Bank Capital Regulation

With Random Audits

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Abstract

1 Introduction

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

2 The Model

$$\frac{dV}{V} = [\mu(V,t) - \delta]\, dt + \sigma dz$$

$$dA = dq$$

2.1 Value of Debt

r C

$D(V, A; B^*, B, \sigma, r, C, \lambda, \delta)$

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r - \delta) V D_V - rD + C + \mathbf{1}_{[B^*,B]}\lambda(V - D) = 0$$

$$\lim_{V\to\infty} D(V) = \frac{C}{r}$$

$$\lim_{V\to B^+} D(B) = \lim_{V\to B^-} D(B)$$

$$\lim_{V\to B^+} D_V(B) = \lim_{V\to B^-} D_V(B)$$

$$D(B^*) = B^*$$

V

$$D_1(V) = \frac{C}{r} + \alpha_1 V^{x_1} + \alpha_2 V^{x_2}$$

$V \geq B$

$$D_2(V) = \frac{C}{r+\lambda} + \frac{\lambda}{\lambda+\delta} V + \beta_1 V^{y_1} + \beta_2 V^{y_2}$$

$B^* \leq V \leq B$

$$x_{1,2} = \frac{\sigma^2 - 2(r-\delta) \pm \sqrt{[\sigma^2 - 2(r-\delta)]^2 + 8r\sigma^2}}{2\sigma^2}$$

$$y_{1,2} = \frac{\sigma^2 - 2(r-\delta) \pm \sqrt{[\sigma^2 - 2(r-\delta)]^2 + 8(r+\lambda)\sigma^2}}{2\sigma^2}$$

x_1 y_1

$\alpha_2 = 0,$

α_1 β_1 β_2

$$\beta_1 = \frac{B^{*y_2}\left[\frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta}\right] - (x_1 - y_2)B^{y_2}\left[\frac{\delta B^*}{\lambda+\delta} - \frac{C}{\lambda+r}\right]}{(x_1 - y_1)B^{y_1}B^{*y_2} - (x_1 - y_2)B^{*y_1}B^{y_2}}$$

$$\beta_2 = -\frac{B^{*y_1}\left[\frac{\lambda C x_1}{r(\lambda+r)} + \frac{\lambda(1-x_1)B}{\lambda+\delta}\right] - (x_1 - y_1)B^{y_1}\left[\frac{\delta B^*}{\lambda+\delta} - \frac{C}{\lambda+r}\right]}{(x_1 - y_1)B^{y_1}B^{*y_2} - (x_1 - y_2)B^{*y_1}B^{y_2}}$$

$$\alpha_1 = \frac{\lambda B^{1-x_1}}{x_1(\lambda+\delta)} + \frac{\beta_1 y_1 B^{y_1 - x_1}}{x_1} + \frac{\beta_2 y_2 B^{y_2 - x_1}}{x_1}$$

B^* B

2.2 Value of Subsidy

τC S

$$\frac{1}{2}\sigma^2 V^2 S_{VV} + (r - \delta) V S_V - rS + \tau C - \mathbf{1}_{[B^*,B]}\lambda S = 0$$

$$\lim_{V\to\infty} S(V) = \frac{\tau C}{r}$$

$$\lim_{V\to B^+} S(B) = \lim_{V\to B^-} S(B)$$

$$\lim_{V\to B^+} S_V(B) = \lim_{V\to B^-} S_V(B)$$

$$S(B^*) = 0$$

V

$$S_1(V) = \frac{\tau C}{r} + \tilde{\alpha}_1 V^{x_1} + \tilde{\alpha}_2 V^{x_2}$$

$V \geq B$

$$S_2(V) = \frac{\tau C}{r+\lambda} + \tilde{\beta}_1 V^{y_1} + \tilde{\beta}_2 V^{y_2}$$

$B^* \cdot V \cdot B$ $x_{1,2}$ $y_{1,2}$

$\tilde{\alpha}_2 = 0$

$$\tilde{\beta}_1 = \frac{\tau C \left[\frac{\lambda x_1 B^{*y_2}}{r} + (x_1 - y_2) B^{y_2}\right]}{(\lambda + r)\left[(x_1 - y_1) B^{y_1} B^{*y_2} - (x_1 - y_2) B^{*y_1} B^{y_2}\right]}$$

$$\tilde{\beta}_2 = - \frac{\tau C \left[\frac{\lambda x_1 B^{*y_1}}{r} + (x_1 - y_1) B^{y_1}\right]}{(\lambda + r)\left[(x_1 - y_1) B^{y_1} B^{*y_2} - (x_1 - y_2) B^{*y_1} B^{y_2}\right]}$$

$$\tilde{\alpha}_1 = \frac{\tilde{\beta}_1 y_1 B^{y_1 - x_1}}{x_1} + \frac{\tilde{\beta}_2 y_2 B^{y_2 - x_1}}{x_1}$$

B^* B

TV

2.3 Total Value of the Bank and Equity Value

$TV(V)$

$$TV(V) = V + S(V)$$

$$E(V) = V + S(V) - D(V)$$

$$E_1(V) = \frac{C(\tau - 1)}{r} + V + (\tilde{\alpha}_1 - \alpha_1)V^{x_1}$$

$V \geq B$

$$E_2(V) = \frac{C(\tau - 1)}{r} + \frac{\delta V}{\lambda + \delta} + (\tilde{\beta}_1 - \beta_1)V^{y_1} + (\tilde{\beta}_2 - \beta_2)V^{y_2}$$

$B^* \leq V \leq B$

B^*

$$E_V(B^*) = 0$$

$$\frac{\delta B^*}{\lambda + \delta} + (\tilde{\beta}_1 - \beta_1)y_1 B^{*y_1} + (\tilde{\beta}_2 - \beta_2)y_2 B^{*y_2} = 0$$

B

$V < B$

$V = B$

B

σ $V \geq B$

$$\frac{\partial E_1(V)}{\partial \sigma^2} \leq 0$$

B σ

$B(\sigma)$ σ

σ

B

σ

$B(\sigma)$ σ

V

$$\tilde{\alpha}_1 = \alpha_1$$

B B^*

$$(\tilde{\beta}_1 - \beta_1) y_1 B^{y_1} + (\tilde{\beta}_2 - \beta_2) y_2 B^{y_2} - \frac{\lambda B}{\lambda + \delta} = 0$$

B B^*

$$\begin{aligned} \left[\tilde{\beta}_1(B, B^*) - \beta_1(B, B^*)\right] y_1 B^{y_1} + \left[\tilde{\beta}_2(B, B^*) - \beta_2(B, B^*)\right] y_2 B^{y_2} - \frac{\lambda B}{\lambda + \delta} &= 0 \\ \left[\tilde{\beta}_1(B, B^*) - \beta_1(B, B^*)\right] y_1 B^{*y_1} + \left[\tilde{\beta}_2(B, B^*) - \beta_2(B, B^*)\right] y_2 B^{*y_2} + \frac{\delta B^*}{\lambda + \delta} &= 0 \end{aligned}$$

$B, B^* \geq 0$

$u = \frac{B}{B^*}$

$$\frac{B^* \left[\delta x_1 (y_1 - y_2) u^{y_1 + y_2} - \lambda x_1 u \left[(1 - y_1) u^{y_1} - (1 - y_2) u^{y_2}\right]\right]}{\lambda + \delta} + \frac{C(\tau - 1)\left[x_1 (y_1 - y_2) u^{y_1 + y_2} + \frac{\lambda}{r} x_1 (y_1 u^{y_1} - y_2 u^{y_2})\right]}{\lambda + r} = 0$$

$$\frac{B^*\left[\delta\left[(x_1-y_1)(1-y_2)u^{y_1}-(x_1-y_2)(1-y_1)u^{y_2}\right]-\lambda u(1-x_1)(y_1-y_2)\right]}{\lambda+\delta}+\frac{C(\tau-1)\left[y_1(x_1-y_2)u^{y_2}-y_2(x_1-y_1)u^{y_1}+\frac{\lambda}{r}x_1(y_1-y_2)\right]}{\lambda+r} = 0$$

[illegible] B^* [illegible] u [illegible] B^* [illegible] u [illegible] u B^*

$$B^*=\frac{C(1-\tau)(\lambda+\delta)}{\lambda+r}\frac{\left[x_1(y_1-y_2)u^{y_1+y_2}+\frac{\lambda}{r}x_1(y_1u^{y_1}-y_2u^{y_2})\right]}{\left[\delta x_1(y_1-y_2)u^{y_1+y_2}-\lambda x_1u\left[(1-y_1)u^{y_1}-(1-y_2)u^{y_2}\right]\right]}$$

[illegible] u

$$\frac{y_1(x_1-y_2)u^{y_2}-y_2(x_1-y_1)u^{y_1}+\frac{\lambda}{r}x_1(y_1-y_2)}{\delta\left[(x_1-y_1)(1-y_2)u^{y_1}-(x_1-y_2)(1-y_1)u^{y_2}\right]-\lambda u(1-x_1)(y_1-y_2)}-\frac{x_1(y_1-y_2)u^{y_1+y_2}+\frac{\lambda}{r}x_1(y_1u^{y_1}-y_2u^{y_2})}{\delta x_1(y_1-y_2)u^{y_1+y_2}-\lambda x_1u\left[(1-y_1)u^{y_1}-(1-y_2)u^{y_2}\right]} = 0$$

[illegible] B^* [illegible] C τ [illegible] u [illegible]

Theorem 1 *i) There exists a solution (B, B^*) with the property that $B > B^* > 0$ iff the equation (30) has a solution $u > 1$.*

ii) B and B^* are linear in the coupon C

iii) B and B^* are linear in the subsidy rate τ

iv) B and B^* do not depend on V.

[illegible] $u > 1$ [illegible] $\delta = 0$ [illegible] $\delta \neq 0$ [illegible]

Proposition 1 *Assume that there are no net cash outflows, i.e.* $\delta = 0$*. Then the non-linear equation (30) has a solution* $u > 1$*.*

Proof: $\delta = 0$

$$
\begin{aligned}
f(u) \equiv\ & \frac{y_1(x_1 - y_2)(1 - y_2)u^{2y_2}}{(1 - x_1)(y_1 - y_2)} + \\
& \frac{y_2(x_1 - y_1)(1 - y_1)u^{2y_1}}{(1 - x_1)(y_1 - y_2)} - \\
& \frac{\left[y_1(x_1 - y_2)(1 - y_1) + y_2(x_1 - y_1)(1 - y_2) - (y_1 - y_2)^2(1 - x_1)\right]u^{y_1+y_2}}{(1 - x_1)(y_1 - y_2)} - \\
& \frac{\lambda(x_1 - y_1)u^{y_1}}{r(1 - x_1)} + \frac{\lambda(x_1 - y_2)u^{y_2}}{r(1 - x_1)} = 0
\end{aligned}
$$

$$
f(1) = \frac{(y_2 - y_1)}{1 - x_1} + \frac{\lambda(y_2 - 1)}{r(1 - x_1)} < 0
$$

$\lim_{u\to\infty} f(u) = \infty$ $f(u)$

$u > 1$ **Q.E.D.**

$a_1, b_1, \tilde{b}_1, b_2, \tilde{b}_2$ C

$$
\begin{aligned}
D_1(V) &= \frac{C}{r}\left[1 - a_1\left(\frac{V}{C}\right)^{x_1}\right] \\
D_2(V) &= \frac{\lambda V}{\lambda + \delta} + \frac{C}{r}\left[\frac{r}{\lambda + r} - b_1\left(\frac{V}{C}\right)^{y_1} - b_2\left(\frac{V}{C}\right)^{y_2}\right]
\end{aligned}
$$

$$TV_1(V) = V + \frac{C}{r}\left[\tau - a_1\left(\frac{V}{C}\right)^{x_1}\right]$$

$$TV_2(V) = V + \frac{C}{r}\left[\frac{\tau r}{\lambda + r} - \tilde{b_1}\left(\frac{V}{C}\right)^{y_1} - \tilde{b_2}\left(\frac{V}{C}\right)^{y_2}\right]$$

$$E_1(V) = V - \frac{C(1-\tau)}{r}$$

$$E_2(V) = \frac{\delta}{\delta+\tau}V - \frac{C}{r}\left[(1-\tau) - (\tilde{b_1} - b_1)\left(\frac{V}{C}\right)^{y_1} - (\tilde{b_2} - b_2)\left(\frac{V}{C}\right)^{y_2}\right]$$

1 2 $[B^*, B]$ $[B, \infty)$

3 Numerical Results

C	
r	
δ	
λ	

V B^*

B $1400(1 - 0.1) = 1260$

V

$B = 1400$

E $V > B$

V

$V < B$

$V > B$

$V > B$

B

3.1 Comparative Statics of Debt Value and Equity Value

B B^*

C C

B

V

B

B^*

[illegible] B [illegible]

[illegible] $B^* < V < B$ [illegible]

[illegible] B^* [illegible]

[illegible]

[illegible]

[illegible]

B

B^*

B

B

$B^* < V < B$

V B

$TV(B)$

C/r

B B^*

τ

τ

3.2 Bank Closure and Capital Adequacy

$\bar{E}$

$$\bar{E} \; ¸ \; k\, TV(V)$$

k

$E(V)$ $\bar{E}$ $E(V)$ $TV(V)$ B

k σ

$k = \alpha\, \sigma$

k 1.56% 3.62%

5.60% 5% 10%

15%

4 Conclusion

λ

λ

References

[illegible]