

Managers, Debt and Industry Equilibrium

By

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Abstract

This paper reconsiders the strategic effect of debt under the assumption that quantity choices are made by managers whose objective is to avoid bankruptcy. The basic result is that quantity choices, which are strategic substitutes under profit maximization, may turn into strategic complements under reasonable assumptions on the profit function. The value of delegation, optimal wage contracts, and empirical implications are discussed. (JEL classification D21, G32, L13)

1 Introduction

In recent years there has been much interest in the way equilibria in oligopolistic markets may be affected when account is taken of the contractual structure inside the firm or of contractual ties with outside investors. This is usually modelled as a two stage game. In the case of Cournot competition, prior to the quantity setting stage, there is a stage in which firm owners can move to write contracts which may affect incentives at the later quantity setting stage. Examples of this literature are Brander and Lewis (1986), Ferstman and Judd (1987), and most recently Clayton and Jorgenson (1997). The common theme of all these papers is that, if goods are substitutes, and therefore are strategic substitutes when chosen by profit-maximizing agents, the possibility of moving prior to the quantity setting stage will be used to commit the firm to more aggressive product market behavior.

Brander and Lewis(1986) analyze the case, where firm owners can write debt contracts with investors in a perfect capital market, before they move again to choose quantities. When there is uncertainty about demand or cost conditions, debt introduces the possibility that the firm may go bankrupt. A positive debt level will therefore make the payoff of shareholders a convex function of the operating profit. Given any quantity choice the shareholders payoff is flat for all realizations of the state of nature such that the firm is bankrupt, but is increasing linearly with profit for good states of nature. Under the assumptions that it is the firm owners who determine quantities and that marginal profit is an increasing function of the unobserved state of nature, it is shown by Brander and Lewis that a positive debt level will cause the firm's reaction function to move out. The intuition is that firm owners are only concerned with those states of nature that leave a positive payoff to them. Since these are the good states, and marginal profit is higher for good states, firm owners will choose higher quantities than they would if no debt had been issued. Given that quantities are strategic substitutes and reaction functions are therefore downward sloping, each firm has an incentive to move

its reaction function out by issuing debt, in order to increase its profits as its own reaction function slides along the rival's downward sloping reaction function. In equilibrium debt levels are positive, quantities are larger and profits are smaller than if the firms could not issue debt.

Both Fershtman and Judd (1987) and Sklivas (1987) study the case where quantities are chosen by managers and firm owners move first to design incentive contracts with their managers. They assume that these contracts can condition both on the realized profit and on sales and restrict the set of admissible contracts to linear combinations of those two variables, so that contracts have the form $b[\alpha + (1 - \alpha)S]$: Under these assumptions they find that the optimal α will be less than one. Managerial incentives are distorted away from profit maximization towards sales maximization. The intuition is that owners want to make their manager more aggressive. When positive weight is on sales, managers will take account less of the costs of an increase in quantities, than they would if their remuneration were based on profit alone. Therefore reaction functions shift out as $(1 - \alpha)$ increases and each owner has an incentive to choose $\alpha < 1$; since this will increase his profit, given that the other firm's reaction function slopes down. In equilibrium both owners choose $\alpha < 1$, so that quantities will be larger and profits will be smaller than if the owners could choose quantities themselves. The commitment available through the possibility of writing an incentive contract worsens the situation of the owners.

Similar results are obtained by Clayton and Jorgensen in a setting, where in a first stage each firm can take an equity position in the rival firm. Denoting by α the share acquired in the competitor's equity firm i will choose its output to maximize $\frac{1}{4} + \alpha \frac{1}{4}$: Clayton and Jorgensen show that when the firms' products are substitutes optimal cross holding involves a short position in the competitor's equity, that is α is optimally negative. The intuition is that when firm i has chosen a negative position in firm j ; firm i gains when firm j 's profits are low. Increasing one's own output will now not only affect

one's own profit but depress the competitors profit and therefore increase firm value more than without crossholdings. By choosing a negative α each firm can give itself additional incentives to raise quantities. Again, reaction functions shift out and the equilibrium is characterized by larger quantities produced, and lower firm and industry profits.

In all of these papers the first stage action is used to commit the firm to a more aggressive output stance. However, since this commitment device is available to both firms, who take actions simultaneously, firms will end up with lower ex ante profits than they would enjoy if first stage actions could not be taken. The possibility of taking these first stage actions exacerbates the prisoner's dilemma, which is already present in the quantity setting stage, where both firms choose higher quantities than would be joint profit maximizing.

In this paper we will go back to the original analysis of Brander and Lewis and reconsider the case of commitment through debt. This case has attracted considerable interest, partly because the major predictions of the Brander and Lewis (1986) analysis have not been validated by the albeit limited empirical evidence, see e.g. Chevalier (1995), Kovenock and Phillips (1995), and Phillips (1995), who found that leverage increases in the 1980's led to softer product-market competition in the industries under study. Also, in the related empirical literature on management buyouts (MBOs) empirical research, (Kaplan (1989) and Smith (1990)) has found increases in operating profits as well as firm value, rather than a decrease of these variables, as the Brander and Lewis (1986) analysis would suggest.

The Brander and Lewis (1986) model has been revisited before us by Glazer (1994), Showalter (1995), and Faure-Grimaud (1997). In a dynamic setting, Glazer (1994) offers some qualification of their basic result. In his model equityholders choose quantities twice, before repayment of "long-term" debt is due. He shows that the behavior in the first quantity setting stage may be quite different from the behavior in the second stage. In the first

stage, there is an incentive to reduce quantities rather than increase quantities beyond the Cournot level. The intuition is that if the firm reduces its quantity in the first stage, this will increase its rival's first stage profit, and thus reduce the net debt burden the rival takes into the second stage. In line with the basic insight of Brander and Lewis (1986), this reduction of indebtedness will make the rival a less aggressive second-stage competitor. Therefore long-term debt may lead to more collusive outcomes in the short-run, while the long-run as well as the average is still characterized by quantities above the Cournot-level.

Showalter (1995) replaces the assumption of Cournot competition by one of Bertrand Competition. When competition is in prices rather than quantities the decision variables are strategic complements when chosen to maximize profit. The cross-partial of the profit function is positive, rather than negative, as was assumed in Brander and Lewis (1986). By assuming Bertrand competition Showalter (1995) reverses yet another crucial assumption on the profit function. Under demand uncertainty, when firms compete in prices, marginal profit is lower, rather than higher for good states of nature. For the case of demand uncertainty Showalter (1995) is then able to find positive debt levels in equilibrium which are associated with profits that are higher than for pure equity firms.

In Faure-Grimaud (1997) the financial investor can observe the quantity choice but neither the realized state of nature, nor the resulting profit. The terms of the contract are determined after the quantity has been chosen and are made conditional on the owner's announcement of the state of nature. To induce truth-telling the contract specifies a probability of granting a reward to the owner, which is increasing in the announced state of nature. When the owner has all the bargaining power vis a vis the investor, the investor has to break even ex ante. Thus both the truth-telling constraint and the break-even constraint are binding. The interplay between these two constraints makes owners choose quantities in equilibrium that are lower than if the owners

were self-financed.

In all of these papers one major assumption of the Brander and Lewis (1986) analysis has been left unquestioned, which is that there is no conflict of interest between the owner (the shareholders) and the manager who chooses quantities. Recall that they assume that quantities are chosen by an agent, whose preferences are perfectly aligned with the owners or, equivalently, that owners choose quantities themselves after having issued debt. Instead we want to follow up the idea that ownership and control over quantity choices may be separated and that therefore quantity choices may be made by a manager whose objective differs from that of the owner. Specifically, we ask what happens if quantity choices are made by a manager whose objective is to avoid bankruptcy. While it clearly is an extreme assumption that this is the only objective of managers in the real world, the threat of bankruptcy arguably is a real concern for managers, who when their firm goes bankrupt almost surely lose their job and most likely much of their reputation. In this paper it is argued therefore that having a manager, whose only objective is not to go bankrupt is at least as natural a starting point as to assume, as Brander and Lewis do, that managers' preferences are perfectly aligned with the shareholders. Indeed, when the manager is risk-averse, or not sufficiently susceptible to monetary incentives, it may be impossible for the shareholders to write an incentive contract that perfectly aligns the manager's preferences to those of the shareholders.

In most settings restrictions on contract design arising from these issues will tend to hurt the principal, since the agent's choices will tend to be inefficient. One of the main results here will be that, by contrast, it may actually help the shareholders when quantity choices are made by a manager whose objectives differ from their own. A similar result has been obtained by Hirshleifer and Thakor (1992). The intuition there is that a manager who cares about his reputation may be more conservative with respect to project choices, which will alleviate the conflict of interest between shareholders and

debtholders over the choice of investment portfolios, as described by Myers (1977). While in our setting also the manager will be more conservative than the shareholders, this is not what will eventually be driving the results. What is important in our case is the strategic interaction between manager controlled firms. To see the basic intuition, recall that when goods are substitutes the choice of quantities is akin to a prisoner's dilemma situation. Both firms would like to reduce their quantities in order to enjoy larger profits. However, when the rival's quantity is low it pays to increase one's own quantity since this increases sales whereas the reduction in price is felt only one's own share of the market. Consider

felt only one's own share of the market. Consider therefore a standard prisoner's dilemma game, such as

1\2	c	d
c	5,5	0,10
d	10,0	3,3

where (d, d) is the only equilibrium. Assume then that the players (the prisoners) can now send agents (their lawyers) to play the game and that the lawyers get a private benefit, or a success premium, whenever the outcome is strictly bigger than a cutoff of say 3. When both players send their lawyers, these will play the following game

1\2	c	d
c	b,b	0,b
d	b,0	0,0

In this example, if one lawyer cooperates, the other lawyer does not increase his payoff from moving to defect. Thus both are happy to play c , so that (c, c) becomes an equilibrium. This illustrates that more collusive and mutually beneficial outcomes can be sustained by delegating play to an agent whose preferences differs from one's own. Of course it then also becomes an issue which cutoff will be chosen and whether these agents are sent in equilibrium if it is the player's choice to either play the game himself or to send an agent. These issues will be looked at more carefully in the framework of the model, below. Section 2 will introduce the model. Section 3 will consider the benchmark case of owner control. Section 4 will explore manager control and give the main results. In section 5 some examples are provided, and section 6 will then endogenize control. Section 7 will offer some discussion and extensions of the results and section 8 will conclude.

2. The Model

Consider two identical firms who compete in quantities in an output market. Each firm's profit is given by $\Pi^i(\theta^i, q_i, q_j)$, where θ^i is an idiosyncratic shock, q_i is the quantity chosen by firm i and q_j is the quantity chosen by its rival. Shocks are

distributed identically and independently, $\theta^i \in (\underline{\theta}, \bar{\theta})$ according to a continuous distribution F .

In line with Bikhchandani and Riley (1991), we assume that the profit function is strictly concave in own quantity and strictly convex in rival quantity.

- (i) $\Pi_i^i(\theta^i, q_i, q_j)$ is strictly concave in q_i and strictly convex in q_j .
- (iv) $\Pi_i^i(\theta^i, q_i, q_j)$ is strictly concave in q_i and strictly convex in q_j .

Assumption 1. For each firm i , there exists a high realization θ^i such that the firm's profit is strictly positive. According to Assumption 1, the firm's profit is strictly positive for high realizations of the shock. To guarantee that the firm's profit is strictly positive for high realizations of the shock, we assume that the firm's profit is strictly positive for high realizations of the shock. (iv) is a consequence of Assumption 1. (v) is a consequence of Assumption 1. (vi) says that the firm's profit is strictly positive for high realizations of the shock.

For future reference, we assume that the firm's profit is strictly positive for high realizations of the shock. For future reference, we assume that the firm's profit is strictly positive for high realizations of the shock.

for firms i, j , and θ^i is an idiosyncratic shock. The firm's profit is given by $\Pi^i(\theta^i, q_i, q_j)$.

In the model, the firm's profit is given by $\Pi^i(\theta^i, q_i, q_j)$. In the first stage, the firm chooses its quantity q_i . In the second stage, the firm chooses its quantity q_j . The firm's profit is given by $\Pi^i(\theta^i, q_i, q_j)$.

the benchmark case of owner control. Section 4 will explore manager control and give the main results. In section 5 some examples are provided, and section 6 will then endogenize control. Section 7 will offer some discussion and extensions of the results and section 8 will conclude.

2 The Model

Consider two identical firms who compete in quantities in an output market. Each firm's profit is given by $\pi^i = \mu^i q_i q_j$; where μ^i is an idiosyncratic shock, q_i is the quantity chosen by firm i and q_j is the quantity chosen by its rival. Shocks are distributed identically across firms. More specifically, μ^i realizes on an interval $[\underline{\mu}, \bar{\mu}]$ according to some distribution function $F(\mu)$ with density $f(\mu)$.

In line with Brander and Lewis (1986) we make the following assumptions on the profit function.

$$\begin{aligned} \text{(i)} \quad \frac{\partial^2 \pi^i}{\partial \mu^i \partial q_i} &> 0; \text{(ii)} \quad \frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j} < 0; \text{(iii)} \quad \frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j} > \alpha(A-1) \\ \text{(iv)} \quad \frac{\partial^2 \pi^i}{\partial q_i^2} &< 0; \text{(v)} \quad \frac{\partial^2 \pi^i}{\partial q_i \partial q_j} < 0; \text{(vi)} \quad \frac{\partial \pi^i}{\partial \mu^i} > 0 \end{aligned}$$

Assumption (i) says that profit is increasing in the shock. This means that high realizations of μ^i result in high profits, and thus are 'good' states of the world. According to assumption (ii) profit of firm i is decreasing in the rival's output. To guarantee interior solutions assumption (iii) postulates that it is worth producing something for any realization of μ^i and any output decision of the firm's rival. (iv) is a concavity assumption while assumption (v) determines the nature of competition between the two firms. It stipulates that quantities are strategic substitutes when both firms are maximizing profit. When firm j increases its output, firm i has an incentive to decrease its output in response. Assumption (vi) says that marginal profit is increasing in μ^i . According to this assumption, good states of the world are associated with higher marginal profits.

For future reference let us state here the equilibrium of the simple game in which owners move once to simultaneously choose quantities. This is given as the solution to

$$\int_{\underline{\mu}}^{\bar{\mu}} \pi^i(\mu^i; q_i; q_j) f^i(\mu^i) d\mu^i = 0$$

for firms i, j ; and we will refer to it as the Cournot equilibrium or the Cournot point $(q^c; q^c)$.

In the model there is a financing stage which precedes the quantity setting stage. In the financing stage the owner of each firm can issue debt against the future earnings of the company. Owners can choose any face value $D_i \geq 0$. The choice of face value is made simultaneously. Once chosen, $(D_i; D_j)$ becomes common knowledge.

After the financing stage, outputs are chosen by the agents who are in charge of making these decisions. Output decisions are taken before the realization of $\mu^i; \mu^j$ is known and are made simultaneously. It is assumed that the output decision taken by this agent is his private knowledge, but that realized operating profit is verifiable.

We make two further technical assumptions. First, $\pi^i(\mu^i; q_i; q_j) \geq 0$ $\forall \mu^i \in [\underline{\mu}; \bar{\mu}]$ and $\pi^i(q_i; q_j)$ in a sufficiently large neighborhood of $(q^c; q^c)$. Under this assumption an all equity firm without limited liability is equivalent to a firm protected by limited liability with a debt level of $D_i = 0$. Second, $\pi^i(\mu^i; q_i; q_j) < 0$ $\forall D_i > 0$ and $\pi^i(q_i; q_j)$ in a sufficiently large neighborhood of $(q^c; q^c)$: This assumption guarantees that debt is risky for all, even very small, positive debt levels¹.

We will analyze two cases. In the benchmark case, following Brander and Lewis (1986), quantities are chosen by the owners of the company. As an alternative we will consider the case, where the manager receives a private benefit when the firm is not bankrupt.

¹These assumptions are easily satisfied by taking a profit function $\pi^i(\mu^i; q_i; q_j)$ which is unbounded for unbounded μ^i ; rescaling it to $A \exp \pi^i(\mu^i; q_i; q_j)$, and letting $\mu^i \in [1; \bar{\mu}]$: Note that such a rescaling preserves A1.

3 A Benchmark: Owner Control

Let us first analyze the case where owners choose quantities after having chosen debt levels at the financing stage. This case has been analyzed by Brander and Lewis (1986) and we rework it here for ease of reference. Consider the subgame that ensues after some arbitrary pair of debt face values, (D_i, D_j) has been fixed at the financing stage. In this subgame shareholders of firm i and firm j simultaneously choose quantities.

Given debt levels (D_i, D_j) ; the owner of firm i will choose q_i to maximize

$$S_i = \int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) - D_i f(\mu^i) d\mu^i \quad (1)$$

where the lower bound of integration β marks the threshold for bankruptcy and is defined implicitly by

$$\int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) - D_i = 0 \quad (2)$$

For given quantity choices the firm defaults for realizations of μ^i such that $\mu^i \leq \beta$. For these realizations the shareholders' payoff is zero, whereas it is $\int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) - D_i$ for all realizations such that $\mu^i > \beta$.

Differentiating one obtains the first-order condition for a maximum as

$$S_i' = \int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) f(\mu^i) d\mu^i + \frac{d\beta}{dq_i} \int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) - D_i f(\beta) = 0 \quad (3)$$

However, since

$$\int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) - D_i = 0$$

the second term vanishes and the first-order condition reduces to

$$S_i' = \int_{\beta}^{\bar{\mu}^i} \mu^i(q_i; q_j) f(\mu^i) d\mu^i = 0 \quad (4)$$

which says that the expected or "average" marginal profit integrated over all non-default states must be zero.

The second-order condition for a maximum is

Consider first the effect of a change of a firm's indebtedness on its optimal quantity choice for any given quantity choice of its rival. In a first step note that by implicitly differentiating (3.1) one finds

$$\frac{d\hat{\theta}}{dD_i} = \frac{1}{\Pi_{\theta}^i(\hat{\theta}, q_i, q_j)} > 0$$

which is intuitive. With a higher face value the firm defaults for higher realizations θ^i , so that the threshold $\hat{\theta}$ moves up with D_i . Implicitly differentiating the first-order condition (3.4) one has

$$\frac{\partial q_i}{\partial D_i} = -\frac{S_{iD_i}^i}{S_{ii}^i}$$

where the denominator is negative by the second-order condition (3.3). The numerator is

$$S_{iD_i} = -\Pi_{\theta}^i(\hat{\theta}, q_i, q_j) f(\hat{\theta}) \frac{d\hat{\theta}}{dD_i}$$

When evaluated at the optimum, by $\Pi_{\theta}^i(\theta^i, q_i, q_j) > 0$ and the first-order condition (3.4) one has that $\Pi_{\theta}^i(\hat{\theta}, q_i, q_j) < 0$; for "average" marginal profit to be zero, it must be that marginal profits are negative at the lower bound of integration. Therefore, $S_{iD_i} > 0$, and $\frac{\partial q_i}{\partial D_i} > 0$. This means that a higher debt level will shift the firm's reaction function out. Intuitively, for any quantity choice of the rival, with a higher debt level states of negative marginal profits are discarded from the calculus, so that overall marginal profits are positive and the quantity choice will increase.

Let us next consider the slope of the reaction functions. Firm i 's optimal response to a change in the quantity of its rival can be found by implicitly differentiating the first-order condition (3.4) to get

$$\frac{\partial q_i}{\partial q_j} = -\frac{S_{ij}^i}{S_{ii}^i}$$

where again the denominator is negative, so that the overall effect will therefore be positive. This can also be evaluated as

$$S_{ij} = \int_{\hat{\theta}}^{\theta^j} \Pi_{ij}^i$$

One sees that there are two effects: the first part of this expression is positive since $\theta^j > \hat{\theta}$ and Π_{ij}^i are substitutes, quantity q_j increases the second part of this expression is negative since again that $\Pi_{ij}^i(\hat{\theta}, q_i, q_j) < 0$.

, since $\Pi_{ij}^i(\hat{\theta}, q_i, q_j) < 0$.

The positive effect of q_j is that its size depends on the relevant range, one will find that the effect is positive in the relevant range, so that the overall effect is positive and the first effect is like the second effect. Lewis and assume that

Given the behavior of the reaction functions in debt levels. Since the reaction functions are downward sloping (shareholder, shareholder + equity) value of the firm is increasing in debt level equilibrium in debt level

to be zero, it must be that marginal profits are negative at the lower bound of integration. Therefore, $S_{iD_i} > 0$, and $\frac{\partial q_i}{\partial D_i} > 0$: This means that a higher debt level will shift the firm's reaction function out. Intuitively, for any quantity choice of the rival, with a higher debt level states of negative marginal profits are discarded from the calculus, so that overall marginal profits are positive and the quantity choice will increase.

Let us next consider the slope of the reaction functions. Firm i 's optimal response to a change in the quantity of its rival can be found by implicitly differentiating the first-order condition (3.4) to get

$$\frac{\partial q_i}{\partial q_j} = -i \frac{S_{ij}^i}{S_{ii}^i}$$

where again the denominator is negative by the second-order condition. The overall effect will therefore have the same sign as the numerator, which can be evaluated as

$$S_{ij}^i = \int_{\underline{\mu}}^{\bar{\mu}} \left[\frac{\partial^2 \pi_i}{\partial q_i \partial q_j}(\mu^i; q_i; q_j) - \mu^i \frac{d}{d\mu^i} \left[\frac{\partial \pi_i}{\partial q_j}(\mu^i; q_i; q_j) \right] \right] f(\mu^i) d\mu^i$$

One sees that there are two opposing effects. Since $\frac{\partial^2 \pi_i}{\partial q_i \partial q_j}(\mu^i; q_i; q_j) < 0$ the first part of this expression is negative. It captures the usual intuition that if goods are substitutes, quantity choice will be strategic substitutes. Observe however that the second part of this expression is positive. This can be established by noting again that $\frac{\partial \pi_i}{\partial q_j}(\mu^i; q_i; q_j) < 0$ and implicitly differentiating (3.2) to get

$$\frac{d}{d\mu^i} \left[\frac{\partial \pi_i}{\partial q_j}(\mu^i; q_i; q_j) \right] > 0$$

, since $\frac{\partial \pi_i}{\partial q_j}(\mu^i; q_i; q_j) < 0$ and $\frac{\partial \pi_i}{\partial \mu^i}(\mu^i; q_i; q_j) > 0$:

The positive effect captures what goes on at the limit of integration. Note that its size depends on the distribution of μ^i . For β small enough over the relevant range, one will have a regular downward sloping curve. If there is a lot of uncertainty, so that the interval $[\underline{\mu}, \bar{\mu}]$ is large and $f(\mu)$ is small

on average, then the positive effect is of second-order importance at least for small levels of debt and the first effect is likely to dominate. For these reasons we follow Brander and Lewis and assume that $S_{ij} < 0$:

Given the behavior at the quantity stage, one can characterize equilibrium in debt levels. Since the debtholder pays the expected value of his claim to the shareholder, shareholders are concerned with maximizing expected overall (debt + equity) value of the firm at the financing stage. One can then analyze the equilibrium in debt levels. Let us define

$$V^i(q_i; q_j) = \int_{\underline{\mu}}^{\bar{\mu}} v^i(\mu^i; q_i; q_j) f^i(\mu^i) d\mu^i$$

as the ex ante value of the firm. Equilibrium is characterized by a pair $(D_i; D_j)$ such that

$$\max_{D_i} V^i(q_i; q_j)$$

$$\text{s.t. } q_i = q_i^S(q_j; D_i)$$

$$q_j = q_j^S(q_i; D_j)$$

$$D_i \geq 0$$

holds for both firms. Each firm owner chooses its firm's reaction function taking the reaction function of its rival as given. To characterize the equilibrium further recall that the Cournot point $(q^c; q^c)$ is defined as the solution to

$$\int_{\underline{\mu}}^{\bar{\mu}} v^i(\mu^i; q_i; q^c) f^i(\mu^i) d\mu^i = 0$$

for firms i and j : Consider the pair of reaction functions that go through $(q^c; q^c)$: In the case of owner control these are given implicitly by

$$\int_{\underline{\mu}}^{\bar{\mu}} v^i(\mu^i; q_i; q_j) f^i(\mu^i) d\mu^i = 0$$

and are characterized by a zero level of debt. One can show that debt levels of zero do not constitute an equilibrium here, but that reactions functions will

be shifted out. To see this start with the reaction functions going through $(q^c; q^c)$; that is, assume that $(D_i; D_j) = (0; 0)$: Given that an increase i

Note that $S_{jD_i}^j = 0$. Using Cramer's rule one can establish that

$$\frac{dq_i}{dD_i} = -\frac{S_{iD_i}^i S_{jj}^j}{S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji}^j} > 0$$

$$\frac{dq_j}{dD_i} = \frac{S_{iD_i}^i S_{ji}^j}{S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji}^j} < 0$$

when $S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji}^j > 0$ and assuming that $S_{ji}^j < 0$.

Total value of the firm is

$$V^i = \int_{\underline{\theta}}^{\bar{\theta}} \Pi^i(\theta^i, q_i(D_i, D_j), q_j(D_i, D_j)) f(\theta^i) d\theta^i$$

where $(q_i(D_i, D_j), q_j(D_i, D_j))$ is the solution to the pair of constraints for any pair (D_i, D_j) . Differentiating with respect to D_i one finds the first-order condition

$$V_{D_i}^i = \left[\int_{\underline{\theta}}^{\bar{\theta}} \Pi_i^i(\theta^i, q_i(D_i, D_j), q_j(D_i, D_j)) f(\theta^i) d\theta^i \right] \frac{dq_i}{dD_i} + \left[\int_{\underline{\theta}}^{\bar{\theta}} \Pi_j^i(\theta^i, q_i(D_i, D_j), q_j(D_i, D_j)) f(\theta^i) d\theta^i \right] \frac{dq_j}{dD_i}$$

Assume first that $D_i = D_j = 0$. Then quantities will be set at the Cournot level, $q_i = q_j = q^c$. At these levels of output the first bracket is zero. The second term is positive however since $\Pi_j^i < 0$ and also $\frac{dq_j}{dD_i} < 0$. Therefore each firm wants to unilaterally increase its debt level. In a symmetric equilibrium therefore $D_i = D_j > 0$, which looking back at (3.3) entails that $q_i = q_j > q^c$. Equilibrium quantities will be beyond the Cournot quantities. Note that this also implies that $V^i = V^j < V^c$. In equilibrium owners will be worse off than they would if they could not issue debt.

4. Manager Control

Let us now consider the case where the output decision is delegated to a manager, whose objective is to avoid bankruptcy. We assume that the manager's quantity

Assume first that $D_i = D_j = 0$. Then quantities will be set at the Cournot level, $q_i = q_j = q^c$: At these levels of output the first bracket is zero. The second term is positive however since $\frac{dq_j}{dq_i} < 0$ and also $\frac{dq_j}{dD_i} < 0$: Therefore each firm wants to unilaterally increase its debt level. In a symmetric equilibrium therefore $D_i = D_j > 0$, which looking back at (3.3) entails that $q_i = q_j > q^c$: Equilibrium quantities will be beyond the Cournot quantities. Note that this also implies that $V^i = V^j < V^c$. In equilibrium owners will be worse off than they would if they could not issue debt.

4 Manager Control

Let us now consider the case where the output decision is delegated to a manager, whose objective is to avoid bankruptcy. We assume that the manager's quantity choice is unobservable to the owner, so that contracts forcing the manager to choose a particular quantity are impossible. For the main part of the analysis we also disallow any other contract which may condition on profit by assuming that the manager does not respond to monetary incentives. This means that manager's preferences cannot be driven away from the goal of avoiding bankruptcy. This assumption is made mainly to have a clear starting point and will be relaxed in a later section. We assume that to produce any positive quantity $q_i > 0$ the manager has to spend some fixed, but small effort cost $\bar{e} > 0$; so that without any other incentives working on the manager the manager would choose $q_i = 0$: The only tool available to owners to motivate their managers is to issue debt against the profits of the firm. We assume that the threat of bankruptcy is the only thing that motivates the manager. In particular, the manager receives a private benefit b whenever the firm is not bankrupt and normalize his payoff in bankrupt states to zero. This is without loss of generality, since we can alternatively think of b as a constant payoff differential between bankrupt states and non-bankrupt states. We also assume $b \gg \bar{e}$; so that the manager will choose

to spend effort if debt has been issued and there is a positive probability of bankruptcy. In the subgame following the choice of debt levels the manager's objective is thus to maximize

$$B^i = \int_{\underline{\mu}}^{\bar{\mu}} b f^i(\mu^i) d\mu^i \quad (6)$$

where again β is given by

$$\beta^i(\beta; q_i; q_j) - D_i = 0 \quad (7)$$

This problem has first order condition

$$B_i^i = \beta^i b f^i(\beta) \frac{\partial \beta}{\partial q_i} = 0 \quad (8)$$

Implicitly differentiating (4.2) one finds

$$\frac{\partial \beta}{\partial q_i} = \beta^i \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \quad (9)$$

and the first-order condition can be written as

$$B_i^i = b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} = 0 \quad (10)$$

The second-order condition is

$$B_{ii}^i = \beta^i b f^i(\beta) \frac{\partial^2 \beta^i}{\partial q_i \partial q_i} - \beta^i b f^i(\beta) \frac{\partial \beta}{\partial q_i} \frac{\partial \beta}{\partial q_i} < 0$$

Again using (4.4) one has

$$\begin{aligned} B_{ii}^i = & b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} + \beta^i b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta}{\partial q_i} \\ & + b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} + \beta^i b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta}{\partial q_i} \\ & + b f^i(\beta) \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta^i(\beta; q_i; q_j)}{\partial \mu^i(\beta; q_i; q_j)} \frac{\partial \beta}{\partial q_i} \end{aligned}$$

, and since $\frac{\partial^3 \beta}{\partial q_i^3} = 0$ by the first-order condition, the second-order condition reduces to

$$B_{ii}^i = \frac{\partial^2 \beta}{\partial q_i^2} = \frac{\partial}{\partial q_i} \left(\frac{\partial \beta}{\partial q_i} \right) < 0$$

One then sees that because $\frac{\partial^2 \beta}{\partial q_i^2} < 0$ and $\frac{\partial \beta}{\partial q_i} > 0$ by assumption, the required inequality holds. Thus, whenever the first-order condition holds the second-order condition will also be satisfied². This implies that for any given debt level and any given rival's output the first-order condition uniquely determines the manager's optimal choice of q_i : The first-order condition therefore implicitly defines a function $q_i^m(q_j; D_i)$ which gives the manager's optimal output choice for any given rival's choice and for any given debt level.

It is useful at this point to compare the manager's problem with the one analyzed in the benchmark case. The manager obtains a positive benefit only when the firm is not bankrupt. He is therefore interested in widening the interval $[\beta; \bar{\mu}]$ as much as possible, since this will minimize the probability of bankruptcy. The manager's problem is therefore equivalent to minimizing β by choice of q_i for any given debt level D_i and any given choice of q_j : Looking back at the first-order condition it is worth noting that it implies that

$$\frac{\partial \beta}{\partial q_i} = 0$$

holds at this minimized β : One can see the intuition for this by assuming that $\frac{\partial \beta}{\partial q_i} = 0$ held for a given D_i , a given q_j , and some choice of q_i ; and that $\frac{\partial \beta}{\partial q_i} > 0$ for the implied β . Then the manager can increase profit by increasing q_i ; which will make $\beta > D_i$ at the old β . But this means that bankruptcy can be avoided for a realization of μ^i below the old β : There will therefore be scope to decrease β by increasing q_i ; and the original choice of q_i can not have been optimal. A reverse argument

²Note that this is true even though the manager's problem may not be globally concave.

can be made for the case that $\frac{\partial^2 \pi}{\partial q_i \partial q_j} < 0$: We therefore must have $\frac{\partial \pi}{\partial q_i} = 0$: This means that the manager choice of q_i is such that he is maximizing profit at the minimized level of β : This is in contrast to the benchmark case where the shareholders were maximizing profit over the interval $\beta; \bar{\mu}$:

As a first comparative static exercise let us analyze how the manager's behavior is influenced by the debt level chosen. One finds that just as in the benchmark case the reaction function shifts out as the debt level increases and state this more formally as

Lemma 1 In the subgame following the choice of debt levels, for given q_j ; with manager control over quantities, a higher debt level D_i will induce the manager to choose a larger output q_i :

Proof:

$$\frac{\partial q_i}{\partial D_i} = \frac{B_{iD}^i}{B_{ii}^i}$$

Since the second-order condition holds, the sign of this will be the same as the sign of B_{iD}^i : One easily obtains

$$\begin{aligned} B_{iD}^i &= \frac{\partial^2 \pi}{\partial q_i \partial D_i} = \frac{\partial}{\partial D_i} \left(\frac{\partial \pi}{\partial q_i} \right) \\ &= \frac{\frac{\partial^2 \pi}{\partial q_i \partial q_j} \frac{\partial q_j}{\partial D_i} + \frac{\partial^2 \pi}{\partial q_i \partial \mu} \frac{\partial \mu}{\partial D_i} + \frac{\partial^2 \pi}{\partial q_i \partial D_i} \frac{\partial D_i}{\partial D_i}}{\frac{\partial^2 \pi}{\partial q_i^2}} \\ &= \frac{\frac{\partial^2 \pi}{\partial q_i \partial q_j} \frac{\partial q_j}{\partial D_i} + \frac{\partial^2 \pi}{\partial q_i \partial \mu} \frac{\partial \mu}{\partial D_i} + \frac{\partial^2 \pi}{\partial q_i \partial D_i}}{\frac{\partial^2 \pi}{\partial q_i^2}} \end{aligned}$$

again using that $\frac{\partial \pi}{\partial q_i} = 0$: All terms in the numerator of this last expression are positive. In particular, implicitly differentiating

$$\frac{\partial \pi}{\partial q_i} = 0$$

gives

$$\frac{d\beta}{dD_i} = \frac{1}{\mu^i \beta; q_i; q_j} > 0$$

Hence

$$B_{iD}^i = \beta \frac{\mu^i \beta; q_i; q_j}{\mu^i \beta; q_i; q_j} > 0$$

so that

$$\begin{aligned} \frac{\partial q_i}{\partial D_i} &= i \frac{B_{iD}^i}{B_{ii}^i} \\ &= i \frac{\mu^i \beta; q_i; q_j}{\mu^i \beta; q_i; q_j} > 0 \end{aligned}$$

The intuition for this result starts by recalling that for any debt level the manager is minimizing β by choice of q_i : Call this minimized value β^m : It is clear that when $D_i^0 > D_i$; then also $\beta^{m0} > \beta^m$: For both levels of debt the manager is maximizing profit at the minimized β : Since marginal profit is increasing in μ^i , $\partial \mu^i / \partial \beta > 0$; when profit is maximized at β^{m0} a higher quantity is called for than when profit is maximized at the lower β^m : The quantity chosen will therefore be increasing in the debt level³.

Since firm i 's output is increasing in its own debt level both for the case where the manager makes decisions and for the benchmark case where quantities are chosen by the owners themselves it may be interesting to compare quantity levels for given debt levels across regimes. The following result is easily obtained:

Proposition 2 For given $(D_i; D_j)$ and given rival's quantity q_j firm i 's quantity choice will be smaller when taken by a manager than when taken by the firm's owner; $q_i^m(q_j; D_i) < q_i^s(q_j; D_i)$

³The intuition here is similar to the case when the manager maximizes the value of the firm and there are exogenous fixed bankruptcy costs, as analyzed in Brander and Lewis (1988).

Proof: The manager chooses q_i at the minimized value $\beta^m = \beta(q_i^m(q_j; D_i); q_j; D_i)$ such that

$$\frac{\partial}{\partial q_i} \beta^m(q_i^m; q_j) = 0$$

is satisfied. Given the same debt level the owner's choice q_i^s would satisfy

$$\int_{\beta}^{\bar{\mu}} \frac{\partial}{\partial q_i} (\mu; q_i^s; q_j) f(\mu) d\mu = 0$$

Clearly, in the latter expression $\beta > \beta^m$; since under owner control the lower bound of integration β is not being minimized: Since $\frac{\partial}{\partial \mu} \mu^i; q_i^s; q_j > 0$ it then follows that $q_i^s > q_i^m$:

For any given debt level the manager is less aggressive than the owner. The manager's objective is to avoid bankruptcy, so that he is looking at the marginal state, where marginal profit is low, whereas the owner will maximize profit over all non-bankrupt states $\mu^i \geq \beta; \bar{\mu}$ where marginal profit is higher. This result confirms the intuition that the manager's output choice will be more conservative than the shareholder's output choice.

Since we have been looking at the subgame only, however, this result cannot be taken to say that the overall equilibrium will be characterized by lower quantities when the manager is in charge of the quantity choice. The owner can choose the debt level before the manager chooses a quantity, so that in principle, the owners can counter the manager's reluctance to choose high quantities by pushing up the debt level at the financing stage.

Before we can characterize equilibrium in debt levels and quantities we need to take into account the strategic interaction between managers. Recall that when quantities are chosen by the owners, an increase in q_j always induces a decrease in q_i along a downward sloping reaction function for appropriate assumptions on the density $f(\mu)$. By contrast, under manager control this need not be the case. Depending on the exact functional form of the profit function the manager's reaction function may be downward sloping or upward sloping. More formally

Lemma 3 In the subgame following the choice of debt levels; when the manager of the rival firm j chooses a higher quantity q_j manager i 's optimal quantity choice q_i may increase, stay the same or decrease.

For the proof note that:

$$\frac{\partial q_i}{\partial q_j} = i \frac{B_{ij}^i}{B_{ii}^i}$$

which again since $B_{ii}^i < 0$ will have the same sign as B_{ij}^i .

$$\begin{aligned} B_{ij}^i &= bf \frac{\frac{\partial^2 \pi_i}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_i}{\partial q_i^2}} \\ &+ bf \frac{\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_j}{\partial q_j^2}} \\ &+ bf \frac{\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} \frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_i}{\partial q_i^2} \frac{\partial^2 \pi_j}{\partial q_j^2}} \\ &= bf \frac{\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} + \frac{\partial^2 \pi_j}{\partial q_i \partial q_j} \frac{d\beta}{dq_j}}{\frac{\partial^2 \pi_i}{\partial q_i^2}} \end{aligned}$$

since $\frac{\partial^2 \pi_i}{\partial q_i \partial q_j} = 0$:

The sign of this is ambiguous. Note that it will be the same as the numerator, which since

$$\frac{d\beta}{dq_j} = i \frac{\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_j}{\partial q_j^2}} > 0$$

we can write as

$$\frac{\frac{\partial^2 \pi_i}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_i}{\partial q_i^2}} + i \frac{\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_j}{\partial q_j^2}} \frac{\frac{\partial^2 \pi_j}{\partial q_i \partial q_j}}{\frac{\partial^2 \pi_j}{\partial q_j^2}}$$

It follows from (A 1) that

$$\frac{\partial q_i}{\partial q_j} \Big|_{\beta} < 0$$

but that

$$\frac{\partial q_i}{\partial \mu} \Big|_{\beta} + \frac{\partial q_i}{\partial \beta} \Big|_{\mu} \frac{\partial \beta}{\partial q_j} > 0$$

As can be seen from this, there are two effects at work.

The first term captures the usual strategic effect. If the other firm increases its quantity, manager i has an incentive to reduce his quantity, and vice versa. This is because, as pointed out before, the manager is maximizing profit at some minimized level of β . At this level the manager's response to a change in the rival's quantity will be profit-maximizing and therefore be of the same sign as when managers behave as shareholders would. Since $\frac{\partial q_i}{\partial q_j} \Big|_{\beta} < 0$; when the rival firm increases its output q_j , manager i has an incentive to reduce his choice of q_i in response.

On the other hand, and captured by the positive part of the expression, a change in q_j will move β : An increase in q_j will depress firm i 's profit, since $\frac{\partial \pi_i}{\partial q_j} < 0$ and therefore move β upward. When β gets pushed up, this will call for a higher q_i ; since marginal profit is higher at higher μ ; $\frac{\partial q_i}{\partial \mu} > 0$: Therefore, when q_j goes up, the manager's response will be to increase his choice of q_i .

When the first effect dominates, quantities are strategic substitutes, as they are under profit-maximization, and reaction functions slope downwards. When the second effect dominates, quantities, which are strategic substitutes under profit maximization, become strategic complements when the probability of bankruptcy is being minimized, and reaction functions slope upwards. Loosely speaking, this is due to profit drain effect. When q_j goes up, this will put a profit drain on firm i : Under the pressure of this profit drain the manager of firm i will have to compete more aggressively to keep up the odds of keeping the company out of bankruptcy. On the other hand, when q_j goes

down, this will bolster firm i's profit and relieve the pressure on the manager of firm i who will then respond by competing less aggressively in order to increase the odds of keeping the company afloat.

Note that the direction of the overall effect no longer depends on the distribution of μ^i over its support. The density no longer enters the expression, and the sign of the expression will be the same for high and low degrees of uncertainty. Which of the two effects will dominate will solely depend on the exact shape of the profit function: From the expression one sees that quantities are more likely to become strategic complements when $\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j}$ and $\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_i}$ are relatively small, but $\frac{\partial^2 \pi^i}{\partial q_i \partial q_j}$ and $\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_i}$ are relatively large. Another way of looking at this is to note that

$$\frac{\partial^2 \pi^i}{\partial q_i \partial q_j} - \frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j} \frac{\partial \mu^i}{\partial q_i} > 0$$

translates into the following condition on the elasticities of the marginal effects of μ^i and q_j on firm profit

$$\frac{\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j} \frac{q_i}{\pi^i}}{\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_i} \frac{q_i}{\pi^i}} > \frac{\frac{\partial^2 \pi^i}{\partial q_i \partial q_j} \frac{q_i}{\pi^i}}{\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j} \frac{q_i}{\pi^i}}$$

Reaction functions will slope upwards whenever the marginal effect of μ^i on firm profit is more elastic with respect to changes in q_i than the marginal effect of q_j : The intuition is that when the rival firm increases its quantity this will increase both q_j and β : When the manager increases his quantity in response this will enlarge the adverse effect on $\frac{\partial^2 \pi^i}{\partial q_i \partial q_j}$. On the other hand it will have a positive impact on $\frac{\partial^2 \pi^i}{\partial \mu^i \partial q_j}$: If the positive effect is stronger than the negative effect, the manager will optimally increase his quantity.

Whether reaction functions slope upwards or downwards will impact decisions at the financing stage. Equilibrium at the financing stage is given by

$$\max_{D_i} V^i(q_i; q_j)$$

$$\begin{aligned} \text{s.t: } q_i &= q_i^m(q_j; D_i) \\ q_j &= q_j^m(q_i; D_j) \\ D_i &\geq 0 \end{aligned}$$

Again replacing the constraints by the first-order conditions and linearizing.

$$\begin{aligned} B_{ii}^i dq_i + B_{ij}^i dq_j + B_{iD_i}^i dD_i &= 0 \\ B_{ji}^j dq_i + B_{jj}^j dq_j + B_{jD_i}^j dD_i &= 0 \end{aligned}$$

Note that $B_{jD_i}^j = 0$: Using Cramer's rule one can establish that

$$\begin{aligned} \frac{dq_i}{dD_i} &= -i \frac{B_{iD_i}^i B_{jj}^j}{B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j} > 0 \\ \frac{dq_j}{dD_i} &= \frac{B_{iD_i}^i B_{ji}^j}{B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j} > (=) (<) 0 \end{aligned}$$

Assuming that the regularity condition $B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j > 0$ holds we can sign the first derivative since $B_{iD_i}^i > 0$; as shown above and $B_{jj}^j < 0$ by second-order condition.

Again under regularity condition $B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j > 0$, the sign of the second derivative will be the same as the sign of B_{ji}^j : This in turn will be of the same sign as

$$\frac{\partial^2 q_i}{\partial q_j \partial q_i} + \frac{\partial^2 q_j}{\partial q_i \partial q_j} - \frac{\partial^2 q_i}{\partial q_j \partial q_i}$$

As explained above, the sign of this expression is ambiguous.

Consider again the total value of the firm

$$V = \int_{\mu}^z \int_{\mu}^i \mu^i(q_i; q_j) f(\mu^j) d\mu^j$$

Differentiating with respect to D_i one finds the first-order condition

$$\begin{aligned}
V_{D_i} &= \sum_i \left[\frac{\partial \pi_i}{\partial q_i} \left(\mu_i^i; q_i(D_i, D_j); q_j(D_i, D_j) \right) \frac{dq_i}{dD_i} \right] \\
&+ \sum_j \left[\frac{\partial \pi_j}{\partial q_j} \left(\mu_j^j; q_i(D_i, D_j); q_j(D_i, D_j) \right) \frac{dq_j}{dD_i} \right] \\
&= 0
\end{aligned}$$

There will be positive debt levels $D_i = D_j > 0$ such that the managers' reaction functions intersect at the Cournot point $(q^c; q^c)$: At the Cournot-level of output, the term in the first bracket is zero. Since $\frac{\partial \pi_j}{\partial q_j} < 0$ the term in the second bracket is negative. The overall sign of the derivative will therefore depend on $\frac{dq_j}{dD_i}$:

If $B_{ji}^j < 0$, the rival's reaction function is downward sloping and one will have $\frac{dq_j}{dD_i} < 0$: Just as in the benchmark case there is an incentive to increase D_i ; since this will lead the rival to reduce its quantity along its reaction curve. This incentive exists for both firms, so that in equilibrium quantities will be higher than Cournot, $(q_i; q_j) = (q^s; q^s) > (q^c; q^c)$, implying ex ante firm values less than Cournot, $V^i = V^j < V^c$.⁴

If $B_{ji}^j = 0$, the rival's reaction function is horizontal. The rival will produce q^c for any quantity firm i produces. Then also $\frac{dq_j}{dD_i} = 0$; and there is no incentive to change the debt level for strategic reasons. The equilibrium quantities will be the Cournot quantities, $(q_i; q_j) = (q^s; q^s) = (q^c; q^c)$: There is no limited liability effect and ex ante firm values will be the Cournot values, $V^i = V^j = V^c$.

If $B_{ji}^j > 0$; the rival's reaction function will be upward sloping and $\frac{dq_j}{dD_i} > 0$: There now is an incentive to decrease D_i ; that is to move the own reaction

⁴Note that one may still have a more collusive quantity choice under manager control as compared with owner control. As shown in the appendix, this will be the case whenever along the line $(q_i; q_j) = (q; q)$ with $(q; q) > (q^c; q^c)$ one has

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

function in, rather than out. This will imply that quantities will be lower than Cournot in equilibrium, $(q_i; q_j) = (q^m; q^m) < (q^c; q^c)$: One can also show that quantities will not be smaller than the joint profit maximizing quantities (see the appendix for a proof), so that here quantities will lie in between the joint profit maximizing and the Cournot quantities. This implies that ex ante firm values will be higher than Cournot, $V^i = V^j > V^c$:

We summarize these results in the following

Proposition 4 In a symmetric equilibrium in debt levels and quantities, when quantities are chosen by managers, equilibrium quantities may be less than, greater than, or equal to Cournot quantities.

The case where equilibrium quantities are (weakly) less than Cournot is intriguing, since it highlights the possibility of sustaining a (weakly) more collusive outcome than would obtain in the simple one-shot game with straight equity value maximization. The intuition for this case is that at the Cournot levels of output both firms want to decrease their debt levels in order to decrease the pressure on the rival firm's manager to generate profits. Less pressure on the rival firm will result in lower rival output and thus benefits the firm which decreases its debt level away from the Cournot level.

5 Examples

Under manager control equilibrium quantities will be equal or below $(q^c; q^c)$ when $B_{ji}^j > 0$: By symmetry this will be the case whenever the profit function satisfies

$$\frac{\partial^2 \pi^i(q_i; q_j)}{\partial q_i^2} < \frac{\partial^2 \pi^i(q_i; q_j)}{\partial q_i \partial q_j} < \frac{\partial^2 \pi^i(q_i; q_j)}{\partial q_j^2} > 0$$

To illustrate that this may well be satisfied take the standard example of a linear demand function and weakly convex costs.

$$\pi^i(q_i; q_j) = [a_i - bq_i - q_j]q_i - cq_i^2$$

where $0 < \sigma < b$ and $\sigma > 1$: In the demand function we allow for the possibility that goods may not be perfect substitutes, in which case $\sigma < b$. Costs are strictly convex when $\sigma > 1$; whereas they are linear when $\sigma = 1$: As it is, the profit function is deterministic. We can make it stochastic by letting its parameters be functions of μ^i : Let us start by looking at cost uncertainty. Replacing c by $c^0 \mu^i$ with $c^0 \mu^i > 0$ and $c^0 \mu^i < 0$ one arrives at a function

$$V^i(\mu^i; q_i; q_j) = [a_i - b q_i - q_j] q_i - c^0 \mu^i q_i^\sigma$$

which satisfies A 1. One finds

$$\begin{aligned} \frac{\partial V^i(\mu^i; q_i; q_j)}{\partial q_j} - \frac{\partial V^j(\mu^j; q_i; q_j)}{\partial q_i} &= -c^0 \mu^i \sigma q_i^{\sigma-1} \frac{q_i}{q_i} \\ &= -\frac{(\sigma q_i^{\sigma-1} q_i)}{q_i} = -(\sigma - 1) > 0 \end{aligned}$$

For the linear cost case, $\sigma = 1$; the two opposing effects exactly cancel. For any given debt level firm i 's response to any output of its rival will be the same fixed quantity, and likewise for firm j : As we have seen, in equilibrium then $(q_i; q_j) = (q^c; q^c)$ and $V^i = V^j = V^c$: When costs are strictly convex, $\sigma > 1$; the second effect dominates. Firm i 's response to a movement in the rival's quantity will go in the same direction as the rival firm's movement. In equilibrium this will lead to $(q_i; q_j) = (q^s; q^s) < (q^c; q^c)$ and $V^i = V^j > V^c$

For demand uncertainty one gets similar results. Let us start by analyzing intercept uncertainty. Then a will be a function of μ^i and one will have

$$V^i(\mu^i; q_i; q_j) = a \mu^i - b q_i - q_j - c q_i^\sigma$$

which, when $a^0 \mu^i > 0$ satisfies A 1. For this function one finds

$$\begin{aligned} \frac{\partial V^i(\mu^i; q_i; q_j)}{\partial q_j} - \frac{\partial V^j(\mu^j; q_i; q_j)}{\partial q_i} &= a^0 \mu^i \frac{(1 - q_i)}{a^0 \mu^i q_i} \\ &= 0 \end{aligned}$$

so that again the two opposing effects exactly cancel. The net effect of a rival's move in quantities on the marginal benefit of a change in quantity is zero, so that when the rival's quantity changes this has no effect on manager i 's choice of quantity. Also, when firm i changes its debt level to move its reaction function, this will have no effect on the quantity chosen by the rival firm, so that in equilibrium debt levels will be chosen such that $(q_i; q_j) = (q^c; q^c)$ and $V^i = V^j = V^c$:

It remains to analyze slope uncertainty. One can think of μ^i entering b_i , the slope of firm i 's residual demand curve, or σ ; the degree of substitutability between the products. If $b = \sigma$ one can analyze a mix of these two types of uncertainty. It turns out that the result is the same for all these cases and we present the analysis for the last of these possibilities only. In this case we have

$$\frac{\partial \pi^i}{\partial q_i} = a_i - b_i \mu^i (q_i + q_j) - c_i$$

where $b_i \mu^i < 0$ to meet assumption A 1. One easily finds

$$\begin{aligned} \frac{\partial \pi^i}{\partial q_j} &= \frac{\partial \pi^i}{\partial q_i} \frac{\partial q_i}{\partial q_j} = \frac{a_i - b_i \mu^i (q_i + q_j) - c_i}{-b_i \mu^i} \frac{b_j \mu^j q_i}{-b_i \mu^i (q_i + q_j)} \\ &= \frac{a_i - b_i \mu^i (q_i + q_j) - c_i}{-b_i \mu^i} \frac{b_j \mu^j q_i}{-b_i \mu^i (q_i + q_j)} \\ &= \frac{a_i - b_i \mu^i (q_i + q_j) - c_i}{-b_i \mu^i} + b_j \mu^j \frac{2q_i + q_j}{q_i + q_j} > 0 \end{aligned}$$

For slope uncertainty a change in the rival's quantity has a positive net effect on marginal profit. As in the case of cost uncertainty with convex costs this will result in equilibrium quantities that are less than Cournot, $(q_i; q_j) = (q^s; q^s) < (q^c; q^c)$ and $V^i = V^j > V^c$:

6 Endogenous Control

So far it was assumed that owners of the firms have to rely on a manager to choose the firm's quantity and cannot choose quantities themselves. One traditional way of justifying such an assumption would be that ownership is dispersed and that free-rider problems lead to the need to employ an outsider to make business decisions on behalf of the shareholders. One could also assume that managers have special skills and expertise for making business decisions and that a manager has to be employed for this reason. Both these explanations are outside the realm of the model we are analyzing here. In this section we want to drop the assumption that shareholders have to employ a manager. Instead we allow the owner a choice as to whether he wants to employ a manager or make the quantity decision himself. These decisions will again be taken in a non-cooperative fashion. We model this choice as a first stage that precedes the financing and quantity setting stages. At this first stage owners simultaneously decide on whether they want to employ a manager to make the quantity decision for them, or whether they want to choose quantities themselves. After this first stage, as before, the owner can choose a debt level. Finally quantities will then be chosen by the manager or the shareholder depending on which decision was taken at the first stage of the game.

In a subgame perfect equilibrium the later play of the game can be collapsed into values associated with the equilibrium payoffs, resulting from the debt and quantity stages, for any pair of first-stage decisions. We therefore need to analyze the following game

inj	m	s
m	$V^i(m; m); V^j(m; m)$	$V^i(m; s); V^j(m; s)$
s	$V^i(s; m); V^i(s; m)$	$V^i(s; s); V^j(s; s)$

where m denotes sending a manager and s means that quantities will be chosen by the owner (a shareholder) himself.

In order to characterize the equilibrium of this game we need to make a further assumption. Given the results in the last section for the main part of this section we want to assume

$$\prod_{j \in N} \mu^j(q_i; q_j) \leq \prod_{i \in N} \mu^i(q_i; q_j)$$

more aggressive and will push its reaction function out and the shareholder-controlled firm who will lose out against a more aggressive rival. In fact, one can show that in this case under A2 the manager-controlled firm becomes a Stackelberg leader and therefore has higher profits than Cournot, whereas the shareholder-controlled firm becomes a Stackelberg follower and will end up with lower profits than Cournot. Thus, when the other firm is sending a shareholder the best response is to send a manager and become a Stackelberg leader in order to enjoy higher than Cournot profits. When the other firm is sending the manager the best response is again to send a manager in order not to become a Stackelberg follower, but again to enjoy higher than Cournot profits. Given the choice between sending a manager and choosing quantities themselves shareholders will therefore want to send the manager, whatever choice is made by the rival firm. In equilibrium both firm owners will therefore employ managers. This will ensure a more collusive outcome in equilibrium than if they made the quantity choice themselves. Intuitively, a more collusive outcome is made possible here, since a manager-controlled firm is soft, when paired with another manager-controlled firm, but highly aggressive when paired with a shareholder-controlled firm. This allows the manager-controlled firm to credibly threaten to punish a deviation to shareholder-control. As a result both firms will use an agent and thus sustain a more collusive outcome in equilibrium.

For the proof note first that for any pair of decisions made at the first stage, (a_i, a_j) , $a_i \in \{m, s\}$, $a_j \in \{m, s\}$ the equilibrium of the financing stage can be characterized by

$$\begin{aligned} & \max_{D_i} V^i(q_i, q_j) \\ \text{s.t. } & q_i = q_i^{a_i}(q_j, D_i) \\ & q_j = q_j^{a_j}(q_i, D_j) \\ & D_i \geq 0 \end{aligned}$$

holding for both firms. At the financing stage each firm chooses its own reac-

enjoy higher than Cournot profits. Given the choice between sending a manager and choosing quantities themselves shareholders will therefore want to send the manager, whatever choice is made by the rival firm. In equilibrium both firm owners will therefore employ managers. This will ensure a more collusive outcome in equilibrium than if they made the quantity choice themselves. Intuitively, a more collusive outcome is made possible here, since a manager-controlled firm is soft, when paired with another manager-controlled firm, but highly aggressive when paired with a shareholder-controlled firm. This allows the manager-controlled firm to credibly threaten to punish a deviation to shareholder-control. As a result both firms will use an agent and thus sustain a more collusive outcome in equilibrium.

For the proof note first that for any pair of decisions made at the first stage, $(a_i; a_j)$; a_i^2 firm; a_j^2 firm; a_j^2 firm; a_j^2 firm the equilibrium of the financing stage can be characterized by

$$\max_{D_i} V^i(q_i; q_j)$$

$$\begin{aligned} \text{s.t: } q_i &= q_i^{a_i}(q_j; D_i) \\ q_j &= q_j^{a_j}(q_i; D_j) \\ D_i &\geq 0 \end{aligned}$$

holding for both firms. At the financing stage each firm chooses its own reaction function taking the rival's debt level, and thus the rival's reaction function as given. Given the other firm's reaction function and the firm's choice of its own debt level a pair of quantities $(q_i; q_j)$ results and determines the expected value of the firm.

Notice also that there is an alternative and more intuitive way of characterizing the equilibrium. Whenever $D_i \geq 0$ is not binding, equilibrium quantities are solutions to

$$\max_{q_i} V^i(q_i; q_j)$$

$$\text{s:t: } q_j = q_j^{aj}(q_i; D_j)$$

for firms i and j : In equilibrium each firm's quantity is value-maximizing given the rival's reaction function. To see that this must hold, let the solution to this problem be q_i^a : Recall also that firm i 's quantity is continuously increasing in its debt level. It is then immediate that if firm i 's choice of debt level were to result in a quantity other than q_i^a given firm j 's reaction function, it would have an incentive to change its debt level in order to move its quantity closer to q_i^a : This means that one can characterize the equilibrium by a tangency condition of the firm's isoprofit curve with the other firm's reaction function. If the rival's reaction function slopes downwards, the tangency will occur at the downward sloping part of the isoprofit curve, so that Cournot quantities can no longer be an equilibrium. If the rival's reaction function slopes upwards, then again Cournot quantities are again no longer an equilibrium, since the tangency must occur at the upward sloping branch of an isoprofit curve. This implies that, as we have seen already, for $(a_i; a_j) = (m; m)$ equilibrium quantities will be less than Cournot, and profits will be higher than Cournot and that for $(a_i; a_j) = (s; s)$ equilibrium quantities are higher than Cournot and profits will be lower than Cournot.

Let us now go on to characterize the equilibrium in the subgame following $(a_i; a_j) = (m; s)$: We claim that this equilibrium is characterized by the Stackelberg quantities. To show this, look at the financing stage and assume that the shareholder controlled firm chooses a debt level such that its reaction function goes through the Cournot-level of output. This involves setting $D_j = 0$ in $q_j^s(q_i; D_j)$ so that $q_j^s(q^c; D_j) = q^c$: Given this reaction function of firm j firm i will choose its reaction function to

$$\max_{D_i} V^i(q_i; q_j)$$

$$\text{s:t: } q_i = q_i^m(q_j; D_i)$$

$$q_j = q_j^s(q_i; 0)$$

$$D_i \leq 0$$

Replacing reaction functions by the first-order conditions and linearizing one has

$$B_{ii}^i dq_i + B_{ij}^i dq_j + B_{iD_i}^i dD_i = 0$$

$$S_{ji}^j dq_i + S_{jj}^j dq_j + S_{jD_i}^j dD_i = 0$$

from which one finds

$$\frac{dq_i}{dD_i} = -i \frac{B_{iD_i}^i S_{jj}^j}{B_{ii}^i S_{jj}^j - B_{ij}^i S_{ji}^j} > 0$$

$$\frac{dq_j}{dD_i} = \frac{B_{iD_i}^i S_{ji}^j}{B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j} < 0$$

since $S_{ji}^j < 0$: Differentiating the value of firm i with respect to D_i one has

$$V_{D_i} = \sum_{i=1}^n \bar{\mu}^i \left[\mu^i(q_i(D_i; 0); q_j(D_i; 0)) \cdot \frac{dq_i}{dD_i} \right. \\ \left. + \sum_{j=1}^n \bar{\mu}^j \left[\mu^j(q_i(D_i; 0); q_j(D_i; 0)) \cdot \frac{dq_j}{dD_i} \right] \right] \\ = 0$$

Start with a debt level D_i such that firm i 's reaction function goes through $(q^c; q^c)$: Given this reaction function the first term is zero and the second term is positive, since

$$\frac{dq_j}{dD_i} < 0$$

and $\sum_{j=1}^n \bar{\mu}^j \mu^j(q_i; q_j) < 0$: Firm i 's best response to firm j 's reaction curve will therefore involve a larger than the hypothesized debt level. Therefore, starting from the Cournot reaction function firm i will have an incentive to move its reaction function out.

Next we need to check that ...rm

If firm j has chosen its reaction curve to go through (q^c, q^c) , and firm i has chosen any reaction curve, it must be that $q_i(D_i, D_j) q_j(D_i, D_j)$ is a point on firm j 's reaction curve. By definition for any such point the first term in brackets is zero. We therefore have

$$V_{D_j}^j \leq 0$$

since $\Pi_i^j(\theta^j, q_j, q_i) < 0$ and $\frac{dq_i}{dD_j} \geq 0$. Because firm i 's reaction function is upward sloping an increase in the debt level of firm j would decrease rather than increase firm j 's profits. Firm j therefore has no incentive to move its reaction function out. Setting $D_j = 0$ is indeed a best response of firm j to firm i 's reaction curve.

It remains to characterize the resulting equilibrium quantities and values. We need to show that for firm i one finds $q_i > q^c$ and $V^i > V^c$ whereas for firm j one has $q_j < q^c$ and $V^j > V^j$.

Start with firm i . Firm i has a positive level of debt, so that the constraint $D_i \geq 0$ is not binding. Equilibrium quantities can therefore be characterized by

$$\max_{q_i} V^i(q_i, q_j)$$

$$s.t. \quad q_j = q_j^*(q_i, 0)$$

which is the program for a Stackelberg leader. Substituting one has

$$\max_{q_i} V^i(q_i, q_j^*(q_i, 0))$$

This problem has first-order condition

$$\begin{aligned} V_{q_i}(q_i, q_j^*(q_i, 0)) &= \left[\int_{\underline{\theta}}^{\bar{\theta}} \Pi_i^i(\theta^i, q_i, q_j) f(\theta^i) d\theta^i \right] \\ &\quad + \left[\int_{\underline{\theta}}^{\bar{\theta}} \Pi_j^i(\theta^i, q_i, q_j) f(\theta^i) d\theta^i \right] \frac{dq_j^*(q_i, 0)}{dq_i} \\ &= 0 \end{aligned}$$

than increase firm j's profits. Firm j therefore has no incentive to move its reaction function out. Setting $D_j = 0$ is indeed a best response of firm j to firm i's reaction curve.

It remains to characterize the resulting equilibrium quantities and values. We need to show that for firm i one finds $q_i > q^c$ and $V^i > V^c$ whereas for firm j one has $q_j < q^c$ and $V^j > V^c$:

Start with firm i: Firm i has a positive level of debt, so that the constraint $D_i \geq 0$ is not binding. Equilibrium quantities can therefore be characterized by

$$\max_{q_i} V^i(q_i; q_j)$$

$$\text{s.t: } q_j = q_j^s(q_i; 0)$$

which is the program for a Stackelberg leader. Substituting one has

$$\max_{q_i} V^i(q_i; q_j^s(q_i; 0))$$

This problem has first-order condition

$$\begin{aligned} V_{q_i}(q_i; q_j^s(q_i; 0)) &= \sum_{i=1}^n \mu^i(q_i; q_j) f'(\mu^i) d\mu^i \\ &+ \sum_{j=1}^n \mu^j(q_i; q_j) f'(\mu^j) d\mu^j \frac{dq_j^s(q_i; 0)}{dq_i} \\ &= 0 \end{aligned}$$

which implies the well-known tangency condition. Looking at the derivative it is easy to see that when evaluated at $q_i = q^c$ one has $V_{q_i}(q_i; q_j^s(q_i; 0)) > 0$; since then the first term in brackets is zero and the second term is positive since $\sum_{j=1}^n \mu^j(q_i; q_j) < 0$ and $\frac{dq_j^s(q_i; 0)}{dq_i} < 0$: One therefore has $V_{q_i}(q_i; q_j^s(q_i; 0)) > 0$, which implies $q_i > q^c$ and $V^i > V^c$:

Moving on to firm j recall that its quantity q_j is the solution to $q_j = q_j^s(q_i; 0)$; which is a downward sloping function. Taking this together with

$q_j^s(q^c; 0) = q^c$ and $q_i > q^c$ one concludes that $q_j < q^c$: Also, since $q_i > q^c$ one has

$$\max_{q_j} V^j(q_j; q_i) < \max_{q_j} V^j(q_j; q^c)$$

which implies $V^j < V^c$:

We have shown $V^i(m; s) > V^c > V^j(m; s)$: To prove that m is a dominant strategy it remains to invoke symmetry to get $V^j(m; s) = V^i(s; m)$; so that $V^i(m; s) > V^c > V^i(s; m)$: Taking this together with $V^i(m; m) > V^c$ and $V^i(s; s) < V^c$ one arrives at $V^i(m; m) > V^i(s; m)$; and $V^i(m; s) > V^i(s; s)$; q.e.d.

Intuitively, since a shareholder-controlled firm has downward sloping reaction functions, starting from the pair of reaction functions going through $(q^c; q^c)$ it pays the firm who has sent a manager for the quantity choice to increase its debt level, since this will lead the shareholder-controlled firm to decrease its quantity. On the other hand, it does not pay the firm who has sent a shareholder to increase its debt level since this would lead to an increase rather than a decrease in the rival's quantity given that the rival is manager-controlled and has upward sloping reaction functions. Therefore only the manager-controlled firm will move its debt level, and it will move it up to the point where its reaction function cuts the reaction function of the shareholder-controlled rival in the Stackelberg point, which is value-maximizing for the manager-controlled firm. Thus, a deviation to shareholder control does not pay, since it will prompt the rival firm to increase its debt level and its resulting quantity, taking advantage of the fact that the firm who has sent a shareholder will have an incentive to decrease its quantity in response.

To complete the analysis let us also briefly look at the case where A2 does not hold and reaction functions are downward sloping both under manager control and under shareholder control. In this case one may still find that delegation to a manager occurs in a dominant strategy equilibrium. As an intuitive extension to the case where the manager's reaction functions are

upward sloping, when they are downward sloping delegation can be shown to be dominant whenever under manager control reaction functions slope downwards less steeply than under shareholder control. More formally we have

Proposition 6 Under assumption A 1, when

$$i \frac{B_{ij}^i}{B_{ii}^i} < 0$$

m is a dominant strategy and $(m; m)$ is the unique Nash equilibrium of the game, whenever along $(q_i; q_j) = (q; q) \rightarrow (q^c; q^c)$ one has

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

To see the intuition behind this result consider the condition on the relative slopes. Notice that it implies that for any given increase in the rival's quantity, under manager control the firm will reduce its quantity by less than it would under shareholder control. When faced with a manager controlled firm the rival firm will therefore have less of an incentive to compete aggressively than when faced with a shareholder controlled firm. As a consequence the manager-controlled firm will be better off than a shareholder controlled firm. Intuitively, since under manager control the firm's response to a rival's increase in quantity is "less elastic", there is less of strategic substitutability, and it pays the rival firm less to increase its quantity either directly or via an increase in its debt level. Note that in this case the equilibrium is less collusive than Cournot, but more collusive than it would be under shareholder control. For a proof see the appendix.

7 Discussion

7.1 The nature of competition

One of the important underlying results of our analysis is that the quantity variables which are strategic substitutes under shareholder control may under

natural assumptions turn into strategic complements, when viewed from the manager's point of view. Under shareholder control, if the rival firm decreases its quantity this has a positive impact on the firm's marginal profit, so that shareholders will respond by increasing their output. The decision variables are therefore strategic substitutes in the terminology of Bulow, Geanakoplos and Klemperer (1985). Under manager control the effect on marginal profit may be dominated by the effect on total firm profit. If the rival firm decreases its output, this will raise total profit for all realizations of the state of the world. This will lower the probability of bankruptcy and allow the manager to compete less aggressively and to reduce the quantity produced. Thus, quantity variables may become strategic complements. The observation that agency problems can turn decision variables that are strategic substitutes under profit maximization into strategic complements has recently also been made by Aghion, Dewatripont and Rey (1997). In their model of R&D competition, R&D effort decisions of two firms are strategic substitutes under profit maximization. If one firm increases its research effort, this will make it more likely that both firms find the innovation, in which case the gain from the innovation will be competed away. Since this will reduce the marginal payoff to research effort, an increase in research effort of one firm will lead the other firm to respond by reducing effort. If, however, running the firm requires a large initial investment which is financed by an outside investor, the effort response may go the other way. The rival firm's increase in research effort will lower total expected profit. The agent running the firm may then have to commit contractually to a higher effort level, in order to increase total expected profit and to ensure that the outside investor still breaks even. Both here and in our model the reversal in the nature of competition stems from the impact the rival's decision has on total rather than marginal profit. In Aghion, Dewatripont, and Rey (1997) total expected profit matters since the outside investor will want to be paid back his investment in expected terms. In our case total profit matters, due to the threshold in the manager's pref-

erences that is drawn in by the bankruptcy level. In both cases the effect on total profit leads to a reversal of the strategic quality of the decision variables and turns strategic substitutes into strategic complements. Note that these results are possibly more general than they might seem at first glance. All we need for the reversal to occur is that the payoff to a variation in the decision variables varies as in A1 and A2. While quantity competition with linear demand and weakly convex costs is an example which meets these assumptions on the profit function, these assumptions may be taken as a reduced form description for a variety of other underlying games. For example, one could reinterpret the decision variable q as investment into plant and equipment or indeed any other activity that exhibits strategic substitutability and model a subsequent stage of competition in prices or quantity. Whenever the payoff structure of such a game maps into the reduced form assumption made our analysis will apply.

7.2 The value of delegation

Our results also point toward the value of delegation in certain noncooperative environments. Here in equilibrium firm owners delegate strategic decisions to an agent whose objectives differ from their own. This alleviates the prisoners' dilemma quality of quantity competition and helps to sustain a more collusive equilibrium outcome. The idea that employing an agent with preferences different from the principal's can be valuable ex ante has been investigated in other contexts. In Schils (1996) delegated bargaining helps to alleviate a hold-up problem that arises when a firm undertakes a relationship with an outside research unit. When the price for an innovation can not be stipulated ex ante there is an incentive for the firm owner to drive a tough bargain ex post and to extract as much of the surplus from the innovation as possible. Anticipating this, the research unit has less of an incentive to invest in innovation generating activity, so that research effort will be inefficiently low. When the firm owner employs a manager whose preferences

different from his own, this inefficiency is reduced. Similarly, in Dessi (1997) the firm-owner has an incentive to breach implicit (nonenforceable) agreements with the workforce to reward high effort whenever the short term gain of doing so exceeds the long term loss of reputation. Employing a manager who is incentivised by issuing short- and long term debt, this problem is reduced, because the marginal gain to the manager of breaching the implicit contract may be zero in situations in which the manager has enough cash to repay the short term debt. Related ideas can also be found in the literature on macroeconomic policy games, where it is suggested that pareto-superior outcomes can be sustained by delegating monetary policy to a conservative and independent central banker, cf. Rogoff (1985) and Walsh (1995). In all of these models it is valuable ex ante to employ an agent whose objectives will ex post be different from the principal's. The contribution of our results is to extend this idea to a symmetric setting with two competing vertical structures. In all of the cited papers there is a single vertical structure, with sequential moves along the structure. Here there are two rival structures that compete with each other in an output market. Delegation is shown to arise in an equilibrium of a simultaneous move game. Both firms would like to delegate play to a manager, since this is valuable ex ante in ensuring softer competition and a more collusive outcome. This can be sustained in equilibrium here, because in an off-equilibrium situation in which one of the firms did not employ a manager, it is the manager-controlled firm who will be aggressive and the shareholder-controlled firm who will lose out. Since deviations away from delegated play will be punished by more aggressive behavior, delegation becomes sustainable as an equilibrium of a noncooperative simultaneous move game.

7.3 Contractual Commitment and Renegotiation

We have seen that with manager control ex post the principal would choose a different quantity than the agent chooses. This feature is shared with most

of the literature on contractual commitment in oligopoly. For example, in Brander and Lewis (1986) the investor, as a debtholder, would choose a different quantity than the shareholder. Likewise, in Sklivas (1987), ex post the owner would choose a different quantity than manager who was incentivised to focus on sales. In each case contractual commitment prevents the principal from letting his preferences govern the quantity choice. The main difference here is that the ability to commit through contractual arrangements is actually valuable ex ante, in that it permits more collusive equilibrium outcomes rather than less collusive outcomes.

One may still ask whether contracts are a good commitment device in our setting. Clearly, the shareholder would, after the manager is sent and the debt levels are chosen, seem to have an incentive to oust his manager and make the quantity choice himself. It is easy to see, however, that when the manager is ousted a conflict of interest will arise between debtholder and shareholder. The shareholder will want to increase the quantity, making the debt more risky. If before the firm had all the bargaining power vis a vis debtholders, then under manager control the debtholders would have broken even. Once the manager is ousted, debtholders will have a negative expected payoff. Anticipating the possibility that the shareholders will have an incentive to take over control from the manager, it is natural to assume that the original debt contract will have offered protection against this. Thus the debt contract will have contained a covenant that made it a condition that the manager would make the quantity decision. It may, of course be possible to renegotiate this debt contract. In a symmetric situation, however, this possibility should be open to both firms. Let us therefore consider an augmented game in which it is possible for both firms to oust their manager after the debt selection stage and then renegotiate the debt contract by making a take-it-or-leave-it offer to the debtholders. It is clear that in the equilibrium of this augmented game none of the firms would want to oust their manager, since, just as before, this would be dominated, given the later play of the

game. Thus, even though each principal would choose a different quantity than the agent chooses, given the choices of the other firm, the equilibrium obtained above clearly is renegotiation-proof when renegotiation is open to both firms and is modelled as a simultaneous move game.

7.4 Managerial Entrenchment

In this model shareholders use capital structure to incentivise their manager and guide his quantity choice. If we think of the manager as having control over the company after the capital structure has been set one might wonder whether the manager may not be able to change the capital structure and reduce the debt level in order to reduce the probability of bankruptcy. While he obviously has an incentive to reduce the debt level, it is easy to see that unless he uses his own personal wealth he will be unable to do so. This is because the capital structure that is in place is value maximizing, given that a manager has been employed and given the reaction function of the rival firm. If the manager does not have any personal wealth, then in order to buy back debt the manager will have to raise the necessary funds by issuing equity. Since such a restructuring will change the managers subsequent quantity choice this must diminish the value of the firm. It will therefore be impossible for the manager to raise sufficient funds for the purpose of buying back debt.

7.5 Wage Contracts

So far we have thought of the manager as an agent who derives a private benefit from not going bankrupt, and who would not depart from the implied behavior when offered a monetary incentive scheme. In the literature, by contrast, managers are often modelled as risk-neutral and highly susceptible to monetary incentives. One may ask therefore, whether our findings are robust to a switch to such an assumption. To examine this, consider a modified game in which as a first stage a managerial compensation scheme is chosen by the owner of each firm, after which in a second stage managers

choose quantities. Let us restrict attention to contracts that condition on the firm's own profits, that is, let us assume that quantities, as well as rival profits are unobservable to the owner. We also want to restrict wage contracts to be either a profit share, an option contracts with a weakly positive exercise profit or a flat wage contracts that condition on some weakly positive cutoff profit level, i.e. a bonus contract.

$$w = \alpha \pi + \beta \max\{\pi - \bar{\pi}, 0\} + \gamma I_{\{\pi \geq \bar{\pi}\}}$$

where $\alpha \geq 0$, $\beta \geq 0$ and $I_{\{\pi \geq \bar{\pi}\}}$ is an indicator function with $I_{\{\pi \geq \bar{\pi}\}} = 1$ if $\pi \geq \bar{\pi}$ and $I_{\{\pi \geq \bar{\pi}\}} = 0$ otherwise. Note that if the reservation utility of the manager is not zero, one can always amend these schemes by paying the manager some fixed base wage, which can be adjusted to give the expected wage the manager requires. When (A1) and (A2) hold with respect to the earlier game and contracts are chosen simultaneously, it follows directly from our earlier analysis that in equilibrium owners will choose a bonus scheme. To see why, note that a bonus scheme is the only contract that will lead the manager's reaction function to slope upwards. Which cutoff is chosen will again be determined by the condition of tangency of the isoprofit line of the owner and the reaction function chosen by the rival. When this condition is met no one of the owners has an incentive to switch to a different cutoff, or indeed to any other contract in the feasible set. This result suggests that low-powered incentive schemes that are not as sensitive to the principal's payoff as they could be optimal when the manager's task is primarily to make strategic decisions. Note also that a bonus scheme is outside the contract domain considered in Fershtman and Judd (1987) and Sklivas (1987), which casts some doubt on the robustness of their results.

8 Conclusion

This paper has reconsidered the strategic effect of debt under the assumption that quantity choices are made by managers whose objective is to avoid

bankruptcy. The basic result is that quantity choices, which are strategic substitutes under profit maximization, may turn into strategic complements under reasonable assumptions on the profit function. Then, in contrast with the benchmark case of owner control over quantity choices, starting from the Cournot level shareholders will want to shift the manager's reaction function back, rather than out. As a result, equilibrium quantities will be less than the Cournot quantities. The prisoners' dilemma inherent in quantity competition is softened. By employing a manager shareholders not only avoid a limited liability-effect of debt, but are able to achieve a more collusive outcome than in the simple model without a financing stage. We have seen that this result is robust when the decision to delegate is endogenized. The intuition is that when one firm does not delegate its quantity choice, it will lose out against a rival who has delegated the quantity choice, but can credibly threaten to use a very aggressive debt policy when faced with a shareholder-controlled firm. Thus, delegation occurs in equilibrium and is associated with a positive ex ante value both on and off the equilibrium path. In contrast with Brander and Lewis (1986) and in line with the empirical evidence, in the equilibrium of our model positive leverage is associated with softer competition than in the standard oligopoly model without a financing stage. The model also implies that given a contract domain including shares, options and bonus schemes, in equilibrium owners would choose simple bonus schemes for their managers, giving a theoretical justification for the kind of managerial preferences assumed.

Appendix 1

We want to show that under manager control equilibrium quantities are always strictly larger than the joint profit maximizing quantities. Recall that equilibrium quantities are characterized by

$$\max_{q_i} V^i(q_i; q_j)$$

$$\text{s.t: } q_j = q_j^m(q_i; D_j)$$

holding for both firms. Substituting the constraint one has

$$\max_{q_i} V^i(q_i; q_j^m(q_i; D_j))$$

from which one finds the first-order condition

$$\begin{aligned} \frac{dV^i}{dq_i} &= \sum_{\mu^i} \frac{\partial V^i}{\partial \mu^i} \frac{d\mu^i}{dq_i} + \sum_{\mu^j} \frac{\partial V^i}{\partial \mu^j} \frac{d\mu^j}{dq_i} \frac{dq_j^m(q_i; D_j)}{dq_i} \\ &= 0 \end{aligned}$$

which can be rearranged to imply the tangency condition

$$\frac{\sum_{\mu^i} \frac{\partial V^i}{\partial \mu^i} \frac{d\mu^i}{dq_i}}{\sum_{\mu^j} \frac{\partial V^i}{\partial \mu^j} \frac{d\mu^j}{dq_i}} = \frac{dq_j^m(q_i; D_j)}{dq_i}$$

or

$$\frac{V_i^i}{V_j^i} = \frac{dq_j^m(q_i; D_j)}{dq_i}$$

Since along $(q_i; q_j) = (q; q)$

$$\frac{V_i^i}{V_j^i} < 0 \text{ if } (q; q) > (q^c; q^c)$$

$$\frac{V_i^i}{V_j^i} = 0 \text{ if } (q; q) = (q^c; q^c)$$

$$i \frac{V_i^i}{V_j^i} > 0 \text{ if } (q_i; q_j) < (q_i^c; q_j^c)$$

the tangency will occur at some $(q_i; q_j) < (q_i^c; q_j^c)$ only if reaction functions are upward sloping,

$$\frac{dq_j^m(q_i; D_j)}{dq_i} = i \frac{B_{ji}^j}{B_{jj}^j} > 0$$

Recall also that we require the intersection of the reaction functions to be stable, that is

$$B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j > 0$$

This is always satisfied for the case of vertical reaction curves with $B_{ij}^i = B_{ji}^j = 0$: If reaction curves are upward sloping, $B_{ij}^i = B_{ji}^j > 0$; this implies

$$i \frac{B_{ii}^i}{B_{ij}^i} > i \frac{B_{ji}^j}{B_{jj}^j}$$

which says that in $(q_i; q_j)_i$ space at the intersection of the reaction curves the reaction curve of firm i is steeper than the reaction curve of firm j :

Next, note that the joint profit maximizing output $(q^p; q^p)$ is given as the solution to

$$\max_{q_i; q_j} V^i + V^j$$

with first-order conditions

$$V_i^i + V_i^j = 0$$

$$V_j^i + V_j^j = 0$$

These imply the tangency condition

$$i \frac{V_i^i}{V_j^i} = i \frac{V_i^j}{V_j^j}$$

If the intersection of the reaction functions were to occur at this joint profit maximizing output one would have

$$i \frac{V_i^i}{V_j^i} = \frac{dq_j}{dq_i} \Big|_{q_j^m(q_i; D_j)} = i \frac{B_{ji}^j}{B_{jj}^j} = i \frac{B_{ii}^i}{B_{ij}^i} = \frac{dq_j}{dq_i} \Big|_{q_i^m(q_j; D_j)} = i \frac{V_i^j}{V_j^j}$$

The joint profit maximizing point is characterized by the tangency of the two isoprofit functions. For this to be an equilibrium reaction functions must be tangent to each other. However, since we require

$$i \frac{B_{ii}^i}{B_{ij}^i} > i \frac{B_{ji}^j}{B_{jj}^j}$$

this would contradict stability.

Next, consider a point $(q_i; q_j) = (q; q) < (q^p; q^p)$: At such a point one will have

$$\begin{aligned} V_i^i + V_i^j &> 0 \\ V_j^i + V_j^j &> 0 \end{aligned}$$

which implies

$$i \frac{V_i^i}{V_j^i} > i \frac{V_i^j}{V_j^j}$$

which means that the isoprofit curve of firm i is steeper than the isoprofit curve of firm j in $(q_i; q_j)_i$ space. If the intersection of the reaction functions were to occur at such a point one would have

$$i \frac{V_i^i}{V_j^i} = \frac{dq_j}{dq_i} \Big|_{q_i^m(q_i; D_j)} = i \frac{B_{ji}^j}{B_{jj}^j} > i \frac{B_{ii}^i}{B_{ij}^i} = \frac{dq_j}{dq_i} \Big|_{q_i^m(q_j; D_j)} = i \frac{V_i^j}{V_j^j}$$

so that the reaction function of firm j would need to be steeper than the reaction function of firm i: This would again contradict

$$i \frac{B_{ii}^i}{B_{ij}^i} > i \frac{B_{ji}^j}{B_{jj}^j}$$

which is required for reaction function stability, q.e.d.

Appendix 2

In this appendix we want to prove that, as claimed in footnote 4, the equilibrium under manager control is more collusive than the equilibrium

under owner control whenever along $(q_i; q_j) = (q; q) \rightarrow (q^c; q^c)$ one has

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

We make use of the fact that both under manager control and under shareholder control equilibrium quantities are characterized by

$$\max_{q_i} V^i(q_i; q_j)$$

$$s.t: q_j = q_j^a(q_i; D_j)$$

holding for both firms. Here $a = m$ for the case of manager control and $a = s$ for the case of owner-control. The first order condition is

$$V_i^i + V_j^i \frac{dq_j^a(q_i; D_j)}{dq_i} = 0$$

Take the equilibrium quantities resulting from owner control and denote them by $(q^s; q^s)$: They will satisfy

$$V_i^i + V_j^i \frac{dq_j^s(q_i; D_j)}{dq_i} = 0$$

$$V_i^i + V_j^i @_i \frac{S_{ji}^j}{S_{jj}^j} \mathbf{A} = 0$$

Since

$$i \frac{S_{ji}^j}{S_{jj}^j} < 0$$

one has

$$V_i^i(q^s; q^s) < 0$$

which implies that $(q^s; q^s) > (q^c; q^c)$:

If the same point $(q^s; q^s)$ were to result in the equilibrium under manager control, one would need

$$V_i^i + V_j^i \frac{dq_j^m(q_i; D_j)}{dq_i} = 0$$

$$V_i^i + V_j^i @_i \frac{B_{ji}^j}{B_{jj}^j} \mathbf{A} = 0$$

satisfied when evaluated at $(q^s; q^s)$:

If however at any point $(q_i; q_j) = (q; q)$ with $(q; q) \succ (q^c; q^c)$

$$i \frac{B_{ji}^j}{B_{jj}^j} > i \frac{S_{ji}^j}{S_{jj}^j}$$

then this is true at $(q^s; q^s)$: This implies

$$V_i^i + V_j^i @_i \frac{B_{ji}^j}{B_{jj}^j} \mathbf{A} < 0$$

at $(q^s; q^s)$ and we need a reduction in the common quantity to make this hold as an equality, q.e.d.

Appendix 3

Proof of Proposition 6.2

According to Proposition 6.2 when

$$i \frac{B_{ij}^i}{B_{ii}^i} < 0$$

m is a dominant strategy and $(m; m)$ is the unique Nash equilibrium of the game, whenever along $(q_i; q_j) = (q; q) \succ (q^c; q^c)$ one has

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

To prove $V^i(s; m) < V^i(m; m)$ consider the equilibrium under $(m; m)$: This is characterized by

$$V_i^i + V_j^i @_i \frac{B_{ji}^j}{B_{jj}^j} \mathbf{A} = 0$$

and

$$V_j^j + V_i^j @_i \frac{B_{ij}^i}{B_{ii}^i} \mathbf{A} = 0$$

holding at the equilibrium quantities $(q^m; q^m)$: Now consider firm i deviating to shareholder control. Since

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

at the old equilibrium point $(q^m; q^m)$ one will have

$$V_i^i + V_j^i @_i \frac{B_{ji}^j}{B_{jj}^j} \mathbf{A} = 0$$

and

$$V_j^j + V_i^j i \frac{S_{ij}^i}{S_{ii}^i} > 0$$

This implies that firm i has no incentive to move its reaction function, whereas firm j ; which is now facing a shareholder controlled firm has an incentive to move its reaction function out. Firm j can do this by moving its debt level up. It follows that in the equilibrium under $(s; m)$ firm i will have to be optimizing along a reaction function of firm j that specifies a higher output for any quantity firm i chooses. Firm i must be worse off in the new equilibrium. This proves $V^i(s; m) < V^i(m; m)$:

To prove $V^i(m; s) > V^i(s; s)$ start with the equilibrium under $(s; s)$: At the equilibrium quantities $(q^s; q^s)$

$$V_i^i + V_j^i @_i \frac{S_{ji}^j}{S_{jj}^j} \mathbf{A} = 0$$

and

$$V_j^j + V_i^j i \frac{S_{ij}^i}{S_{ii}^i} = 0$$

hold. Consider a deviation of firm i to manager control. Given

$$i \frac{B_{ij}^i}{B_{ii}^i} > i \frac{S_{ij}^i}{S_{ii}^i}$$

at $(q^s; q^s)$ one now has

$$V_i^i + V_j^i @_i \frac{S_{ji}^j}{S_{jj}^j} \mathbf{A} = 0$$

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