

**Bank Capital Regulation**

**With Random Audits**

**By**

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## Abstract

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# 1 Introduction

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## 2 The Model

Consider a system of particles in a volume  $V$ . The energy of the system is given by

$$\frac{dV}{V} = [\mu(V, t) + \delta] dt + \sigma dz$$

where  $\mu(V, t)$  is the chemical potential,  $\delta$  is a constant, and  $\sigma$  is a parameter. The volume  $V$  is a function of time  $t$  and position  $z$ .

The system is in a state of equilibrium, and the energy is conserved.

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$$dA = dq$$

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## 2.1 Value of Debt

$C$

$$D(V, A; B^*, B, \sigma, r, C, \lambda, \delta)$$

$$\frac{1}{2}\sigma^2V^2D_{VV} + (r - \delta)V D_V - rD + C\mathbf{1}_{[B^*, B]} - \lambda(V - D) = 0$$

$$\lim_{V \rightarrow \infty} D(V) = \frac{C}{r}$$

$$\lim_{V \rightarrow B^+} D(B) = \lim_{V \rightarrow B^-} D(B)$$

$$\lim_{V \rightarrow B^+} D_V(B) = \lim_{V \rightarrow B^-} D_V(B)$$

$$D(B^*) = B^*$$

$V$

□

$$D_1(V) = \frac{C}{r} + \alpha_1 V^{x_1} + \alpha_2 V^{x_2}$$

$V \cdot B$

$$D_2(V) = \frac{C}{r + \lambda} + \frac{\lambda}{\lambda + \delta} V + \beta_1 V^{y_1} + \beta_2 V^{y_2}$$

$B^* \cdot V \cdot B$

$$x_{1,2} = \frac{\sigma^2 \pm 2(r + \delta) \pm \sqrt{[\sigma^2 \pm 2(r + \delta)]^2 + 8r\sigma^2}}{2\sigma^2}$$

$$y_{1,2} = \frac{\sigma^2 \pm 2(r + \delta) \pm \sqrt{[\sigma^2 \pm 2(r + \delta)]^2 + 8(r + \lambda)\sigma^2}}{2\sigma^2}$$

$$\alpha_1 = \frac{B^{*y_2} \left[ \frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta} \right] \pm (x_1 \pm y_2) B^{y_2} \left[ \frac{\delta B^*}{\lambda+\delta} \pm \frac{C}{\lambda+r} \right]}{(x_1 \pm y_1) B^{y_1} B^{*y_2} \pm (x_1 \pm y_2) B^{*y_1} B^{y_2}}$$

$$\alpha_2 = 0$$

$$\beta_1 = \frac{B^{*y_1} \left[ \frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta} \right] \pm (x_1 \pm y_1) B^{y_1} \left[ \frac{\delta B^*}{\lambda+\delta} \pm \frac{C}{\lambda+r} \right]}{(x_1 \pm y_1) B^{y_1} B^{*y_2} \pm (x_1 \pm y_2) B^{*y_1} B^{y_2}}$$

$$\beta_2 = \frac{B^{*y_2} \left[ \frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta} \right] \pm (x_1 \pm y_2) B^{y_2} \left[ \frac{\delta B^*}{\lambda+\delta} \pm \frac{C}{\lambda+r} \right]}{(x_1 \pm y_1) B^{y_1} B^{*y_2} \pm (x_1 \pm y_2) B^{*y_1} B^{y_2}}$$

$$\alpha_1 = \frac{\lambda B^{1-x_1}}{x_1(\lambda + \delta)} + \frac{\beta_1 y_1 B^{y_1-x_1}}{x_1} + \frac{\beta_2 y_2 B^{y_2-x_1}}{x_1}$$

□

## 2.2 Value of Subsidy

The value of the option  $S$  is determined by the following partial differential equation (PDE) in the region  $V > B$ :
 
$$\frac{1}{2}\sigma^2 V^2 S_{VV} + (r - \delta)V S_V - rS - \tau C \mathbf{1}_{[B^*, B]} \lambda S = 0$$
 where  $\tau C$  is the subsidy rate and  $\mathbf{1}_{[B^*, B]}$  is the indicator function for the interval  $[B^*, B]$ .

$$\frac{1}{2}\sigma^2 V^2 S_{VV} + (r - \delta)V S_V - rS - \tau C \mathbf{1}_{[B^*, B]} \lambda S = 0$$

The boundary conditions for the option value  $S$  are:

$$\lim_{V \rightarrow \infty} S(V) = \frac{\tau C}{r}$$

$$\lim_{V \rightarrow B^+} S(B) = \lim_{V \rightarrow B^-} S(B)$$

$$\lim_{V \rightarrow B^+} S_V(B) = \lim_{V \rightarrow B^-} S_V(B)$$

$$S(B^*) = 0$$

The option value  $S$  is a function of  $V$  and is denoted by  $S(V)$ .

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$$S_1(V) = \frac{\tau C}{r} + \tilde{\alpha}_1 V^{\tilde{\alpha}_1} + \tilde{\alpha}_2 V^{\tilde{\alpha}_2}$$

The option value  $S$  is a function of  $V$  and is denoted by  $S(V)$ .

□

$$S_2(V) = \frac{\tau C}{r + \lambda} + \tilde{\beta}_1 V^{y_1} + \tilde{\beta}_2 V^{y_2}$$

$$B^* V \cdot B \quad x_{1,2} \quad y_{1,2}$$

$$\tilde{\alpha}_2 = 0$$

$$\tilde{\beta}_1 = \frac{\tau C \left[ \frac{\lambda x_1 B^{*y_2}}{r} + (x_1 \mid y_2) B^{y_2} \right]}{(\lambda + r) [(x_1 \mid y_1) B^{y_1} B^{*y_2} \mid (x_1 \mid y_2) B^{*y_1} B^{y_2}]}$$

$$\tilde{\beta}_2 = \frac{\tau C \left[ \frac{\lambda x_1 B^{*y_1}}{r} + (x_1 \mid y_1) B^{y_1} \right]}{(\lambda + r) [(x_1 \mid y_1) B^{y_1} B^{*y_2} \mid (x_1 \mid y_2) B^{*y_1} B^{y_2}]}$$

$$\tilde{\alpha}_1 = \frac{\tilde{\beta}_1 y_1 B^{y_1 - x_1}}{x_1} + \frac{\tilde{\beta}_2 y_2 B^{y_2 - x_1}}{x_1}$$

$$B^* \quad B$$

*TV*

### 2.3 Total Value of the Bank and Equity Value

$$TV(V)$$

$$TV(V) = V + S(V)$$

$$E(V) = V + S(V) - D(V)$$

□□









$$\frac{B^* [\delta [(x_1 | y_1)(1 | y_2)u^{y_1} | (x_1 | y_2)(1 | y_1)u^{y_2}] | \lambda u(1 | x_1)(y_1 | y_2)]}{\lambda + \delta} + \frac{C(\tau | 1) [y_1(x_1 | y_2)u^{y_2} | y_2(x_1 | y_1)u^{y_1} + \frac{\lambda}{r}x_1(y_1 | y_2)]}{\lambda + r} = 0$$

$B^*$   $u$   $B^*$

$$B^* = \frac{C(1 | \tau)(\lambda + \delta)}{\lambda + r} \frac{[x_1(y_1 | y_2)u^{y_1+y_2} + \frac{\lambda}{r}x_1(y_1u^{y_1} | y_2u^{y_2})]}{[\delta x_1(y_1 | y_2)u^{y_1+y_2} | \lambda x_1u[(1 | y_1)u^{y_1} | (1 | y_2)u^{y_2}]]}$$

$u$   $B^*$   $\tau$   $u$

$$\frac{y_1(x_1 | y_2)u^{y_2} | y_2(x_1 | y_1)u^{y_1} + \frac{\lambda}{r}x_1(y_1 | y_2)}{\delta [(x_1 | y_1)(1 | y_2)u^{y_1} | (x_1 | y_2)(1 | y_1)u^{y_2}] | \lambda u(1 | x_1)(y_1 | y_2)} | \frac{x_1(y_1 | y_2)u^{y_1+y_2} + \frac{\lambda}{r}x_1(y_1u^{y_1} | y_2u^{y_2})}{\delta x_1(y_1 | y_2)u^{y_1+y_2} | \lambda x_1u[(1 | y_1)u^{y_1} | (1 | y_2)u^{y_2}]} = 0$$

- Theorem 1**
- i) There exists a solution  $(B, B^*)$  with the property that  $B > B^* > 0$  iff the equation (30) has a solution  $u > 1$ .
  - ii)  $B$  and  $B^*$  are linear in the coupon  $C$
  - iii)  $B$  and  $B^*$  are linear in the subsidy rate  $\tau$
  - iv)  $B$  and  $B^*$  do not depend on  $V$ .

$u > 1$   $\delta = 0$   $\delta \neq 0$

**Proposition 1** Assume that there are no net cash outflows, i.e.  $\delta = 0$ . Then the non-linear equation (30) has a solution  $u > 1$ .

**Proof:**  $\delta = 0$

$$f(u) = \frac{y_1(x_1 - y_2)(1 - y_2)u^{2y_2}}{(1 - x_1)(y_1 - y_2)} + \frac{y_2(x_1 - y_1)(1 - y_1)u^{2y_1}}{(1 - x_1)(y_1 - y_2)} - \frac{[y_1(x_1 - y_2)(1 - y_1) + y_2(x_1 - y_1)(1 - y_2) - (y_1 - y_2)^2(1 - x_1)]u^{y_1+y_2}}{(1 - x_1)(y_1 - y_2)} - \frac{\lambda(x_1 - y_1)u^{y_1}}{r(1 - x_1)} + \frac{\lambda(x_1 - y_2)u^{y_2}}{r(1 - x_1)} = 0$$

$$f(1) = \frac{(y_2 - y_1)}{1 - x_1} + \frac{\lambda(y_2 - 1)}{r(1 - x_1)} < 0$$

$$\lim_{u \rightarrow \infty} f(u) = 1$$

$f(u)$

$u > 1$

**Q.E.D.**

$$D_1(V) = \frac{C}{r} \left[ 1 - a_1 \left( \frac{V}{C} \right)^{x_1} \right]$$

$$D_2(V) = \frac{\lambda V}{\lambda + \delta} + \frac{C}{r} \left[ \frac{r}{\lambda + r} - b_1 \left( \frac{V}{C} \right)^{y_1} - b_2 \left( \frac{V}{C} \right)^{y_2} \right]$$

□

$$\begin{aligned}
TV_1(V) &= V + \frac{C}{r} \left[ \tau + a_1 \left( \frac{V}{C} \right)^{x_1} \right] \\
TV_2(V) &= V + \frac{C}{r} \left[ \frac{\tau r}{\lambda + r} + \tilde{b}_1 \left( \frac{V}{C} \right)^{y_1} + \tilde{b}_2 \left( \frac{V}{C} \right)^{y_2} \right] \\
E_1(V) &= V + \frac{C(1 + \tau)}{r} \\
E_2(V) &= \frac{\delta}{\delta + \tau} V + \frac{C}{r} \left[ (1 + \tau) + (\tilde{b}_1 + b_1) \left( \frac{V}{C} \right)^{y_1} + (\tilde{b}_2 + b_2) \left( \frac{V}{C} \right)^{y_2} \right]
\end{aligned}$$

1
2
[B\*, B]
[B, 1)

### 3 Numerical Results

C
r
δ
λ

$C$	
$r$	
$\delta$	
$\lambda$	

$B^*$

$$1400(1 + 0.1) = 1540$$

$V$

$$E = V > B$$

$$V < B$$

$$V > B$$

### 3.1 Comparative Statics of Debt Value and Equity Value

Debt value  $B$  and debt face value  $B^*$

$C$

□



1.  $V$  是  $B$  的子空间,  $V$  的基为  $\{v_1, v_2, \dots, v_r\}$ ,  $B$  的基为  $\{b_1, b_2, \dots, b_n\}$ .  
 2.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  可以扩展为  $B$  的基  $\{v_1, v_2, \dots, v_r, b_{r+1}, \dots, b_n\}$ .  
 3.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 4.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 5.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 6.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 7.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 8.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 9.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .  
 10.  $V$  的基  $\{v_1, v_2, \dots, v_r\}$  与  $B$  的基  $\{b_1, b_2, \dots, b_n\}$  的秩相同, 即  $r = \text{rank}(B)$ .













### 3.2 Bank Closure and Capital Adequacy

The bank's capital adequacy is determined by the ratio of its capital to its risk-weighted assets. The capital adequacy ratio (CAR) is defined as:

$$CAR = \frac{E}{k TV(V)}$$

where  $E$  is the bank's capital,  $k$  is the risk-weighting factor, and  $TV(V)$  is the total value of the bank's assets.

$$\bar{E} = k TV(V)$$

The bank's capital adequacy is determined by the ratio of its capital to its risk-weighted assets. The capital adequacy ratio (CAR) is defined as:

$$E(V) = \bar{E} = k TV(V) \quad B$$

The bank's capital adequacy is determined by the ratio of its capital to its risk-weighted assets. The capital adequacy ratio (CAR) is defined as:

$$k = \alpha \sigma$$

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## 4 Conclusion

The first part of the paper is devoted to the study of the asymptotic behavior of the solutions of the Cauchy problem for the wave equation in the case of a non-compact manifold. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

In the second part of the paper, we study the asymptotic behavior of the solutions of the wave equation in the case of a compact manifold. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

The third part of the paper is devoted to the study of the asymptotic behavior of the solutions of the wave equation in the case of a non-compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

The fourth part of the paper is devoted to the study of the asymptotic behavior of the solutions of the wave equation in the case of a non-compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

In the fifth part of the paper, we study the asymptotic behavior of the solutions of the wave equation in the case of a compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

The sixth part of the paper is devoted to the study of the asymptotic behavior of the solutions of the wave equation in the case of a non-compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

In the seventh part of the paper, we study the asymptotic behavior of the solutions of the wave equation in the case of a compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

The eighth part of the paper is devoted to the study of the asymptotic behavior of the solutions of the wave equation in the case of a non-compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

In the ninth part of the paper, we study the asymptotic behavior of the solutions of the wave equation in the case of a compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.

The tenth part of the paper is devoted to the study of the asymptotic behavior of the solutions of the wave equation in the case of a non-compact manifold with a boundary. The main result is the theorem on the asymptotic expansion of the solutions in terms of the distance from the source. The proof is based on the method of stationary phase and the theory of pseudodifferential operators.



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