undervalues public spending.

Applying an SOC approach to ‘choice of technique’ analysis, with a discount rate much higher than an STP rate, biases such choices significantly towards higher than optimal future spending.

The financial economics approach of adding equity market risk premiums to social discount rates reduces the weight given to future welfare, while overlooking profound differences between tax-funding and equity financing.

Special cases such as comparing consumption streams from a proposed regulation or estimating the present value social cost of private financing, defy rigorous analysis without use of an STP rate.

6.3. Potential for better practice

Nearly all institutions and economists concerned with these issues have a settled view. Major change is generally brought about by big exogenous change, such as the need forty years ago to address more seriously the very long term, or big changes in key personnel or a national political climate.

There may however be some scope for development *within* existing STP and SOC regimes. The financial economics approach lacks authoritative research on any relevance of equity market fluctuations to the cost of tax-funding.

Wider understanding of all these issues might develop if teaching presented the main global inconsistencies of approach as an unresolved problem.

7. Conclusions and recommendations

7.1. Conclusions

The past sixty years have seen impressive advances in microeconomic analysis of public spending and regulation. However developments in the 1960s in social discounting and in financial economics have led to mutually inconsistent global approaches to discounting and the cost of public funding. All current approaches present significant problems.

The social cost of a marginal, tax-funded dollar can be usefully framed as a multiple (≥ 1) of a consumption dollar, but not as a rate of return. Tax-burden costs cannot compound indefinitely at a rate higher than the economic growth rate and so have a finite present value at an STP discount rate.

Arrow's approach of monetising costs and benefits in consumption dollars and discounting at an STP rate for consumption is sound in principle. However reliable, explicit conversion of public revenue or spending dollars to consumption dollars appears to be impossible. This cost is thus often overlooked or applied by shadow prices that are probably too low. A rigorous method that avoids an explicit conversion factor is rarely applied.

The SOC approach has intuitive appeal and the great virtue of simplicity, but it is not well suited for some applications. This is now widely recognised for very long-term discounting, but less widely for 'choice of technique' analysis such as optimising engineering design or choice of technology.

Many financial economists believe that social discount rates should be set at competitive financial market rates for investment in a similar activity. However studies of the equity market risk premium suggest that it is specific to equity markets and not materially relevant to public funding. This global situation has barely changed over half a century and is set to continue.

The lack of convergence stems largely from differences in framing the cost of public funding. Once a framing is set in an institutional or disciplinary culture it can become a baseline or starting point, to be asserted and explained, but not to be questioned.

Lack of change means continued global teaching of inconsistent methodologies to successive generations; to widespread, hidden inefficiencies in microeconomic appraisal; and continued unsupported beliefs about the relevance of equity market risk to public funding. There may however be some limited scope for convergence as outlined below.

7.2. Recommendations

Applications of STP discounting to CBA should give more weight to spending dollars than to consumption dollars. If an explicit shadow price is used this should preferably be set at no less than 1.5. But, if possible, the weighting should be applied by ranking CBAs by their ratios of net consumption benefit to net public spending from the politically constrained budget.

Finance ministries or their equivalent, to whom large projects are submitted for approval, should require that, with STP discounting, CBA benefit cost ratios are defined in that way.

SOC regimes should more widely recognise the distinction between public spending to obtain consumption benefits and ‘choice of technique’ to achieve a given output. They should, if possible, consider adopting lower rates for choice of technique and some other applications, as does the current US OMB guidance followed by the US EPA.

STP discount rates should not include adjustments for income-correlated risk derived from equity market risk premiums unless authoritative, interdisciplinary research demonstrates a material relationship between the cost of equity market fluctuations to shareholders and the social cost of public funding.

Teaching of microeconomic appraisal techniques, whatever the institution’s preferred approach, should encourage students to see the continuing wide, global inconsistencies of practice as an issue for concern.

Notes

1. Two main approaches being those described as ‘social opportunity cost’ (SOC) and ‘social time preference’ (STP). The third, rarely applied but sometimes influential, is costing public funding as if it were private sector capital financing.
2. For a supplier of major publicly funded infrastructure; in government spending agencies and a finance ministry, including a period as overseer and joint reformer of government appraisal guidance; and subsequently continuing to work with public sector agencies on appraisal guidance and application. I am deeply indebted to Mark Moore, David Maddison, Peter Abelson, Tim Dalton, Mark Freeman, Ben Groom, Stuart Allen, Iven Stead, to two anonymous reviewers and the Editor for invaluable advice, but all views, errors and omissions are the author’s alone.
3. Bradford (1975, 807–898) emphasised the need for this shadow price, but suggested a low value of about 1.1 on the basis only of displaced private investment.
4. As in Harberger (2007), Harberger and Just (2012) and Harberger and Jenkins (2015). Its development is also illustrated by Edwards (1986), following the expansion of international financial markets, and Harrison (2010).
5. Both drafts specify STP discounting plus a default systematic risk premium of 1.1%. The draft on spending overlooks the cost of public funds. That on regulation says the public funding dollars cost no more than consumption dollars. The compromises in the current guidance are forgotten or rejected.
6. An STP rate estimated from the Ramsey equation will generally be significantly higher than the economic growth rate.
7. Hence internal rates of return are rarely recommended as a metric in public sector appraisal guidance.
8. The New Zealand’s SOC regime does however apply a λ of 0.2 to public spending (Abelson 2020, 28).
9. The point is noted in Arrow (1966, 26) and Arrow and Kurz (1973, xxv) and made rigorously in Feldstein (1970).
10. The MCPF would also be unimportant if the costs and benefits both include public spending and consumption in similar proportions, as may be the case in some long term environmental analysis.
11. Governments of developed economies hold financial investments only for specific policy needs such as some pension funds or a sovereign wealth fund.
12. Although market data also support a discount rate of this order for the very long term (Giglio, Maggiori, and Stroebel 2015).
13. Values of around 1.25 are unsurprising from a behavioural perspective. Pressure for a shadow price generally comes from spending agencies, who might see a public funding premium of 10% (an MCPF of 1.1) as too small to be worthwhile but a 40% premium as too harsh.
14. Boardman et al also includes (pp 15–17) a cogent critique of Bos, van der Pol, and Romijn (2019) which proposes an MCPF of 1.0.
15. Feldstein was also concerned that lack of public and political awareness of the cost of marginal taxation distorted political judgements about aggregate public spending.
16. Dahlby’s estimate reflects a tax’s efficiency as a revenue raiser. If a 1% increase in a tax rate increases revenue by 0.8% the Dahlby MCF is $1/0.8 = 1.25$.
17. EPA guidance mentions the MCPF but does not recommend its use (EPA 2010, 6–19).
18. These include premiums paid to shareholders to compensate them for equity market risk. The social *benefits* of private enterprise investment are complex, including tax revenue, producer and consumer surplus, and promotion of competition and innovation.
19. The MCPF may vary across programmes, partly because setting high level budgets is a political process.
20. The term ‘analytically preferred’ is used by the US OMB for STP discounting with CBA conditional on there being a robust MCPF (OMB 1992, section 8b(3)).

21. Gollier (2002) also expresses concern that failure to include such a premium leads to excessive public spending. However aggregate government spending in developed economies is set at a very high political level. This is influenced by market interest rates, but generally not by the SDR.
22. The *perception* of gains or losses against a clear baseline is fundamental to the ‘prospect theory’ of Kahneman and Tversky (1979), showing that neoclassical expected utility analysis understates the cost of risk in circumstances such as that of active equity market investors.

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