A Dilution Cost Approach to Financial Intermediation and Securities Markets

By

Patrick Bolton
and
Xavier Feixas

DISCUSSION PAPER 305

October 1998

FINANCIAL MARKETS GROUP
AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

ISSN 0956-8549-305

Dilution Cost pproach to Financial Intermediation and Securities Markets[*]

Patrick Bolton[†] Xavier Freixas[‡]

December 1997

Abstract

This paper proposes a model of financial markets and corporate finance, with asymmetric information and no taxes, where equity issues, Bank debt and Bond financing may all co-exist in equilibrium. The paper emphasizes the relationship Banking aspect of financial intermediation: firms turn to banks as a source of investment mainly because banks are good at helping them through times of financial distress. The debt restructuring service that banks may offer, however, is costly. Therefore, the firms which do not expect to be financially distressed prefer to obtain a cheaper market source of funding through bond or equity issues. This explains why bank lending and bond financing may co-exist in equilibrium. The reason why firms or banks

[*]We are grateful to Helmut Bester, Richard Brealey, Sudipto Bhattacharya, Mathias Dewatripont, Bernard Dumas, Gerard Genotte, Mark Gertler, Denis Gromb, Martin Hellwig, Bengt Holmström, Jose Marin, William Perraudin, Enrico Perotti, Rafael Repullo, Jean- Charles Rochet, Jean Tirole, Ernst-Ludwig von Thadden and David Webb as well as seminar participants at Studienzentrum Gerzensee,WWZ Basel, Wharton, the CEPR conference on Banking held in Madrid (February 1994), the Tel-Aviv conference on Financial intermediation (January 1994), the IGIER conference on the design of the banking system held in Milan (February 1995), the FMG-LBS conference in honour of Fisher Black held in Alghero (September 1996) and the CEPR conference on Financial intermediation and the structure of capital markets held at INSEAD (April 1997) for helpful comments. This paper originated at the Financial Markets Group of the London School of Economics, when Xavier Freixas was a visiting fellow. Financial support from DGICYT grant no. PB93-0388 is gratefully acknowledged.

[†]ECARE, Université Libre de Brusselles and CentER, Tilburg University

[‡]Universitat Pompeu Fabra

also issue equity in our model is simply to avoid bankruptcy. Banks have the additional motive that they need to satisfy minimum capital adequacy requirements. Several types of equilibria are possible, one of which has all the main characteristics of a "credit crunch". This multiplicity implies that the channels of monetary policy may depend on the type of equilibrium that prevails, leading sometimes to support a "credit view" and other times the classical "money view".

1 Introduction

The main purpose of this paper is to build an equilibrium model of the capital market, comprising a banking sector as well as a primary securities market, which is consistent with the main stylized facts that are known about stock and credit markets.

The observations we are primarily interested in are the following: All developed market economies have a capital market composed of both intermediated finance and direct finance. The relative size of direct and intermediated finance varies considerably across countries as well as over time. The size of the banking sector also seems to vary with the business cycle. In addition, the composition of bank finance and direct finance varies considerably across firms. Outside equity and bond financing is found predominantly in mature and relatively safe firms, while bank finance (or other forms of intermediated finance) is the only source of funding for start-up firms and risky ventures. The model we build is consistent with these stylized facts as well as a number of empirical regularities uncovered or corroborated by recent research. Namely that, young riskier firms rely more on bank loans than on financial markets (see Petersen and Rajan (1994) and (1995)); bank loan renegotiation tends to be easier than bond restructurings (see Lummer and McConnel (1989) and Gilson and Lang (1990)); at the beginning of a downturn firms tend to switch from bank lending to commercial paper (see Kashyap, Stein and Wilcox (1993)).

Besides suggesting a plausible and unified explanation for all these observations, the main motivation for building such a model is to improve our understanding of the effects of financial regulation on the structure of the financial system and the effects of monetary policy on the real sector.

Our paper is by no means the first attempt at building such a framework. Recently, there has been renewed interest in the questions of what determines the structure of the capital market and how the financial and the real sector in the economy interact. Our paper adds to a small recent theoretical literature concerned with the coexistence of bank lending and bond financing (most notably, Besanko and Kanatas (1993), Hoshi, Kashyap and Scharfstein (1993), Holmstrom and Tirole (1994) and Repullo and Suarez (1994)). Our main contribution to this literature is to introduce outside equity financing by both firms and banks alongside bank loans and bonds and thus bring our model closer to reality. Although our model is still too stylized to adequately capture the main interactions between financial and real sectors we believe that the introduction of outside equity financing is a significant step forward.

Admittedly, outside equity financing by non-financial firms is small at least in flow terms. For example, from 1946 to 1987 equity financing accounted for only 7 percent of all external financing by non-financial firms in the US. Building a model which excludes equity financing for these firms could thus be justified as a useful simplifying approximation. However, we believe that excluding outside equity from banks is a much stronger assumption. Indeed, banks rely in an essential way on outside equity financing to meet their capital requirements and to expand their lending activities. In attempting to understand the effects of, say, a reduction of bank capital on lending and aggregate investment it is important to consider the possibility that banks can offset a reduction in their current capital base with new equity issues. Alternatively, since changes in the capital base of banks may have amplifying effects on aggregate investment it is important to understand when and how banks determine an increase (or decrease) of their capital base through new equity issues.

In our model there is asymmetric information between firms and investors, so that firms raising equity bear an informational dilution cost (as in Myers and Majluf (1984)). Bank lending may involve a lower dilution cost for a firm, but because banks themselves must bear a dilution cost when they issue equity there is an intermediation cost to be borne by borrowers.

In equilibrium firm financing is segmented as follows: riskier firms take out bank loans (since they have a greater demand for flexible financing) while the safer ones prefer to tap the securities markets to avoid paying the intermediation cost. Firms resort to issuing equity only when their underlying risk is so high that the expected cost of bankruptcy under bond financing and the cost of bank lending are so high that they outweigh the added dilution cost of equity financing.

In our model banks' equity base (and internally generated funds) is a key variable in determining the total supply of loans. Because of dilution costs in issuing equity the funds banks raise in the financial markets cannot be perfect substitutes for internally generated funds. In this respect our model captures the existence of a *credit channel* of monetary policy (see e.g. Kashyap and Stein (1994)).

The paper is organized as follows: Section 2 outlines the model. Section 3 derives the optimal mode of financing for firms. Section 4 examines the banks' funding strategies. Sections 5 and 6 analyse the credit market equilibrium. Section 7 discusses some implications of our model for monetary policy and financial regulation. Finally, section 8 offers some concluding comments. The proofs of most results are given in an appendix.

2 The model

We consider an economy composed of a continuum of risk neutral agents, firms and banks. Both firms and banks are run by wealth-constrained owner-managers, who need to raise outside funds to cover their investment outlays. Firms' investments can be funded either by issuing securities (bonds or shares) or through a bank loan, while banks can be funded by deposits and equity or bond issues. We begin by describing the characteristics of firms' projects and then we turn to the funding options available to firms and banks.

2.1 Firms' investment projects

Each firm has a project requiring an investment I at date $t = 0$ and yielding returns at $t = 1$ and $t = 2$. For simplicity we assume that profits at dates $t = 1$ and $t = 2$ can only take two values, π_H and π_L, with $\pi_H > \pi_L$, so that there are only four possible states of nature, $\{H, H\}, \{H, L\}, \{L, H\}$ and $\{L, L\}$. The project can be liquidated at $t = 1$ and a resale value $A \in (\pi_L, \pi_H)$ obtained. Of course, in that case the returns of period $t = 2$ are foregone. We assume, again for simplicity, that interest rates are normalized to zero and that the liquidation value of the project at $t = 2$ is zero.

Firms' owner-managers can invest at most $w < I$ in the firm and must raise at least $I - w$ from the financial markets or a financial intermediary. We shall assume, without loss of generality, that owner-managers invest all their wealth in the firm and we normalize all our variables so that $I - w = 1$. We also introduce private benefits of control $B > 0$ which owner-managers obtain at date $t = 2$ if the firm is not liquidated. By definition these benefits are not transferable to outside investors.

Firms differ in the probabilities p_1 and p_2 of obtaining high cash flow realizations in respectively periods 1 and 2. The range of possible values for p_1 is $[\underline{p}_1, 1]$, where $\underline{p}_1 < \frac{1}{2}$, and that for p_2 it is simply $\{0, 1\}$. We shall label firms according to their second period return: L-firms are said to be "bad" firms and have a second period return of π_L ($p_2 = 0$), while H-firms are "good" and obtain π_H during the second period ($p_2 = 1$). We assume that the value of p_2 is drawn independently of the value of p_1[1].

Agents have different information on the value of p_1 and p_2. We assume that p_1 is publicly observable, but that p_2 is private information to the firm at $t = 0$. The probability p_1 can be thought of as a credit rating. The value of p_2 is only revealed at $t = 1$ to a bank who has lent to the firm at $t = 0$, and only at $t = 2$ to other security holders[2]. At $t = 0$,

2. **equity issue**: an equity issue specifies a share $a \in [0,1]$ that outside shareholders are entitled to. It also specifies shareholder control rights, but we shall assume that shareholder dispersion is such that outside shareholders never exert any effective control over the owner-manager. We also assume that the private benefits of control are large enough that the owner-manager always decides to continue at date $t = 1$ if given the choice.

3. **bank debt**: a bank loan specifies a repayment schedule $\{\widehat{R}_1, \widehat{R}_2\}$. If the firm defaults on its first period repayment the bank is able to observe the type of firm (through monitoring) and decides whether to liquidate or let the firm continue. If it lets the firm continue it appropriates all last period returns (through, say, a debt/equity swap). Since the bank observes the firm's type at date $t = 1$ it lets the firm continue if and only if the firm is "good". Thus, the main distinction between bank debt and bonds is that bank debt is more flexible (or easier to restructure).

If firms choose to combine these different instruments we assume that the priority structure in bankruptcy is such that bonds have the highest priority, followed by bank debt and equity.

2.3 Banks' screening technology and objective function

A central assumption of our model is that banks face similar informational problems as firms when they seek to obtain outside financing for their loans. Just as there are "good" and "bad" firms there are also "good" and "bad" banks. While firms differ in the quality of the projects available to them, banks may differ in their ability to screen "good" projects from "bad" ones. To keep things as simple as possible, we assume that high screening ability banks (or H-banks) can perfectly discriminate "good" and "bad" firms, while low ability banks (L-banks) cannot distinguish between "good" and "bad" projects at all. Although L-banks do not know the type of the firm at date $t = 0$ they learn the firm's type at date $t = 1$. Therefore, at that point they can make an efficient liquidation/continuation decision. Banks' outside investors do not know the bank's type; all they know is that there is a mass M of H-banks and m of L-banks. So, their prior belief about a bank's type is that they face an L-bank with probability $m/(M+m)$ and an H-bank with probability $M/(M+m)$.

For most of the remainder of the paper we shall assume that banks are subject to minimum capital requirements and that deposits are insured. Thus, the dilution cost of banks is essentially concentrated in the cost of outside equity necessary to meet minimum capital requirements. This is only a simplifying assumption and it will become clear that our model also applies to other environments with different forms of bank regulation.

Having specified banks' screening technology it remains to determine their objective function. As with firms, we assume that banks are run by risk-neutral owner-managers who have an equity stake w in the bank. We assume, in addition that these owner-managers may want to liquidate their stake in the bank at date $t = 1$ with probability $\lambda \in (0, 1)$. As a result, these owner-managers care about both the bank's accumulated profits in period $t = 2$, and the bank's share price in period $t = 1$. More formally, if we denote by q the share price of the bank and by Π_2 the bank's accumulated profit up to period $t = 2$, the bank manager's objective is to maximize $\lambda q + (1-\lambda)\Pi_2$[3].

In the next section we shall determine firms' choice of financing for an exogenously given intermediation cost, $\rho > 0$. This allows us to derive the aggregate demand for bank credit. We then proceed to derive the aggregate supply of bank credit, and to determine the equilibrium cost of intermediation.

3 Firms' choice of financing: equit , bonds or bank loans

Having defined each instrument in the previous section, we begin our analysis of the choice of capital structure by outlining the main tradeoffs involved in the three modes of funding.

- Under equity financing there are no bankruptcy costs. But there may be higher dilution costs for good firms since the market tends to un-

[3] Note that this objective function is similar to that considered by Myers and Majluf (1984). However, it is not vulnerable to the criticisms voiced against their specification (see e.g. Dybvig and Zender (1989)). Note also that bank managers' private benefits are not explicitly modeled. The reason is that banks never fail in our model so that the issue of bank managers' objectives regarding liquidation or continuation of the bank never arises explicitly.

dervalue their stock.

- Under bond financing dilution costs may be lower. But, when the firm's debt is high it may be forced into bankruptcy and liquidation. It is efficient to liquidate the firm when it is bad ($p_2 = 0$), but not when it is good ($p_2 = 1$) . Under bond financing, however, the firm is always liquidated following default, so that there is a bankruptcy cost for good firms in making large bond issues[4].

- Under a bank loan the firm may also be forced into bankruptcy. But

3.1 The H-optimal contingent contract

We shall consider the optimal contracting problem from the perspective of an H-firm who knows that any contract it offers to financiers will be mimicked by L-firms, so that it is always pooled with L-firms in the same observable risk class[5].

In order to compare these contracts not only to bonds but also to loans, we will consider both non-monitored contingent contracts, related to bond contracts and monitored contingent contracts akin to bank loans.

The optimal non-monitored contracting problem for an H-firm is to offer:

1. a feasible repayment schedule, $\left\{R_1^H, R_1^L, R_2^H, R_2^L\right\}$ with $R_1^H \leq \pi_H$, $R_1^L \leq \pi_L$, $R_2^H \leq \pi_H$, $R_2^L \leq \pi_L$, where, R_t^K is the second period repayment of a firm with a π_K return at time t.

2. a continuation decision at date $t = 1$ which is given by the probability of continuation x_1, to solve:

$$\left\{\begin{array}{l} \max\ p_1(\pi_H - R_1^H) + (1-p_1)(\pi_L - R_1^L) + x_1(\pi_H - R_2^H + B) \\ \text{subject to:} \\ p_1 R_1^H + (1-p_1)R_1^L + \nu x_1 R_2^H + (1-\nu)x_1 R_2^L + (1-x_1)A \geq 1 \end{array}\right\}$$

It is easy to see that the firm's non-monitored optimal choice is $x_1 = 1$ if $\nu\pi_H + (1-\nu)\pi_L > A$.

Determining the optimal monitored contingent contract leads to a similar problem, except for the fact that the continuation decision is taken after observing the firms type. In the optimal contract only L-type firms will be liquidated, and the bank will obtain the liquidation value A ($A > \pi_L$). In addition there is a monitoring cost ρ.

Whether under monitored or non-monitored finance it is obvious (and easy to show) that the optimal contract is such that $R_1^H = \pi_H$, $R_1^L = \pi_L$ and

[5] The reason why L-firms imitate H-firms is that a different strategy would reveal they are L-firms with negative NPV projects. Moreover, we assume that it is not possible to bribe L-firms to reveal themselves ex-ante since any positive bribe would be a 'free lunch' for anyone pretending to be an L-firm.

In principle H-firms could attempt to partially reveal themselves by offering a menu of contracts which would support a semi-separating equilibrium. We shall not consider this possibility since such outcomes can only be supported by ad-hoc beliefs.

$R_2^L = \pi_L$, setting $R_2^H - R_2^L$ equal to the smallest possible value satisfying the individual rationality constraint of the investor. Indeed, this is the contract that minimizes dilution costs. For future reference we highlight the optimal contract under monitored and non-monitored finance in the two propositions below:

Proposition 1 *In the H-Optimal Financial Contract with no monitoring the firm sets $R_1^H = \pi_H; R_1^L = \pi_L, R_2^L = \pi_L$ and:*

1. *If $p_1\pi_H + (1 - p_1)\pi_L + \pi_L < 1$ and $\nu\pi_H + (1 - \nu)\pi_L > A$ then the firm sets $R_2^H = \frac{1 - p_1\pi_H - (1-p_1)\pi_L - (1-\nu)\pi_L}{\nu}$ and $x_1 = 1$. In this case the firm incurs a positive dilution cost of $(R_2^H - \pi_L)(1 - \nu)$.*

2. *If $p_1\pi_H + (1 - p_1)\pi_L + \pi_L < 1$ and $\nu\pi_H + (1 - \nu)\pi_L \leq A$ then the firm must set $x_1 = 0$ and incurs no dilution cost. However, in this case the H-type firm pays a positive bankruptcy cost.*

Proposition 2 *Optimal Financial Contract with monitoring: The firm sets $R_1^H = \pi_H$, $R_1^L = \pi_L, R_2^L = \pi_L$ and*
$R_2^H = \frac{1+\rho - p_1\pi_H - (1-p_1)\pi_L - (1-\nu)A}{\nu}$. It incurs a positive dilution cost of $\left(R_2^H - A\right)(1 - \nu)$ in addition to the intermediation cost ρ.

The comparison of the optimal contracts under respectively monitored and non-monitored finance immedately reveals that monitored finance reduces dilution costs, but implies paying the endogenous cost ρ. Depending on the relative importance these costs a firm may favour monitored or non-monitored finance.

It is also clear from the description of the optimal contract under non-monitored finance that it cannot be replicated by any combination of equity, bank debt or bonds. Indeed, to replicate the contract the firm must: i) issue safe debt worth $2\pi_L$, ii) issue 100% outside equity, and iii) give the manager a call option on all the outside equity to be exercised at date $t = 2$ at the exercise price $R_2^H - \pi_L$. Only managers of "good" firms will then exercise this option, and get a payoff $\pi_H - R_2^H$ as under the optimal contract. In the same way it is impossibe to replicate the optimal monitored finance contract with a bank loan.

In the main body of the paper we shall only allow firms to choose between equity, bonds and bank debt. Thus, we do not allow firms to exploit the best available financial options. However, it will become clear from the analysis below that the loss in efficiency from ruling out exotic financial instruments is small in our model, so that our restriction to standard financial instruments is not very strong. Moreover, the results we obtain under this restriction are easier to relate to empirical evidence[6].

3.2 The mix between equity, bonds and bank loans

As above we first characterize the optimal capital structure without monitoring and then ask which firms would prefer monitored (bank) finance.

3.2.1 The optimal Bond-Equity ratio

It is clear from the above analysis that firms should issue no less than $2\pi_L$ in riskless debt no matter what form of additional financing they choose to obtain. If the firm's primary consideration is to avoid bankruptcy at date $t = 1$, it has two financial alternatives: either issue safe bonds worth $2\pi_L$ and raise the remaining funds with equity, or issue the maximum amount of debt subject to no default at date $t = 1$, $2\pi_L + \nu(\pi_H - \pi_L)$, and pay a dilution cost both on equity and on the $\nu(\pi_H - \pi_L)$ component of the debt. The next lemma determines under what conditions the first mode is preferable to the second.

Lemma 3 *If equity is issued then it is optimal for an H-firm to issue the maximum amount of first period default free bonds, $2\pi_L + \nu(\pi_H - \pi_L)$ if $\nu \geq 1 - p_1$. If $\nu < 1 - p_1$, then it is optimal for the firm to only issue an amount of debt $2\pi_L$.*

Proof. : See the appendix. ■

The intuition behind this lemma is as follows. A change in ν induces both a change in expected return and risk. Now, debt tends to misprice risk more than equity, while equity misprices expected returns more. Depending on which factor is more important, risk or return, the market price of bonds is a better or worse reflection of the underlying value of the firm than equity. The above lemma gives a necessary and sufficient condition for risky bond financing to have higher dilution cost than equity.

[6] In practice there may be many reasons why firms do not fully optimize their choice of financing mix. It is beyond the scope of this paper to address the question why standard financial instruments such as equity, bonds and bank loans are so widely used.

For the remainder of the paper we shall restrict attention to the case where dilution costs are higher under equity financing by assuming:

$$\text{A2: } \nu > 1 - \underline{p}_1$$

We will also focus on the case of inefficient liquidation, by assuming

$$\text{A3: } \nu\pi_H + (1-\nu)\,\pi_L > A$$

All H-firms issuing equity then also issue $2\pi_L + \nu(\pi_H - \pi_L)$ worth of bonds. Under this asssumption we can reduce the choice of an H-firm's non-monitored financial structure to two options: either issue only risky bonds (B) which the firm may default on in period $t = 1$, or issue equity with maximum first period default free debt, $2\pi_L + \nu(\pi_H - \pi_L)$ (E).

The choice between B and E generally depends on the first period probability of success, p_1. Issuing risky bonds implies that in the event of bankruptcy the firm is liquidated, and incurs a deadweight loss of $\nu\,(\pi_H - \pi_L) - (A - \pi_L)$. Alternatively, issuing equity with safe debt involves an additional dilution cost for the funds raised above $\hat{I} = 2\pi_L + \nu\,(\pi_H - \pi_L)$. The choice

3.2.2 Direct vs Intermediated Finance

When we introduce the additional option of bank financing, then firms' choice of capital structure is as follows: The choice between bond financing and bank loans for firms with p_1 close to 1 is simple: since the expected bankruptcy cost is negligible and since the cost of intermediation is positive these firms prefer bond financing over bank lending. For all other firms the bank loan option may be attractive provided that intermediation costs are not too high. As in our comparison between bonds and equity, one potential difficulty that we face is determining which mode of financing involves higher dilution costs. Bank loans, just as bond financing, may actually involve higher dilution costs when ν is low. We shall again restrict attention to the more plausible parameter values where dilution costs are higher under equity financing than under debt financing and assume that the following assumption holds:

$$\text{A4: } \nu \geq (\pi_H - A) / (\pi_H - \pi_L)$$

Under assumptions A1, A2, A3 and A4, we can show that only the firms with low p_1 choose bank loans over direct financing.

Proposition 5 *Under assumptions A1, A2, A3 and A4 the demand for bank loans is the measure of* p_1-*firms in the interval* $[1-\nu, p_1^*(\rho)]$ *which we denote by* $\mathcal{M}([1-\nu, p_1^*(\rho)])$*;* $p_1^*(\rho)$ *is decreasing in* ρ*, with* $p_1^*(0) = 1$ *and* $p_1^*(\rho) = 1-\nu$ *for any* $\rho \geq \rho_c$*.*

Proof. See appendix. ■

In other words, the demand for bank loans comes from the firms with the greatest underlying cash-flow risk. If the intermediation cost is zero ($\rho = 0$) all firms in the economy seek bank financing and $\mathcal{M}([1-\nu, p_1^*(0)]) = \mathcal{M}([1-\nu, 1])$. As intermediation costs rise the demand for bank lending goes down: $\frac{d\mathcal{M}([1-\nu, p_1^*(\rho)])}{d\rho} < 0$. Furthermore, for a positive intermediation cost the safest firms prefer to issue bonds. Equity may or may not be issued by some firms depending on whether $p_1^*(\rho) \leq \hat{p}_1$ or $p_1^*(\rho) > \hat{p}_1$ where $\hat{p}_1$ is the threshold level defined in proposition 4 . Finally, the demand for bank loans may not be entirely met by the banking sector. There may be a positive interval of firms $\left[1-\nu, \underline{p}_1(\rho)\right]$ which are not sufficiently profitable for banks. These firms either get no financing at all or are financed by equity (and bonds). These firms can be thought of as receiving venture capital financing.

In sum, for any given intermediation cost $\rho > 0$, firms can be partitioned into the following four financial classes (as illustrated in figure 1):

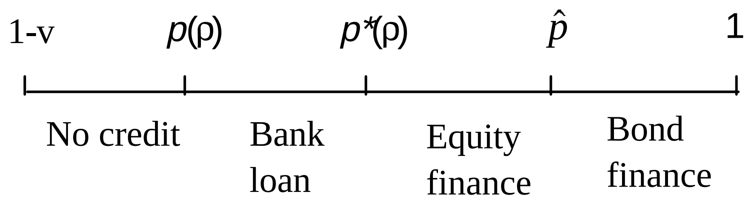

Figure 1:

1. firms with $p_1 \in \left[1-\nu, \underline{p}_1(\rho)\right]$ get no funding or are financed by equity and default free bonds,
2. firms with $p_1 \in \left[\underline{p}_1(\rho), p_1^*(\rho)\right]$ are bank financed,
3. firms with $p_1 \in [p_1^*(\rho), \widehat{p}_1]$ are financed by equity and default free bonds,
4. firms with $p_1 \in [\widehat{p}_1, 1]$ are bond financed.

Note that these predictions of our model are roughly in line with observed stylized facts[7].

4 Banks' Liabilit Structure

Having determined the firms' choice of funding we now turn to the banks' source of funding. Recall that banks are run by managers who invest an amount w of their personal wealth in the bank. They can use their own invested funds w together with (insured) deposits D to fund a total amount of loans $w+D$, subject to meeting the capital adequacy requirements $\frac{w}{w+D} \geq \kappa$[8]. Even if the supply of deposits is infinitely elastic, a bank's supply of credit is limited to $\frac{w}{\kappa}$ unless it gets further outside equity. We shall assume, for simplicity, that deposits are infinitely elastic so that the banks' financing

[7] One financial class we have excluded is risky (low p_1-) firms financed with junk bonds. This financial class could be obtained in our model if we do not make assumptions A2 and A3. Still, within the framework of our model, if junk bonds could be renegotiated they will be closer to bank loans than to publicly issued bonds.

[8] The BIS capital adequacy rules in our highly simplified model are that $\kappa = 0.08$ for standard unsecured loans.

problem boils down to the question whether to issue more equity and if so by how much[9]. As with firms we shall consider this problem in an adverse selection setting, where investors cannot distinguish banks with a high screening ability (H-banks) from those with a low screening ability (L-banks).

Once again, we consider this problem from the perspective of H-banks, who know that their actions are mimicked by L-banks. The reason why L-banks always mimick H-banks is the same as with firms: L-banks are negative NPV institutions, which do not get any outside equity funding once they are identified. Below, we give sufficient conditions guaranteeing that L-banks are not profitable[10].

An H-bank contemplating an equity issue faces the following tradeoff. If it issues equity it can increase lending and thus raise profits, but since it's equity is undervalued in the financial market the bank's owner-manager does not appropriate the entire increase in profits. Depending on the profitability of loans and the extent of the undervaluation of equity the H-bank may or may not decide to relax it's lending constraint by issuing more equity. Thus, to determine an H-bank's choice we need to specify the profitability of loans and the extent of dilution.

In equilibrium the lending terms that H or L-banks offer are the same. Otherwise any deviation by an L-bank would reveal it's type and hit it's share price. We give sufficient conditions guaranteeing that the cost of a fall in share price outweighs any benefit to L-banks from undercutting H-banks' terms in the loan market.

Now, recall that the optimal lending terms are such that banks obtain all the firm's (expected) first period revenues, $p_1\pi_H + (1-p_1)\pi_L \equiv \overline{\pi}$, plus a

[9]The more realistic assumption that deposits are inelastic would complicate the analysis without producing qualitatively different results. With inelastic deposits banks' lending capacity is not only constrained by its' equity base but also by its' "financial slack", which includes deposits, repayments on previous loans, and more generally all liquid assets in its' portfolio. It is then possible for banks to fund their investments by themselves issuing bonds, so that a more realistic liability structure for banks could be obtained. Also, when deposits are assumed to be interest-inelastic a new set of issues arises concerning the banks' liability management policy which is beyond the scope of this paper.

[10]The reason why we restrict attention to situations where L-banks always mimick H-banks is to keep the analysis as tractable as possible. It should be clear that even if equilibria where H-banks can separate themselves from L-banks may exist for other parameter values these equilibria would also have positive intermediation costs and therefore would be qualitatively similar to the pooling equilibria characterized in the body of the paper.

second period repayment, $\max\{R_2^H(p_1), \pi_L\}$. Given these (identical) lending terms, H-banks get a higher return per loan than L-banks since H-banks perfectly screen H-firms from L-firms, while L-banks cannot distinguish between the two types of firms.

If we denote by $\widehat{\nu}$ the probability that an L-bank lends to an H-firm[11] and by (ρ_H, ρ_L) the return per loan of respectively H and L-banks, then given equal lending terms we have:

$$\rho_H = -1 + \overline{\pi}(p_1) + R_2^H(p_1) \text{ and} \tag{2}$$

$$\rho_L = -1 + \overline{\pi}(p_1) + \widehat{\nu} R_2^H(p_1) + (1 - \widehat{\nu})A \tag{3}$$

The returns of L-banks can, thus, be written as a function of H-banks returns:

$$\rho_L = (1 - \widehat{\nu})(\overline{\pi}(p_1) - 1 + A) + \widehat{\nu}\rho_H \tag{4}$$

Denoting by ρ_F the return per loan that the market expects a generic bank to obtain following the bank's decision to issue new equity of F, and by ρ_0 the return the market expects from a bank issuing no equity, we can write the market value of a bank's equity as follows:

$$(1 + \rho_F)(\frac{w + F}{\kappa}) - (w + F)(\frac{1}{\kappa} - 1) \ = (\frac{\rho_F}{\kappa} + 1)(w + F)$$

where, $(w + F)(\frac{1}{\kappa} - 1)$ is the maximum amount of deposits the bank can use given it's equity base $w + F$.

The new shareholders then get a fraction of shares in the bank, α, equal to:

$$\alpha[(\frac{\rho_F}{\kappa} + 1)(w + F)] = F \tag{5}$$

Therefore, an H-bank manager's expected payoff from issuing F is:

$$(1 - \alpha)\{\lambda[(\frac{\rho_F}{\kappa} + 1)(w + F)] + (1 - \lambda)[(\frac{\rho_H}{\kappa} + 1)(w + F)]\}$$

or, substituting for the value of α in (5) and rearranging, the manager's payoff is:

[11] In equilibrium, we have $\widehat{\nu} \leq \nu$ since some H-firms obtain funding from H-banks, while any L-firm can only obtain funding from an L-bank.

$$\frac{(\frac{\rho_F}{\kappa}+1)w+\frac{\rho_F}{\kappa}F}{(\frac{\rho_F}{\kappa}+1)}(\frac{\lambda\rho_F+(1-\lambda)\rho_H}{\kappa}+1)$$

Thus, given market expectations ρ_F, an H-bank manager is better off issuing equity $F>0$ than issuing no additional equity if and only if,

$$\frac{(\frac{\rho_F}{\kappa}+1)w+\frac{\rho_F}{\kappa}F}{(\frac{\rho_F}{\kappa}+1)}(\frac{\lambda\rho_F+(1-\lambda)\rho_H}{\kappa}+1)\ \geq \frac{[\lambda\rho_0+(1-\lambda)\rho_H}{\kappa}+1]w \quad (6)$$

Under the same market expectations, an L-bank manager decides to issue equity $F>0$ if and only if,

$$\frac{(\frac{\rho_F}{\kappa}+1)w+\frac{\rho_F}{\kappa}F}{(\frac{\rho_F}{\kappa}+1)}(\frac{\rho_F}{\kappa}+1)\ \geq [\frac{\rho_0}{\kappa}+1]w \quad (7)$$

Conditions (6) and (7) differ because a manager of an L-bank is better off selling his equity stake at date $t=1$ than holding on to it until date $t=2$, irrespectively of whether he has a liquidity need at date $t=1$ or not. The point is that the market always (weakly) overvalues the shares of an L-bank and therefore the manager would make a capital loss by holding on to his shares. This is why the relevant return on loans for an L-bank manager is always the return expected by the market ρ_F.

We can simplify condition (7) and obtain:

$$\rho_F(w+F)\geq\rho_0 w$$

To summarize, the banks' source of funding problem reduces to the question of whether and by how much to expand lending capacity by raising more capital. For an H-bank manager new equity issues are costly since the market tends to undervalue H-bank stocks. Thus, an H-bank manager will issue new equity only if the expected return on new loans outweighs the dilution cost of equity.

5 Partial equilibrium in the banking sector

In this section we establish existence of a Bayesian-Nash Equilibrium in the banking sector, where banks are playing a sequential game with the following timing:

1. In a first stage, banks of each type set the lending terms they are willing to offer to firms, $\mathbf{R}^H = \{R_t^j(H)\}$, $\mathbf{R}^L = \{R_t^j(L)\}$, where $t = 1, 2$ and $j = \pi_H, \pi_L$. Then those firms who prefer bank lending apply for loans. If an L-Firm applies to an H-bank it's application is turned down. An L-firm whose application has been denied can apply to another bank until it finds a bank that is willing to lend[12].

2. In a second stage, banks decide how to structure their asset portfolio. In particular, they must decide what proportion of available funds to invest in new loans and what proportion in treasury bills or bonds[13].

3. Finally, banks choose the amount of new equity they want to issue. The amount of total capital they end up with, $w + F$ determines their total lending capacity. We assume that banks choose an amount of equity to issue within the interval $\left[0, \bar{F}\right]$, where $\overline{F} < \infty$[14].

(See Figure 2 for an illustration of the time line).

Several remarks are in order about this game.

First, this is a price setting game among (potentially) capacity constrained financial intermediaries. There are a number of potential difficulties with analysing such games. For example, firms applying for a loan at a bank offering better terms than others are not sure to get a loan since the bank may have a limited lending capacity and there are too many applicants. So, one difficult question in this set-up is to determine how firms should respond to a more attractive offer. Another difficult problem is the characterization of a bank's best response function in stage 1.

[12] We assume that a firm that has been denied credit is indistinguishable from a first applicant. An applicant that has been denied credit cannot communicate that information to others.

[13] In the absence of any regulation banning equity investments by banks, it is conceivable that banks may want to invest in firms by taking an equity stake rather than by writing a debt contract. However, in our model this is not the case. First, a bank can always replicate an equity stake by writing a debt contract with the same repayment stream as dividends. Second, a bank's informational advantage over other investors in pricing equity is the same as its' informational advantage in lending to firms.

[14] We justify the existence of an upper bound on F because of the following potential incentive problem between bank mangers and bank shareholders: if the bank raises an amount superior to $\bar{F}$ then bank managers would have an incentive to use the money they collect in their own interest, because the expected private benefits for the managers outweights the opportunity cost of their loss in reputation, while shave-holders monitoring is insufficient to identify whether the funds collected are properly invested.

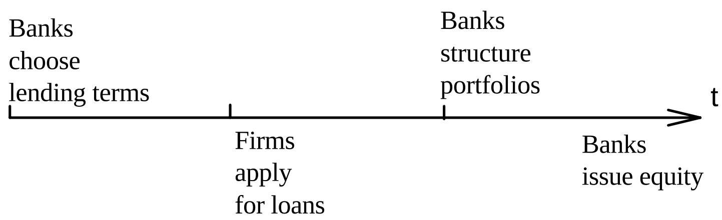

Figure 2:

Second, the game banks play here also has elements of a multidimensional signalling game, since banks may signal their type either through their terms of lending $\mathbf{R}$ or through the amount of equity they issue F. Again, there are potential difficulties associated with the analysis of such multidimensional signalling games.

Before describing this equilibrium in greater detail we need to give a more precise definition.

Definition 6 *A Bayesian-Nash Equilibrium in the banking sector is characterized by:*

- *banks' best pricing strategies at stage* 1, $\mathbf{R}^H, \mathbf{R}^L$.
- *banks' best investment strategies at stage* 2
- *banks' best equity issue strategies at stage* 3, $F^i \in [0, \overline{F}]$, $i = H, L$.
- *The conditional beliefs of the capital market about the bank's type,* $(\delta_H(\mathbf{R}, F)$ *and* $\delta_L(\mathbf{R}, F))$.

Each bank type's equilibrium strategy must be a best response to the other banks' equilibrium strategies, given the market's conditional beliefs. Moreover, the market's conditional beliefs must be consistent with Bayesian updating.

The only pure strategy Bayesian-Nash equilibria we obtain are pooling equilibria, where both types of banks offer the same lending terms, $\mathbf{R}^H = \mathbf{R}^L = \mathbf{R}$, and both types of banks make the same equity issue decisions, $F^H = F^L = F$. In any of these equilibria, neither firms nor equity investors are able to identify a bank's type from it's lending policy. As usual, such a pooling equilibrium can be supported by out-of-equilibrium beliefs such that $\delta_H(\widehat{\mathbf{R}}, \widehat{F}) = 0$ and $\delta_L(\widehat{\mathbf{R}}, \widehat{F}) = 1$ for all $\widehat{\mathbf{R}} \neq \mathbf{R}$ and $\widehat{F} \neq F$. In addition, a mixed strategy semi-separating equilibrium exists if the return of an L-bank is allowed to be negative, which is the case under our assumptions[15].

In order to determine the equilibria of this game, we proceed backwards in two steps. First we fix banks' lending terms $\mathbf{R}$ and determine a bank's equilibrium equity issue given $\mathbf{R}$. Note that once the lending terms $\mathbf{R}$ have been fixed a bank's expected return per loan is given by, equations 2 and 3.

In deriving the bank's equilibrium equity issue we assume that the bank is able to lend all its' available funds at an expected return per loan of ρ_H for an H-bank and ρ_L for an L-bank.

In a second step we determine the terms $\mathbf{R}^*$ where the aggregate supply of bank credit is equal to aggregate demand for bank loans. At these terms each bank is justified in assuming that it is able to lend all its' available funds. This $\mathbf{R}^*$ is then a full general equilibrium if neither an H-bank nor an L-bank have an incentive to deviate by offering different terms[16].

In this section we focus on the equity issue decision and consider only the partial equilibrium in the banking sector for a fixed $\mathbf{R}$. We deal with the choice of $\mathbf{R}^*$ in the next section, where we consider the general equilibrium in the capital market. So, for now we take an arbitrary $\mathbf{R}$ and determine the (pooling) best-response at stage 1 given $\mathbf{R}$. As explained above, we shall derive this best response $F(\mathbf{R})$ by fixing out-of equilibrium beliefs such that $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for all $\widehat{F} \neq F(\mathbf{R})$.

As one might expect, because out of equilibrium beliefs can be chosen arbitrarily, we may obtain infinitely many pooling equilibria. The one we single out is the best pooling equilibrium for an H-Bank. That is, we focus on the equilibrium where, for a given $\mathbf{R}$, the amount of equity issued, F, is optimal for H-banks (given that L-banks mimick this choice and, thus, dilute the value of H-banks' equity).[17] It turns out that the optimal equity issue

[15] Note that the timing of moves specified here is crucial to obtain semi-separating equilibria. If the bank's investment choice was made at a later stage or was not observable, L-banks would invest in bonds and obtain at least a zero return. This would then upset the semiseparating equilibrium.

[16] Another necessary condition to obtain an equilibrium is that the demand for bank loans by H-firms is greater than or equal to the total supply of bank credit by H-banks. As long as the proportion of H-banks is small enough and the proportion of H-firms high enough this condition will always be satisfied.

[17] Note that a common refinement criterion, such as Cho and Kreps' intuitive criterion would select this equilibrium over all other pooling equilibria in our game (see Cho and

for H-banks in a pooling equilibrium is either 0 or $\overline{F}$. This is established in the following lemma:

Lemma 7 *For a given perceived return ρ_F, the optimal equity issue for an H-bank given lending terms $\mathbf{R}$ is either $\overline{F}$ if ρ_F is positive, 0 if ρ_F is negative, or undetermined if $\rho_F = 0$.*

Proof. See appendix. ■

In light of lemma 7, we are able to derive a particularly simple aggregate bank-credit supply schedule. Below a given cut-off ρ_H^1 choosing $F = 0$ is the best response for H-banks. If we denote by M the total number of H-banks and m the total number of L-banks then the total supply of funds from the banking sector for $\rho_H \leq \rho_H^1$ is $(M+m)\frac{w}{\kappa}$. Similarly, above a cut-off ρ_H^2 an equity issue $F = \overline{F}$ is the best response for H-banks. Then the total supply of bank credit may be $(M+m)\frac{w+\overline{F}}{\kappa}$. The next proposition establishes that $\rho_H^2 < \rho_H^1$, so that multiple equilibria may exist on the interval $[\rho_H^2, \rho_H^1]$. There may then be multiple pooling equilibria as well as a semiseparating equilibrium in which H-banks are indifferent between setting $F = \overline{F}$ or $F = 0$; only a fraction of H-banks choose $F = \overline{F}$ while all L-banks choose $F = \overline{F}$. In this semiseparating equilibrium total bank credit supply lies between $(M+m)\frac{w}{\kappa}$ and $(M+m)\frac{w+\overline{F}}{\kappa}$, with the fraction of H-banks choosing $F = \overline{F}$ being chosen so that aggregate bank credit supply equals aggregate demand.

Proposition 8 *Let $\rho_F = \delta_H(\mathbf{R}, \overline{F})\rho_H + \delta_L(\mathbf{R}, \overline{F})\rho_L$ denote the equity market's equilibrium expected return per loan of a bank issuing new equity worth $\overline{F}$. And let out-of-equilibrium beliefs be such that $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for all $\widehat{F} \neq F(\mathbf{R})$.*

Then the aggregate pooling bank-credit supply function $\Psi()$ is:

$$\Psi(\rho_H) = (M+m)\frac{w}{\kappa} \quad \text{for} \quad \rho_H \leq \rho_H^2$$
$$\Psi(\rho_H) = (M+m)\frac{w+\bar{F}}{\kappa} \quad \text{for} \quad \rho_H^1 \leq \rho_H$$

and aggregate supply in a semiseparating equilibrium is given by:

$$\Psi(\rho_H) \in \left[(M+m)\frac{w}{\kappa}, (M+m)\frac{w+\bar{F}}{\kappa}\right] \text{ for } (\rho_H, \rho_L) \text{ such that } \frac{M}{M+m}\,\rho_H + \frac{m}{M+m}\,\rho_L = 0$$

Proof. See appendix ■

Kreps (1987)).

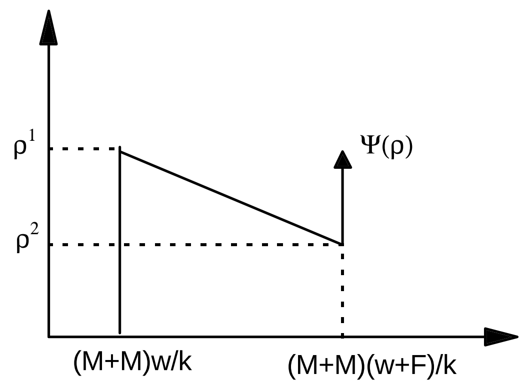

Figure 3:

The source of multiplicity of equilibria in the interval $[\rho_H^2, \rho_H^1]$ is, as in all signalling games, due to the degree of freedom in specifying of out-of-equilibrium beliefs[18]. An equilibrium best-response of $\overline{F}$ is supported by out-of-equilibrium beliefs $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for all $\widehat{F} \neq \overline{F}$; in particular, $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for $F = 0$. On the other hand, an equilibrium best-response of 0 is supported by out-of-equilibrium beliefs $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for all $\widehat{F} \neq 0$; in particular, $\delta_H(\mathbf{R}, \widehat{F}) = 0$ and $\delta_L(\mathbf{R}, \widehat{F}) = 1$ for $F = \overline{F}$. Given that we have different out-of equilibrium beliefs supporting each equilibrium it is not entirely surprising that we should obtain two cut-offs $\rho_H^1 > \rho_H^2$.

One response to this result could be that equilibrium is really unique once a complete theory for belief formation and updating is formulated. Indeed, this is how Spence understood signalling equilibria in the first place. We favour such an interpretation and would like to argue that the out-of-equilibrium beliefs supporting the respective equilibria where $F = 0$ and $F = \overline{F}$ are natural since in each equilibrium L-banks gain more from a deviation than H-banks for any given market beliefs.

The aggregate bank-credit supply schedule is represented in the figure below. In the next section we provide sufficient conditions under which nei-

[18] If $\rho_L \geq 0$ we have $\rho_F > 0$ and the mixed strategy semiseparating equilibrium disappears. In that case the equilibrium set only contains the two pooling equilibria. The aggregate bank credit supply correspondence is then discontinuous, giving rise potentially to problems of existence of equilibrium.

ther H-banks nor L-banks gain from deviating from the equilibrium strategy $\{\mathbf{R}^*, F(\mathbf{R}^*)\}$ by setting different lending terms.

6 General Equilibrium in the capital market

In this section we provide sufficient conditions for the existence of equilibrium lending terms, $\mathbf{R}^*$, or (ρ_H^*, ρ_L^*), such that:

1. the aggregate demand for bank credit is equal to aggregate supply:

$$\Psi(\rho_H^*) = \mathcal{M}([1 - \upsilon, p_1^*(\rho_H^*)])$$

   and,

2. neither H-banks nor L-banks have an incentive to deviate in either stage 1, by offering different terms $\mathbf{R} \neq \mathbf{R}^*$, or in stage 2, by making an equity issue $F \neq F(\mathbf{R}^*)$.

If $\rho_H^* \geq \rho_H^1$ or $\rho_H^* \leq \rho_H^2$ then a general (pooling) equilibrium exists as long as neither type of bank has an incentive to deviate by offering different lending terms. When out-of-equilibrium beliefs are such that $\delta_H(\mathbf{R}, F(\mathbf{R}^*)) = 0$ and $\delta_L(\mathbf{R}, F(\mathbf{R}^*)) = 1$ for all $\mathbf{R} \neq \mathbf{R}^*$, then deviation does not pay for either type of bank if the following condition holds:

$$A5 : (\nu - \widehat{\nu})[R_2^H(p_1) - A]\frac{w}{\kappa} < \frac{M}{M+m}(1 - \widehat{\nu})[\rho_H - \overline{\pi}(p_1) + 1 - A]\frac{w}{\kappa}$$

The LHS of $A5$ is the maximum gain for an L-bank of a deviation: by slightly undercutting $\mathbf{R}^*$ the L-bank is able to improve the average quality of it's loan applicants from a proportion $\widehat{\nu}$ of H-firms to a proportion ν and, thus, to raise its' payoff. The RHS is the minimum cost to an L-bank from such a deviation: it represents the capital loss on its' equity from being identified as an L-bank.

This condition is sufficient to guarantee existence because when it holds neither L-banks nor H-banks have an incentive to deviate. By construction H-banks choose the equilibrium lending terms, $\mathbf{R}^*$, so that they would be worse off deviating. Under assumption $A5$ undercutting cannot benefit L-banks either.

For any given equilibrium bank spread, ρ_H^*, firms are sorted into the different p_1-classes, with some p_1-firms being denied credit[19], others being financed by banks, some issuing equity, and the least risky ones issuing only bonds.

If there is an excess supply of loanable funds, ρ_H will be zero and all firms will be financed by banks. But as soon as there is a positive spread in the credit market, the best firms prefer to issue public debt. Hence, except when bank spreads are equal to zero we always have bank loans alongside bond and equity financing in equilibrium.

7 General Implications

Although the primary aim of our paper is to provide a model of equilibrium in credit and securities markets that is consistent with the known stylized facts about the interaction between the financial and real sectors, a secondary objective is to also briefly spell out the implications of our analysis for monetary policy and bank regulation.

7.1 Monetary Policy

To understand the effects of monetary policy on the real sector in our model it is helpful to think of the variant of our model where deposits are interest inelastic and where banks' lending is constrained not only by their equity base but also by their financial slack, denoted here by S[20]. In this variant of our model monetray policy through open market operations works by changing banks' financial slack or liquidity[21].

The effects of a change in monetary policy on bank lending and aggregate activity then depend on whether it induces a change in banks' equity issue decisions.

[19] This does not mean that they are "rationed": simply they do not have enough cash flows to offer in exchange for credit under the prevailing information structure for the lenders.

[20] See the discussion in section 4, for a broad definition of a bank's financial slack.

[21] In the variant with perfectly elastic deposits, banks never have liquidity shortages. In this case monetary policy can only be understood as a policy of puchasing more or less existing loans in the secondary market for bank loans. While central banks have been conducting monetary policy through this channel in the past, this is no longer a central instrument of monetary policy in most economies.

When a change in monetary policy does not imply a switch from an equilibrium with $F^* = 0$ to one with $F^* = \overline{F}$, monetary policy has all the expected effects on bank lending and real activity: an expansion of the monetary base through open market operations increases the liquidity of the banking sector S and aggregate supply of credit. As a result, the equilibrium spread decreases. In our model, the decrease in interest rates mainly results in an increase in lending to the riskiest firms in the economy.

The prediction that the spread (and external finance premium) decreases in response to a monetary expansion is in line with the so called money-view. But our explanation differs from that view in predicting that monetary policy mostly affects the smallest, youngest or riskiest firms, which rely on bank lending.

This latter prediction is consistent with the empirical evidence of Gertler and Gilchrist (1994), who found that small firms' inventory investment is significantly more sensitive to monetary policy than other forms of investment. It is also consistent with the results of Oliner and Rudebush (1994) who identify large differences in responses to monetary shocks accross different size classes of fixed investment. Our model can explain why the effect of monetary shocks on small firms' investment tends to be larger. As Bernanke and Gertler (1995) have argued these observations cannot be explained entirely through changes in interest rates.

The main novel implications of our analysis, however, concern the effects of monetary shocks when they induce a switch in the type of equilibrium. This will occur when in a $F^* = 0$ pooling equilibrium the external finance premium ρ_H rises above ρ_H^1 and when in a $F^* = \bar{F}$ pooling equilibrium the external finance premium falls below ρ_H^2. The overall effect can be established simply by analyzing Figure 2. Consider first a tightening of monetary policy starting from an equilibrium with $F^* = 0$. The increase in the equilibrium spread that results may trigger a change in the equilibrium from $F^* = 0$ to $F^* = \overline{F}$, offsetting the tightening effect of monetary policy. Vice versa when the initial position is an equilibrium with $F^* = \bar{F}$, and money supply increases, then as a result of the expansion of the monetary base ρ_H decreases and banks may be induced to switch to an equilibrium with a smaller equity base by buying back outside equity, thus again offsetting the effects of monetary policy. In sum, the effects of monetary policy could be (partially) reversed when it leads to changes in equity issues by banks, or more generally to changes in lending capacity expansion decisions.

esting insight on the credit market. With the same parameter constellation we may obtain an equilibrium with $F^* = 0$ or $F^* = \bar{F}$. That is, two otherwise identical economies may end up in two different equilibria, one with a low credit supply and a high external finance premium, the other with a high credit supply and a low external finance premium. In this multiple-equilibrium scenario we could interpret the $F^* = 0$ pooling equilibrium as one exhibiting a credit crunch since the state of the economy is compatible with an equilibrium with more bank lending and more favorable credit terms. As argued by Bernanke and Lown (1991), the source of a credit crunch is the lack of capital. Yet in our view, it is not only risk based capital regulation that may trigger a credit crunch but also the dilution cost of issuing equity that H-banks face.

overall soundness of the banking system. However, if the restructuring benefits are large enough and if the difference in screening ability is not too large then it may be welfare improving to subsidize banking even when the proportion of L-banks is large.

Regulatory changes such as increases in capital adequacy requirements, (partial or total) suppression of deposit insurance, or even ceilings on the remuneration of bank deposits tend to reduce the overall size of the banking sector by effectively increasing the costs of raising funds for banks. As a result such measures tend to increase the cost of bank loans (the equilibrium spread) and the overall failure rate of firms in the economy, as more firms tend to get bond financing instead of bank financing. In addition, such measures may have the effect of shutting out of the credit market a larger and larger fraction of firms. The firms that are likely to be shut out of the credit market altogether are those which would not be funded under bond or equity financing and which would only obtain funding from a bank if spreads are not too large.

These regulations may improve the solvency of the banking system by increasing profit margins for banks, but they do not necessarily improve the overall efficiency of the banking sector by reducing the proportion of L-banks.

The most direct way of improving the efficiency of the banking sector, of course, is to step up bank rating activities and thus identify more precisely the types of individual banks. If bank rating is fully effective then no further supporting regulation is required in our model. But since in practice monitoring of banks is unlikely to work perfectly there will in general be a need for further regulation.

It is interesting to contrast these results with the welfare implications derived in the related equilibrium models of bank loan and bond markets of Holmstrom and Tirole (1994) and Repullo and Suarez (1994). These models are built on the idea that banks provide costly monitoring services while bond markets do not. As in our model the credit market equilibrium in these models may generate an inefficient mix of bond and bank financing, if banks are initially capital constrained. In these models bank lending is limited by the bank's capital base since banks have an incentive to monitor firms only if enough of their own capital is at stake. Unfortunately, since bank capital is exogenously given and since banks cannot increase their capital base through outside equity issues in these models the only regulatory response that can be considered is a direct recapitalization by the central bank.

8 Conclusion

This paper proposes a simple model of the capital market and the interaction between the real and financial sectors built around two general observations: i) firms as well as banks face an informational dilution cost when they issue equity; they can reduce that cost by issuing bonds or taking out a bank loan ii) bank lending is more flexible and more expensive than bond financing (because of intermediation costs); as a result, only those firms with a sufficiently high demand for flexibility choose bank lending over bond financing.

These two observations are widely accepted and a growing body of empirical evidence supports these two hypotheses. It is remarkable that the simple model developed here, which abstracts from many other relevant considerations generates qualitative predictions about the equilibrium in the capital market and the effects of monetary shocks on the real sector which are broadly consistent with all the stylized facts on the effects of monetary policy on investment and firm financing uncovered by recent empirical studies.

The basic structure of the model proposed here, thus seems to be a good basis for exploring further the interface between corporate financing decisions and monetary policy. An important avenue for further research, in particular, is to explore in greater detail the effect on aggregate activity of changes in bank liquidity. Another important direction to explore is to address the effects of different forms of monetary policy on the real sector in a fully closed general equilibrium model, which would trace the effects of monetary policy on both firms and households. Finally, an interesting question to consider is whether the different overall structures of the financial systems of Germany or Japan versus the US and UK have important consequences for how monetary shocks get transmitted to the real sector.

A Mathematical Appendix

Proof of lemma 3: We have to compare the cost of funds under the two financing modes for a firm that has already issued the amount of riskless debt $2\pi_L$.

Raising 1\$ beyond $2\pi_L$ costs

$$1 = p_1 + (1 - p_1)\nu.K$$

so that the repayment K has to equal $\frac{1}{\nu}$. The dilution cost is that the

H-firm has to repay $K-1$, an event that occurs with probability $(1-p_1)$ .Therefore the dilution cost equals $(1-p_1)\left(\frac{1}{\nu}-1\right)=\frac{(1-p_1)}{\nu}(1-\nu)$.

On the other hand, for a firm with debt $2\pi_L$ raising 1$ in equity implies handing over a percentage Δa of the firm's equity such that

$$1=\Delta\ a\left[p_1\left(\pi_H-\pi_L\right)+\nu\left(\pi_H-\pi_L\right)\right]$$

which for an H-firm amounts to giving up

$$\Delta\ a\left[p_1\left(\pi_H-\pi_L\right)+\left(\pi_H-\pi_L\right)\right]$$

in expected profits, so that the dilution cost of this alternative mode of financing is

$$\frac{p_1+1}{p_1+\nu}-1=\frac{1-\nu}{p_1+\nu}$$

Simplifying, we obtain that raising equity with safe debt $2\pi_L$ is dominated by equity with maximum safe debt $2\pi_L+\nu(\pi_H-\pi_L)$ if and only if:

$$\frac{1}{p_1+\nu}>\frac{1-p_1}{\nu}$$

that is, iff $\nu>1-p_1$. ■

Proof of Proposition 4:

- We first compute the maximum repayment $R_1+\pi_L$ for which the firm faces no default risk. With probability $1-p_1$ the firm has cash-flow π_L in period 1 and can raise at most $\nu\left(\pi_H-\pi_L\right)$ in new bonds to cover the cash shortfall $R_1-\pi_L$, so that

$$\nu\left(\pi_H-\pi_L\right)=R_1-\pi_L \tag{A1}$$

and the investors' zero profit condition is:

$$\hat{I}=2\pi_L+p_1\left(R_1-\pi_L\right)+\left(1-p_1\right)\left(R_1-\pi_L\right) \tag{A2}$$

An $H-$firm's profit when issuing a fraction a of equity (and $\hat{I}$ in bonds) then is:

$$W_E=(1-a)\left(p_1\left(\pi_H-R_1\right)+p_1\left(\pi_H-\pi_L\right)\right) \tag{A3}$$

replacing R_1 we obtain:

$$W_E = (1-a)\ \ p_1 (2-\nu)(\pi_H - \pi_L) \tag{A4}$$

where a is such that exactly the additional amount $1-\hat{I}$ is raised:

$$1-\hat{I} = a\left[p_1(\pi_H - R_1) + p_1\nu(\pi_H - \pi_L)\right] \tag{A5}$$

Again replacing R_1 we obtain

$$1-\hat{I} = a\left[p_1(\pi_H - \pi_L)\right] \tag{A6}$$

and therefore,

$$W_E = p_1(2-\nu)(\pi_H - \pi_L) - \left(1-\hat{I}\right)(2-\nu) \tag{A7}$$

- An H-firm issuing bonds has to promise a repayment R_1 in period 1 such that:

  $$1 = p_1 R_1 + (1-p_1)A + \pi_L \tag{A8}$$

  Therefore, the $H-$firm's profits under bond financing are:

  $$W_B = p_1(\pi_H - R_1) + p_1(\pi_H - \pi_L) \tag{A9}$$

  Replacing R_1, we obtain:

  $$W_B = 2p_1(\pi_H - \pi_L) + (1-p_1)(A-\pi_L) - (1-2\pi_L) \tag{A10}$$

- The optimal funding mode is then determined by the sign of $\Delta = W_E - W_B$:

  $$\begin{aligned}\Delta &= -\nu p_1(\pi_H - \pi_L) - \left(1-\hat{I}\right)(2-\nu) + \\ &\quad + \left(1-\hat{I}\right) + \nu(\pi_H - \pi_L) + (1-p_1)(A-\pi_L)\end{aligned} \tag{A11}$$

  that is:

  $$\Delta = -\left(1-\hat{I}\right)(1-\nu) + (1-p_1)\left[\nu(\pi_H - \pi_L) - (A-\pi_L)\right] \tag{A12}$$

  Therefore, under Assumption A3, Δ is decreasing with p_1 and in addition we have $\Delta < 0$, for $p_1 = 1$. ■

Proof of Proposition 5:

We compare the best security funding a firm is able to obtain with a bank loan BL, and show that the difference in the firm's profit Δ is always a decreasing function of p_1. We consider in turn bond financing and equity financing.

I <u>Bonds vs. bank loans</u>

A bond is simply defined by its first period amount R_1 while the second period repayment π_L is fixed. A bank loan is characterized by two repayments $\hat{R}_1$ and $\hat{R}_2$, where either $\hat{R}_1 \leq \pi_H$ and $\hat{R}_2 = \pi_L$ or else $\hat{R}_1 = \pi_H$ and $\hat{R}_2 > \pi_L$.

Case 1 : $\hat{R}_1 \leq \pi_H$; $\hat{R}_2 = \pi_L$.

The bond holders break even condition is:

$$1 = p_1 \left(R_1 + \pi_L\right) + \left(1 - p_1\right) \left(A + \pi_L\right) \tag{A13}$$

Let $E(L)$ denote the expected continuation value in the event of default when renegotiation is possible:

$$E(L) = \nu \pi_H + (1 - \nu) A$$

Or, rearranging,

$$\pi_H - E(L) = (1 - \nu)(\pi_H - A)$$

and

$$E(L) - A = \nu(\pi_H - A) > 0$$

The equivalent break-even condition under bank lending is:

$$(1 + \rho) = p_1 \left(\hat{R}_1 + \pi_L\right) + \left(1 - p_1\right) \left(E(L) + \pi_L\right) \tag{A14}$$

so that

$$(1-p_1)(E(L)-A)+p_1\left(\hat{R}_1-R_1\right)=\rho \tag{A15}$$

The corresponding objective functions for an H-firm are:

$$W_B=p_1\left(\pi_H-R_1\right)+p_1\left(\pi_H-\pi_L\right) \tag{A16}$$

and

$$W_{BL}=p_1\left(\pi_H-\hat{R}_1\right)+p_1\left(\pi_H-\pi_L\right) \tag{A17}$$

Consequently,

$$\Delta=W_{BL}-W_B=-p_1\left(\hat{R}_1-R_1\right) \tag{A18}$$

and using (A15), we obtain:

$$\Delta=-\rho+\left(1-p_1\right)\left(E(L)-A\right) \tag{A19}$$

so that Δ is decreasing in p_1 and $\Delta<0$ for $p_1=1$.

- **Case 2** : $\hat{R}_1=\pi_{H;}\hat{R}_2>\pi_L$.

  We have

$$(1+\rho)=p_1\left(\pi_H+\pi_L\right)+p_1\nu\left(\hat{R}_2-\pi_L\right)+\left(1-p_1\right)\left(E(L)+\pi_L\right) \tag{A20}$$

and

$$W_{BL}=p_1\left(\pi_H-\hat{R}_2\right) \tag{A21}$$

and the corresponding expressions for bonds (A13) and (A16), are the same as in case 1. We therefore obtain:

$$\Delta=-p_1\left(\pi_H-R_1\right)-p_1\left(\hat{R}_2-\pi_L\right) \tag{A22}$$

and

$$\rho = p_1 (\pi_H - R_1) + p_1 \nu \left(\hat{R}_2 - \pi_L \right) + (1 - p_1) [E(L) - A] \quad \text{(A23)}$$

which yields

$$\Delta = -\rho - p_1 (1 - \nu) \left(\hat{R}_2 - \pi_L \right) + (1 - p_1) (E(L) - A) \quad \text{(A24)}$$

From (A20) we obtain:

$$\frac{d\left(p_1 \left(\hat{R}_2 - \pi_L\right)\right)}{dp_1} = \frac{1}{\nu} (-\pi_H + E(L)) \quad \text{(A25)}$$

and

$$\begin{aligned} \frac{d\Delta}{dp_1} &= \frac{1-\nu}{\nu} (\pi_H - E(L)) - (E(L) - A) = \\ &= \frac{1}{\nu} \left[(1-\nu)^2 (\pi_H - A) - \nu^2 (\pi_H - A) \right] = \\ &= \frac{\pi_H - A}{\nu} (1 - 2\nu) \end{aligned} \quad \text{(A26)}$$

and by assumption $A2$ $\nu > \frac{1}{2}$
so that $\frac{d\Delta}{dp_1} < 0$ and $\Delta < 0$ for $p_1 = 1$.

II Equity vs bank loans

Again we have to distinguish two cases.

Case 1 :$\hat{R}_1 \leq \pi_H$; $\hat{R}_2 = \pi_L$

Equations (A14) and (A17) remain unchanged. As we have shown in the proof. of Proposition 4 (in equation A7), the objective function of an equity financed H-firm is:

$$W_E = (2-\nu)\left(p_1(\pi_H - \pi_L) - \left(1 - \hat{I}\right)\right) \tag{A27}$$

so that,

$$\frac{dW_E}{dp_1} = (2-\nu)(\pi_H - \pi_L) \tag{A28}$$

On the other hand, from (A17) we obtain

$$\frac{dW_{BL}}{dp_1} = \pi_H - \frac{d\left(p_1 \hat{R}_1\right)}{dp_1} + \pi_H - \pi_L \tag{A29}$$

substituting for the value of $\hat{R}_1$ in (A14) we obtain:

$$\frac{dW_{BL}}{dp_1} = \pi_H - E(L) + \pi_H - \pi_L \tag{A30}$$

Thus for $\Delta = W_{BL} - W_E$ we have

$$\frac{d\Delta}{dp_1} = -(A - \pi_L)(1-\nu) < 0 \tag{A31}$$

- **Case 2** $:\hat{R}_1 = \pi_H \ ; \ \hat{R}_2 > \pi_L$.

We now have to use (A25) to derive

$$\frac{d\left(p_1 \hat{R}_2\right)}{dp_1} = \pi_L - \frac{(\pi_H - E(L))}{\nu} \tag{A32}$$

which by (A21) implies that:

$$\frac{dW_{BL}}{dp_1} = \pi_H - \pi_L + \frac{\pi_H - E(L)}{\nu} \tag{A33}$$

Combining with (A28) we obtain:

$$\frac{d\Delta}{dp_1} = \pi_H - \pi_L + \frac{\pi_H - E(L)}{\nu} - (2-\nu)(\pi_H - \pi_L) \qquad \text{(A34)}$$

implying:

$$\frac{d\Delta}{dp_1} = [(1-\nu)]\left[\frac{\pi_H - A}{\nu} - (\pi_H - \pi_L)\right] \qquad \text{(A35)}$$

which is negative under assumption A4: $\frac{\pi_H - A}{\pi_H - \pi_L} < \nu$.
■

Proof of lemma 7:

To simplify our notations, we first define

$$\hat{\rho} = \lambda\rho_F + (1-\lambda)\rho_H$$

$$K_H = \left(\frac{\hat{\rho}}{\kappa} + 1\right) w \qquad \text{(A36)}$$

and

$$K = \left(\frac{\rho_F}{\kappa} + 1\right) w \qquad \text{(A37)}$$

The objective function of H-banks can then be written as

$$W_H(F) = \frac{K + \frac{\rho_F}{\kappa}F}{K + \left(\frac{\rho_F}{\kappa} + 1\right)F}\left(K_H + \left(\frac{\hat{\rho}}{\kappa} + 1\right)F\right) \qquad \text{(A38)}$$

differentiating with respect to F we then obtain:

$$W_H'(F) = \frac{1}{D^2}\left\{-KK_H + K\left(K + \frac{\rho_F}{\kappa}F\right)\left(1 + \frac{\hat{\rho}}{\kappa}\right) + \right.$$

$$+\frac{\rho_F}{\kappa}F\left(K+\left(1+\frac{\rho_F}{\kappa}\right)F\right)\left(1+\frac{\hat{\rho}}{\kappa}\right)\Bigg\} \qquad \text{(A39)}$$

where,

$$D = K + \left(\frac{\rho_F}{\kappa}+1\right)F$$

but (A36) and (A37) together imply that:

$$-KK_H + KK\left(1+\frac{\hat{\rho}}{\kappa}\right) = Kw\left(1+\frac{\hat{\rho}}{\kappa}\right)\frac{\rho_F}{\kappa}$$

so that, $W'_H(F)$ has the same sign as ρ_F. ■

Proof of Proposition 8:

We first prove the existence of the three types of equilibria (two pooling and one semiseparating) and then proceed to show that the two pooling equilibria satisfy $\rho_H^2 < \rho_H^1$.

Let ρ_F and ρ_0 be the equilibrium market beliefs for a bank that issues an amount of equity respectively equal to $\bar{F}$ and 0. We define two functions:

$$\begin{aligned}\psi_H(\rho_F,\rho_0) &= \left(w+\frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)\left[\frac{\lambda\rho_F+(1-\lambda)\rho_H}{\kappa}+1\right] \\ &\quad -\left[\frac{\lambda\rho_0+(1-\lambda)\rho_H}{\kappa}+1\right]w\end{aligned}$$

and

$$\psi_L(\rho_F,\rho_0) = \rho_F(w+\bar{F}) - \rho_0 w$$

which represent the net payoff of issuing equity worth $F = \overline{F}$ instead of $F = 0$, for respectively an H and an L-bank.

Our different types of equilibria are then characterized by the values of the functions ψ_H and ψ_L, as follows:

1. The pooling equilibrium with $F = 0$ is characterized by $\psi_H(\rho_F, \rho_0) \leq 0$ and $\psi_L(\rho_F, \rho_0) \leq 0$.

2. The pooling equilibrium with $F = \overline{F}$ by,

   $\psi_H(\rho_F, \rho_0) \geq 0$ and $\psi_L(\rho_F, \rho_0) \geq 0$

3. Separating equilibria by different signs of the functions ψ_H and ψ_L.

4. The semi-separating equilibria by a mixed strategy of one type of agents which implies either $\psi_H(\rho_F, \rho_0) = 0$ or $\psi_L(\rho_F, \rho_0) = 0$.

Now, to prove Proposition 8, we first establish a preliminary lemma:

Lemma 9 *:*
i) If $\rho_F > 0$,*then* $\psi_L(\rho_F, \rho_0) \geq 0$ *implies* $\psi_H(\rho_F, \rho_0) > 0$
ii) If $\rho_F < 0$,*then* $\psi_L(\rho_F, \rho_0) \leq 0$ *implies* $\psi_H(\rho_F, \rho_0) < 0$
iii) $\psi_L(\rho_F, \rho_0) = 0$ *implies* $\psi_H(\rho_F, \rho_0) = \frac{(1-\lambda)\rho_F}{\kappa}\left(\frac{\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)\left(\frac{\rho_H}{\kappa}+1\right)$, *so that if* $\rho_F = 0$, $\psi_H(\rho_F, \rho_0) = 0$

Proof of lemma 9: To show *i)*, notice that $\psi_L(\rho_F, \rho_0) \geq 0$ implies $\rho_F(w + \bar{F}) \geq \rho_0 w$. We can thus replace $\rho_0 w$ in ψ_H , and obtain:

$$\begin{aligned}\psi_H(\rho_F, \rho_0) \quad &\geq \quad \left(w + \frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)\left[\frac{\lambda\rho_F + (1-\lambda)\rho_H}{\kappa} + 1\right] \\ &\quad -\frac{\lambda\rho_F(w+\bar{F})}{\kappa} - \left[\frac{(1-\lambda)\rho_H}{\kappa} + 1\right] w\end{aligned}$$

Rearranging and simplifying we have:

$$\psi_H(\rho_F, \rho_0) \geq -\frac{\lambda\rho_F}{\kappa}\left(\frac{\bar{F}}{\frac{\rho_F}{\kappa}+1}\right) + \left[\frac{(1-\lambda)\rho_H}{\kappa} + 1\right]\left(\frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)$$

so that, rearranging again, we obtain:

$$\psi_H(\rho_F, \rho_0) \geq \frac{(1-\lambda)\rho_F}{\kappa}\left(\frac{\bar{F}}{\frac{\rho_F}{\kappa}+1}\right) + \frac{(1-\lambda)\rho_H}{\kappa}\left(\frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)$$

or, equivalently,

$$\psi_H(\rho_F, \rho_0) \geq \frac{(1-\lambda)\rho_F}{\kappa}\left(\frac{\bar{F}}{\frac{\rho_F}{\kappa}+1}\right)\left(\frac{\rho_H}{\kappa}+1\right) > 0$$

The proof of *ii)* and *iii)* is exactly the same.

■

We now proceed to prove proposition 8

- First, if $\psi_L(\rho_F, \rho_0) > 0$, the only possible equilibrium is a pooling equilibrium with a maximum equity issue. Assume by way of contradiction that we had instead $\psi_H \leq 0$. The equilibrium beliefs would imply $\rho_0 = \rho_H \geq 0$, which using $\psi_L > 0$, implies $\rho_F > 0$. But then, because of lemma 9, we know that the pooling equilibrium is obtained.

- Next, we show that if $\psi_L(\rho_F, \rho_0) < 0$, the only possible equilibrium is a pooling equilibrium with no equity issue. Indeed, because of lemma 9, we know that if $\rho_F < 0$ we obtain the pooling equilibrium. If instead we had $\rho_F \geq 0$, then $\psi_L < 0$ implies $\rho_F < \rho_0$. If, in addition, $\psi_H < 0$ did not hold, the equilibrium beliefs would imply $\rho_F = \rho_H$ which yields a contradiction, since $\rho_0 \leq \rho_H$ will always hold true.

- Finally, in the case

$$\psi_L(\rho_F, \rho_0) = 0$$

a mixed strategy equilibrium with $\rho_F = 0$ is obtained. A priori, an equilibrium satisfying the above equality will be characterized by one of the three following conditions:

i) $\psi_H(\rho_F, \rho_0) > 0$

ii) $\psi_H(\rho_F, \rho_0) < 0$

iii) $\psi_H(\rho_F, \rho_0) = 0$

Using *iii)* of the previous lemma allows us to determine that,in each case *i), ii)* and *iii)*, the sign of ρ_F is the same as the sign of ψ_H.

To exclude case *i)* , notice that the equilibrium beliefs imply $\rho_0 = \rho_L$ so that $\psi_L(\rho_F, \rho_0) = \rho_F \bar{F} - (\rho_F - \rho_L)\, w > 0$ yields a contradiction.

As for *ii)*, the equilibrium expectations imply $\rho_F = \rho_L < 0$. But $\psi_L(\rho_F, \rho_0) = 0$ would imply $\rho_L \left(\bar{F} + w\right) = \rho_0 w$, so that $\rho_L > \rho_0$, which is a contradiction since ρ_0 is an average of ρ_L and ρ_H.

In sum, $\psi_L(\rho_F, \rho_0) = 0$ implies $\rho_0 = 0$ and $\psi_H(\rho_F, \rho_0) = 0$. Since $\rho_H > \rho_L$, this implies the necessary condition $\rho_L < 0$.

Now, in order to check that these equilibria do exist, we only have to replace the equilibrium values for ρ_F and ρ_0 in the functions ψ_H and ψ_L. It is easy to prove that both the zero pooling equilibrium conditions and the maximum equity pooling equilibrium conditions are met for $\rho_L = 0$, so that for the relevant range of parameters these equilibria do exist, proving the first part of proposition 8.

We now proceed to show that the two pooling equilibria satisfy $\rho_H^2 < \rho_H^1$.

Consider first the limiting point ρ_H^1. We first show that it occurs when $\psi_H = 0$ and not when $\psi_L = 0$.

In order to do this notice first that since for $\rho_L \leq 0$ both zero pooling equilibrium conditions are satisfied, the limiting value ρ_H^1 has to be such that $\rho_L > 0$. But this implies $\rho_F > 0$, and, by lemma 9, it also implies that if $\psi_L = 0$ the zero pooling condition for the H-banks would not be met since $\psi_H > 0$. Therefore the limiting value ρ_H^1 has to be such that $\psi_H = 0$. Introducing our assumption on the out-of-equilibrium beliefs for the zero pooling equilibrium, that is $\rho_F = \rho_L$ and $\rho_0 = \bar{\rho} \equiv \frac{M}{M+m}\rho_H + \frac{m}{M+m}\rho_L$ we have

$$\begin{aligned}\psi_H(\rho_L, \bar{\rho}) \quad &= \quad \left(w + \frac{\frac{\rho_L}{\kappa}\bar{F}}{\frac{\rho_L}{\kappa}+1} \right) [\frac{\lambda\rho_L + (1-\lambda)\rho_H}{\kappa} + 1] - \\ &\quad [\frac{\lambda\bar{\rho} + (1-\lambda)\rho_H}{\kappa} + 1]w = 0\end{aligned}$$

Also, the value for ψ_H at the equilibrium where $F = \overline{F}$ is given by:

$$\begin{aligned}\psi_H(\bar{\rho}, \rho_L) \quad &= \quad \left(w + \frac{\frac{\bar{\rho}}{\kappa}\bar{F}}{\frac{\bar{\rho}}{\kappa}+1} \right) [\frac{\lambda\bar{\rho} + (1-\lambda)\rho_H}{\kappa} + 1] - \\ &\quad [\frac{\lambda\rho_L + (1-\lambda)\rho_H}{\kappa} + 1]w\end{aligned}$$

Substracting the term

$$\left(w + \frac{\frac{\rho_L}{\kappa}\bar{F}}{\frac{\rho_L}{\kappa}+1} \right) [\frac{\lambda\rho_L + (1-\lambda)\rho_H}{\kappa} + 1] - [\frac{\lambda\bar{\rho} + (1-\lambda)\rho_H}{\kappa} + 1]w = 0$$

we can simplify the expression of $\psi_H(\bar{\rho}, \rho_L)$ to:

$$\psi_H(\bar{\rho}, \rho_L) = \left(\frac{\frac{\bar{\rho}}{\kappa}\bar{F}}{\frac{\bar{\rho}}{\kappa}+1} - \frac{\frac{\rho_L}{\kappa}\bar{F}}{\frac{\rho_L}{\kappa}+1} \right) [\frac{(1-\lambda)\rho_H}{\kappa} + 1] + \frac{\lambda}{\kappa} \left(\frac{\frac{\bar{\rho}}{\kappa}\bar{F}}{\frac{\bar{\rho}}{\kappa}+1}\bar{\rho} - \frac{\frac{\rho_L}{\kappa}\bar{F}}{\frac{\rho_L}{\kappa}+1}\rho_L \right)$$

so that $\psi_H(\bar{\rho}, \rho_L) > 0$. Since, in addition, ψ_L is positive whenever $\bar{\rho} > 0$, this establishes that a pooling equilibrium with $F = \overline{F}$ exists in a neighborhood of the limiting point ρ_H^1, proving the multiplicity of equilibria.

Now, to show that the limiting point ρ_H^2 is below ρ_H^1, we prove that, in the pooling equilibrium with $F = \overline{F}$, ψ_H is an increasing function of ρ_H.

To see this compute first the derivative $\frac{d\psi_H}{d\rho_H}$

$$\begin{aligned}\frac{d\psi_H}{d\rho_H} &= \left\{ \frac{\frac{\bar{F}}{\kappa}}{\left[\frac{\rho_F}{\kappa}+1\right]^2} \left(\frac{\lambda\rho_0 + (1-\lambda)\rho_H}{\kappa} + 1 \right) + \left(w + \frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1} \right) \frac{\lambda}{\kappa} \right\} \frac{d\rho_F}{d\rho_H} + \\ &+ \left(w + \frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1} \right) \frac{1-\lambda}{\kappa} - \left(\frac{\lambda}{\kappa}\frac{d\rho_0}{d\rho_H} + \frac{1-\lambda}{\kappa} \right) w\end{aligned}$$

Since in the pooling equilibrium with $F = \overline{F}$ we have $\frac{d\rho_F}{d\rho_H} = \frac{M}{M+m} + \frac{m}{M+m}\hat{\nu}$ and $\frac{d\rho_0}{d\rho_H} = \hat{\nu}$ we obtain:

$$\frac{d\psi_H}{d\rho_H} \geq \left(\frac{\frac{\rho_F}{\kappa}\bar{F}}{\frac{\rho_F}{\kappa}+1} \right) \frac{1-\lambda}{\kappa} + \frac{\lambda}{\kappa} w \left(\frac{d\rho_F}{d\rho_H} - \hat{\nu} \right) > 0$$

which establishes that since the maximum equity equilibrium exists for ρ_H^1, it will also exist for any higher value. The limiting point ρ_H^2 has therefore to satisfy $\rho_H^1 > \rho_H^2$. ■

References

[1] Bernanke, B.S. and C.S. Lown (1991) "The Credit Crunch", Brooking Papers on Economic Activity, pp. 204-239.

[2] Bernanke, B.S. and M. Gertler (1995) "Inside the Black Box: the Credit Channel of Monetary Policy Transmission", *New York University Working Paper.*

[3] Besanko, D. and G. Kanatas (1993) "Credit Market Equilibrium with Bank Monitoring and Moral Hazard", *Review of Financial Studies* 6, pp.213-32.

[4] Cho, I. K. and D. M. Kreps (1987) "Signalling games and stable equilibria" , *Quarterly Journal of Economics* , 102: 179-221

[5] Dybvig, P. and Zender, J. (1991) "Capital Structure and Dividend Irrelevance with Asymmetric Information", *Review of Financial Studies* 4, pp.201-19.

[6] Gertler, M. and S. Gilchrist (1994) "Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms", *Quarterly Journal of Economics 109,* May, pp. 309-40.

[7] Gilson, S.K.J. and L. Lang (1990) "Troubled Debt Restructurings: an Empirical Study of Private Reorganization of Firms in Default", *Journal of Financial Economics 27*, pp. 315-53.

[8] Holmstrom, B. and J. Tirole (1994) "Financial Intermediation, Loanable Funds and the Real Sector" mimeo, MIT.

[9] Hoshi, T., Kashyap, A., and D. Scharfstein (1993) "The Choice between Public and Private Debt: an Analysis of Post-Deregulation Corporate Financing in Japan" mimeo, MIT.

[10] Kashyap, A., Stein, J.C., and D.W. Wilcox (1993) "Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance", *American Economic Review 83*, pp. 78-98.

[11] Kashyap, A. and J.C. Stein (1994) "Monetary Policy and Bank Lending", in N. Gregory Mankiw, ed. *Monetary Policy*, University of Chicago Press for National Bureau of Economic Research.

[12] Lummer, S. and J. McConnel (1989) "Further Evidence on the Bank Lending Process and the Reaction of the Capital Market to Bank Loan Agreements", *Journal of Financial Economics 25*, pp. 99-122.

[13] Myers, S.C. and N.S. Majluf (1984) "Corporate Financing and Investment Decisions when Firms have Information that Investors do not have", *Journal of Financial Economics 13*, pp. 187-221.

[14] Oliner, S. and G. Rudebush (1994) "Is there a Broad Credit Channel?" Board of Governors, mimeo.

[15] Petersen, M. and R. Rajan (1994) "The Benefits of Lending Relationships: Evidence from Small Business Data", *Journal of Finance, 49*, March, pp. 3-37.

[16] Petersen, M. and R. Rajan (1995) "The Effect of Credit Market Competition on Lending Relationships", *Quarterly Journal of Economics, 110*, May, pp. 407-43.

[17] Repullo, R. and J. Suarez (1994) "Entrepreneurial Moral Hazard and Bank Monitoring: a Model of Financial Intermediation", mimeo, CEMFI, Madrid.