Informed Trading, Investment, and Welfare

By

James Dow and
Rohit Rahi

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James Dow

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ABSTRACT

This paper studies the welfare economics of informed trading in a stock market. We provide a model in which all agents are rational and trade either to exploit information or to hedge risk. We analyze the e®ect of more informative prices on investment, given that this dependence will itself be re°ected in equilibrium prices. Agents understand that asset prices may a®ect corporate investment decisions, and condition their trades on prices. We present both a general framework, and a parametric version that allows a closed-form solution. We show that in rational expectations equilibrium with price-taking competitive behaviour, and in the presence of risk-neutral uninformed agents, uninformed traders cannot lose money on average to informed traders. However, some agents with superior information may be willing to lose money on average in order to improve their hedging possibilities. While a higher incidence of informed speculation always increases ¯rm value through a more informative trading process, the e®ect on agents' welfare depends on how revelation of information changes risk-sharing opportunities in the market. Greater revelation of information that agents wish to insure against reduces their hedging opportunities (the Hirshleifer e®ect). On the other hand, early revelation of information that is uncorrelated with hedging needs allows agents to construct better hedges.

Journal of Economic Literature Classi¯cation Numbers: G14, G18, D82, D60

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1. Introduction

This paper is about the analysis of security markets in terms of their e®ect on overall economic welfare, in a setting where agents in the economy have asymmetric information. We address the following important question: is a securities market trader, who trades with the motive of making pro¯ts on the basis of superior private information, socially bene¯cial? Does he confer a bene¯t on the economy, or are his pro¯ts made merely by draining resources from others? We do not attempt an unequivocal answer to this question, since the answer in any given instance must depend on the relative importance of a number of di®erent pros and cons, but we do attempt to provide an improved framework for analyzing the issue. Indeed, our key objective in this paper is to integrate various e®ects that have been noted in the literature within a standard rational expectations model of asset price formation. We do this both in a general setting, without speci¯c functional forms, and in the CARA-normal setting which is the standard parametric framework for analyzing these models.

Financial economists have expressed di®erent views about the costs and bene¯ts of informed trading. We start by discussing these views in general terms, rather than going into the details of actual models. This is because, in many cases, these views were not expressed in the context of precise models of securities markets and, conversely, many of the models in the literature are not well adapted to address questions of welfare economics. Later we shall review these questions in the context of the models in the literature.

The traditional analysis of information in general economic settings assumed that, because it is hard to establish property rights over information, the social bene¯ts of private information should normally exceed the private bene¯ts. This view was challenged by Hirshleifer (1971) who argued that, on the contrary, the private bene¯ts could be large even in situations where the social bene¯ts were minimal. An example of such a situation, which Hirshleifer called \foreknowledge," is when a stock market trader is able to make trading pro¯ts based on his superior forecast of a company's earnings shortly before its public announcement. In a somewhat di®erent vein, Akerlof's (1970) analysis of the used car market introduced the extremely in°uential \lemons" or \adverse selection" paradigm in which private information is viewed as having a negative e®ect on the operation of a market.

In line with these arguments, there is now a quite widely accepted view among ¯nancial economists that the pro¯ts made by informed traders are like a tax on other investors (although they di®er as to the nature of any o®setting bene¯ts). To cite two examples that are explicitly concerned with public policy: Tobin (1978) has been in°uential in promoting the view that speculative pro¯ts in the foreign exchange market do not have any o®setting bene¯t; King and RÄoell (1988) suggest that pro¯ts to better-informed traders cause a corresponding reduction in the returns of other investors and hence depress investment by reducing market participation. Manove (1989) expresses this \adverse selection" view clearly:

> Insider traders and informed speculators appropriate some part of the returns to corporate investments at the expense of other shareholders. This misappropriation : : : tends to discourage corporate investment and reduce the e±ciency of corporate behaviour.

In contrast, Manne (1966, p. 61) explicitly rejects this argument:

> The insider's gain is not made at the expense of anyone. The occasionally-voiced objection to insider trading | that someone must be losing the speci¯c money the insiders make | is not true in any relevant sense.

In this paper, we show that Manne's argument, quixotic as it may seem, can indeed be reconciled with a rational expectations model of the stock market. We shall return to this point below to explain how this is possible.

A second welfare e®ect of informed trading is that more informative security prices may improve productive decisions. The presence of informed traders in a market tends to make the price more e±cient, i.e. to re°ect their information (Roberts (1967)). A very large body of empirical research has investigated the e±ciency of prices, and the view that more informative prices are also economically more e±cient has often been held as axiomatic (Fama (1976, p. 133)):

> An e±cient capital market is an important component of a capitalist system : : : if the capital market is to function smoothly in allocating resources, prices of securities must be good indicators of value.

However, it has proved harder to provide standard models where e±cient prices lead to better investment and allocative decisions. This point is made by HolmstrÄom and Tirole (1993):

> There is a vast literature in ¯nance devoted to the analysis of information °ows in stock markets, including how completely and how fast information is incorporated into prices. But in almost no model is information collection socially useful.

In this paper, we model how stock prices guide investment decisions (in contrast to Holmstrom and Tirole, who model the role of stock prices in improving managerial incentives). The e®ect may initially appear straightforward: if the stock price rises, this signals good information to the ¯rm and investment will rise. However, if traders know that investment will respond in this way to prices, then prices themselves must adjust to re°ect this dependence. This is the \feedback e®ect" described by Bresnahan, Milgrom and Paul (1992, p. 213, fn 16):

> we assume ::: there are no tricky gaming issues between management and the outsider traders. Suppose, for example, that the manager will withdraw the project if the stock market reaction is adequately adverse. Then the value of the security re°ects this prospect :::

This is an example, but the general point is that security values will be endogenous since, if prices do convey valuable information, then security values will themselves depend on their own prices. In this paper, we model the feedback e®ect in equilibrium. Stock prices in°uence investment, and this dependence is incorporated into the equilibrium price formation process.

Another role of the stock market is to allocate risk, and the theory of ¯nancial markets has emphasized this aspect (Arrow (1953), Diamond (1967), Allen and Gale (1994)). However, the e®ect of informed trading on the welfare of uninformed hedgers d077 [36 0

So far we have discussed three aspects of the impact of informed trading: the \adverse selection e®ect," the \feedback e®ect" whereby informative prices in°uence production decisions, and the e®ects on hedging opportunities (the \Hirshleifer e®ect" and the \spanning e®ect"). This discussion was in general terms, rather than in relation to speci¯c models, to which we now turn.

The earlier models of asset price formation with asymmetric information are not well adapted to address these questions. For example, Grossman and Stiglitz (1980), Glosten and Milgrom (1985) and Kyle (1985) introduced an exogenous random component to asset trade, often interpreted as emanating from \liquidity" traders. This random element to trade was introduced to prevent prices from being fully revealing. In these models the uninformed liquidity traders systematically lose money to the informed traders, suggesting an adverse selection or \lemons" view of the consequences of informed trading. However, a complete welfare analysis is not possible without modelling the utilities of all agents.

The exogenous liquidity trades can be endogenized by, instead, introducing risk-averse agents with endowment shocks. This was done by Diamond and Verrecchia (1981) in the framework of the competitive rational expectations equilibrium models of Grossman (1977) and Grossman and Stiglitz (1980). Similarly, Spiegel and Subrahmanyam (1992) and Glosten (1989) introduced endowment shocks to endogenize the noise trade of the market-maker models of Kyle (1985) and Glosten and Milgrom (1995), respectively. In these papers the hedgers face a trade-o®: they lose money on average by participating in the market, but they are able to obtain some insurance against their endowment risk.

In this paper, the role of informational e±ciency in altering risk-sharing opportunities for hedgers is di®erent in two ways: ¯rst, adverse selection, in the sense of uninformed traders systematically losing money to uninformed traders, is absent in our model; and second, we examine instead the trade-o® between the Hirshleifer e®ect and the spanning e®ect. The latter di®erence, i.e. the trade-o® between the two risk-sharing e®ects, has already been described above. However, the absence of adverse selection deserves some comment here since, in the literature on asset price formation, partial revelation of information has become closely associated with adverse selection (see, for example, Spatt's (1991) comments in the introduction to the symposium issue of the Review of Financial Studies: \adverse selection is certainly one of the central issues in market microstructure").

The model we use is a competitive, Walrasian, rational expectations equilibrium model with a mass of risk-neutral uninformed agents. In other words, it di®ers from standard REE models such as Diamond and Verrecchia (1981) merely by the addition of risk-neutral uninformed agents who are analogous to the market-maker in Kyle (1985). Likewise, it di®ers from standard microstructure models such as Kyle (1985) and Spiegel and Subrahmanyam (1992) only insofar as it allows limit orders, as opposed to market orders, and assumes competitive price-taking behaviour. Since, as we explain below, adverse selection is absent even if informed traders are noncompetitive, the key di®erence is the ability to use limit orders to learn from prices.

We use this model of price formation since it seems reasonable to allow agents to use limit orders, and also to challenge the assumption that has developed in the literature that adverse selection is synonymous with partially-revealing price formation. Of course, the extent to which strategies such as limit orders and conditioning on market information can be used to reduce adverse selection is ultimately an empirical question.

None of the above papers model the feedback e®ect. However, several other papers in the literature have been concerned with the e®ect of stock prices on investment. Leland (1992) addresses this issue, but does so in a setting where the ¯rm does not learn anything from the stock price, and where investment is not chosen to maximize shareholder value: hence, there is no feedback e®ect. Henrotte (1992) models the e®ect of security prices on a ¯rm's output decisions, in the spirit of our feedback e®ect. However, his security is a futures contract on the ¯rm's output and hence a change in ¯rm value does not directly a®ect the security value. Fishman and Hagerty (1992) model an industry where potential entrants are guided by incumbents' stock prices, and this may have a feedback e®ect on the incumbents' pro¯tability and hence their share price. Boot and Thakor (1997) and Dow and Gorton (1997) do model the feedback e®ect fully (Dow and Gorton (1997) also incorporate a managerial incentive problem that can be remedied with stock-price based compensation). However, they use exogenous liquidity traders rather than rational hedgers. Habib et al. (1996) model a feedback e®ect without noise traders, and with full revelation with respect to the asset payo®. Their model (an analysis of spin-o®s) is not adapted to address the welfare questions that are the focus of this paper.

Finally, we mention Manove (1989), Dennert (1990), Ausubel (1990) and Bhattacharya and Nicodano (1995), who study the adverse selection e®ect where uninformed traders lose to informed, and this depresses ex ante investment levels that are chosen before trading in shares takes place. In contrast, in our model, apart from the absence of the adverse selection e®ect, investment responds to contemporaneously determined share prices.

To summarize the discussion so far, this paper presents a model where informed traders make superior pro¯ts, but these pro¯ts are not at the expense of the uninformed. The presence of informed traders makes prices more informative and this improves investment e±ciency, while altering risk-sharing opportunities.

To analyze the welfare e®ects of informed trading, we use a model where all agents are rational utility-maximizers. We ¯rst set out the model in a general form: it features a ¯rm owned by risk-neutral shareholders, and traders who may have private information as well as hedging needs. We analyze rational expectations equilibria of the model, in which all agents are competitive price-takers. We show that in our setting, uninformed agents who trade for hedging reasons cannot lose money on average to informed agents. We then consider a parametric speci¯cation of the model with one type of informed agent and two types of hedgers. We compute the equilibrium of the model and the ex ante expected utilities of all agents. This allows us to study the e®ect of increased informed trading on investment policy and ¯rm value, and on risk-sharing opportunities for uninformed traders.

Before proceeding with the analysis, we make a brief comment on the interpretion of \private" or \asymmetric" information. This paper is about traders with superior private information, and not speci¯cally about insider traders. We use the term \private information" to represent any situation where a trader or fund manager has a better insight than the market into general economic conditions, prospects for the interest rate term structure, the ¯nancial situation of an individual company, etc. Often this comes about as a result of superior modelling or analytical skills, as is recognized by practitioners' usage of the term \model risk" for some securities (for example, pre-payment risk and default risk on tranches of asset-backed securities). The term \analyst" also has the same connotation. In other words, \informed" traders are not necessarily in possession of privileged information. Much of the time their informational advantage is something that they themselves have created, in contrast to \insider" traders. Insider trading is a special case of informed trading, which also has the feature that a trader may pro¯t from information even though he does not have property rights over the information that allow him to do so. The problem of insider trading is, in large part, a problem of misappropriating property rights. Unfortunately, much of the academic literature has failed to appreciate the distinction between insider trading and other kinds of informed trading and simply uses \insider trading" as a generic term to describe all informed trading.

We proceed as follows. In Section 2 we set out a general model of a security market with agents who trade for informational and hedging motives, that includes the feedback effect. We prove the "no-loss" result: hedgers who have no information other than publicly available market information cannot lose money to informed traders (Proposition 2.1). Section 3 presents the parametric model with the equilibrium computed in Section 4. The feedback effect is analyzed in Section 5. In Section 6, we compute equilibrium ex ante expected utilities (Proposition 6.1) and present comparative statics results (Proposition 6.2). Section 7 concludes. All proofs, except for that of Proposition 2.1, are in the Appendix.

2. A General Model

We consider a firm, the value of whose productive assets is given by

$$v = f(k; y);$$

where k represents the level of investment, and y is a random variable affecting profitability. We normalize the number of outstanding shares to one. In addition to these shares a riskless bond is available for trade, which we take to be the numeraire, normalizing the interest rate to zero. The original owner of the firm (agent 0) is risk-neutral. There are n other agents who trade to exploit superior information or to hedge their risk exposures. All agents are competitive price-takers (i.e. each should be interpreted as a continuum of infinitesimal traders). Agent i $(i = 1; \ldots; n)$ has a von Neumann-Morgenstern utility function U_i, and a stochastic endowment e_i. He privately observes $(s_i; x_i)$, where the signal s_i is correlated with the firm's profitability parameter y, and x_i parameterizes the agent's risk exposure to a random variable z. Taking an asset position t_i at the market price p leaves him with terminal wealth

$$w_i = e_i(x_i; z) + t_i(v - p): \qquad (1)$$

Definition 1. A rational expectations equilibrium is a price function $p(s_1; \ldots; s_n; x_1; \ldots; x_n)$, order flow $t(s_1; \ldots; s_n; x_1; \ldots; x_n)$, a trade t_i for each agent $i = 1; \ldots; n$, and an investment level k, such that:

(a) $t_i \in \arg\max E[U_i(w_i) | s_i; x_i; p; t]; \quad (i = 1; \ldots; n)$,

(b) $t = \sum_{i=1}^{n} t_i$,

(c) $p = E(v | p; t)$, and

(d) $k \in \arg\max E(v | p; t)$:

Note that we allow uninformed traders to make inferences from the order °ow or volume of trade, in addition to the equilibrium price. As will be seen below when we discuss the parametric model, this assumption is made partly for technical reasons since it is necessary to retain linearity in the CARA/Normal setting in the presence of the feedback e®ect. However, a reasonable notion of equilibrium should permit agents to condition on any publicly available information, and trading volume is an obvious candidate.

Agents know the price and order °ow functions and learn from their observation of prices and order °ows. In particular the ¯rm is guided in its investment decisions by the information aggregated and conveyed by prices and order °ows. Simultaneously the price and order °ow themselves re°ect this dependence. Since agent 0 is risk-neutral and competitive, he determines the price through condition (c), and absorbs the aggregate trade of the other agents. This ensures market-clearing.

We use the terms \order °ow" and \trading volume" interchangeably. This de¯nition of trading volume as the aggregate trade of agents 1 through n excludes agent 0 who can be viewed as a market-maker à la Kyle (1985). We will ¯nd it convenient to adopt this market-maker interpretation in what follows.

Proposition 2.1. Suppose an agent i's private information signals $(s_i; x_i)$ are degenerate random variables. Then in equilibrium his expected trading pro¯ts are zero on each trade. Any agent whose ex ante expected trading pro¯ts are negative must have better information in equilibrium than the market-maker.

Proof. An agent with null private information nevertheless learns from prices and the order °ow. His expected trading pro¯t on a position t_i is $t_i[E(v|p;t) - p]$ which is zero by condition (c) in De¯nition 1. It immediately follows that any agent with nonzero expected trading pro¯ts has more information than is revealed by $(p;t)$, which is the market-maker's information. ∎

An agent with null private information has exactly the same information as the market-maker. Paradoxically, a trader does not lose money unless he has better information than the market-maker. Indeed, traders who lose money do so deliberately because this allows them to construct a better hedging strategy than by simply breaking even on each trade. In contrast traders with no private information cannot make (expected) losses no matter how hard they try. This \no-loss" result clearly holds whether or not informed traders behave strategically. Hence the result, which may initially seem counter-intuitive, holds in a framework that is very similar to the standard Kyle (1985) model. The key difference is that in our model traders are allowed to use limit orders rather than market orders, which seems a reasonable assumption.

In order to develop the intuition of these results in greater detail, and to carry out a complete welfare analysis that includes both the feedback effect of stock prices on investment and the effect of asymmetric information on hedgers' utilities, we now study a parametric version of the model.

3. A Parametric Model

In this section we consider specific forms for the functions and random variables of the model just described. The value of the firm is given by

$$v = ky - \frac{c}{2}k^2; \tag{2}$$

where y denotes profitability per unit of investment, and c is a (positive) investment cost parameter. All traders are infinitesimal price-taking agents. There is a measure $q_S \in (0; \infty)$ of identical privately informed speculators who observe a signal s that is correlated with y. A speculator has no endowment. Taking an asset position t_S at the market price p leaves him with terminal wealth

$$w_S = t_S(v - p):$$

In addition there are two types of hedgers who are exposed to the random variable z. The risk exposure of a hedger of type 1 is itself random: his initial endowment is $e_1 = xz$ (where x is random). After privately observing x, he trades an amount t_1 which results in net wealth

$$w_1 = xz + t_1(v - p):$$

A hedger of type 2 has a constant risk exposure with endowment $e_2 = z$; and trades t_2 to realize terminal wealth

$$w_2 = z + t_2(v - p):$$

There is a measure $q_1 \in (0; \infty)$ of type 1 hedgers and we normalize the mass of type 2 hedgers to be one. For convenience we will henceforth refer to an individual speculator as "the speculator" and likewise to a hedger of type i as "hedger i."

Agent i ($i = S; 1; 2$) has constant absolute risk aversion r_i and has information I_i, i.e. I_S is the partition generated by observing $(s; p; t)$, and similarly I_1 is induced by $(x; p; t)$

and I_2 by $(p; t)$. All random variables are joint normally distributed. Without loss of generality we can take $y = s + \epsilon$ where s is independent of ϵ. We assume that

$$\begin{pmatrix} s \\ \epsilon \\ z \\ x \end{pmatrix} \sim N\left[0; \begin{pmatrix} V_s & 0 & V_{zs} & 0 \\ 0 & V_\epsilon & V_{z\epsilon} & 0 \\ V_{zs} & V_{z\epsilon} & V_z & 0 \\ 0 & 0 & 0 & V_x \end{pmatrix}\right].$$

We use the following notational convention: for random variables g and h, $V_{gh} := \mathrm{cov}(g; h)$. Also ρ_{gh} denotes the correlation coefficient between g and h, and $\beta_{gh} := V_{gh} V_h^{-1}$ is the coefficient from the regression of g on h (the "beta" of g with respect to h).

In general, the risk z may be correlated with both s (the predictable component of y) and ϵ (the residual), and these correlations may be different. The magnitude of hedger 1's risk exposure, x, is independent of all other random variables. We assume that the covariance matrix above is positive definite, a necessary and sufficient condition for which is that all variances be strictly positive and $\rho_{zs}^2 + \rho_{z\epsilon}^2 < 1$. We also take V_{zs} to be nonnegative, which entails no loss of generality. Finally, to ensure that equilibria are not always fully revealing, we assume that $V_{z\epsilon}$ is nonzero.

As we shall see, the "noise" in this model that prevents equilibrium from being fully revealing arises from the trading of hedger 1. This agent trades a random amount which depends on his privately observed endowment shock x. The endowment shock could equally well be interpreted as a liquidity shock suffered by the agent resulting in a need to rebalance his portfolio. Unlike the usual "noise-trader" or "liquidity-trader" model, hedger 1 maximizes utility and makes inferences like any other rational trader.

The specification of hedger 1 and hedger 2 requires some comment. Why do we need both hedgers, and why is their risk exposure not symmetric? If we only had hedger 1, then, as we shall see below, in equilibrium there would be almost full revelation. The market-maker would not be able to distinguish separately the trade of informed and uninformed, but each would be able to subtract his own demand from the total and infer the other's trade (and private information). Hence all traders, apart from the market-maker, would be fully informed and the equilibrium would be rather degenerate.

Given this, it would seem natural and more elegant to have two hedgers both with different endowment shocks. However, the equilibrium for this model cannot be solved in closed form. Hence the formulation we have chosen, which is the simplest one that admits a non-degenerate closed-form solution.

4. Equilibrium

We now proceed to compute the equilibrium. The market-maker sets the price equal to his conditional expectation of the asset payoff given the order flow, i.e. $p = E(v|p, t)$, where

$$t = q_S t_S + q_1 t_1 + t_2. \tag{3}$$

Agents observe the price and order flow. From this observation they can infer the firm's investment level k (k is (p, t)-measurable since the firm's owner, agent 0, has no private information.) We see from (2) that

$$p = kE(s|p, t) - \frac{c}{2}k^2. \tag{4}$$

We look for a linear equilibrium where

$$E(s|p, t) = \lambda s + \mu x \tag{5}$$

for some parameters λ and μ that will be determined below. Note that it is clear from (4) and (5) that (provided λ and μ are both nonzero) the speculator and hedger 1 have the same information in equilibrium: $I_S = I_1$, which is the partition induced by knowing both s and x, while the firm and hedger 2 are unable to isolate s from x.

We can now apply the standard mean-variance certainty-equivalent analysis to the agent's optimization problem, since interim wealth is normal conditional on his information. Agent i's expected utility is

$$\begin{aligned} E[-\exp(-r_i w_i)] &= -E\Big[E[\exp(-r_i w_i)|I_i]\Big] \\ &= -E\left[\exp\left(-r_i\Big[E(w_i|I_i) - \frac{r_i}{2}\mathrm{Var}(w_i|I_i)\Big]\right)\right]. \end{aligned} \tag{6}$$

Let

$$E_i := E(w_i|I_i) - \frac{r_i}{2}\mathrm{Var}(w_i|I_i). \tag{7}$$

The agent's optimization problem reduces to choosing a position t_i to maximize E_i given his information. From the expression (1) for w_i:

$$E_i = E(e_i|I_i) + t_i\Big[E(v|I_i) - p\Big] - \frac{r_i}{2}\Big[\mathrm{Var}(e_i|I_i) + t_i^2\mathrm{Var}(v|I_i) + 2t_i\mathrm{cov}(v, e_i|I_i)\Big]. \tag{8}$$

The optimal portfolio is therefore

$$t_i = \frac{E(v|I_i) - p - r_i\mathrm{cov}(v, e_i|I_i)}{r_i\mathrm{Var}(v|I_i)}. \tag{9}$$

Proposition 4.1. There exists a unique linear equilibrium. The price function is

$$p = \frac{1}{2c}(\lambda s + \mu x)^2;$$

the equilibrium investment is

$$k = \frac{1}{c}(\lambda s + \mu x);$$

the equilibrium holdings of the agents are given by

$$t_S = \frac{(1 - \lambda)s - \mu x}{r_S k V_\epsilon};$$

$$t_1 = \frac{(1 - \lambda)s - (\mu + r_1 V_{z\epsilon})x}{r_1 k V_\epsilon};$$

$$t_2 = -\frac{(1 - \lambda)V_{zs} + V_{z\epsilon}}{k[(1 - \lambda)V_s + V_\epsilon]};$$

and the order flow is

$$t = cq_1 V^2$$

lost. This feature of the model arises from the feedback effect. Alternatively, we could assume that agents can condition only on prices if there were some other way of revealing the sign of $(\lambda s + \mu x)$, for example a futures contract on the firm's output (i.e. an asset with payoff y), whose equilibrium price could also be used to make inferences.

5. The Feedback Effect

From Proposition 4.1 we see that the level of investment is more responsive to the share price the lower is the adjustment cost (measured by the parameter c). This feeds back into the equilibrium share price. The lower is c, the stronger is the feedback effect. We can easily calculate the equilibrium volatility of investment as well as the mean and variance of the share price.

Proposition 5.1. In equilibrium, the variance of the level of investment is

$$\mathrm{Var}(k) = \frac{\lambda V_s}{c};$$

the mean and variance of the share price are, respectively,

$$E(p) = \frac{\lambda V_s}{2c}$$

and

$$\mathrm{Var}(p) = \frac{\lambda^2 V_s^2}{2c^2};$$

and the expected value of the firm is

$$E(v) = \frac{\lambda V_s}{2c}.$$

Note that the expected value of the firm is equal to the expected share price (since $E(v) = E[E(v|p,t)] = E(p)$). With a greater intensity of informed trading and/or a lower cost of investment, both the average share price and the volatility of the share price are higher. Investment also is more volatile. The increased volatility, here, is beneficial from the point of view of the firm's shareholders. It reflects a more efficient price that leads to a better corporate investment policy.

6. Welfare Analysis

We measure agents' welfare in equilibrium in terms of their certainty-equivalent wealth. We denote this by U_i for agent i and for convenience we refer to it as the agent's payoff:

$$
\begin{aligned}
U_i &:= -\frac{1}{r_i}\ln\Big[-EU_i(w_i)\Big] \\
&= -\frac{1}{r_i}\ln\Big[E[\exp(-r_i w_i)]\Big] \qquad (10)
\end{aligned}
$$

where expectations are taken over the ex ante distribution of wealth in equilibrium. Notice that, for agents S and 1, wealth is not normally distributed ex ante, and therefore certainty-equivalent wealth cannot be computed by the usual mean-variance formula (the welfare analysis in Leland (1992) is therefore incorrect). In the expression for agent i's terminal wealth,

$$
w_i = t_i(v - p) + e_i(x_i; z);
$$

t_i is the ratio of two normals (for agents S and 1), v and p are both the product of two normals, while e_i is either zero (in the case of agent S) or the product of two normals (agent 1).

Proposition 6.1. The payoffs of the agents are:

$$
\begin{aligned}
U_S &= \frac{1}{2r_S}\ln\left[1 + (1-\lambda)V_s V_\epsilon^{-1}\right] \\
U_1 &= \frac{1}{2r_1}\ln\left[(1 - r_1^2 V_x V_z)[1 + (1-\lambda)^2 V_s V_\epsilon^{-1}] + (\mu + r_1[(1-\lambda)V_{zs} + V_{z\epsilon}])^2 V_x V_\epsilon^{-1}\right] \\
U_2 &= \frac{r_2}{2}\left(\frac{[(1-\lambda)V_{zs} + V_{z\epsilon}]^2}{(1-\lambda)V_s + V_\epsilon} - V_z\right):
\end{aligned}
$$

We now wish to assess the welfare impact of changing q, the relative intensity of informed trading.

Proposition 6.2. The speculator's payoff U_S is decreasing, with respect to q, while the uninformed hedger's payoff U_2 is

(a) decreasing if and only if $|\beta_{zs} - \beta_{z\epsilon}| \leq \beta_{zs}$;

(b) increasing if and only if $|\beta_{zs} - \beta_{z\epsilon}| \geq V_y V_\epsilon^{-1}\beta_{zs}$; and

(c) strictly convex and attains a minimum if and only if $\beta_{zs} < |\beta_{zs} - \beta_{z\epsilon}| < V_y V_\epsilon^{-1}\beta_{zs}$:

Here we use the terms increasing and decreasing in the strict sense. Since λ is increasing in q, the statement regarding the speculator's payoff is immediate from Proposition 6.1. The interpretation is straightforward: an individual speculator's payoff U_S is decreasing in q since a more revealing trading process means less favourable opportunities for speculative profit.

The comparative statics for the uninformed hedger are more subtle. Recall that β_{gh} is the regression coefficient from the regression of g on h. Whether hedger 2 prefers to be less or more informed in equilibrium depends on the relative size of the two betas, β_{zs} and $\beta_{z\epsilon}$. A bigger β_{zs} means a stronger Hirshleifer effect: observing a signal that is highly informative about endowments reduces risk-sharing opportunities in the market. On the other hand, the bigger is the magnitude of $\beta_{z\epsilon}$, the more desirable it is to obtain a good estimate of s so that the endowment risk associated with ϵ can be hedged more effectively. If $\beta_{z\epsilon}$ is very small relative to β_{zs} (case (a)), the Hirshleifer effect dominates and the hedger is worse off as informed trading increases and more information is revealed by the market. In case (b) the opposite is true: the hedger prefers more revelation to less since the speculator's information resolves a lot of uncertainty regarding the asset payoff and not much regarding the endowment. In the intermediate case (c), the hedger prefers the equilibrium to be either fully revealing or not revealing at all.

It has been observed that the typical daily pattern of trading volume in financial markets is U-shaped, with heavy trading in the morning and late afternoon and relatively little activity in the middle of the day. This is consistent with case (c) above: if prices are more revealing as the trading day progresses, uninformed hedgers would prefer to trade either at the open or the close. A theory of intraday patterns that exploits this idea is presented in Martín and Rahi (1997b).

7. Conclusions

In this paper, we have presented a general model of a security market with agents who trade for informational and hedging motives. The model also incorporates the feedback effect of investment policy (as a function of the price) back onto price formation.

We first prove the "no-loss" result: in a model with limit orders, hedgers who have no information other than publicly observed market signals cannot lose money to informed traders. To analyze the welfare effects of informed trading, we use a parametric model where all agents are rational utility-maximizers and we compute explicit closed-form solutions for their equilibrium utility levels. We obtain a continuous parameterization of equilibrium with respect to the intensity of informed trading. A more informative price is always bene¯cial with regard to real investment decisions, even though this entails higher volatility of the share price and investment, while reducing the returns from informed speculation. For uninformed hedgers, the answer is not unambiguous: it depends on the whether the information being revealed is primarily about the hedger's endowment risk which he wants to insure (the Hirshleifer e®ect), or information that improves hedging e±ciency by resolving asset payo® uncertainty (the spanning e®ect).

APPENDIX

Lemma A.1. Suppose A is a symmetric $m \times m$ matrix, b is an m-vector, d is a scalar, and w is an m-dimensional normal variate: $w \sim N(0;\Sigma)$; Σ positive definite. Then $E[\exp(w^\top Aw + b^\top w + d)$ is well-defined if and only if $(I - 2\Sigma A)$ is positive definite, and

$$E[\exp(w^\top Aw + b^\top w + d) = |I - 2\Sigma A|^{-\frac{1}{2}} \exp[\frac{1}{2} b^\top (I - 2\Sigma A)^{-1}\Sigma b + d]:$$

Proof.

$$\begin{aligned}
&E[\exp(w^\top Aw + b^\top w + d)] \\
&= \int_{\mathbb{R}^m} \exp(w^\top Aw + b^\top w + d)(2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}}\exp(-\frac{1}{2}w^\top\Sigma^{-1}w)\,dw \\
&= \int_{\mathbb{R}^m} (2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}}\exp[-\frac{1}{2}w^\top(\Sigma^{-1} - 2A)w + b^\top w + d]\,dw \\
&= \int_{\mathbb{R}^m} (2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}}\exp[-\frac{1}{2}(w - \overline{w})^\top(\Sigma^{-1} - 2A)(w - \overline{w}) + \frac{1}{2}b^\top(\Sigma^{-1} - 2A)^{-1}b + d]\,dw \\
&= |\Sigma|^{-\frac{1}{2}}|(\Sigma^{-1} - 2A)^{-1}|^{\frac{1}{2}}\exp[\frac{1}{2}b^\top(\Sigma^{-1} - 2A)^{-1}b + d];
\end{aligned}$$

where $\overline{w} = (\Sigma^{-1} - 2A)^{-1}b$: The result follows immediately. ∎

Proof of Proposition 4.1. The firm solves the problem:

$$\max_{k\in\mathbb{R}} \quad kE(s|p;t) - \frac{c}{2}k^2;$$

giving $k = c^{-1}E(s|p;t) = c^{-1}(\lambda s + \mu x)$, using (4). Also, from (4) and (5),

$$p = k(\lambda s + \mu x) - \frac{c}{2}k^2:$$

By substituting in the equilibrium k we obtain the desired expression for the price function.

For the speculator, using (2) and (9), and standard properties of the normal distribution (see, for example, Anderson (1984)), we get

$$\begin{aligned}
t_S &= \frac{E(v|s) - p}{r_S \mathrm{Var}(v|s)} \\
&= \frac{ks - \frac{c}{2}k^2 - [k(\lambda s + \mu x) - \frac{c}{2}k^2]}{r_S k^2 V_\epsilon} \\
&= \frac{(1-\lambda)s - \mu x}{r_S k V_\epsilon}:
\end{aligned}$$

Similarly for the hedgers

$$
\begin{aligned}
t_1 &= \frac{(1-\lambda)s-(\mu+r_1V_{z\epsilon})x}{r_1kV_\epsilon};\\
t_2 &= -\frac{\mathrm{cov}(z;s|p;t)+V_{z\epsilon}}{k(\mathrm{Var}(s|p;t)+V_\epsilon)}.
\end{aligned}
\tag{11}
$$

Substituting into (3) we can write the aggregate order flow as

$$t = t_2 + \frac{q_1}{kV_\epsilon}\cdot\tau;$$

where

$$\tau := q(1-\lambda)s-(q\mu+V_{z\epsilon})x;$$

and q is as defined in the statement of the proposition. We proceed under the assumption that observing prices and the order flow is equivalent to observing $(\lambda s+\mu x)$. As we shall see, this will turn out to be true in equilibrium. Then

$$
\begin{aligned}
E(s|p;t) &= E(s|\lambda s+\mu x)\\
&= \frac{\lambda V_s}{\lambda^2V_s+\mu^2V_x}\cdot(\lambda s+\mu x).
\end{aligned}
$$

It follows from (5) that

$$\lambda^2V_s+\mu^2V_x = \lambda V_s. \tag{12}$$

We conjecture that τ is proportional to $(\lambda s+\mu x)$. Then

$$\frac{\lambda}{\mu} = -\frac{q(1-\lambda)}{q\mu+V_{z\epsilon}}.$$

Crossmultiplying and simplifying, we get

$$\frac{\lambda}{\mu} = -\frac{q}{V_{z\epsilon}}. \tag{13}$$

Equations (12) and (13) can now be solved for λ and μ.

The conditional moments for hedger 2, who observes only the price and order flow, are equivalent to the moments conditional on $(\lambda s+\mu x)$. Using the standard properties of the normal, together with (12), we get:

$$
\begin{aligned}
\mathrm{Var}(s|p;t) &= (1-\lambda)V_s\\
\mathrm{cov}(z;s|p;t) &= (1-\lambda)V_{zs}.
\end{aligned}
\tag{14}
$$

Substituting into (11) we obtain the desired formula for t_2. The equilibrium order flow can now be readily computed. ∎

Proof of Proposition 5.1. The variance of k is immediate from the expression for k in Proposition 4.1 and equation (12). From the moment generating function of the normal distribution, if $X \sim N(0, \sigma^2)$, then $E(X^2) = \sigma^2$ and $\mathrm{Var}(X^2) = 2\sigma^4$. Now we obtain the mean and variance of the share price by using the expression for the price function from Proposition 4.1 and equation (12). Finally, note that $E(v) = E[E(v|p, t)] = E(p)$. ∎

Proof of Proposition 6.1. From (6), (7) and (10),

$$U_i = -\frac{1}{r_i} \ln \Big[E[\exp(-r_i E_i)] \Big].$$

Using (8) and (9), in equilibrium,

$$\begin{aligned} E_i &= E(e_i|I_i) - \frac{r_i}{2}\mathrm{Var}(e_i|I_i) + t_i \Big[E(v|I_i) - p - r_i \mathrm{cov}(v, e_i|I_i) \Big] - \frac{r_i}{2} t_i^2 \mathrm{Var}(v|I_i) \\ &= E(e_i|I_i) - \frac{r_i}{2}\mathrm{Var}(e_i|I_i) + \frac{r_i}{2} t_i^2 \mathrm{Var}(v|I_i). \end{aligned} \tag{15}$$

Setting $e_S = 0$ in (14), substituting for the equilibrium holding of the speculator from Proposition 4.1, and using Lemma A.1, we obtain

$$U_S = \frac{1}{2r_S} \ln \Big[1 + V_z^{-1}[(1-\lambda)^2 V_s + \mu^2 V_x] \Big].$$

The

Proof of Proposition 6.2. Note that λ is increasing in q. The comparative statics for $E(v)$ and U_S are immediate from Proposition 6.1. From the expression for U_2 we see that if $V_{zs} = 0$, U_2 is increasing in q. This case is covered by item (c) in the proposition. Henceforth we restrict V_{zs} to be strictly positive (note our convention that $V_{zs} \geq 0$). Differentiating U_2 with respect to λ, we obtain two critical points:

$$\lambda^* = 1 + \frac{V_2}{V_s} \cdot \frac{\beta_{z^2}}{\beta_{zs}}$$

$$\lambda^{**} = 1 + \frac{V_2}{V_s}\left(2 - \frac{\beta_{z^2}}{\beta_{zs}}\right):$$

Also we see that

$$\operatorname{sgn}\left[\frac{\partial^2 U_2}{(\partial\lambda)^2}\right]_{\lambda=\lambda^*} = -\operatorname{sgn}\left[\frac{\partial^2 U_2}{(\partial\lambda)^2}\right]_{\lambda=\lambda^{**}} = \operatorname{sgn}(\beta_{zs} - \beta_{z^2}):$$

The comparative statics for U_2 can now be verified by considering each case in turn and restricting λ to the unit interval $(0;1)$. ∎

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