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Modelling a Housing and Mortgage Crisis*

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Abstract

The purpose of this paper is to explore financial instability in this case due to a housing crisis and defaults on mortgages. The model incorporates heterogeneous banks and households. Mortgages are secured by collateral, which is equal to the amount of housing which agents purchase. Individual default is spread through the economy via the interbank market. Several comparative statics illustrate the directional effects of a variety of shocks in the economy.

Keywords: Mortgage default; Financial fragility; Competitive banking; General equilibrium; Monetary policy

JEL Classification: D52; E4; E5; G1; G2

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1 Introduction

The current crisis has centred on borrower defaults on mortgages, and the knock-on effects of that on banks' own credit standing (and in several cases their own default), and hence on tightened conditions for lending to new (mortgage) borrowers. No model which does not incorporate such key elements, or at least most of them, can possibly hope to capture the defining features of the current crisis, certainly not standard DSGE models which (mostly) assume away the possibility of default altogether!

What we have done in this paper is to build on our previous model of a system in which default plays a central role for both borrowers and banks, and in which financial intermediation and money, therefore, have a necessary real function, to include both an additional good, housing, in addition to the prior composite basket of other goods and services, and an additional agent, a new entrant to the housing market. Our previous papers on this include Goodhart, Sunirand and Tsomocos (2004, 2005, 2006a and b).

Dealing with a model with default and heterogeneous agents is not straightforward, which is why standard DSGE models abstract from such concerns, despite their resulting lack of realistic micro-foundations. We, therefore, regard this paper as a preliminary step in a longer exercise. In particular, the shocks that we model in the second period of our two period model, (the first being an initial predetermined set-up period), can be categorised as supply shocks, in which the endowment of our agents declines greatly in the case of adverse shocks. Nevertheless, our model is general enough to allow for the examination of a wide variety of shocks, which can lead to financial instability.

In practice, the main adverse shock in 2007/8 was a sharp decline in housing prices (in the USA), whereas previously they had been expected to continue rising, or to hold steady at worst. In a future version of this paper we will experiment with this and other financial shocks. The main reason for proceeding with the current model is simply shortage of time, (with this kind of simulation model the authors learn as they go along, and we started with the shocks that we had used previously in our prior work). But there were adverse supply shocks facing Western economies, in the guise of rising energy and commodity prices, in 2007/8, and these played a role in worsening the current downturn. Furthermore we can, and do, include in our simulations examples of changes in financial conditions, e.g. changes in the money stock (interest rates) and in bank capital endowments, so we can potentially explore how financial policy measures (e.g. government recapitalisation of banks) may affect the outcome. Nevertheless this should be treated as a preliminary exercise.

The plan of the paper is as follows. In Section II we briefly reprise the basic structure of the GST model and detail the innovations that we have made here. In Section III we set out how the model works and its clearing conditions. In Section IV we report our choice of exogenous parameters for our numerical simulation and describe the resulting equilibrium values. Then in Section V, we report on the comparative statics of changes in the parameters chosen. Section VI concludes.

With such a large, and alas complicated, model there are a vast number of such exercises that could be run, each with an accompanying set of tables and diagrams. While in one sense this is a strength of this approach, since one can examine a huge variety of potential shocks and policy responses, both individually and in conjunction, it can also lead to a mind-boggling multiplication of detail.

In pursuit of focus and comparative simplicity, we are focusing here on just 4 examples. These are a decrease in the money supply in the initial period, an increased desire to take on risk, (as occurred in 2003-6, and leads to adverse shocks having a stronger effect on the system), a (foreseen) intervention by the authorities to provide liquidity assistance in very bad states, and finally a compound combination of the former two simultaneously, (partly to examine how non-linear are the resulting effects).

2 The background set-up

Goodhart, Sunirand and Tsomocos (2004, 2006) and Tsomocos (2003) have developed models of financial stability that are rich enough to include defaultable consumer loan, deposit and interbank markets. In their models consumers maximize their expected utility from consumption of goods and banks maximize their expected profits. The main financial imperfection in Goodhart *et al.* (2004, 2006) is that they assume that individual bank borrowers are assigned , by history or by informational constraints, to borrow from a single bank. Money is introduced by a cash in advance constraint, whereby a private agent needs money to buy commodities from other agents; commodities cannot be used to buy commodities. Similarly they assume that agents needing money can always borrow cheaper from their (assigned) bank than from other agents; banks have an informational (and perhaps scale) advantage that gives them a role as an intermediary. The amount of loans they repay is a choice variable for consumers, thus default in these models is endogenous.

In their general model (2006) there are a set of heterogeneous private sector agents with initial endowments of both money and commodities; it is an endowment model without production. There is also a set of heterogeneous banks, who similarly have differing initial allocations of capital (in the form of government bonds). There are two other players, a Central Bank which can inject extra money into the system through open market operations (OMOs), and a Financial Supervisory Agency, which can set both liquidity and capital minimum requirements and imposes penalties on failures to meet such requirements and on defaults.

The main purpose of this paper is to model the market for mortgages and to examine the implications of default in bank lending and of a housing market crisis. To do so, we alter the above framework in the following ways.

1. We introduce another good into the economy which is durable and gives utility in every period. The utility of consuming this good resembles the utility from buying a house. For tractability the durable good (house) is assumed to be infinitely divisible.

2. We explicitly model a market for mortgages. Consumers enter a mortgage contract to buy housing which they pledge as collateral. They default on their mortgage when the endogenous value of collateral is less than the amount they have to repay (Geanakoplos, 2003, Geanakoplos and Zame, 1995). When they default the bank seizes the amount of housing pledged as collateral and immediately offers it in the next period housing market. In this sense default is highly discontinuous as consumers do not choose the exact amount they want to default (as in the model discussed above), but only decide on whether to default or not[1].

3. We introduce a new agent λ who is only "born" in period two. The motivation behind this is that the healthy functioning of the housing market generally depends on the existence of first

[1] Shubik and Wilson (1977) and Dubey, Geanakoplos and Shubik (2005) analyse continuous default.

time buyers.

4. We allow for short-term loan markets operating within each period. This was not necessary in the Goodhart et al. models, but in our analysis it plays a crucial role to provide credit to first time buyers, i.e. Mr λ. For consistency, all agents can borrow short-term. In this market there is no uncertainty regarding repayment. We have the Central Bank intervening in the short-term loan markets in the second period to keep the interest rates (in the good state) at reasonable levels.

5. Since we are not considering wider asset markets, we exclude capital requirements for banks from our analysis.

3 The Model

Given the limited participation in the loan markets in our model, we need at least four agents-households (α,β,ϕ,λ) and two commercial banks (γ,δ). There are two periods and S states of the world. All agents maximize their utility over the consumption of the good and of housing in every period $t \in T = (0,1)$ and state $s \in S$. Banks maximize their expected profits in the second period. The set of all states is given by $s \in S^* = \{0\} \cup S$.

Agent $h \in (\alpha, \beta)$ is endowed with the good at every $s \in S^*$, whereas agent λ at every $s \in S$, since he enters the economy only in the second period. Agent ϕ is endowed with houses only at $t = 0$. Agents α and ϕ interact with bank γ, whereas agents β and λ with bank δ. All households are also given government cash free and clear of any obligations ($m_{s^*} \geq 0$). Both endowments and cash are allowed to vary across states of nature.

The Central Bank acts in the interbank market at t=0 by providing liquidity M^{CB} and in the short-term loan markets at t=1 by providing liquidity $M^{CB}_{\gamma s}$ and $M^{CB}_{\delta s}$ in those markets organized by banks γ and δ respectively.

In the following we give the timeline of our model and specify the optimization problems for all the participants in the economy.

3.1 The Time Structure of Markets

In each period $t \in T$, six markets meet: the short-term (intraperiod) loan, mortgage, deposit and interbank (intertemporal) markets meet simultaneously, and then the good and housing markets. Finally, short-term loans come due at the end of the period. This set-up maximizes the number of transactions possible and allows agents to borrow in the short term money market in order to invest in the long term bond or asset market. It also allows for an explicit speculative motive for holding money. Agents have the option of investing cash in the short loan market, and then carrying it over to the next period. The only reason why they may not do this is because they believe that they will get a higher return from holding deposits. While preserving Keynesian motives on the uses of money, it also provides a rationale for an upward sloping term structure.

Figure 1 indicates our time line, including the moments at which the various loans and assets come due. We make the sequence precise when we formally describe the budget set.

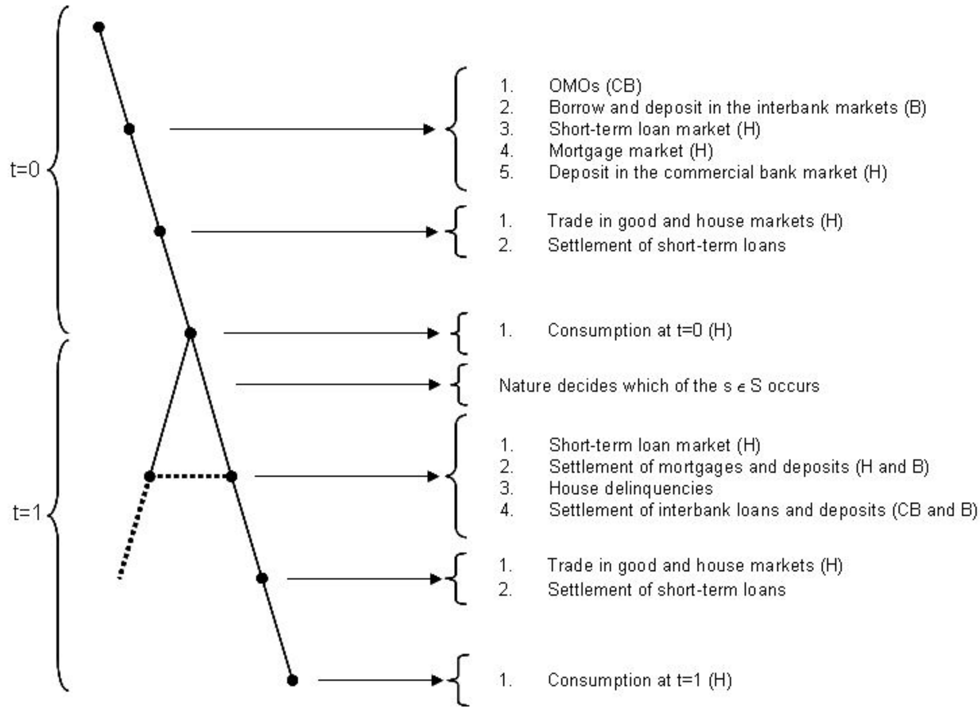

Figure 1: CB: Central Bank, B: Commercial Banks, H: Households

3.2 The agents' optimisation problems and market clearing conditions

3.2.1 Household $\alpha's$ and $\beta's$ optimization problem

Each consumer $h \in (\alpha, \beta)$ maximizes his payoff, which is his utility from consumption of the good and the house[2]. In order to acquire housing he enters a mortgage contract, which he has to repay in the last period. The amount of housing that he purchases is pledged as collateral. He honors his mortgage when the value of housing that he has bought is greater than the amount he has to repay. If it is lower, then he defaults on his mortgage and the bank that extended the mortgage seizes the collateral. In essence he repays the minimum between the two values, i.e. $\min(\text{ value of collateral, mortgage amount })$. We denote by $S_1^h \subset S$ the set of states that agent h does not default on his mortgage, i.e. $S_1^h = \{s \in S : \text{ value of collateral} \geq \text{mortgage amount}\}$. The maximization problem is as follows:

[2]In our simulations we use a CRRA utility function to account for wealth effects

$$\max_{q^h_{s1},b^h_{s2},\mu^h_s,\overline{\mu}^h} \Pi^h = u(e^h_{01}-q^h_{01})+u\left(\frac{b^h_{02}}{p_{02}}\right)+\sum_{s\in S}\theta_s u(e^h_{s1}-q^h_{s1})+\sum_{s\in S^1}\theta_s u\left(\frac{b^h_{02}}{p_{02}}+\frac{b^h_{s2}}{p_{s2}}\right)+\sum_{s\notin S^1}\theta_s u\left(\frac{b^h_{s2}}{p_{s2}}\right)$$

$$s.t. \quad b^h_{02} \leq \frac{\mu^h_0}{1+r^k_0}+\frac{\overline{\mu}^h}{1+\overline{r}^k}+m^h_0 \qquad (1)$$

$(i.e.,\quad \text{expenditure for housing at t} = 0 \ \leq \ \text{amount borrowed short} - \text{term at t} = 0 + \text{mortgage amount} \ + \ \text{initial private monetary endowment})$

$$\mu^h_0 \leq p_{01}q^h_{01} \qquad (2)$$

$(i.e.,\quad \text{short} - \text{term loan repayment} \ \leq \ \text{good sales at t} = 0)$

$$b^h_{s2}+\overline{\mu}^h \leq \frac{\mu^h_s}{1+r^k_s}+m^h_s, \ \forall \text{ s} \in \text{S}^\text{h}_1 \qquad (3)$$

$(i.e.,\quad \text{expenditure for housing in the second period , state s} \in \text{S}^\text{h}_1 \ + \ \text{mortgage repayment} \leq \text{amount borrowed short} - \text{term} \ + \ \text{private monetary endowment in s} \in \text{S}^\text{h}_1)$

$$\mu^h_s \leq p_{s1}q^h_{s1}, \ \forall \text{ s} \in \text{S}^\text{h}_1 \qquad (4)$$

$(i.e.,\quad \text{short} - \text{term loan repayment} \ \leq \ \text{good sales in s} \in \text{S}^\text{h}_1)$

$$b^h_{s2} \leq \frac{\mu^h_s}{1+r^k_s}+m^h_s, \ \forall \text{ s} \notin \text{S}^\text{h}_1 \qquad (5)$$

$(i.e.,\quad \text{expenditure for housing in the second period , state s} \notin \text{S}^\text{h}_1 \ \leq \ \text{amount borrowed short} - \text{term} \ + \ \text{private monetary endowment in s} \notin \text{S}^\text{h}_1)$

$$\mu^h_s \leq p_{s1}q^h_{s1}, \ \forall \text{ s} \notin \text{S}^\text{h}_1 \qquad (6)$$

$(i.e.,\quad \text{short} - \text{term loan repayment} \ \leq \ \text{good sales in s} \notin \text{S}^\text{h}_1)$

$$q^h_{s1} \leq e^h_{s1}, \ \forall \text{ s} \in \text{S}^* \qquad (7)$$

$(i.e.,\quad \text{quantity of goods sold in s} \ \leq \ \text{endowment of goods in s})$

where:
$k=\gamma$ for $h=\alpha$ and $k=\delta$ for $h=\beta$
$b^h_{s2} \equiv$ amount of fiat money spent by $h\in H$ to trade in the housing market in $s\in S^*$
$q^h_{s1} \equiv$ amount of goods offered for sale by $h\in H$ in $s\in S^*$
$\overline{\mu}^h \equiv$ mortgage amount that $h\in H$ takes out
$\mu^h_s \equiv$ short-term borrowing by $h\in H$ in $s\in S^*$
$\overline{r}^k \equiv$ mortgage rate offered by bank k
$r^k_s \equiv$ short-term rate offered by bank k in $s\in S^*$
$p_{s1} \equiv$ price of the good in $s\in S^*$
$p_{s2} \equiv$ price of housing in $s\in S^*$
$e^h_{s1} \equiv$ endowment of goods of $h\in H$ in $s\in S^*$
$m^h_s \equiv$ monetary endowment of $h\in H$ in $s\in S^*$

3.2.2 Household $\phi's$ optimization problem

Agent ϕ is endowed with housing at t=0, some (much) of which he sells to buy goods for consumption . He then deposits interperiod a part of the sales' receipts for use in the second period. His maximization problem is as follows:

$$\max_{q^{\phi}_{s2},b^{\phi}_{s1},\overline{d}^{\phi},\mu^{\phi}_{s}} \Pi^{\phi} = u\left(\frac{b^{\phi}_{01}}{p_{01}}\right) + u(e^{\phi}_{02} - q^{\phi}_{02}) + \sum_{s\in S}\theta_s u\left(\frac{b^{\phi}_{s1}}{p_{s1}}\right) + \sum_{s}\theta_s u\left(e^{\phi}_{02} - q^{\phi}_{02} - q^{\phi}_{s2}\right)$$

$$s.t.\quad b^{\phi}_{01} + \overline{d}^{\phi} \leq \frac{\mu^{\phi}_0}{1+r^{\gamma}_0} + m^{\phi}_0 \qquad (8)$$

$(i.e.,\ \ \text{expenditure for goods} + \text{interperiod deposits} \leq \text{amount borrowed short} - \text{term} + \text{private monetary endowment at t} = 0)$

$$\mu^{\phi}_0 \leq p_{02}q^{\phi}_{02} \qquad (9)$$

$(i.e.,\ \ \text{short} - \text{term loan repayment} \leq \text{housing sales at t} = 0)$

$$q^{\phi}_{02} \leq e^{\phi}_{02} \qquad (10)$$

$(i.e.,\ \ \text{quantity of housing sold at t} = 0 \leq \text{endowment of housing at t} = 0)$

$$b^{\phi}_{s1} \leq \frac{\mu^{\phi}_s}{1+r^{\gamma}_s} + \overline{d}^{\phi}(1+\overline{r}_d) + m^{\phi}_s \ \ \forall\ \text{s} \in \text{S} \qquad (11)$$

$(i.e.,\ \ \text{expenditure for goods} \leq \text{amount borrowed short} - \text{term} + \text{deposits and interest payment} + \text{private monetary endowment in s})$

$$\mu^{\phi}_s \leq p_{s2}q^{\phi}_{s2} \ \ \forall\ \text{s} \in \text{S} \qquad (12)$$

$(i.e.,\ \ \text{short} - \text{term loan repayment} \leq \text{housing sales in s})$

$$q^{\phi}_{s2} \leq e^{\phi}_{02} - q^{\phi}_{02} \ \ \forall\ \text{s} \in \text{S} \qquad (13)$$

$(i.e.,\ \ \text{quantity of housing sold in s} \leq \text{endowment of housing at t} = 0 - \text{quantity of housing sold at t} = 0)$

where:
$b^{\phi}_{s1} \equiv$ amount of fiat money spent by ϕ to trade in the goods market in $s \in S^*$
$q^{\phi}_{s2} \equiv$ amount of housing offered for sale by ϕ in $s \in S^*$
$\overline{d}^{\phi} \equiv$ deposit amount for ϕ
$\mu^{\phi}_s \equiv$ short-term borrowing by ϕ in $s \in S^*$
$\overline{r}_d \equiv$ deposit rate
$r^{\gamma}_s \equiv$ short-term rate offered by bank γ in $s \in S^*$
$e^{\phi}_{02} \equiv$ endowment of housing of ϕ at t=0
$m^{\phi}_s \equiv$ monetary endowment of ϕ in $s \in S^*$

3.2.3 Household $\lambda' s$ optimization problem

Agent λ enters the economy in the second period and is endowed with goods. His maximization problem is as follows:

$$\max_{q^\lambda_{s1}, b^\lambda_{s2}, \mu^\lambda_s} \Pi^\lambda = \sum_{s\in S} \theta_s u\left(e^\lambda_{s1} - q^\lambda_{s1}\right) + \sum_{s\in S} \theta_s u\left(\frac{b^\lambda_{s2}}{p_{s2}}\right)$$

$$s.t. \quad b^\lambda_{s2} \leq \frac{\mu^\lambda_s}{1+r^\delta_s} + m^\lambda_s \ \ \forall \text{ s} \in \text{S} \qquad (14)$$

$$(i.e., \quad \text{expenditure for housing} \ \leq \ \text{amount borrowed short} - \text{term} + \text{private monetary endowment in s})$$

$$\mu^\lambda_s \leq p_{s1} q^\lambda_{s1} \ \ \forall \text{ s} \in \text{S} \qquad (15)$$

$$(i.e., \quad \text{short} - \text{term loan repayment} \ \leq \ \text{good sales in s})$$

$$q^h_{s1} \leq e^h_{s1} \ \ \forall \text{ s} \in \text{S} \qquad (16)$$

$$(i.e., \quad \text{quantity of goods sold in s} \ \leq \ \text{endowment of goods in s})$$

where:
$b^\lambda_{s2} \equiv$ amount of fiat money spent by λ to trade in the housing market in $s \in S$
$q^\lambda_{s1} \equiv$ amount of goods offered for sale by λ in $s \in S$
$\mu^\lambda_s \equiv$ short-term borrowing by λ in $s \in S$
$r^\delta_s \equiv$ short-term rate offered by bank δ in $s \in S$
$e^\lambda_{s1} \equiv$ endowment of goods of λ in $s \in S$
$m^\lambda_s \equiv$ monetary endowment of λ in $s \in S$

3.2.4 Bank $\gamma' s$ optimization problem

Bank γ (as also bank δ) maximizes its expected profits in the second period. In the first period it borrow from the interbank market, since it is relatively poor in initial capital, and extends credit in the short-term loan and mortgage markets. It also receives deposits from ϕ. In the second period it receives the repayment on the mortgage it extended (full repayment for $s \in S^\alpha_1$, partial elsewhere since the value of the collateral is less than the amount of the mortgage), repays its interbank and deposit borrowing and extends credit short-term. Its maximization problem is as follows[3]:

[3] Banks' risk-aversion is captured via a quadratic objective function, as in essence they are facing a portfolio problem and we want to capture diversification effects as closely as possible

$$\max_{\pi_s^\gamma, m_s^\gamma, \overline{m}^\gamma, \mu_I^\gamma, \overline{\mu}_d^\gamma} \Pi^\gamma = \sum_{s\in S} \theta_s \left(\pi_s^\gamma - c^\gamma(\pi_s^\gamma)^2\right)$$

$$s.t. \quad m_0^\gamma + \overline{m}^\gamma \leq \frac{\mu_I^\gamma}{1+\rho} + \frac{\overline{\mu}_d^\gamma}{1+\overline{r}_d} + e_0^\gamma \qquad (17)$$

$(i.e.,.\ \ \text{short}-\text{term lending}\ +\ \text{mortgage extension}\ \leq\ \text{interbank loans}\ +\ \text{consumer deposits}\ +$ $\text{initial capital endowment at t} = 0)$

$$m_s^\gamma + \overline{\mu}_d^\gamma + \mu_I^\gamma \leq \overline{m}^\gamma(1+\overline{r}_s^\gamma) + m_0^\gamma(1+r_0^\gamma) + e_s^\gamma \ \ \forall\ \text{s} \in \text{S} \qquad (18)$$

$(i.e.,\ \ \text{short}-\text{term lending}\ +\ \text{deposit repayment}\ +\ \text{interbank loan repayment}\ \leq\ \text{effective}$ $\text{mortgage repayment}\ +\ \text{first period short}-\text{term loan repayment}+\ \text{capital endowment in s} \in \text{S})$

$$\pi_s^\gamma = m_s^\gamma(1+r_s^\gamma) \ \ \forall\ \text{s} \in \text{S} \qquad (19)$$

$(i.e.,\ \ \text{profits}\ =\ \text{short}-\text{term loans repayment s} \in \text{S})$

where:
$\pi_s^\gamma \equiv$ bank $\gamma' s$ profits at state $s \in S$
$\overline{m}^\gamma \equiv$ mortgage extension by bank γ
$m_s^\gamma \equiv$ short-term loan extension by bank γ at state $s \in S^*$
$\mu_I^\gamma \equiv$ interbank borrowing by bank γ
$\overline{\mu}_d^\gamma \equiv$ amount borrowed from consumers in the form of deposits by bank γ
$\overline{r}_s^\gamma \equiv$ effective repayment rate on the mortgage at state $s \in S$
$r_s^\gamma \equiv$ short-term rate offered by bank γ in $s \in S^*$
$\rho \equiv$ interbank rate
$e_s^\gamma \equiv$ capital endowment of bank γ at state $s \in S^*$

3.2.5 Bank $\delta' s$ optimization problem

Bank δ maximizes its expected profits in the second period. In the first period it deposits in the interbank market, since it is relatively rich in initial capital, and extends credit in the short-term loan and mortgage markets. In the second period it receives the repayment on the mortgage it extended (full repayment for $s \in S_1^\beta$, partial elsewhere since the value of the collateral is less than the amount of the mortgage), receives payment from depositing in the interbank market and extends credit short-term. Its maximization problem is as follows:

$$\max_{\pi_s^\delta, m_s^\delta, \overline{m}^\delta, d_I^\delta} \Pi^\delta = \sum_{s \in S} \theta_s \left(\pi_s^\delta - c^\delta (\pi_s^\delta)^2 \right)$$

$$s.t. \quad m_0^\delta + \overline{m}^\delta + d_I^\delta \leq e_0^\delta \qquad (20)$$

$(i.e., \quad \text{short} - \text{term lending} \ + \ \text{mortgage extension} \ + \ \text{interbank deposits} \ \leq \ \text{initial capital endowment at t} = 0)$

$$m_s^\delta \leq \overline{m}^\delta (1 + \overline{r}_s^\delta) + m_0^\delta (1 + r_0^\delta) + d_I^\delta (1 + \rho) + e_s^\delta \ \ \forall \ \text{s} \in \text{S} \qquad (21)$$

$(i.e., \quad \text{short} - \text{term lending} \ \leq \ \text{effective mortgage repayment} + \text{first period short} - \text{term loan repayment} \ + \ \text{interbank deposits and interest payment} + \text{capital endowment in s} \in \text{S})$

$$\pi_s^\delta = m_s^\delta (1 + r_s^\delta) \ \ \forall \ \text{s} \in \text{S} \qquad (22)$$

$(i.e., \quad \text{profits} \ = \ \text{short} - \text{term loans repayment s} \in \text{S})$

where:
$\pi_s^\delta \equiv$ bank $\delta' s$ profits at state $s \in S$
$\overline{m}^\delta \equiv$ mortgage extension by bank δ
$m_s^\delta \equiv$ short-term loan extension by bank δ at state $s \in S^*$
$d_I^\delta \equiv$ interbank deposits by bank δ
$\overline{r}_s^\delta \equiv$ effective repayment rate on the mortgage at state $s \in S$
$r_s^\delta \equiv$ short-term rate offered by bank δ in $s \in S^*$
$e_s^\delta \equiv$ capital endowment of bank δ at state $s \in S^*$

3.3 Market clearing conditions

There are six market categories in our model (goods, housing, mortgage, short-term loan, consumer deposit and interbank markets). Each of these markets determines a price that equilibrates demand and supply in equilibrium).

3.3.1 Goods Market

The goods market clears when the amount of money offered for goods is exchanged for the quantity of goods offered for sale.

$$p_{01} = \frac{b_{01}^\phi}{q_{01}^\alpha + q_{01}^\beta} \qquad (23)$$

$$p_{s1} = \frac{b_{s1}^\phi}{q_{s1}^\alpha + q_{s1}^\beta + q_{s1}^\lambda} \ \ \forall \ \text{s} \in \text{S} \qquad (24)$$

3.3.2 Housing Market

The housing market clears when the amount of money offered for housing is exchanged for the quantity of housing offered for sale. When agent $h \in H$ defaults on his mortgage the amount of housing pledged as collateral is offered for sale by the bank that seizes it.

$$p_{02} = \frac{b^{\alpha}_{02} + b^{\beta}_{02}}{q^{\phi}_{02}} \qquad (25)$$

$$p_{s2} = \frac{b^{\alpha}_{s2} + b^{\beta}_{s2} + b^{\lambda}_{s2}}{q^{\phi}_{s2}} \text{ for } s \in S^{\alpha}_{1} \cap S^{\beta}_{1} \qquad (26)$$

$$p_{s2} = \frac{b^{\alpha}_{s2} + b^{\beta}_{s2} + b^{\lambda}_{s2}}{q^{\phi}_{s2} + \frac{b^{\alpha}_{02}}{p_{02}}} \text{ for } s \in S^{\beta}_{1} \backslash S^{\alpha}_{1} \cap S^{\beta}_{1} \qquad (27)$$

$$p_{s2} = \frac{b^{\alpha}_{s2} + b^{\beta}_{s2} + b^{\lambda}_{s2}}{q^{\phi}_{s2} + \frac{b^{\beta}_{02}}{p_{02}}} \text{ for } s \in S^{\alpha}_{1} \backslash S^{\alpha}_{1} \cap S^{\beta}_{1} \qquad (28)$$

$$p_{s2} = \frac{b^{\alpha}_{s2} + b^{\beta}_{s2} + b^{\lambda}_{s2}}{q^{\phi}_{s2} + \frac{b^{\alpha}_{02}}{p_{02}} + \frac{b^{\beta}_{02}}{p_{02}}} \text{ for } s \notin S^{\alpha}_{1} \cup S^{\beta}_{1} \qquad (29)$$

When agent $h \in (\alpha, \beta)$ defaults on his mortgage, the amount of housing his has pledged as collateral will be offered by his bank for sale in the market. This amount is equal to the amount of housing he purchases in the initial period, i.e. $\dfrac{b^{h}_{02}}{p_{02}}$. For example, in state $s \in S^{\beta}_{1} \backslash S^{\alpha}_{1} \cap S^{\beta}_{1}$ agent α (but not β) defaults, thus the amount of housing he purchases in the initial period and pledged as collateral will be offered for sale by bank γ.

3.3.3 Mortgage Market

$$1 + \bar{r}^{k} = \frac{\bar{\mu}^{h}}{\bar{m}^{k}} \qquad (30)$$

The effective return on the mortgage is

$$\frac{\min(\text{ value of collateral, mortgage amount })}{\textit{initial} \text{ credit extension}} \quad \text{or} \quad 1 + \bar{r}^{k}_{s} = \frac{\min\left(\frac{b^{h}_{02}}{p_{02}} p_{s2}, \bar{\mu}^{h}\right)}{\bar{m}^{k}}$$

where $k = \gamma$ for $h = \alpha$ and $k = \delta$ for $h = \beta$. Thus, we get the following clearing conditions for effective returns on mortgages.

$$1 + \bar{r}^{k}_{s} = 1 + \bar{r}^{k} \text{ for } s \in S^{h}_{1} = \left\{ s \in S : \bar{\mu}^{h} \leq \frac{b^{h}_{02}}{p_{02}} p_{s2} \right\} \qquad (31)$$

$$1 + \bar{r}^{k}_{s} = (1 + \bar{r}^{k}) \frac{b^{h}_{02}}{\bar{\mu}^{h}} \frac{p_{s2}}{p_{02}} \text{ for } s \notin S^{h}_{1} \qquad (32)$$

3.3.4 Short-term Loan Market

$$1 + r^{\gamma}_{0} = \frac{\mu^{\alpha}_{0} + \mu^{\phi}_{0}}{m^{\gamma}_{0}} \qquad (33)$$

$$1 + r^{\gamma}_{s} = \frac{\mu^{\alpha}_{s} + \mu^{\phi}_{s}}{m^{\gamma}_{s} + M^{CB}_{\gamma s}} \qquad (34)$$

$$1+r_0^{\delta}=\frac{\mu_0^{\beta}}{m_0^{\delta}} \qquad (35)$$

$$1+r_s^{\delta}=\frac{\mu_s^{\beta}+\mu_s^{\lambda}}{m_s^{\delta}+M_{\delta s}^{CB}} \qquad (36)$$

3.3.5 Consumer Deposit Market

$$1+\bar{r}_d=\frac{\bar{\mu}_d^{\gamma}}{\bar{d}^{\phi}} \qquad (37)$$

3.3.6 Interbank Market

$$1+\rho=\frac{\mu_I^{\gamma}}{d_I^{\delta}+M^{CB}} \qquad (38)$$

3.4 Definition of Equilibrium

Let $\sigma^h=(q_{s1}^h,b_{s2}^h,\mu_s^h,\bar{\mu}^h)\in\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}$ for $h\in\{\alpha,\beta\}$;
$\sigma^{\phi}=(q_{s2}^{\phi},b_{s1}^{\phi},\mu_s^{\phi},\bar{d}^{\phi})\in\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}$;
$\sigma^{\lambda}=(q_{s1}^{\lambda},b_{s2}^{\lambda},\mu_s^{\lambda})\in\mathbb{R}^s\times\mathbb{R}^s\times\mathbb{R}^s$;
$\sigma^{\gamma}=(\pi_s^{\gamma},m_s^{\gamma},\bar{m}^{\gamma},\mu_I^{\gamma},\bar{\mu}_d^{\gamma})\in\mathbb{R}^s\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R}$;
$\sigma^{\delta}=(\pi_s^{\delta},m_s^{\delta},\bar{m}^{\delta},d_I)\in\mathbb{R}^s\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R}$.

Also, let $\eta=(p_{s1},p_{s2},r_s^{\gamma},r_s^{\delta},\bar{r}^{\gamma},\bar{r}_s^{\gamma},\bar{r}^{\delta},\bar{r}_s^{\delta},\rho)\in\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R}^s\times\mathbb{R}\times\mathbb{R}^s\times\mathbb{R}$, $B^h(\eta)=\{\sigma^h:(1)-(7)\text{ hold}\}$ for $h\in\{\alpha,\beta\}$, $B^{\phi}(\eta)=\{\sigma^{\phi}:(8)-(13)\text{ hold}\}$, $B^{\lambda}(\eta)=\{\sigma^{\lambda}:(14)-(16)\text{ hold}\}$, $B^{\gamma}(\eta)=\{\sigma^{\gamma}:(17)-(19)\text{ hold}\}$, $B^{\delta}(\eta)=\{\sigma^{\delta}:(20)-(22)\text{ hold}\}$.

We say that $(\sigma^{\alpha},\sigma^{\beta},\sigma^{\phi},\sigma^{\lambda};p_{s1},p_{s2},r_s^{\gamma},r_s^{\delta},\bar{r}^{\gamma},\bar{r}_s^{\gamma},\bar{r}^{\delta},\bar{r}_s^{\delta},\rho)$ is a monetary equilibrium with commercial banks, collateral and default iff:

(i)

$$(a)\ \sigma^{\mathrm{n}}\in\ \mathrm{Argmax}_{\sigma^{\mathrm{n}}\in\mathrm{B}^{\mathrm{n}}(\eta)}\Pi^{\mathrm{n}}(\chi^{\mathrm{n}}),\ \mathrm{n}\in\{\alpha,\beta,\phi,\lambda\}$$
$$(b)\ \sigma^{\mathrm{k}}\in\ \mathrm{Argmax}_{\sigma^{\mathrm{k}}\in\mathrm{B}^{\mathrm{t}}(\eta)}\Pi^{\mathrm{k}}(\chi^{\mathrm{k}}),\ \mathrm{k}\in\{\gamma,\delta\}$$

(ii) All markets (23)-(38) clear.

4 Discussion of Equilibrium

Hereafter, we investigate a parametrized version of the model whereby in the second period only three states of nature are possible. We have chosen the exogenous parameters in our model in such a way as to be able to illustrate a housing and mortgage crisis. Their initial values are presented in Table 1. The initial equilibrium which our model yields is presented in Table 2 and analysed below.

In the initial equilibrium we examine three different scenarios which can occur in the second period. State 1 occurs with the highest probability and state 2 is more probable than state 3. State 1 is the good period in which neither borrower defaults. In state 2 one of the two agents, Mr β, defaults on his mortgage debt, but the other does not. In state 3 both default. Agent α is richer in endowments of the good in the first period, whereas agent β is relatively richer in the second state in the second period. Bank γ has less initial capital than bank δ, while it has more capital in the second period. The capital of both banks in the second period can be interpreted as outside banking profits or capital injections obtained in the second period and will play a crucial role in the comparative statics we perform. We have chosen the parametrization to motivate lending in the interbank market and in particular to motivate bank δ to deposit in the interbank rate.

The level of default on the mortgages depends on the relative -second period- differential between the value of houses that each agent has bought and the mortgage amount they have to repay. Agent α, who is richer in the first period, needs to take a comparatively lower loan to value mortgage for the amount of houses he wants to purchase than agent β, since he can finance the purchase through the sale of goods in the first period. As a result the effective return to the lending bank on the mortgages in state 3 when both agents default will be higher for α than β. In combination with the fact that α does not default in state 2, this results in a lower interest rate on the mortgage for α than for β, since rational expectations are assumed throughout.

Table 1: Exogenous variables

Coefficient of risk aversion	Endowment Goods	Houses	Money	Capital	Others
$c^\alpha = 1.3$	$e^\alpha_{01} = 11$	$e^\phi_{02} = 5.5$	$m^\alpha_0 = 0.1$	$e^\gamma_0 = 4$	$M^{CB} = 65$
$c^\beta = 1.3$	$e^\alpha_{11} = 10$		$m^\alpha_1 = 0.1$	$e^\gamma_1 = 0.7$	$M^{CB}_{\gamma 1} = 10.9$
$c^\phi = 1.3$	$e^\alpha_{21} = 10$		$m^\alpha_2 = 4.4$	$e^\gamma_2 = 0.7$	$M^{CB}_{\gamma 2} = 8$
$c^\lambda = 1.3$	$e^\alpha_{31} = 0.7$		$m^\alpha_3 = 0.1$	$e^\gamma_3 = 0.7$	$M^{CB}_{\gamma 3} = 0.5$
$c^\gamma = 0.005$	$e^\beta_{01} = 2$		$m^\beta_0 = 5.8$	$e^\delta_0 = 13$	$M^{CB}_{\delta 1} = 2.4$
$c^\delta = 0.005$	$e^\beta_{11} = 7$		$m^\beta_1 = 0.1$	$e^\delta_1 = 1$	$M^{CB}_{\delta 2} = 0.8$
	$e^\beta_{21} = 3$		$m^\beta_2 = 0.1$	$e^\delta_2 = 1$	$M^{CB}_{\delta 3} = 0.5$
	$e^\beta_{31} = 0.1$		$m^\beta_3 = 0.1$	$e^\delta_3 = 1$	$\theta_1 = 0.90$
	$e^\lambda_{11} = 4$		$m^\phi_0 = 0.1$		$\theta_2 = 0.075$
	$e^\lambda_{21} = 4$		$m^\phi_1 = 0.1$		$\theta_3 = 0.025$
	$e^\lambda_{31} = 3$		$m^\phi_2 = 0.1$		
			$m^\phi_3 = 0.1$		
			$m^\lambda_1 = 0.1$		
			$m^\lambda_2 = 0.1$		
			$m^\lambda_3 = 0.1$		

e_{s1}: Endowment of goods in state $s \in S^*$
e_{02}: Endowment of houses at t=0
m_s: Private money held by households in state $s \in S^*$
e_0: Initial capital of banks
e_s: Additional capital of banks in state $s \in S$
M^{CB}: Money supply at t=0
M^{CB}_{ks}: Money injection by the Central Ban in the short-term loan market organized by bank $k \in \{\gamma, \delta\}$ in state $s \in S$
θ_s: Probability of state $s \in S$

In our simulation the prices of the good and of the house move in a reverse way in the second period. The good is relatively more expensive in state 2 than 1 and in state 3 than 2. The opposite holds for the price of the house. The intuition behind the result is quite simple, since the model is driven by adverse supply shocks to goods' endowments, worse in state 3 than in state 2. Agents default on their mortgages when the value of the house is low. This happens when the endowments of goods are low, an adverse supply shock, since agents will not have enough income to allocate to the housing market. In turn this implies that the price of the good should rise.

In order to buy the house, agents α and β sell goods in the first period and take out a mortgage as well. This creates income for ϕ, the initial owner of the housing stock, who uses it to purchase goods and deposits the rest in the interperiod deposit market. In state 1 when α and β do not default on their mortgages, and then find themselves with more houses than they want, they sell some of the amount they bought in the previous period (house prices are high relative to goods prices, and utility maximization leads α and β to switch out of housing into goods)[4]. This is possible as the economy is going well, endowments of goods are high and there is a strong demand for housing from agent λ a first time buyer who enters the economy in the second period. Agent ϕ also finds it profitable to sell some of the housing he is left with at those prices.

However, in state 2 in period 2 when Mr β defaults on his mortgage and essentially loses his house, he finds himself in a situation when he still wants to purchase some housing. Although the supply of houses due to delinquencies is high, his demand in combination with $\alpha's$ and $\lambda's$ prevents the price of houses from collapsing. This gives on incentives to ϕ to sell some of the housing he owns, as in state 1.

One would expect the same scenario to occur in state 3. However, since both agents α and β become extremely poor in their endowments of goods in that state, so their demand for houses is drops precipitously. As a result the housing market should collapse and agent λ, who is the only one endowed with a sufficient amount of the scarce good, would enjoy the services from the purchase of housing at a very low price. The reason that this cannot happen is twofold. First, agent ϕ finds it profitable to purchase back some of the houses he sold in the first period[5], thus the demand for housing is maintained as does the price. The second and most important one lies in the liquidity constraints that all agents face. In state 3 banks are short of liquidity. Hence, they are only willing to lend money short-term at a very high interest rate. Although the good is very expensive, agent λ can only find credit at an extremely high interest rate, which prevents him from enjoying the full benefits of the falling housing market. This is not the case for ϕ as he has money at hand from depositing in the first period.

Finally, lower prices is the last period are outweighed by the high real interest rates. Thus, by the Fischer effect short-term interest rates rise.

[4]Their cash-in-advance constraints have been adjusted to include housing sales as well as goods' sales

[5]Since ϕ does not sell any houses in state 3 he does not demand a short-term loan

Table 2: Initial Equilibrium

Prices	Interest rates	Loans/Deposits *Households*		Loans/Deposits *Banks*	Repayment rates on mortgages	Goods	Houses
$p_{01} = 4.10$	$\bar{r}^{\gamma} = 0.079$	$\bar{\mu}^{\alpha} = 12.33$	$\mu_1^{\lambda} = 6.23$	$\overline{m}^{\gamma} = 11.43$	$v_1^{\alpha} = 100\%$	$q_{01}^{\alpha} = 7.20$	$b_{02}^{\alpha} = 49.78$
$p_{11} = 2.60$	$\bar{r}^{\delta} = 0.123$	$\mu_0^{\alpha} = 29.47$	$\mu_2^{\lambda} = 6.97$	$\bar{\mu}_d^{\gamma} = 25.50$	$v_2^{\alpha} = 100\%$	$q_{11}^{\alpha} = 4.42$	$q_{12}^{\alpha} = 0.09$
$p_{21} = 3.10$	$r_0^{\gamma} = 0.043$	$\mu_1^{\alpha} = 12.80$	$\mu_3^{\lambda} = 18.46$	$\bar{\mu}_I^{\gamma} = 69.07$	$v_3^{\alpha} = 62\%$	$q_{21}^{\alpha} = 4.48$	$b_{22}^{\alpha} = 5.12$
$p_{31} = 12.31$	$r_1^{\gamma} = 0.047$	$\mu_2^{\alpha} = 13.65$		$m_0^{\gamma} = 83.24$	$v_1^{\beta} = 100\%$	$q_{31}^{\alpha} = 0.35$	$b_{32}^{\alpha} = 1.24$
$p_{02} = 26.12$	$r_2^{\gamma} = 0.047$	$\mu_3^{\alpha} = 4.35$		$m_1^{\gamma} = 5.28$	$v_2^{\beta} = 62\%$	$q_{01}^{\beta} = 0.29$	$b_{02}^{\beta} = 17.58$
$p_{12} = 15.17$	$r_3^{\gamma} = 2.83$	$\bar{\mu}^{\beta} = 11.96$		$m_2^{\gamma} = 5.28$	$v_3^{\beta} = 28\%$	$q_{11}^{\beta} = 4.54$	$q_{12}^{\beta} = 0.04$
$p_{22} = 10.96$	$r_0^{\delta} = 0.043$	$\mu_0^{\beta} = 1.18$		$m_3^{\gamma} = 0.64$		$q_{21}^{\beta} = 1.71$	$b_{22}^{\beta} = 5.08$
$p_{32} = 5.04$	$r_1^{\delta} = 0.048$	$\mu_1^{\beta} = 12.43$		$\overline{m}^{\delta} = 10.65$		$q_{31}^{\beta} = 0.04$	$b_{32}^{\beta} = 0.29$
	$r_2^{\delta} = 0.049$	$\mu_2^{\beta} = 5.22$		$d_I^{\delta} = 1.22$		$b_{01}^{\phi} = 30.64$	$q_{02}^{\phi} = 2.20$
	$r_3^{\delta} = 1.58$	$\mu_3^{\beta} = 0.50$		$m_0^{\delta} = 1.13$		$b_{11}^{\phi} = 29.55$	$q_{12}^{\phi} = 0.27$
	$\bar{r}_d = 0.043$	$\overline{d}^{\phi} = 24.44$		$m_1^{\delta} = 15.41$		$b_{21}^{\phi} = 25.84$	$q_{22}^{\phi} = 0.03$
	$\rho = 0.043$	$\mu_0^{\phi} = 57.36$		$m_2^{\delta} = 10.82$		$b_{31}^{\phi} = 23.30$	$b_{32}^{\phi} = 2.30$
		$\mu_1^{\phi} = 4.14$		$m_3^{\delta} = 6.84$		$q_{11}^{\lambda} = 2.40$	$b_{12}^{\lambda} = 6.05$
		$\mu_2^{\phi} = 0.25$				$q_{21}^{\lambda} = 2.29$	$b_{22}^{\lambda} = 6.75$
						$q_{31}^{\lambda} = 1.50$	$b_{32}^{\lambda} = 7.25$

p_{s1}: Price of goods in state $s \in S^*$	$\bar{r}^k$: Mortgage rate offered by bank $k \in \{\gamma, \delta\}$
p_{s2}: Price of houses in state $s \in S^*$	r_s^k: Short-term loan rate offered by bank $k \in \{\gamma, \delta\}$ in state $s \in S^*$
$\bar{r}_d$: Consumer deposit rate	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
ρ: Interbank rate	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
$\bar{\mu}$: Mortgage amount	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
μ_s: Short-term borrowing by households in state $s \in S^*$	m_s^k: Short-term loan extension by bank $k \in \{\gamma, \delta\}$ in state $s \in S^*$
$\overline{d}^{\phi}$: Deposit by agent ϕ at t=0	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
$\overline{m}^k$: Mortgage extension by bank $k \in \{\gamma, \delta\}$	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
$\bar{\mu}_I^{\gamma}$: Interbank borrowing by bank γ	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
d_I^{δ}: Interbank deposits by bank δ	$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ
v_s^h: Effective repayment on mortgage by agent $h \in \{\alpha, \beta\}$ in state $s \in S$	q_{s1}: Amount of goods offered for sale in state $s \in S^*$
b_{s1}, b_{s2}: Money spent in the goods and housing markets in $s \in S^*$	q_{s2}: Amount of houses offered for sale in state $s \in S^*$

5 Comparative Statics

This section shows the effects of changes in the exogenous variables/parameters of the model. Tables 3 & 4 in the Appendix describe the directional effects on endogenous variables of changing various parameters. Although we have performed a number of comparative statics, we discuss in detail only those that we reckon that are the most interesting. The analysis involves the following principles, which derive from the model structure[6]:

1. The determination of interest rates:
   Since base money is fiat and the horizon is finite, in the end no household will be left with fiat money. Thus, all households will finance their loan repayments to commercial banks via their private cash endowment and the initial capital endowments of banks. However, since we allow for defaults, the total amount of interest rate repayments is adjusted by the corresponding anticipated default rates. In sum, aggregate ex post interest rate payments, adjusted for default to commercial banks, is equal to the total amount of outside money (i.e. sum of cash monetary and initial commercial banks' endowments). In this way, the overall liquidity of the economy and endogenous default co-determine the structure of interest rates.

2. Quantity theory of money proposition:
   The model possesses a non-mechanical quantity theory of money. Velocity will always be less than or equal to one (one if all interest rates are positive). However, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's *sell-all* assumption), the real velocity of money, that is how many real transactions can be undertaken by money per unit of time, is endogenous. The upshot of this analysis is that nominal changes (i.e. changes in monetary policy) affect both prices and quantities.

3. Fisher effect:
   The nominal interest rate is equal to the real interest rate plus the expected rate of inflation.

5.1 Decrease in the money supply

Let the Central Bank engage in contractionary monetary policy by decreasing the money supply (M^{CB}) in the interbank market in the initial period (or equivalently increasing the interbank interest rate-ρ). The effects on the endogenous variables are summarized in Table 3.

Increasing the interbank rate induces bank γ to borrow less from the interbank market and therefore to reduce its supply of short-term loans and mortgages to Mr. α and Mr. ϕ, pushing up the corresponding lending rates r_0^{γ} and $\bar{r}^{\gamma}$. Consequently, Mr. α reduces his short-term and mortgage borrowing, similarly Mr. ϕ reduces his short-term borrowing and subsequent deposits in bank γ. Since bank δ increases its deposits in the interbank market, Mr. β faces stricter credit conditions in the short-term. So, he will switch towards mortgages, which will induce bank δ to reallocate its portfolio and supply slightly more mortgages to him. Finally, from the liquidity structure of interest rates, last period short-term interest rates decrease except for bank γ in the second state.

[6]The reader should bear in mind that the qualitative structure of the initial equilibrium does not change. For example, an increase in the price of goods in state 3 does not mean that this price has become higher than the prices of goods in states 1 and 2.

Given a higher interest rate, trade becomes less efficient[7]. Quantities of goods and houses traded in the initial period go down and so do prices. We see the quantity theory of money in action in our model. The reduction of the money supply, given that the velocity of money is at most one, leads, typically, to lower prices and quantities traded. Agent heterogeneity, and positive volumes of trade, are necessary for this result to hold. Given the low price of housing and the increased mortgage extension by bank δ, Mr. β is led to demand more mortgages, which results in higher mortgage rate ($\bar{r}^{\delta}$) for him as well. Since less money chases the same amount of goods, by the quantity theory of money proposition, prices in the last period will go down as well. Recall that agents default on their mortgages when the value of their housing holdings is less than that of their mortgage. Thus, lower house prices in the second period will result in lower effective returns on the mortgages (which can be interpreted as higher defaults) and even higher initial mortgage rates, given rational expectations. An increase in the interbank rate resulted not only in higher mortgage extension by the rich bank, but also in lower effective returns (higher levels of effective default) when the bad states materialize. Although the poor bank reduces its mortgage extension, it does not find itself in a better situation, since the effective return on its mortgages goes down as well when the very bad state obtains.

The higher mortgage extension and mortgage rates for bank δ are not enough to outweigh the lower effective returns due to default on mortgages in the bad states of the world (i.e., states 2 and 3). The impact on bank γ is the same. Contractionary monetary policy, therefore, results in lower expected profits for the banking sector.

The effect on households differs. For Mr. α an increase in the interbank rate has a negative effect on his welfare. The opposite holds for Mr. β and λ. The welfare of Mr. ϕ remains almost unaffected (fig. 2). The decrease in $\alpha's$ welfare is mainly due to the fact that he borrows less since he is affiliated with the poor bank. Although the price of housing at t=0 goes down, the price of goods decreases even more (fig. 3). Mr. β is affiliated with the rich bank and undertakes a bigger mortgage to take advantage of the falling housing prices in the initial period. In conjunction with the falling short-term rates in the last period (fig. 4), Mr. $\beta's$ welfare goes up. Mr. λ benefits as well from the lower short-term rates and enjoys an increase in his utility.

In sum, according to the Goodhart-Tsomocos financial stability measure, contractionary monetary policy results into higher financial instability since lower banking profits and higher default lead to welfare loses in the bad states of nature.

5.2 Liquidity assistance to banks in the very bad state of the world

Let there be an increase in both banks' capital in the third state of the world, which participants in the economy perfectly anticipate (Table 3). This increase can be of the form of liquidity assistance by the government or new equity capital. An increase in the money endowments in the third state of the world will result in a price increase in goods and housing at that state as it is expected from the quantity theory of money. A price increase in housing results in a higher effective return for both banks when both Mr. α and Mr. β default on their mortgages. Finding themselves with more money at the very bad state of the world, the banks will increase their extension of mortgages at the initial period. This will drive the mortgage rates down and the demand for mortgages up. Bank δ will switch its portfolio from interbank deposits towards mortgages, since the latter become less risky. Since there is overall more activity and higher prices, when government help is anticipated in

[7]The reason is that agents encounter a higher transaction cost

(very) bad states of the world, interest rates in the short loan market go up in the initial period due to increased money demand by households.

Although the effective returns on the mortgages go up and the overall default in absolute terms goes down, both banks will sustain a drop in their expected profitability. The reason is that the rates on mortgages to which they switch their portfolios go down (fig. 5). In addition, bank γ has to pay a higher interest for the money it borrows from depositors and the interbank market, and bank δ does not fully take advantage of the higher interbank rate, since it reallocates its portfolio towards mortgages that obtain higher effective returns.

The welfare of Mr. ϕ decreases because the liquidity injection occurs in state 3 where he is relatively rich and he suffers a negative wealth effect due to higher prices in that state. Apart from Mr. ϕ, the effect on household welfare is positive (fig. 6). All agents α, β and λ are better-off since the first two benefit from the lower mortgage rates and all three from the lower short-term rates in the last period, which translates into cheaper credit.

Liquidity assistance, unlike contractionary monetary policy, not only reduces aggregate default but also improves the real sector of the economy since it eases credit conditions households and first-time buyers face.

5.3 Banks become less risk-averse

Assume that both become more risk-loving (Table 4). The change in risk-aversion is anticipated in the first period. Their first response will be to switch from safer to riskier investments. Consequently, extension of mortgages goes up and short-term lending goes down, which means lower mortgage and higher short-term rates in the initial period. Bank δ also reduces its interbank deposits, which results in bank γ having less funds to extend credit. Mr. α takes advantage of the lower mortgage rates and demands more mortgages. He also reduces his sales of goods in the initial period, since he can finance his housing purchases with more mortgages, and the transaction cost for selling his goods (short-term interest rate) has gone up due to banks' funds reallocation. Mr. β, who is poorly endowed in the initial period, will reduce his sales of goods and short-term lending as well. However, he will not demand more mortgages as the fall in the mortgage rate he faces allows him to maintain his housing purchases. The reason that the mortgage rate falls more for Mr. β than for Mr. γ is that he is affiliated with bank δ which has more funds to allocate to mortgages, since it reduces its interbank and short-term lending. The demand for housing has increased. However, its initial price will fall. The reason is that the initial supply of goods onto the market by Mr. α and Mr. β has fallen and Mr. ϕ has to sell more of his housing endowment to fund his purchase of goods. Thus, Mr. $\phi's$ disposal income falls and he allocates less money into the goods markets forcing their initial price to go down as well.

Lower housing prices and higher mortgage extension translate into lower effective returns on mortgages because of higher aggregate default in the economy in the bad states. Depending on the severity of the reduction in risk-aversion and its initial level, aggregate default may increase a lot. In our exercise we have chosen a relatively low initial risk-aversion (in order to capture in the initial equilibrium the conception about banks' risk-aversion before the crisis), so an even a relatively small increase in the appetite for risk results in a 0.5% increase in aggregate default. Of course, what matters is the directional effect and not the absolute number. Unlike our other comparative statics, a change in risk-aversion, although exogenous in the model, is in reality a choice variable of the banks. The reason that they might adopt a more risk-loving behaviour is that they expect higher profits. This is consistent with what our model yields.

Households' welfare moves in different directions (fig. 7). Mr. α and Mr. λ are worse-off, whereas Mr. β and Mr. ϕ are better-off. The reason that Mr. ϕ is better-off is that houses are a durable commodity and their price should be affected positively by an decrease in the overall risk-aversion in the economy. As a result, the price of housing in the initial period decreases less than the price of goods (fig. 8) that generates a slightly positive effect on Mr. $\phi's$ welfare. Mr. β, who is poorly endowed in the initial period, will benefit from the lower mortgage rates and enjoy an increase in his utility. Mr. α on the other hand is worse-off, since he faces a higher interest rate for short-term loans in the initial period, which is his main source for funding his housing purchases. Mr. λ has a decrease in his welfare due to the fact that short-term interest rates in the last period go up in the presence of higher aggregate default (fig. 9) and the higher real rates of interest.

5.4 Compound comparative static

The comparative statics we examine above do not fully exhibit what we might expect to observe in a severe mortgage crisis. For that reason we have performed an exercise of letting more than one adverse shocks occur at a time. So we allow for contractionary monetary policy and for a decrease in banks' risk-aversion simultaneously . The results are summarize in Table 4.

The reduction in the money supply yields a first order effect pushing the interbank rate up. Bank δ increases its interbank lending and reduces its mortgage extension. The reduction in risk-aversion will moderate this pressure. The trade-off between these two effects will determine whether bank δ will extend more mortgages or not. In our simulation we find that mortgage extension by bank δ increases. The reduction is more severe for bank γ, since it is more dependent on monetary injections. Mortgage rates go up (fig. 10), since the demand does not decrease much due to the higher cost of short-term borrowing. Prices of goods and housing in all periods and states go down, as predicted by the Quantity Theory of Money. The pressure is greater due to lower risk-aversion (as discussed above). The result is lower expected returns on mortgages, which translate into higher defaults in conjunction with the fact that mortgage rates were higher to start with.

Higher default should mean higher mortgage rates, other things being equal. However, a higher appetite for risk pushes mortgage rates down. Nevertheless, these second order effects are outweighed by the increased default due to a lower money supply, as analysed in the relevant section. An interesting result is that default increases disproportionally when contractionary monetary policy is combined with a higher appetite for risk by banks. When these adverse shocks occur at the same time, expected repayment on mortgages falls more than the aggregate change when they happen independently. In particular, nonlinear effects are not trivial as shown in fig. 11.

Expected banking profits go up. On the one hand lower money supply and increased default put downward pressure on expected profits and on the other lower risk-aversion pushes them up. In our exercise the latter forces prevail, but the effect of the former becomes more intense as the money supply continues to decrease.

The effect on households' welfare differs. Agents that are affiliated with the poor bank, i.e. Mr. α and Mr. ϕ, observe a decrease in their expected welfare. This is due to the fact that the stricter credit environment affects poorly capitalized banks more severely. In addition, the initial price of goods falls more that of housing (fig. 12) affecting negatively the purchasing power of Mr. α, who mainly finances his housing purchase though the sale of goods in the initial period. Mr. β is able to benefit from the falling price of housing in the initial period via entering a mortgage contract, since he is affiliated with a well capitalized -more risk-loving- bank, and his welfare increases. Housing prices in state 1 fall more than goods' prices (fig. 13) due to the fact that Mr. ϕ decreased its deposits in the initial period and increases his sales of housing in that state to finance the purchase of goods. The lower demand for money by Mr. β in the last period (partially due to lower prices and higher defaults) and the well-capitalized position of bank δ put downward pressure on the short-term interest rates at the states in which agents default. Mr. λ benefits from the looser credit condition and enjoys a higher utility.

However, lower banking profits in the two bad states combined with relatively much higher aggregate default (as compared to contractionary monetary policy only) result into higher welfare losses in these states. Hence, we see that contractionary monetary policy coupled with an attempt to gamble to resurrect on behalf of the banks exacerbates the mortgage crisis and increases financial instability.

6 Conclusions

Central Bank officials are prone to describe the months since August 2007 as being akin to war-time, as contrasted with more normal peace-time. In longer run time-series econometric exercises stretching back, say, to 1900, the war years of WWI, 1914-18, and WWII, 1939-45, are frequently omitted (or dummied out) as involving regimes and structures too a-typical for normal analysis. By analogy the years 2007/8 may also become excluded from standard econometric analysis as too extraordinary to fit with our standard models. After all such standard models abstract from counter-party risk, from default, from endogenous risk premia, and even from financial intermediation.

If, however, we do want to address, and to model, current events, then we need a model which incorporates default as a central feature, and in which credit risk is endogenous (not an exogenous add-on). The model which we have explored above is such a model. It is, however, an initial, preliminary attempt. Much more needs to be done.

For example, it is an endowment model. Thus the economy has a given time path of goods, houses, capital and fiat money. With such predetermined endowments, the resulting time-path of prices, interest rates and quantities just redistributes goods and assets between agents. The welfare implications are never clear-cut since some gain and others lose. In order to explore the welfare implications of financial crises, they have to be incorporated into a production economy, wherein a credit crunch adversely impacts upon output and employment, so that real incomes become generally reduced, not just redistributed. This can be done. As an additional step we do not think that that will be too difficult.

In general, the results of our simulations are more or less what most economists would have imagined. Tight money reduces prices and quantities traded. Government support to banks in crises stabilises the economy. When banks become risk-loving, a subsequent crisis becomes even more extreme. We are encouraged that our model reproduces common-sense outcomes. The direction of effects seems correct.

This raises the question whether such a model as this can be taken beyond numerical solution and simulation to the actual data. Could it be used to try to match and to calibrate the actual time path of the major data series in existing countries and to explore alternative policy options in real time? We believe that it can, though it will not be straightforward to do so.

Running simulations often provides the authors with more illumination than to the readers of the resulting paper. One of the lessons that this exercise has taught us concerns the limitations of a strict rational expectations (RE) model. In such an RE model, an event in some subsequent period, such as a change in risk aversion, or a change in government policy, may be regarded at the outset as a low probability event, but in a fully rational world it cannot by definition have been entirely unexpected. One cannot run simulations, in an RE world, in which the completely unexpected occurs. This makes it rather harder to simulate extraordinary time periods such as 2007/8. Thus, for example, the risk-seeking behaviour of financial intermediaries in 2004-6, gave way to strong risk aversion in 2007/8 in a way that was entirely unexpected in 2004-6. Had it been anticipated, it would have been discounted in an RE system. So what one has to do is assume that unexpected changes of behaviour were actually previously expected, but with an extremely small probability, for example that there would be a generalised fall in US housing prices. When we started on this exercise, we had not appreciated this. It also raises the philosophical question whether the subjective probability distribution of actual expectations, based on some combination of the accidents of human history and the limited stretch of our imaginations, can ever approximate to the true underlying objective probability distribution. If that approximation is partial at best, in what sense can expectations be held to be 'rational'? Keynes and Shackle would have appreciated that question.

But our purpose is not so much to query the current RE methodology as to demonstrate that within the format of existing model best-practice, it should be possible to model a combined collapse of the housing and financial markets.

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8 Appendix

We illustrate the results from the comparative statics exercise in two tables. The directional effect of a change in the respective(s) exogenous variable(s) is presented. The diagrams used in the above analysis are included. Other comparative statics we have performed include a decrease in the liquidity in the short-term loan markets in the last period, a decrease in banks' initial capital, a decrease in banks' capital in the last period, a change in agents' expectations regarding the occurrence of each state of the world and a production shock in the goods market. The results can be found in our working paper (Goodhart, Tsomocos and Vardoulakis (2009)).

Table 3: Comparative Statics A

Endogenous variables	Decrease in Money Supply t=0	Liquidity assistance to banks State 3	Endogenous variables	Decrease in Money Supply t=0	Liquidity assistance to banks State 3
p_{01}	-	-	$\overline{d}^{\phi}$	-	+
p_{11}	-	-	μ_0^{ϕ}	-	+
p_{21}	-	-	μ_1^{ϕ}	+	-
p_{31}	-	+	μ_2^{ϕ}	+	-
p_{02}	-	-	μ_1^{λ}	-	-
p_{12}	-	-	μ_2^{λ}	-	-
p_{22}	-	-	μ_3^{λ}	-	+
p_{32}	-	+	q_{01}^{α}	-	-
$\overline{r}^{\gamma}$	+	-	q_{11}^{α}	+	+
$\overline{r}_3^{\gamma}$	-	+	q_{21}^{α}	+	+
$\overline{r}^{\delta}$	+	-	q_{31}^{α}	-	-
$\overline{r}_2^{\delta}$	-	-	q_{01}^{β}	-	-
$\overline{r}_3^{\delta}$	-	+	q_{11}^{β}	+	+
$\rho, \overline{r}_d, r_0^{\gamma}, r_0^{\delta}$	+	+	q_{21}^{β}	-	-
r_1^{γ}	-	+	q_{31}^{β}	-	+
r_2^{γ}	+	-	b_{01}^{ϕ}	-	-
r_3^{γ}	-	-	b_{11}^{ϕ}	-	-
r_1^{δ}	-	-	b_{21}^{ϕ}	-	-
r_2^{δ}	-	-	b_{31}^{ϕ}	-	+
r_3^{δ}	-	-	q_{11}^{λ}	-	+
$\overline{m}^{\gamma}$	-	+	q_{21}^{λ}	-	-
$\overline{m}^{\delta}$	+	+	q_{31}^{λ}	-	-
m_0^{γ}	-	-	b_{02}^{α}	-	+
m_1^{γ}	+	-	q_{12}^{α}	-	+
m_2^{γ}	+	-	b_{22}^{α}	-	-
m_3^{γ}	+	+	b_{32}^{α}	+	+
m_0^{δ}	-	-	b_{02}^{β}	-	+
m_1^{δ}	+	-	q_{12}^{β}	+	+
m_2^{δ}	+	-	b_{22}^{β}	+	-
m_3^{δ}	-	+	b_{32}^{β}	-	+
$\overline{\mu}_d^{\gamma}$	-	+	q_{02}^{ϕ}	-	+
$\overline{\mu}_I^{\gamma}$	-	+	q_{12}^{ϕ}	+	-
d_I^{δ}	+	-	q_{22}^{ϕ}	+	-
$\overline{\mu}^{\alpha}$	-	+	b_{32}^{ϕ}	-	-
$\overline{\mu}^{\beta}$	+	+	b_{12}^{λ}	+	-
μ_0^{α}	-	-	b_{22}^{λ}	+	-
μ_1^{α}	-	+	b_{32}^{λ}	-	+
μ_2^{α}	-	+	U^{α}	-	+
μ_3^{α}	-	+	U^{β}	+	+
μ_0^{β}	-	-	U^{ϕ}	-	-
μ_1^{β}	-	-	U^{λ}	+	+
μ_2^{β}	-	-	$\gamma's$ profits	-	-
μ_3^{β}	-	+	$\delta's$ profits	-	-

Table 4: Comparative Statics B

Endogenous variables	Decrease in both banks' risk-aversion	Compound comparative static	Endogenous variables	Decrease in both banks' risk-aversion	Compound comparative static
p_{01}	-	-	$\overline{d}^{\phi}$	+	-
p_{11}	-	-	μ_0^{ϕ}	+	-
p_{21}	-	-	μ_1^{ϕ}	-	+
p_{31}	-	-	μ_2^{ϕ}	-	+
p_{02}	-	-	μ_1^{λ}	-	-
p_{12}	-	-	μ_2^{λ}	-	-
p_{22}	-	-	μ_3^{λ}	-	-
p_{32}	-	-	q_{01}^{α}	-	-
$\overline{r}^{\gamma}$	-	+	q_{11}^{α}	+	+
$\overline{r}_3^{\gamma}$	-	-	q_{21}^{α}	+	+
$\overline{r}^{\delta}$	-	+	q_{31}^{α}	+	Up&Down
$\overline{r}_2^{\delta}$	-	-	q_{01}^{β}	-	-
$\overline{r}_3^{\delta}$	-	-	q_{11}^{β}	+	+
$\rho, \overline{r}_d, r_0^{\gamma}, r_0^{\delta}$	+	+	q_{21}^{β}	+	-
r_1^{γ}	-	-	q_{31}^{β}	-	-
r_2^{γ}	-	+	b_{01}^{ϕ}	-	-
r_3^{γ}	+	Up&Down	b_{11}^{ϕ}	-	-
r_1^{δ}	+	+	b_{21}^{ϕ}	+	-
r_2^{δ}	+	-	b_{31}^{ϕ}	-	-
r_3^{δ}	+	-	q_{11}^{λ}	+	-
$\overline{m}^{\gamma}$	+	-	q_{21}^{λ}	+	-
$\overline{m}^{\delta}$	+	+	q_{31}^{λ}	-	-
m_0^{γ}	-	-	b_{02}^{α}	+	-
m_1^{γ}	-	+	q_{12}^{α}	+	+
m_2^{γ}	-	+	b_{22}^{α}	-	-
m_3^{γ}	-	Up&Down	b_{32}^{α}	-	Up&Down
m_0^{δ}	-	-	b_{02}^{β}	+	-
m_1^{δ}	-	+	q_{12}^{β}	+	-
m_2^{δ}	-	+	b_{22}^{β}	-	+
m_3^{δ}	-	-	b_{32}^{β}	-	-
$\overline{\mu}_d^{\gamma}$	+	-	q_{02}^{ϕ}	+	-
$\overline{\mu}_I^{\gamma}$	+	-	q_{12}^{ϕ}	-	+
d_I^{δ}	-	+	q_{22}^{ϕ}	-	+
$\overline{\mu}^{\alpha}$	+	-	b_{32}^{ϕ}	+	-
$\overline{\mu}^{\beta}$	-	+	b_{12}^{λ}	-	+
μ_0^{α}	-	-	b_{22}^{λ}	-	+
μ_1^{α}	+	-	b_{32}^{λ}	-	-
μ_2^{α}	+	-	U^{α}	-	-
μ_3^{α}	+	-	U^{β}	+	+
μ_0^{β}	-	-	U^{ϕ}	+	-
μ_1^{β}	-	-	U^{λ}	-	+
μ_2^{β}	-	-	$\gamma' s$ profits	+	+
μ_3^{β}	-	-	$\delta' s$ profits	+	+

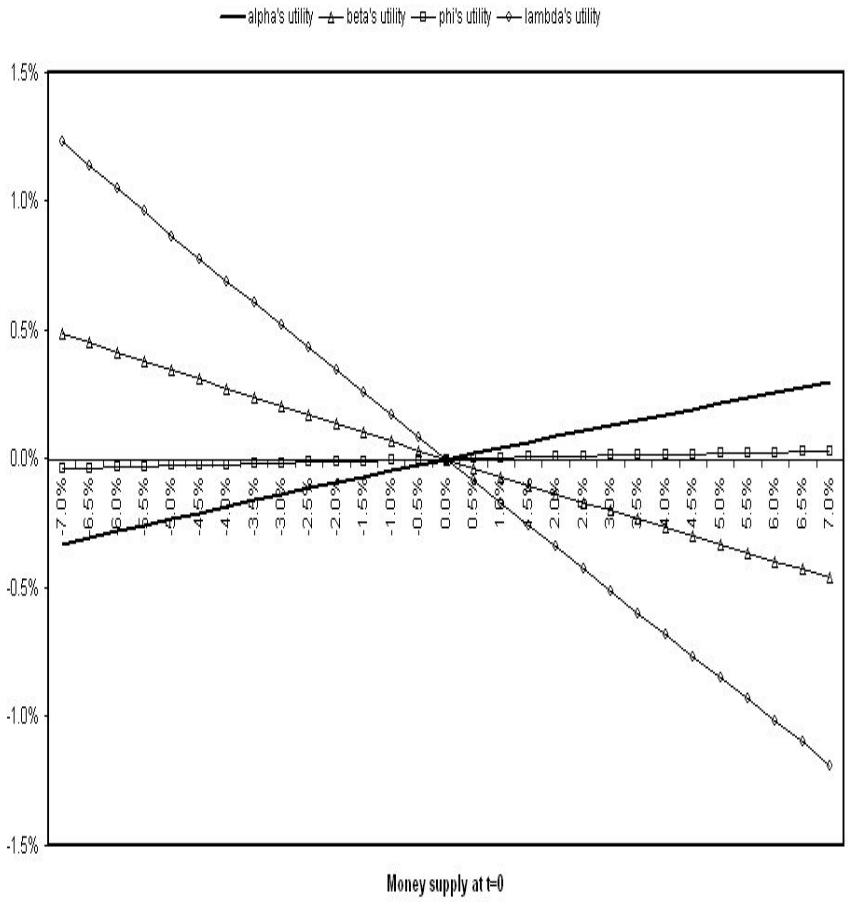

Figure 2: Household welfare vs money supply

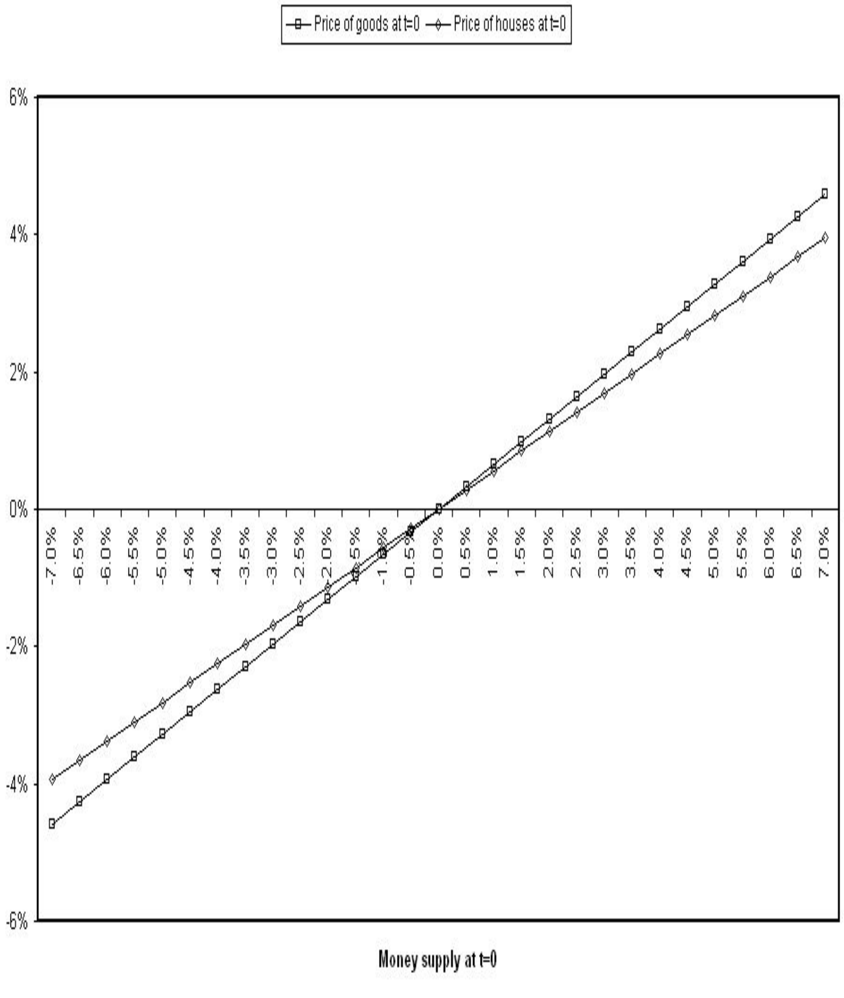

Figure 3: Housing and goods prices vs money supply

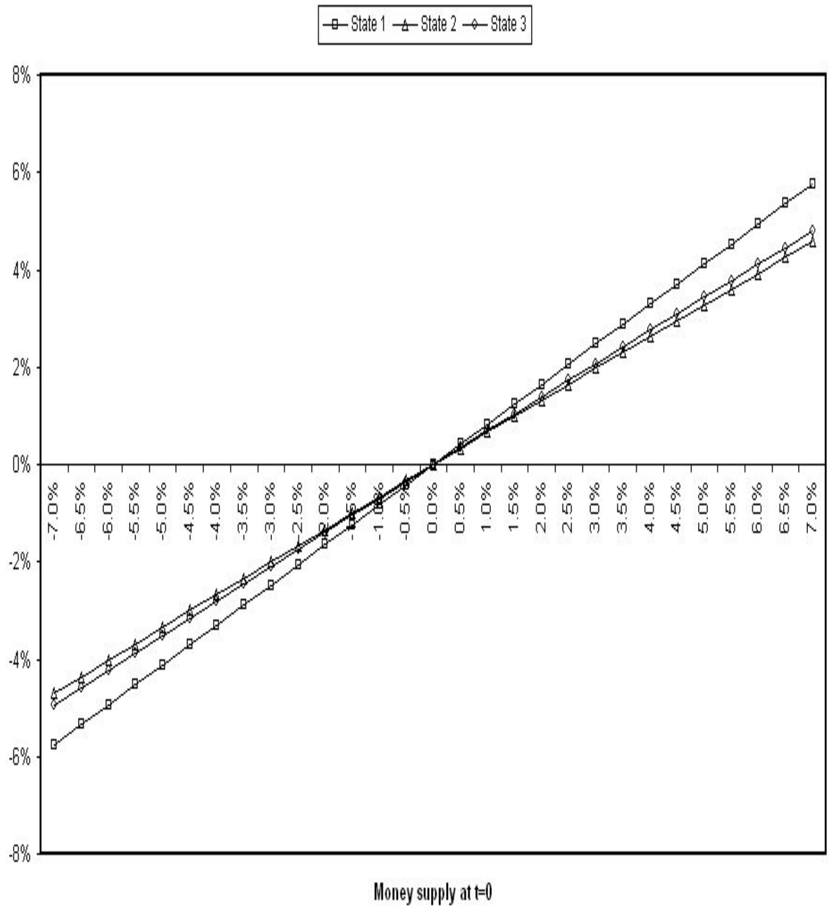

Figure 4: Short-term interest rates by bank δ vs money supply

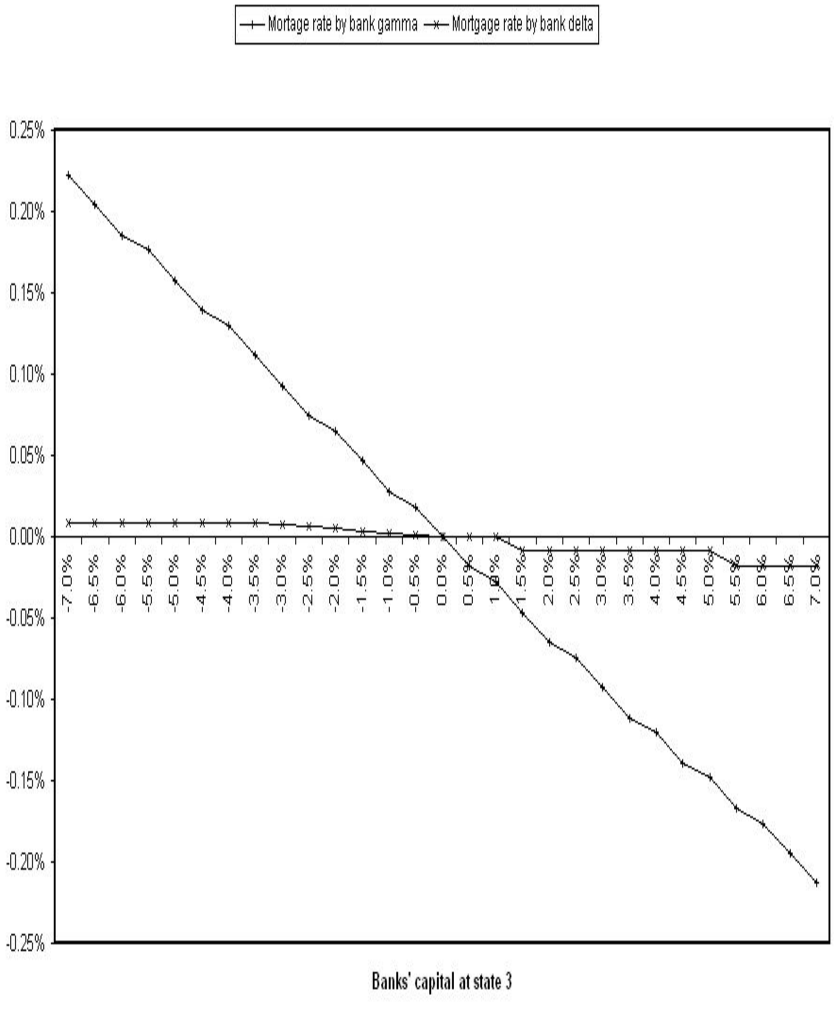

Figure 5: Mortgage rates vs banks' capital in state 3

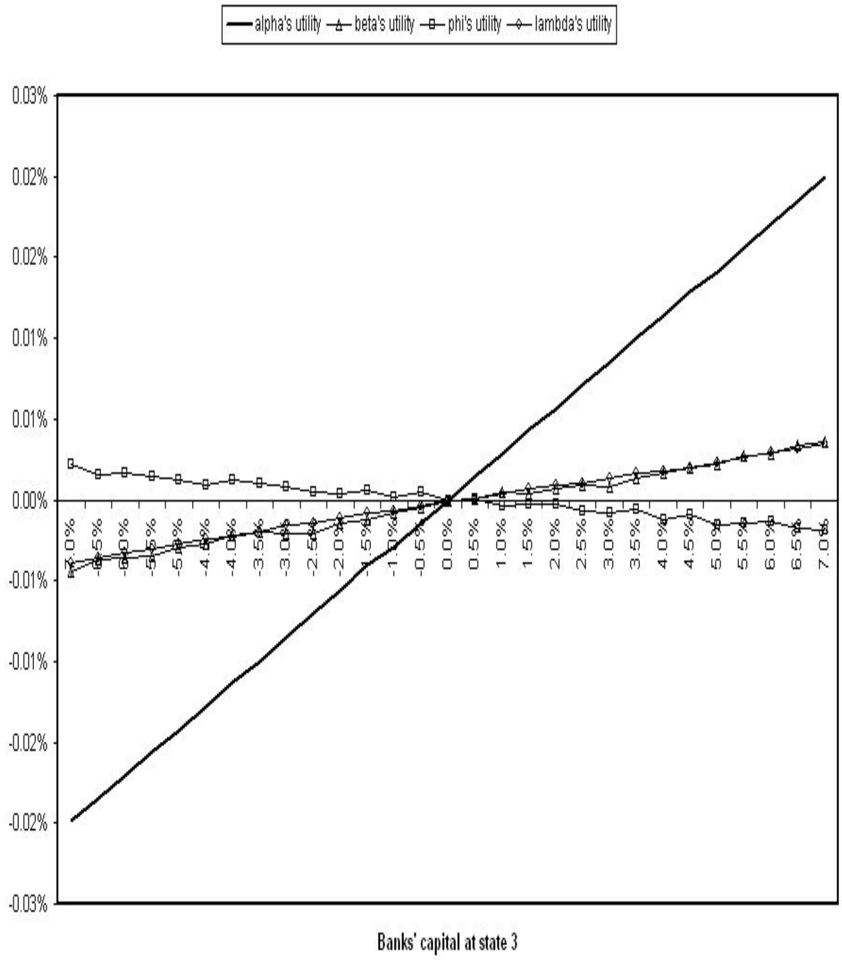

Figure 6: Household welfare vs banks' capital in state 3

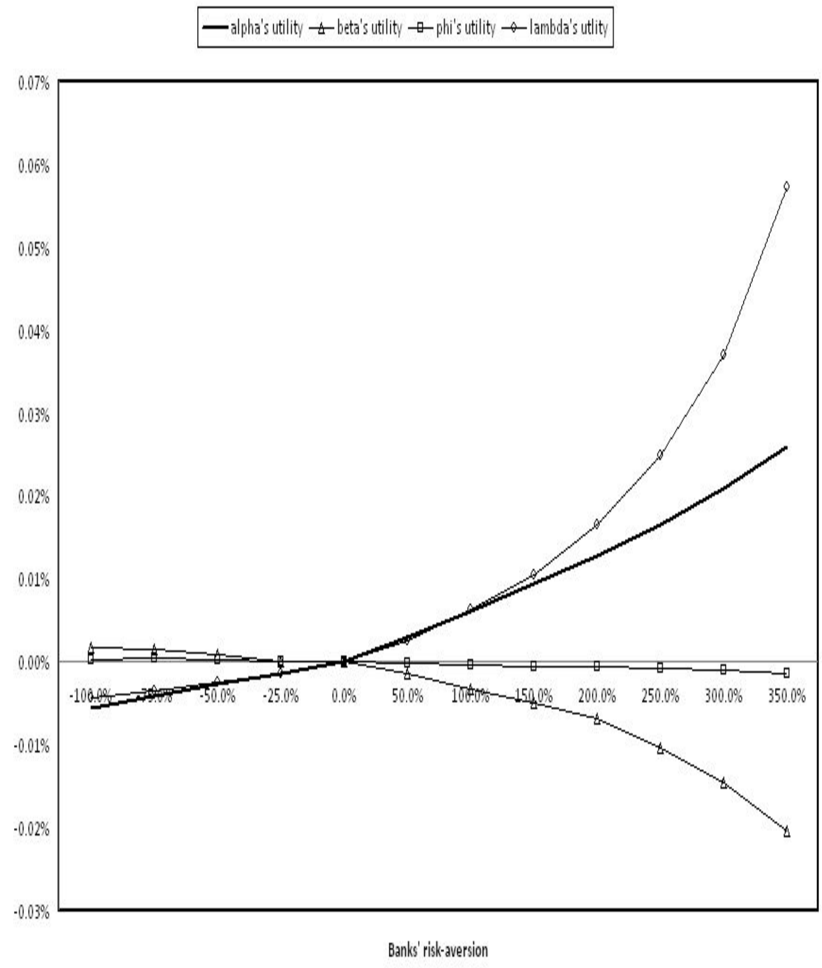

Figure 7: Household welfare vs banks' risk-aversion

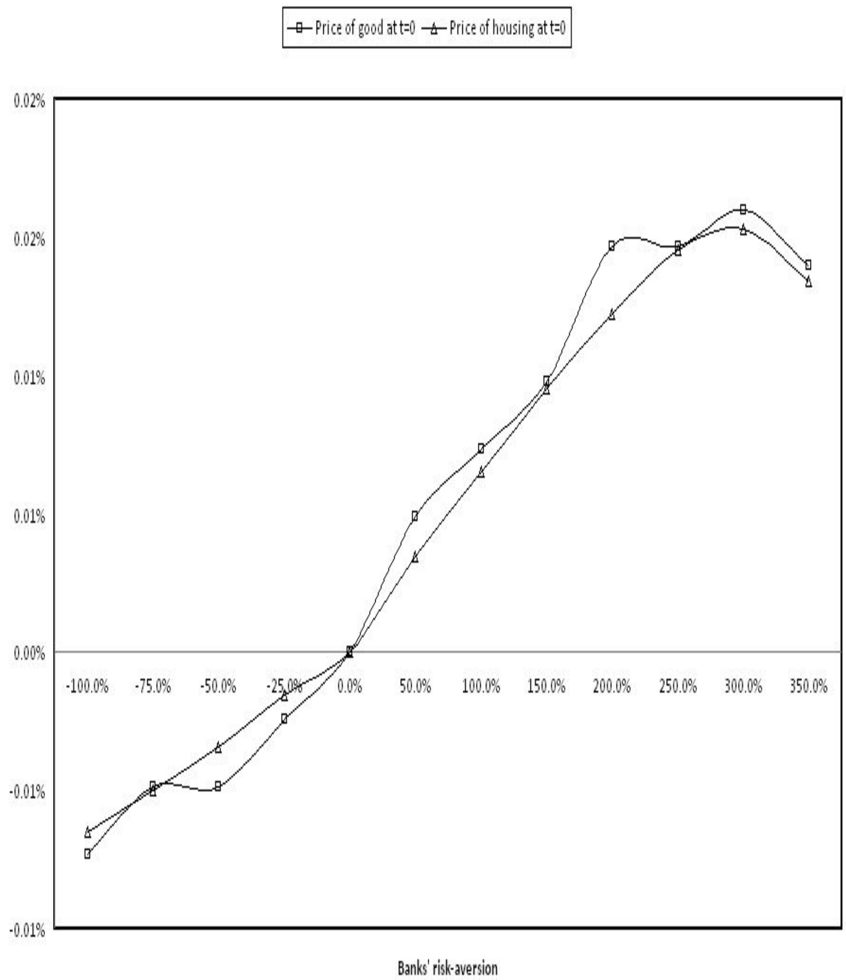

Figure 8: Housing and goods prices at t=0 vs banks' risk-aversion

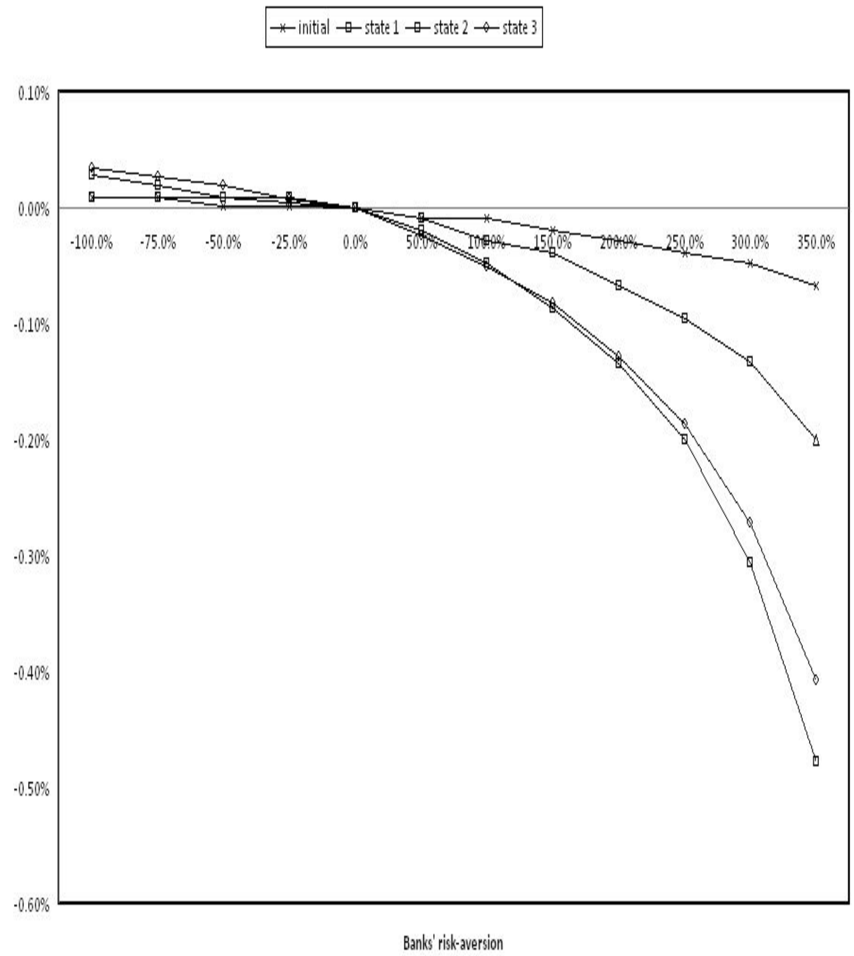

Figure 9: Short-term interest rates by bank δ vs banks' risk-aversion

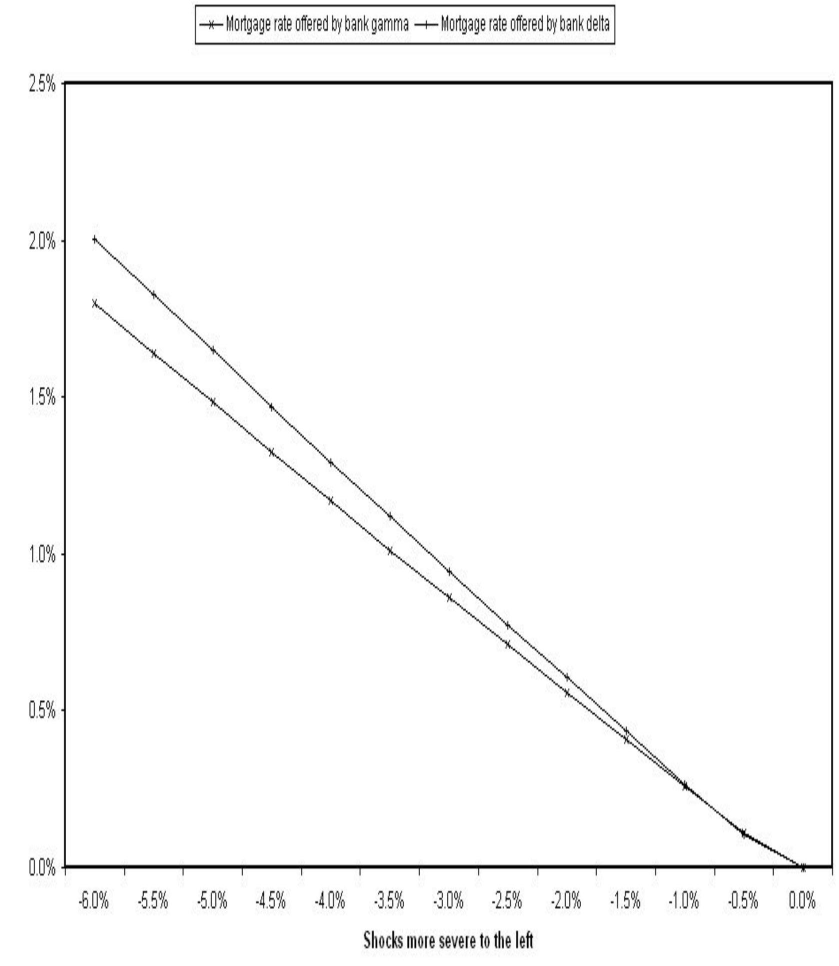

Figure 10: Mortgage rates vs compound decrease in money supply and in banks' risk-aversion

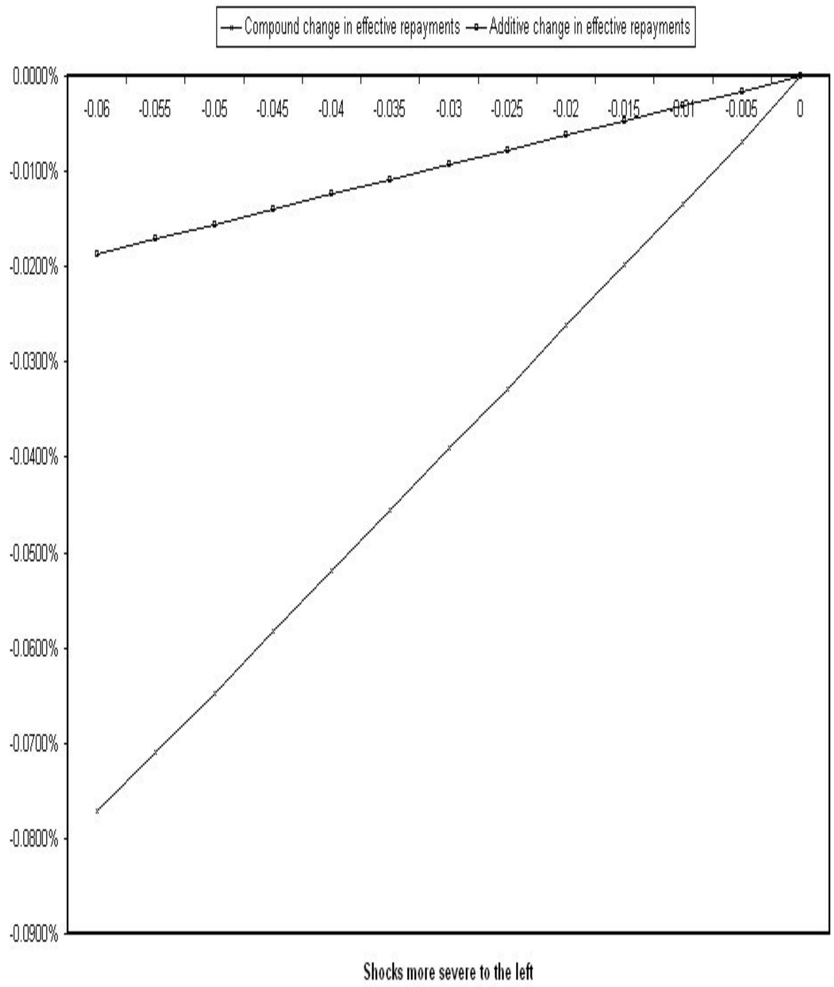

Figure 11: Nonlinear effects on mortgage repayment vs compound decrease in money supply and in banks' risk-aversion

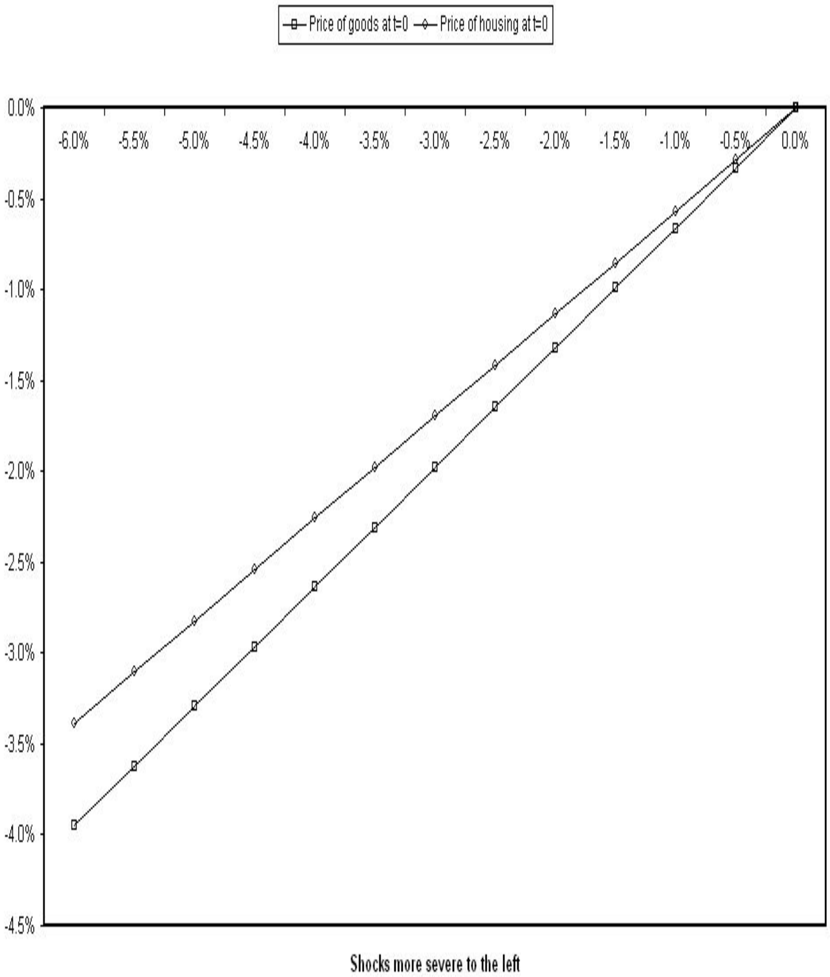

Figure 12: Housing and goods prices at t=0 vs compound decrease in money supply and in banks' risk-aversion

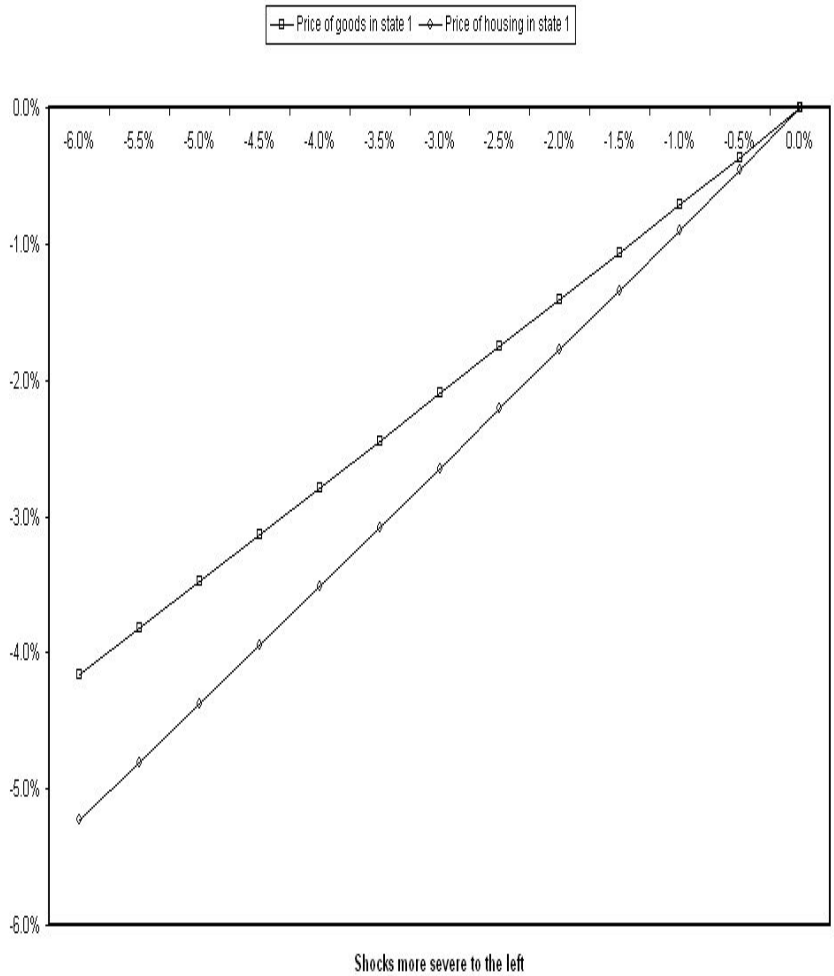

Figure 13: Housing and goods prices in state 1 vs compound decrease in money supply and in banks' risk-aversion