Financial Constraints, Precautionary Saving and Firm Dynamics

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Abstract

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1 Introduction

2 Evidence of the effects of financing constraints.

3 A model of investment with private income and collateral constraint.

θ

$$y_t^T = p_t \theta_t k_t^{\alpha}$$

$$y_t^{NT} = \eta y_t^T$$

$$0 < \alpha < 1; \eta > 0$$

k p y^T y^{NT} η

γ θ

$1-\gamma$ a_t

ra_t

$$a_t = (1+\eta)y_t^T + p_t^i(1-\delta)k_t - b_t$$

$$w_t = a_t - y_t^{NT}$$

$$0 < \delta < 1$$

a_t w_t *total* *financial* p^i δ b_t t

$$V_t\left(w_t, k_t, \theta_t\right) = \underset{k_{t+1}, b_{t+1}}{MAX}\ \gamma E_t\left\{x_t + \frac{1}{R}E_t\left[V_{t+1}(w_{t+1}, k_{t+1}, \theta_{t+1})\right]\right\} + (1-\gamma)a_t$$

$$x_t + k_{t+1} = a_t + \frac{b_{t+1}}{R}$$

$$x_t \geq y_t^{NT}$$

$$b_{t+1} \leq \tau p_{t+1}^I k_{t+1}$$

$$0 < \tau \leq 1-\delta; 0 < \gamma < 1$$

x_t t k_{t+1} b_{t+1} $R = 1 + r$ r τ w_t, a_t,

$$V_0\left(w_0, k_0, \theta_0\right) = \underset{\{k_{t+1}\}_{t=0}^{\infty}, \{b_{t+1}\}_{t=0}^{\infty}}{MAX}\ E_t\left\{\sum_{t=0}^{\infty}\left(\frac{\gamma}{R}\right)^t\left\{\gamma x_t + (1-\gamma)a_t + \mu_t\left(x_t - y_t^{NT}\right) + \lambda_t\left[\tau p_{t+1}^I k_{t+1} - b_{t+1}\right]\right\}\right.$$

---

t $i_t = k_{t+1} - (1-\delta)k_t$ i_t k_{t+1}

μ_t

λ_t $x_t,$ t

b_{t+1}

$$\mu_t = R\lambda_t + \gamma E_t\left(\mu_{t+1}\right)$$

$$\frac{\mu_t}{R} = E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+j}\right)$$

$\mu_t > 0$ $E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+j}\right) > 0$

$E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+j}\right) > 0$

$x_t = y_t^{NT}$ t

$t-1.$

after $y_0^{NT},$ k_1 b_1

w_0 θ_0

$$V_0\left(w_0,\theta_0\right) = \underset{\{k_{t+1}\}_{t=0}^{\infty},\{b_{t+1}\}_{t=0}^{\infty}}{MAX} E_0\left\{\sum_{t=0}^{\infty}\left(\frac{\gamma}{R}\right)^t\left\{\frac{1}{R}\left[\gamma y_{t+1}^T + (1-\gamma)a_{t+1}\right] + \lambda_t\left[\tau p_{t+1}^I k_{t+1} - b_{t+1}\right]\right\}\right\}$$

$$p_t^i k_{t+1} = w_t + \tfrac{b_{t+1}}{R} \quad \forall t \geq 0$$

θ_t

3.1 The precautionary saving effect.

total *tradable*

ϕ_t b_{t+1} k_{t+1},

$$\phi_t = RE_t\left[\sum_{j=0}^{\infty}\gamma^j\lambda_{t+j}\right] + 1$$

$$\lambda_t\tau p^i_{t+1} - \phi_t p^i_t + \frac{1}{R}\left(1+\eta-\gamma\right)E_t\left(MPK_{t+1}\right) + \frac{1}{R}\left(1-\gamma\right)\left(1-\delta\right)p^i_{t+1} +$$

$$+\frac{\gamma}{R}E_t\left(\phi_{t+1}\right)\left[E_t\left(MPK_{t+1}\right)+\left(1-\delta\right)p^i_{t+1}\right] + \Psi_{t+1} = 0$$

$E_t\left(MPK_{t+1}\right) = \alpha p_{t+1}E_t\left(\theta_{t+1}\right)k_{t+1}^{\alpha-1}$ *tradable*

k_{t+1}

$$\frac{(1+\eta)}{R}E_t\left(MPK_{t+1}\right) = \frac{U_t}{R} + \left[Rp^i_t - \tau p^i_{t+1}\right]\lambda_t + \gamma\left[U_t - E_t\left(MPK_{t+1}\right)\right]E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right) - \Psi_{t+1}$$

$$S_t \geq k_{t+1}$$

$$\lambda_t \geq 0$$

$$[S_t - k_{t+1}]\,\lambda_t = 0$$

$U_t = Rp_t^i - p_{t+1}^i\,(1-\delta)$ $S_t = w_t/[p_t^i - \frac{\tau}{R}p_{t+1}^i]$

Ψ_{t+1} $\Psi_{t+1} = \frac{\alpha\gamma}{R} p_{t+1} k_{t+1}^{\alpha-1} cov\left(\phi_{t+1}, \theta_{t+1}\right)$

k_{t+1}

$\lambda_t = 0\ \forall t \geq 0.$

$$(1+\eta)\,E_t\,(MPK_{t+1}) = U_t$$

$(1+\eta)\,E_t\,(MPK_{t+1})$ *total* $k_{t+1}^{PM}(\theta_t)$

k_{t+1}^{PM}

$$k_{t+1}^{PM} = \left[\frac{(1+\eta)\,\alpha p_{t+1} E_t\,(\theta_{t+1})}{U_t}\right]^{\frac{1}{1-\alpha}}$$

Proposition 1 *Conditional on continuation, a level of investment of* k_{t+1}^{PM} *generates negative expected marginal tradable profits.*

$\eta > 0$ $E_t\,(MPK_{t+1}) < U_t$

$k_{t+1}^{*}(\theta_t)$

$E_t\,(MPK_{t+1}) = U_t$

$$k_{t+1}^{*} = \left[\frac{\alpha p_{t+1} E_t\,(\theta_{t+1})}{U_t}\right]^{\frac{1}{1-\alpha}}$$

$k_{t+1}^{PM} > k_{t+1}^{*}$ $k_{t+1}^{PM} - k_{t+1}^{*}$ η

$U_t - E_t\,(MPK_{t+1}) > 0$ k_{t+1}^{PM}

$\lambda_t = 0$

$E_t\left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j}\right) > 0.$

$$\frac{(1+\eta)}{R} E_t\,(MPK_{t+1}) = \frac{U_t}{R} + \gamma\,[U_t - E_t\,(MPK_{t+1})]\,E_t\left(\sum_{j=0}^{\infty} \gamma^j \lambda_{t+1+j}\right) - \Psi_{t+1}$$

---

w_t

k_{t+1}^{PM}

$\gamma\left[U_t - E_t\left(MPK_{t+1}\right)\right] E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right)$.

$U_t - E_t\left(MPK_{t+1}\right) > 0$

$\left(E_t\left(\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right)\right)$

$k_{t+1}^{o}\left(w_t, \theta_t\right)$ $k_{t+1}^{o}\left(w_t, \theta_t\right)$

w_t θ_t,

$k_{t+1}^{*} < k_{t+1}^{o} < k_{t+1}^{PM}$ $E_t\left[\sum_{j=0}^{\infty}\gamma^j\lambda_{t+1+j}\right]$

k_{t+1}^{o}

k_{t+1}^{*}

k_{t+1}^{*}

$\widetilde{k}_{t+1}\left(w_t, \theta_t\right)$

$$\widetilde{k}_{t+1}\left(w_t, \theta_t\right) = \begin{cases} k_{t+1}^{o}\left(w_t, \theta_t\right) & k_{t+1}^{o}\left(w_t, \theta_t\right) < S_t\left(w_t, \theta_t\right) \\ S_t\left(w_t, \theta_t\right) & \end{cases}$$

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k_{t+1}^{o} k_{t+1}^{PM} k_{t+1}^{o}

w_t $E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+j+1}\right\} \to 0$ $\Psi_{t+1} \to 0$ $\lim_{E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+1+j}\right\}\to 0} k_{t+1}^{o} = k_{t+1}^{PM}$

$E_t\left\{\sum_{j=0}^{\infty}\lambda_{t+1+j}\right\} \to \infty$ $E_t\left(L_{t+1}\right) = 0$ $\lim_{E_t\left\{\sum_{j=1}^{\infty}\lambda_{t+j}\right\}\to\infty}$

$\widetilde{k}_{t+1} = k_{t+1}^{*}$.

4 Numerical solution and simulation.

θ $\theta_t \in \{\theta_L, \theta_H\}$ $\theta_H > \theta_L$

	$\theta_{t+1} = \theta_L$	$\theta_{t+1} = \theta_H$
$\theta_t = \theta_L$	ρ	$1 - \rho$
$\theta_t = \theta_H$	$1 - \rho$	ρ

$.5 < \rho < 1$

$$\gamma = 0.95 \quad \alpha = 0.6 \quad R = 1.01 \quad p_t = p_t^i = p_{t+1}^i = 1$$
$$\delta = 0.7 \quad \tau = 0.3 \quad \rho = 0.7 \quad \eta = 0.81$$
$$\theta_L = 1.7 \quad \theta_H = 2.3$$

δ η η $k_{t+1}^{PM} = k_{t+1}^*$ k_{t+1}^{PM} k_{t+1}^{PM} $\theta_{t+1} = \theta_H$. $k_{t+1}^o < k_{t+1}^{PM}$ $\theta_{t+1} = \theta_H$ $\theta_{t+1} = \theta_L$ $\widetilde{k}_{t+1}(w_t \mid \theta_t = \theta_H)$ $\widetilde{k}_{t+1}(w_t \mid \theta_t = \theta_L)$ w_t $\widetilde{k}_{t+1} = S_t$ $1/\left(p_t^i - \frac{\tau}{R} p_{t+1}^i\right)$ $\widetilde{k}_{t+1} = k_{t+1}^o < k_{t+1}^{PM}$. $\widetilde{k}_{t+1}$ k_{t+1}^{PM} θ_t θ θ_t

4.1 Simulation: single firm.

$\widetilde{k}_t$ θ_H θ_L

θ_H w_0

w_0

$\widetilde{k}_t\left(w_t,\theta_t\right) \simeq k_t^{PM}\left(\theta_t\right)\ \forall t$

w_0

[illegible]

4.2 Simulation: aggregate.

[illegible]

θ_{it}

[illegible]

$E(\theta_{t+1}) = (\theta_H + \theta_L)/2 = \overline{\theta}$

$\overline{\theta}$

$\overline{\theta}$.

$$\textit{Type1:}\ \overline{\theta}^1 = 2 \qquad \theta_L^1 = 1.7 \qquad \theta_H^1 = 2.3 \qquad k^{PM1}\left(\theta_L^1\right) = 14 \qquad k^{PM1}\left(\theta_H^1\right) = 18.9$$

$$\textit{Type 2:}\ \overline{\theta}^2 = 2.25 \quad \theta_L^2 = 1.9125 \quad \theta_H^2 = 2.5875 \quad k^{PM2}\left(\theta_L^2\right) = 18.8 \quad k^{PM1}\left(\theta_H^2\right) = 25.4$$

$\overline{\theta}$

$$\frac{\left(\theta_H^1 - \theta_L^1\right)}{\overline{\theta}^1} = \frac{\left(\theta_H^2 - \theta_L^2\right)}{\overline{\theta}^2}$$

$\theta_t = \widehat{\theta}_t \overline{L}^{\beta}$ $\widehat{\theta}_t$ $\overline{L}$

$\alpha + \beta \geq 1$

$$w_0 = \frac{k^{PM_z}\left(\theta_H^z\right)}{\left(p_t^i - \frac{\tau}{R}p_{t+1}^i\right)}$$

$z \in \{1,2\}$

w_0

cumulative rate of growth of capital

cumulative rate of growth of the capital/sales ratio

τ

τ

k^o_{t+1} U_t

$\widetilde{k}_{t+1}$

S_t $p^i_t - \frac{\tau}{R} p^i_{t+1}$

$\widetilde{k}_{t+1}$

k^{PM}_{t+1} k^*_{t+1}

5 Conclusions

References

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Figure 1
Policy functions

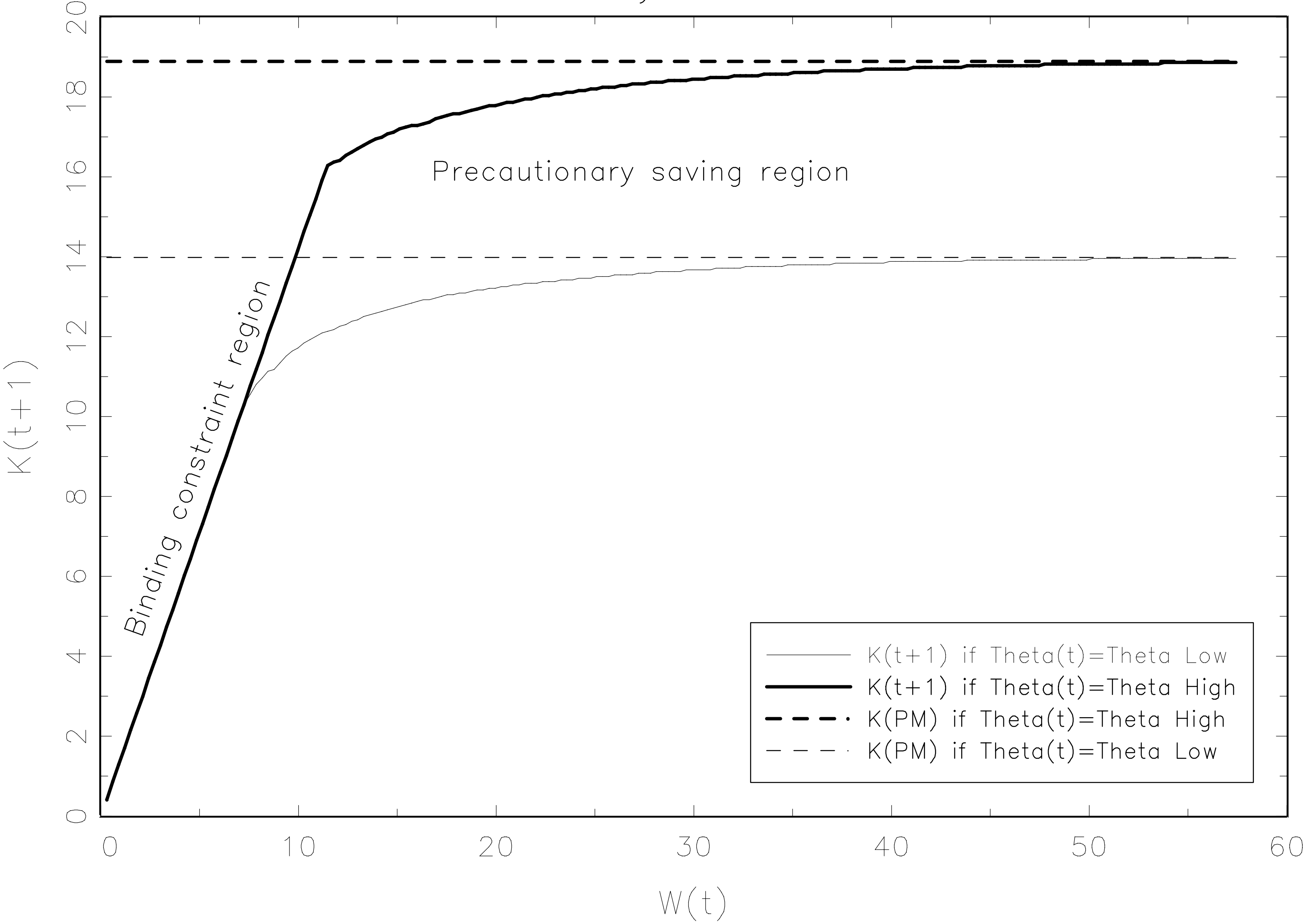
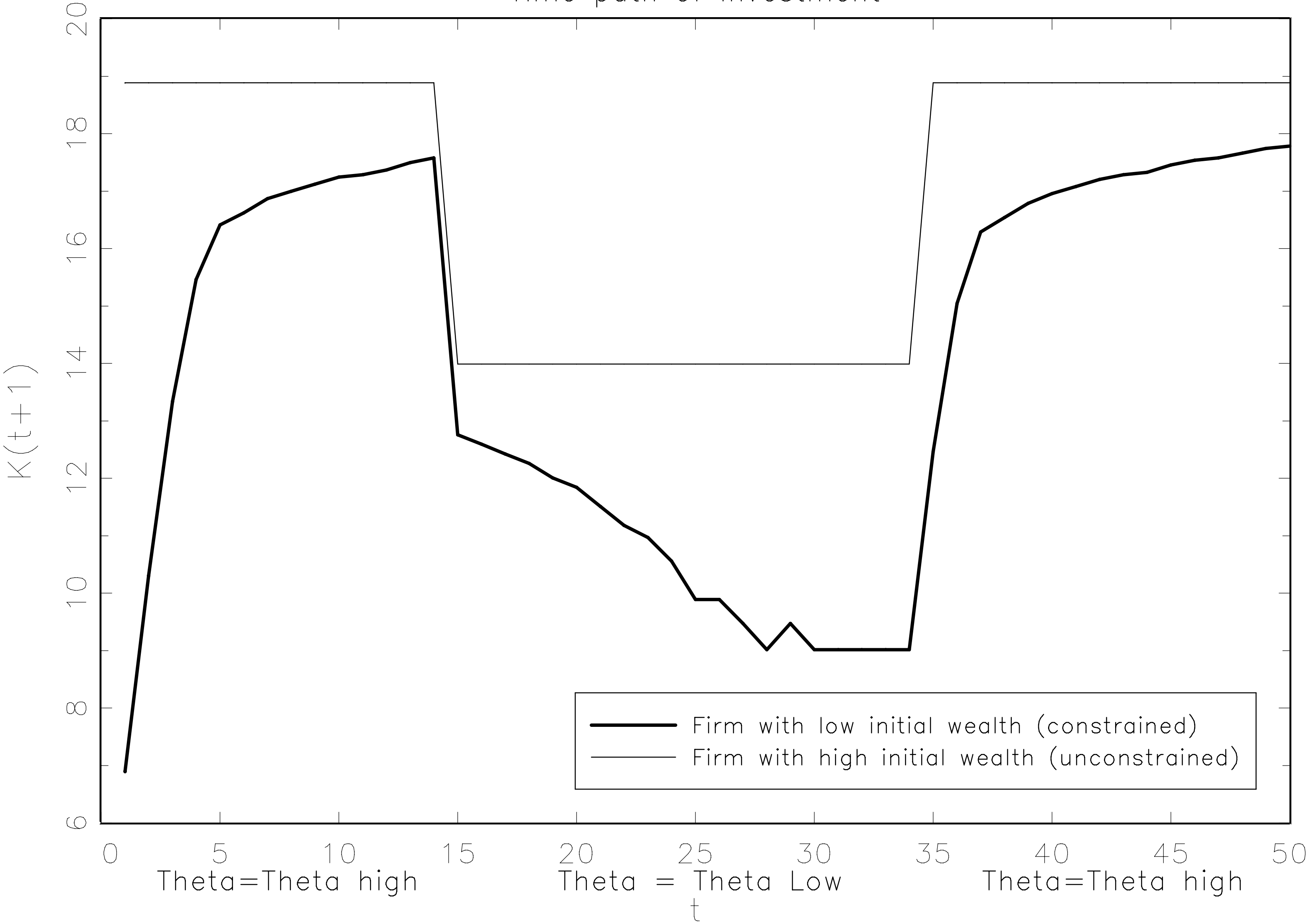


Figure 2

Time path of investment



Figure

Time paths of investment and net financial wealth

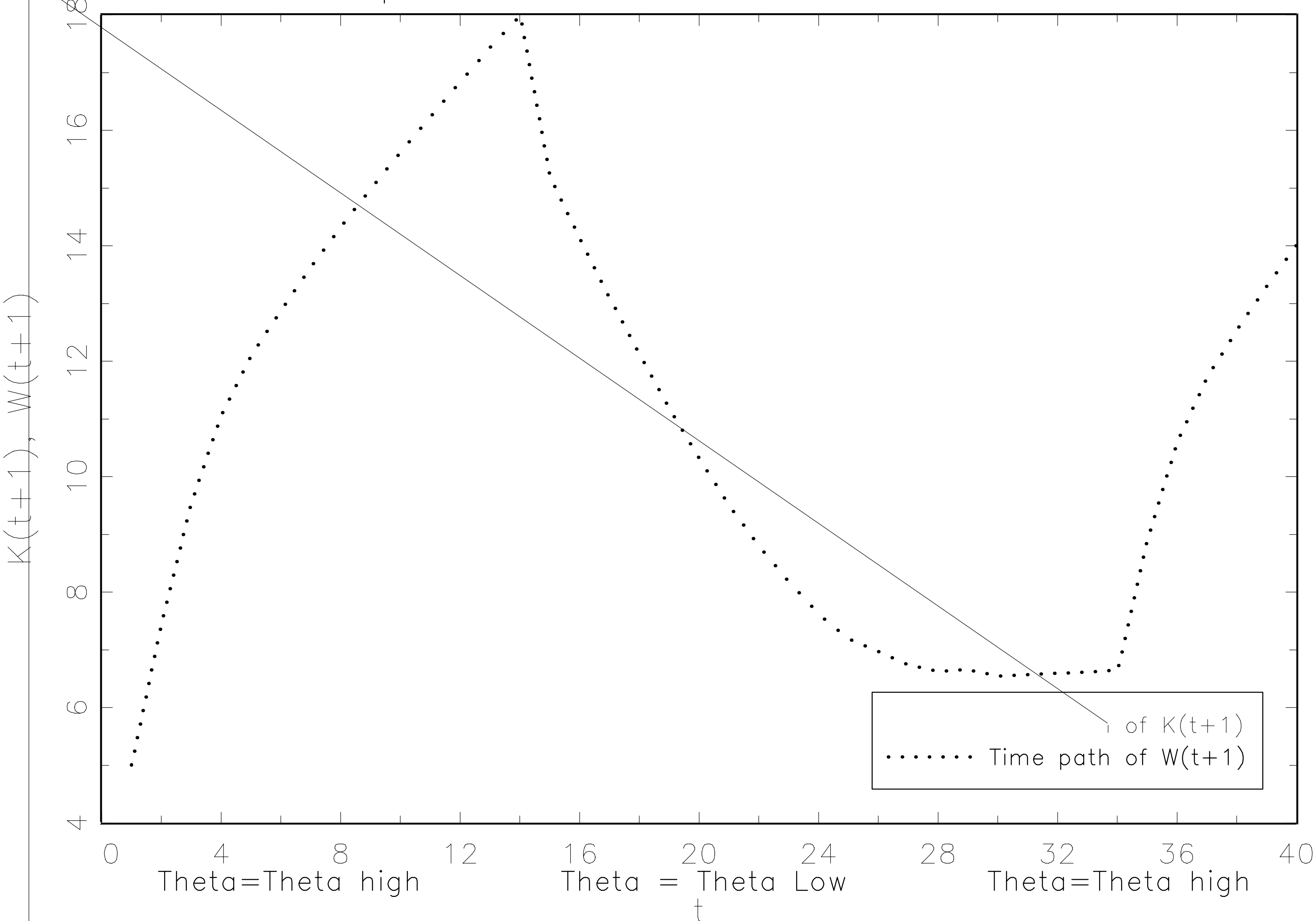


Figure 4

Unanticipated temporary reduction in output of 10%

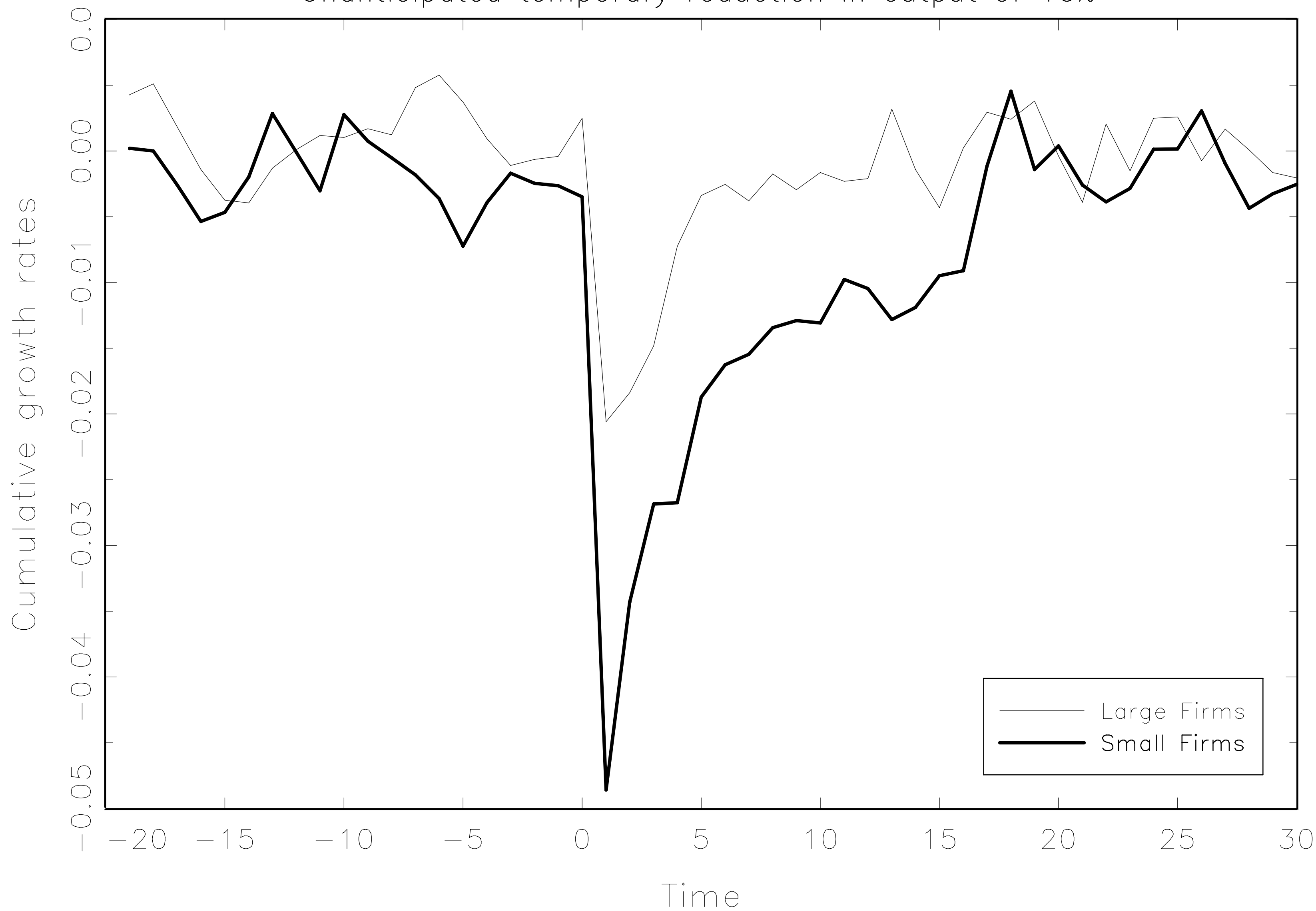


Figure 5

Unanticipated temporary increase in output of 10%

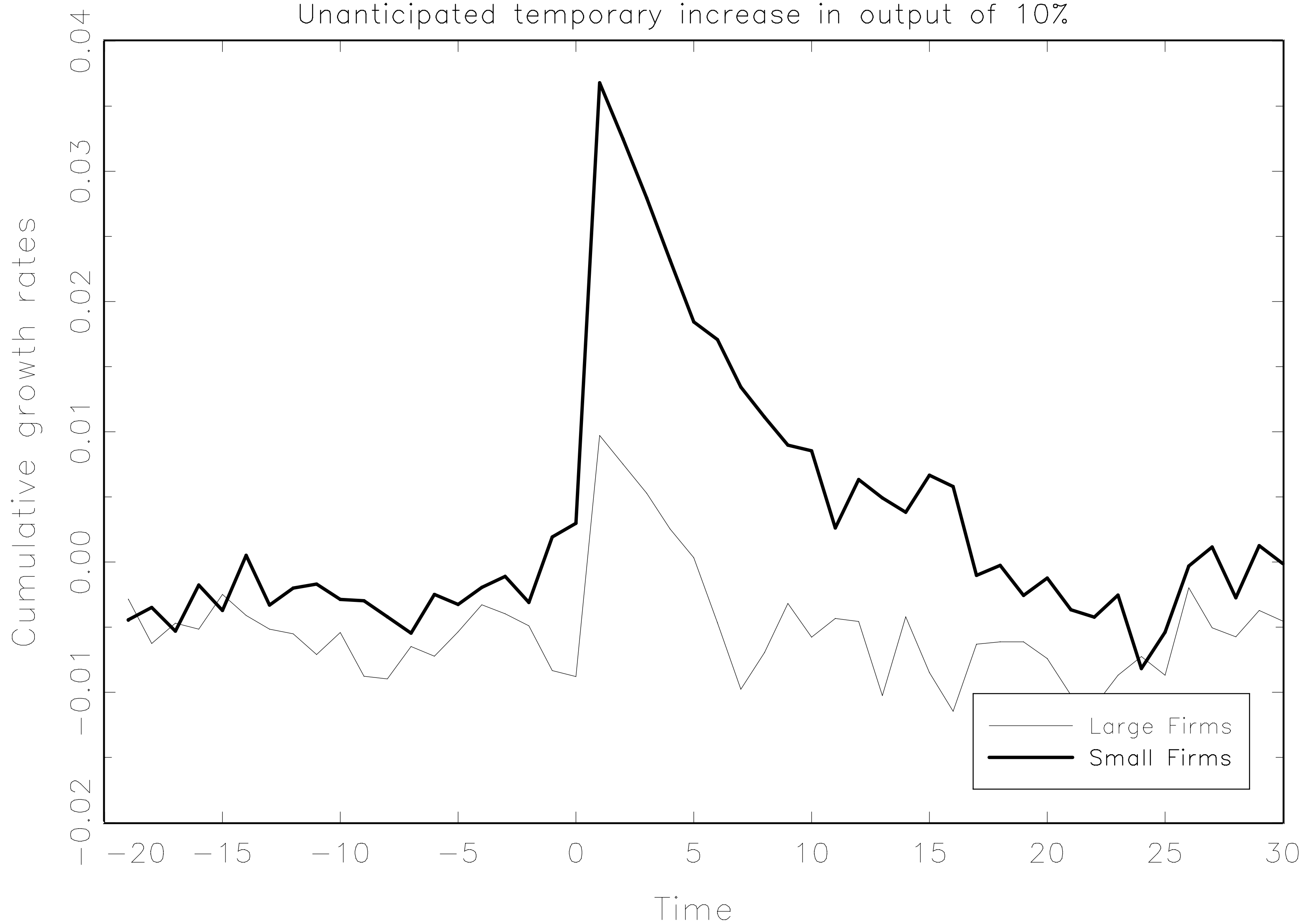


Figure 6

Unanticipated permanent reduction in tau from 0.3 to 0.2

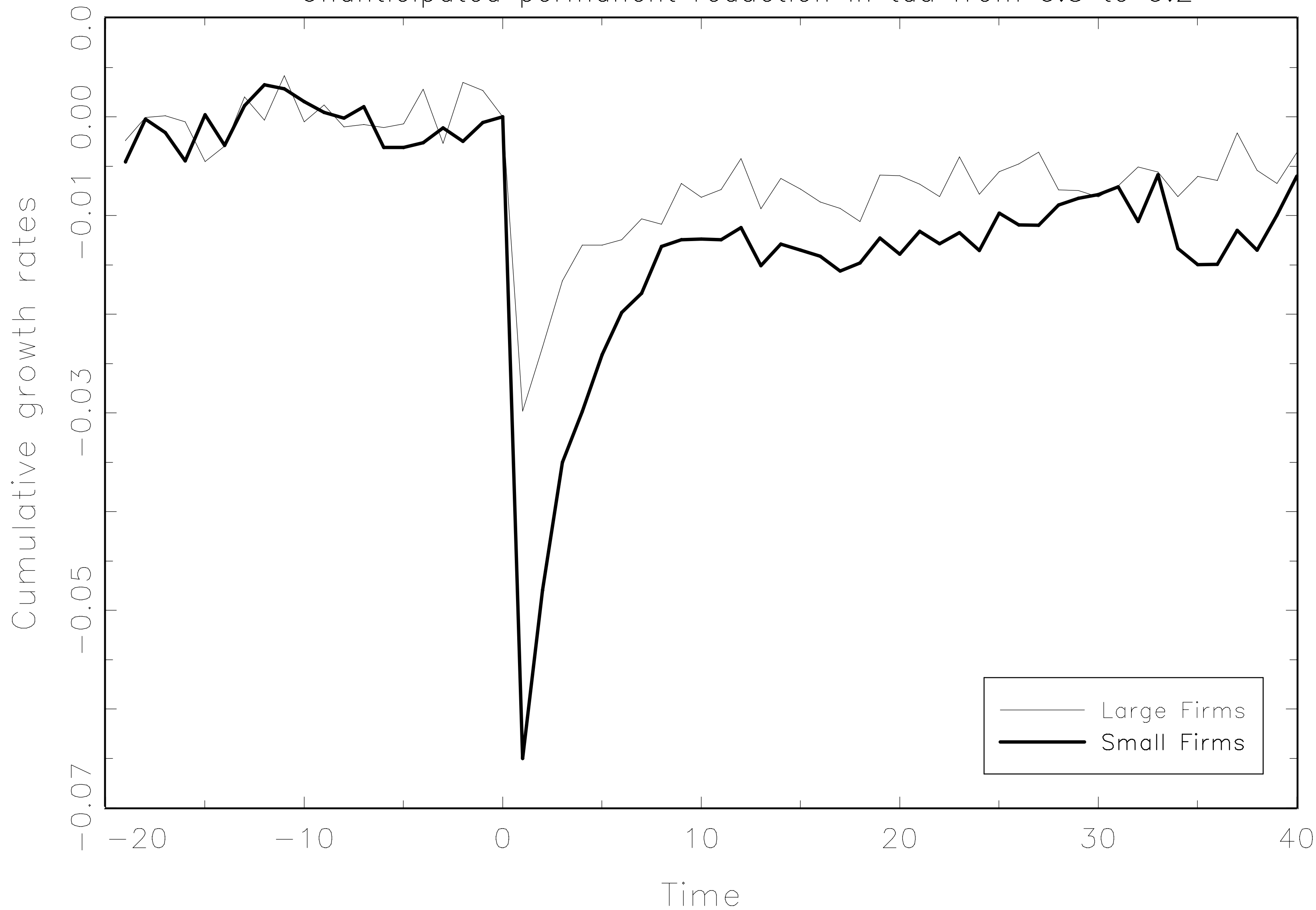


Figure 7

ɔd permanent increase in interest rate from 1% to 3%

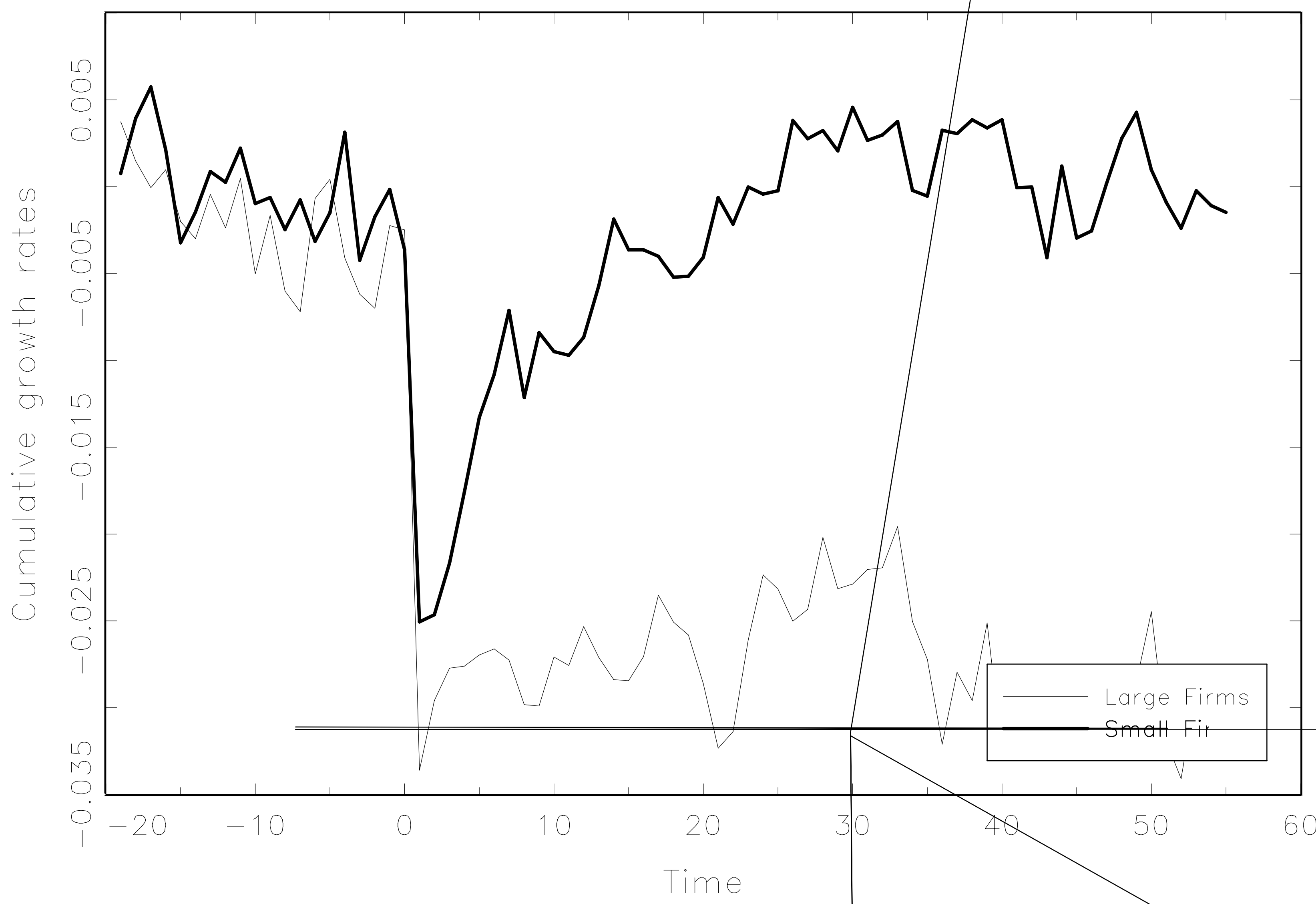


Figure 8

increase in interest rate from 1% to 3% – high leverage economy

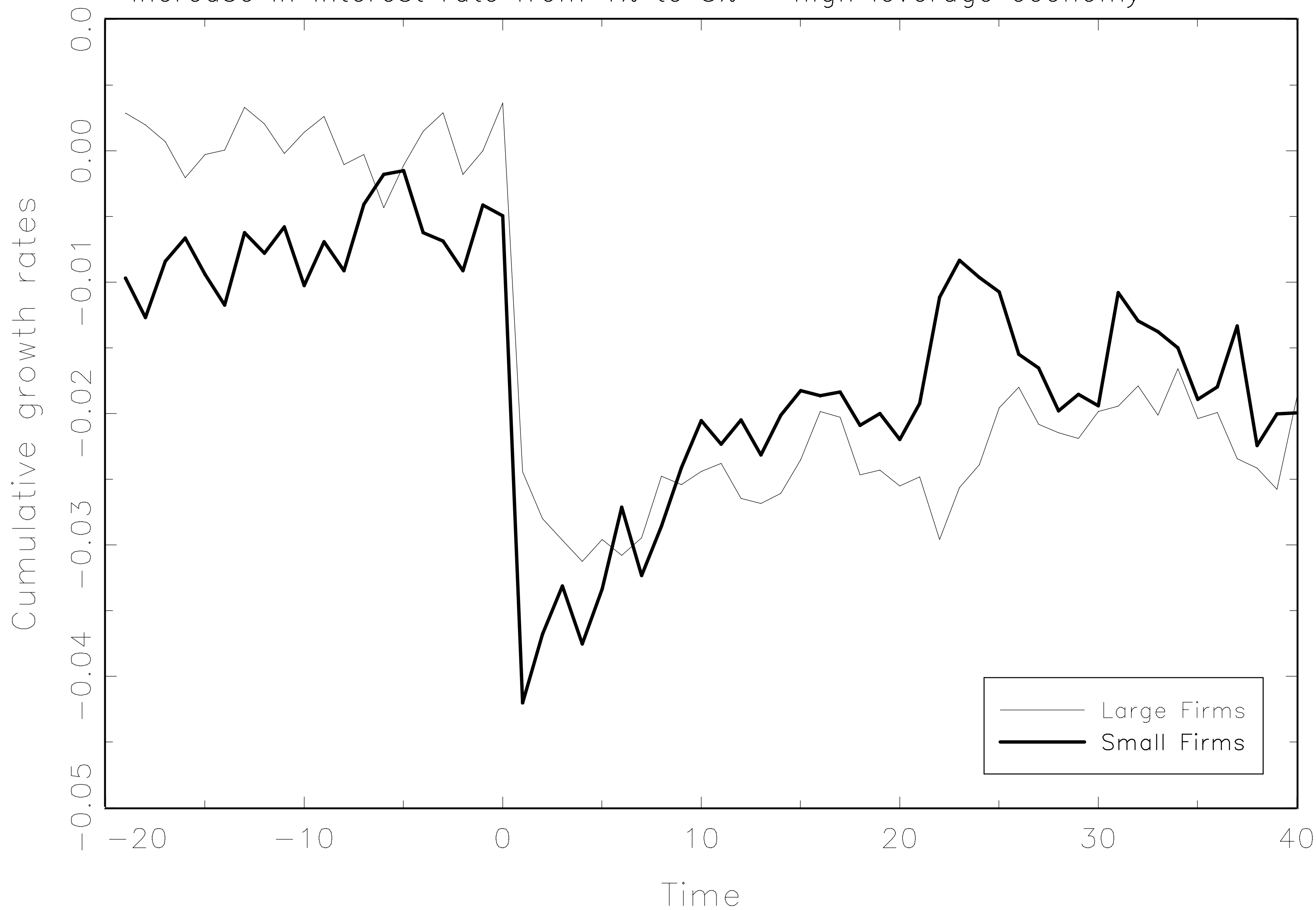


Figure 9

Unanticipated temporary decrease in output of 10%

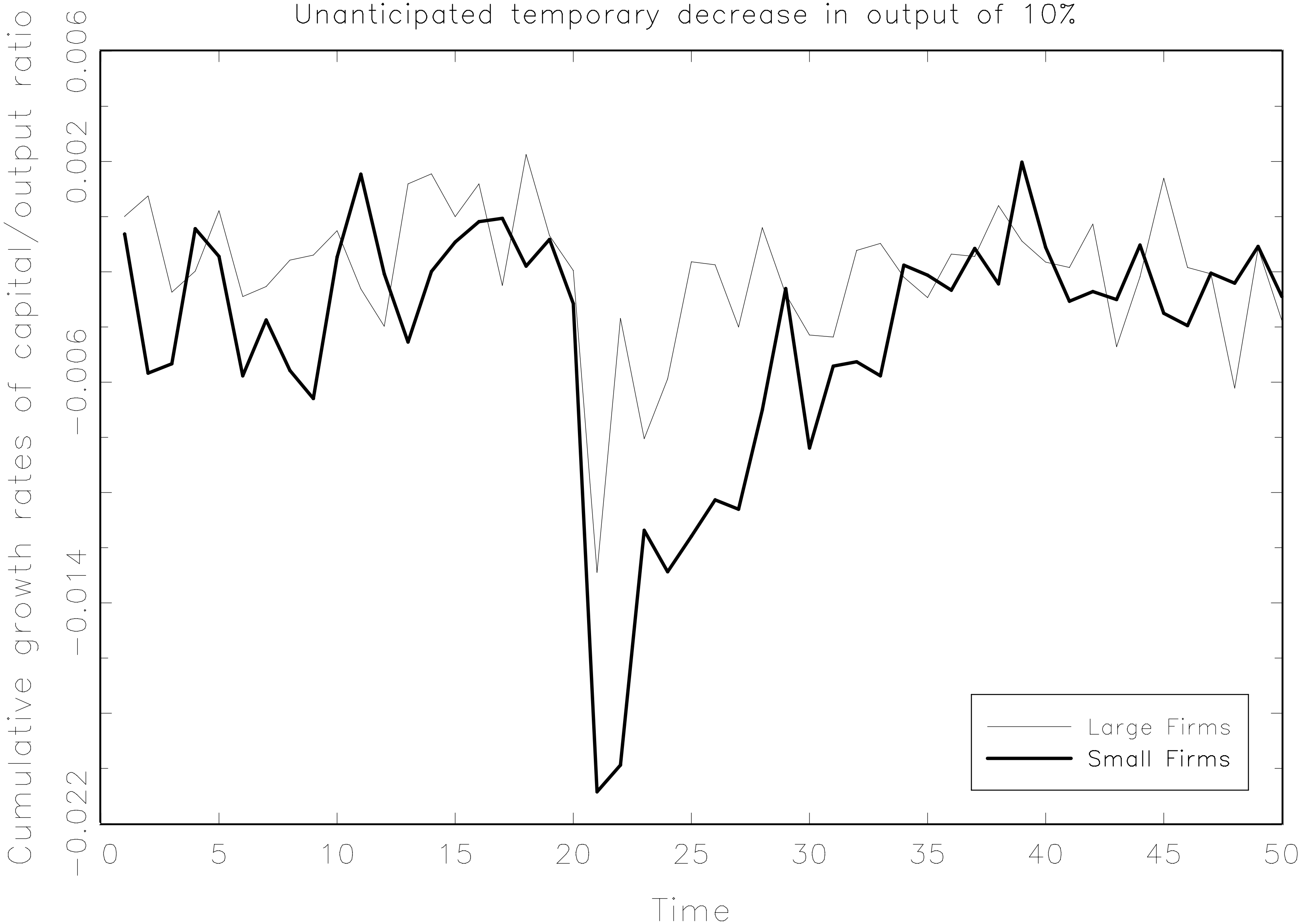
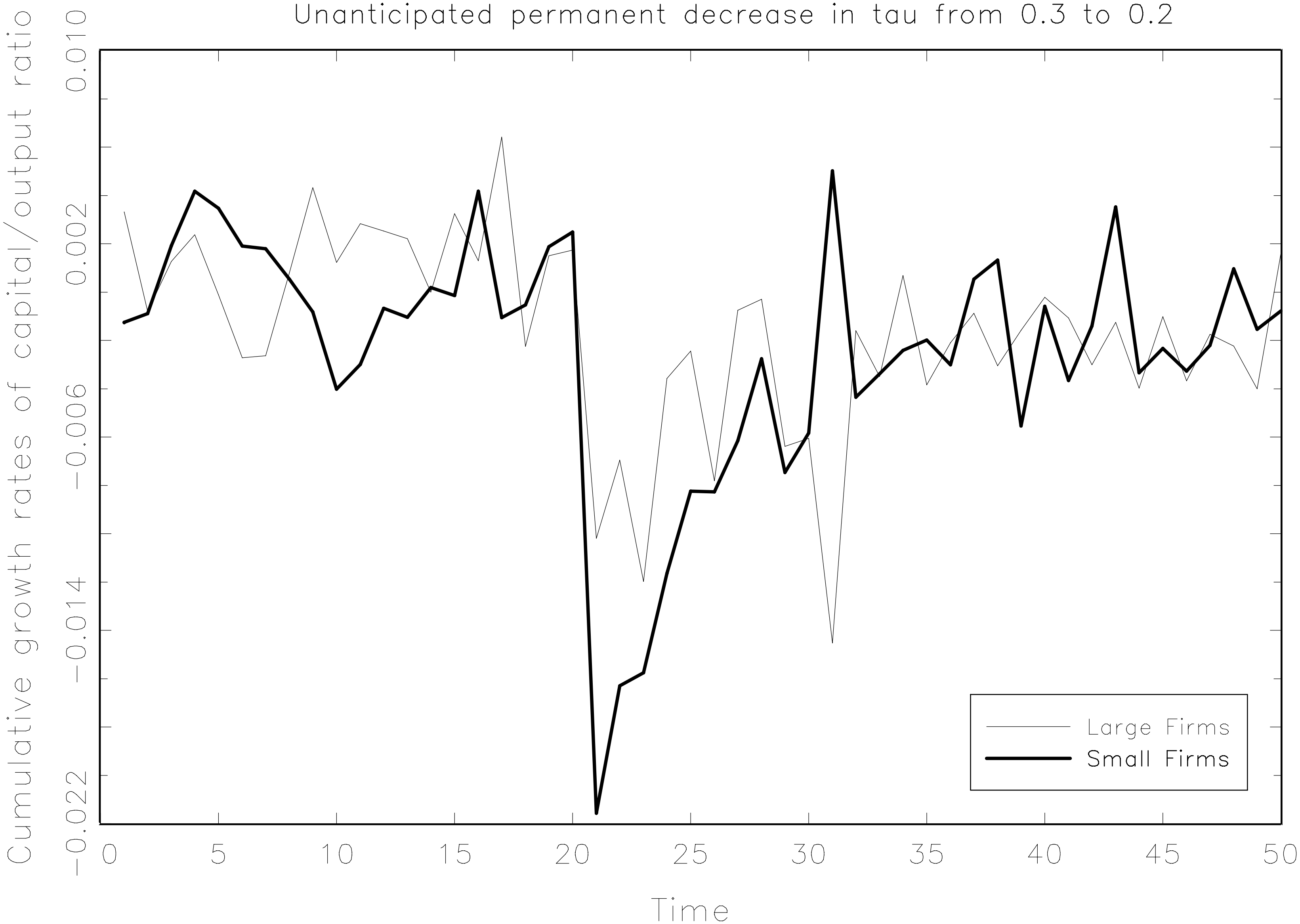


Figure 10

Unanticipated permanent decrease in tau from 0.3 to 0.2



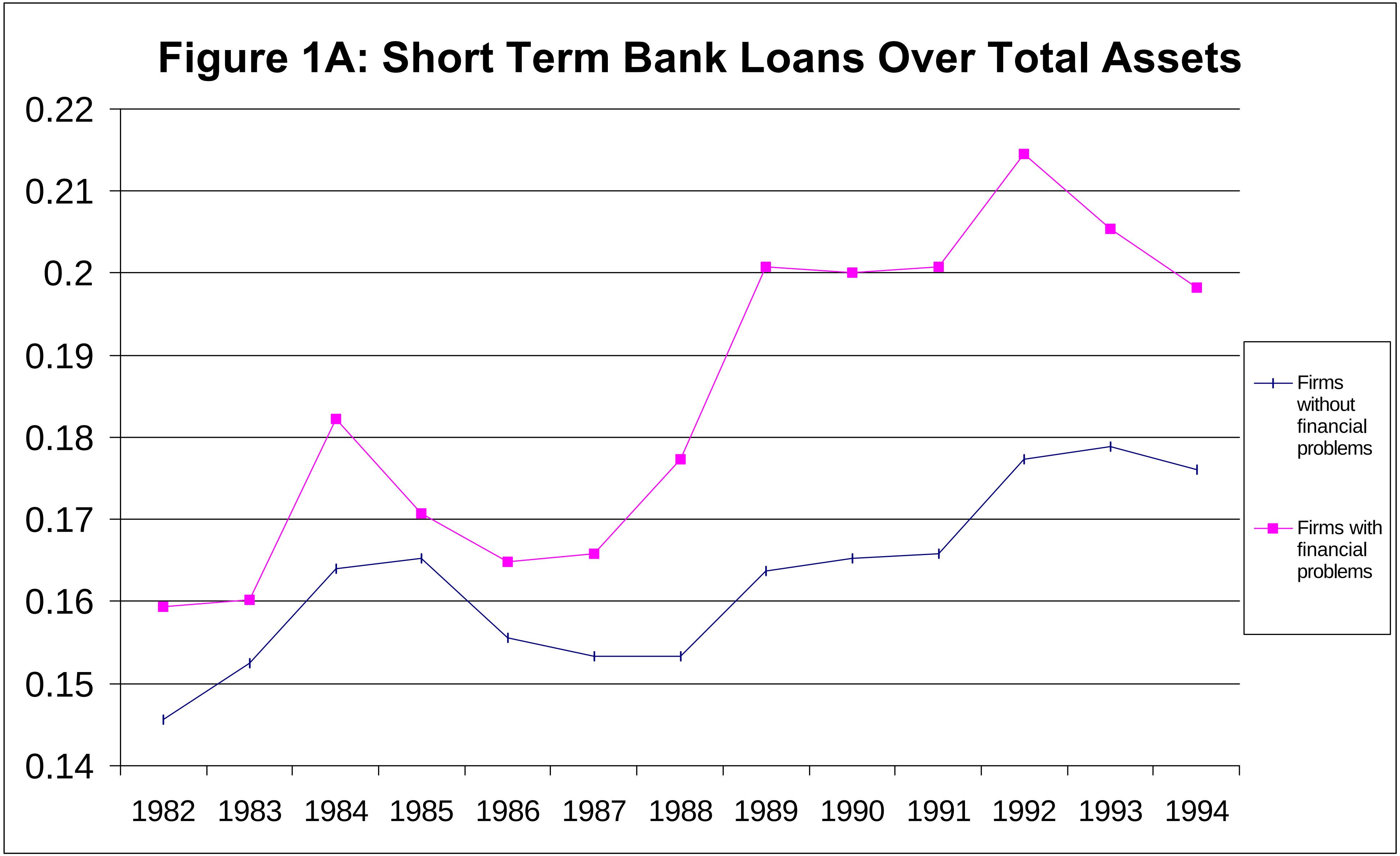


Figure 1A: Short Term Bank Loans Over Total Assets
0.22
0.21
0.2
0.19
0.18
0.17
0.16
0.15
0.14
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
Firms without financial problems
Firms with financial problems

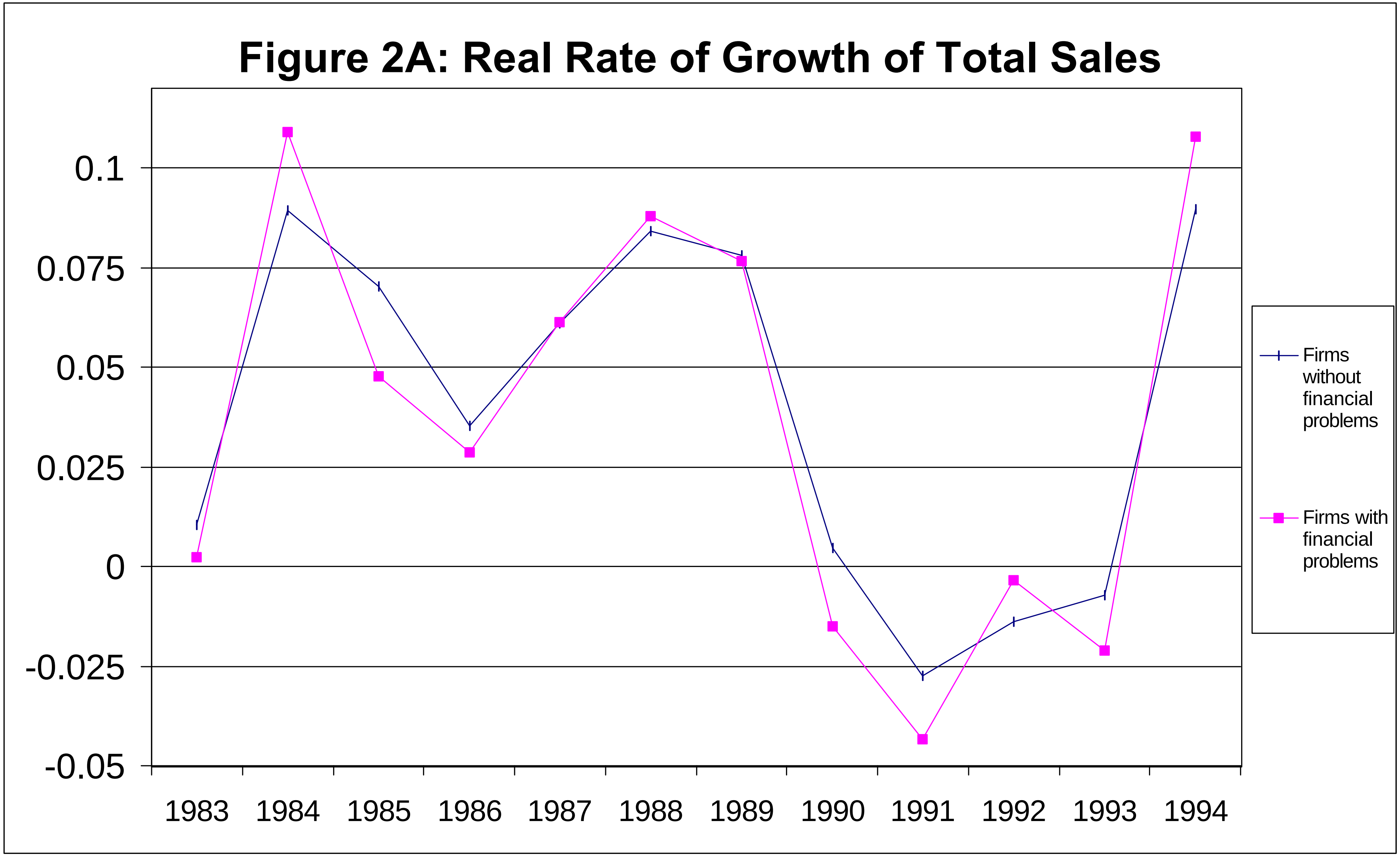


Figure 2A: Real Rate of Growth of Total Sales
0.1
0.075
0.05
0.025
0
-0.025
-0.05
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
Firms without financial problems
Firms with financial problems

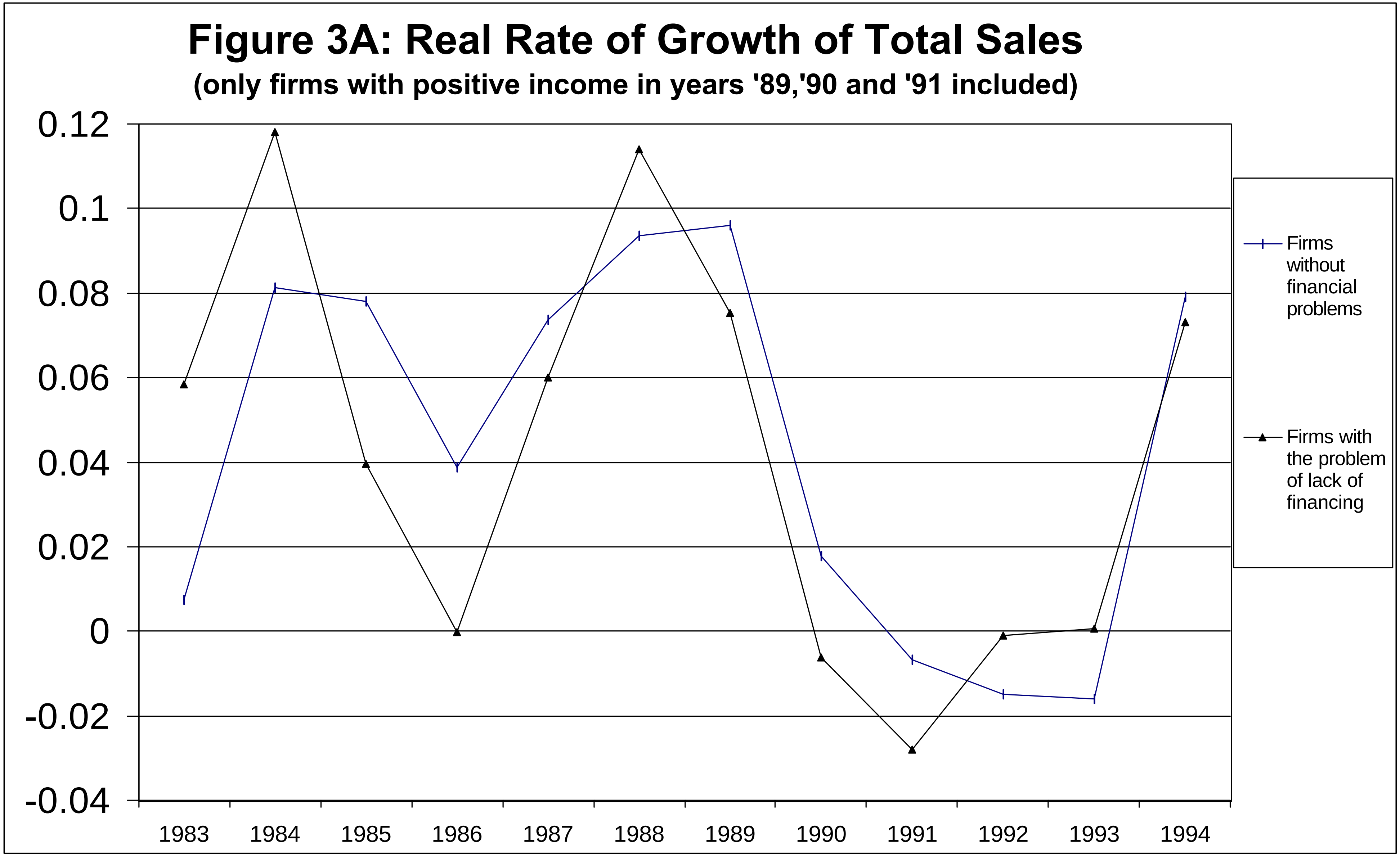


Figure 3A: Real Rate of Growth of Total Sales
(only firms with positive income in years '89,'90 and '91 included)
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
Firms without financial problems
Firms with the problem of lack of financing