Bank Capital Regulation

With Random Audits

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DISCUSSION PAPER 354

August 2000

FINANCIAL MARKETS GROUP AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS



Any opinions expressed are those of the author and not necessarily those of the Financial Markets Group.

ISSN 0956-8549-354

With Random Audits

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vember 1998, Revised May 2000

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Abstract

of optimal bank closure rules (cum capital replenishment by banks), ted audits of the bank's asset value by the regulator, with the goal orating) the incentives of levered bank shareholders/ managers to take ir choice of underlying assets. The roles of (tax or other) subsidies on ents by the bank, and of the auditing frequency are examined.

1 Introduction

al regulation of banking has received much attention in both theoretical er the last decade or so. On the policy side, we have progressed from the controls on banks' asset portfolios and deposit interest rates in the early bout sensible regulations especially on bank capital requirements as well closure rules in the 1990s, epitomized by the Basle accords on capital ded to "market risks" on traded asset portfolios. On the theoretical e issues has been rekindled by recent advances in the microeconomic ee Bhattacharya and Thakor (1993) for a survey), and also on capital I (contingent) control rules for corporate governance of a levered ...tm; Dewatripont and Tirole (1994) for applications to bank regulation.

bank regulation — whereby regulatory authorities serve as a "proxy" bank deposit holders subject to free-rider problems in monitoring their and returns performances, and (b) the general public concerned with " of a bank's failure on other related banks and the payment system — e backdrop of heightened instabilities in the banking systems of many the 1980s, and Japan today) .The market environment facing banks has ramatically since the 1970's, beginning with disintermediation arising oney market funds for short-maturity deposits, to increased interbank eregulated environment of the 1980s, through the explosive growth of kets that allow banks much greater ‡exibility in hedging their asset Iso to speculate on such risks in the economy as whole.

ave emerged in the realm of bank regulation over this period, and een retected in policies or in proposals for regulatory reform. First,

m rents in the banking market, both in protecting banks from default importantly, in creating incentives for bank managers to wisely choose isk, has acquired credence since the empirical ..ndings of Keeley (1990). importantly in the policy domain, the role of bank capital (regulations) ult risks on deposits, as well as in incentivising sensible risk-taking, ead recognition, as a minimally intrusive and veri..able instrument of . Third, other controls and regulations on interbank competition -/ entry rules, ceilings on bank deposit interest rates, and even portfolio n to be closely examined as potentially welfare-enhancing components Fourth, various aspects of the regulators' functioning itself, such as commit to tough closure rules given their instinct for self-preservation t taking strong actions early, have also received signi..cant research (1996).

progress has occurred along qualitative lines, "multi-instrument and ntial regulation which incorporate detailed quantitative criteria for the ments of regulation, have not been advanced in great numbers. In the al regulation, Hellwig (1998) has observed that the Basle Accord capital no account of dynamic risks, and hence the (regulated) readjustment multi-period context. He has also noted that for the "non-market" uidated assets the precise levels of capital regulations are not carefully ather than 4% or indeed 50%?", he asks.

he past two decades has sought to address these structural and quanto prudential bank regulation and the several alternative instruments arya (1982) it was pointed out that future rents in banking, arising n risk-adjusted lending and deposit interest rates, would help curtail

managers') incentives to take excessive risks in the short-run, and that rents for e¢ cient risk-taking need not be preserved in a competitive h raising its deposit interest rate would cause a higher volume of funds iven bank. More recently, Hellman, Murdock, and Stiglitz (1997) have nquiry to incorporate bank equity capital ratio regulations as well. Emsumption that the (risk-adjusted) cost of bank equity capital exceeds estment opportunities available to the bank, they reach the conclusion through capital controls alone is never optimal, and controls (ceilings) rest rates paid on bank deposits are called for to encourage e¢ cient

g (1997) and Matutes and Vives (1997) also model interbank compeon banks' default probabilities, with a focus on regulatory restriction If are impact of the market outcomes. Gehrig (1997) models imperfect nally dimerentiated competition across banks in both their asset/loan , with entry and rents determining bank survival probabilities given d systemic default risks on loans. However, there is no further margin rentially risky assets/ loans by the banks in his model. Given this, reguch as deposit interest rate controls have no role in terms of enhancing prudent current risk-taking by banks - instead these may be used to by new banks when this is too low in market equilibrium. In the paper (1997), only the deposit-market competition is explicitly modeled, but mong alternative mean-preserving asset return distributions. Default exogenously given social costs which their model does not endogenize. may have a role in reducing such costs, given the level of default risks uilibrium. However, instruments such as deposit interest rate controls ks directly by creating pro.t bupers for banks, which are nevertheless

eir most risky investment strategies.

ave taken a more explicitly quantitative "valuation approach" to the I regulation of banks. In two early papers, Merton (1977, 1978) estabm between the value of a deposit insurer's liabilities and that of a put assets, ..r.st without and then with random and costly interim moniasset value. However, these models omitted the possibility of ongoing ishment as well as regulatory choice of the level of the Closure Rule, lue at which the bank would be closed. Issues concerning endogenous portfolio risks by banks were also not fully addressed. These issues of a bank closure rule, and its impact on the choice of risk-taking by ers / managers, has received more attention in the recent research of nd Perraudin (1997). However, their bank closure rule is predicated rganization cost criterion which has the feature that, given the scale of , the regulator's bankruptcy/closure costs are lower if the bank's asset time of its closure! Hence, the regulator "waits for the bank to shrink given monitoring cost rate while doing so. With a more sensible bank n cost function whereby the regulator is better on having a higher bank e time of closure, and continous monitoring of the continuously-evolving es of default risk on bank deposits would simply not arise.

ek to address and quantify the issues that have been raised in prior atic way, to incorporate (a) ongoing deposit liabilities (perpetual debt) a bank's reorganization following default by one set of equityholders; replenishment of bank capital - although not as instantaneously as in Stiglitz (1997) - by equityholders subject to the constraint of a closure egulator; (c) subsidies (from tax-shield or controlled interest rates, for

..nance relative to the costs of bank equity capital; and (d) stochastic iable intensity by a bank regulator when the underlying asset value of tinuous-time di¤usion process. The regulatory controls are chosen to ve risk-taking incentives of the levered bank equityholders, at least in bank would not be closed on audit. Our model enables us to quantify various policy instruments in attaining this goal. We hope that this e further research on richer models that incorporate considerations of d bene.ts of the alternative policy instruments such as bank closure ubsidies/ ceilings on bank deposit interest rates, the levels of capital e speed of replenishment thereof, and regulatory audit frequency. A is should take into account the implications of various bank regulations zation costs, and the e¤ects on the volume of savings and its channeling ed ..nancing.

d as follows. In the next section, we describe the structure of our model liabilities of a bank subject to regulatory controls on closure, and derive ghly non-linear in some parameters) solutions for these as well as for y closure rule, which seeks to induce neutrality towards risk-taking by In section 3, we calibrate the model numerically, for a range of values ables such as the maximal payout ratio (which determines the speed of ment as well), the regulatory audit frequency, the di¤erentially lower ce, and the (overall value-maximizing) choice of the level of riskiness bank's asset portfolio.In section 4 we conclude, with suggestions for e regulatory issues that we attempt to highlight.

2 The Model

B

Black and Cox (1976), and Leland (1994), we assume that the bank's e V follows a continuous-time divided process characterised by the interential equation:

$$\frac{dV}{V} = \left[\mu(V,t) \mathbf{i} \ \delta\right] dt + \sigma dz \tag{1}$$

 $\mu(V,t)$ otal expected rate of return on asset value V, δ is the constant fraction id out to security holders, σ is the constant proportional volatility of it time, and dz is the increment of a standard Brownian motion. This ntinues without any time limit unless the asset value V falls below a lue B^* chosen by the bank's equityholders, or there occurs an audit and rity decides to close the bank because V is below a prespeci..ed value regulatory body. The regulator's audits of the bank's asset value are a Poisson process where the mean number of audits per unit time is λ Ily this is described by the following stochastic process

$$dA = dq \tag{2}$$

A udit occurs and zero otherwise. Note that under the above assumptions n audit occurs in the time interval dt is λdt , the probability of no audit bability of more than one audit is of order O(dt). It is assumed that the ses dz and dq are independent. Later on it will be shown that the two rs B^* and B can be determined analytically as a part of the regulatory

2.1 Value of Debt

debt of the bank let us assume that a riskless asset exists that pays erest r and that the bank continuously pays a non-negative coupon Cts creditors (depositors), unless it declares default or is closed by the The value of the outstanding debt $D(V, A; B^*, B, \sigma, r, C, \lambda, \delta)$ is given following non-linear ordinary dimerential equation

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r_i \ \delta) V D_{Vi} \ r D + C + \mathbf{1}_{[B^*,B]} \lambda(V_i \ D) = 0$$
(3)

her (a) all agents are risk-neutral, or (b) agents other than the regulator informed of these asset and liability values and can trade continuously e, the following economic boundary conditions must also be satis..ed:

 $\lim_{V \to \infty} D(V) = \frac{C}{r}$

 $\lim_{V \to B^+} D(B) = \lim_{V \to B^-} D(B)$

 $\lim_{V \to B^+} D_V(B) = \lim_{V \to B^-} D_V(B)$

 $D(B^*) = B^*$

ecause default becomes irrelevant as *V* becomes large and the value of alue of the capitalized coupon and therefore the value of risk-free debt. iii) are the common "smooth pasting" conditions, and condition (iv) – rm in equation (3) – guarantee that in the case of default – or closure e value of debt is equal the asset value of the bank. The solution of the ation is given by:

$$D_1(V) = \frac{C}{r} + \alpha_1 V^{x_1} + \alpha_2 V^{x_2}$$
(3)

V B it is

$$D_2(V) = \frac{C}{r+\lambda} + \frac{\lambda}{\lambda+\delta}V + \beta_1 V^{y_1} + \beta_2 V^{y_2}$$
(4)

 $B^* \cdot V \cdot B$

ulations show that

$$x_{1,2} = \frac{\sigma^2 \mathbf{i} \ 2(r \mathbf{i} \ \delta) \$ \sqrt{[\sigma^2 \mathbf{i} \ 2(r \mathbf{i} \ \delta)]^2 + 8r\sigma^2}}{2\sigma^2}$$
(5)

$$y_{1,2} = \frac{\sigma^2 \left[2(r \left[\delta \right] \$ \sqrt{[\sigma^2 \left[2(r \left[\delta \right])^2 + 8(r + \lambda)\sigma^2 \right]} \right]}{2\sigma^2}$$
(6)

enerality we can assume that x_1 and y_1 are negative. It is immediately i) implies that $\alpha_2 = 0$, and the remaining conditions yield the following $\alpha_1 \ \beta_1$ and β_2 :

$$\beta_1 = \frac{B^{*y_2} \left[\frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta} \right] \mathbf{i} \quad (x_1 \mathbf{i} \quad y_2) B^{y_2} \left[\frac{\delta B^*}{\lambda+\delta} \mathbf{i} \quad \frac{C}{\lambda+r} \right]}{(x_1 \mathbf{i} \quad y_1) B^{y_1} B^{*y_2} \mathbf{i} \quad (x_1 \mathbf{i} \quad y_2) B^{*y_1} B^{y_2}}$$
(7)

$$\beta_2 = \mathbf{i} \frac{B^{*y_1} \left[\frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta} \right] \mathbf{i} \quad (x_1 \mathbf{i} \quad y_1) B^{y_1} \left[\frac{\delta B^*}{\lambda+\delta} \mathbf{i} \quad \frac{C}{\lambda+r} \right]}{(x_1 \mathbf{i} \quad y_1) B^{y_1} B^{*y_2} \mathbf{i} \quad (x_1 \mathbf{i} \quad y_2) B^{*y_1} B^{y_2}}$$
(8)

$$\alpha_1 = \frac{\lambda B^{1-x_1}}{x_1(\lambda+\delta)} + \frac{\beta_1 y_1 B^{y_1-x_1}}{x_1} + \frac{\beta_2 y_2 B^{y_2-x_1}}{x_1}$$
(9)

 B^* B e exogenously given parameters. In what follows, we shall show how ised.

2.2 Value of Subsidy

the fact that the banking industry may not be perfectly competitive usion, interest ceilings on deposits, or government (tax-shield on intercally, we assume that the size of this rent is proportional to the interest τC . This implies that the value of the subsidy, S, obeys the following i¤erential equation:

$$\frac{1}{2}\sigma^2 V^2 S_{VV} + (r_i \ \delta) V S_{Vi} \ rS + \tau C_i \ \mathbf{1}_{[B^*,B]} \lambda S = 0$$
(10)

conomic boundary conditions must be satis..ed:

 $\lim_{V \to \infty} S(V) = \frac{\tau C}{r}$ $\lim_{V \to B^+} S(B) = \lim_{V \to B^-} S(B)$ $\lim_{V \to B^+} S_V(B) = \lim_{V \to B^-} S_V(B)$ $S(B^*) = 0$

ecause default becomes irrelevant as V becomes large and the value of es its riskless capitalized present value. Conditions (ii) and (iii) are the ing conditions, and condition (iv) retects the loss of the (tax-) subsidy current owners of the bank are concerned if the bank declares default solution of the above di¤erential equation is given by:

$$S_1(V) = \frac{\tau C}{r} + \tilde{\alpha_1} V^{x_1} + \tilde{\alpha_2} V^{x_2}$$
(11)

 $V \ B$ it is

$$S_2(V) = \frac{\tau C}{r+\lambda} + \tilde{\beta}_1 V^{y_1} + \tilde{\beta}_2 V^{y_2}$$
(12)

 $B^* \cdot V \cdot B$ re $x_{1,2}$ and $y_{1,2}$ are given as before.

ion (i) implies that $\tilde{\alpha_2} = 0$ and the remaining ones yield the following

$$\tilde{\beta}_{1} = \frac{\tau C \left[\frac{\lambda x_{1} B^{*y_{2}}}{r} + (x_{1} \mathbf{i} \ y_{2}) B^{y_{2}} \right]}{(\lambda + r) \left[(x_{1} \mathbf{i} \ y_{1}) B^{y_{1}} B^{*y_{2}} \mathbf{j} \ (x_{1} \mathbf{j} \ y_{2}) B^{*y_{1}} B^{y_{2}} \right]}$$
(13)

$$\tilde{\beta}_{2} = \mathbf{i} \frac{\tau C \left[\frac{\lambda x_{1} B^{*y_{1}}}{r} + (x_{1} \mathbf{j} \ y_{1}) B^{y_{1}} \right]}{(\lambda + r) \left[(x_{1} \mathbf{j} \ y_{1}) B^{y_{1}} B^{*y_{2}} \mathbf{j} \ (x_{1} \mathbf{j} \ y_{2}) B^{*y_{1}} B^{y_{2}} \right]}$$
(14)

$$\tilde{\alpha_1} = \frac{\tilde{\beta_1} y_1 B^{y_1 - x_1}}{x_1} + \frac{\tilde{\beta_2} y_2 B^{y_2 - x_1}}{x_1}$$
(15)

eters B^* and B are exogenous but they will be determined endogenously are now in a position to de. ne the total value, TV, of the bank.

2.3 Total Value of the Bank and Equity Value

bank TV(V) is the bank's asset value plus the value of the subsidy of

$$TV(V) = V + S(V) \tag{16}$$

s the total value of the bank minus the value of its debt:

$$E(V) = V + S(V) \, \mathbf{i} \, D(V)$$
 (17)

ave:

$$E_1(V) = \frac{C(\tau_i \ 1)}{r} + V + (\tilde{\alpha}_1_i \ \alpha_1)V^{x_1}$$
(18)

V , B

$$E_{2}(V) = \frac{C(\tau \mathbf{i} \ 1)}{r} + \frac{\delta V}{\lambda + \delta} + (\tilde{\beta}_{1} \mathbf{i} \ \beta_{1})V^{y_{1}} + (\tilde{\beta}_{2} \mathbf{i} \ \beta_{2})V^{y_{2}}$$
(19)

 $B^* \cdot V \cdot B$

 σ

-triggering asset value level B^* is chosen by the bank equityholders by a covenant), then as pointed out by Merton (1973) this value is llowing "low contact" condition:

$$E_V(B^*) = 0 \tag{20}$$

the following equation:

$$\frac{\delta B^*}{\lambda + \delta} + (\tilde{\beta}_1 \mathbf{i} \ \beta_1) y_1 B^{*y_1} + (\tilde{\beta}_2 \mathbf{i} \ \beta_2) y_2 B^{*y_2} = 0$$
(21)

re rule B, whereby the bank is closed and reorganized if a situationV < Bred during a (Poison-distributed) regulatory audit, implicitly de..nes
adequacy standard. For empirical plausibility, the modeled regulatory
hetotal value of the bank assets relative to the face value of its liabilitiesV = Bnot exceed unity. We require the regulatory authority to choose the
at the bank's equityholders become indi¤erent with respect to the risk
all asset values V = B. Mathematically, this implies:

$$\frac{\partial E_1(V)}{\partial \sigma^2} \stackrel{\sim}{_{\sim}} 0 \tag{22}$$

I *B* which satisi..es this condition will depend on the level of σ .¹ One the regulator announces the function $B(\sigma)$, and then the bank picks σ . Iosure rule implied by the model accords quite well with the regulatory y OECD countries, for example, by the EU capital adequacy directives. e bank's equity requirements are directly proportional to its value at ework is proportional to σ .²

the requirement that risk shifting incentives are completely eliminated this paper models a stylized objective function of the regulator. In a e one modeled here, regulators should take into account their monitoring eight costs of deposit insurance and bank reorganiation. In a more es of the riskiness of a bank's operations would also in‡uence other ts asset value. If this e¤ect is very strong, then eliminating risk-shifting from the overall value-maximizing investment choice would be most , the implementation of any regulatory strategy will require that it is for the banks' equityholders not to alter the investment strategies on are predicated.

he ..r.st derivative of the equity value with respect to a change in the

ital requirements on a given day are actually calculated on the basis of the maximum nt value at risk and a historic average value at risk e.g. over the past 60 days.

stify the regulatory closure rule adopted in the following analysis is to assume that the s observable but not veri..able in a court ex post. The regulator can choose *B* based on bank risk ex post. More extreme sanctions for excessive risk taking, such as jail terms feasible. Complications would be introduced when the riskiness of a bank's asset/loan eriorate owing to luck, rather than via explicit asset substitution by its managers or equity k's regulator can not distinguish that from unlucky shifts in σ then the optimal regulatory our model should always be implemented, if the potential societal losses from σ -choices maximize *V* are su¢ ciently large, relative to any costs arising from the bank managers' e (unmodeled) costs of bank closure or reorganisation.

s if and only if:

$$\tilde{\alpha_1} = \alpha_1 \tag{23}$$

ation we need to determine the model parameters B and B^* endogeons (7) and (13) above, the above condition (23) can be rewritten as

$$(\tilde{\beta}_{1} \ \mathbf{j} \ \beta_{1})y_{1}B^{y_{1}} + (\tilde{\beta}_{2} \ \mathbf{j} \ \beta_{2})y_{2}B^{y_{2}} \ \mathbf{j} \ \frac{\lambda B}{\lambda + \delta} = 0$$
(24)

Igebraic manipulations, we can show that B and B^* are the solutions ly) non-linear two-dimensional system of equations:

$$\begin{bmatrix} \tilde{\beta}_{1}(B,B^{*}) \\ i \\ \beta_{1}(B,B^{*}) \end{bmatrix} y_{1}B^{y_{1}} + \begin{bmatrix} \tilde{\beta}_{2}(B,B^{*}) \\ i \\ \beta_{2}(B,B^{*}) \end{bmatrix} y_{2}B^{y_{2}} i \\ \frac{\lambda B}{\lambda + \delta} = 0$$
(25)

$$\left[\tilde{\beta}_{1}(B,B^{*}) \, \mathbf{j} \, \beta_{1}(B,B^{*})\right] y_{1}B^{*y_{1}} + \left[\tilde{\beta}_{2}(B,B^{*}) \, \mathbf{j} \, \beta_{2}(B,B^{*})\right] y_{2}B^{*y_{2}} + \frac{\delta B^{*}}{\lambda + \delta} = 0 \quad (26)$$

r or not a solution exists, and if so if it has the economic property that $B \ B^* \ 0$ stence of such a solution has to be proved, of course. In order to solve quations it turns out that a substitution of the form $u = \frac{B}{B^*}$ is useful. le the above equations have the following functional form:

$$\frac{B^* \left[\delta x_1(y_1 \mathbf{i} \ y_2) u^{y_1+y_2} \mathbf{i} \ \lambda x_1 u \left[(1 \mathbf{i} \ y_1) u^{y_1} \mathbf{i} \ (1 \mathbf{i} \ y_2) u^{y_2}\right]\right]}{\lambda + \delta} + (27)$$

$$\frac{C(\tau \mathbf{i} \ 1) \left[x_1(y_1 \mathbf{i} \ y_2) u^{y_1+y_2} + \frac{\lambda}{r} x_1(y_1 u^{y_1} \mathbf{i} \ y_2 u^{y_2})\right]}{\lambda + r} = 0$$

$$\frac{B^* \left[\delta \left[(x_1 \ \mathbf{i} \ y_1)(1 \ \mathbf{j} \ y_2)u^{y_1} \ \mathbf{i} \ (x_1 \ \mathbf{j} \ y_2)(1 \ \mathbf{j} \ y_1)u^{y_2} \right] \mathbf{i} \ \lambda u(1 \ \mathbf{j} \ x_1)(y_1 \ \mathbf{j} \ y_2)}{\lambda + \delta} + (28)$$

$$\frac{C(\tau \ \mathbf{j} \ 1) \left[y_1(x_1 \ \mathbf{j} \ y_2)u^{y_2} \ \mathbf{j} \ y_2(x_1 \ \mathbf{j} \ y_1)u^{y_1} + \frac{\lambda}{r}x_1(y_1 \ \mathbf{j} \ y_2) \right]}{\lambda + r} = 0$$

linear in B^* and non-linear in u. Therefore we can write down B^* as a ouple u form B^* . Doing this we get:

$$B^{*} = \frac{C(1 \mathbf{i} \tau)(\lambda + \delta)}{\lambda + r} \frac{\left[x_{1}(y_{1} \mathbf{i} y_{2})u^{y_{1} + y_{2}} + \frac{\lambda}{r}x_{1}(y_{1}u^{y_{1}} \mathbf{i} y_{2}u^{y_{2}})\right]}{[\delta x_{1}(y_{1} \mathbf{i} y_{2})u^{y_{1} + y_{2}} \mathbf{i} \lambda x_{1}u \left[(1 \mathbf{i} y_{1})u^{y_{1}} \mathbf{i} (1 \mathbf{i} y_{2})u^{y_{2}}\right]\right]}$$
only one non-linear equation in *u*:
$$(29)$$

$$\frac{y_1(x_1 \mathbf{i} \ y_2)u^{y_2} \mathbf{i} \ y_2(x_1 \mathbf{i} \ y_1)u^{y_1} + \frac{\lambda}{r}x_1(y_1 \mathbf{i} \ y_2)}{\delta[(x_1 \mathbf{i} \ y_1)(1 \mathbf{i} \ y_2)u^{y_1} \mathbf{i} \ (x_1 \mathbf{i} \ y_2)(1 \mathbf{i} \ y_1)u^{y_2}] \mathbf{i} \ \lambda u(1 \mathbf{i} \ x_1)(y_1 \mathbf{i} \ y_2)} \mathbf{i} \qquad (30)$$

$$\frac{x_1(y_1 \mathbf{i} \ y_2)u^{y_1+y_2} + \frac{\lambda}{r}x_1(y_1u^{y_1} \mathbf{i} \ y_2u^{y_2})}{\delta x_1(y_1 \mathbf{i} \ y_2)u^{y_1+y_2} \mathbf{i} \ \lambda x_1u \left[(1 \mathbf{i} \ y_1)u^{y_1} \mathbf{i} \ (1 \mathbf{i} \ y_2)u^{y_2}\right]} = 0$$

at the left hand side of (29) is always positive and hence that B^* is equation (30) is independent of C and τ and therefore also u does not meters. These observations yield immediately the following result.

- **Theorem 1** i) There exists a solution (B, B^*) with the property that $B > B^* > 0$ iff the equation (30) has a solution u > 1.
 - ii) B and B^* are linear in the coupon C
 - iii) B and B^* are linear in the subsidy rate τ
 - iv) B and B^* do not depend on V.

u

ce of a solution u > 1 for the case $\delta = 0$. (The case $\delta \Leftrightarrow 0$ is similar but tions are rather cumbersome.)

Proposition 1 Assume that there are no net cash outflows, i.e. $\delta = 0$. Then the non-linear equation (30) has a solution u > 1.

Proof: $\delta = 0$ ation (30) attains the following form

$$\begin{aligned} f(u) &\stackrel{}{\leftarrow} \frac{y_1(x_1 \mid y_2)(1 \mid y_2)u^{2y_2}}{(1 \mid x_1)(y_1 \mid y_2)} + \\ & \frac{y_2(x_1 \mid y_1)(1 \mid y_1)u^{2y_1}}{(1 \mid x_1)(y_1 \mid y_2)} \mathbf{i} \\ \frac{[y_1(x_1 \mid y_2)(1 \mid y_1) + y_2(x_1 \mid y_1)(1 \mid y_2) \mid (y_1 \mid y_2)^2(1 \mid x_1)]u^{y_1 + y_2}}{(1 \mid x_1)(y_1 \mid y_2)} \mathbf{i} \\ & \frac{\lambda(x_1 \mid y_1)u^{y_1}}{r(1 \mid x_1)} + \frac{\lambda(x_1 \mid y_2)u^{y_2}}{r(1 \mid x_1)} = 0 \end{aligned}$$

shows that

$$f(1) = \frac{(y_2 | y_1)}{1 | x_1} + \frac{\lambda(y_2 | 1)}{r(1 | x_1)} < 0$$

 $\lim_{u\to\infty} f(u) = 1$. These two properties of the function f(u) together with its there exists a u > 1 which solves the equation. Q.E.D.

quence of Theorem 1 is that there exists (it is easy to write down for these, but this is not done due to the length of the expressions) $a_1, b_1, \tilde{b_1}, b_2, \tilde{b_2}$ which are independent of the coupon rate C such that the following

$$D_1(V) = \frac{C}{r} \left[1_i \ a_1 \left(\frac{V}{C} \right)^{x_1} \right]$$
(31)

$$D_2(V) = \frac{\lambda V}{\lambda + \delta} + \frac{C}{r} \left[\frac{r}{\lambda + r} \mathbf{i} \ b_1 \left(\frac{V}{C} \right)^{y_1} \mathbf{i} \ b_2 \left(\frac{V}{C} \right)^{y_2} \right]$$
(32)

$$TV_1(V) = V + \frac{C}{r} \left[\tau_i \ a_1 \left(\frac{V}{C} \right)^{x_1} \right]$$
(33)

$$TV_2(V) = V + \frac{C}{r} \left[\frac{\tau r}{\lambda + r} \,_{\mathbf{i}} \, \tilde{b_1} \left(\frac{V}{C} \right)^{y_1} \,_{\mathbf{i}} \, \tilde{b_2} \left(\frac{V}{C} \right)^{y_2} \right]$$
(34)

$$E_1(V) = V_i \frac{C(1_i \tau)}{r}$$
(35)

$$E_2(V) = \frac{\delta}{\delta + \tau} V_{\mathbf{i}} \frac{C}{r} \left[(1_{\mathbf{i}} \tau)_{\mathbf{j}} (\tilde{b_1}_{\mathbf{i}} b_1) \left(\frac{V}{C}\right)^{y_1}_{\mathbf{i}} (\tilde{b_2}_{\mathbf{i}} b_2) \left(\frac{V}{C}\right)^{y_2} \right]$$
(36)

1 nd 2 indicate the appropriate domain $[B^*, B]$ and [B, 1), respectively.

3 Numerical Results

umerical comparative static results. For this purpose we have calibrated e model to represent realistic values.³ Unless otherwise stated, all sults in this section are based on the following parameter values:

Parameter	Value
C	70
r	5 %
δ	4.2 %
λ	•

equityholders would stop the interest payments and declare default, B^* , he asset value at which the regulator should choose to close the bank, 4. The latter value would have equaled 1400(1 j 0.1) = 1260 in the 4), in which continuous observability of the asset value V is assumed; debt interest payments the optimal closure rule in that model would B = 1400, to keep the debt claim riskless. Note that, unlike in Leland's model, ly ensure risk-invariance of the equity value E for V > B, provided he level of risk that the bank's equityholders would choose (optimally, e the overall asset value V). The reason for this is that the chosen pact on the bank's equityholders' choice between continuing to make sus declaring default in the region where V < B, but this is as yet not ator's audits, and this choice would in turn a pect their equity value in V > Bhus, our criterion for the regulator's choice of a closure rule presumes k level is chosen irrevocably when the bank is still "solvent" (V > B), knowledge of the bank's risk-choice and thus she can adapt her closure choice by equityholders, so as to induce the optimal (the overall asset oice of asset return risk. Such a scenario is relevant when the bank's able (by the regulator) but not veri. able", so that stronger contractual nfeasible. It also presumes a certain degree of regulatory discretion re, exercised in the interest of overall value maximization, which may lator is separated from an agency that insures the (par) value of the

V

B

B

3.1 Comparative Statics of Debt Value and Equity Value

igure 1, both B (solid line) and B^* (dotted line) are linearly increasing on rate C. The equity value is a decreasing function of C, since a aises the debt (deposit) value, and the increased default cum closure with a higher coupon reduces the value of the subsidy in proportion

Figure 1: Closure, Default, and the Debt Coupon

ayout ratio increases, the regulatory closure level B which eliminates s..r.st increases and then decreases. The latter exect occurs because the e drift rate on the bank's asset value, so that equityholders voluntarily her levels of V. The resulting loss of the value of the subsidy makes costly to equityholders, thereby allowing for a lower regulatory closure cts are depicted in Figure 2 below, in which it is remarkable that the payout ratio on the optimal regulatory closure rule, B, is miniscule ct on B^* , for example).

Figure 2: Closure, Default, and the Payout Ratio

perspective, an important policy instrument aside for capital controls is r its Poisson intensity. Since auditing the bank more frequently lowers and debt value losses, the regulator optimally chooses a lower closure r frequency of audits. The opposite is true for equityholders' choice nce a higher audit frequency increases the probability of losing future is insolvent, when $B^* < V < B$, the equityholders optimally default levels, B^* s, as the audit frequency increases. Figure 3 illustrates the s of these two exects.

igure 3: Closure, Default, and the Audit Frequency

B

of regulatory audit frequency on the bank's debt value is therefore not is value would impact on the value of the regulator's liabilities if she , as in Merton (1977, 1978). Our numerical calibrations suggest that higher audit frequency on the debt value is small for a solvent bank, ...cial impact on the debt value in a seriously insolvent bank, as depicted 5 below. The equity value of a solvent bank also shows no signi..cant frequency.

re 4: Debt Value and Audit Frequency (Solvent Bank)

e 5: Debt Value and Audit Frequency (Insolvent Bank)

n with respect to capital requirements and closure rules is how these retect changes in the underlying (and overall value-maximizing) level k's asset portfolio. Our numerical results show that the optimal closure

ing function of the asset return volatility, \ddagger owing from the related obserer volatility equityholders would voluntarily continue coupon payments thus lowering B^* . It is also interesting to note, in Figure 6 below, that een B and the asset returns volatility is almost linear, as suggested by proach for marketable assets, even though the closure criterion in our d by .xing the probability of insolvency (over a given horizon) as such.

ure 6: Closure, Default, and Asset Return Volatility

s of the two exects noted above, increased asset returns volatility still of a solvent bank. It leaves the equity value of a solvent bank unaxected

B adjusted appropriately. Nevertheless, the equity value of an insolvent ncreasing in its asset return volatility (Figure 7), so that the regulatory e can not eliminate bank equityholders "gambling for resurrection" via he range $B^* < V < B$; only intrusive monitoring of the bank's activities *B* in recent audits) might rule this out.

V

Equity Value and Asset Return Volatility (Insolvent Bank)

mpare the value of the bank at the closure point TV(B), with the face , C/r. Numerical simulations show that this ratio is less than one over nge of parameter values. Figure 8 gives the comparative static results pect to changes in the volatility.

k Value, Face Value of Debt, and Asset Return Volatility at Closure

e the relationship between the closure points *B* and *B*^{*} and the subsidy 9 shows that both the point at which the regulator wishes to close the t which the equityholders abandon the bank decrease signi..cantly with license becomes more valuable, the equityholders have less incentives to a lower closure point su¢ ces to eliminate any risk shifting incentives.

Figure 9: Closure, Default, and Subsidy

au

au

3.2 Bank Closure and Capital Adequacy

y Directive regulates the bank's equity requirements in such a way that capital, \overline{E} , must exceed a certain percentage of the market value of its

\bar{E} , $k \ TV(V)$

kepends on the risk of the bank's assets. If we use the equity value,E(V) \bar{E} and evaluate the ratio of E(V) to TV(V) at the closure level B, ourhat the factor k increases (almost) linearly with the asset volatility σ ,ith the Value at Risk approach, where $k = \alpha \sigma$ (Figure 10). For theage of 3 per year) in our base case, k increases from 1.56%, to 3.62%,5.60%the annual standard deviation of asset returns grows from 5% to 10%15%of which is more likely), well within the range of the recommended extios in the Capital Adequacy Directive.

0: Market Equity Ratio at Closure and Return Volatility

e of capital requirements a Scola Raiu Rr or Rsd in V

4 Conclusion

main contributions to the literature on intertemporal bank regulation. sly consider several regulatory policy instruments. More speci..cally, ork is speci..ed by (i) a capital replenishment rule or an equity value at re required to contribute more capital; (ii) a closure rule or an equity e bank is closed if audited; and (iii) a frequency at which a bank is analysis demonstrates how capital adequacy, bank auditing and closure signed to eliminate or mitigate bank equityholders' incentives to take r choice of bank assets. In contrast to existing literature we demonstrate ion of capital replenishment, closure and auditing regulation completely incentives as long as the bank is solvent.

reveal how the model parameters in tuence the regulatory policy which entive compatible risk choices. First, we ..nd that closure levels are early with the risk of the underlying assets. This provides a rationale n should be linked to the bank's market risk, possibly quanti..ed by inodels. Second, somewhat surprisingly, there is no clear cut relationship nishment rules implied by dividend constraints and optimal closure.

ble, namely the frequency of bank audits, signi..cantly a¤ects the optiher frequency of bank auditing allows the regulator to close the bank at thout creating adverse risk-shifting incentives. Our analysis also yields capital adequacy regulation. We provide solutions for equity market apital replenishment is required, as well as equity market values below lace upon an audit. For reasonable parameter values our model procy levels which are signi..cantly higher than the eight percent currently tandards. This framework may be extended in several ways. First, the ould be made stochastic. This would introduce another source of risk into account both by equityholders and regulators. Second, λ could after an audit. In particular, the regulator may want to increase λ s that the bank is close to bankruptcy. Third, an objective function be speci..ed so that an optimal mix of regulatory instruments can be uld require making assumptions about costs of audits, bankruptcy etc.

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