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OPTIMAL BAIL OUT POLICY, CONDITIONALITY AND CONSTRUCTIVE AMBIGUITY*

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Abstract

This paper addresses the issue of the optimal behaviour of the Lender of Last Resort (LOLR) in its microeconomic role regarding individual financial institutions in distress. It has been argued that the LOLR should not intervene at the microeconomic level and let any defaulting institution face the market discipline, as it will be confronted with the consequences of the risks it has taken. By considering a simple cost benefit analysis we show that this position may lack a sufficient foundation. We establish that, instead, under reasonable assumptions, the optimal policy has to be conditional on the amount of uninsured debt issued by the defaulting bank.Yet in equilibrium, because the rescue policy is costly, the LOLR will not rescue all the banks that fulfill the uninsured debt requirement condition, but will follow a mixed strategy. This we interpret as the confirmation of the "creative ambiguity" principle, perfectly in line with the central bankers claim that it is efficient for them to have discretion in lending to individual institutions. Alternatively, in other cases, when the social cost of a bank's bankruptcy is too high, it is optimal for the LOLR to bail out the institution, and this gives support to the "too big to fail" policy. *Journal of Economic Literature* Classification Numbers: G10, G18, E58

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1 INTRODUCTION

Although there is nowadays a clear consensus on the role of the Lender of Last Resort's (LOLR) interventions at the macroeconomic level, its intervention at the microeconomic level, providing assistance to individual financial institutions is clearly controverted. The behavior of the LOLR has been widely criticized in the countries confronted with a financial institutions crisis. The US Saving and Loans crisis, Mexico's 1994 crisis, the Credit Lyonnais and the Long Term Credit Bank of Japan are spectacular[1] illustrations of the controversial bail-outs and give support to the view that any rescue operation will ultimately be borne by taxpayers. Yet, to our knowledge, there has not been any attempt to characterize the LOLR optimal policy regarding individual bank crisis on the basis of a cost benefit analysis. This is the main motivation behind this paper.

The criticisms of the LOLR bail-outs emerge from the *non-interventionist* view that bailing out banks distorts the incentives and leads bank managers to excessive risk taking. This view has been advocated, among others, by Goodfriend and King(1988), T.Humphrey(1986), and Schwartz(1995) and is partially reflected in the Central Bankers doctrine. Their basic argument is that, given today's well developed and liquid financial markets, the LOLR should intervene only at the macroeconomic level and through the open market operations. Only when systemic risk is at stake and the standard funding mechanisms are not available to the banks, for instance, because of a gridlock on the interbank market, should the LOLR lend directly to the banks[2]. The reasons not to intervene are to be found in the negative incentive effects that, supposedly, outweigh the positive short run benefits. From that perspective, this view is related to Bagehot's argument that the LOLR "should never lend to unsound people"(p.97) because it would encourage excessive risk taking. In addition, an alternative justification of the *non-interventionist* view, stating that peer monitoring could be more efficient than central bank monitoring, has been put forward. These are two forceful arguments that any model addressing the LOLR policy has to take into account.

The opposed view, arguing that in some cases bailing out banks may be efficient is also widely held, for instance by Mishkin(1995), Santomero and Hoffman(1998) or Freixas, Parigi and Rochet(1998). It acknowledges two important facts: first, that a bank's bankruptcy generates externalities and has a social cost. Second, that if the bank continues operating, the bank's charter value[3]

[1] The cost of a financial bail out are quite high. The estimates of the cost of the bank crisis reach 30% of GDP for Japan and 27% in the case of Mexico.(See respectively The Financial

and, more generally, its *growth opportunities* (Herring and Vankudre(1987)) are preserved.

But the main argument against the *non-interventionist* thesis may be, simply, that central banks do bail out financial institutions on the grounds that it eliminates potential contagion problem leading to systemic risk. Also,they do not commit to a specific policy but implement a "constructive ambiguity"doctrine, a doctrine that, to our knowledge, has no substantial support from the theoretical perspective, whereby there is no public announcement on what the bailing out policy will be. Finally, it is well known that when the bank in financial distress is sufficiently large the "constructive ambiguity" ceases to operate and leaves precedence to the "too big to fail" principle.

Consequently, the issues at stake are clearly more involved than what the *non-interventionist* free-market view suggests. Our paper addresses the cost benefit analysis of bail-outs with the objective to identify under what set of (mutually exclusive) conditions are the *non-interventionist,* the constructive ambiguity and the "too big to fail" policy justified.

The empirical analysis of bank default resolutions backs up the idea that liquidation is not the rule but the exception. Goodhart and Schoenmaker(1995) study support that view by gathering evidence on the effective bailing out policy of central bankers all over the world. Out of the sample they construct of 104 failing banks 73 resulted in rescue and 31 in liquidation. Santomero and Hoffman's(1998) review of financial institutions resolution establishes, similarly, that access to the discount window in the US between 1985 and 1991 was often granted to institutions that would end up failing. This is in clear contrast with the radical *non-interventionist* view of the LOLR policy. In Santomero and Hoffman's(1998) own words:

> "Often economists are quick to argue that failure should have a rapid and brutal response. Failed private institutions should pay the private penalty for default. However, while this result may be viable in theory, it is never employed in practice. In reality, the options open to the regulator will depend not only on the state of the institutions involved, but also on the s

Whatever the form of resolution, closure policy by regulatory authorities has been characterized by the "constructive ambiguity" and the "too big to fail" policies. Yet, to our knowledge, the first of these doctrines has never been sufficiently justified. Why should not the LOLR commit to a given line of action, for instance supporting solvent illiquid banks? Bagehot's (1873) position was precisely the opposite, that the LOLR's behavior should be publicly announced. The argument developed by central bankers, stating that a publicly announced behavior would lead the banks to take full benefit of the LOLR support and to increase their risks, is inconsistent with the standard assumptions of rational expectations on behalf of the agents.

Our analysis of the LOLR policy starts from a general model of the banking activity in order to identify the costs and benefits of a bail-out policy. This allows us to establish how the LOLR's intervention implicitly subsidizes the banking activity. This is not surprising, since the bail-out of distressed financial institutions is equivalent to the extension of the benefits of deposit insurance to the uninsured claim holders. This subsidy therefore introduces a distortion in the resource allocation process.

We begin by considering, as a benchmark, the non-commitment case where the behavior of the LOLR is determined as a perfect Bayesian equilibrium. In the non-commitment case, the LOLR cannot credibly commit to a given policy, and the policy will be obtained as the perfect equilibrium of the game between the LOLR and the banks. The resulting strategy of the LOLR will be simply a pure strategy where it will either rescue or liquidate the bank depending on the costs and benefits of the two options. As a consequence the banks will anticipate this policy and will choose their financial structure in such a way as to maximize the value of the bail-out option.

In the commitment case, where the bank identifies, ex ante, the effects of its bail-out policy on the uninsured funding of the banks the results are more involved. Our main result is that, depending on the bank's characteristics, the optimal policy may be either a systematic bail-out or a mixed strategy. We argue that the fact that the optimal policy is a mixed strategy may be the foundation of the "constructive ambiguity" doctrine. In this way, our result draws a bridge between two completely different strands of the literature on LOLR, the bailing out policy, which has been studied from a microeconomic perspective, and the Central Bank preference for "constructive ambiguity" which, has only been justified at the macroeconomic level. We also obtain that the optimal policy will make use of *conditionality* in the following sense: absent moral hazard on behalf of investors, support to defaulting banks will be conditional on their having a low level of uninsured funding. Still, because the optimal policy is a

are less costly than others, and therefore should be preferred. For this reason we disregard cases d) "modified purchase and assumption" and g)"life boat" where the financial crisis can be solved without bearing the full cost of paying the uninsured depositors back. The other forms of resolution may be either part of a rescue package or lead to liquidation, which are the simplified cases we consider here. I would like to thank Alistair Milne for suggesting that the link between the model and the actual alternative policies should be articulated more explicitly.

mixed strategy, this is purely a necessary condition and not all banks satisfying the necessary condition will have access to the LOLR support.

We then proceed to extend our result to the case of moral hazard. In the context of the financial industry, moral hazard may stem from two different opposed sources: first, it may be brought about by bank managers engaging in risk-shifting activities, and secondly, it may result from lack of monitoring on behalf of the lenders. Both types of moral hazard result in a welfare loss. Our main result carries over to the case of risk-shifting on behalf of bank managers, where, again, conditionality and constructive ambiguity are justified. Still if moral hazard on behalf of the investors is the issue at stake, our result holds true only in the sense that the optimal policy will still be either a pure strategy bail-out or a mixed strategy between bail-out and liquidation. Yet, when external monitoring has an important disciplinary effect, our result is inverted because support to defaulting banks may be conditional on their having a **sufficiently high level** of uninsured debt. (From a more technical point of view, it should be remarked that although the result holds true with probability one, for some set of parameters (which have a zero measure) a systematic liquidation could be the optimal strategy).

As a consequence, our model establishes that the case for a strict *non-interventionist* policy appears very weak, because if the LOLR is able to commit, then a mixed strategy conditional on the amount of uninsured debt will always dominate systematic liquidation. Thus, if anything, the justification of the *non-interventionist* position has to be based on aspects that are outside the scope of a cost-benefit analysis.

There have been some interesting contributions in this area in recent years. The non-commitment case has been studied by Mailath and Mester(1994) and Repullo(1993). Still, in such a framework the issues at stake in the debate at hand cannot be addressed , because by definition in the non-commitment case no policy can be defined. Addressing this issue from a macromonetary perspective, Goodhart and Huang(1998) have built an interesting model of the LOLR, showing why monetary authorities may be interested in following a "too big to fail policy", and how a dynamic frameworks leads the LOLR to consider using a "constructive ambiguity" policy. On the other hand, Aghion, Bolton and Fries (1998) consider the issue of financial distress from the ex ante asymmetric information perspective. They argue that if bank managers can delay insolvency by hiding the extent of their bank loan losses, the optimal bank closure rules should be obtained as the solution to a problem of asymmetric information. They do not consider the possibility of a "constructive ambiguity" but focus on the effect of asymmetric information.

The structure of the paper is the following. In the next section, we provide some intuition on the issues at stake in a bank rescue by means of a simple example. Section 3 is devoted to the description of our model and to the treatment of the non-commitment case. Section 4 characterizes the optimal bail-out policy in the absence of moral hazard. Section 5 introduces moral hazard on behalf of the banks managers and, symmetrically, section 6 examines the issue of moral hazard on behalf of the banks uninsured creditors. Finally section 7 is devoted to the policy implications of the model.

2 A PRELIMINARY EXAMPLE

In order to briefly present our argument we consider a simple example in which the lender of last resort has to choose between the liquidation (payoff) and the bail-out (purchase and assumption) of a financial institution, which we will refer to hereafter as a bank. The LOLR's considers the total expected[5] cost of bail-out and liquidation. The bank has three types of liabilities: insured deposits, uninsured debt, and equity. The bank's assets, with initial value 100, are worth 80 if the institution is sold as a going concern but only 50 if it is liquidated, so that the value of growth opportunities is 30, which for the sake of the example we take to be an exaggeratedly large amount. Still, if the institution is to continue in operation, the LOLR has to compensate all the creditors, so that the uninsured claim holders will end up obtaining the face value of their claims.

The costs of liquidating or rescuing the bank will therefore depend on its liability structure. We consider that the bank's equity has a book value of 10, and focus on two different funding strategies: in the 100% *deposit funding case*, the bank has 90 of insured deposits; while in the *deposits/uninsured debt funding case*, the bank funding combines 45 in deposits and 45 in uninsured liabilities.

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Assuming away any administrative costs, the LOLR faces the following costs (liabilities net of realized assets)[6]

	LIQUIDATION	BAIL-OUT
INSURED DEPOSIT FUNDING	90-50=40	90-80=10
DEPOSIT AND UNINSURED DEBT FUNDING	Max(45-50,0)=0	90-80=10

COST OF BANK FAILURE

Because the LOLR need not compensate the uninsured debt holders, in the case where they have funded 50% of the bank the cost is lower if the bank

[5] It may be argued that having precise information on these values requires time, which is always a scarce resource when a bank faces a crisis (e.g. Barings). Because we consider that the rational expectations and risk neutrality this important practical dimension is secondary here.

[6] Obviously this example assumes implicitly that uninsured debt holders cannot step in and renegotiate the initial contract, for instance agreeing to a charge off. Indeed, by reducing their claim from 40 to 30 the bank may be bailed out and the bond holders will get 30. If instead it is liquidated, bond holders obtain 0. We assume away this possibility which requires that the uninsured debt holders are able to coordinate and have as good information as the central bank.

is liquidated and the uninsured debt holders receive 5. Therefore, the LOLR will bail out the bank in the deposit funding case and will liquidate it in the deposit/uninsured debt funding case.

The example illustrates, first, that the liabilities structure of a bank will be a crucial element for the decision to be taken by the LOLR. Second, it shows that the uninsured debt holders expected return depends on the liabilities structure of the bank. As a consequence, the external cost of the debt is non linear, because up to a given threshold, (equal in this example to a funding with 30 in uninsured debts and 60 in deposits), the uninsured debt holders may be confident that the LOLR will fully reimburse their claims. Beyond this threshold, the LOLR will not bail out the bank and the ex ante cost of uninsured debt financing will be higher[7].

Obviously, when we want to extend this example we have to be aware that cost is not the only issue at stake, because the behavior of the LOLR may take into account the effect of its decision on the banks behavior (moral hazard). Also, we have considered only the ex-post decision (non-commitment) while in the rest of the paper we will consider also the commitment case in a more general welfare setting.

3 THE DISTORTIVE EFFECT OF THE LOLR POLICY: THE NON-COMMITMENT CASE

We will consider first the case of a LOLR that is unable to stick to a pre-specified policy in a credible way. So, even if the LOLR may be perfectly aware of the ineffectiveness of its behavior regarding banks liquidations and bail-outs, (for instance, because of moral hazard) it cannot correct it. In this context, the LOLR cannot credibly affect the strategic choices of the bank regarding size, risk and the proportion of uninsured liabilities, while it will be able to do so in the commitment case.

3.1 THE SUBSIDIZATION EFFECT OF THE LOLR POLICY

Since the LOLR bail-out policy is an (ex post) subsidy, it will be reflected in the ex ante borrowing cost. But it is worth emphasizing that the main characteristic of the LOLR support, is that the subsidy is conditional on the bank being in financial distress. So, as we will see, absent complementary mechanisms (as taxes or insurance premia) the higher the probability of default, the higher the expected value of the subsidy. To establish rigorously this point, we develop a model of bank failure and LOLR intervention.

[7] If the LLR and the Deposit Insurance Company are distinct entities, the choice between liquidation and bail-out should be the same, even if the costs will not be borne by the same institutions. The implications, would therefore, be the same.

Consider a bank that is facing a liquidity shortage and asking for a loan to the LOLR. We assume the bank has a probability p to be solvent in which case it will obtain an expected cash flow x and a complementary probability, $1-p, (p > 1-p)$ to be in financial distress. In this case, if the bank is bailed out, the expected value of its assets will be V_C;

The bank has an initial value for its assets A, with $A = E + B + D$. Since we are not concerned with the asset side of the bank activities, we will simply describe the financial intermediary transformation function as by a production function exhibiting decreasing returns. This is justified by the fact that in the short run the bank monitoring and screening abilities are a quasi-fixed resource. We will denote by $x(B)$ the gross return a successful bank obtains on a portfolio of loans.

We will assume that for the relevant range of values of B, in case of success the bank is able to repay all its debts

$Assumption 1 : x > \ B(1 + R^l) + D(1 + r_D)$

The LOLR choice will be the result of comparing the costs and benefits of liquidation versus continuation. The LOLR may be concerned only with the cost or it may maximize a social welfare function. We assume there is a social cost, λ, (which may originate in the cost of taxation), for transferring funds from the public to the private sector. In this way we acknowledge that, ultimately, bank failures have a cost for taxpayers (as well documented by Goodhart and Schoenmaker(1993)). If a bank is bailed out, the amount

$$S = B(1 + R^l) - Max\left[V_L - D(1 + r_D), 0\right] \qquad (2)$$

is the subsidy the bank's uninsured debt holders receive. This subsidy is computed with respect to the liquidation benchmark, V_L; the cost of this subsidy will be $S - (V_C - V_L)$. Computing the subsidy with respect to the continuation value V_C would be equivalent for some purposes, but would lead to a different equilibrium rate for uninsured funds, R^l. Taking V_C as the benchmark is justified if, in case of financial distress, there is no way for the uninsured depositors

$2: B(1+R^l)+D(1+r_D)-V_C>0$

Assumption 2 implies that S is positive.

We denote by C the total social cost of closing down a bank, which include all types of externalities[10]. In particular, we include in C the administrative cost of liquidation, the cost of selling illiquid assets and the cost of breaking up relationships with the borrowers which has been found empirically relevant[11]. These costs are assumed to increase with the defaulting bank's size.

Assumption3 : C is an increasing function of the bank's assets

The reason why we emphasize this apparently innocuous assumption is because it will be crucial in deriving our main results. The justification of this assumption is that, in addition to the administrative costs of liquidation, the cost of illiquidity or fire-sale liquidation implies that, possibly because of information asymmetries, some of the bank's assets have to be sold at a price inferior to their present value.

Still, we will require the stronger following assumption in order to derive some of our results. It seems a natural assumption if we think of the cost C as the cost of contagion in the financial markets:

Assumption3.1 : C is an increasing function of the bank's assets A satisfying: $\frac{\partial C}{\partial A}>\lambda(1+r)$

The interpretation of assumption 3.1 is that the marginal social cost of a bank failure is larger than the cost of a marginal transfer of principal plus interest from the public to the private sector.

Ex post, that is, when facing the choice between liquidation and continuation, the LOLR will weight the cost of its two possible lines of action:

- Liquidation, with a cost equal to $(1+\lambda)C$
- Continuation with a cost equal to

$$\lambda(B(1+R^l)-Max\,[V_L-D(1+r_D),0])-(1+\lambda)(V_C-V_L)=$$
$$=\lambda S-(1+\lambda)(V_C-V_L)$$

The decision depends therefore on Δ defined as the difference between the former and the later, that is

$$\Delta=\lambda S-(1+\lambda)(V_C-V_L)-(1+\lambda)C \tag{3}$$

This expression is simply the algebraic sum of three terms: the cost of transferring resources from the public to the private sector, λS, the goodwill value of the bank when transferred to the public sector $(1+\lambda)(V_C-V_L)$, and the expected cost of liquidation $(1+\lambda)C$. If $\Delta<0$, a bank will be bailed out

[10] An interesting example of an externality is the case of Johnson-Matthey bank. The main reason for bailing out Johnson-Matthey bank was its prominent role in London Gold Market.

[11] this is well documented at least in the case of Continental Illinois

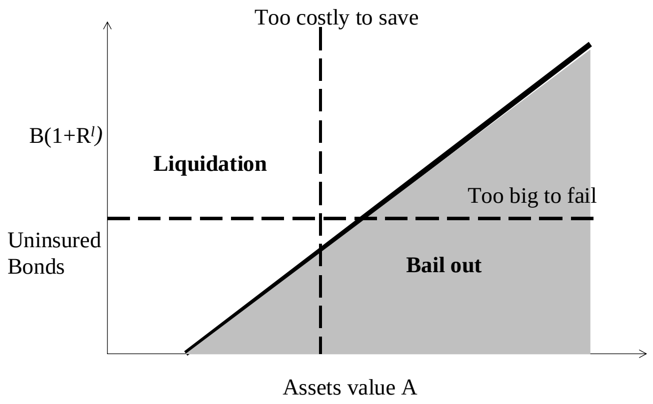

Figure 1:

because it is less costly to do so: the total expected cost of liquidation is higher than the cost of making a full repayment to the uninsured debt holders net of the efficiency gains from keeping the bank as a going concern.

We obtain the following result:

Proposition 1 *In the non-commitment case, the LOLR policy will be to bail out a bank if and only if* $\Delta < 0$. *In addition, if assumption 3.1 is satisfied, the* ***too big to fail property*** *holds for the LOLR policy, in the following sense that if it is optimal to rescue a bank, then it would be optimal to rescue any larger bank.*

Proof. see appendix ■

As illustrated in Figure 1, under non-commitment, the LOLR decision depends on Δ and this, in turn, involves the level of assets, and the level of uninsured funds that will determine the subsidy.

Once the LOLR bailing out strategy is defined, the return on uninsured debt, R, will be given, ex ante, by the equality of the net expected returns on uninsured debt to the riskless rate. In particular, $R^l = r \quad$ if $\Delta < 0$, and the bank uninsured debt is riskless. But in the general case, where the LOLR may play a mixed strategy assigning a probability z to continuation, the expression of R^l is given as the solution to the following equation[12]

$$(p + (1-p)z)B(1 + R^l) + (1-p)(1-z)Max(V_L - D(1 + r_D), 0) = (1 + r)B \tag{4}$$

Denoting by $R^l(z)$ this solution as a function of the probability of the bank being rescued z, we compute the subsidy the bank obtains as the difference of the expected cost of funds, $pB(R^l(0) - R^l(z))$. Replacing and simplifying this expression yields:

$$pB(R^l(0) - R^l(z)) = (1-p)zS$$

Thus, as expected, the expected value of the subsidy, will be the larger the higher the probability of the bank going bankrupt.[13]

3.2 THE DISTORTION EFFECT

Whatever the ability to commit of the LOLR, the very existence of a bail-out policy, implies that the ex ante behavior of the agents involved will be modified. Concerning the banks strategy, the bail-out policy will affect their choice of **size** and its choice of the portfolio of **assets riskiness**. We will focus here on the size effect in order to keep notations simple, but the effect of the change on the degree of risk of the bank's assets, as well as the effect of monitoring on the cost of funds constitute a straightforward extension.

Ex ante, the banks expected profits will be given by:

$$\Pi = p\left[x(B) - B(1 + R^l) - D(1 + r_D)\right] - m \tag{5}$$

where the cash flows in case of success, x, depend on the size, and ex ante choice of the size $A = E + D + B$ is here reduced to the choice of a level B of uninsured funds.

Replacing $pB(1 + R^l)$ in (4) and rearranging these terms, we obtain, under the fairly priced deposit insurance condition (1) the more intuitive expression:

[12] The empirical evidence backs up this idea, with riskier institutions paying higher interest rates (Park and Peristiari(1998). Also, the market includes the probability of a bail-out in the cost of the banks uninsured debt, as the analysis of the Too Big To Fail banks in the US has shown (O'hara and Shaw(1990) and Hughes and Mester(1993))

[13] If there is a too big to fail policy, as it will occur under assumption 3.1, larger institutions with a higher probability to be bailed out in case of financial distress, will choose more risky investments. Obviously, this implies that there is no other dimension as reputation building, conservation of somo oligopolistic rents, etc. that enters the managers objective function. Still, the idea that, within a limited range of size, larger banks take more risks is consistent with Barth, Hudson and Jahera(1995) findings in the S&L industry. Their results show that larger institutions invested more heavily in non-traditional activities and were more exposed to financial distress.

$$\Pi = \Pi_0 + (1-p)zS \tag{6}$$

where

$\Pi_0 = px(B) + (1-p)V_L - B(1+r) - D(1+r_D)$ is the expected profit the bank would obtain absent any subsidy. The non-committed LOLR will take into account both the size and the cost of uninsured debt into its decision. If assumption 3.1 holds, then it is easy to show (using the same type of proof as in Proposition 1), that the effect of size will prevail over the cost of uninsured debt. As a consequence, the non-commitment policy will be characterized by a threshold $\hat{B}$ such that

$$\begin{aligned} z &= 0 \text{ for } B < \hat{B} \\ z &= 1 \text{ for } B > \hat{B} \end{aligned}$$

(In the general case the non-commitment policy could be much more involved, since, depending on the level of B, the Δ could be locally increasing or decreasing, reflecting which one, of size considerations and cost of uninsured debt dominates)

Expression (6) allow us to identify the effect of the existence of the implicit subsidy in the general case. As it has often been argued, a subsidy on a banks' activity will have a distortive effect giving the bank incentives to structure its funding so as to maximize the value of its profits which include the subsidy. If assumption 3.1 holds, (too big to fail case), then we have an unambiguous effect of the LOLR rescue. It leads large banks to expand and small banks either to expand or to disregard the possibility to be bailed out.

As shown in Figure 2 the objective function of a small bank and illustrates how its discontinuity will affect its choice.

Let B_0 be the level of uninsured borrowing that maximizes the banks profit absent any subsidy, i.e. the level that maximizes Π_0 . The bank's marginal profit at B_0, once the bail-out subsidy is introduced, becomes:

$$\frac{\partial \Pi}{\partial B} = \frac{\partial \Pi_0}{\partial B} \qquad B < \hat{B} \tag{7}$$

$$\frac{\partial \Pi}{\partial B} = \frac{\partial \Pi_0}{\partial B} + (1-p)(1+r) \qquad B > \hat{B} \tag{8}$$

- If $B_0 < \hat{B}$, the bank will have to choose between the two local maxima which are B_0, and the maximum of Π for $B > \hat{B}$, as illustrated in Figure 2. If the maximum is reached at B_0 (*no-subsidy no-distortion case*), then the benefits of the LOLR bailing out policy are too small in regard with the negative distortion on size and on profitability the subsidy requires, so that it is not worth it for the (small) bank to go for the subsidy. If, instead, the maximum is reached at point $\hat{B}$ or beyond, the distortive effect of the subsidy leads the bank to choose an inefficiently large size.

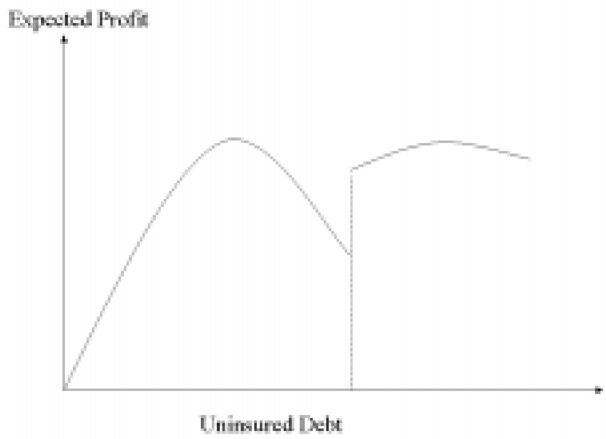

Figure 2:

- If $B_0 > \hat{B}$, then the bank size will be inefficiently large. This is clear from (8): the bank will choose a level of uninsured debt B^* that makes the first order condition hold, so that we obtain $B_0 < B^*$.

In the non-commitment case, the extension to the case where managers choose the level of **assets riskiness** of their portfolio is straightforward, since the LOLR cannot introduce any type of discipline. Hence, the only difference is that the thick profit schedule on Figure 2 will shift upwards.

As it is obvious, this has a strong effect on the conditions of economic competition among banks, that is the on the *playing field.* Indeed, large banks benefit from better funding conditions while small banks falling under the first case do

The ability to commit implies that we reformulate the LOLR problem so as to take into account the also the indirect effects of the policy in terms of the equilibrium values of the different relevant variables, whether observable or non observable (pure moral hazard). There are three main relevant variables that we have to consider: the effect on risk shifting activities, the effect on the level of monitoring and the effect on the level of uninsured funds, which depends on the equilibrium cost of funds. The first two are moral hazard variables chosen by the banks without any possible intervention on behalf of the LOLR. As it has been often argued, the introduction of either the risk shifting activities of the bank managers or the monitoring effort level on behalf of investors will make the LOLR policy under commitment tougher, simply because the additional costs of the policy are internalized. But the interesting issue is here to introduce these relevant variables jointly with the effect of the LOLR on the equilibrium size of the financial institutions.

As we will see, the analysis is then more complex. The commitment hypothesis allows to analyze whether the LOLR has to implement a "constructive ambiguity" policy as it has sometimes been advocated in a variety of contexts.

4.1 The optimal policy

In order to cover all possible types of LOLR policies, we do not restrict the analysis to pure strategies. Thus, if a bank is in financial distress, the LOLR will consider bailing it out with some probability, z, where this probability will typically depend on the bank's liabilities structure. It is implicitly assumed that the policy z is not only credible but perfectly anticipated by the market[15].

The LOLR will choose the probability z which maximizes its welfare under the restriction that the different agents play their best strategy. The main relevant variable is the level of noninsured finance, which we assume to be observable and on which we will center the analysis. Still, in the last two subsections we will extend the results in order to take into account two important phenomena: moral hazard on behalf of banks, via an increase of the riskiness of the banks' assets in response to a lenient LOLR, and moral hazard on behalf of the borrowers.

For the time being, we assume that the banks managers cannot react to the announcement of a LOLR policy by modifying the risk level, p, a point we postpone to the next section.

The objective function of the LOLR can be defined as a function of the total surplus created by the financial intermediary minus the expected cost of

Some of the criticisms to the LOLR intervention can be tracked back to the view that the LOLR is in fact unable to commit to a policy. We implicitly assume that the necessary institutional arrangements for the LOLR to act as an independent institution pursuing a long run well defined objective are satisfied.

[15]The empirical evidence supports rather a dynamic model in which the agents update their beliefs. For instance,the liquidation of the BCCI in the UK led to a flight to quality . Also, the effect of the announcement by the Comptroller of the Currency of the Too Big to Fail policy in the US, generated abnormal positive returns on the large banks stocks and abnormal negative returns on the small banks'ones (O'hara and Saw(1990));

transferring resources to the uninsured debt holders[16]

$$\begin{aligned} W(B,z) &= px(B) + (1-p)\left[zV_C + (1-z)V_L\right] - B(1+r) - D(1+r_D) - \quad (9) \\ &-(1-p)\{z\left[\lambda(B(1+R^l) + D(1+r_D) - V_C)\right] + (1-z)(1+\lambda)C\} \end{aligned}$$

- The first part of this expression,

$$px(B) + (1-p)\left[zV_C + (1-z)V_L\right] - B(1+r) - D(1+r_D) = \Pi_0 + (1-p)z(V_C - V_L)$$

is the expected surplus generated by the bank, which, obviously, depends on the bailing out policy, and takes into account the fact that with probability $(1-p)z$ the continuing franchise value of the bank will not be lost.

- The absolute value of the second term, $(1-p)z\left[\lambda(B(1+R^l) + D(1+r_D) - V_C)\right]$ is the expected cost of the bail-out. This expression can be rewritten as the difference between $(1-p)z\left[\lambda(B(1+R^l) + D(1+r_D) - V_L)\right] = (1-p)z\lambda S$, which is the expected cost of the subsidizing the uninsured debt holders and $(1-p)z\lambda(V_C - V_L)$ which is the cost reduction due to the efficiency gains that originate from the institution continuing franchise value.
- The third term, $(1-p)(1-z)(1+\lambda)C$, is simply the expected cost of liquidating the bank.

Using the definition of Δ, the objective function $W(z)$ can be rewritten as

$$W(z) = \Pi_0 - (1-p)\left\{z\Delta + (1+\lambda)C\right\} \quad (10)$$

or, alternatively, using $\Pi = \Pi_0 + (1-p)z\lambda S$ as:

$$W(B,z) = \Pi - (1+\lambda)(1-p)\left[(zS - z(V_C - V_L) + (1-z)C\right] \quad (11)$$

which shows explicitly the impact of the bank expected profits Π and the subsidy, S, on the regulator's objective function.[17]

Using the function $z(.)$, which associates to any level of B the corresponding probability of being rescued, z,the LOLR problem can be formulated as

[16] This is a general formulation since any weighted algebraic sum of the bank's generated surplus and the social cost of bankruptcy can be rewritten as in 9

[17] Notice that as λ increases, the weight of the expected monetary costs for the LLR increases, so the pure "cost of intervention" welfare function considered e.g. by Mailath and Mester(1994) is obtained in the limit as λ goes to infinity.

$$\max_{B^*,z} W(B^*, z(.))$$

$$B^* \in \arg\max p\left[x(B) - B(1+R^l) - D(1+r_D)\right] \qquad (12)$$

$$\Pi(B^*, z(B^*)) \geq \max_B \Pi(B, 0) \qquad (13)$$

with

$$B(1+R^l) = \frac{B(1+r) - (1-p)(1-z(B))(V_L - D(1+r_D))}{p+(1-p)z(B)} \qquad (14)$$

where, for the sake of simplicity we will assume hereafter that V_L is sufficient to cover for insured deposits, $D(1+r_D)$ (the result carries over to the general case, provided the fair deposit premium assumption is fulfilled).

In order to analyze this problem, it is important to notice that, absent any constraint on the function $z(.)$, only the individually rational constraint (13) may be binding. To establish this it is only necessary to see that the above problem is equivalent to

$$\max_{B,z} W(B, z) \qquad (15)$$

$$\Pi(B, z) \geq \max_B \Pi(B, 0) \qquad (16)$$

The objective function of this problem turns out to be non concave, but since we are able to show (see Lemma 7 of the Appendix) that the second order partial derivative $\frac{\partial^2 W}{\partial z^2}$ is positive, this implies that the only unconstrained local maxima of $W(B^*, z^*)$are to be found for extreme values of z, that is $z = 0$ and $z = 1$. Because of the individual rationality constraint the solution may, still be an interior one.

The two cases $z = 1$ and $z = 0$ are illustrated, respectively, in Figures 2 and 3 below. The thick curve represents the ex ante individually rational isoprofit constraint for the bank. The LOLR objective function is increasing with z in Figure 3 and decreasing with z in Figure 4.

The analysis of this non-concave optimization problem leads to the following proposition:

Proposition 2 *If C is increasing with B, then the optimal policy is either to bail out the bank or to use a mixed strategy, where the bank is liquidated with some probability.In the mixed strategy case, the optimal policy implies liquidation of any bank that holds an excessive amount of uninsured debt **(Out of equilibrium conditionality)***

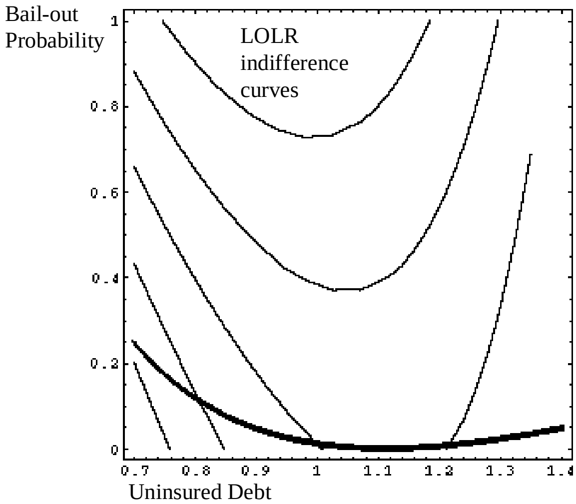

Figure 3:

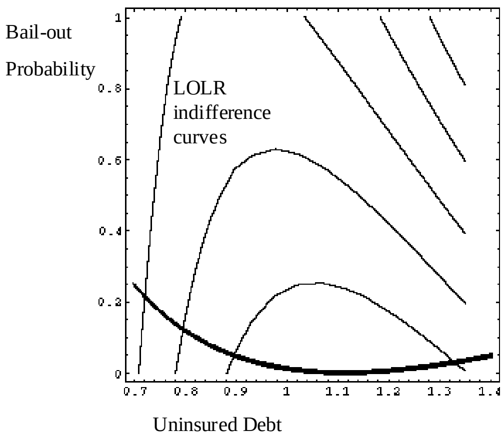

Figure 4:

Proof. See appendix ■

Figures3 and 4 provide the key for the interpretation of this result. In Figure 3 both the LOLR and the bank objective functions are increasing with z, as it is the case when the sufficient condition $\Delta < 0$ holds (As it appears in expression 21 of the appendix). As a consequence, since both agents prefer $z = 1$, there is no room for a mixed strategy. In Figure 4, the LOLR and the bank have conflicting views on z and on B. Absent any intervention from the LOLR, the bank will choose the level of uninsured debt B_0. If the LOLR wants the bank to deviate from this level, it will have to lure the bank into issuing less uninsured debt and this will be done with the only possible instrument available, that is the bailing out policy which enables the banks to obtain funds at a lower cost.

From a technical point of view, the optimality of a mixed strategy is obviously related to the non-concavity of the objective function. The origin of the non-concavity resides in the fact that the expected cost of the policy depends upon the term $zB(1 + R^l(z))$ which is concave. The economic interpretation is straightforward: an increase in the probability of rescue would, for a given repayment imply a proportional increase in the expected cost of rescuing a firm. But the market reaction to an increased bail-out probability would allow to decrease the promised repayment. As a consequence, when the bail-out probability increase the corresponding expected cost increases less than proportionally.

To provide a better intuition for Proposition 1, we have to notice, first, (as it derives from the proof in the appendix) that were the cost of liquidation constant, the optimal LOLR policy towards a specific bank would be a pure strategy, where either the bank is always bailed out or it is always liquidated. The reason why we obtain a mixed strategy is therefore fundamentally related to the fact that the LOLR takes into account the effect of B on the total cost of liquidation C. Because of this effect, the LOLR would prefer to have a lower level of B,thus decreasing possible contagion effects, and liquidation costs (recall that we are assuming no moral hazard effects at this stage). But in order to do so, the only instrument the LOLR can use is precisely its bailing out policy. The LOLR will induce the bank to lower its level of uninsured funding by promising a higher probability of bail-out. Still, since a systematic bail-out policy would be excessively costly for a LOLR that prefers a liquidation policy ($z = 0$), the solution is a mixed strategy which is sufficient to discourage the bank from taking a large uninsured lending but not too costly in terms of the expected cost of intervention.

4.2 Implementation

Proposition 2 establishes that the optimal policy requires the introduction of endogenous uncertainty. It is therefore tempting to interpret it as the *raison d'être* of "constructive ambiguity", because an outside observer facing a mixed strategy would see no clear pattern in the LOLR bail-out behavior, so that no set of contingent rules could possibly reproduce it. Yet, this is insufficient, because if the optimal policy is a mixed strategy, we still have to justify why this policy cannot be publicly announced.

In our view the main justification for the policy not to be announced is that the variables on which it is based are not publicly observable nor verifiable within the time interval during which the decision has to be taken(usually a decision has to be taken before the next day market opening). As a consequence, if the mixed strategy probabilities were publicly available and stated in a detailed way, this would limit the ability of the LOLR to take a decision on the basis of all hard and soft information available.

The way out is then to delegate the decision to an institution that would credibly follow the rules. This implies, on the one hand, that the institution should have a strong reputation to commit and act in consideration of a long term horizon, and, on the other hand, that it has to be a public institution, since profit maximizing would not give the right incentives.

4.3 Comparison with the non-commitment case

Notice that, as in the non-commitment case, the optimal policy depends on Δ, but here an additional effect appears, so that the comparison with the non-commitment case yields a prima facie surprising result. In fact, this is quite a natural result in this context where moral hazard is absent (In the next section we will see that this result is not robust to the introduction of moral hazard).

Remark 3 *Absent risk shifting opportunities and monitoring on behalf of investors, the optimal policy of a LOLR with ability to commit is more lenient than the non-commitment solution.*

As a consequence, here again, it is optimal to rescue every solvent illiquid institution, since for those banks Δ is negative.

Proof. Since in the non-commitment case a bank would be rescued only if $\Delta < 0$, and liquidated in the opposite case, a direct implication of Proposition 2, is that, under non-commitment, banks will be rescued more often. ■

To provide an intuition for this result, it is worth recalling that the difference between the commitment and the non-commitment cases comes from the effect of the LOLR policy on a)the cost of funds and b) the amount of uninsured debt issued by the banks. The first of these effects implies that the LOLR subsidy will not be as large because the market interest rate has already accounted for the probability of an LOLR bail-out. When compared to the non-commitment case, this first effect will favor the rescue of the banks.

5 ENDOGENOUS RISK LEVEL

For expository reason, we have, so far, skipped the moral hazard issue, although it is an important aspect in the debate on the LOLR's role. Clearly, if a lenient behavior of the LOLR has to incentive higher risk taking on behalf of the banks, this will reduce the ex ante benefits of bailing out banks. The importance of the banks' risk taking behavior has been long recognized, and Bagehot(1973) himself warned against the perverse effects of rescuing banks on the behavior of the whole banking industry. For these reasons it is crucial to examine if our previous results carry over to the case of endogenous risk, where the degree of riskiness of a bank's portfolio results from the banks' own strategic decision.

Assume, therefore, that the probability p is now chosen by the bank at some cost $\varphi(p)$, which is assumed to be strictly increasing and convex and twice differentiable. This probability is not observable by the investors in financial markets, so that they will infer it as being the one that maximizes the bank's expected profits, as it is standard in a moral hazard set up. The banks profit function becomes[18]:

$$\Pi = p\left[x(B) - B(1+R^l) - D(1+r_D)\right] - m - \varphi(p)$$

where m and R^l, the market required rate, are adjusted for the rationally expected banks' choice of the probability $\hat{p}$. Notice that, because the bank is largely indebted, in case of financial distress its charter value will go to its creditors, thus our framework does not allow to consider the effect of the existence of a charter value on the risk behaviour of the banks. Still, the extension is straightforward, since we deal here with the marginal effects on the chosen probability, it does not preclude that the optimal level may depend on the charter value. The first order condition lead to

$$\frac{\partial \Pi}{\partial p} = x(B) - B(1+R^l) - D(1+r_D) - \frac{d\varphi(p)}{dp} = 0 \tag{17}$$

As it is standard, under debt financing, the resulting level of $\hat{p}$ is inferior to its first best level p^*, (characterized by $x(B) = \frac{d\varphi(p)}{dp}$) so that debt financing incentives a higher level of risk on behalf of the banks. This is so because the indebtness $B(1+R^l) + D(1+r_D)$ level does not appear in the first best.

The implications[19] on the optimal bail-out policy of making the bank unobservable choice of the risk level endogenous are summarized in the following proposition:

Proposition 4 *When the banks riskiness is endogenous, the optimal policy will still be either a systematic bail-out or a mixed strategy (constructive ambiguity). The effect of the LOLR bail-out policy is **(i)** to increase the banks riskiness, and **(ii)**to decrease the marginal benefits of rescuing banks.*

Proof. See appendix ∎

[18] Obviously, the deposit risk premium is now adjusted to the endogenously determined probability $\hat{p}$.

[19] Notice that in a dynamic setting, the risk choice for the banks that benefit from a bail-out, in particular for the too-big-to-fail banks, may be more conservative than what is obtained in our static setting, because bankruptcy implies the loss of the net present value of future subsidies implicit in the too policy

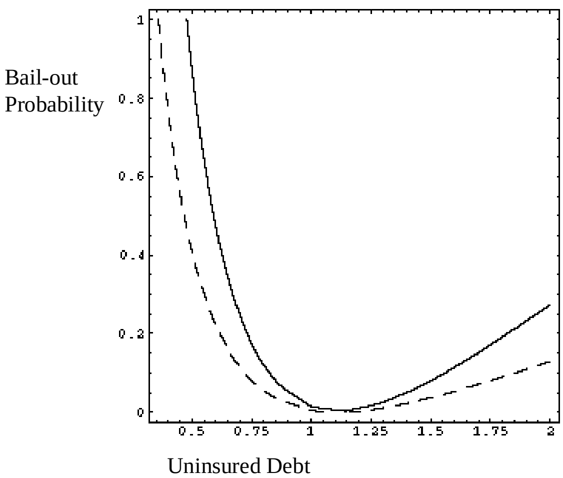

Figure 5:

The first point, (i) means that, except for $z = 0$, the bank will react to the announcement of a bailing out policy $z(B)$ by choosing a higher degree of risk, switching from the continuous to the dashed line in Figure 5. Thus, for any policy z, the bank choice of its risk level will erode some of the benefits from the LOLR policy.

Considering a different curve for the individual rationality constraint will not change the main qualitative conclusions previously obtained. Since the same analysis will apply to another indifference curve, it is intuitive that Proposition 2 carries over to the endogenous risk case. This is point (iii) of Proposition4, that states that the systematic liquidation policy will never be optimal thus justifying, the "constructive ambiguity" doctrine.

On the other hand, as intuition also suggests, risk shifting on behalf of the banks managers implies a higher cost of the LOLR interventions (as stated in point (ii) of Proposition4). As a consequence the LOLR policy may become tougher and liquidation will be more frequent when risk is chosen by the bank. This implies why, in the absence of additional assumptions, Remark 3 will not be true anymore, which is consistent with the standard view of the effects of the LOLR policy: the only reason to liquidate solvent illiquid banks is because of the incentive effects it creates on the institution intuitions risk taking behaviour. To understand why, it is worth recalling that in the endogenous risk case, the derivative of the welfare function has to introduce a term that will reflect the rescuing a bank is given by:

$$\frac{\partial W}{\partial z} = (1-p)\left[-\Delta + \lambda z \frac{(1-p)S}{p+(1-p)z}\right] + \frac{dp}{dz}\frac{\partial W}{\partial p} \tag{18}$$

Therefore, if a bank would be bailed out under non-commitment ($\Delta < 0$), the first term in $\frac{\partial W}{\partial z}$ is obviously positive, but the second one is negative, so that the net effect is undetermined, and it is not true anymore that a LOLR with ability to commit will be more lenient.

6 MONITORING ON BEHALF OF INVESTORS

We now turn to the symmetric source of moral hazard by assuming that investors determine the level of effort e, with $e \in [\underline{e}, \bar{e}]$ they allocate to the screening of projects. A higher level of screening on behalf of investors results in a limited access to funds for the banks with a higher probability of default. As a consequence, the ex ante probability of success of a bank $p(e)$ increases with the monitoring effort of investors, affecting welfare. Exerting an effort has a non pecuniary cost $\phi(e)$ that is borne by investors. As a result, the investors will solve

$$Max_e \Psi(e)$$

where the value of $\Psi(e)$ is obtained from (4)[20]:

$$\Psi(e) = p(e)\left[B(1+R^l) + D(1+r_D) - V_L\right] - \phi(e) \tag{19}$$

We take the standard assumptions to guarantee that $\Psi(e)$ has an interior maximum. In particular, we assume $\frac{d^2\Psi}{de^2} < 0$, $lim_{e\rightarrow\underline{e}} \frac{d\Psi}{de} < 0, lim_{e\rightarrow\underline{e}} \frac{d\Psi}{de} > 0$

Using the first order conditions it is easy to show that, as intuition suggests, the level of monitoring effort will decrease with the probability z of the debtor being rescued by the LOLR.

Proposition 5 *When investors exert effort in monitoring the banks, **(i)** the* *holds, the optimal policy implies liquidation of any bank that holds an excessive amount of uninsured debt; and if*

$$\frac{dp}{dB}\frac{\partial W}{\partial p}+\frac{de}{dB}\frac{\partial W}{\partial e}>(1+\lambda)(1-p)\frac{\partial C}{\partial B}$$

the optimal policy implies liquidation of any bank that holds too low a level of uninsured debt. For liquidation to be the optimal pure strategy the following condition is required.

$$\frac{dp}{dB}\frac{\partial W}{\partial p}+\frac{de}{dB}\frac{\partial W}{\partial e}=(1+\lambda)(1-p)\frac{\partial C}{\partial B}$$

(ii)*the effect of the bail-out policy is to decrease monitoring effort.* ***(iii)****This, in turn, decreases the marginal benefits of rescuing banks.*

The intuition behind proposition 5 is that in the presence of an investor's costly effort, there are two opposite forces at work. On the one hand, as we have established in the previous sections, the existence of a social cost of a financial institution bankruptcy implies that it is optimal to rescue only the banks that have a small level of uninsured debt. But here this first effect may be counterbalanced by the moral hazard effect: indeed, a larger amount of uninsured debt generates a closer monitoring of the bank by its creditors, and this will force it to choose safer projects. If moral hazard effect dominates the cost effect, the optimal bail-out policy may imply to use a mixed strategy in order to stimulate the banks to have a larger proportion of uninsured debt, and thus to be more tightly monitored. This is therefore consistent with the point of view expressed by Kaufman and Benston(1993) that the issue of subordinated debt allows the banking industry to have a better banking discipline. Proposition 5 clarifies this point by establishing not only that it requires that the effect of moral hazard has to overtake the effect on the bankruptcy cost, but also that this occurs when the LOLR is able to commit to bail-out institutions with some posiitive probability.

Still, except for a zero probability subset of parameters, the optimal LOLR policy is to implement a mixed strategy. Also, as intuition suggests, the bail-out policy has a negative effect on the monitoring effort and therefore the marginal benefits of rescuing banks decrease.

7 POLICY IMPLICATIONS

The main policy implication of our model is that whenever the LOLR is able to define a policy, it should make use of *conditionality.* This implies defining a critical level of uninsured debt beyond which the LOLR will not rescue any institution. As a consequence, the institutions will choose the second best level of uninsured funding, thus improving efficiency in the banking industry. As the optimal policy depends on the cost of raising taxes, on the risk of contagion and on the cost of liquidation, it will change both through time and across countries.

First, it is natural that the LOLR bail-out policy changes over the business cycle. When the bank margins have been eroded, either by an economic recession or by the conditions on interest rates, the probability of contagion is higher and the cost of liquidation is increased leading to a more lenient policy on behalf drawback is that the private institution has to implement a policy on the basis of some variables such as the social cost of liquidation that may not be observable by a third party. If so, the profit maximizing behavior of the insurance institutions may create important distortions with respect to the social welfare function.

8 REFERENCES

Aghion, P., P.Bolton and S. Fryes (1998)"Optimal Design of Bank Bailouts: The Case of Transition Economies", mimeo, EBRD,

Barth, J., C. Hudson and J. Jahera(1995) "S&L Closures and Survivors:

Kaufman, G. and G. Benston, (1993) "The ilntellectual history of the Federal Deposit insurance Corporation Act of 1991" in Kaufman and Litna, eds: *Assessing Bank Reform: FDICIA one year later*, The Brookings Institution, Washington D.C., 19-32

James, C. (1991) " The Losses realized in Bank Failures" *The Journal of Finance,* 46(4), 1223-42

Mailath G. and L. Mester(1994) "A positive analysis of bank closure" *Journal of Financial Intermediation, 3(3) 272*-99

Mishkin F. (1995) "Comments on Systemic Risk" in G.Kaufman ed. "Banking Financial Markets and Systemic Risk", Research in Financial Services, Private and Public Policy, Vol. 7,Hampton, JAI press Inc

O'hara and Shaw(1990) "Deposit Insurance and Wealth Effects" *Journal of Finance, 45(5),293−315*

Park, S. and S. Peristiari(1998) "Market Discipline y Thrift Depositors", Journal of Money Credit and Banking, August, Part I, 347-64

Repullo, R.(1993) "Who should decide on bank closures? An incomplete contract model. Working Paper. CEMFI, Madrid.

Santomero and Hoffman(1998) "Problem Bank Resolution: Evaluating the Options", D.P 98-05, Financial Institutions Center, The Wharton School, University of Pennsylvania.

Schwartz, A. (1995) "Systemic Risk and the Macroeconomy" in G.Kaufman ed. "Banking Financial Markets and Systemic Risk", Research in Financial Services, Private and Public Policy, Vol. 7,Hampton, JAI press Inc.

9 APPENDIX

Proof. of Proposition1 Since deposits and equity are optimally chosen, the firm choice of the size will depend on the amount of uninsured debt B. For any A such that $\Delta = 0$, the right hand derivative of Δ with respect to the level of assets A is given by the following expression :

$$\frac{\partial \Delta}{\partial A} = \lambda \left(1 + r - \frac{\partial V_L}{\partial A} \right) - (1 + \lambda) \left(\frac{\partial V_C}{\partial A} - \frac{\partial V_L}{\partial A} + \frac{\partial C}{\partial A} \right)$$

Recalling that $\frac{\partial V_C}{\partial A} - \frac{\partial V_L}{\partial A}$ is negative, under assumption 3.1, we have $\frac{\partial \Delta}{\partial A} < 0$ and therefore if there exists a threshold $\bar{A}$ such that $\Delta(A) = 0$, then for any A such that $A > \bar{A}$ we have $\Delta < 0$. ■

Lemma 6 *:*

$$\frac{\partial \left[B(1 + R^l) \right]}{\partial z} = \frac{\partial S}{\partial z} = -\frac{(1-p)S}{p + (1-p)z}$$

Proof. *The first equality is obvious from the definition of S. Regarding the second one, differentiating* $(p+(1-p)z)B(1+R^l)+(1-p)(1-z)(V_L-D(1+r_D)) = (1+r)$
we obtain
$(p+(1-p)z)d\left[B(1+R^l)\right] = -(1-p)\left[B(1+R^l)-(V_L-D(1+r_D))\right]dz$
yielding the result. ■

Lemma 7

$$\frac{\partial^2 W}{\partial z^2} > 0 \tag{20}$$

Proof. Using (10) and the previous Lemma yields:

$$\frac{\partial W}{\partial z} = (1-p)\left[-\Delta + \lambda z \frac{(1-p)S}{p+(1-p)z}\right] \tag{21}$$

and taking again the derivative with respect to z, yields:

$$\frac{\partial^2 W}{\partial z^2} = (1-p)\left(-\frac{\partial \Delta}{\partial z} + \lambda(1-p)\frac{\partial\left(zS/(p+(1-p)z)\right)}{\partial z}\right)$$

Using Lemma 6 and $\frac{\partial(zS/(p+(1-p)z))}{\partial z} = \frac{(p-(1-p)z)S}{(p+(1-p)z)^2}$ yields:

$$\frac{\partial^2 W}{\partial z^2} = \lambda\frac{2p(1-p)^2 S}{(p+(1-p)z)^2} > 0$$

■

Lemma 7 implies that unconstrained maxima of W are to be found for $z=0$ or for $z=1$

Proof. of Proposition 2
First, $z=1$, bail-out, is a possible solution to the problem 15. The Lagrangian for 15 is

$$\mathfrak{L}(B,z,\lambda,\mu_0,\mu_1) = W(B,z) + \lambda\left(\Pi(B,z) - \Pi(B_0,0)\right) + \mu_0 z + \mu_1(1-z)$$

where $B_0 = \arg\max_B \Pi(B,0)$ and Kuhn and Tucker conditions(K-T) imply

$$\frac{\partial W}{\partial B} + \lambda\frac{\partial \Pi}{\partial B} = 0$$

and

$$\frac{\partial W}{\partial z}+\lambda\frac{\partial \Pi}{\partial z}+\mu_0-\mu_1=0$$

If $\Delta<0$, then expression 21 implies $\frac{\partial W}{\partial z}>0$. Since we also have $\frac{\partial \Pi}{\partial z}>0$ this implies $\mu_1>0$, and $z=1$, proving (i).

Second, to establish the existence of a mixed strategy we will establish that $z=0$ does not satisfies the Kuhn-Tucker necessary conditions, so that when $\frac{\partial W}{\partial z}<0$ there will be a tangency solution for some z^* such that $0<z^*<1$.

Consider first, the point $z=0$. The only point satisfying the constraint with $z=0$ is B_0. At B_0 we have $\frac{\partial \Pi}{\partial B}=\frac{\partial \Pi_0}{\partial B}=0$. But computing $\frac{\partial W}{\partial B}$ and recalling that

$$\frac{\partial W}{\partial B}\mid_{B=B_0}=\frac{\partial \Pi}{\partial B}-(1+\lambda)(1-p)\left(z\frac{1+r}{p+(1-p)z}+\frac{\partial C}{\partial B}\right)<0$$

we obtain:

$$\left(\frac{\partial W}{\partial B}+\lambda\frac{\partial \Pi}{\partial B}\right)\mid_{B=B_0}<0$$

implying $B^*<B_0$, a contradiction. ■

Lemma 8 $\frac{\partial W}{\partial p}>0$.

Proof. : From (11) we have:

$$\frac{\partial W}{\partial p}=\frac{\partial \Pi}{\partial p}+z(\Delta+S)+(1+\lambda)C$$

Now, $\frac{\partial \Pi}{\partial p}=x(B)-V_L-zS>x(B)-V_L-S=x(B)-\left[B(1+R^l)+D(1+r_D)\right]>0$ using Assumption 1.

As a consequence, if $\Delta>0$ the lemma is proved. On the other hand, in the case $\Delta<0$, we have

$$z(\Delta+S)+(1+\lambda)C>\Delta+S+(1+\lambda)C=(1+\lambda)(B(1+R^l)+D(1+r_D)-V_C)>0$$

because of Assumption 2, thus implying $\frac{\partial W}{\partial p}>0$. ■

Proof. Proposition4

The derivative $\frac{dp}{dz}$is obtained from the first order condition (17)

$$\frac{\partial^2\Pi}{\partial p\partial z} = -\frac{(1-p)S}{p+(1-p)z} < 0$$

and $\frac{dp}{dz} = -\frac{\frac{\partial^2\Pi}{\partial p\partial z}}{\frac{\partial^2\Pi}{\partial p^2}}$ jointly with the concavity of Π implies $\frac{dp}{dz} < 0$, proving (i).

The marginal welfare will be

$$\frac{\partial W}{\partial z} = (1-p)\left[-\Delta + \lambda z\frac{(1-p)S}{p+(1-p)z}\right] + \frac{dp}{dz}\frac{\partial W}{\partial p} \tag{22}$$

and using expression (8) we obtain that the effect of the bank risk taking is to reduce $\frac{\partial W}{\partial z}$, proving (ii).

To prove (iii), it suffices to establish that the first order K-T conditions are not met for $z = 0$. For $z = 0$ we have

$$\frac{\partial W}{\partial B} = \frac{\partial\Pi}{\partial B} - (1+\lambda)(1-p)\frac{\partial C}{\partial B} + \frac{dp}{dB}\frac{\partial W}{\partial p}$$

and $\frac{dp}{dB} = -\frac{\frac{\partial^2\Pi}{\partial p\partial B}}{\frac{\partial^2\Pi}{\partial p^2}}$

Using (17) we obtain

$$\frac{\partial^2\Pi}{\partial p\partial B} = \frac{\partial x}{\partial B} - \frac{1+r}{p}$$

recalling that $\frac{\partial\Pi}{\partial B} = \frac{\partial\Pi_0}{\partial B} = \frac{\partial x}{\partial B} - \frac{1+r}{p} = 0$ we obtain $\frac{\partial W}{\partial B} < 0$ ∎

Proof. of Proposition5

(**i**) The first order condition with respect to B computed at the point $z = 0$ is the following:

$$\frac{\partial W}{\partial B} = \frac{\partial\Pi}{\partial B} - (1+\lambda)(1-p)\frac{\partial C}{\partial B} + \frac{de}{dB}\frac{\partial W}{\partial e}$$

The last term is positive because, on the one hand, $\frac{de}{dB}$ has the sign of $\frac{\partial^2\Psi}{\partial e\partial z}$ and

$$\frac{\partial^2\Psi}{\partial e\partial B} = \frac{dp}{de}\left[\frac{1+r}{p+(1-p)z}\right] > 0$$

and, on the other hand, we have:

$$\frac{\partial W}{\partial e} = \frac{dp}{de}\frac{\partial W}{\partial p}$$

with $\frac{dp}{de} > 0$ implying $\frac{\partial W}{\partial e} > 0$.

As a consequence, at point $(B_0, 0)$ we have three possible cases:
$\frac{\partial W}{\partial B} < 0, \frac{\partial W}{\partial B} > 0, \frac{\partial W}{\partial B} = 0.$

$$\frac{dp}{dB}\frac{\partial W}{\partial p} + \frac{de}{dB}\frac{\partial W}{\partial e} = (1+\lambda)(1-p)\frac{\partial C}{\partial B}$$

corresponding to the three optimal policies described.

(**ii**)The necessary and sufficient first order condition for (19) to have a maximum at $e^*$are given by:

$$\frac{d\Psi}{de} = \frac{dp}{de}(1-z)\left[B(1+R^l) + D(1+r_D) - V_L\right] - \frac{d\phi}{de} = 0$$

From this expression we are able to compute the sign of $\frac{de^*}{dz}$ which, because of the concavity conditions equals the sign of $\frac{\partial^2\Psi}{\partial e \partial z}$ given by:

$$\frac{\partial^2\Psi}{\partial e \partial z} = -\frac{dp}{de}\left[B(1+R^l) + D(1+r_D) - V_L + (1-z)\frac{(1-p)S}{p+(1-p)z}\right] < 0$$

(**iii**) We want to prove that when e is introduced we have:

$$\frac{\partial W(z)}{\partial z} > \frac{\partial W(z, e^*(z))}{\partial z}$$

Where the first term is the welfare function for an exogenously given level of monitoring and the second is for a monitoring decision optimally chosen by the investors. The difference between the two is that the right hand side includes the term $\frac{\partial W}{\partial e}\frac{de^*}{dz}$. We know $\frac{de^*}{dz} < 0$,and in addition But lemma (8) implies $\frac{\partial W}{\partial p} > 0$, so that $\frac{\partial W}{\partial e}\frac{de^*}{dz}$ is negative, yielding the result. ■