

**Financial Constraints, Precautionary Saving  
and Firm Dynamics**

**By**

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# 1 Introduction

The first part of the paper discusses the importance of the problem and the related work. The second part describes the proposed method and its implementation. The third part presents the experimental results and the analysis. The fourth part concludes the paper and discusses the future work.

The proposed method is based on the idea of using a neural network to learn the mapping from the input data to the output data. The neural network is trained on a set of data and is then used to predict the output for new data. The results show that the proposed method is effective and efficient. The analysis shows that the proposed method is robust and can handle noisy data. The future work will focus on improving the performance of the proposed method and extending it to other applications.

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## 2 Evidence of the effects of financing constraints.

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### 3 A model of investment with private income and collateral constraint.

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$\theta$

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1. 假设生产函数为柯布-道格拉斯形式，即  $y_t = p_t \theta_t k_t^\alpha$ ，其中  $y_t$  为产出， $p_t$  为价格， $\theta_t$  为技术效率， $k_t$  为资本投入， $\alpha$  为资本弹性系数。

2. 假设折旧率为  $\delta$ ，则资本的更新方程为  $\dot{k}_t = \eta y_t^{NT} - \delta k_t$ ，其中  $\eta$  为储蓄率， $y_t^{NT}$  为净产出。

3. 假设初始资本为  $k_0$ 。

$$y_t^T = p_t \theta_t k_t^\alpha \quad \square \square$$

$$y_t^{NT} = \eta y_t^T \quad \square \square$$

$$0 < \alpha < 1; \eta > 0$$

4. 假设生产函数为  $y_t = k_t^\alpha$ ，其中  $k_t$  为资本投入， $y_t$  为产出。

5. 假设折旧率为  $\delta$ ，则资本的更新方程为  $\dot{k}_t = \eta y_t^{NT} - \delta k_t$ 。

6. 假设初始资本为  $k_0$ 。

7. 假设生产函数为  $y_t = k_t^\alpha$ ，其中  $k_t$  为资本投入， $y_t$  为产出。

8. 假设折旧率为  $\delta$ ，则资本的更新方程为  $\dot{k}_t = \eta y_t^{NT} - \delta k_t$ 。

9. 假设初始资本为  $k_0$ 。

10. 假设生产函数为  $y_t = k_t^\alpha$ ，其中  $k_t$  为资本投入， $y_t$  为产出。

11. 假设折旧率为  $\delta$ ，则资本的更新方程为  $\dot{k}_t = \eta y_t^{NT} - \delta k_t$ 。

$ra_t$

$$a_t = (1 + \eta)y_t^T + p_t^i(1 - \delta)k_t - b_t$$

$$w_t = a_t - y_t^{NT}$$

$$0 < \delta < 1$$

$a_t$      $w_t$      $\delta$      $k_t$      $k_{t+1}$      $b_t$      $b_{t+1}$      $t$

$$V_t(w_t, k_t, \theta_t) = \underset{k_{t+1}, b_{t+1}}{MAX} \gamma E_t \left\{ x_t + \frac{1}{R} E_t [V_{t+1}(w_{t+1}, k_{t+1}, \theta_{t+1})] \right\} + (1 - \gamma)a_t$$

$$x_t + k_{t+1} = a_t + \frac{b_{t+1}}{R}$$

$$x_t \geq y_t^{NT}$$

$$b_{t+1} \leq \tau p_{t+1}^I k_{t+1}$$

$$0 < \tau \leq 1 - \delta; 0 < \gamma < 1$$

$$R = 1 + r$$

$$V_0(w_0, k_0, \theta_0) = \underset{\{k_{t+1}\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty}}{MAX} E_t \left\{ \sum_{t=0}^{\infty} \left( \frac{\gamma}{R} \right)^t \left[ \gamma x_t + (1 - \gamma)a_t + \mu_t (x_t - y_t^{NT}) + \lambda_t [\tau p_{t+1}^I k_{t+1} - b_{t+1}] \right] \right\}$$

□

$\mu_t$

$\lambda_t$   
 $b_{t+1}$

$t$

$$\mu_t = R\lambda_t + \gamma E_t(\mu_{t+1})$$

$$\frac{\mu_t}{R} = E_t\left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j}\right)$$

$$\mu_t > 0$$

$$E_t\left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j}\right) > 0$$

$$E_t\left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+j}\right) > 0$$

$t$

$k_t$      $b_t$

$$V_0(w_0, \theta_0) = \underset{\{k_{t+1}\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty}}{MAX} E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{\gamma}{R}\right)^t \left\{ \frac{1}{R} [\gamma y_{t+1}^T + (1-\gamma)a_{t+1}] + \lambda_t [\tau p_{t+1}^I k_{t+1} - b_{t+1}] \right\} \right\}$$

$$p_t^i k_{t+1} = w_t + \frac{b_{t+1}}{R} \quad \forall t \geq 0$$

$\theta_t$

□



### 3.1 The precautionary saving effect.

The total amount of resources available to the household in period  $t+1$  is the sum of the return on capital and the return on bonds. The return on capital is  $Rk_{t+1}$  and the return on bonds is  $b_{t+1}$ . The total amount of resources available to the household in period  $t+1$  is  $Rk_{t+1} + b_{t+1}$ .

The household's budget constraint in period  $t+1$  is given by:

$$\lambda_t \tau p_{t+1}^i - \phi_t p_t^i + \frac{1}{R} (1 + \eta - \gamma) E_t (MPK_{t+1}) + \frac{1}{R} (1 - \gamma) (1 - \delta) p_{t+1}^i + \frac{\gamma}{R} E_t (\phi_{t+1}) [E_t (MPK_{t+1}) + (1 - \delta) p_{t+1}^i] + \Psi_{t+1} = 0$$

where  $\phi_t$  is the Lagrange multiplier on the budget constraint in period  $t$ .

$$\phi_t = RE_t \left[ \sum_{j=0}^{\infty} \gamma^j \lambda_{t+j} \right] + 1$$

$$\lambda_t \tau p_{t+1}^i - \phi_t p_t^i + \frac{1}{R} (1 + \eta - \gamma) E_t (MPK_{t+1}) + \frac{1}{R} (1 - \gamma) (1 - \delta) p_{t+1}^i + \frac{\gamma}{R} E_t (\phi_{t+1}) [E_t (MPK_{t+1}) + (1 - \delta) p_{t+1}^i] + \Psi_{t+1} = 0$$

$$E_t (MPK_{t+1}) = \alpha p_{t+1} E_t (\theta_{t+1}) k_{t+1}^{\alpha-1}$$

The first-order conditions for the household's problem are:

$$\frac{(1 + \eta)}{R} E_t (MPK_{t+1}) = \frac{U_t}{R} + [Rp_t^i - \tau p_{t+1}^i] \lambda_t + \gamma [U_t - E_t (MPK_{t+1})] E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right) - \Psi_{t+1}$$

$$\frac{(1 + \eta)}{R} E_t (MPK_{t+1}) = \frac{U_t}{R} + [Rp_t^i - \tau p_{t+1}^i] \lambda_t + \gamma [U_t - E_t (MPK_{t+1})] E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right) - \Psi_{t+1}$$

$$S_t \geq k_{t+1}$$

$$\lambda_t \geq 0$$

□

$$[S_t - k_{t+1}] \lambda_t = 0$$

$$U_t = R p_t^i - p_{t+1}^i (1 - \delta)$$

$$S_t = w_t / [p_t^i - \frac{\tau}{R} p_{t+1}^i]$$

$$\Psi_{t+1}$$

$$\Psi_{t+1} = \frac{\alpha \gamma}{R} p_{t+1} k_{t+1}^{\alpha-1} cov(\phi_{t+1}, \theta_{t+1})$$

$$k_{t+1}$$

$$(1 + \eta) E_t(MPK_{t+1}) = U_t$$

$$(1 + \eta) E_t(MPK_{t+1}) = total$$

$$k_{t+1}^{PM}(\theta_t)$$

$$k_{t+1}^{PM} = \left[ \frac{(1 + \eta) \alpha p_{t+1} E_t(\theta_{t+1})}{U_t} \right]^{\frac{1}{1-\alpha}}$$

**Proposition 1** Conditional on continuation, a level of investment of  $k_{t+1}^{PM}$  generates negative expected marginal tradable profits.

$$\eta > 0 \quad E_t(MPK_{t+1}) < U_t$$

$$k_{t+1}^*(\theta_t)$$

$$E_t(MPK_{t+1}) = U_t$$

$$k_{t+1}^* = \left[ \frac{\alpha p_{t+1} E_t(\theta_{t+1})}{U_t} \right]^{\frac{1}{1-\alpha}}$$

$$k_{t+1}^{PM} > k_{t+1}^*$$

$$k_{t+1}^{PM} - k_{t+1}^*$$

$$\eta$$

$$U_t - E_t(MPK_{t+1}) > 0$$

$$k_{t+1}^{PM}$$

$$\lambda_t = 0$$

$$E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right) > 0$$

$$\frac{(1 + \eta)}{R} E_t(MPK_{t+1}) = \frac{U_t}{R} + \gamma [U_t - E_t(MPK_{t+1})] E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right) - \Psi_{t+1}$$

$w_t$

□

$$\gamma [U_t - E_t(MPK_{t+1})] E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right)$$

$U_t -$

$$E_t(MPK_{t+1}) \gg 0 \quad \left( E_t \left( \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right) \right)$$

$$k_{t+1}^o(w_t, \theta_t)$$

$w_t$

$$k_{t+1}^o(w_t, \theta_t)$$

$\theta_t,$

$$E_t \left[ \sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j} \right]$$

$k_{t+1}^o$

$$k_{t+1}^* \ll k_{t+1}^o < k_{t+1}^{PM}$$

$$k_{t+1}^*$$

$$k_{t+1}^*$$

$$\tilde{k}_{t+1}(w_t, \theta_t)$$

$$k_{t+1}^o(w_t, \theta_t) \quad k_{t+1}^o(w_t, \theta_t) < S_t(w_t, \theta_t)$$

$$\tilde{k}_{t+1}(w_t, \theta_t) =$$

$$S_t(w_t, \theta_t)$$

$$k_{t+1}^o \quad k_{t+1}^{PM}$$

$$k_{t+1}^o$$

$$E_t \left\{ \sum_{j=0}^{\infty} \lambda_{t+j+1} \right\} \rightarrow$$

$$\mathbf{0} \quad \Psi_{t+1} \rightarrow 0$$

$$\lim_{E_t \left\{ \sum_{j=0}^{\infty} \lambda_{t+1+j} \right\} \rightarrow 0} k_{t+1}^o = k_{t+1}^{PM}$$

$$E_t \left\{ \sum_{j=0}^{\infty} \lambda_{t+1+j} \right\} \rightarrow \infty$$

$$E_t(L_{t+1}) = 0$$

$$\lim_{E_t \left\{ \sum_{j=1}^{\infty} \lambda_{t+j} \right\} \rightarrow \infty}$$

$$\tilde{k}_{t+1} = k_{t+1}^*$$

□

## 4 Numerical solution and simulation.

$$\theta_t \in \{\theta_L, \theta_H\} \quad \theta_H > \theta_L$$

$$\begin{aligned} \theta_{t+1} &= \theta_L & \theta_{t+1} &= \theta_H \\ \theta_t = \theta_L & \rho & 1 - \rho \\ \theta_t = \theta_H & 1 - \rho & \rho \end{aligned}$$

$$0.5 < \rho < 1$$

$$\begin{aligned} \gamma &= 0.95 & \alpha &= 0.6 & R &= 1.01 & p_t &= p_t^i = p_{t+1}^i = 1 \\ \delta &= 0.7 & \tau &= 0.3 & \rho &= 0.7 & \eta &= 0.81 \\ \theta_L &= 1.7 & \theta_H &= 2.3 \end{aligned}$$

$\eta$

$\eta$

$$\begin{aligned} k_{t+1}^{PM} &= k_{t+1}^* & k_{t+1}^{PM} & & k_{t+1}^{PM} & & k_{t+1}^o < k_{t+1}^{PM} \\ k_{t+1}^{PM} & & \theta_{t+1} &= \theta_H & & & \theta_{t+1} = \theta_H \end{aligned}$$

$$\theta_{t+1} = \theta_L$$

$$\tilde{k}_{t+1}(w_t | \theta_t = \theta_H) \quad \tilde{k}_{t+1}(w_t | \theta_t = \theta_L)$$

$$\tilde{k}_{t+1} = S_t$$

$$1 / \left( p_t^i - \frac{\tau}{R} p_{t+1}^i \right)$$

$$\tilde{k}_{t+1} = k_{t+1}^o < k_{t+1}^{PM}$$

$$\tilde{k}_{t+1} < k_{t+1}^{PM}$$

$$\theta_t$$

$\theta$

$$\theta_t$$

### 4.1 Simulation: single firm.

$$\tilde{k}_t$$

$\theta_H$

$\theta_L$

$\theta_{it}$

$w_0$

$$\tilde{k}_t(w_t, \theta_t) \simeq k_t^{PM}(\theta_t) \quad \forall t$$

$w_0$

### 4.2 Simulation: aggregate.

$\theta_{it}$

$$E(\theta_{t+1}) = (\theta_H + \theta_L) / 2 = \bar{\theta}$$

$\bar{\theta}$

$\bar{\theta}$

$$\text{Type 1: } \bar{\theta}^1 = 2 \quad \theta_L^1 = 1.7 \quad \theta_H^1 = 2.3 \quad k^{PM1}(\theta_L^1) = 14 \quad k^{PM1}(\theta_H^1) = 18.9$$

$$\text{Type 2: } \bar{\theta}^2 = 2.25 \quad \theta_L^2 = 1.9125 \quad \theta_H^2 = 2.5875 \quad k^{PM2}(\theta_L^2) = 18.8 \quad k^{PM1}(\theta_H^2) = 25.4$$

$\bar{\theta}$

$$\frac{(\theta_H^1 - \theta_L^1)}{\bar{\theta}^1} = \frac{(\theta_H^2 - \theta_L^2)}{\bar{\theta}^2}$$

$\bar{L}$

$$\alpha + \beta \geq 1$$

□

$$w_0 = \frac{k^{PMz} (\theta_H^z)}{\left(p_t^i - \frac{\tau}{R} p_{t+1}^i\right)}$$

$z \in \{1, 2\}$

$w_0$

*cumulative rate of growth of capital*

*cumulative rate*

*of growth of the capital/sales ratio*

... ..

$\tau$

... ..

... ..  $k_{t+1}^o$  ... ..

$U_t$

... ..  $\tilde{k}_{t+1}$  ... ..  $S_t$  ... ..

$P_t^i - \frac{\tau}{R} P_{t+1}^i$

... ..  $\tilde{k}_{t+1}$  ... ..

... ..  $k_{t+1}^{PM}$  ... ..  $k_{t+1}^*$  ... ..

... ..



the first part of the paper, we have seen how the theory of the firm can be used to explain the existence of the firm. In the second part, we have seen how the theory of the firm can be used to explain the structure of the firm. In the third part, we have seen how the theory of the firm can be used to explain the behavior of the firm. In the fourth part, we have seen how the theory of the firm can be used to explain the growth of the firm. In the fifth part, we have seen how the theory of the firm can be used to explain the failure of the firm.

## 5 Conclusions

In this paper, we have seen how the theory of the firm can be used to explain the existence, structure, behavior, growth, and failure of the firm. The theory of the firm is a powerful tool for understanding the firm and its role in the economy. It is a theory that is constantly being refined and improved, and it is a theory that is essential for understanding the firm and its role in the economy.

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## References

1. *The Theory of the Firm*, by Oliver E. Williamson, Harvard University Press, 1985.

2. *The Theory of the Firm*, by Ronald S. Gertler, Cambridge University Press, 1995.





Figure 1  
Policy functions

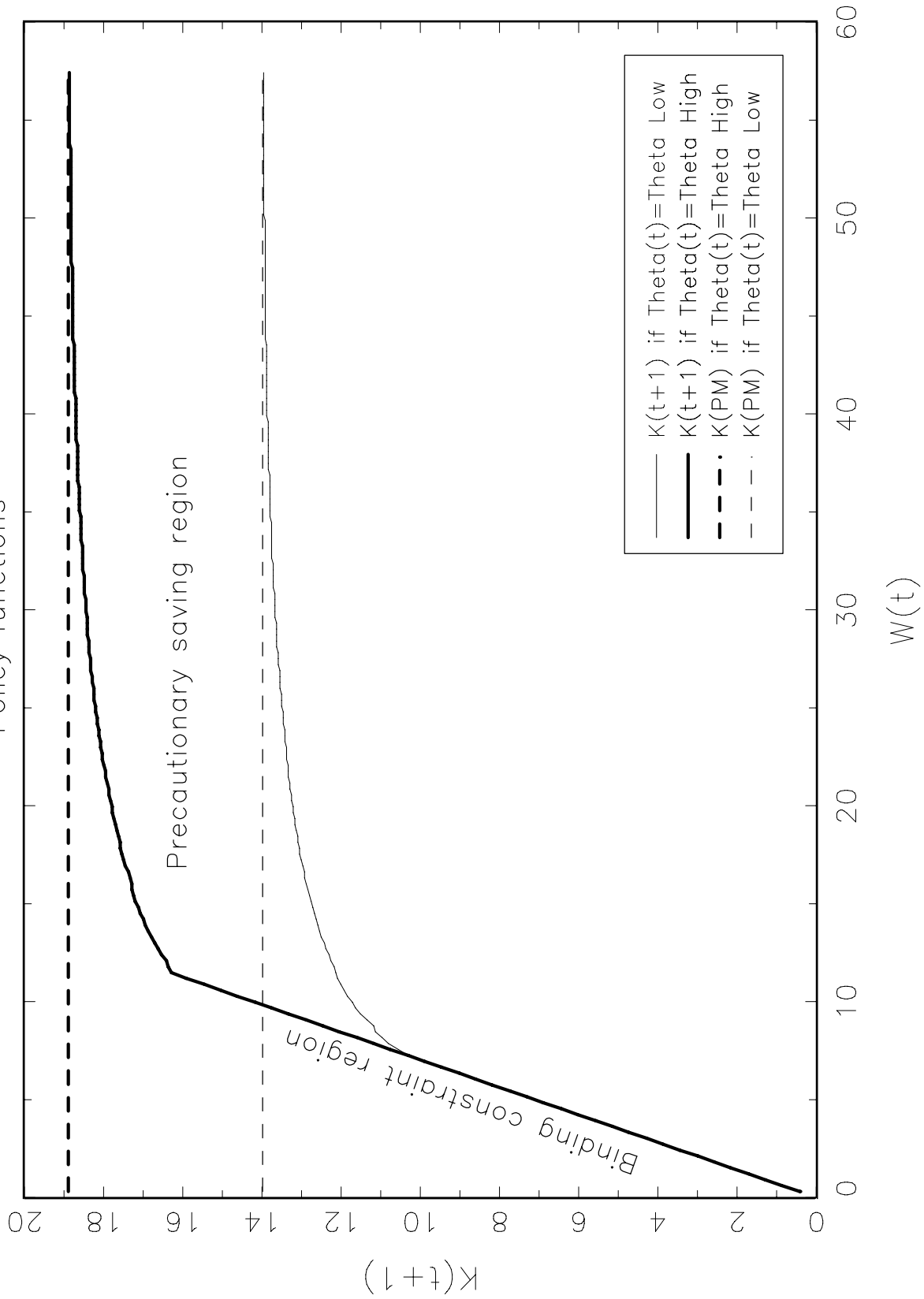
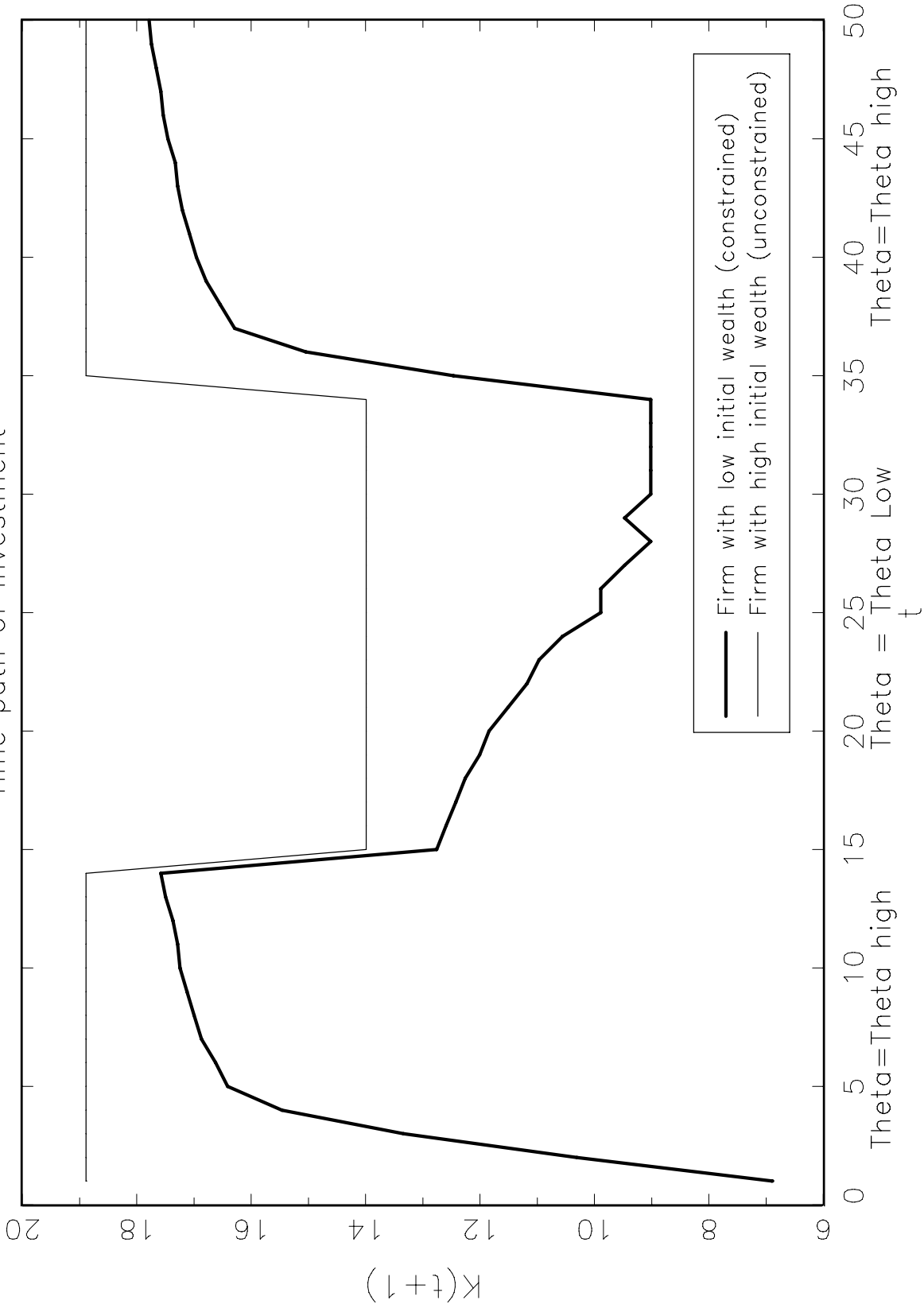


Figure 2  
Time path of investment



Figure

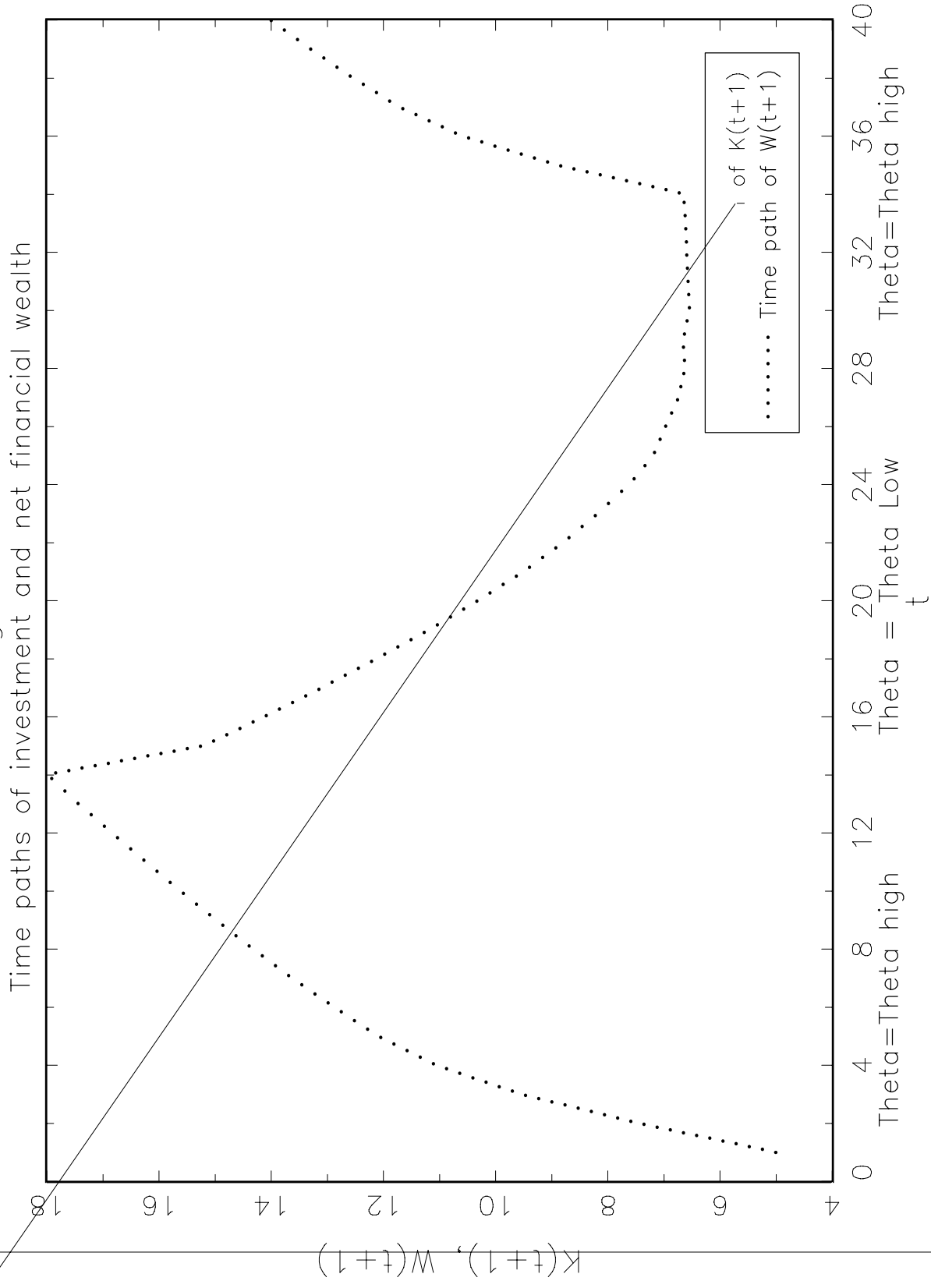


Figure 4

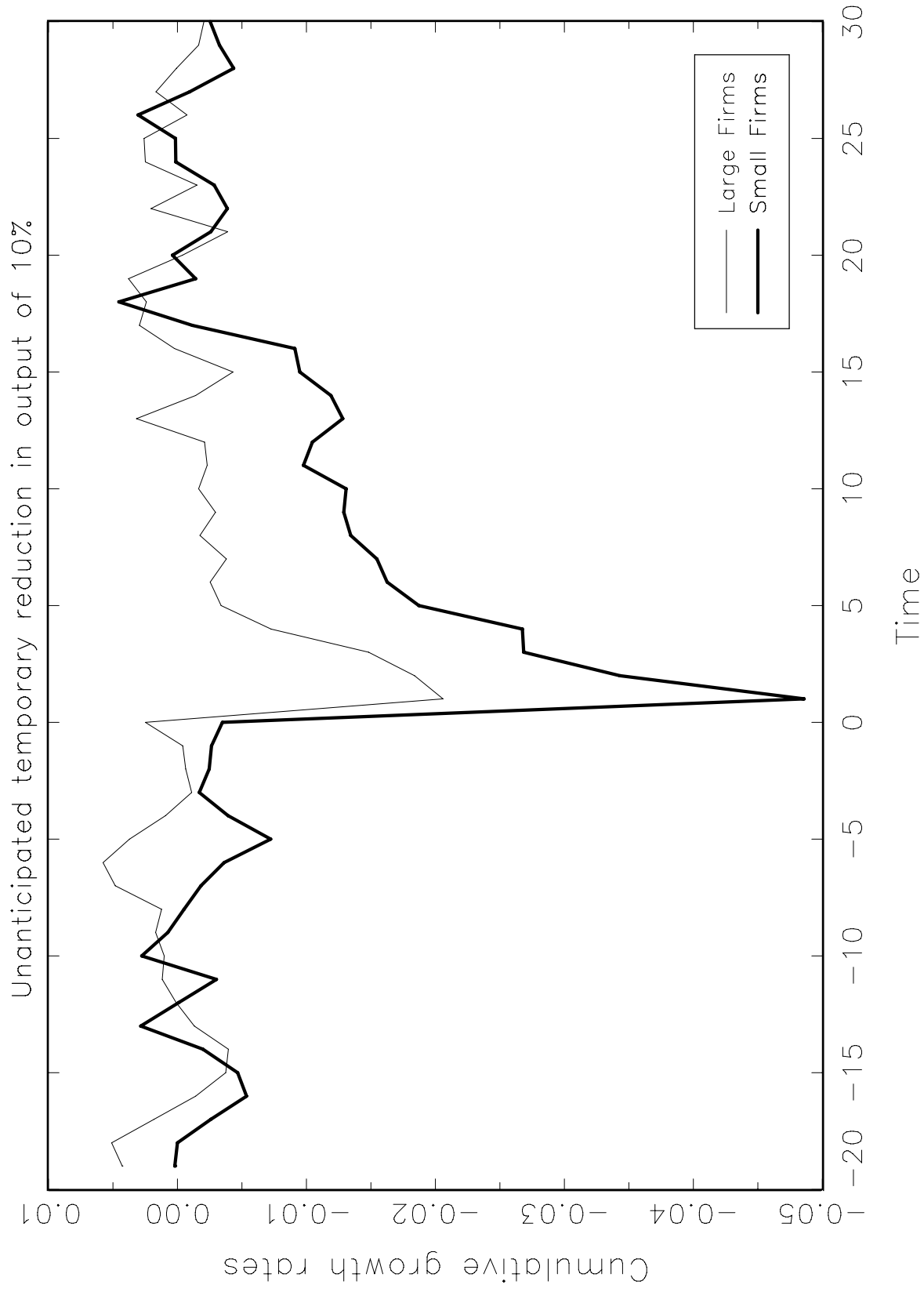


Figure 5

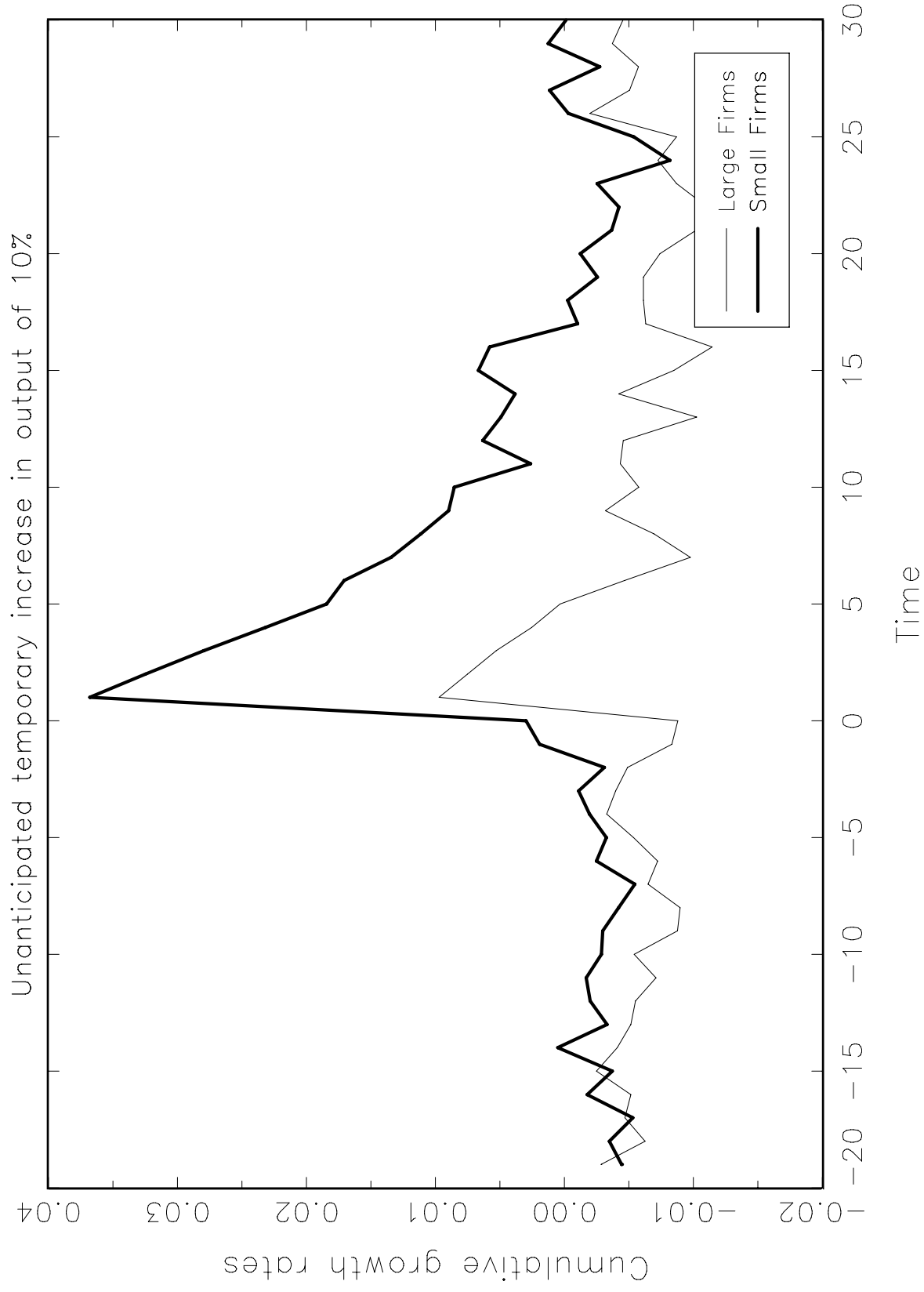




Figure 6

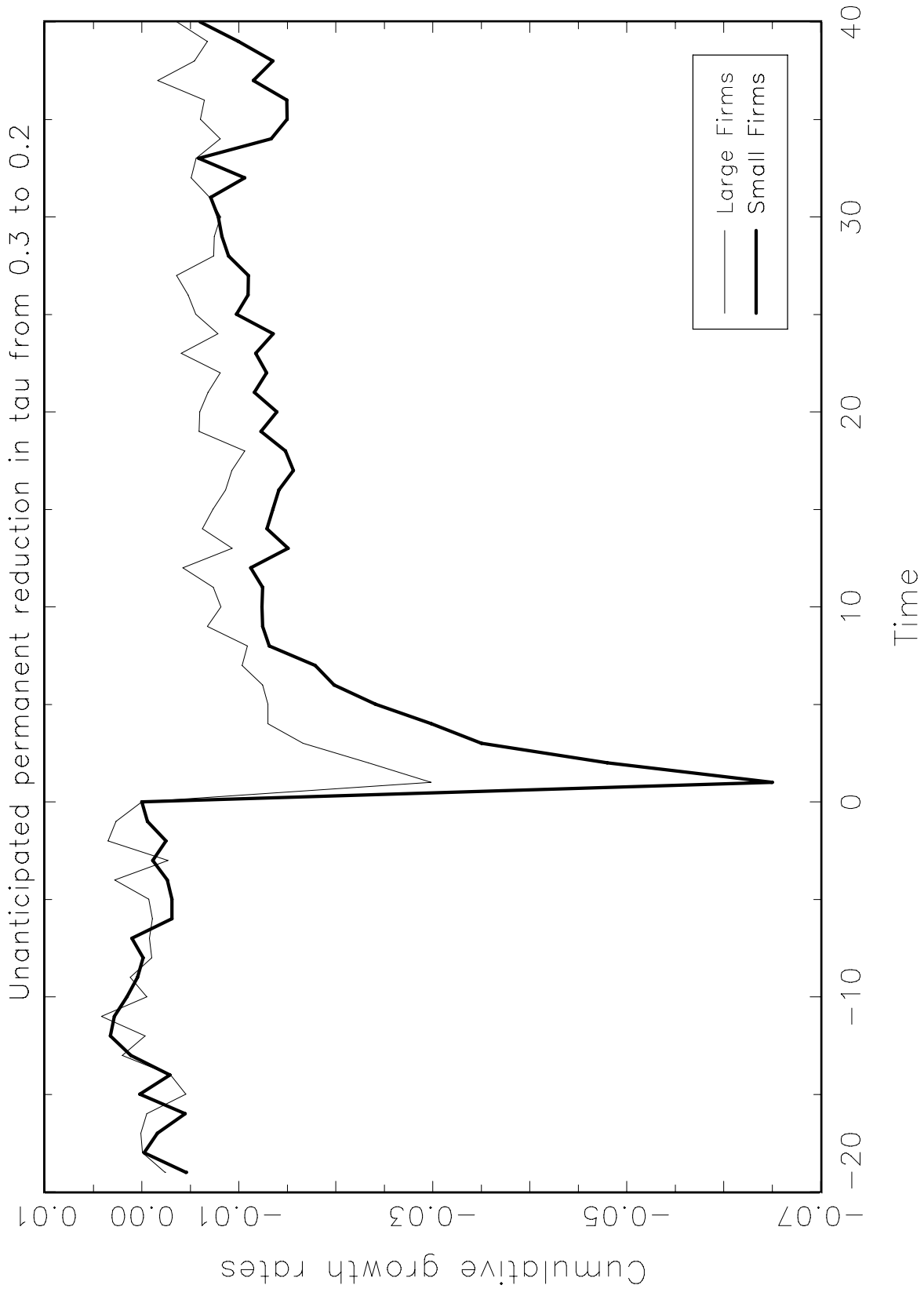


Figure 7

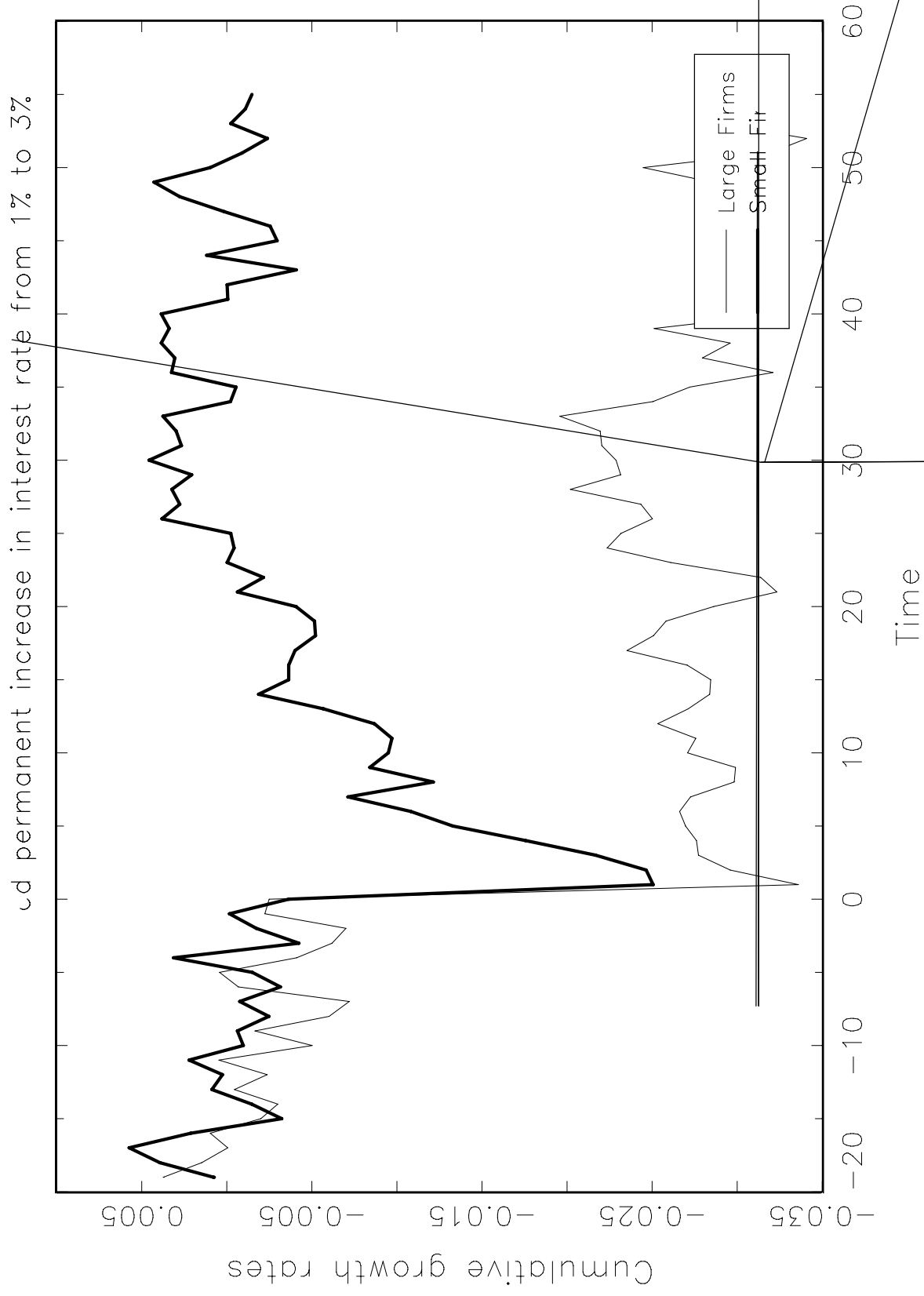


Figure 8

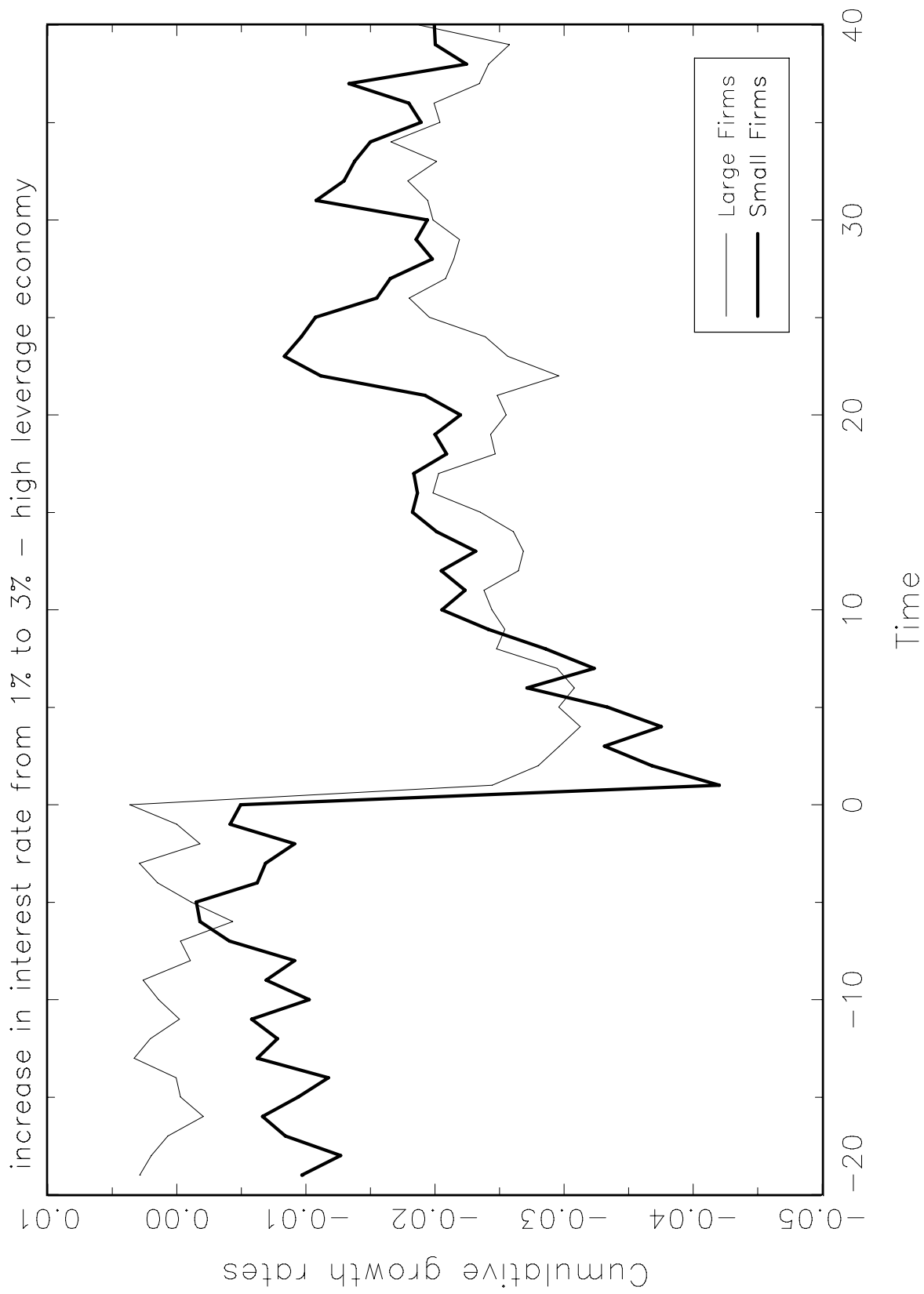


Figure 9

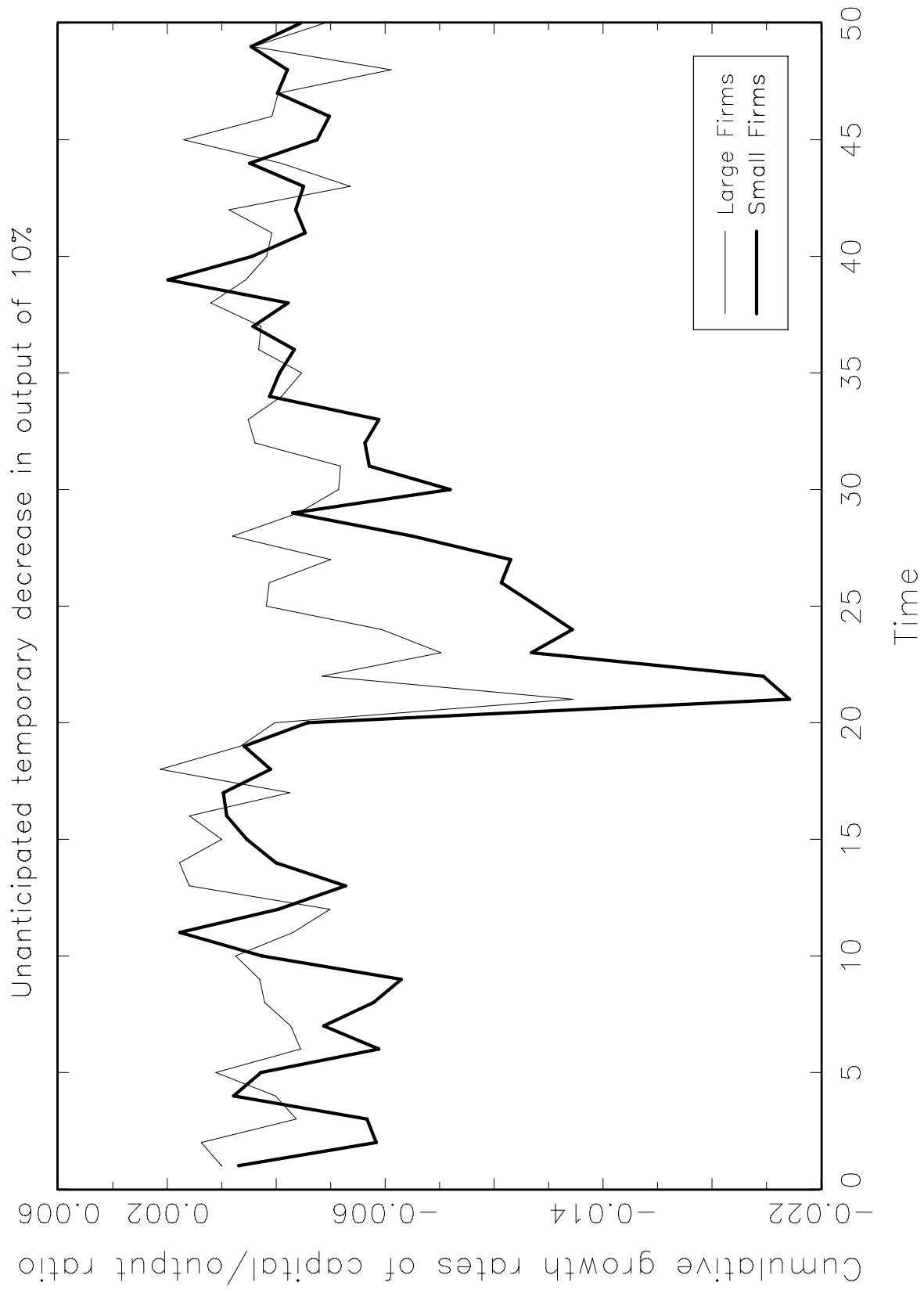
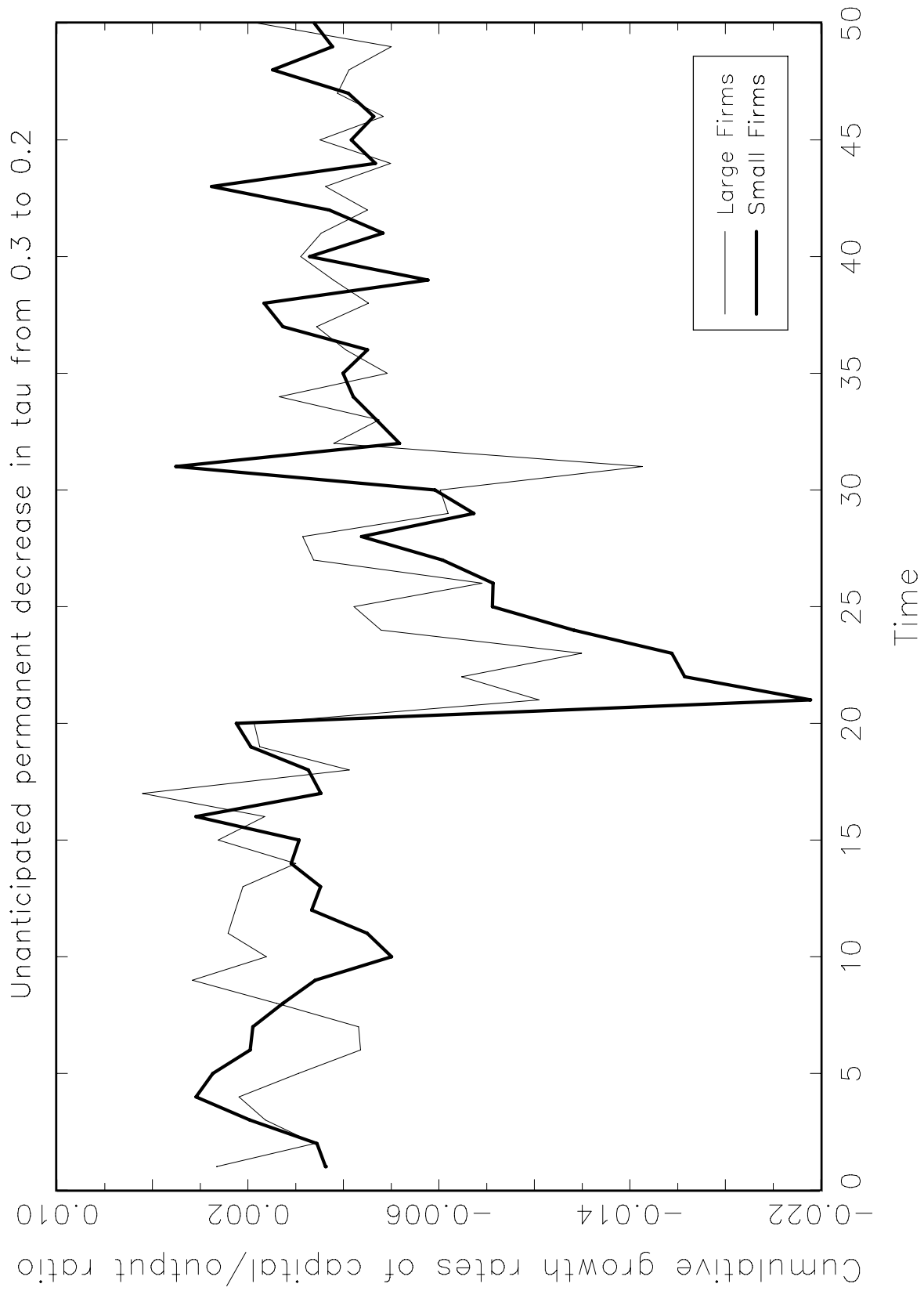
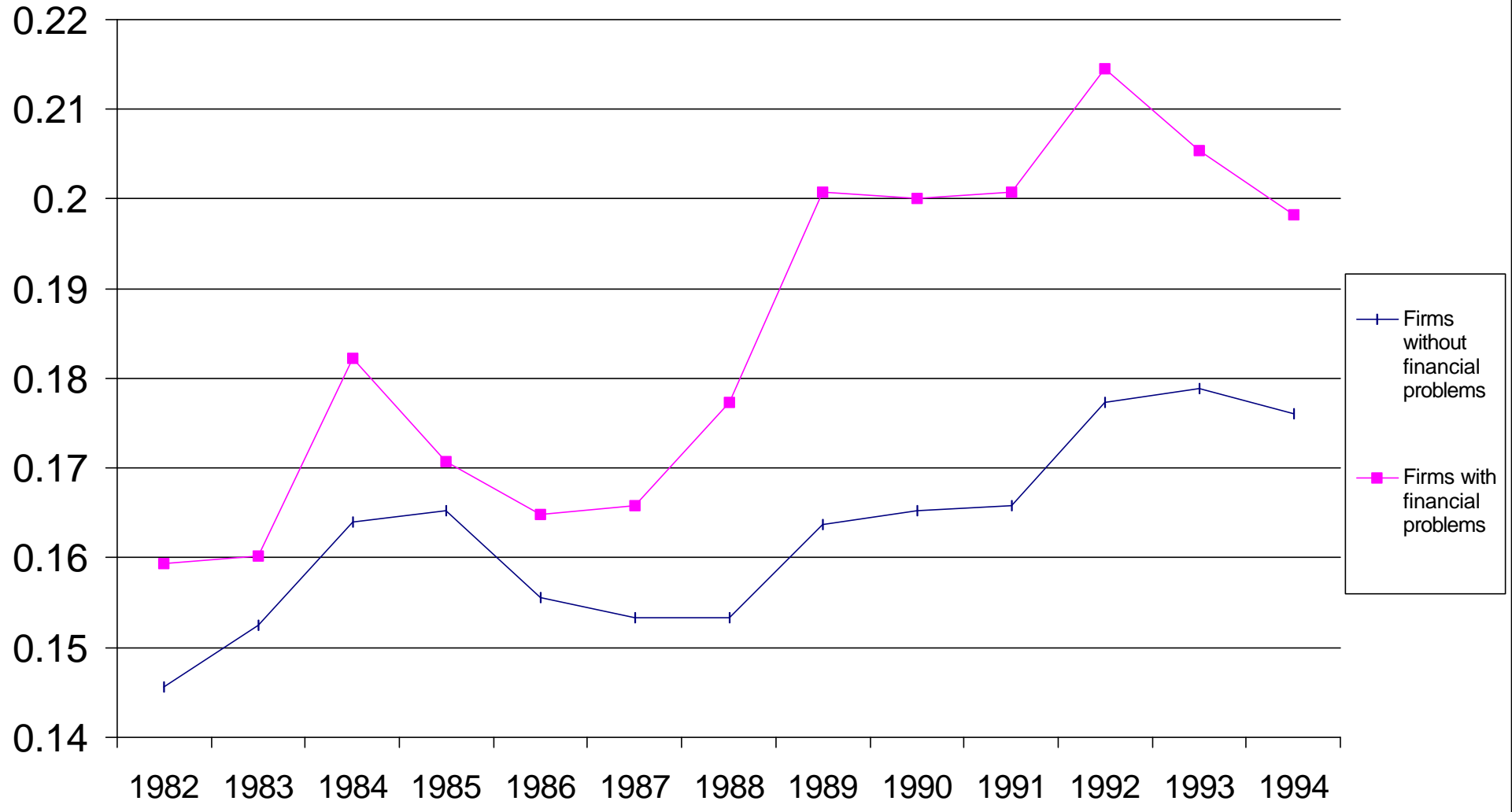


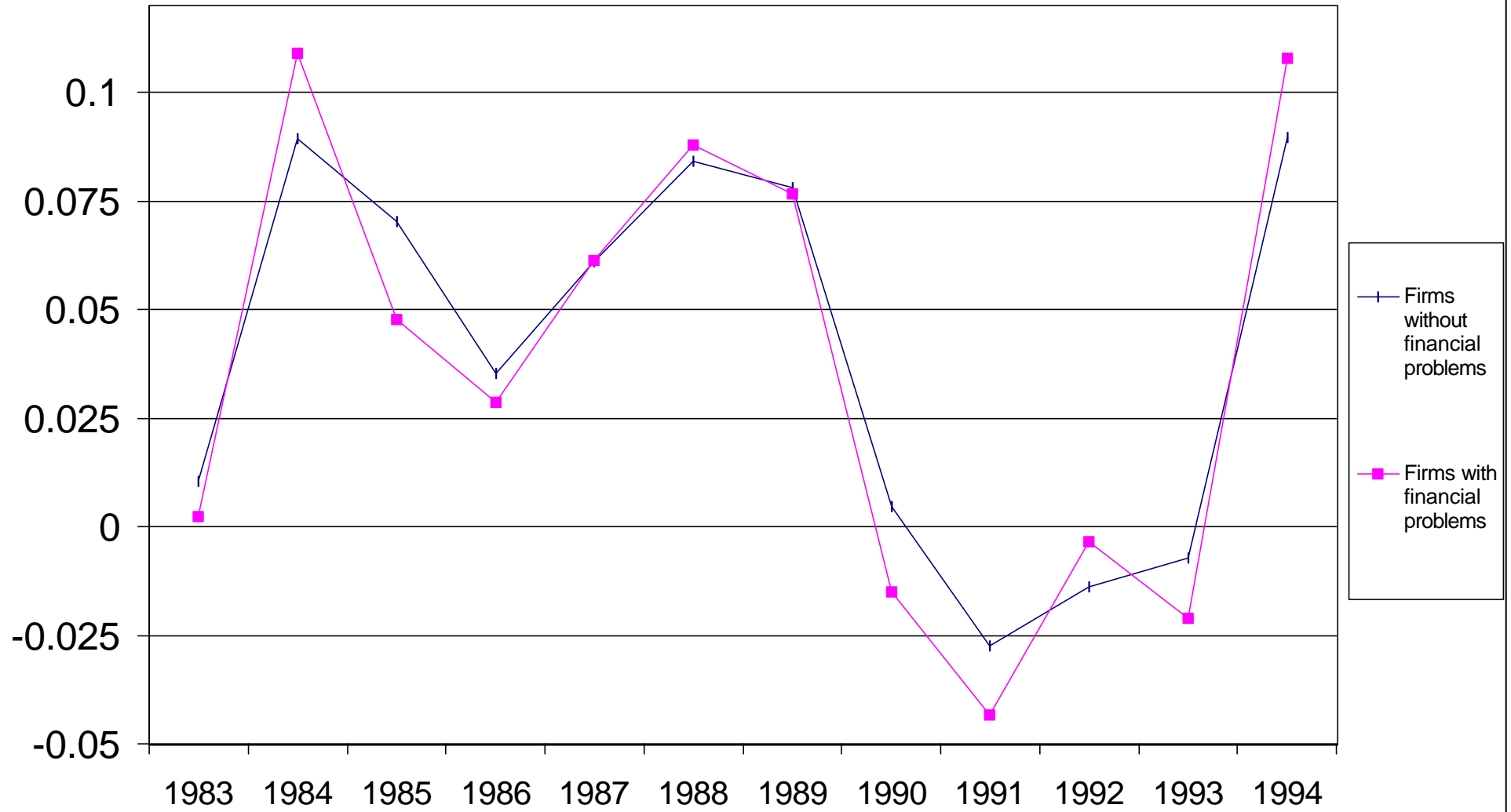
Figure 10



# Figure 1A: Short Term Bank Loans Over Total Assets



### Figure 2A: Real Rate of Growth of Total Sales



# Figure 3A: Real Rate of Growth of Total Sales

(only firms with positive income in years '89,'90 and '91 included)

